



## Preface

ROBERT DENK, YOSHIKAZU GIGA, HIDEO KOZONO, JÜRGEN SAAL,  
GIERI SIMONETT AND EDRISS TITI



Matthias Hieber, June 2020

Matthias Hieber studied Mathematics and Physics at the Universities of Tübingen, Besançon and Tulane in the early 1980s. In 1989, he received his doctoral degree in Mathematics under the supervision of Wolfgang Arendt and Helmut H. Schaefer from the University of Tübingen. Matthias Hieber's Ph.D. thesis was on integrated semigroups and differential operators on  $L_p$ . On the one side, he set sights on Banach space-valued Laplace transforms and spectral theory [1,4,5] and on the other side on functional analytic tools for partial differential equations, like the Schrödinger equation [2]. A highlight in this direction is the monograph "Vector Valued Laplace Transforms and Cauchy Problems" [B1], published jointly with Wolfgang Arendt, Charles Batty and Frank Neubrander in the Birkhäuser Monographs Series.

From 1990 to 1995, he held a position as Oberassistent at the University of Zurich. There, his research interests shifted more and more toward properties of elliptic operators arising in partial differential equations of evolution type. His publications during this period demonstrate the significance of heat kernel and Gaussian estimates as well of the  $H^\infty$ -calculus in the treatment of evolution systems [3,6,8,9]. In 1995, he completed his habilitation under the mentorship of Herbert Amann at the University of Zurich.

He then held positions as Professeur Associé at the Université de Franche-Comté at Besançon and as Hochschuldozent at the Karlsruhe Institute of Technology. During his years in Besançon and Karlsruhe, great advances in the theory of maximal regularity were made. From the beginning, Matthias was significantly involved in this

development, resulting in the influential results [7,9] that sectorial operators on  $L_q$ -spaces satisfying a heat kernel bound (or more generally a Poisson bound) admit the property of maximal  $L_p$ -regularity or on functional calculi for operators on vector-valued  $L_q$ -spaces via the transference principle [8].

In 1999, he was appointed full professor at the Technische Universität Darmstadt and he became head of the Applied Analysis Group there. He continued to work on maximal regularity properties of evolution equations. In fact, his joint publications [12,17,B2] with Robert Denk, Giovanni Dore, Jan Prüss and Alberto Venni on parabolic systems were quite influential and became highly cited sources for the theory of maximal regularity,  $H^\infty$ -calculus and its applications to partial differential equations. Extensions to  $L_p - L_q$ -estimates for parabolic systems with only  $VMO$ -coefficients were obtained jointly with Horst Heck and Robert Haller in [16]. A new characterization of the growth bound of a semigroup based on Fourier multiplier methods was given by him in [10].

At the same time, his research interests started to shift to the field of mathematical fluid dynamics. In [11], jointly with Jan Prüss and Wolfgang Desch, he investigated the Stokes operator in a half space and proved the surprising result that the Stokes operator generates an analytic semigroup on the solenoidal subspace of  $L^\infty(\mathbb{R}_+^n)$ , but not on  $L^1(\mathbb{R}_+^n)$ . Further results concerned the Navier–Stokes equations with linearly growing data ([13] jointly with Okihiko Sawada) or in the exterior of moving domains ([14] jointly with Matthias Geissert and Horst Heck).

During the first decade in Darmstadt, his research group grew continuously, in spite of the fact that he was occupied as Dean of the Department of Mathematics. In subsequent years, Matthias played a leading role in various collaborative research initiatives. From 2007 to 2014, he was Principal Investigator and member of the Steering Committee of the Center of Excellence “Smart Interfaces” at TU Darmstadt (EXC259). From 2009 to 2018, he was the Spokesperson of the “International Research Training Group (IRTG1529)” on Mathematical Fluid Dynamics, a joint cooperation between TU Darmstadt, Waseda University and The University of Tokyo, funded by DFG and JSPS. On the Japanese side, this program was coordinated by Yoshihiro Shibata and Hideo Kozono.

The rich scientific outcome of these research initiatives, especially of the IRTG, contributed to a deeper understanding of fluid dynamical phenomena and related topics. Let us mention here results on Bogovskii’s operator in Sobolev spaces of negative order [15], fluid structure interaction problems [21], complex fluids as described, e.g., by Oldroyd-B models [23,34], as well as on the Navier–Stokes equations in the rotational setting [18].

A special focus lied on the theory of the Stokes equation: Remarkable results jointly with Ken Abe, Yoshikazu Giga and Paolo Maremonti show that the Stokes operator generates an analytic semigroup on the solenoidal subspace of  $L^\infty(\Omega)$  [24,25] for certain classes of domains  $\Omega \subset \mathbb{R}^n$ . Further results connect the weak Neumann problem to the Navier–Stokes equations [20]; we refer also the survey article [32] jointly with Jürgen Saal.

Another focal point was the analysis and modeling of nematic liquid crystal flows. A new thermodynamically consistent understanding of these flows established in collaboration with Jan Prüss in [31] paved the way for a thorough understanding of the general Ericksen–Leslie model subject to general Leslie stress by means of the theory of quasilinear evolution equations. The beautiful articles [26, 30, 31, 35] present far reaching local and small data global existence results for strong solutions for this system in the compressible and incompressible situation. For the first time, no structural conditions on the Leslie coefficients, as, e.g., Parodi’s relation, were needed for the analysis of the general Ericksen–Leslie system.

Matthias also obtained important results on periodic solutions for incompressible fluid flow problems. A general framework was developed in [27], and the situation of arbitrarily large forces in the context of the primitive equations is described in [29] in joint work with Paolo Galdi and Takahito Kashiwabara.

In the last years, he was particularly interested in geophysical flows and the primitive equations. A fundamentally new approach to the latter equations was developed jointly with Takahito Kashiwabara in [28]. The situation of bounded data and its relationship to the uniqueness problem for weak solutions was analyzed in a German–Japanese team including Yoshikazu Giga in the articles [36, 37]. The stability of Ekman layers arising in geophysical flows was described in the deterministic and stochastic setting in [19] and [22].

His broad range of interests is also documented by recent regularity and global existence results for nonlocal equations, like the bidomain equations [33, 38], jointly with Jan Prüss, and through new contributions in geometric analysis concerning the Helmholtz–Weyl decomposition of  $L_r$ -vector fields in exterior domains [39], jointly with Hideo Kozono, Senjo Shimizu and others.

Over the years, Matthias Hieber received several offers from renowned German Universities; however, he decided to stay at TU Darmstadt. He held visiting professor or visiting scholar positions at highly ranked mathematical institutes all over the world, among them the University of California at Berkeley, UCLA, the Courant Institute and the University of Tokyo. In addition, he received several honorable distinctions: Since 2012, he is adjunct professor at the University of Pittsburgh; from 2016 to 2019, he was guest professor at Waseda University in Tokyo. Since 2020, he serves as the Vice-Director of the Mathematical Research Institute in Oberwolfach.

As a trend-setting mathematician, Matthias made numerous important contributions to the analytical understanding of evolution equations. He published more than one hundred research papers, co-authored three monographs [B1, B2, B5], three special volumes [V1, V2, V3] and wrote two textbooks [B3, B4].

His mathematical expertise comprises the development of evolution equations and also its application to concrete problems arising in the applied sciences. For his Ph.D. students, he has been a steady source of motivation and encouragement. One reason for attracting about twenty Ph.D. students, up to now, lies in his ability to share and to transfer his enthusiasm for mathematical research. Like this, he was very pleased to host Okihito Sawada and Huy Nguyen as Humboldt Research Fellows as well as

numerous scholars in Darmstadt. He also acted as a mentor for five habilitations at TU Darmstadt.

We also would like to mention here the organization of several workshops at Oberwolfach on Geophysical Flows, jointly with Yoshikazu Giga and Edriss Titi. He also serves as a member of the editorial boards of various journals including “Differential and Integral Equations,” “Advances in Differential Equations” and “Journal of Mathematical Fluid Mechanics” as well as for the “Springer Lectures Notes in Mathematical Fluid Dynamics.”

His scientific achievements, his broad interests, his contagious enthusiasm for mathematics, his sense of humor and his communication skills all contribute to the high standing Matthias Hieber enjoys in the mathematical community. It is, then, not surprising that he was invited to numerous summerschools as a lecturer.

The collection of original research papers in this volume reflects the wide-ranging scientific interests of Matthias Hieber. It is inspired by the Conference “Evolution Equations: Applied and Abstract Perspectives,” held on October 28–November 1, 2019, at CIRM in Luminy, France, on the occasion of his 60th birthday, and which was organized by Karoline Disser, Robert Haller-Dintelmann, Horst Heck, Mads Kyed, Jürgen Saal, Okihiko Sawada and Ian Wood. We express our gratitude to the organizers and participants of the conference and, of course, to all contributors of this volume.

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Robert Denk  
Fachbereich Mathematik und Statistik  
Universität Konstanz  
78457 Konstanz  
Germany  
E-mail: robert.denk@uni-konstanz.de

*Yoshikazu Giga*  
*Graduate School of Mathematical Sciences*  
*The University of Tokyo*  
*Komaba 3-8-1 Meguro*  
*Tokyo 153-8914*  
*Japan*  
*E-mail: ygiga@ms.u-tokyo.ac.jp*

*Hideo Kozono*  
*Department of Mathematics*  
*Waseda University*  
*Tokyo 169-8555*  
*Japan*  
*E-mail: kozono@waseda.jp ;*  
*hideo.kozono.c7@tohoku.ac.jp*

*and*

*Research Alliance Center of Mathematical Sciences*  
*Tohoku University*  
*980-8578 Sendai*  
*Japan*

*Jürgen Saal*  
*Heinrich-Heine-Universität Düsseldorf*  
*Mathematisches Institut*  
*40204 Düsseldorf*  
*Germany*  
*E-mail: juergen.saal@hhu.de*

*Gieri Simonett*  
*Department of Mathematics*  
*Vanderbilt University*  
*Nashville TN 37240*  
*USA*  
*E-mail: gieri.simonett@vanderbilt.edu*

*Edriss Titi*  
*Department of Mathematics*  
*Texas A&M University*  
*College Station TX 77843*  
*USA*  
*E-mail: titi@math.tamu.edu ;*  
*edriss.titi@damtp.cam.ac.uk*

*and*

*Department of Applied Mathematics and Theoretical*  
*Physics*  
*University of Cambridge*  
*Cambridge CB3 0WA*  
*UK*

*and*

*Department of Computer Science and Applied Mathematics*

*Weizmann Institute of Science*

*Rehovot 7610001*

*Israel*