Communicating Centrality in Policy Network Drawings

Ulrik Brandes, Patrick Kenis, and Dorothea Wagner

Abstract—We introduce a network visualization technique that supports an analytical method applied in the social sciences. Policy network analysis is an approach to study policy making structures, processes, and outcomes, thereby concentrating on relations between policy actors. An important operational concept for the analysis of policy networks is the notion of centrality, i.e., the distinction of actors according to their importance in a relational structure. We integrate this measure in a layout model for networks by mapping structural to geometric centrality. Thus, centrality values and network data can be presented simultaneously and explored interactively.

Index Terms—Information visualization, graph drawing, force-directed placement, social network analysis, centrality.

1 Introduction

Graphs or networks are an essential class of data models and their visualization is an active area of research [9], [19], [21]. For graphs from some specific domain of application, utilization of general graph layout methods often yields diagrams that are readable, but fail to communicate the important, domain-specific information represented in the graph. Encountered in different applications, the same abstract structure can have quite different meanings and different layouts may suggest different interpretations. Note, however, that "graphical excellence requires telling the truth about the data" [30, p. 51] so that open or ambiguous presentation must be avoided.

We introduce a method to visualize networks in such a way that important information specific to networks analyzed in the social sciences is conveyed. It is a unique approach to social network visualization that combines graphical presentation of the network with a distinct analytical perspective. Ultimately, it can serve as a visual component [13] for network analysis.

The original motivation for our work is a comparative study of local drug policies based on an analysis of policy networks. In the last decade, the policy network approach has become particularly prominent in the analysis of public policies. It developed as a criticism to previous policy analysis approaches, which were considered to be too instrumental, mechanistic, and rationalistically oriented. In contrast, the network-based approach has a much more realistic perspective of how policies develop and considers policies principally as a result of a collaboration of a differentiated set of actors (public and private, local, regional, and national, etc.). Unlike previous models, it does not assume a priori, e.g., that state actors are more important in public policy making than private actors or that national actors are more important than local actors. Thus, the policy network approach conceptualizes policy-making as the result of interactions between policy actors and does not make theoretical assumptions about the structure of the network of interactions. Rather, it is assumed that the structure of these interactions explains policy outcomes [22].

A number of structural characteristics are taken into account in the analysis and explanation of social networks and policy networks in particular. The most prominent ones are structural notions of centrality since public policy analysis is a part of political science in which traditionally the principal question has been: "Who has the power?" Centrality is considered a fairly good indicator for power in networks and formalizations of this notion are typically based on nodal degree, shortest paths, network flow, or eigenvectors of graph-related matrices. See [31] for a comprehensive overview of methods and applications of network analysis.

While structural centrality has an immediate geometric counterpart, we know of no previous attempt to automatically produce network visualizations that facilitate the exploration and communication of centrality. Typically, node centralities are tabulated while the network is visualized independently, if at all. Though some layout methods that are popular for social networks, such as multidimensional scaling or spring embedder variants, sometimes happen to place structurally central nodes close to the center of the diagram, they are by no means reliable in this respect. In fact, they are more often than not misleading. We therefore constrain our visualizations to represent exactly the aggregate values of a structural centrality index by mapping them to geometric centrality.

Background on the rationale behind our approach to graphical presentation is provided in the following subsection. Select formal definitions of centrality are given in

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Manuscript received 31 Jan. 2001; revised 19 Oct. 2001; accepted 24 Oct. 2001.
For information on obtaining reprints of this article, please send e-mail to: tvcg@computer.org, and reference IEEECS Log Number 113559.

graphical features	graphical variables	
point	*positional*	x-, y-, z-coordinate
curve	*retinal*	size, shape, orientation,
area		brightness, color,
volume		transparency, texture

Fig. 1. Primitives of graphical presentations and their properties (see [2], [23]).

Section 2. A visualization model for centrality in policy networks is developed in Section 3 and an algorithm to determine the layouts of such visualizations is presented in Section 4. Our implementation of the algorithm facilitates graphical exploratory centrality analysis by allowing several forms of interaction described in Section 4.4. In Section 5, some examples from the above-mentioned study serve to demonstrate results and usage of our approach.

1.1 Graphical Presentation

Policy network diagrams are to aid the exploration and communication of substantive network content. Their automatic and reliable production is essentially an information visualization task. See [5] for an overview of the field of information visualization. We take a thoroughly formal approach that is similar to the usual approach in scientific visualization.

Following Bertin [2], we think of graphical presentations as being composed of primitive graphical objects (the *graphical features*) that represent the data elements. The properties of these objects (the *graphical variables*) are either prescribed by some form of representation, or varied according to the data. Fig. 1 lists graphical features for two and three-dimensional presentations and their properties in common media. Note that we confine ourselves to static graphical presentations.

Effective visualization of social networks has three main aspects: substance to be conveyed, a graphical design, and an algorithm. Since the following sections are organized according to these aspects, we briefly summarize them here. See [4] for background and details.

Substance. Graphs are used to model relational information in an abstract to the application of general methods and derivation of general statements amenable way. However, the underlying domain-specific meaning of the graph is the essence of what is to be conveyed through visualization. This particular information is called the network's *substance*. Consequently, any graphical presentation should be prepared in close accordance with the substance to be conveyed.

Substance can be divided into *syntactic* (intrinsic) information comprised solely of the graph's structure, no matter what it actually represents, and *semantic* (extrinsic) information not captured in the relation itself. In Section 2, we define the syntactic substance we are interested in. Semantic substance relevant to our particular application is described briefly in Sections 3.2 and 5.

Design. A graphical design has to specify which graphical features are to represent which data elements (*representation*) and how values shall be assigned to positional and retinal graphical variables (*layout* and *rendering*). The design specification should result in a nonambiguous presentation of substance, but should also obey ergonomic criteria to ease perception.

We decided to represent policy networks in the traditional form of a *sociogram* [24], in which nodes are represented as points and edges between them as curves. Our other design decisions are explained in Section 3.

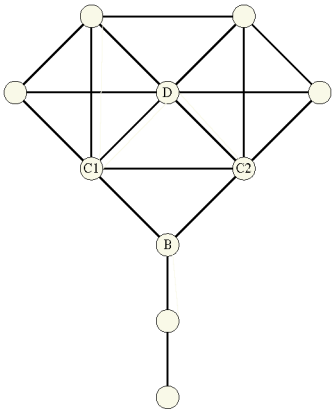
Algorithm. Many criteria for effective layout (crossing minimization, uniform edge length, angle maximization, etc.) are only approximately satisfiable and possibly conflicting. Besides running time and stability, the artifacts a particular algorithm may introduce are therefore important to know about and to keep in mind when interpreting the diagrams it yields. The most relevant algorithmic aspects in our setting are discussed in Section 4.

2 Structural Centrality

Centrality is considered a crucial characteristic of policy networks since it gives an indication of the most important, the important, and the unimportant actors in the network. This knowledge is particularly useful mainly for two reasons: First, it tells us something about the social or political structure of policy making and, second, it is assumed to be a relevant factor for the policy outcomes that networks produce. The social or political structure of a network indicates which type of actor is involved in which way in the policy-making process. Who has access and control over resources and who has a brokerage position? From this perspective, it makes a considerable difference whether there is a most important actor in a network and, if this is the case, what type of actor it is. In terms of legitimacy, accountability, justice, etc., it makes a difference whether, for example, in the health policy field, the most important actor is a state or a private actor and how important the actor is relative to the others. Moreover, there is evidence that the centrality structure of the network explains why a network was particularly successful in producing certain outcomes or why policies have failed to come about.

A number of measures have been devised to operationalize "importance," addressing a different aspect of the intuitive notion (for an overview, see [15], [17]). Although we make use of others as well, we limit our exposition to three exemplary measures that are used widely: degree, closeness, and betweenness centrality. Moreover, we confine ourselves to undirected networks $G = (V, E)$, where V is the set of nodes (*vertices*) representing policy actors, and E is a set of undirected *edges*, representing a certain type of linkage between actors.

The *degree centrality* of a vertex $v \in V$ is simply the degree of that vertex,



vertex	C'_D	C'_C	C'_B
D	0.67	0.60	0.05
C1, C2	0.56	0.64	0.20
B	0.33	0.60	0.39

Fig. 2. The kite graph has different centers under degree (D), closeness ($C1, C2$), and betweenness centrality (B).

$$C_D(v) = d_G(v),$$

i.e., the number of incident edges, the idea being that the degree to which actors are active in relating to other actors is relevant. Actors with high degree centrality are where the action in the network is. In policy networks, these actors are highly visible for the other actors and are recognized by the others as major channels of information.

This very local measure can be extended in several ways to take into account the whole graph. One is by taking the *closeness*, i.e., the sum of the distances to all other vertices, as a basis. *Closeness centrality* [28], defined as inverse closeness,

$$C_C(v) = \frac{1}{\sum_{t \in V} d_G(v,t)},$$

thus focuses on how close an actor is to all the other actors in the network. If we consider a policy network where a certain actor has information which is crucial to all other actors, one would expect this actor to have high closeness centrality for the network to function effectively.

Finally, *betweenness centrality* [14], [1] is defined as the sum of the fractions of shortest paths between other actors that an actor sits on,

$$C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}},$$

where σ_{st} is the number of shortest paths between vertices s and t and $\sigma_{st}(v)$ is the number of shortest paths between s and t passing through v. Betweenness indicates the extent to which an actor has the ability to control the interaction of two nonadjacent actors. In policy networks, these actors are considered important because they control the spread of information between actors or sets of actors and thus influence decision-making processes.

A framework to obtain normalized and network level centrality measures from a given actor level measure is described in [15]. Any centrality measure C is normalized to lie between zero and one by dividing its values by the maximum possible score in any graph with the same number of vertices. The above three measures thus yield *normalized measures*

$$C'_D(v) = \frac{C_D(v)}{n-1},$$
$$C'_C(v) = \frac{C_C(v)}{1/(n-1)}, \text{ and}$$
$$C'_B(v) = \frac{C_B(v)}{(n-1)(n-2)/2}.$$

Network *centralization*, on the other hand, quantifies the range of variability of the individual node indices. A low network centralization in a policy network is thus an indication that there is no distinct power center. Centralization is formalized as the accumulated differences between all node centralities and their maximum attained in the present network, normalized by the maximum possible such sum. For all of the above measures, the star is a maximally centralized graph, whereas cliques and circles are not centralized at all.

The network in Fig. 2 shows that these measures actually differ. Each identifies a different set of maximally central vertices, marked by corresponding labels.

3 Visual Centrality

3.1 Layout

Several aspects of a policy network are of interest in the kind of data analysis we want to support. While semantic substance is contingent on the particular study, syntactic substance is inherent to the method of analysis. It can and should therefore be incorporated into the design of visualizations.

The principal syntactic substance we want to convey is centrality. Positions not only appear to be the most accurate means for visually representing numbers [7], but they also provide an immediate counterpart for structural centrality —geometric centrality. We therefore place vertices such that their distance from the center of the diagram is proportional to their centrality score.

Although discovered only after the model described below had been set up, there is an historic precursor for this idea. A *target diagram* [25] displays social choice within a group by placing actors inside of rings corresponding to centrality quartiles (where centrality is defined by the sum of received choices). See Fig. 3 and note that only the

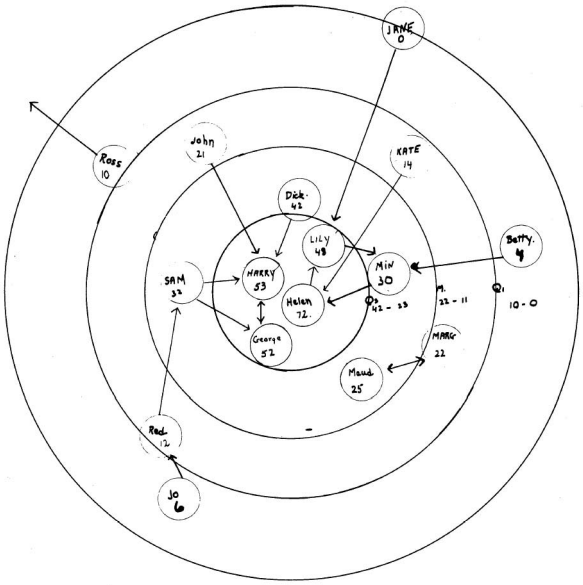


Fig. 3. Target diagram [25]. Vertices are constrained to lie in rings representing centrality quartiles.

strongest choice is shown for each actor. The placement was performed manually by arranging labeled poker chips.

In our model, every vertex is constrained to lie on the circumference of a circle centered at the center of the diagram. The radius $r(v)$ of the circle for vertex v is determined from its structural centrality and it appears most natural to use distances from the center that reflect the closeness of each vertex. However, the straightforward mapping soon proved to result in a resolution problem, illustrated by Fig. 4. After experimenting with several other mappings, we propose scaling the range of attained normalized centrality scores under some measure C to the unit disk according to

$$r(v) = 1 - \frac{C'(v) - \min\limits_{u \in V} C'(u)}{\max\limits_{u \in V} C'(u) - \min\limits_{u \in V} C'(u) + c(G)},$$

where $c(G)$ is an offset used to avoid overlap if there is more than one vertex of maximum centrality. For closeness centrality, the offset is chosen such that the number of closeness levels is increased by one less than the number of maximally central vertices. Since there are no underlying integer levels for betweenness centrality, we instead use

$$c(G) = \min\left\{\frac{1}{2}, \frac{n_B}{n-1}\right\},$$

where n_B is the number of vertices with maximum betweenness. Note that this definition of the offset works with any centrality index.

By constraining vertices to have a distance from the center exactly representing their centrality scores, we have proposed a design that displays the crucial substance. The central problem, however, is to actually compute the positions of vertices, subject to these centrality constraints, such that the network is displayed in a readable manner. We address this problem in Section 4.

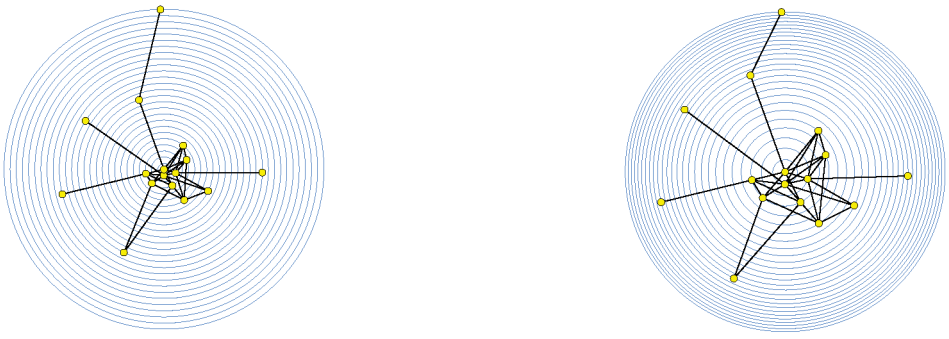
3.2 Rendering

Several other aspects of network data are incorporated in the design to increase the information density of our visualizations. The following elements of our design map substance to retinal graphical variables so that they are trivially realizable and nonconflicting and therefore do not require sophisticated algorithms.

Centrality levels. Showing levels as thin circles allows us to compare centrality scores exactly so that tabular presentation is no longer needed. For closeness centrality, an appropriately scaled circle is shown for each integer value between the minimum and maximum closeness scores. Hence, the hyperbolically decreasing differences between radii in our closeness examples.

For betweenness centrality, we can at least indicate the range of scores attained and, hence, the significance of radius differences. A circle is displayed for every multiple of $\frac{1}{10}$ within the range $[\min_{v \in V} C'_B(v), \max_{v \in V} C'_B(v)]$ and for the minimum and maximum betweenness centrality score.

Degree centrality. While the closeness or betweenness centrality of a vertex is determined by the entire network structure, its degree centrality is a local property. It thus seems appropriate to treat it differently from the other centrality indices and visualize it locally. Since degree centrality is an indicator of "activity" or "visibility" of an actor, we use the size of the graphical element representing the actor to visualize it, rather than, e.g., the color. To



plain inverse

Fig. 4. Applied to realistic data, plain closeness levels tend to emphasize the less interesting periphery. Vertices are therefore positioned according to their closeness centrality value (inverse closeness).

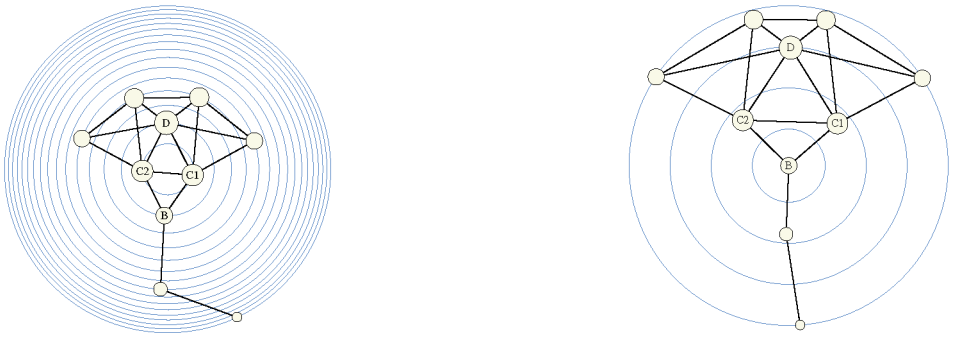


Fig. 5. Layouts showing closeness and 2betweenness centrality of the kite graph, where vertex size is proportional to degree centrality.

facilitate comparison across networks, we do not let the degree but the normalized degree centrality $C'_D(v)$ determine the size of a vertex v.

Fig. 5 shows visualizations of the kite graph from Fig. 2 with the graphical design described so far. This design applies to arbitrary networks for which centrality is a relevant substance. In the application at hand, however, there is additional information the analysts are interested in.

Reciprocation. Policy network data often consist of the relationships policy actors perceive to have with other actors and are collected by means of questionnaires. A consequence of the subjectivity of such data is the presence of unconfirmed relations. Consider, say, a network of informal communication between organizations. While one organization may claim to communicate informally with another organization, the latter may not reciprocate this claim. For the analysis of the power structure, usually a decision is made whether the edge is considered or not, but the fact that a claim is not reciprocated is useful information by itself.

We therefore render reciprocated edges by solid black lines, whereas every nonreciprocated edge is rendered in gray, slightly thinner to maintain the dominant visual impression of the subgraph of reciprocated edges, and with an arrowhead indicating which actor actually claimed to be engaged with which other actor in the respective relation.

Unconfirmed degree centrality. Another refinement motivated by the presence of nonreciprocated edges is to visualize the ratio and number of made and received claims of relationship. An actor claiming many relationships without being reciprocated is clearly different from one claiming few relationships while receiving many nominations. The ratio of in and outdegree thus indicates a potential discrepancy of perception between an actor and its alters.

Instead of only the normalized degree centrality, we here use both the ratio of in and outdegree and the total degree in the directed unconfirmed network to determine the size of an actor's representation. Assume, an actor v is represented by a point feature of rectangular shape and let h and w denote its height and width, respectively. Furthermore, denote by $d^{in}_G(v)$ and $d^{out}_G(v)$ its normalized in and outdegree. Then, we want the ratio of height and width to equal the ratio of in and outdegree and the area of the rectangle to equal the sum of the degrees,

$$\frac{h}{w} = \frac{d^{in}_G(v)}{d^{out}_G(v)}$$
$$h \cdot w = d^{in}_G(v) + d^{out}_G(v).$$

From these equations, we obtain

$$h = \sqrt{\left(d^{in}_G(v) + d^{out}_G(v)\right) \cdot \frac{d^{in}_G(v)}{d^{out}_G(v)}}$$
$$w = \sqrt{\left(d^{in}_G(v) + d^{out}_G(v)\right) \cdot \frac{d^{out}_G(v)}{d^{in}_G(v)}},$$

with straightforward adaptations to guarantee minimum height and width in the case of zero in or outdegree and scaling factors for actors represented by other shapes.

Semantic attributes. Crucial semantic information in the present case study indicates whether organizations act supportive or repressive toward drug users. The legal status of the organization is another important characteristic of the organization, but is, in the present research context, secondary to the previous one. Both are nominal-scale attributes that we express by choosing different colors and shapes to represent organizations. Since the attitude toward drug users constitutes the more important piece of information, the visually more prominent [7] variable "color" is used for it.

The complete design of our visualization is illustrated by the example in Fig. 6. In this instance, centrality is computed on the basis of only the reciprocated relationships of informal communication between organizations. Note that the two repressive organizations (red vertices) are not at all involved in this network. Another interesting observation are the two organizations on the left of which one is making many nonreciprocated claims, whereas the other receives many nominations it does not confirm. See the examples in Section 5 for more specific hints on how to read such diagrams.

4 Layout Computation

The design described in the previous section constrains vertices to lie on the circumference of circles in order to depict the crucial syntactic substance in a network. To produce a network visualization, we still have to determine the actual position of vertices subject to this constraint. The

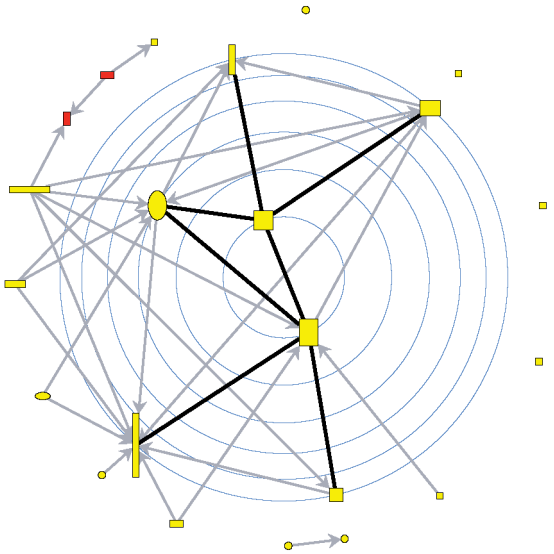


Fig. 6. Closeness centrality in a network of informal communication among organizations based on reciprocated relationships only.

remaining degree of freedom should be used to improve the readability of the diagram. Our algorithm is based on techniques from the field of graph drawing. For an overview of graph drawing, see [9], [19], [21].

4.1 Related Graph Layout Problems

The most closely related form of graphical presentation is the *ring diagram* introduced in [27], where vertices are constrained to the circumferences of circles with radii corresponding to levels in a hierarchy. After fixing the ordering of vertices on the inner (outer) level, vertices on lower (higher) levels are successively placed in the angular barycenter of their already placed neighbors. Unfortunately, this elegant technique only works because the graphs considered in [27] do not exhibit edges that span a layer. Note that the above placement ignores overlap between nodes and edges passing them by.

The circular layering is also reminiscent of horizontally layered drawings of directed graphs and, consequently, there are approaches using radial layers [6] and recurrent hierarchies [11]. In the popular framework of [29], such layouts are computed according to the following steps:

1. Assign a layer to each vertex,
2. Compute an ordering of the vertices in each layer,
3. Assign x and y-coordinates respecting the relative ordering of vertices and layers, respectively.

In our case, levels and radii (corresponding to layers and y-coordinates) are already determined by the centrality constraint.

The ordering of vertices serves to reduce the number of edge crossings, which is a crucial requirement for readability [26]. Common approaches introduce dummy vertices that subdivide edges spanning more than one layer, fix the ordering in one, say, the topmost layer, and successively compute orderings for the lower layers. Then, the ordering in the bottom layer is fixed and the computation proceeds upward again. This iteration is alternated until no improvement is made. Note that even the basic step of minimizing the number of crossings between two layers with the order in one layer fixed is $\mathcal{NP}$-hard [12].

A major problem in adapting the approach for horizontally layered graphs is that the number of crossings between two layers in a circular layering is not uniquely determined by the cyclic orderings of vertices in these layers. This holds also if the positions of vertices in one layer are fixed. To avoid this difficulty, we may impose the additional constraint that edges may not pass through the circles of inner layers. We say that a drawing is *outward* if edges are entirely outside the circle of the layer of their inner vertex. For outward drawings, the crossing minimization problem is essentially the same as in the case of horizontal layers. In conclusion, even this very restricted ordering problem is $\mathcal{NP}$-hard.

Moreover, crossing minimization is not only a hard problem, but bend edges resulting from the dummy vertices that are necessarily introduced by two-layer methods tend to be rather confusing in circular layouts.

Since the above methods seemed difficult to adapt and we wanted to have the ability to try out different designs before deciding on the prevailing criteria for suitable placement, we resorted to the more flexible energy-based layout approaches.

4.2 Energy-Based Placement

Physical modeling for graph layout is introduced in [10]. The idea of the so-called *spring embedder* is to regard vertices of a graph to be repelling objects, held together by springs instead of edges. An equilibrium configuration of such a system generally corresponds to a pleasing layout, with well-distributed vertices and uniform edge lengths. It is obtained by iteratively moving the vertices in the direction of the total force acting on them. An overview of layout approaches based on physical analogies is given in [3].

Notable refinements of the basic spring embedder can be found in [20], [18], [16], [8]. In particular, the approaches of [20], [8] explicitly define objective functions for readable layout. These objective functions correspond to an energy function in the underlying, physically inspired model. They are composed of sums of potential functions that weigh the distortion of a layout locally. Since we make extensive use of potentials, some useful ones are presented next.

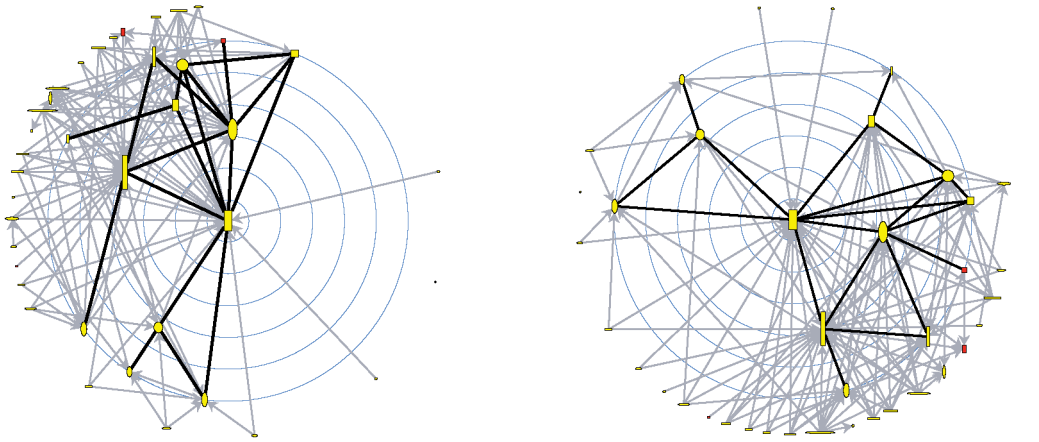
It is often desirable to specify that two vertices should be placed at a given distance. The potential energy of a spring is the squared difference of its natural and actual lengths, multiplied with its stiffness constant. For a layout $x = (x_v)_{v \in V}$ that has two vertices $u, v \in V$ with desired distance λ_{uv} at Euclidean distance $d(x_u, x_v)$, Kamada and Kawai [20] hence introduce the potential

$$\text{Distance}(x_u, x_v \,|\, \lambda_{uv}) = \frac{c}{\lambda_{uv}^2} \cdot (d(x_u, x_v) - \lambda_{uv})^2,$$

where c is a fixed constant. In fact, their energy function is composed solely of such distance potentials,

$$U_{\text{KK}}(x) = \sum_{u,v \in V} \text{Distance}(x_u, x_v \,|\, d_G(u, v)),$$

where $d_G(u, v)$ denotes the length of a shortest path from u to v in G.



uniform treatment of all edges

preferential treatment of confirmed edges

Fig. 7. The nonuniform visual impact of edges necessitates a nonuniform layout model.

Instead of long springs between pairs of distant vertices, repelling potentials

$$\text{Repulsion}(x_u, x_v \mid c_\lambda) = \frac{c_\lambda^4}{d(x_u, x_v)^2}$$

are used between every pair of vertices in [8]. To keep adjacent vertices close, additional attraction potentials

$$\text{Attraction}(x_u, x_v) = d(x_u, x_v)^2$$

are introduced for pairs of adjacent vertices. Note that these correspond to springs of natural length zero and that the combination of a repulsion and an attraction potential yields a potential that is minimized when $d(x_u, x_v) = c_\lambda$. Parameter c_λ thus controls the desired length of an edge. Let $d(x_v, x_{\{u,w\}})$ denote the smallest distance between x_v and any point on the straight line through x_u and x_w. Similar to [8], we define

$$\text{Repulsion}(x_v, x_{\{u,w\}}) = \begin{cases} \frac{c_\varrho}{d(x_v, x_{\{u,w\}})^2} & \text{if } d(x_v, x_{\{u,w\}}) \le d(x_v, x_u), d(x_v, x_w) \\ 0 & \text{otherwise,} \end{cases}$$

penalizing edges that closely pass by vertices. Finally, a potential $\text{Crossings}(x_{e_1}, x_{e_2})$ that counts the number of crossings in a layout [8] will be used.

In the next section, we describe how these potentials are combined into an energy function for readable realization of our design.

4.3 A Three-Phase Layout Model

The visual dominance of reciprocated edges implied by our design renders uniform treatment of the whole network inappropriate (see Fig. 7). To cope with this difference in visual impact, we split the layout process into three phases. First, we lay out the core subgraph induced by reciprocated edges, then introduce nonreciprocated edges (while moderately changing previously determined positions), and, finally, position those vertices incident only to nonreciprocated edges on the periphery of the drawing. Each of these steps is based on a tailor-made layout model.

The energy function in each of these phases is approximately minimized using simulated annealing, which is known to produce satisfactory layouts for small to medium size graphs. New candidate layouts are generated by moving a single vertex v along the perimeter of the circle with radius $r(v)$ so that the centrality constraints are maintained throughout the annealing. To reduce the number of candidates that will be rejected and to guarantee convergence, the vertex is moved by an angle that is drawn uniformly at random from the interval $[-\frac{T}{T_0}\pi, \frac{T}{T_0}\pi]$, where T is the temperature parameter of the annealing and $T_0 > 0$ is a fixed initial temperature. After trying a new position for each vertex, T is decreased to $c \cdot T$ for some constant $0 < c < 1$ (geometric cooling schedule) until $T < \varepsilon$, where $\varepsilon > 0$ is a fixed threshold. We typically use $T_0 = 10$, $\varepsilon = 0.001$, and $c = 0.97$.

We next describe the energy functions used in each of the three phases of the overall layout. For the given graph $G = (V, E)$, denote the set of reciprocated edges by $E_R \subseteq E$ and let $E_N = E \setminus E_R$ be the set of nonreciprocated edges. Furthermore, let $V_C \subseteq V$ be the set of vertices in the *core* of G, i.e., vertices in the subgraph induced by E_R, and let $V_P = V \setminus V_C$ be the set of vertices in the *periphery* of G. Finally, we denote by E_C the edges in the subgraph induced by V_C. The entire layout algorithm is summarized in Algorithm 1 shown in Fig. 8 and its phases are illustrated in Fig. 9.

Core layout. Fig. 7 illustrates the importance of a readable layout of the core, i.e., the subgraph induced by reciprocated edges. Owing to its visual dominance, this may even be at the expense of other parts of the overall layout. Therefore, it is initially placed all by itself and positions are modified only slightly in the latter phases.

For vertices with sufficiently different centrality scores, radial edges are achieved using a target edge length corresponding to the difference in radius. However, vertices of similar centrality should not be placed at angularly close positions since edges become too short to be recognized. Moreover, it turns out that, when a combination of attraction and repulsion potentials is used, as in [8], the different repelling contributions are difficult to control.

```
function anneal(V, U, T)
begin
    while T > ε do
        foreach v ∈ V do
            x' ← x;
            draw α ∈ [−(T/T0)π, (T/T0)π] uniformly at random;
            x ← x' with x_v rotated by α;
            Δ ← U^(T)(x) − U^(T)(x');
            if Δ > 0 then with probability 1 − e^(−Δ/T) reset x ← x';
        T ← c · T;
end
// 1. phase: core layout
anneal(V_C, U_1, T_0);
// 2. phase: introduce non-reciprocated edges
anneal(V_C, U_2, 1);
// 3. phase: periphery layout
anneal(V_P, U_3, T_0);
```

Fig. 8. Algorithm 1.

Instead of vertex-vertex repulsion, the distance potentials of [20] are therefore used.

The desired distance between two vertices is computed in the following way: Each edge $\{u, w\}$ of the core graph is assigned a length $\lambda_{uv} = |r(u) - r(v)|$, if $|r(u) - r(v)| \geq \lambda$ for some minimum length λ. To determine the target length of an edge between vertices with radial difference smaller than the minimum, we first count the number $n(i)$ of short edges that span roughly the same level i, i.e., we compute

$$n(i) = \left| \left\{ \{u, v\} \,:\, |r(u) - r(v)| < \lambda \text{ and } \left\lceil 10 \cdot \frac{r(u) + r(v)}{2} \right\rceil = i \right\} \right|.$$

Recall that $0 \leq r(v) \leq 1$ for all vertices v of the core. For short edges $\{u, v\}$, λ_{uv} is set to

$$\min \left\{ \frac{r(u) + r(v)}{\sqrt{2}}, \frac{2\pi}{n(\lceil 10 \cdot \frac{r(u)+r(v)}{2} \rceil)} \right\}.$$

This yields short edges of length slightly longer than the average radius of their endpoints, except when there are too many short edges spanning roughly the same levels, which are then assigned a target length equally dividing the perimeter of the unit circle among them. Desired vertex distances for the distance potentials of [20] are then computed by solving an all-pairs shortest paths problem in the core graph with the above-defined positive edge lengths.

Besides radial edges, two other ergonomic criteria are important for readable layout of the core graph. First, there should be sufficient spacing between vertices and edges and, second, there should be as few as possible edge crossings. We therefore add vertex-edge Repulsion and Crossing potentials.

Some caveats are in order, though. Vertex-edge Repulsion potentials make it difficult for a vertex to "jump" over an edge spanning its radius, while the discrete nature of crossing potentials may destroy fine-tuned parts of a layout by drastically reducing the energy when a significant movement of a vertex eliminates a crossing. Moreover, Crossing potentials are expensive to evaluate and typically redundant at low temperatures of the annealing. Hence, Crossing potentials are evaluated only while $T > 1$ and the contribution of vertex-edge Repulsion potentials is considered and, in fact, gradually increased only when $T \leq 1$.

At temperature T, the layout model for the core graph $G_C = (V_C, E_R)$ of a network thus reads

$$U_1^{(T)}(x) = \sum_{u,v \in V_C} \text{Distance}(x_u, x_v \,|\, d_{G_C^\lambda}(u, v)) + \begin{cases} \sum_{e_1, e_2 \in E_R} \text{Crossing}(x_{e_1}, x_{e_2}) & \text{if } T > 1 \\ \sum_{v \in V_C, e \in E_R : v \notin e} \frac{1}{T} \cdot \text{Repulsion}(x_v, x_e) & \text{if } T \leq 1, \end{cases}$$

where G_C^λ corresponds to G_C with edge weights that are determined as described above.

Nonreciprocated edges. During this phase, only edges corresponding to nonreciprocated links between actors already represented in the core graph are introduced. With unchanged vertex positions, these edges would often pass through vertices and form many small angles.

To accommodate additional edges while only moderately changing the layout, the second layout phase starts annealing at temperature $T = 1$ and uses an objective function consisting of vertex-edge Repulsion potentials only. Thus, the energy function for the core graph augmented by nonreciprocated edges is

$$U_2^{(T)}(x) = \sum_{v \in V_C, e \in E_C : v \notin e} \frac{1}{T \cdot \text{Repulsion}}(x_v, x_e).$$

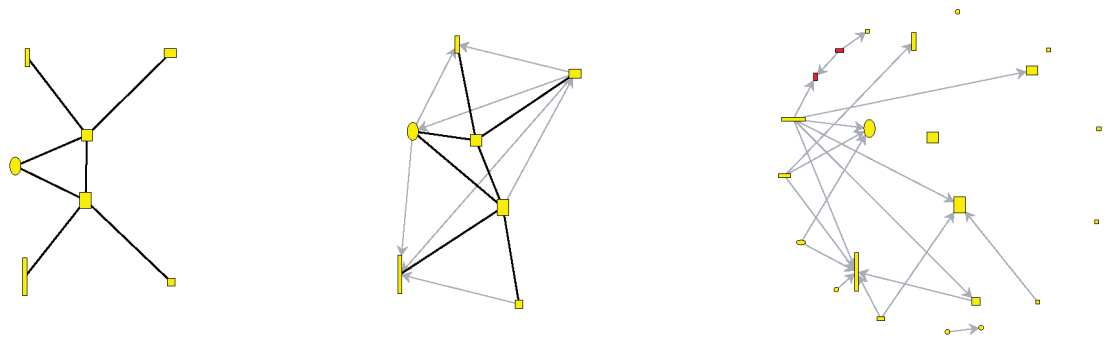


Fig. 9. Layout computation for the network shown in Fig. 6. The three phases reflect the relative importance of induced subgraphs.

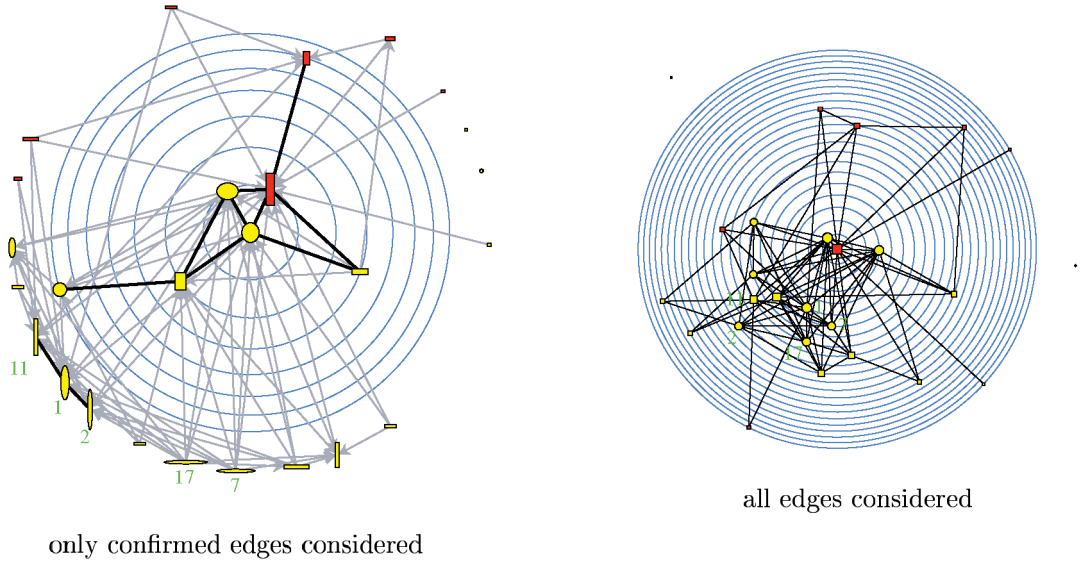


Fig. 10. Closeness centrality in network informal communication among organizations involved in local drug policy making in Stuttgart.

Periphery layout. Finally, vertices corresponding to actors with no reciprocated relation at all have to be positioned. They are called the *periphery* of the network and constrained to an orbit around the augmented core graph by setting $r(v) = 1.2$ for all peripheral vertices $v \in V_P$. Naturally, they should be close to core actors they claim to have a relation with, but should not cluster too much. Clearly, these nonreciprocated edges should also pass by vertices of the core at some reasonable distance.

This time, the Attraction and vertex-vertex Repulsion potentials come in handy. To place peripheral vertices close to their core neighbors, we introduce an Attraction potential between every pair of a peripheral vertex that is adjacent to a core vertex. On the other hand, we let every pair of peripheral vertices repel each other. If two of them are adjacent, we simply add another Attraction potential. Using λ^* as the repulsion constant, we require peripheral nonreciprocated edges to have minimum distance λ^*. In summary, the objective function of the third phase is

$$\begin{aligned} U_3^{(T)}(x) = &\sum_{u,v \in V_P} \text{Repulsion}(x_u, x_v \,|\, \lambda^*) \\ &+ \sum_{\{u,v\} \in E_P \,:\, u \in V_P} \text{Attraction}(x_u, x_v) \\ &+ \sum_{v \in V, e \in E \,:\, v \notin e} \frac{1}{T} \cdot \text{Repulsion}(x_v, x_e), \end{aligned}$$

where core vertices are constrained to retain their positions.

4.4 User Interaction

We have implemented the above model for readable centrality layouts in an interactive system. We decoupled the centrality constraints from the readability criteria represented in the three-phase layout model. As a consequence, we were able to provide the following forms of interaction which proved particularly useful for exploratory centrality analysis.

Snap to levels. By choosing a specific centrality index, the user initiates that all vertices are moved to have a distance from the center of the layout that is determined by the index. This movement is along the ray emanating from the center and passing through the current position of the vertex. If the vertex is currently in the center, a randomly oriented ray is chosen.

By default, only the confirmed edges are considered in the centrality computation. The user can indicate that certain nonreciprocated edges should be considered, too, simply by selecting them.

Layout. The three steps described in the previous section are carried out, where vertices are constrained to retain their distance from the center of the layout area, no matter whether these correspond to actual centralities or not.

Even without fine-tuning our implementation, layout times are interactive (i.e., under two seconds for graphs with around 40 nodes and 70 edges typically arising in our application) on a common PC, but, to give users an intuitive understanding of why the algorithm yields a particular layout, we animated the iterative layout process. Though slowing down the computation, this helps build confidence in the result and also provides an indication where manual repositioning might further improve readability. We do, however, update the display not after each, but only after a small number of iterations since the impression of hectic relocation proved counter-productive.

Layout adjustment. Instead of computing a layout from scratch, users can choose to locally increase readability by selecting this option. It is implemented by setting the initial temperature of the annealing $T_0 = 1$ in each of the three phases of the layout. The effect is that the circular ordering of vertices is largely preserved (since crossings are not considered in the objective function), but vertex-vertex and vertex-edge distances are improved.

This option is particularly useful to generate similar drawings of the same network for different centrality indices.

Users may move vertices freely to investigate relations or locally improve the readability of a layout. Using the above options, they can always set vertices back to their centrality-

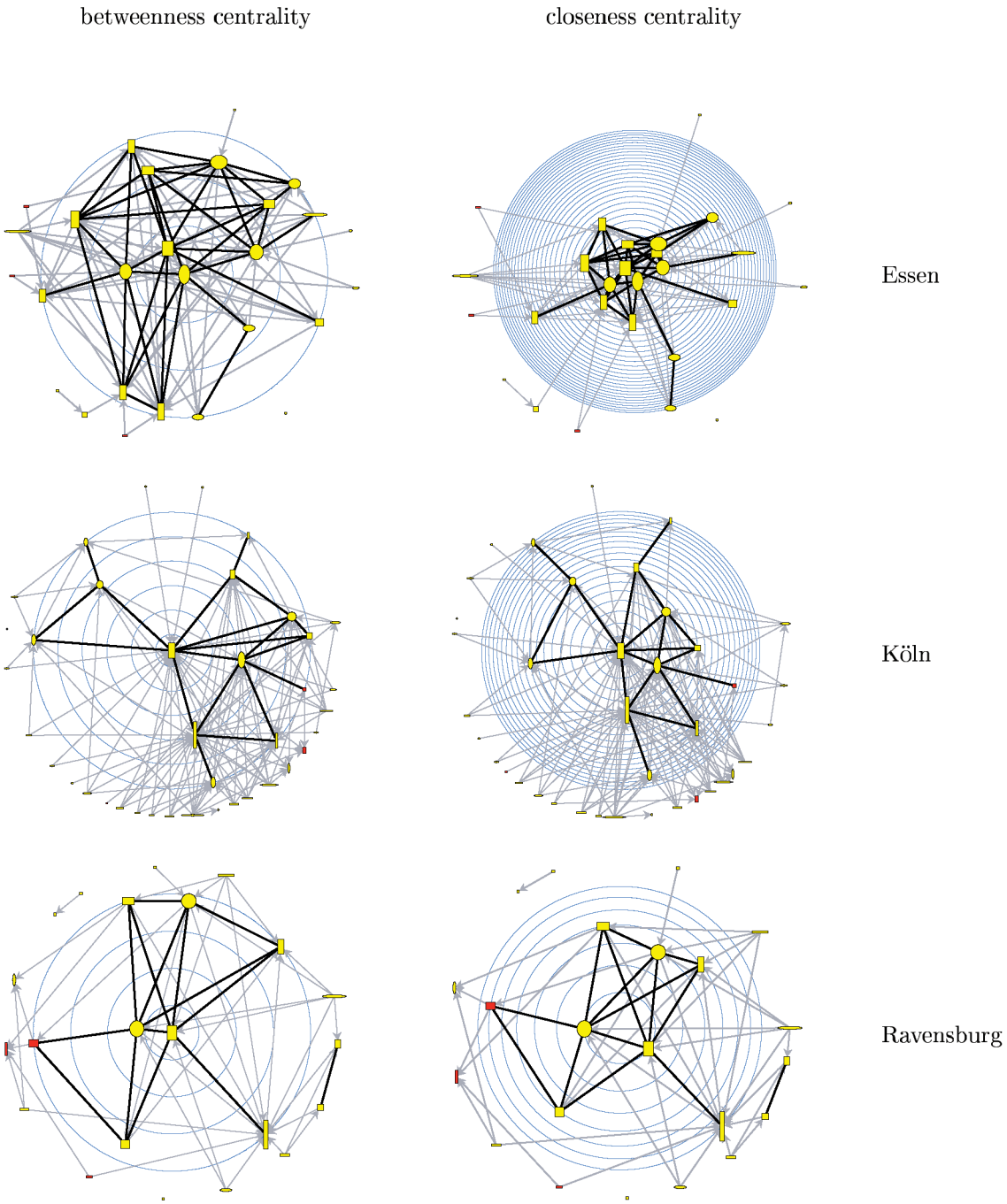


Fig. 11. Informal communication among organizations involved in drug policy making.

induced radii and adjust the layout to a local equilibrium of attracting and repelling forces. Finally, there is an option to scale the height and width of vertices to indicate their in and outdegrees and to proportionally shrink or enlarge them.

5 Application

The usage of our network visualization approach for centrality analyses is demonstrated using data from a project that studies the incidence of HIV-preventive measures for IV-drug users in nine selected German municipalities. The research question underlying the project is: Why do these municipalities differ so much in the provision of HIV-preventive measures (such as methadone substitution and needle exchange), given the fact that the problem load (i.e., the number of IV drug users and the HIV epidemiological situation) are very similar?

The study tests the hypothesis of whether the difference in the provision in HIV-preventive measures can be explained by the structure of the policy networks. Given the amount of controversy and complexity involved in the provision of such measures, the hypothesis is that they are contingent on the relations among the different policy actors. The policy networks studied here are therefore comprised of all local organizations directly or indirectly involved in the provision of such measures. This includes

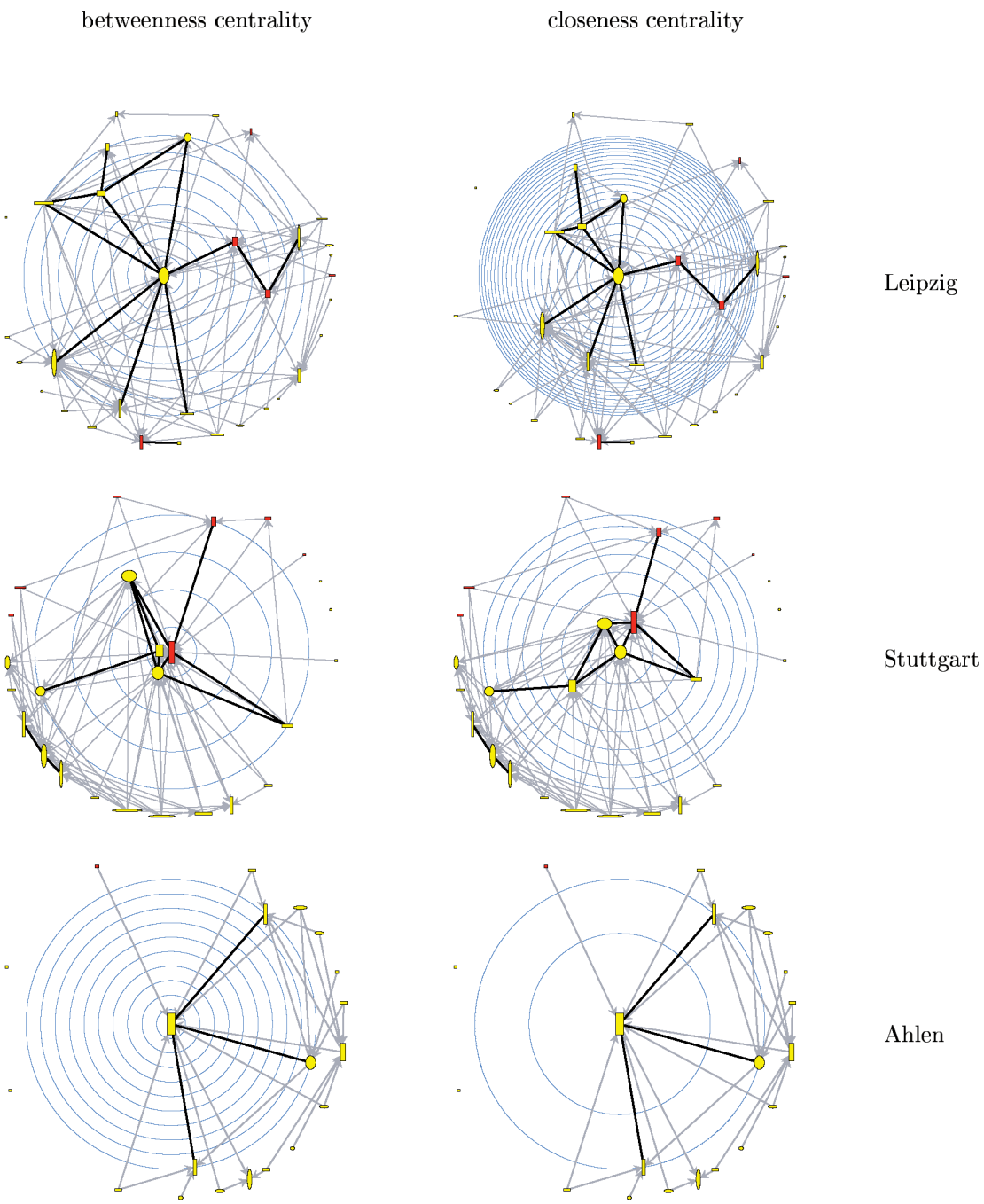


Fig. 12. Informal communication among organizations involved in drug policy making.

organizations who actually provide these measures to their clients (e.g., by administering methadone substitution), but also those organizations who have an indirect effect on the availability of such measures, such as financing authorities or police authorities. For instance, the frequency with which the police patrol an area in which a needle exchange program is run has considerable impact on the effectiveness of the program.

In each of the nine municipalities, the 22-38 organizations included in the study were queried about their relations with other organizations in the same municipality. The types of relations included, e.g., strategic collaboration, common activities, informal communication. For each set of organizations and each relation, a single network was constructed and analyzed. The number of edges in these networks was never larger than 120, typically more than 50 percent of them unconfirmed.

As pointed out in Section 2, centrality indices are particularly relevant to the study of policy networks because they provide insight into the social or political structure of policy making and help understanding the policy outcomes that policy networks produce. The visualization approach proved to be an excellent instrument for answering these two types of questions. Important insight is gained merely on the basis of centrality drawings, i.e., without any additional calculation or tabulated data. This

point is illustrated by first discussing a single network whose structural characteristics are elucidated by its visualization and then a set of drawings that help analyze the relationship between network structure and policy outcome.

Fig. 10 shows the network of informal communication among organizations in the city of Stuttgart. Informal communication is an undirected relationship in principle, but pairs of actors often have a different perception of whether they are engaged in this respect.

While policy network analysts often make a principal decision to either include or ignore nonreciprocated nominations, the visualization emphasizes that the incidence of nonreciprocation can be useful information in itself. Consider, e.g., actors 1, 2, and 11 who are peripheral in the network of confirmed relationships. It can be seen that they are fairly central in the unconfirmed network, but seem reluctant to indicate relationships. Another interesting story is actor 7, a small nonprofit organization, since it names many other organizations, but is not reciprocated by any of them. It is this kind of observation that explains the story behind aggregate network indices and helps come to an adequate evaluation of who actually the most important actor is.

Another type of insight which can be gained on the basis of these visualizations has to do with the relationship between the outcome of the networks (i.e., in the present case, the provision of HIV-preventive measures) and properties depicted in the diagrams. The six networks presented in Figs. 11 and 12 (informal communication in the cities of Essen, Köln, Leipzig, Stuttgart, Ravensburg, and Ahlen) differ a lot in their outcome effectiveness (Essen being the most effective and Ahlen being the least effective), but also differ a lot in their structural characteristics. Observe, e.g., the presence of repressive (red) actors near the center of some networks and the fact that the sparser networks seem prone to have some actors that are either particularly high or wide, thus indicating highly unbalanced perceptions.

The reason that the two visualizations of each network are similar is that one of them has been obtained using the full layout algorithm, while the other is generated from the first by scaling vertices to their new radii and applying the layout adjustment function described in Section 4.4.

On the basis of the drawings, a number of relationships become visible which can be further developed regarding the effectiveness of networks: the number of rather active actors (i.e., high degree centrality); the degree of discrepancy between confirmed and unconfirmed links; the fact of whether a mixture of different types of actors (public and private and repressive and supportive) is found close the center of the network; the fact of whether there is one clearly central actor in the network, a couple of central actors, or no clearly central actors in the network. It is this type of inductive observations which are a direct result of the graph drawing techniques and which contribute substantially to the analysis of the questions "Who has the power?" and "What are the consequences of the power structure?"

ACKNOWLEDGMENTS

The authors thank Vanessa Kääb for help with the implementation of an earlier prototype and Jörg Raab and Volker Schneider for valuable feedback and helpful discussions. A preliminary version of this work appeared in the *Proceedings of the Seventh International Symposium on Graph Drawing*, pp. 250-258, 1999. Part of this research was done while Ulrik Brandes was with the Department of Computer Science at Brown University. He gratefully acknowledges financial support from the German Academic Exchange Service (DAAD/Hochschulsonderprogramm III).

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