Cavity nano-optomechanics inside a fiber-based micro-cavity

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Abstract

The field of cavity optomechanics studies the interaction between electromagnetic radiation and a macroscopic mechanical mode enhanced by an optical cavity. The use of low-dimensional objects in the field of cavity optomechanics is limited by their low scattering cross section compared to the size of the optical cavity mode. Fiber-based Fabry-Pérot micro-cavities can feature tiny mode cross sections and still maintain a high finesse, boosting the light-matter interaction and thus enabling the sensitive detection of the displacement of minute objects. This work presents such a fiber-based microcavity that features a micrometer sized mode cross section in combination with an ultrahigh finesse.

For a proof-of-principle demonstration of dynamical backaction in the system, we study stripes made out of a low-stress silicon nitride (SiN) membranes. We are able to position-tune the static optomechanical coupling to the stripe, reaching frequency pull parameters of up to $G_\omega/2\pi = 3\text{GHznm}^{-1}$. We also demonstrate the optical spring effect on the fundamental out-of-plane flexural mode of the stripe. The measurements match established theory and we obtain a single-photon coupling of $g_\omega/2\pi = 2.5\text{kHz}$. With the switch to stripes made out of high-stress silicon nitride ($\text{Si}_3\text{N}_4$), absorption in the stripe is reduced and and the loaded finesse reaches values up to $\mathcal{F} = 195000$. This is the largest finesse reported in loaded fiber cavities so far. The optomechanical coupling strength is reduced compared to the SiN sample due to a reduced refractive index. We demonstrate the optical spring effect and optical induced damping on two out-of-plane flexural modes of the stripe with a single-photon coupling of $g_\omega/2\pi = 575\text{Hz}$.

Towards the realization of dynamical backaction on vibrations of carbon nanotubes (CNTs), we study two sample geometries that allow to position the CNT inside the cavity mode. In a tuning-fork configuration, the presence of the CNT results in a increase of the cavity linewidth. CNTs grown between adjacent SiN stripes allow more flexibility in the sample positioning and alignment. For the first time, we map a static dispersive coupling and are able to position-tune the optomechanical coupling to the CNT with frequency pull parameters of up to $G_\omega/2\pi = 120\text{kHznm}^{-1}$. At a sample position with $g_\omega \approx 10\text{kHz}$ and a loaded cavity finesse of $\mathcal{F} = 95000$, we observe a peak in the cavity transmission spectrum that we attribute the a flexural vibrational mode of the CNT. We measure first hints of the optical spring effect and of optical induced damping on the vibrations of the CNT.
Zusammenfassung

Das Gebiet der Kavitäts-Optomechanik untersucht die Wechselwirkung zwischen elektromagnetischer Strahlung und einer makroskopischen mechanischen Mode, die durch eine optische Kavität verstärkt wird. Die Verwendung von niedrigdimensionalen Objekten auf dem Gebiet der Kavitäts-Optomechanik ist durch ihren geringen Streuquerschnitt im Vergleich zur Größe der optischen Kavitätsmode begrenzt. Faserbasierte Fabry-Pérot-Mikrokavitäten können winzige Modenquerschnitte aufweisen und dennoch eine hohe Finesse beibehalten, was die Licht-Materie-Wechselwirkung verstärkt und damit die empfindliche Detektion der Auslenkung kleinsten Objekte ermöglicht. In dieser Arbeit wird eine solche faserbasierte Mikrokavität vorgestellt, die einen mikrometergroßen Modenquerschnitt in Kombination mit einer ultrahohen Finesse aufweist.

Zum prinzipiellen Nachweis der dynamischen Rückkopplung im unserem System untersuchen wir Streifen, die aus einer freistehenden Siliziumnitrid (SiN)-Membran hergestellt werden. Wir sind in der Lage, die statische optomechanische Kopplung über die Probenposition zu verändern und erreichen Kopplungsparamater von bis zu $G_\alpha/2\pi = 3\,\text{GHz}\,\text{nm}^{-1}$. Wir zeigen den "optical spring effect" für die fundamentale Biegemode des Streifens. Die Messungen stimmen mit der gängigen Theorie überein und wir erhalten eine Einzelphotonenkopplung von $g_\text{0}\omega/2\pi = 2.5\,\text{kHz}$. Mit dem Wechsel zu Streifen aus stochiometrischem Siliziumnitrid ($\text{Si}_3\text{N}_4$) wird die Absorption im Streifen reduziert und die Finesse der Kavität mit eingebauter Probe erreicht Werte von bis zu $\mathcal{F} = 195\,000$. Die optomechanische Kopplungsstärke reduziert ist im Vergleich zur SiN-Probe aufgrund eines geringeren Brechungsindexes. Wir demonstrieren den "optical spring effect" und die optisch induzierte Dämpfung für zwei Biegemoden des Streifens mit einer Einzelphotonenkopplung von $g_\text{0}\omega/2\pi = 575\,\text{Hz}$. Es wird außerdem das Abkühlen der mechanischen Mode von Raumtemperatur bis auf etwa 12 K durch dynamische Rückkopplung gezeigt.

Zur Realisierung der dynamischen Rückkopplung auf Schwingungen von Kohlenstoff-Nanoröhren (CNTs) untersuchen wir zwei Probengeometrien. Diese erlauben es, die CNT innerhalb der Kavität zu positionieren. In einer Stimmgabel-Konfiguration führt die Anwesenheit der CNT zu einer Vergrößerung der Linienbreite der Kavität. CNTs, die zwischen benachbarten SiN-Streifen aufgewachsen sind, erlauben mehr Flexibilität bei der Positionierung und Ausrichtung der Probe. Wir weisen erstmals eine statische dispersive Kopplung nach und erreichen Kopplungsparameter von bis zu $G_\alpha/2\pi = 120\,\text{kHz}\,\text{nm}^{-1}$. Bei einer Probenposition mit einer Einzelphotonenkopplung von $g_\text{0}\omega \approx 10\,\text{kHz}$ und einer Finesse der Kavität von $\mathcal{F} = 95\,000$ beobachten wir einen Peak im Transmissionsspektrum der Kavität. Diesen ordnen wir einer Biegeschwingungsmoden des CNT zu. Wir messen erste Hinweise auf den "optical spring effect" und auf eine optisch induzierte Dämpfung der Schwingungen des CNTs.
0. Zusammenfassung
Chapter 1

Introduction

Light exerts forces on matter. In 1609, Johannes Kepler witnessed the passing of a comet in the European sky. He observed that a part of the comet tail always points away from the sun, which he explained by sun rays interacting with the celestial object [1]. Later, James Clerk Maxwell described light as electromagnetic radiation that carries a momentum [2]. He predicted arising radiation-pressure forces if light is reflected from a mirror. To test this hypothesis, Sir William Crookes invented the light mill [3]. Although the first observations were dominated by thermal effects, later experiments in light mill configuration represent the first experimental demonstration of radiation-pressure [4, 5]. The linear and angular momentum transfer of photons as predicted by Max Planck and Albert Einstein was shown by Otto Frisch [6] and Richard Beth [7] respectively. Nowadays, radiation pressure forces are commonly used in a variety of experimental techniques e.g. to trap particles in optical tweezers [8] or for the laser cooling of atomic motions [9].

Introducing an optical resonator (cavity) in the study of radiation-pressure forces boosts the light matter interaction and adds a retardation due to the finite lifetime of the cavity photons. In an electro-magnetic cavity with a suspended end-mirror, this force retardation leads to damping or antidamping of the harmonic motion of the end-mirror. Vladimir Braginsky studied those frictional effect in a microwave cavity in his pioneering works [10, 11]. First demonstrations in optical system were carried out in the group of Herbert Walther [12] studying the modification of the mechanical potential of a cavity end-mirror by the radiation pressure force.

Since then, the field of cavity optomechanics [13, 14] is prospering. Possible applications range from exploring quantum signatures of macroscopic objects [15, 16], over generating quantum states of light and matter [17, 19], to realizing nonreciprocal devices [20, 21] or acceleration or force sensors [22, 25]. Those applications are enabled by the extreme detection sensitivities of optomechanical systems [26] and often enhanced by minimal thermally induced decoherence at cryogenic temperatures or via reservoir engineering [27, 28].

A convenient way to optimize the performance of the optomechanical system is to separate the mechanical and the optical cavity mode in a membrane-in-the-middle type configuration [29]. Typically,
the mechanical resonators under investigation are macroscopic objects such as membranes with almost millimeter-scale lateral dimensions \([30,34]\). However, studying mechanical objects with dimensions in the lower nanometer regime can be an interesting alternative. In this work we aim towards cavity optomechanics with mechanical resonators formed by carbon nanotubes (CNTs). CNTs \([35,37]\) are microscopic tubes made out of carbon atoms arranged in a honeycomb lattice similar to the structure of graphene. We study single-walled carbon nanotubes (SWCNTs) with diameters in the in the single-digit nanometer range contrary to so-called multi-walled carbon nanotubes (MWCNTs) consisting of several nanotubes nested inside one another. As a result of their outstandingly low masses, systems employing CNT resonators are extremely sensitive to environmental changes enabling mass and force sensing with remarkable resolutions \([38,39]\). At the same time CNT resonators exhibit large zero-point fluctuations \(z_{zpf}\), enabling large single-photon coupling strengths \(g_0\omega\) when coupled to an electro-magnetic cavity mode. That makes them ideal candidates to reach the single-photon strong coupling regime \((g_0\omega > \kappa, \\kappa\) the cavity energy decay rate\) which allows to resolve the frequency shift induced by a single photon-phonon scattering process. Other promising properties include an exceptional large Young’s modulus \([40]\) and a low defect density enabling very high mechanical quality factors at low temperatures \([41,42]\).

Optical cavities with small mode volumes \([43]\) are perfect candidates to detect the vibrations of such mesoscopic objects. Reducing the mode volume increases the light-matter coupling \([44]\). This boosts the frequency pull parameter \(G_\omega = -\frac{\partial \omega_{\text{cav}}}{\partial z}\), which translates the displacement of the mechanical resonator to a frequency shift of the cavity. While the frequency pull parameter is set by the geometry and the optical properties of the system, the cavity finesse \(\mathcal{F}\) boosts the intracavity photon number \(n_{\text{circ}}\) and therefore the effective coupling \(g_\omega = \sqrt{n_{\text{circ}} g_0 \omega} = \sqrt{n_{\text{circ}} G_\omega z_{zpf}}\). In addition, it increases the magnitude of phase fluctuations in the output field and thus improves the sensitivity of the measurement.

This work introduces a platform for cavity optomechanics that is optimized for ultrasensitive optical detection of single-digit nanometer size mechanical objects. This is achieved with a fiber-based Fabry-Pérot microcavity (FFPC) \([43,45,47]\) that features a small mode cross section while maintaining the highest finesse reported so far in loaded FFPCs. While these systems are being studied in many groups \([48,51]\), they are either operated at low finesse or at low temperatures where parasitic vibrations are frozen out. Here, we demonstrate the operation of an ultrahigh finesse FFPC cavity at room temperature, which is enabled by a very rigid cavity gluing scheme combined with a thorough acoustic shielding. We note, that the choice of the mechanical resonator in this resonator-in-the-middle scheme is flexible, and conveniently allows for the investigation of, e.g., tethered membranes \([34,52]\), nanowires \([49]\) or low-dimensional materials such as carbon nanotubes (CNT) \([51,53,55]\) or two-dimensional crystals \([56]\). Especially CNTs are discussed as one possible path towards quantum optomechanics at room temperature \([54]\).

We present a proof of principle of radiation pressure backaction using free-standing silicon nitride membrane stripes which are inserted into the cavity. We demonstrate the optical spring effect and dynamical backaction cooling and heating on flexural vibrational modes of the stripes. Towards cavity optomechanics with CNTs, we introduce two sample geometries, that allow to insert free-standing CNTs into the cavity mode. First cavity optomechanical measurements on a CNT are shown.

This work is organized as follows. Chapter 2 covers the theoretical fundamentals. We consider important properties of Fabry-Pérot cavities and introduce the FFPC platform. A brief summary of important
nanomechanical basics is followed by a theoretical treatment of different cavity optomechanical models and resulting effects assuming both dispersive and dissipative optomechanical coupling. The experimental setup is described and a full characterization of the FFPC is given in Ch. 3.

We present the membrane stripe geometry in Ch. 4 and discuss the findings on low-stress SiN stripes. We map the dispersive and the dissipative couplings as a function of the position of the stripe inside the standing-wave cavity mode and we measure the optical spring effect on the flexural mechanical motion of the membrane stripe. In Ch. 5 we switch to high-stress Si$_3$N$_4$ stripes that show improved mechanical and optical properties compared to the low-stress SiN stripes. Again, we measure the position-dependant couplings and the optical spring effect. Additionally, we show dynamical backaction cooling of the vibrational mode temperature. First measurements on CNTs inside the setup can be found in Ch. 6. We measure a dispersive coupling of the CNT to the cavity mode and we find hints at dynamical backaction on the CNT vibrations.

A first generation of the setup used a different cavity wavelength. Due to absorption in the SiN membrane material at that wavelength, photothermal forces arose. We discuss those photothermal effects that finally led to a switch of the cavity wavelength in Ch. 7. Finally, Ch. 8 summarizes the findings of this work and suggests future improvements and directions for the experiment.
1. Introduction
Chapter 2

Fundamentals

In this chapter, we discuss the theoretical concepts necessary for the experiments in this manuscript. First, we focus on optics and discuss a Fabry-Pérot cavity while considering influences of losses on the cavity modes. We consider stable cavity modes and briefly discuss particularities of fiber-based Fabry-Pérot cavities.

The second part explains the vibrational dynamics of nanoresonators. We discuss the spectral response of a harmonic oscillator subjected to an external driving force. In a continuum mechanics treatment, the framework of Euler-Bernoulli beam theory describes the deflection patterns of nanomechanical systems. We use it to estimate their resonance frequencies.

Part three discusses the interaction between a mechanical mode and an optical cavity mode. The textbook cavity optomechanical model considering dispersive coupling is presented and we derive observable phenomena, namely the optical spring effect and optomechanical damping. Additionally, the influence of dissipative effects on the dynamical backaction is calculated. Finally, we discuss different system geometries and consider a nano-object inside a cavity.

We choose a notation that is in accordance with the textbook literature if possible. An overview of the symbols that we use is given in Ap. A.

2.1 Optics

The experiment in this work uses light to measure and manipulate the mechanical motion of a nano-object. The central optical element is a fiber-based Fabry-Pérot cavity as discussed in the following.

2.1.1 Fabry-Pérot cavity

A Fabry-Pérot cavity (FPC) is formed between two opposing mirrors with separation $L$ as shown in Fig. 2.1(a). Light that enters the cavity forms a standing wave pattern, if an integer multiple $m$ of the half-wavelength of the light $\lambda/2$ matches the cavity length: $m\lambda/2 = L$. Therefore, the cavity output
2. Fundamentals

Figure 2.1: (a) Schematics of a Fabry-Pérot cavity. An electro-magnetic field with amplitude $E_{\text{in}}$ impinges on the first mirror. We calculate the light field amplitudes in reflection $E_r$, in transmission $E_t$ and inside the cavity $E_{\text{cav}}$. The mirrors are described by amplitude reflection coefficients $r$, transmission coefficients $t$ and losses $l$. (b) Transmission spectrum $I_t$ normalized by the light intensity $I_0$ of a FPC for a mirror reflectivity of $R = 0.8$ (blue) and $R = 0.3$ (black).

spectrum displays equally spaced modes as plotted in Fig. 2.1 (b). In the following, we derive the electro-magnetic fields of the cavity output ports and inside the cavity.

To start, we assume that light enters with normal incidence. The mode matching between the input mode and the cavity mode is assumed to be perfect and clipping losses due to the finite size of the mirrors are neglected. We discuss imperfect mode matching in Sec. 2.1.4. The electro-magnetic fields are neither filtered spectrally nor spatially. These filtering types play a role in fiber Fabry-Pérot cavities (FFPC) as discussed in Sec. 2.1.2.

Both mirrors are assumed to be beam splitters with amplitude coefficients $t_{1,2}$, $r_{1,2}$ and $l_{1,2}$ for the transmission, reflection and the losses [57] of mirror 1 or 2 respectively. The arbitrary loss mechanisms may include e.g. absorption in the mirror material or scattering losses from surface roughnesses [43]. We assume symmetric properties for light passing the mirror in both directions: $t'_{1,2} = t_{1,2}$, $r'_{1,2} = r_{1,2}$ and $l'_{1,2} = l_{1,2}$. Because of energy conservation the reflectivity $\mathcal{R}_{1,2} = |r_{1,2}|^2$, the transmissivity $\mathcal{T}_{1,2} = |t_{1,2}|^2$ and the losses $\mathcal{L}_{1,2} = |l_{1,2}|^2$ of mirror 1 or 2 add up to unity: $\mathcal{R}_{1,2} + \mathcal{T}_{1,2} + \mathcal{L}_{1,2} = 1$. The calculations presented below follow Ref. [58].
Transmission

To calculate the light field measured in transmission, we sum up the fields circulating inside the cavity:

\[ E_t = E_{in} t_1 t_2 \exp(ikL) \sum_{p=0}^{\infty} (r_1 r_2 \exp(2ikL))^p \]

\[ = E_{in} \frac{t_1 t_2 \exp(ikL)}{1 - r_1 r_2 \exp(2ikL)} \]

\[ = E_{in} \frac{t_1 t_2 \exp(ikL)}{1 - \sqrt{1 - (L_1 + T_1 + L_2 + T_2) + O(\|t_{1,2}\|^4) \exp(2ikL)}}. \quad (2.1) \]

In the last step energy conservation is used and we assume that \( r_1 r_2 \) is real. Now we identify the total losses \( L_{tot} = L_1 + T_1 + L_2 + T_2 \) of the cavity. In the limit of highly reflective mirrors \( R_{1,2} \approx 1 \), terms of the order of \( O(\|t_{1,2}\|^4) \) can be neglected. The remaining square root in the denominator can be Taylor expanded:

\[ \sqrt{1 - L_{tot}} \bigg|_{L_{tot} \ll 1} \approx 1 - \frac{L_{tot}}{2}. \]

On resonance the wave vector is given by \( k_0 = \frac{\pi m}{L} \) with the mode number \( m \). For large mirror reflectivities it is sufficient to evaluate Eq. (2.1) around resonance \( k = k_0 + \delta k \). Here \( \delta k \approx 0 \) and \( \exp(ikL) \approx (-1)^m \).

We can identify the detuning \( \Delta = c \delta k \) of the incoming light with respect to the cavity resonance using the speed of light in vacuum \( c \).

The free spectral range (FSR) \( \omega_{FSR} = \pi c \) corresponds to the frequency spacing of two adjacent cavity modes \( (m, m+1) \). Now the exponential function in the denominator can be simplified to \( \exp(2ikL) \big|_{\Delta \ll \omega_{FSR}} \approx 1 + \frac{2\pi i \Delta}{\omega_{FSR}} \).

Finally the transmitted field can be approximated by

\[ E_t \approx E_{in} (-1)^m \frac{t_1 t_2}{1 - (1 - \frac{L_{tot}}{2})} \left( 1 + \frac{2\pi i \Delta}{\omega_{FSR}} \right) \]

\[ \approx E_{in} (-1)^m \frac{\sqrt{\kappa_1 \kappa_2}}{\frac{\pi}{2} - i\Delta} \quad (2.2) \]

Here the external coupling \( \kappa_{e1,2} = \frac{\omega_{FSR}|t_{1,2}|^2}{2\pi} \) specifies the rate of photons entering or leaving the cavity through mirror 1 or 2, respectively. \( \kappa \) is the total loss rate of the cavity \( \kappa = \kappa_0 + \kappa_{e1} + \kappa_{e2} \), with the internal losses \( \kappa_0 = \frac{\omega_{FSR}(|l_1|^2 + |l_2|^2)}{2\pi} \). It corresponds to the full width at half maximum of the Lorentzian frequency response of the cavity resonance. The finesse \( \mathcal{F} \) describes the spectral resolution of a FPC. It is given by

\[ \mathcal{F} = \frac{\omega_{FSR}}{\kappa} = \frac{2\pi}{L_{tot}}. \quad (2.3) \]

The power measured at the transmission port of the cavity under the assumption of a large finesse and for small detunings compared to the FSR is

\[ P_t \approx P_{in} \frac{\kappa_{e1} \kappa_{e2}}{\left( \frac{\pi}{2} \right)^2 + \Delta^2}. \quad (2.4) \]
Reflection

With calculations similar to the ones above we can derive the reflected field:

\[
E_r = E_{\text{in}} \left( r_1 + t_1 t_2 \exp(i\pi) \sum_{p=1}^{\infty} (r_1 \exp(2ikL) r_2)^p \right) \\
= E_{\text{in}} \left( r_1 - t_1 t_2 \exp(2ikL) \right) \\
\approx E_{\text{in}} \left( 1 - \frac{|t_1|^2}{2\pi \frac{\omega}{\text{FSR}}} \left( \frac{\kappa}{2} - i\Delta \right) \right) \\
= E_{\text{in}} \left( 1 - \frac{\kappa t_1}{\left( \frac{\kappa}{2} - i\Delta \right)} \right). 
\tag{2.5}
\]

With the same assumptions as above, the power measured at the reflection port of the cavity is

\[
P_r \approx P_{\text{in}} \left( 1 - \frac{\kappa^2}{\left( \frac{\kappa}{2} + \Delta \right)^2} \right). \tag{2.6}
\]

Intracavity field

We can calculate the field inside the cavity the same way we derived the transmitted field but without considering the transmission through the second mirror:

\[
E_{\text{cav}} = E_{\text{in}} \sqrt{\frac{\kappa t_1}{\frac{\kappa}{2} - i\Delta}}. \tag{2.7}
\]

The circulating power in the cavity is given by

\[
P_{\text{cav}} \approx P_{\text{in}} \frac{\kappa t_1}{\left( \frac{\kappa}{2} \right)^2 + \Delta^2}. \tag{2.8}
\]

To calculate the number of photons \( n_{\text{circ}} \) inside the cavity we have to consider the energy \( U_{\text{cav}} \) of light stored inside the cavity. The energy is given by the stored power times the cavity round-trip time \( \tau = \frac{2\pi}{\omega_{\text{FSR}}} \):

\[
U_{\text{cav}} = P_{\text{in}} \frac{\kappa t_1}{\left( \frac{\kappa}{2} \right)^2 + \Delta^2}. \tag{2.9}
\]

The photon number can be obtained by dividing the stored energy \( U_{\text{cav}} \) by the energy of a single photon \( h\omega \) with the photons angular frequency \( \omega \) and the reduced Planck’s constant \( \hbar \):

\[
n_{\text{circ}} = \frac{P_{\text{in}}}{h\omega} \frac{\kappa t_1}{\left( \frac{\kappa}{2} \right)^2 + \Delta^2}. \tag{2.10}
\]
2.1 Optics

2.1.2 Fiber-based Fabry-Pérot cavities

Miniaturization of FPCs allows to reduce the cavity mode cross-section and to boost light-matter coupling [44]. Promising systems are fiber Fabry-Pérot cavities (FFPCs) [43, 46, 59]. FFPCs consist of mirrored optical fibers. The fibers are cleaved and the resulting fiber end faces are shaped by CO2 laser ablation. Short and powerful laser pulses allow to form atomically smooth mirror surfaces with radii of curvatures \( R \) in the range of tens to hundreds of \( \mu \text{m} \). The resulting mirror surfaces are then ion beam sputter (IBS) coated with a distributed Bragg reflector (DBR) coating. The resulting coatings show optical absorption losses in the coating wavelength range of the order of tens of ppm. A subsequent annealing process increases the designed transmission because it decreases the absorption in the DBR material. With careful alignment of two fiber mirrors, very short cavities with lengths and mode cross-sections in the single-digit \( \mu \text{m} \) range [43, 60] and finesse values of up to around \( F \approx 250000 \) are achievable [61].

Compared to open cavities, the fibers of FFPCs lead to a non-Lorentzian cavity response in reflection [46]. Spatial filtering of the light leaving the cavity leads to two effects specific to FFPCs. First, a misalignment of the mirror curvature with the fiber core leads to a reduction of the off-resonant transmission, termed prompt reflection in the following. This misalignment is considerable especially for fiber end faces with imperfect cleave angle. A portion of the light is reflected with an angle bigger than the compliance angle of the fiber and is guided out of the fiber core. This effect is particularly noticeable for single-mode (SM) optical fibers due to their small numeric apertures. Additionally, the magnitude of the dip that is observed in reflection depends on the relative phase between the prompt reflection and the light that leaves the cavity through the input mirror and is guided in the fiber core. This leads to an asymmetric lineshape of the reflection that is specific to FFPCs. A complete treatment of this case is given in Ref. [46]. The physics is similar to the asymmetric lineshape in microwave cavity resonances due to reflections in the feedline circuits [62–64].

2.1.3 Gaussian modes

The paraxial Helmholtz equation describes the propagation of electromagnetic waves. A cavity formed between two spherical mirrors 1 and 2 with radii of curvature \( R_{1,2} \) and spacing \( L \) forms stationary boundary conditions for the light mode inside the cavity. Depending on the coordinate system Hermite-Gaussian modes or Laguerre-Gaussian modes provide an orthogonal set of solutions to this problem. We consider only TEM\(_{00}\) modes in the following. We assume that the wavefront curvature \( R(z) \) must match the mirror profiles at the mirror interfaces. This leads to a set of equations for the Gaussian beam parameters in terms of the mirror curvatures and the cavity length [58].

It is helpful to calculate those parameters as a function of the \( g \) parameter of the cavity mirrors defined as:

\[
g_{1,2} = 1 - \frac{L}{R_{1,2}}.
\]  

(2.11)

The \( g \) parameter depends on the curvature of the mirror and on the cavity length. Originally introduced to describe laser resonators, these parameters allow to simplify the treatment of the Gaussian beam inside the cavity.
Important parameters that we use here are the mode waist which measures the beam size at the focus

\[ w_0^2 = \frac{L\lambda}{\pi} \sqrt{\frac{g_1g_2(1 - g_1g_2)}{(g_1 + g_2 - 2g_1g_2)^2}}, \]  \hspace{1cm} (2.12)

and the mode radii at the mirrors

\[ w_{1,2}^2 = \frac{L\lambda}{\pi} \sqrt{\frac{g_{2,1}}{(g_{1,2}(1 - g_1g_2))}}. \]  \hspace{1cm} (2.13)

The divergence of the beam is described by the Rayleigh range \( z_R = \frac{\pi w_0^2}{\lambda} \) with the input light wavelength \( \lambda \) as depicted in Fig. 2.2. Light that passes the waist acquires an additional phase factor called the Gouy phase \[ \Theta(z) = \arctan \left( \frac{z}{z_R} \right). \]  \hspace{1cm} (2.14)

Here, \( z \) describes the distance from the waist.

**Figure 2.2:** Electro-magnetic field distribution of a Gaussian beam propagating in \( z \) direction. The Rayleigh range \( z_R \) is the distance along the propagation direction to the place where the cross section area is doubled compared to the waist. Here this results in \( w(z_R) = \sqrt{2}w_0 \).
2.1 Optics

2.1.4 Cavity stability and mode matching

The $g$ parameter of the cavity can also be used to estimate whether a cavity is in a stable configuration. For a stable cavity the following relation is satisfied:

$$0 \leq g_1 g_2 \leq 1.$$  \hfill (2.15)

In Fig. 2.3 the product of the stability parameters $g_1 g_2$ is depicted for curvature ranges accessible with fiber mirrors and for a fixed cavity length of $L = 50 \mu m$. Relevant stable configurations are near-planar ($R_1, R_2 \gg L$), hemispherical ($R_1 = L, R_2 \gg L$), confocal ($R_1 = R_2 = L$) and concentric ($R_1 = R_2 = L/2$) cavities. Near-planar cavities are favorable for very short cavities because of their insensitivity to length fluctuations. In cases where a small beam waist is favorable, concentric cavities are used.

![Diagram showing cavity stability $g_1 g_2$. Stable cavity configurations are displayed in blue for different mirror curvatures $R_1$ and $R_2$ for a fixed cavity length of $L = 50 \mu m$. Relevant configurations are near-planar ($R_1, R_2 \gg L$), hemispherical ($R_2 = L, R_1 \gg L$), confocal ($R_1 = R_2 = L$) and concentric ($R_1 = R_2 = L/2$). The black lines encircle exemplary parameters that correspond to the shown cavity configurations.](image)

The mirror transmission defines the input coupling $\kappa_e$ of the cavity as discussed above. Additionally, the efficiency to inject light in and to extract light from the cavity is determined by the mode matching between the cavity mode and the input or output mode. For perfectly aligned Gaussian modes, the input
The coupling efficiency $\varepsilon$ through the input mirror 1 is given by

$$\varepsilon = \frac{4}{\left(\frac{w_f}{W_f} + \frac{w_f}{W_1}\right)^2 + \left(\frac{2\eta_f w_f}{R_1 A}\right)^2}.$$  

(2.16)

$w_f$ is the mode diameter of the input fiber, $n_f$ the refractive index of the fiber core. In the formula we include the lensing effect by the curved fiber surface and a phase mismatch due to wavefront curvature [43]. Similarly, one can calculate the efficiency to out-couple light from the cavity through mirror 2.

Figure 2.4 displays $\varepsilon$ for a fixed cavity length of $L = 50\mu m$ and varying mirror curvatures. For the input fiber we assume a SMF28 fiber with parameters $w_f = 4.6\mu m$ and $n_f = 1.46$. For near-planar cavities, input coupling efficiencies of above 80% are expected. In the experiment we are unable to measure this mode matching parameter and thus include it in the input coupling $\kappa_{e,1}$ of the input SM fiber. In the output port, a graded index multimode fiber with a fiber core of $50\mu m$ provides a mode matching of close to unity.

**Figure 2.4:** Input coupling efficiency $\varepsilon$ for different mirror curvatures $R_1$ and $R_2$ for a fixed cavity length of $L = 50\mu m$. The contour lines visualize the input coupling efficiencies.
2.2 Nanomechanics

This work studies two different kinds of nanomechanical resonators: silicon nitride (SiN) membrane stripes with incorporated tensile pre-stress and free-standing carbon nanotubes (CNTs). The deflectional vibrations of both types of mechanical resonators can be modelled in the framework of continuum mechanics. However, in physical experiments often the time-dependent deflection of any point along the beam is irrelevant and the problem can be mapped to a simple harmonic oscillator with one degree of freedom. Details on the calculations in this chapter can be found in standard textbooks and review articles [65–69].

2.2.1 Lumped-element model

We model a single mechanical mode of a mechanical resonator as a point mass placed at the point of maximum beam deflection. The mode is described by an effective mass \( m_{\text{eff}} \) oscillating with an effective spring constant \( k_{\text{eff}} \) and subjected to damping described by the damping constant \( \Gamma \). To calculate the effective parameters, we compare the kinetic energies in the lumped-element model to the kinetic energies of the continuum mechanics structure [69]. For a doubly-clamped, stressed beams, we obtain

\[
m_{\text{eff}} = \frac{m_0}{2}
\]

with \( m_0 \) the mass of the mechanical structure. The eigenfrequency of the mode is defined as

\[
\Omega_0 = \sqrt{k_{\text{eff}} m_{\text{eff}}}
\]

This results in the equation of motion for a periodically driven (time-dependent driving force \( F(t) \)), damped harmonic oscillator with the displacement \( \delta z \):

\[
F(t) = m_{\text{eff}} \frac{\partial^2 \delta z(t)}{\partial t^2} + \Gamma m_{\text{eff}} \frac{\partial \delta z(t)}{\partial t} + k_{\text{eff}} \delta z(t).
\] (2.17)

In the Fourier space we obtain the relation between the one-sided power spectral density (PSD) for the mechanical motion \( S_z \) and the spectral density of the driving force \( S_F \)

\[
S_z(\omega) = |\chi(\omega)|^2 S_F(\omega),
\] (2.18)

with the mechanical susceptibility

\[
\chi(\omega) = \frac{1}{m_{\text{eff}} (\Omega_0^2 - \omega^2 - i\omega \Gamma)}.
\] (2.19)

We assume a thermal driving force that can be approximated as white noise in the frequency range of interest. With the equipartition theorem we can write \( S_F = 4 k_B T \Gamma m_{\text{eff}} \) with the temperature of the phonon bath \( T \) and the Boltzmann constant \( k_B \). This allows us to express the PSD of the thermally driven mechanical mode under weak damping as

\[
S_z(\omega) = \frac{4 k_B T \Gamma}{m_{\text{eff}} \left( (\omega - \Omega_0)^2 + (\Gamma)^2 \right)}.
\] (2.20)

In a real-world experiment, we measure the PSD \( S_{z_{\text{exp}}} \) given by \( S_{z_{\text{exp}}}(\omega) = \beta S_z(\omega) + S_{w, V} \) with the calibration factor \( \beta \) with units \( V^2/m^2 \) and assuming a white noise background \( S_{w, V} \). Thus, we can calibrate the sensitivity of the measurement from a Brownian motion PSD [69].
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The mechanical quality factor \((Q\text{-factor})\) is defined as \(2\pi\) time the stored vibrational energy related to the energy dissipated per cycle:

\[
Q = \frac{\Omega_0}{\Gamma}.
\] (2.21)

Relevant damping mechanisms of nanomechanical systems are e.g. thermoelastic effects [70], built-in two-level systems [71] or surface coatings and defects [72, 73]. Extrinsic dissipation stems from e.g. gas damping [74] or clamping losses [75, 76].

2.2.2 Euler-Bernoulli beam theory

In a continuum mechanics treatment, we consider a free-standing beam between two clamps. The beam has dimensions of length \(L_{\text{beam}}\), width \(w_{\text{beam}}\) and thickness \(t_{\text{beam}}\) as sketched in Fig. 2.5. We consider transversal flexural vibrations perpendicular to the resonator surface (out-of-plane (oop) modes) and in the resonator plane (in-plane (ip) modes).

The Euler-Bernoulli beam theory is based on two main assumptions: Areas oriented perpendicular to the neutral plane remain planar areas perpendicular to the neutral plane under deflection of the beam and deflections are small. Those assumption hold for resonators of linear, isotropic and homogeneous material under elastic deformations. We consider the bending moment, shear, inertial forces, and tensile stress acting on an infinitesimal segment \(dx\). The generalized Euler-Bernoulli equation can be derived:

\[
\rho A \frac{\partial^2 \delta z(x,t)}{\partial t^2} + EI \frac{\partial^4 \delta z(x,t)}{\partial x^4} - \sigma A \frac{\partial^2 \delta z(x,t)}{\partial x^2} = 0,
\] (2.22)

where \(A\) is the cross section area of the beam, \(\rho\) the mass density, \(E\) the Young’s modulus, \(\sigma\) the incorporated tensile stress and \(I\) the area moment of inertia. The solutions are given by the product of a time-dependent amplitude and the position dependent mode shape function. The clamping conditions of the beam determine the boundary conditions of the mode shape function. For our doubly-clamped beam with simply supported clamps, the boundary conditions lead to the eigenfrequency of the \(j\)-th mode:

\[
\Omega_{0,j} = \frac{\pi j^2}{L^2} \sqrt{\frac{EI}{\rho A}} \sqrt{1 + \frac{\sigma A L^2}{EI^2 \pi^2}}.
\] (2.23)
For large tensile stress or long beams compared to their lateral dimensions \( \frac{\sigma A L^2}{EI_j} \gg 1 \) the
eigenfrequencies can be approximated by the string model

\[
\Omega_{0,j} = \frac{\pi j}{L} \sqrt{\frac{\sigma}{\rho}}.
\] (2.24)

Even though it is not obvious that basic assumptions made in the Euler-Bernoulli framework are valid
for CNTs, continuum mechanics is found to well describe their vibrational behavior at least for small
displacements [78].

### 2.3 Cavity optomechanics

The field of cavity optomechanics studies the parametric coupling between an optical cavity mode
and a mechanical mode [13, 14]. Interest in the field increased when advances in micro- and
nanofabrication enabled high-Q nanomechanical resonators. Those could be coupled to high finesse
optical resonators accessible through commercially available dielectric mirror coatings with low losses.
First implementations were vibrating cavity end mirrors on cantilevers [79, 80] or micromechanical beam
resonators [81]. We will use such a flexible end-mirror system as a theoretical model in the following.

It should be noted that similar formulas can be derived for different systems and the same physics can
be found in e.g. membrane-in-the-middle configurations (MIM) [29], optomechanical disc resonators
[82], photonic optomechanical crystals [15], microwave cavity optomechanical systems [16] or levitated
nanoparticles [28] inside an optical cavity. All systems have their own advantages and disadvantages
and allow to study different regimes and phenomena. Because MIM systems are relevant for this work,
we will discuss them later. Not only the modes are completely different for all those systems but also
the coupling mechanisms differ greatly [13]. We consider the theory for radiation pressure dynamical
backaction arising from a dispersive or dissipative coupling. In principle, photothermal forces can also
give rise to backaction.

#### 2.3.1 Dispersive optomechanics

Most experiments in cavity optomechanics can sufficiently be described with a linear dispersive coupling.
A theoretical toy model of such a dispersively coupled system is depicted in Fig. 2.6. It consists of an
optical FPC formed by two opposed mirrors. The cavity is resonant with a certain TEM\(_{00,m}\) mode with
frequency \( \omega_{\text{cav}} = m \frac{2 \pi}{L} \) and damping constant \( \kappa \). One of the mirrors has a rigid mount, while the other
one is suspended on a spring (mounted on a vibrating beam in this case). It vibrates with its mechanical
eigenfrequency \( \Omega_0 \). We neglect other mechanical modes of the mirror. The mechanical damping constant
is denoted by \( \Gamma \).
Figure 2.6: Common optomechanical model. The cavity optomechanical model consists of an optical cavity with flexible end mirror. The end mirror is mounted on a beam resonator. Taken from Ref. [14].

### Dispersive coupling

A displacement $z_m$ of the end mirror changes the cavity length $L \rightarrow L + z_m$. For small displacements the cavity resonance becomes

$$\omega_{\text{cav}}(z_m) \approx \omega_{\text{cav}}(0) + \frac{\partial \omega_{\text{cav}}(z_m)}{\partial z_m}.$$  \hspace{1cm} (2.25)

One defines the so called frequency pull parameter $G_{\omega}$ as

$$G_{\omega} = -\frac{\partial \omega_{\text{cav}}(z_m)}{\partial z_m}. \hspace{1cm} (2.26)$$

On the other hand, the optical cavity mode exerts a radiation pressure force on the mechanical mode as

$$F_{\text{rad}} = n_{\text{circ}} \hbar G_{\omega}, \hspace{1cm} (2.27)$$

with the number of circulating cavity photons $n_{\text{circ}}$. Thus, $G_{\omega}$ sets the magnitude of optomechanical effects. It can be calculated in the framework of cavity perturbation theory (see e.g. Refs. [44, 83])

$$G_{\omega} = \frac{\omega_{\text{cav}}(0)}{2} \frac{\partial}{\partial z_m} \left( \iint_{\text{dielectric}} \varepsilon_0 \left( \varepsilon_r(\vec{r}) - 1 \right) \left| \vec{E}(\vec{r}) \right|^2 d\vec{r} \right) \iint \varepsilon_0 \left| \vec{E}(\vec{r}) \right|^2 d\vec{r}, \hspace{1cm} (2.28)$$

with the electro-magnetic field amplitude $\vec{E}(\vec{r})$, the dielectric constant $\varepsilon_0$ and the permittivity $\varepsilon_r(\vec{r})$. The parametric coupling is mediated by a shape or material perturbation of the cavity. We assume that $\varepsilon_r(\vec{r}) - 1$ vanishes outside the dielectric element that causes this perturbation.

In the case of our one-dimensional toy model, $G_{\omega}$ scales inversely with $L$. In the following we consider a linear dispersive coupling which sufficiently describes systems where the position dependence of the optical resonance is weak. However, in some experimental configurations higher order coupling terms may become dominant [84].
Classical dynamics

In the classical regime, the mechanical displacement can be described by the modified equation of motion
\[
m_{\text{eff}} \frac{\partial^2 z_m(t)}{\partial t^2} + m_{\text{eff}} \Gamma \frac{\partial z_m(t)}{\partial t} + m_{\text{eff}} \Omega_0^2 z_m(t) = F_{\text{rad}}(t) + F_{\text{ext}}(t),
\]
(2.29)
including the radiation pressure force \(F_{\text{rad}}\). We calculate the complex light field amplitude \(\alpha\) in the input-output formalism for a two-sided cavity. In a frame rotating at the frequency of the laser drive, the dynamics of the cavity light field amplitude is given by
\[
\frac{\partial \alpha(t)}{\partial t} = i(\Delta - G_\omega z_m) \alpha(t) - \frac{\kappa}{2} \alpha(t) + \frac{\kappa}{2} \alpha_{\text{max}}.
\]
(2.30)
The detuning of the laser frequency with respect to the cavity resonance is defined as \(\Delta = \omega_l - \omega_{\text{cav}}\). The imaginary term describes the change of cavity resonance frequency due to the parametric coupling. The latter two terms describe the cavity decay and the laser drive. The constant amplitude of the laser drive \(\alpha_{\text{max}}\) is defined in a way that in the uncoupled regime (\(G_\omega = 0\)) Eq. (2.10) is recovered.

Equations (2.29) and (2.30) are a set of nonlinear equations coupled through the complex light field amplitude \(\alpha\) that is related to the photon number as \(|\alpha|^2 = n_{\text{circ}}\). To solve those equations, we assume that the system has a steady solution (\(\frac{\partial z_m(t)}{\partial t} = \frac{\partial \alpha(t)}{\partial t} = 0\)) with small fluctuations around this steady state. In this case we can linearize both amplitudes
\[
\begin{align*}
z_m &= \bar{z} + \delta z(t), \\
\alpha &= \bar{\alpha} + \delta \alpha(t).
\end{align*}
\]
(2.31)
(2.32)
Since the fluctuations are small, we keep only terms of first order in \(\delta z\) and \(\delta \alpha\). The resulting equations of motion read
\[
m_{\text{eff}} \frac{\partial^2 \delta z(t)}{\partial t^2} + m_{\text{eff}} \Gamma \frac{\partial \delta z(t)}{\partial t} + m_{\text{eff}} \Omega_0^2 \delta z(t) = hG_\omega (\bar{\alpha}^* \delta \alpha(t) + \bar{\alpha} \delta \alpha^*(t)) + F_{\text{ext}}(t),
\]
(2.33)
\[
\frac{\partial \delta \alpha(t)}{\partial t} = i\Delta_{\text{eff}} \delta \alpha(t) - \frac{\kappa}{2} \delta \alpha(t) + iG_\omega \delta \alpha^*(t).
\]
(2.34)
The constant radiation pressure term leads to a shifted equilibrium position \(\bar{z} = F_{\text{rad}}/m_{\text{eff}}\Omega_0^2\). This static mechanical displacement modifies the detuning, resulting in an effective detuning \(\Delta_{\text{eff}} = \Delta + G_\omega \bar{z}\).

The coupled partial differential equations (2.33) and (2.34) can be solved straight-forwardly in Fourier space (for details see Ref. [13]). The resulting mechanical response is
\[
\delta z(\omega) = \frac{F_{\text{ext}}(\omega)}{m_{\text{eff}}(\Omega_0^2 - \omega^2 - i\omega\Gamma) + \Sigma(\omega)},
\]
(2.35)
Optomechanical backaction leads to a modification of the mechanical susceptibility by the optomechanical self-energy \(\Sigma(\omega)\)
\[
\Sigma(\omega) = -iG_\omega |\bar{\alpha}|^2 \left( \frac{i(\omega + \Delta_{\text{eff}}) + \frac{\kappa}{2}}{(\omega + \Delta_{\text{eff}})^2 + (\frac{\kappa}{2})^2} - \frac{i(\omega - \Delta_{\text{eff}}) + \frac{\kappa}{2}}{(\omega - \Delta_{\text{eff}})^2 + (\frac{\kappa}{2})^2} \right).
\]
(2.36)
The prefactor can be written in terms of the zero-point fluctuations $z_{\text{zpf}} = \sqrt{\frac{\hbar}{2m_{\text{eff}}\Omega_0}}$, the single-photon coupling strength $g_{0\omega} = G_\omega z_{\text{zpf}}$ and the photon-number enhanced coupling $g_{\omega} = g_{0\omega}|\alpha|$ as $\hbar G_\omega|\alpha|^2 = 2m_{\text{eff}}\Omega_0 g_{\omega}^2$.

Overall, we showed that optomechanical coupling modifies the mechanical response. The real part of the optomechanical self-energy shifts the mechanical resonance position (optical spring effect), the imaginary part adds an additional dissipation channel (optomechanical damping). Those two effects are discussed in the following.

### Optical spring effect

First, we consider the change in the mechanical spring constant described by $\text{Re} \Sigma(\omega)$. We evaluate Eq. (2.36) at the mechanical resonance and define the resulting frequency shift $\delta(\Omega^2)$ as

$$\delta(\Omega^2) = \frac{1}{m_{\text{eff}}} \text{Re} \Sigma(\Omega_0) \quad (2.37)$$

$$\approx 2\Omega_0 g_{\omega}^2 \left( \frac{\Omega_0 + \Delta_{\text{eff}}}{(\Omega_0 + \Delta_{\text{eff}})^2 + (\frac{\kappa}{2})^2} - \frac{\Omega_0 - \Delta_{\text{eff}}}{(\Omega_0 - \Delta_{\text{eff}})^2 + (\frac{\kappa}{2})^2} \right). \quad (2.38)$$

The mechanical response can be expanded for small frequency shifts as

$$\Omega_m = \sqrt{\Omega_0^2 + \delta(\Omega^2)}$$

$$\approx \Omega_0 + \frac{1}{2\Omega_0} \delta(\Omega^2). \quad (2.39)$$

Note that the light field amplitude is included in the coupling strength $g_{\omega}$. When the mechanical response is plotted against effective detuning ($\Omega(\Delta_{\text{eff}})$), one has to consider that the photon number $n_{\text{circ}} = |\alpha|^2$ also depends on the detuning as described in Eq. (2.10). In the resolved sideband regime ($\Omega_0/\kappa > 1$), the mechanical motion that is mixed in the cavity output spectrum results in resolved motional sidebands. In this situation the amplitudes of the motional sidebands have to be considered when calculating the photon number. Their magnitude strongly depends on the experimental implementation (e.g. thermal motion amplitude of the mechanical resonator).

Figure 2.7 displays the mechanical resonance frequency as a function of the laser detuning in different regimes. In the bad cavity regime ($\Omega_0/\kappa < 1$, Fig. 2.7(a)) the influence of the dependence of the photon number is illustrated. For the case that the cavity photon number follows a Lorentzian response (blue curve), the maximum frequency shift is reached for $\Delta_{\text{eff}} = \pm \kappa/4$. If the photon number is fixed (black curve), the maximum frequency shift is obtained for $\Delta_{\text{eff}} = \pm \kappa/2$. Figure 2.7(b) displays the optical spring effect in a resolved-sideband system for a fixed photon number. Here the maximum frequency shift is achieved around the mechanical Stokes and anti-Stokes sidebands $\Delta_{\text{eff}} = \pm \Omega_0$. 
2.3 Cavity optomechanics

Figure 2.7: Dispersive optical spring effect. The mechanical resonance frequency normalized by the natural eigen-frequency $\Omega_\text{m}/\Omega_0$ for different effective detunings $\Delta_{\text{eff}}$ is plotted. (a) The simulation parameters in the bad cavity regime are $\Omega_0/\kappa = 0.1$ and on resonance $g_\omega = \Omega_0$. We compare the response for a constant photon number (constant $g_\omega$, black line) and for a Lorentzian response of the photon number (blue line). The constant coupling plots corresponds to what is often depicted in textbooks and research articles, while the blue line corresponds to a more realistic experimental situation where the photon number in the cavity is not controlled and thus depends on the chosen detuning. (b) In the resolved-sideband regime we use simulation parameters $\Omega_0/\kappa = 10$ and a constant coupling of $g_\omega = \Omega_0/100$.

**Optomechanical damping**

Cavity retardation leads to friction that we define as

$$\Gamma_{\text{opt}} = -\frac{1}{m_{\text{eff}}\Omega_0} \text{Im}(\Sigma(\Omega_0))$$

$$= g_\omega^2 \kappa \left( \frac{1}{(\Omega_0 + \Delta_{\text{eff}})^2 + \left(\frac{\kappa}{2}\right)^2} - \frac{1}{(\Omega_0 - \Delta_{\text{eff}})^2 + \left(\frac{\kappa}{2}\right)^2} \right).$$

(2.40)

In total the effective damping of the mechanical resonator is $\Gamma_{\text{eff}} = \Gamma + \Gamma_{\text{opt}}$. Depending on the sign of the optomechanical damping, the damping of the mechanical resonator is either increased (cooling of the mechanical mode) or decreased (heating of the mechanical mode). If the effective damping is tuned towards zero, parametric instabilities arise and the mechanical resonator starts to perform self-induced oscillations [85]. The optomechanical damping is plotted in Fig. 2.8.

**Quantum treatment and limit to cooling**

In the classical treatment we showed that optomechanical backaction can be used to effectively cool the mechanical mode. To estimate the limit of this cooling scheme, a quantum treatment of our model system is required.

First, we quantize the mechanical and the optical mode. We write the position operator $\hat{z}_m$ in terms of
2. Fundamentals

Figure 2.8: Dispersive optomechanical damping. The effective mechanical damping normalized by the internal damping $\Gamma_{\text{eff}}/\Gamma$ for different effective detunings $\Delta_{\text{eff}}$ is plotted. (a) The simulation parameters in the bad cavity regime are $\Omega_0/\kappa = 0.1$ and on resonance $g_\omega = \Omega_0$. We compare the response for a constant photon number (constant $g_\omega$, black line) and for a Lorentzian response of the photon number (blue line). (b) In the resolved-sideband regime we use simulation parameters $\Omega_0/\kappa = 10$ and a constant coupling of $g_\omega = \Omega_0/100$.

the phonon creation and annihilation operators $\hat{b}^\dagger, \hat{b}$

\[ z_{\text{zpf}} = z_{\text{zpf}} \left( \hat{b} + \hat{b}^\dagger \right). \]  

(2.41)

$z_{\text{zpf}}$ represents the zero-point fluctuations introduced above. The optical field is expressed with the photon creation and annihilation operators $\hat{a}^\dagger, \hat{a}$. The complex light field amplitude is defined as $\alpha = \langle \hat{a} \rangle$. The resulting Hamiltonian for linear dispersive coupling is

\[ \mathcal{H} = \hbar \Omega_0 \hat{b}^\dagger \hat{b} + \hbar \alpha \langle 0 \rangle \hat{a}^\dagger \hat{a} - \hbar G_\omega z_{\text{zpf}} \left( \hat{b} + \hat{b}^\dagger \right) \hat{a}^\dagger \hat{a}. \]  

(2.42)

It neglects dissipation and driving of the two modes. Those are better described via the input-output formalism. To simplify this cubic Hamiltonian, we linearize the light field as before: $\hat{a} = \bar{\alpha} + \delta \hat{a}$. $\bar{\alpha}$ describes the average light field amplitude and $\delta \hat{a}$ are quantum fluctuations. Again we shift the energy reference to account for the new equilibrium position due to the constant radiation pressure force term and we neglect small fluctuations of the order $\delta \hat{a}^\dagger \delta \hat{a}$. Finally we switch to the frame rotating at the frequency of the laser drive and we include the optomechanical coupling strength $g_\omega = g_0 \omega |\alpha|$ to obtain the "linearized" optomechanical Hamiltonian

\[ \mathcal{H} = \hbar \Omega_0 \hat{b}^\dagger \hat{b} - \hbar \Delta \delta \hat{a}^\dagger \delta \hat{a} - \hbar g_\omega \left( \hat{b} + \hat{b}^\dagger \right) \left( \delta \hat{a} + \delta \hat{a}^\dagger \right). \]  

(2.43)

This is a standard Hamiltonian of two coupled harmonic oscillators, coupled through linear dispersive coupling. To calculate the temporal dynamics of the system, equations of motion in the form of quantum Langevin equations are used. The experiment presented in the scope of this dissertation is operated purely in the classical regime. For that reason, the calculations are not presented in detail here and the interested reader is referred to Ref. [13].
2.3 Cavity optomechanics

One important result of the quantum treatment that we will use in this work is the minimal phonon number $N_{\text{min}}$ as calculated in Refs. [85, 86]

$$N_{\text{min}} = \left(\frac{\kappa}{4\Omega_0}\right)^2. \quad (2.44)$$

Hence, in the resolved-sideband regime phonon occupations $N_{\text{min}} \ll 1$ are accessible, which allows cooling close to the quantum-mechanical ground state. Ground state cooling in the bad cavity limit is possible when an additional dissipative coupling mechanism is present in the system [13].

Classical static bistability

Another effect of radiation pressure in optomechanical systems is static bistability as first reported in Ref. [12]. It is a result from the modification of the mechanical potential energy $V_m = \frac{1}{2}m_{\text{eff}}\Omega_m^2z_m^2$ by the radiation pressure potential $V_{\text{rad}} = -\hbar\kappa|n_{\text{max}}|^2\arctan(2(G_{\omega_m} - 2\kappa)/\kappa)$. If the light intensity inside the cavity is large enough, two stable local minima in the effective mechanical potential $V_{\text{eff}} = V_m + V_{\text{rad}}$ are generated [14]. This results in a hysteresis behaviour under parameter sweep as shown in Fig. 2.9. The data is obtained by numerically solving the coupled equations of motion.

![Figure 2.9: Static bistability. We numerically calculate a hysteresis behaviour for a cavity sweep in the regime of static bistability. The black line is obtained for a sweep with increasing laser detuning, the blue line corresponds to a sweep with decreasing detuning. The dotted lines connect the two stable solutions.](image)

Note that photothermal forces may also give rise to bistabilities. One possible mechanism is absorption in the mirror substrates [87]. Heating and expansion of the mirror coatings change the cavity length and thus lead to a parametric coupling between the circulating light power and the cavity resonance. While in our system the static bistability only occurs in the presence of a mechanical resonator, this photothermal bistability can be observed in the empty cavity if the input power is big enough.
2. Fundamentals

2.3.2 Dissipative optomechanics

In the situation above, dynamical backaction effects are enabled by a finite cavity response time. Depending on the value and on the sign of the detuning, the feedback is in phase or out of phase with the mechanical displacement. The underlying force gradient arises from the change in intracavity light field intensity due to a change in resonance frequency with mechanical displacement. Another mechanism that leads to such a change in light field amplitude and thus to a radiation pressure force gradient, is a modulation of the cavity dissipation with mechanical displacement. This so called dissipative coupling can be described by a corresponding frequency pull parameter

\[ G_\kappa = \frac{\partial \kappa(z_m)}{\partial z_m}. \] (2.45)

We term the resulting single-photon coupling \( g_{0\kappa} \) and the effective coupling \( g_\kappa \). Dissipative coupling arises e.g. in MIM systems. Possible mechanisms are absorption in the dielectric membrane material \[88\] or position dependent scattering of the cavity photons \[53\]. Effects of membrane absorption are discussed in the following, the implications of a nanoscatterer inside an optical cavity are described in Sec. 2.3.4. Another effect leading to (small) dissipative couplings is the modulation of the input coupling \( \kappa(z_m) \) \[89\].

Although the underlying coupling mechanism is dissipative in nature, the resulting effects arise from radiation pressure dynamical backaction. This is in contrast to backaction mediated by photothermal forces.

To include a dissipative coupling in the standard dispersive optomechanical theory, we use a complex refractive index \( \text{Im}(n) \neq 0 \) for the mechanical element. The imaginary part of the relative permittivity \( \varepsilon_r \approx (\text{Re}(n) + i\text{Im}(n))^2 \) leads to a small imaginary part of the frequency shift in Eq. (2.28). As a consequence, the cavity dissipation acquires a component that depends on the mechanical position, \( \kappa = \kappa_0 + \kappa(z_m) \). This nonlinear dissipation process leads to an additional input noise terms in the Heisenberg equations of motion. To solve them, again the system is linearized and the set of coupled equations is solved. These troublesome calculations are presented in Ref. \[88\]. We write the resulting equations for the optical spring effect and for the optical damping in the form of Eqs. (2.38) and (2.40). The resulting optical spring effect reads:

\[ \delta(\Omega^2) = 2\Omega_0 g_{0\omega}^2 \left[ \frac{\Omega_0 + \Delta_{\text{eff}}}{(\Omega_0 + \Delta_{\text{eff}})^2 + \left(\frac{\Delta_{\text{eff}}}{2}\right)^2} - \frac{\Omega_0 - \Delta_{\text{eff}}}{(\Omega_0 - \Delta_{\text{eff}})^2 + \left(\frac{\Delta_{\text{eff}}}{2}\right)^2} \right] \]
\[ - 2\Omega_0 g_{0\omega} g_\kappa \kappa \left[ \frac{1}{(\Omega_0 + \Delta_{\text{eff}})^2 + \left(\frac{\Delta_{\text{eff}}}{2}\right)^2} + \frac{1}{(\Omega_0 - \Delta_{\text{eff}})^2 + \left(\frac{\Delta_{\text{eff}}}{2}\right)^2} \right]. \] (2.46)

We see, that compared to the purely dispersive case, a second term is added. This term depends on the sign of \( g_{0\omega} g_\kappa \), is symmetric around \( \Delta_{\text{eff}} = 0 \) and scales linearly with \( \kappa \). Figure 2.10 displays the influence of this additional term.
2.3 Cavity optomechanics

![Figure 2.10](image)

**Figure 2.10:** Dissipative optical spring effect. The mechanical resonance frequency normalized by the natural eigenfrequency $\Omega_m/\Omega_0$ for different effective detunings $\Delta_{\text{eff}}$ are plotted. (a) In the bad cavity regime ($\Omega_0/\kappa = 0.1$) we compared the dissipative case with couplings $g_\omega = \Omega_0$ and $g_\kappa = g_\omega/8$ on resonance (blue line) with a purely dispersive case with resonant coupling $g_\kappa = 0$ (black line). In both cases, the photon number exhibits a Lorentzian dependence on the effective detuning. (b) In the resolved-sideband regime ($\Omega_0/\kappa = 10$) the plotted couplings are $g_\omega = \Omega_0/100$ and $g_\kappa = g_\omega/2$ in the dissipative case (blue line) and in the purely dispersive case $g_\kappa = 0$ (black line). In both cases we calculate with constant couplings.

Similarly, the optical damping is expanded by a dissipative term

$$
\Gamma_{\text{opt}} = g_\omega^2 \kappa \left[ \frac{1}{(\Omega_0 + \Delta_{\text{eff}})^2 + \left( \frac{\kappa}{2} \right)^2} - \frac{1}{(\Omega_0 - \Delta_{\text{eff}})^2 + \left( \frac{\kappa}{2} \right)^2} \right] - g_\omega g_\kappa \left[ \frac{\Omega_0 - \Delta_{\text{eff}}}{(\Omega_0 - \Delta_{\text{eff}})^2 + \left( \frac{\kappa}{2} \right)^2} + \frac{\Omega_0 + \Delta_{\text{eff}}}{(\Omega_0 + \Delta_{\text{eff}})^2 + \left( \frac{\kappa}{2} \right)^2} \right].
$$

Again this leads to an asymmetry between the Stokes and the anti-Stokes sideband, but the influence on the optical damping is far less pronounced than on the optical spring (see Fig. 2.11).

### 2.3.3 Membrane-in-the-middle

The "suspended mirror" geometry as introduced before has some major technical drawbacks. In order to boost the single-photon coupling, small mass mechanical resonators are favorable. On the other hand, in these configurations, the cavity mode cross section poses a limit on the lateral dimensions of the movable mirror [90]. Further reducing those dimensions leads to increased clipping losses and limits the attainable optical finesse. High quality dielectric mirror coatings add to the effective mechanical mass and increase mechanical dissipation.

Because of that, different optomechanical configurations with specific advantages and applications are studied. In this work we focus on the so-called membrane-in-the-middle [29, 32, 91] (MIM) geometry. MIM systems are formed by a rigid cavity with a movable dielectric sheet (membrane) inside the cavity. The system can be thought of as a cavity formed by three mirrors with a semi-transparent middle mirror.
2. Fundamentals

The two cavity modes are coupled by the membrane transmission. A displacement of the membrane will change the length of the cavities, resulting in a frequency shift of the coupled system. A displacement of the membrane by half a wavelength will translate to a cavity frequency shift of one free spectral range for a perfectly reflecting membrane. In the case of high membrane transmission, the cavity TEM\(_{00}\) modes of the cavities exhibit an avoided crossing, reducing the frequency shift per membrane displacement [92].

This dispersive coupling stems from the introduction of the dielectric sheet inside the cavity mode. Depending on its relative position with respect to the cavity mode, the membrane changes the phase of photons circulating in the cavity and thus effectively changes the cavity mode frequency. We can calculate the resulting cavity modes in the transfer-matrix formalism [88, 92, 93]. A degenerate perturbation theory treatment [82, 94] on the other hand describes the full spectrum of cavity modes even for an arbitrary alignment of the membrane and the optical axis of the cavity.

The main advantages of MIM systems it that by separating the mechanical resonator from the cavity, one can achieve a stable, rigid cavity with high finesse. Additionally, one can change the relative position of the membrane with respect to the cavity mode. This allows to tune the optomechanical coupling strength. On top of that, a quadratic dispersive coupling that is dominant with the membrane positioned close to a node or anti-node of the cavity field, can be enhanced by tilting the membrane with respect to the cavity mode axis [84, 95] (avoided crossings between different optical modes increase the quadratic coupling). A quadratic dispersive coupling is expected to allow quantum non-demolition measurements by position squared readout [30, 94, 96]. A quartic coupling as demonstrated in Ref. [84] is expected to allow the preparation of Schrödinger Cat states [97].
2.3 Cavity optomechanics

2.3.4 Nano-objects inside cavity

Instead of dielectric membranes, nanomechanical objects with sub-wavelength lateral dimension can be coupled to a rigid optical cavity as illustrated in Fig. 2.12. If the scattering dipoles of those objects are aligned with the cavity mode, this allows cavity optomechanics with dimensions below the optical diffraction limit \[^{[13, 53]}\]. Since the dimensions of such a nanomechanical object are small compared to the mode cross section, this scattering can be modelled by the insertion of a thin plane of point-like dipoles. The plane is described by a transmittance and reflectance with an imaginary conductance \(\tilde{\Sigma} = \Sigma_1 + i\Sigma_2\). The real part describes a dissipative interaction, the imaginary part gives rise to a dispersive interaction. One can derive analytical expressions for the resonant cavity transmission \(P_{\text{res}}\) and frequency shift:

\[
\begin{align*}
P_{\text{res}}(\bar{z}) &= \frac{1}{\left(1 + \frac{4\pi}{\bar{z}} \Sigma_1 \sin^2 (k\bar{z})\right)^2}, \\
\Delta(\bar{z}) &= \frac{2c\Sigma_2}{L} \sin^2 (k\bar{z}).
\end{align*}
\]

Figure 2.13 plots the resonant transmission and the cavity frequency shift as a function of the nano-scatterer position for a purely dispersive case (\(\Sigma_1 = 10^6, \Sigma_2 = 0\)) and for a purely dissipative case (\(\Sigma_1 = 0, \Sigma_2 = 10^6\)).
Figure 2.13: Static coupling of a nanoscatterer to a FPC. (a) The resonant transmission as a function of the sample position $z$ is calculated from Eq. (2.48) using $F = 10^5$. In the dispersive case ($\Sigma_1 = 10^6$, $\Sigma_2 = 0$, blue line), the transmission is independent of the sample position. We observe a modulation of the transmission with the period of the standing-wave cavity field for a dissipative case ($\Sigma_1 = 0$, $\Sigma_2 = 10^6$, black line). (b) The purely dispersive case leads to a modulation of the cavity resonance frequency. In the purely dissipative case the cavity resonance frequency is unaltered. Simulation parameters and color code are the same as in panel (a).

The magnitude of those effects depends on the nanomechanical scatterer but also on its interaction with the cavity mode. Again, it is calculated via cavity perturbation theory. Reference [53] lists analytical approximations for a singly-clamped bar, a doubly-clamped bar and a strongly stressed bar.

Experimental realizations use a variety of nanomechanical objects ranging from cold atoms [98], nanorods [49, 59], levitated nanoparticles [100, 101] and pressure waves in He [48, 102] to CNTs [51]. Many of those systems can be engineered with incorporated optical dipole transitions. This would enable the realization of hybrid optomechanical systems.
Chapter 3

Fiber-based microcavity setup

This chapter describes the experimental setup that has emerged around the FFPC in the last few years. We start with the discussion of the main optical and electrical components of the setup. Afterwards the cavity layout is depicted and we take a look on the cavity fabrication process. The last part shows measurements that characterize the cavity. The measurements allow to extract properties of the cavity mode and mirrors.

3.1 Experimental setup

During my work on this project, the experimental setup went through countless numbers of iterations and advances compared to the state described in [77]. At the original wavelength of $\lambda = 780$nm, photothermal effects in the mirror substrates as well as in the mechanical resonator materials led to experimental difficulties. Chapter [7] describes some of the effects that can be observed in such a system. The switch to telecom wavelengths eliminated those experimental difficulties.

At telecom wavelengths optical fiber-based components are cheap and available in a variety of configurations. Together with new measurement devices (especially a digital lock-in amplifier with sufficient bandwidth) this allowed to simplify and greatly improve the setup. The addition of a second laser line at the cavity wavelength enables to probe the cavity with a defined detuning.

Figure 3.1 (a) shows an overview of the experimental setup. Especially the electrical part changes significantly depending on the measurement that is performed. Different configurations are detailed in the following. We operate the setup at room temperature at a pressure of $1 \cdot 10^{-6}$mbar to avoid gas damping. We maintain the vacuum by an ion pump to avoid pump vibrations. For acoustical shielding, the cavity is placed inside a wooden box on a damped optical table. A photograph of the inside of this wooden box is displayed in Fig. 3.1 (b).
3. Fiber-based microcavity setup

3.1. Optical setup

The main component of the setup is the FFPC. This device is intrinsically fiber-coupled. Therefore we use fiber-based components with single-mode fibers (SMF28) on the input side of the cavity and graded index multimode fibers (GI50D) on the output side of the cavity. Fiber connections are either fusion spliced or use angled (FC/APC) connectors to minimize back-reflections and to eliminate standing waves inside the fibers. Optical fibers are depicted as orange connections in Fig. 3.1 (a).

We use an ultra-low noise fiber laser (Koheras Basik E15) at \( \lambda = 1550.12 \text{nm} \) to generate light at the cavity wavelength. A Pockels cell in an electro-optic modulator (EOM, iXblue MPZ-LN-10) allows to phase modulate sidebands that we use for heterodyne measurements and to generate a Pound-Drever-Hall signal for cavity stabilization [103, 104]. A MEMS based variable optical attenuator (VOA, Thorlabs V1550A) attenuates the lock tone by up to 60dB. A fiber polarization controller (depicted as three fiber loops) enables us to select a polarization mode of the cavity.

The probe tone is generated by a second laser (Koheras Basik E15 FM). The frequency modulation option of this laser allows to sweep the tone over the locked cavity resonance. The laser maintains its low noise profile while it can be frequency detuned by a total of around 500MHz. We attenuate the probe tone and select one polarization. Afterwards, a 90:10 coupler combines the probe tone and the lock tone. The two tones pass a circulator and travel to the cavity input port. The reflection from the cavity is measured at a
3.1 Experimental setup

photodetector (PD1)\textsuperscript{4}

An additional laser tone at $\lambda = 780\text{ nm}$ is used to measure sample position and vibrations. To this end, the light of an external cavity diode laser (ECDL) in Littrow configuration is locked to an optical transition in a rubidium vapor cell [105]. We discuss this laser in Appendix D. Since the wavelength is outside the coating band of the FFPC mirrors and is not propagated in the SM fiber, the MM fiber mirror and the sample surface form a low finesse interferometer.

Light that exits the cavity at the output port passes a wavelength-division multiplexer (WDM) that splits the 780 nm light from light at the cavity wavelength. PD2 measures the cavity transmission. The slow output allows to measure the DC transmission to extract e.g. the cavity photon number, the fast output is demodulated for the heterodyne measurements. The 780 nm interferometric signal passes a 50:50 splitter and is measured by PD3.

3.1.2 Electrical setup

Since the system has a lot of parameters to monitor and adjust, the electrical setup is quite complex and the ability to remote control all the relevant parameters is crucial.

Depending on the measurement, we select from four different photodetectors to convert light to electrical signals. Two commercial photodetectors and one homebuilt detector work at 1500 nm, a second homebuilt detector at 780 nm. A commercial photodetector (Femto OE200) is used as power meter and for low light applications (e.g. mapping of the static optomechanical couplings). It possesses a switchable gain of up to $10^4 \text{V}/\mu\text{W}$, but its bandwidth is very limited especially at higher gain settings.

A homebuilt photodetector is designed to feature a bandwidth of more than 30 MHz. While that does not allow to resolve a sideband-resolved (phase-)modulation, it is fast enough to measure the mechanical frequencies of interest. It features two outputs, one is DC coupled with a gain of around 0.95 V/μW and one fast AC port with a gain of the same magnitude. That allows to lock the cavity with down to around 10 nW of transmitted light power. A second homebuilt detector with comparable specifications uses a silicon based photodiode and allows to measure the light from the 780 nm interferometer. The heterodyne measurement of the cavity quadratures requires even higher modulation frequencies. For that purpose a commercial detector (Thorlabs PDB435C-AC) with a bandwidth of 350 MHz is used. Since the gain of this detector is limited to 0.01 V/μW, it is not suited to measure in low light situations.

The heterodyne measurements and the generation of the error signal for the cavity feedback are enabled by a high frequency lock-in amplifier (Zurich Instruments HF2-LI) as sketched in Fig. 3.2. The lock-in drives the EOM to phase modulate the light that passes our system (device under test, DUT). We measure the resulting amplitude modulation of the transmitted light with a sufficiently fast photodetector (PD, we either use the Thorlabs or the homebuilt detector). The lock-in amplifier digitizes and demodulates the AC output of the PD. The resulting quadratures $X$ and $Y$ are output from the auxiliary output of the lock-in and recorded on an oscilloscope.

We measure the spectral response of the nanomechanical resonators with a spectrum analyzer

\textsuperscript{4}Depending on the exact measurement we use one of four different photodetectors. Their specifications are summarized in Sec. 3.1.2
3. Fiber-based microcavity setup

Figure 3.2: Schematic of heterodyne measurements. The EOM enables to modulate the phase of the light field with a sinusoidal signal. The light interacts with the device under test (DUT) and is converted back to an electrical signal at the PD. A dual demodulation extracts the quadratures $X$ and $Y$. From that quadratures the amplitude $R$ and the phase $\Theta$ are calculated.

Figure 3.3: Piezoelectric tuning and stabilization of the cavity length. A DC voltage source generates an offset that is amplified in the high voltage amplifier (HV Amp) and is used to change the position of the input mirror. The achievable length change is enough to tune the cavity by almost one FSR. The lockbox is used to either scan across a cavity mode or to stabilize the cavity length. The lockbox output fine controls the input mirror position.

(Rhode & Schwarz FSVR7 or Rhode & Schwarz FSV4) with either the 780nm interferometer, the cavity transmission or the quadratures from the heterodyne measurement. Mechanical ring-down measurements allow to extract the energy dissipation more precisely, especially for low loss mechanical resonators. To this end, we actuate the mechanical mode of interest with the piezo on the sample holder. The interferometric signal is demodulated (either with the lock-in or with a spectrum analyzer) around the mechanical mode of interest to obtain the ring-down traces.

We control the cavity length with an electrical signal sent to the piezos of the cavity fibers. Figure 3.3 details this: a constant high voltage offset sent to the piezo of the MM fiber brings the cavity close to resonance of the desired optical mode. To this end, a DC signal is output from the lock-in aux output and amplified in a homebuilt high voltage amplifier. The amplifier can output $\pm 450$V (gain of 40V/V) and features a bandwidth of around 20kHz. To stabilize the output and to protect the amplifier from shorts, we low-pass filter the output with a cut-off frequency of 16Hz.

A analog lock-box (designed and built by Anton Scheich, chair of T. Hänsch at LMU Munich) allows to scan the cavity across resonance or to lock the cavity. A dither signal consisting of a ramp with an
3.1 Experimental setup

Figure 3.4: Electrical setup for sample positioning. We move the five positioner axes \((X, Y, Z, \Theta, \Phi)\) with the Attocube controller. A gamepad interfaces the Matlab program that steps the corresponding positioner axis. Two relays are used to switch between the goniometers and the Z axis. Additionally, DC voltages can be added to the individual axes allowing for piezoelectric fine-tuning of the sample position.

The lockbox outputs up to \(\pm 10\) V which detunes the cavity by around \(\pm 25\) GHz. If a bigger scan range is desired, we amplify the lockbox output in the high voltage amplifier. This results in a detuning range of slightly more than one free spectral range. Additionally, we can send the resulting HV scan to both cavity piezos simultaneously to further increase the scan range and to obtain a symmetric scan that keeps the spatial distribution of the cavity field constant.

Finally, the sample is positioned with the help of a nanopositioner stack (Attocube consisting of two horizontal positioners ANPx101, one vertical positioner ANPz101 and two goniometers ANG101). A piezo controller (Attocube ANC350) drives the positioner stack. The controller unit houses three modules. Each module can output ramps to step one positioner axis (step sizes in the \(\mu m\) range) or a constant voltage offset to scan the piezo (nm resolution). This voltage offset can either be generated internally or an external DC can be amplified in the module with a gain of 15 V/V. Two relays allow to control the five positioner axis with only three modules in the controller. This is sketched in Fig. 3.4. We remote control and interface the controller and the relays with a Xbox gamepad. This, together with camera access from two orthogonal directions, allows to intuitively position the sample. Once the sample is inserted and aligned to the cavity axis, we ground all axis and position the sample with respect to the cavity mode with a DC offset on the z axis. An internal 16Hz low-pass filter in the controller significantly decreases coupling of vibrations from the nanopositioner stack into the system. We change the DC offset to move the sample across several nodes and anti-nodes of the intracavity field.

An ultra-stable multichannel voltage source (Stahl-Electronics BS14) generates DC voltages that control the VOAs and the DC offset that adjusts the sample position \(\bar{z}\).
3.2 Cavity layout

Figure 3.5 (a) shows photographs of the fiber-based microcavity used for most of the measurements in this work. We start from commercial fibers: for the input fiber (top) we use a single mode fiber (art photonics GmbH) with a core of 9µm, a cladding of 125µm and a copper-alloy coating of 160µm. The output fiber (bottom) is a high NA graded index multimode fiber with a core of 50µm, a cladding of 125µm and a coating of 170µm. We etch the copper coating with a FeCl$_3$ solution and clean the fiber with an arc in the fusion splicer afterwards. This removes a remaining thin graphite layer. We cleave the fibers to form flat end surfaces.

The cavity itself consists of two mirrors concavely shaped on the fiber end faces by CO$_2$ laser ablation (center part of the image in Fig. 3.5 (a)). We collaborated with Jacob Reichel’s group at LKB Paris for the fibers at 780nm and with David Hunger’s group at KIT Karlsruhe for the 1550nm cavities. A white-light interferometer in Mirau configuration allows to measure the resulting curvatures of the mirror profiles. In this chapter we discuss measurements on a telecom wavelength cavity. For this cavity we obtain curvatures of 191µm and 140µm for the SM and MM fiber, respectively. This yields a near-planar cavity configuration [58]. The processed fibers are then IBS coated with a highly-reflective DBR centered around 1550nm with a designed transmission of 10ppm by Laseroptik GmbH. To form the cavity, the fibers are glued to silicon v-groove chips (top and bottom in Fig. 3.5 (b)) on top of shear piezo elements (cavity piezos). During the gluing process all degrees of freedom are aligned and fixed, resulting in a very rigid cavity configuration. Reference [77] describes the gluing process in great detail. Finally, we mount the finished cavity on an Aluminum frame, install it inside a rigid vacuum chamber.
and fusion splice it to connectorized fibers. Figure 3.5(b) shows the cavity as it is mounted inside the vacuum chamber.

We choose the v-groove cavity geometry over a ferrule based approach [47] because of its inherent mechanical and thermal stability. One main disadvantage of this geometry is the fact that we cannot open the cavity for sample insertion. With a designed cavity length of \( L = 30 \mu m \), this raises limitations on possible sample geometries. The mechanical resonators under study have to be inserted into this small gap between the optical fibers. Different sample geometries that comply with this limitation will be presented in the following chapters. We define the sample position \( \vec{z} \) with respect to its nearest field node as illustrated in the inset in Fig. 3.5(a).

### 3.3 Cavity characterization

In the following we discuss measurements to characterize the cavity and to determine important cavity parameters.

#### 3.3.1 Power calibrations

Some optomechanical measurements require the knowledge of e.g. the number of photons circulating inside the cavity or the amount of light leaving the cavity through one of its ports. To this end, we measure all the losses in the detection ports and we calibrate the gains of the detectors.

In the first step, we calibrate the amount of light sent to the cavity. We fix the laser to the maximum output power (to make sure it operates with the specified noise performance) and make sure it is well thermalized. Figure 3.6 shows the measured light power at the output port of the circulator as a function of VOA voltage. The power can be adjusted between around 250 nW and 2 mW. We term the measured power as cavity input power \( P_{in} \).

Next, we measure optical losses in the system and minimize them. The most dominant loss sources in the setup are the optical components and fiber connectors. After every mating process, we clean the fiber connector surfaces with a dry dust-free cloth. Under inspection with the fiber microscope, the contact surface should be free of debris and scratches. To minimize mating losses, the transmitted power is monitored and maximized during the mating process. Resulting losses are of the order of a few percent. Additionally, we find that losses in fiber splices are smaller than power fluctuations of the laser and can be neglected if the splicing process succeeds.

To sum up, we consider the relevant losses in the cavity and the detection ports: firstly, light measured in reflection passes through the optical circulator with measured combined losses (losses in the connectors and losses in the device) of 14%. Light in transmission passes the WDM with combined losses of 5%. Finally, not all light sent to the cavity enters the cavity. Light is lost due to imperfect input coupling efficiency (see Ch. 2.1.4) and due to misalignment of the cavity mirrors with the fiber core (see e.g. Ref. [46]). We expect the mode matching to be very good and therefore neglect it. The prompt reflection considers the fact that some of the light impinging on the fiber mirror is scattered out of the fiber core. This is due to imperfect alignment of the mirror curvature with the fiber core. It strongly depends on the cavity geometry and varies for the two polarizations modes.
Lastly, we calibrate the detectors. If possible, detector offsets are nulled. Afterwards we shine a known light power to the detector and measure the voltage response to obtain the detector gain. Note that depending on the detector, the gain can be frequency dependent. For example, for the homebuilt detectors, the gain decreases from $0.95 \text{V}/\mu\text{W}$ to $0.19 \text{V}/\mu\text{W}$ when a cavity scan with a frequency of $10.4 \text{Hz}$ and scan range of $\pm 25 \text{GHz}$ is recorded with the DC output of the detector.

The knowledge of all the relevant losses and the gains in the setup allows us to extract the light power reflected $P_r$ by and transmitted $P_t$ through the cavity. Together with the light power sent to the cavity, we can calculate the scattering parameters of the cavity. We term the cavity input port (SM fiber side) as port 1 and the output port (MM fiber side) as port 2. The relative field amplitudes at the two ports are $S_{11} = E_r/E_{\text{in}}$ and $S_{21} = E_t/E_{\text{in}}$. Consequently, the measured powers we obtain are $S_{11}^2 = P_r/P_{\text{in}}$ and $S_{21}^2 = P_t/P_{\text{in}}$. In the following, those will be referenced as normalized reflection ($S_{11}^2$) and normalized transmission ($S_{21}^2$).

### 3.3.2 Cavity scan

As mentioned earlier, we scan the cavity length while the laser wavelength is fixed to measure the cavity response. As the cavity resonance frequency depends inversely on the cavity length, we can assume a linear relation for small mirror displacements. In order to calibrate a voltage sent to the cavity piezo to a cavity resonance frequency, we phase modulate sidebands onto the frequency response spectrum of the cavity. A Lorentzian fit allows to extract the frequency axis. Figure 3.7 shows the normalized reflection and transmission of a TEM$_{00}$ mode of the cavity for a moderate input power of $P_{\text{in}} = 1.2 \mu\text{W}$. The two curves correspond to the two non-degenerate polarization modes of the cavity. Two polarization modes are visible because of deviations of the mirror profile from a perfect concentric shape. From
the frequency splitting, the relative tilt between the two mirrors can be calculated. Another possible mechanism leading to such a frequency splitting is mirror birefringence [106].

![Figure 3.7: Scattering parameters of the two polarization modes. Normalized reflection $S_{11}^2$ (solid lines) and transmission $S_{21}^2$ (dashed lines) of the empty FFPC for an input power of $P_{in} = 1.2 \mu W$ show two resonances. The lower frequency polarization mode (red) and the higher frequency polarization mode (blue) show a frequency splitting of 123.8 MHz.](image)

Now the prompt reflection $\eta_{pr}$ can be extracted directly from the off-resonant reflection. We obtain values of $\eta_{pr} \approx 0.6$ with slight deviations between the two polarization modes.

We fit the normalized reflection and transmission to extract the total cavity loss rate $\kappa$ and the input couplings $\kappa_{e1}$ and $\kappa_{e2}$ of the two mirrors (see Fig. 3.8). The deviation from the typical Lorentzian lineshape of free space cavities in the normalized reflection in Fig. 3.8 is striking. It stems from interference of light leaving the cavity with light that is prompt reflected. Together with the spatial filtering of the fiber core, this requires an extensive theoretical treatment that is presented in Ref. [46]. Since we are not able to extract all the relevant parameters for our system with sufficient accuracy, we fit the asymmetry with an additional heuristic phase factor. Because the dip height depends on the phase difference between the interfering beams, we assume to slightly underestimate the input coupling $\kappa_{e1}$.

The cavity can either be undercoupled ($\frac{\kappa_{/c}}{\pi} < 0.5$) or overcoupled ($\frac{\kappa_{/c}}{\pi} > 0.5$). For our two-sided cavity with intrinsic losses we assume that the cavity is undercoupled (we verify this assumption in the following). The obtained parameters are: $\kappa/2\pi = 16.8(1)$ MHz, $\kappa_{e1}/2\pi = 2.01(1)$ MHz and $\kappa_{e2}/2\pi = 3.17(2)$ MHz.

---

bThe heuristic fit function we use is: $S_{11}^2 = \left| \text{\sqrt{\eta_{pr}} - \frac{\kappa_{e1}}{\pi/2 + i\kappa} + V_{bg} \exp(i\Phi)} \right|^2$. 

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3. Fiber-based microcavity setup

Figure 3.8: Scattering parameters of the lower frequency polarization mode. Normalized reflection $S_{21}$, transmission $S_{22}$ and fits (black lines) of the empty FFPC for an input power of $P_{in} = 1.2\mu W$ are used to extract the loss rates. The fits of the reflection using Eq. (2.6) and the transmission using Eq. (2.4) result in $\kappa/2\pi = 2.01(1) \text{ MHz}$ and $\kappa/2\pi = 16.8(1) \text{ MHz}$.

3.3.3 Input coupling

To distinguish between the overcoupled and the undercoupled situation experimentally, we have to measure the quadratures of the cavity reflection. To this end the heterodyne detection scheme is used. In order to clearly resolve the heterodyne sidebands, we use a modulation frequency of $\omega_{het}/2\pi = 74.8\text{ MHz}$. Because of the modulation frequency, we require the high bandwidth of the Thorlabs detector. Additionally, this frequency is faster than the lock-in bandwidth. To compensate for that, we measure the attenuation of the lock-in at that specific frequency and correct for that. We output the quadratures $X$ and $Y$ on the oscilloscope and record a cavity scan. Finally the quadratures are normalized by the gains and the losses in the ports. Here we have to consider that the lock-in produces rms voltages.

Figure 3.9 (a-b) compares the obtained quadratures to the theoretical lines as calculated according to reference [61]. We highlight the good agreement with the theoretical curves. Deviations in the peak positions result from fluctuations of the cavity length during the scan. Both quadratures show a small offset compared to the theoretical curve. We explain them by the fact, that the phase between the local oscillator and the measured signal was not exactly set to zero, so the $X$ quadrature still has a small out-of-phase component. In the phase-space plot in Fig. 3.9 (c) this results in a small rotations of the measurement compared to the theory.
3.3 Cavity characterization

Figure 3.9: Quadratures $X$ (a) and $Y$ (b) of the lower frequency polarization mode measured via heterodyne detection (blue). The black solid lines represent the theory for the undercoupled case ($\kappa_1/2\pi = 2.0$ MHz). The black dashed lines are the overcoupled theory ($\kappa_1/2\pi = 14.7$ MHz) as extracted from an overcoupled fit of $S_{11}$ for illustration. (c) Phase space plot of the cavity response with theory curves for the overcoupled (black dashed line) and the undercoupled (black solid line) case. The small rotation of the measurement compared to the theory is due to imperfect phase reference.
3.3.4 Photothermal bistability

When the cavity is pumped more strongly, absorption in the mirror coating material leads to a thermal expansion of the coating. As a result, the effective cavity length changes. This non-linear mechanism leads to a bistability in the cavity frequency response as shown in Fig. 3.10. The cavity lineshape changes depending on the scan direction and on the scan speed. This effect can be used to passively stabilize the cavity on resonance ("self-locking") [43, 46].

![Graph showing scattering parameters](image)

**Figure 3.10:** Scattering parameters in the presence of strong pumping. Normalized reflection $S_{11}^2$ (blue solid line) and transmission $S_{21}^2$ (blue dotted line) show a non-linear response for an input power of $P_{in} = 35\mu W$.

3.3.5 Free spectral range

The FSR $\omega_{FSR}$ is inversely proportional to the cavity mirror separation $L$. From a measurement of the FSR, we can calculate the cavity length with high precision. In the cavity gluing process, this helps to determine the cavity length and enables to built cavities with the desired geometries.

For cavities at 780nm it turned out convenient to measure the spectrum under broadband light source illumination with a spectrometer. Due to the lack of access to a spectrometer at telecom wavelengths, we use a tuneable laser (TL) that can be tuned over more than 100nm to scan over several FSRs. Figure 3.11 (a) sketches the experimental setup to measure the FSR. We ground both cavity piezos and record the transmission and the reflection during a wavelength sweep of the laser. Additionally, a voltage can be applied to one of the piezos. If one tracks the change in FSR as a function of piezo voltage, one can in principle calibrate the piezo travel.

![Graph showing transmission](image)

**Figure 3.11 (b):** Transmission of the cavity for laser wavelengths $\lambda$ between 1520nm and 1630nm. The frequency axis is calculated from the exact wavelength of the laser at each measured point as $\omega = 2\pi c / \lambda$. Four equally spaced fundamental TEM$_{00}$ resonances can be identified. The left-most shows a higher transmission than the other resonances because the laser output power is not constant over the scanned wavelength range. Smaller resonances arise from higher order modes. Those modes are present due to the non-perfect alignment of the fiber mirrors.
3.3 Cavity characterization

![Diagram of cavity spectroscopy]

Figure 3.11: Cavity spectroscopy. (a) A tuneable laser (TL) allows to scan across several FSRs of the cavity. The cavity response is recorded in reflection (PD1) and in transmission (PD2). Both cavity piezos are grounded for this measurement. (b) Cavity transmission for a wavelength scan of the tuneable laser. The laser wavelength is scanned between $\lambda = 1520$nm and 1630nm. Resonances at $\omega/2\pi \approx 186.3$, 189.7, 193.1 and 196.5THz are attributed to TEM$_{0,0}$ modes, smaller peaks are higher order modes. Note that the laser output power is stronger for smaller frequencies.

The FSR extracted from this measurement is $\omega_{FSR}/2\pi = 3.42(2)$ THz. From the FSR we can calculate the effective cavity length as $L = \frac{\pi c}{2\omega_{FSR}} = 43.84(3)\mu$m. The camera suggests a cavity length of around 35$\mu$m. In the camera images the indentation of the mirror surfaces can not be seen. The indentation is a few $\mu$m on each mirror which explains the deviation between the two methods.

3.3.6 Mirror Properties

To sum up the cavity characterization, we calculate all accessible mirror properties. All the important figures are summarized in Tab. 3.1 for both polarization modes. The extracted values for the mirror transmittivities are close to the design value of 10ppm. In fact for the mirror curvatures and the given cavity length we expect values of $\varepsilon > 0.85$ as discussed in Ch. 2.1.4. In the evaluation process the mode matching is assumed to be perfect. That results in a lowered observed input coupling $\kappa_{e1}$. Additionally, we assume to underestimate the SM fiber input coupling as we neglect interference effects due to the misalignment of the fiber mirrors. Combined, this explains the slightly lower transmittivity of the SM mirror coating compared to the MM mirror coating.
3. Fiber-based microcavity setup

<table>
<thead>
<tr>
<th></th>
<th>Lower frequency polarization</th>
<th>Higher frequency polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prompt reflection $\eta_{pr}$</td>
<td>0.60(1)</td>
<td>0.62(1)</td>
</tr>
<tr>
<td>$\kappa/2\pi$</td>
<td>16.8(1) MHz</td>
<td>18.5(1) MHz</td>
</tr>
<tr>
<td>Finesse $\mathcal{F}$</td>
<td>203000</td>
<td>185000</td>
</tr>
<tr>
<td>Total losses $\mathcal{L}_{tot}$</td>
<td>31 ppm</td>
<td>34 ppm</td>
</tr>
<tr>
<td>$\kappa_1/2\pi$</td>
<td>2.01(1) MHz</td>
<td>2.57(1) MHz</td>
</tr>
<tr>
<td>$T_1$ (SM)</td>
<td>4.3 ppm</td>
<td>5.5 ppm</td>
</tr>
<tr>
<td>$\kappa_2/2\pi$</td>
<td>3.17(4) MHz</td>
<td>3.12(4) MHz</td>
</tr>
<tr>
<td>$T_2$ (MM)</td>
<td>9.2 ppm</td>
<td>5.8 ppm</td>
</tr>
<tr>
<td>Internal losses $\mathcal{L}_0$</td>
<td>17 ppm</td>
<td>23 ppm</td>
</tr>
</tbody>
</table>

Table 3.1: Mirror properties calculated from the calibration measurements for the two measured polarization modes.

3.4 Discussion

We present a setup that is built around a fiber-based micro-cavity. The cavities are glued on v-groove chips and thus feature a high rigidity. Combined with thorough acoustic shielding, the system provides a promising platform for cavity optomechanics at room temperature that is optimized for ultrasensitive optical detection of single-digit nanometer size mechanical objects.

We fully characterize a cavity with effective cavity length $L = 43.8 \mu$m corresponding to a free spectral range of $\omega_{fsr}/2\pi = 3.42$ THz. The mode volume is as small as $V \approx 250\lambda^3$ with a mode waist of 5.2 \mu m. The finesse of the empty cavity exceeds $\mathcal{F} = 200000$ and is only slightly lower compared to the largest values reported for FFPCs in literature (Ref. [61] reports values of up to $\mathcal{F} = 250000$). An annealing run before gluing the fibers has been shown to further decrease this absorption and increase the transmission of the coatings [61]. This should be taken into account for future cavities.

Spatial filtering of the optical fibers and misalignment of the fiber mirrors with respect to the fiber core lead to interference effects visible in the asymmetric lineshape of the cavity reflection. Measuring the transmittivity of the fiber mirror coating before gluing the cavity, the input coupling of the SM fiber could be extracted from a fit of the reflection to the theory presented in Ref. [46]. As we have no access to the transmittivity of the mirror coatings of the already glued cavity, we are unable to extract the input coupling with sufficient accuracy. Although we neglect this mechanism, we obtain reasonable values.
Optical spring effect on SiN membrane stripes

As a proof-of-principle, we demonstrate the optical spring effect on the vibrational motion of a membrane stripe made out of silicon-rich SiN in the following. Firstly, our system allows us to determine the static optomechanical coupling. The measurement of the cavity frequency shift per displacement of the mechanical resonator yields the frequency pull factor and thus enables us to calculate important figures of the system. Those results are compared to transfer matrix simulations.

We demonstrate radiation-pressure backaction by measurement of the optical spring effect on the fundamental oop mode of the membrane stripe. To the best of our knowledge, this is the first demonstration using a fiber cavity with high finesse. Experimental challenges arise because the system is driven close to the onset of static bistability. Those challenges are discussed as we evaluate the measurements.

4.1 Sample geometry and fabrication

The cavity length and the fiber dimensions pose limitations on the geometry of samples accessible in our setup. The mechanical resonator has to be free-standing over the effective lateral extension of the cavity mode. We found that scattering effects at edges and clipping losses limit the cavity finesse unless the mechanical resonator is free-standing over a distance of at least 30µm corresponding to a multiple of the beam waist. The substrate has to be thin enough to enter between the fibers and a clear aperture at least the dimensions of the optical fibers is needed. Additionally, the mechanical element should be transparent at the cavity wavelength. Figure 4.1 illustrates the geometrical constraints on the sample and presents our sample layout.

Our approach to this limitations is the use of free-standing silicon nitride (SiN) membranes. The membranes with thicknesses between 30nm and 200nm easily fit inside the cavity. A window opening of 500 × 500µm allows for some movement between the optical fibers. Since our cavity is glued and we cannot open it to insert the membrane, we mechanically break off one side of the substrate frame. Thus
4. Optical spring effect on SiN membrane stripes

Figure 4.1: Drawing of the geometry of the fiber cavity adopted from \[77\]. The optical fibers (blue, diameter of 125µm) and the cavity length (30µm) limit the sample geometries we can use inside the cavity. Our samples consist of free-standing SiN stripes (red). The stripes are etched from a membrane on top of a Si frame (grey, thickness 200µm) which is cleaved to form a u-shape. The window opening dimensions (500 × 500µm) allow to position the stripe inside the cavity.

we cleave the frame on one side, which results in a u-shape that allows to insert the membrane into the cavity. In order for the membrane to survive this mechanically cleaving process, we pattern and etch the membrane into stripes. Those stripes serve as mechanical resonators for the first few experiments presented in Ch. 4 and Ch. 5. Later, we use the stripes as holders for free-standing CNTs that are grown between two adjacent stripes (see Ch. 6).

4.1.1 SiN membrane patterning

We fabricate our membrane stripes with established nanofabrication techniques. We start from commercially available membrane chips (Norcada Inc.). Those chips consist of a Si frame of dimensions 5 × 5 × 0.2mm with a window opening of 500 × 500µm. A silicon-rich SiN membrane is free-standing over the window opening. While Si-rich SiN with low tensile stress turned out stable during fabrication, we were unable to etch stripes into stochiometric Si$_3$N$_4$ membranes with high stress because we cannot create a plasma that is gentle enough in our dry etcher.

The membrane chips come packed in gel capsules and require no pre-cleaning. We pattern the stripes via optical lithography using AZ MIR 701 photoresist with a HMDS adhesion layer. A custom holder block helps to mount the chips on the chuck of the spin coater. The use of a projection photolithography system (Smart Print from microlight3d) avoids the membrane from being in contact with a mask and allows for easier alignment and rapid prototyping. Either we use the photoresist as an etch mask for the reactive ion etch (RIE) of the SiN stripes, or we evaporate 30nm of Al. In both recipes, heat transfer fluid enhances the thermal contact to the substrate holder in the inductively coupled plasma (ICP) RIE. Afterwards, we remove the etch mask in NaOH and clean the samples in piranha solution. Process parameters are stated in Appendix B.

To cleave the frame, we mount the sample in a custom holder block and use a diamond scribe to scratch
the backside of the frame. The holder enables to position the scratch in a reproducible way with a precision of around 100µm. We cleave the chip from the backside to protect the sample surface. Finally, the sample is glued to a sample holder and mounted on the nanopositioner stack.

4.2 Static spectroscopy

The measurements presented in this chapter are performed on a 30nm low-stress SiN membrane (Norcada NX5050X) etched into wide stripes of dimensions 80µm×500µm×30nm. Figure 4.2 shows a micrograph of the sample. We select a comparatively thin membrane in order to reduce absorption losses inside the dielectric material. The width of the stripes prevents scattering losses on the stripe edges and facilitates fabrication. We glue the sample to the sample holder, insert it in the cavity and align it with the cavity axis. Finally, the MIM configuration allows to scan the sample across the standing-wave pattern of the optical cavity mode. Therefore, we can track the cavity resonance frequency as a function of the sample position and directly measure the frequency pull factor $G_\omega$ of the optomechanical system. Similarly, we can monitor the cavity losses and extract the dissipative pull factor $G_\kappa$.

The sample alignment is challenging. A well proven procedure is to coarsely align the sample by eye to the axis of the optical fibers. Afterwards, we rapidly scan the sample height (in $z$ direction along the cavity axis) by around 1µm and note down the maximum and minimum transmission powers. We then slightly tilt the sample in one direction and repeat the height scan. The alignment improves, if the minimum transmission increases. Iteratively we can adjust both tilt angles. Another measure of the sample alignment is the magnitude of higher order modes in the optical spectrum of the cavity [94]. A different possible alignment procedure is thus to suppress those higher order modes.

4.2.1 Experimental setup

As mentioned before, in a MiM setup we can directly measure the static optomechanical couplings $G_\omega$ and $G_\kappa$. The setup used for this measurement is shown in Fig. 4.3. We use a single laser tone and record the cavity transmission to track the cavity detuning and the linewidth $\kappa$. Modulated sidebands (EOM) allow for the calibration of the cavity frequency axis. With the VOA we attenuate the laser tone

![Figure 4.2:](image)

Figure 4.2: Optical micrograph of the low-stress SiN sample used for the measurements presented in this chapter. The Si frame (grey) is cleaved to form a u-shape. Two free-standing SiN stripes have been etched from a 500µm×500µm large, 30nm thick membrane. Parts where the membrane or its supporting frame have been removed appear as a black background. The third and uppermost stripe broke during fabrication. Triangular shapes at the clamping points help during lift-off.
4. Optical spring effect on SiN membrane stripes

Figure 4.3: Schematic of the setup to measure the static optomechanical coupling. Compared to the layout described before, we generate a voltage ramp (Dither) to scan the cavity. In the lockbox we offset and attenuate the scan to adjust the scan range and position. The lockbox output is amplified (HV) with a fixed gain. The resulting signal scans both cavity piezos simultaneously. Phase modulated sidebands (SB) allow to calibrate the resulting cavity length change.

enough to obtain a linear cavity response. We observe a static bistability (see Sec. 4.2.5) especially for sample positions with large optomechanical coupling. Therefore, we work with rather small input powers and use the Femto detector in transmission. The detector gain is adjusted to resolve the peak even in the presence of large dissipative couplings, but not saturate the detector for positions with small cavity linewidths.

Depending on the sample properties and position, we expect detunings of the cavity resonance of magnitude THz. To scan the cavity over such a broad frequency range, we amplify the scan output of the lockbox in the high-voltage amplifier. The resulting HV signal is split and sent to both cavity piezos simultaneously. That allows to scan the cavity over a range of around three FSR. The piezo elements we use are not rated for those large scan voltages. Particularly for negative voltages and for very large positive voltages, the expansion and contraction of the piezos is strongly nonlinear in the applied voltage. For that reason, we apply an offset to the scan with the lockbox to keep the cavity close to resonance with the desired mode. The scan voltage is attenuated to only scan the mode of interest to obtain a cavity length scan as linear as possible.

Finally, we can vary the sample position with respect to the optical field. With the optical mode centered on one membrane stripe, we ground all positioner axes except for the \( z \) axis. A 16Hz low-pass filter reduces electronic noise and greatly enhances the cavity resonance shape with the sample inserted. Now we apply a DC voltage to the positioner \( z \) axis to move the sample across several nodes of the cavity field. As a cross check, we use the 780nm interferometer to track and calibrate the sample position.

4.2.2 Calibrations

First, the sample position and the cavity detuning must be calibrated. We scan the sample position by applying a voltage to the respective positioner piezo. We will calibrate the resulting displacement of the piezo in the following.
4.2 Static spectroscopy

Figure 4.4: Calibration of the sample height and the cavity detuning. (a) The measurement of the probe reflection shows a periodicity with respect to the sample position. Fitting with a Fourier fit (black line) with period $780\,\text{nm}/2$ yields the calibrated height axis indicating the sample position. The axis is shifted so that $\bar{z} = 0$ corresponds to the sample sitting in a node of the intracavity field. (b) The cavity transmission is fitted (black line) including the two phase-modulated markers to obtain the frequency axis. A detuning of $\Delta = 0$ corresponds to the empty cavity resonance.

The 780nm interferometer signal carries information on the spacing between the sample surface and the MM fiber mirror. Figure 4.4(a) plots the fringe pattern of the 780nm interferometer as a function of sample position. The scan of the cavity fibers is resolved in the interferometer. Therefore every circle corresponds to the mean interferometer signal during the cavity scan for a given sample position. The fringe pattern shows a clear periodicity with respect to the applied positioner voltage. From a sinusoidal fit (black line) it is obvious that the piezo movement is slightly nonlinear. With increasing voltage, the piezo expansion per voltage difference increases. We obtain a total sample position scan range of around $2.5\,\mu\text{m}$. The nonlinearity of the piezo leads to deviations of up to 5%. Since they are small, we do not correct for those deviations. Finally, we shift the sample position axis to comply with our notation, e.g. the axis is shifted so that $\bar{z} = 0$ corresponds to the sample sitting in a node of the cavity field.

For small cavity scan ranges compared to the FSR, we calibrate the cavity detuning with the use of sidebands. As the cavity linewidth is increased considerably by the presence of the stripe compared to the empty cavity measurements presented in Ch. 3, we modulate sidebands of at least $\omega_{sb}/2\pi = 2\,\text{GHz}$ and record the cavity scans with high sampling rates to resolve the sidebands for all sample positions. A zoom of a cavity scan close to the resonance is shown in Fig. 4.4(b). The measurement is fitted with the sum of three Lorentzian resonances with shifted resonance frequencies (black line). We define the cavity detuning axis in a way that a detuning of zero corresponds to the situation where the sample is placed at a node of the cavity field. Note that the insertion of the dielectric membrane into the cavity leads to a shift of the cavity resonance compared to the empty cavity due to the change in effective path length.

4.2.3 Dispersive coupling

With all the relevant axes calibrated, we plot the resulting map of the normalized cavity transmission for different sample positions and cavity detunings in Fig. 4.5. The map reveals a clear periodicity in the
4. Optical spring effect on SiN membrane stripes

The intracavity field as expected for a Fabry-Pérot cavity. The observed period matches a half wavelength of the 1550 nm laser. This confirms our calibration of the sample position with the 780 nm interferometer. The maximum cavity detuning is almost $\Delta_{\text{max}}/2\pi = 600$ GHz and is reached when the sample is placed at an anti-node. We can numerically reproduce our measurements via the transfer matrix method. The membrane stripe is simulated as a dielectric slab with a membrane thickness $t = 32$ nm and a refractive index of $\text{Re}(n) = 2.2$. Although this value seems large at first, values of the same order are reported for low-stress SiN in Ref. [107] with increasing refractive index with decreasing tensile stress in the SiN film. Absorption in the dielectric membrane is included by a non-zero imaginary part of the refractive index: $\text{Im}(n) = 1.4 \cdot 10^{-4}$.

To extract the dispersive coupling, we extract the resonance position from the cavity scan at each sample position. This can be accomplished by Lorentzian fits if the response is linear for all sample positions. Otherwise, we have to numerically extract the maximum of the transmission. The extracted detuning as a function of the sample position is plotted in Fig. 4.5(a). The measurement shows a small asymmetry and thus deviates from the expected sinusoidal shape. Contrary to the general procedure described in Sec. 4.2.1, we only scan one cavity piezo for this measurement while the other one is fixed. This causes the optical field to move relative to the sample during the cavity scan. If we consider this in the simulation, we can reproduce the shape of the cavity detuning (black line in Fig. 4.5(a)).

The slight deviations from the measurement again stem from the nonlinear piezo movement. Furthermore, the magnitude of the detuning curve slowly increases. We can explain this behavior by a membrane tilt that depends on the sample position. Due to our geometry, the cavity axis is aligned to the positioner $z$ axis only by eye. Any small misalignment together with an observed crosstalk with other positioner
4.2 Static spectroscopy

Figure 4.6: Cavity detuning and dispersive coupling. (a) The cavity detuning as a function of sample position is extracted from the measurement presented in Fig. 4.5. Measurement and numerical simulation (black line) show a slight asymmetry which results from the asymmetric cavity scan. The shape as well as the values show very good agreement to the simulation. (b) Measurement and simulation of the dispersive frequency pull parameter $G_{\omega}$ is extracted from the data in panel (a). (c) Quadratic dispersive coupling $G_{\omega^2}$.

axes will change the relative lateral position of the membrane stripe inside the cavity when the sample position is varied. For wide stripes as the one under investigation here, we observe that the edges of the stripes slope down due to missing tensile stress in this direction. While the tilt increases with increasing sample position, the effective membrane thickness and therefore the detuning increases. On top of that, the reflection and transmission coefficients of the membrane also depend on this tilt angle. Combined, we assume that those two effects explain the deviation between the simulation and the experiment.

The derivative of the cavity detuning with respect to the sample position corresponds to the frequency pull factor $G_{\omega}$. It is depicted for different sample positions in Fig. 4.6 (b). To calculate the derivative it turns out helpful to fit a spline to the data. Otherwise small fluctuations of the cavity resonance lead to huge errors in the couplings. Both the magnitude and the shape agree well with the simulation. Deviations can be explained by the error in the sample position calibration and by the gradual increase of
4. Optical spring effect on SiN membrane stripes

the magnitude of the detuning modulation as discussed above. With the sample positioned at a node of the cavity field, the frequency pull factor vanishes. Displacing the sample by some tens of nm results in moderate frequency pull factors of a few hundred MHz nm\(^{-1}\). At positions of high field gradients, values of up to \(|G_\omega|/2\pi = 3 \text{ GHz nm}^{-1}\) can be reached. From the sample geometry we calculate the zero-point fluctuations \(z_{\text{zp}} = 4.4 \text{ fm}\). Therefore, we expect single-photon coupling rates of around \(|\tilde{g}_{0}\omega|/2\pi = 2.5 \text{ kHz}\) close to the node. Overall, values of up to \(|\tilde{g}_{0}\omega|/2\pi = 11 \text{ kHz}\) are accessible.

We also determine the second derivative to obtain the quadratic frequency pull parameter \(G_\omega = \frac{\partial^2 \omega_{\text{cav}}}{\partial \bar{z}^2}\). We plot \(G_\omega\) in Fig. 4.6(c). Again the magnitude and the shape agree with the simulations. The quadratic coupling is strongest around the nodes or the anti-nodes of the intracavity fields. It reaches values of up to \(|G_\omega^2|/2\pi = 25 \text{ MHz nm}^{-2}\).

4.2.4 Dissipative coupling

The absorption coefficient of Si-rich SiN at telecom wavelengths is non-negligible \([108]\), leading to an expected modulation of the cavity losses with respect to the sample position \([30, 53, 109]\). Additionally, we assume that the alignment of the sample is non-ideal. That results in scattering of photons out of the cavity mode. Because the amount of scattered light depends on the field intensity at the sample surface, scattering also contributes to a dissipative coupling in our system.

Despite the small number of significant data points we can extract the linewidth from fitting the cavity scans close to the node and anti-node. Scans over the cavity resonance at positions with higher dispersive coupling show a broadened response. This broadening originates from the optomechanically induced static bistability for stronger input powers. But even for smaller coupling or small laser powers, fluctuations in the sample position lead to a spectral broadening of the cavity resonance due to considerable optomechanical coupling. Those fluctuations arise from acoustical noise and vibrations of the surrounding, that couple in the system through the relatively floppy stack of nanopositioners. Vibration damping and acoustic shielding of the setup help to minimize those vibrations in the low frequency regime below approximately 1 kHz. To extract the intrinsic linewidth, we assume that the lifetime of cavity photons is much shorter than the timescale of those fluctuations. Therefore, the magnitude of the resonant transmission is set by the cavity linewidth, but the peak appears as a Gaussian with a broadened linewidth. Figure 4.7(a) shows the resonant transmission \(S_{21}^2(\Delta = 0)\) as a function of sample position. Again, we observe a clear periodicity in the cavity field. With the sample placed at a node or anti-node, absorption losses are minimized and the resonant transmission is maximized. In between the nodes and anti-nodes, the transmission drops drastically. The black line corresponds to a spline that is used to numerically calculate the derivative later on.

We assume constant input couplings and we calculate the cavity linewidth \(\kappa(\bar{z})\) from the resonant transmission as

\[
\kappa(\bar{z}) = \sqrt{2\kappa_1 \kappa_2 / S_{21}^2(\Delta = 0, \bar{z})}.
\]

Close to the node we obtain values down to \(\kappa/2\pi \approx 31 \text{ MHz}\) which corresponds to a loaded finesse of \(\mathcal{F} \approx 110000\). Figure 4.7(b) displays the cavity linewidth as a function of sample position. It is apparent that \(\kappa\) depends on the sample position with a periodicity in the cavity field. Especially away from the field nodes, the measured linewidths are significantly larger than what we expect from a simulation with a literature value of \(\text{Im}(n) = 7.1 \cdot 10^{-6}\) at 1550 nm \([109]\). We numerically fit the linewidth as a function
4.2 Static spectroscopy

Figure 4.7: Cavity resonant transmission, calculated linewidth and dissipative coupling. (a) Resonant transmission $S_{21}^2(\Delta = 0)$ as a function of sample position and spline (black line) used for the numerical derivative. (b) Calculated cavity linewidth extracted from the resonant transmission in panel (a) using Eq. (4.1). The black line corresponds to the simulation with $\text{Im}(n) = 1.4 \cdot 10^{-4}$. (c) Dissipative coupling $G_\kappa$ extracted using the spline from panel (a) and simulation (black line).

of the sample position to obtain $\text{Im}(n) = 1.4 \cdot 10^{-4}$. Thus, the optical dissipation in the membrane is approximately 20 times larger than expected from the material properties of the stripe. The black line in Fig. 4.7(b) shows the simulation with this greatly enhanced dissipation. While the asymmetry from the asymmetric cavity scan is clearly visible for the simulations, the measurements show a more symmetric shape. We explain the enhanced optical dissipation by residual misalignment. We observe that the measured linewidth greatly depends on the quality of the alignment of the membrane surface with respect to the optical axis of the cavity. We assume that even a small misalignment leads to considerable scattering of light out of the cavity. Another reason could be an increased absorption due to surface imperfections arising from the fabrication process.

The dissipative frequency pull parameter $G_\kappa = \frac{\partial \kappa}{\partial \bar{z}}$ can be calculated from the linewidths obtained both from the measurements and from the numerical simulations (see Fig. 4.7(c)). The periodic shape of the dissipative coupling is apparent for both the simulation and the measurement. The exact shape of
4. Optical spring effect on SiN membrane stripes

Figure 4.8: Static bistability. The cavity transmission measured at a sample position of around \( z = 0.5 \mu m \) and a strong input power of \( P_{in} = 1.8 \text{mW} \) shows a strongly non-linear response. The bistability is also visible for the first order sidebands.

the measured coupling strongly depends on the smoothing parameter of the spline used to calculate the derivative. Nevertheless, both the measurement and the simulation show a dissipative coupling of the order of MHz nm\(^{-1}\). Thus, for the measurements presented here, the dissipative coupling is two orders of magnitude smaller than the dispersive coupling. However, it cannot be completely neglected in our system.

4.2.5 Static bistability

As discussed in the last section, for some sample positions the resonant transmission is reduced by several orders of magnitude. Depending on the gain of the detector, we need to use a considerable amount of light power to resolve the peaks at those sample positions. Together with quite large effective dispersive coupling strengths this leads to cavity responses with an apparent nonlinear shape. We show an exemplary measurement at a sample position of \( z = 0.5 \mu m \) and a very strong input power of \( P_{in} = 1.8 \text{mW} \) in Fig. 4.8. The bistable response is apparent even for the first order sidebands with a spacing of \( \omega_{sb}/2\pi = 5 \text{GHz} \). It is impossible to extract the cavity resonance frequency or the linewidth. The shape of the nonlinear cavity response matches qualitatively the simulated response in the static bistability regime as plotted in Fig. 2.9.
4.3 Dynamical backaction

We demonstrate dynamical backaction in our system by measuring the optical spring effect. For this measurement, we switch to the setup as described in Fig. 3.1. We use the Koheras Basik E15 laser to generate the lock tone that we use to lock the cavity. The lock tone is phase modulated at $\omega_{\text{PDH}}/2\pi = 18.29 \text{MHz}$.

The amplitude of the cavity transmission extracted from the quadratures as $R = \sqrt{X^2 + Y^2}$ obtained with the homebuilt detector serves as error signal for the cavity stabilization. In order to minimize backaction from the lock tone, we lock the cavity length so that the lock tone is red-detuned with a detuning of around $\Delta_l/2\pi \approx 150 \text{MHz}$. The error signal is fed to the lockbox, which directly controls the piezo of the output MM fiber. A DC offset is amplified in the high-voltage amplifier and positions the input SM fiber so that the laser is close to resonance with a cavity TEM$_{00}$ mode.

In order to operate the cavity with an ultra-high finesse, we position the sample at the node of the cavity field, then the high voltage offset is adjusted. Finally, an offset voltage on the z positioner places the sample at the desired working position. For the measurement presented in the following, we place the sample around 30 nm above a node of the cavity field.

We generate the probe tone from a tuneable laser (Agilent 81940A) and scan its wavelength across the cavity resonance with steps of $\delta \lambda = 0.1 \text{pm}$. For every detuning, we acquire the transmission together with a mechanical spectrum. The mechanical spectra are measured with the 780 nm interferometer and are acquired using a spectrum analyzer. The mechanical mode of interest is the fundamental out-of-plane mode of the membrane stripe with a resonance frequency of $\Omega_m/2\pi = 238 \text{kHz}$ and a linewidth of $\Gamma_m/2\pi = 990 \text{Hz}$, resulting in a mechanical $Q$ factor of $Q = 240$.

4.3.1 Cavity linewidth and number of circulating photons

First, we require knowledge of the circulating photon number $n_{\text{circ}}$ for every detuning. For that purpose we plot the mean cavity transmission as a function of probe detuning in Fig. 4.9 (a). We observe that the response deviates from the expected Lorentzian lineshape. A Lorentzian fit for the effective linewidth (with knowledge of all the other parameters) fails. Corresponding to the observed broadening, a linewidth of $\kappa/2\pi = 120 \text{MHz}$ would lead to a greatly reduced overall transmission (black dotted line). The linewidth calculated from the resonant transmission using Eq. (4.1) is $\kappa/2\pi = 38 \text{MHz}$. We explain the deviation from the expected lineshape (solid black line) with a broadened response due to the frequency noise of the probe laser. If the frequency jitter of the laser happens on a timescale that is longer than the lifetime of cavity photons but shorter than the timescale of the measurement, we average over several displaced Lorentzians. Therefore, the magnitude of the resonant transmission is set by the cavity linewidth, but the peak will appear as a broadened Gaussian shape. The data can be fitted with a Gaussian (red line) with linewidth $\kappa/2\pi = 119 \text{MHz}$.

Since the cavity shows a non-Lorentzian response, we extract the number of circulating cavity photons $n_{\text{circ}}$ directly from the measured transmitted power $P_t$ as

$$n_{\text{circ,p}}(\Delta) = \frac{P_{1p}(\Delta)}{\hbar \omega_p \kappa_2}. \quad (4.2)$$

We fit a spline to the data (black line in Fig. 4.9 (b)) instead of using the Gaussian fit to be able to fit the measurements even when later the transmission is not Gaussian any more. To account for a static
4. Optical spring effect on SiN membrane stripes

Figure 4.9: Cavity response in the two-tone measurement, calculated intracavity photon number and effective detuning. (a) The cavity transmission as a function of probe detuning shows a Gaussian response (red line: Gaussian fit). The response cannot be approximated with Eq. (2.4) (only free parameter: $\kappa$) as either the linewidth or the magnitude of the transmission do not match the measurement (black solid line: $\kappa/2\pi = 38$ MHz to match the resonant transmission and black dashed line: $\kappa/2\pi = 120$ MHz to match the linewidth). (b) The intracavity photon number $n_{\text{circ}}$ is calculated from the transmission using Eq. (4.2). A spline is fitted to the data (black line). It serves to plot the optical spring theory curve for a non-Lorentzian cavity response. (c) The shift of the equilibrium position of the mechanical resonator due to a static radiation pressure force leads to an effective detuning. This correction is plotted for the cavity scan.

displacement of the equilibrium position of the resonator we calculate the effective detuning $\Delta_{\text{eff,p}}$. For our experimental condition we obtain a correction below 1 MHz (Fig. 4.9 (c)) which is much smaller than detuning steps of 12.5 MHz that we use to probe the system. Because of that, we still operate the system outside of the static bistability regime.

Note that the extracted linewidth of $\kappa/2\pi = 38$ MHz is an upper limit, because it is calculated from the measured transmitted power. It corresponds to a loaded cavity finesse of $\mathcal{F} > 90000$. Closer to the node, we perform similar measurements and reach values of up to $\mathcal{F} > 126000$. 

4.3 Dynamical backaction

4.3.2 Mechanical response

Once the cavity is locked, we measure the mechanical motion with the 780 nm interferometer. We fit the measured mechanical PSDs to Eq. (2.20) to extract the resonance frequencies and the effective mechanical linewidths. Figure 4.10 (a) displays two amplitude spectral densities (ASDs) of the stripe resonator and their corresponding fits, showing the thermomechanical motion of the fundamental oop mode. The black curve is measured with an off-resonant probe tone with $\Delta_{\text{eff},p}/2\pi = -193$ MHz. In this case the dynamical backaction from the probe tone is negligible. In order to rule out backaction effects from the lock tone, the lock tone is always far detuned from the cavity resonance. The effective linewidth $\Gamma_{\text{eff}} = \Gamma + \Gamma_{\text{opt}}$ thus corresponds to the natural linewidth of the mode (as confirmed by a simple interferometric ringdown measurement, not shown), where only the eigenfrequency is slightly shifted. The blue curve corresponds to a blue detuned probe with $\Delta_{\text{eff},p}/2\pi = 44$ MHz. In this case we expect optomechanical heating to decrease the mechanical linewidth. However, in the case of strong dynamical backaction, laser frequency fluctuations lead to an effective broadening of the mechanical linewidth. This effect dominates the mechanical response of our resonator as shown in Fig. 4.10 (b). An observed frequency broadening of up to 7 kHz disguises the much smaller expected optical damping of the order of $\Gamma_{\text{opt}}/2\pi = \pm 20$ Hz. The theory curve is shown as black line in Fig. 4.10 (b).

4.3.3 Optical spring effect

We are able to resolve the optical spring effect, although the laser frequency noise obscures dynamical backaction cooling and heating. In our measurements we effectively average the mechanical response and the mechanical frequency matches on average the theoretical value. This occurs because the fluctuations are symmetric with respect to the applied effective detuning. Figure 4.11 depicts the measured mechanical frequency as a function of the effective detuning. Negative detunings lead to a softening, positive detunings to a stiffening of the spring constant, resulting in a mechanical frequency
shift $\delta \Omega_m = \Omega_m - \Omega_{m,0}$ of up to $|\delta \Omega_m|/2\pi \approx 4.5$ kHz. The black line corresponds to the theory curve including the correct detuning dependence of the intracavity photon number. The fitted single-photon couplings are $|g_{0\omega}|/2\pi = 2.5$ kHz and $|g_{0\kappa}|/2\pi = 30$ Hz. Measurement and theory agree very well. Those values are of the same magnitude as the coupling strengths obtained from the static measurements at a sample position of 30 nm: in Sec. 4.2.3 we obtained $|g_{0\omega}|/2\pi = 2.5$ kHz from the simulations and $|g_{0\kappa}|/2\pi = 2.0$ kHz from the measurement. Repeated measurements with different probe powers and slightly varying sample positions showed single photon couplings of the same magnitude. Overall, we extract a mean value over 20 measurements of $|g_{0\omega}|/2\pi = 2.4$ kHz with a standard deviation of 450 Hz.

### 4.3.4 Fitting the theory

A few details on the fitting routine are given in the following: the optical spring measurement is fitted to Eq. (2.46). The formula includes the dispersive and the dissipative effective coupling strengths $g_\omega$ and $g_\kappa$, both of them include the actual number of cavity photons as $g = g_0 \sqrt{n_{\text{circ.p}}}$. As discussed before, the photon number can show an arbitrary dependence on the probe detuning. For that reason, we determine the photon number from the transmission measurement as explained above and use the photon number as a second input variable for the fit routine. The optical spring measurement is fitted as a surface on the $n_{\text{circ.p}} - \Delta_{\text{eff.p}}$ phase space. The only free fit parameters are the single photon couplings $g_{0\omega}$ and $g_{0\kappa}$ and the effective cavity linewidth $\kappa$. Figure 4.12 illustrates the fitting routine. The black circles represent the fit input variables $n_{\text{circ.p}}$ and $\Delta_{\text{eff.p}}$ extracted from the transmitted power. The contour lines illustrate the mechanical detuning $\delta \Omega_m = \Omega_m - \Omega_0$ for the respective areas in the $n_{\text{circ.p}} - \Delta_{\text{eff.p}}$ phase space.

The effective cavity linewidth obtained from the fit corresponds to a broadened linewidth that scales the
4.4 Discussion

We present a proof of principle of radiation pressure backaction using a free-standing SiN membrane stripe inserted into the cavity. The insertion of the membrane stripes affects the cavity linewidth and the cavity resonance frequency. We observe a modulation of both values that matches the standing-wave intracavity field pattern. The static spectroscopy measurement allows us the estimate both the dispersive and the dissipative optomechanical coupling. We report dispersive frequency pull parameters of up to $|G_\omega|/2\pi = 3\text{GHz} \text{nm}^{-1}$ corresponding to single-photon couplings of up to $|g_{0\omega}|/2\pi = 11\text{kHz}$. The dissipative coupling we measure is around three orders of magnitude smaller. The static measurements are supported by transfer matrix simulations with reasonable parameters. Due to the small mode volume, we obtain large frequency pull parameters comparable to the values reported e.g. in Ref. [49]. Using thicker SiN membranes, we could in principle further increase those values at the cost of increased absorption losses. Due to the macroscopic dimensions of the stripes, the zero-point fluctuations are small and the resulting single-photon coupling cannot compete with values obtained in systems with much lighter mechanical modes. The largest single-photon couplings to date are achieved in cold atom systems [110, 111] and in sliced photonic crystal nanobeams [112] with values up to $g_{0\omega}/2\pi = 25\text{MHz}$. The low mechanical quality factor limits the attainable backaction cooling. This could be overcome...
4. Optical spring effect on SiN membrane stripes

by reducing the lateral dimensions of the stripe. For narrower stripes, we have found $Q$ factors of up to $Q = 300000$. In addition, frequency noise in the system broadens the cavity response in the two tone measurements. This laser noise leads to a significant increase of the effective mechanical linewidth especially for probe detunings that lead to strong dynamical backaction. Thus, we are unable to resolve optomechanical damping in the measurements.

Nevertheless, we measure the optical spring effect on the fundamental oop flexural mode of the stripe inside a ultra-high finesse fiber cavity. The measurement is supported by the theoretical model, allowing to quantitatively extract the parameters of the system. To the best of our knowledge, the cavity finesse of up to $\mathcal{F} = 126000$ is the highest reported so far in a loaded FFPC system. We obtain a single-photon coupling of $|g_{0\omega}|/2\pi = 2.5\text{kHz}$ consistent with the values measured in the static spectroscopy at that certain position of the membrane stripe in the cavity mode.
Dynamical backaction on stochiometric Si$_3$N$_4$ membrane stripes

Stoichiometric Si$_3$N$_4$ shows largely reduced optical absorption compared to Si-rich SiN in the near-infrared (NIR) range [108]. Additionally it is grown with increased tensile prestress on Si substrates. Therefore, mechanical eigenfrequencies of resonators made of Si$_3$N$_4$ are higher, leading to increased mechanical Q factors as a result of dissipation dilution [113][114].

In this chapter we present measurements performed on Si$_3$N$_4$ membrane stripes. Those devices overcome the main limitations of the measurements presented in the previous chapter, namely the low mechanical Q factor and the strong absorption losses of the cavity mode. Together with a laser with improved noise performance, this enables us to observe dynamical backaction cooling and heating.

**Figure 5.1:** Sample under investigation in this chapter. Several Si$_3$N$_4$ membrane stripes (light grey) are patterned over the u-shaped window opening (dark grey). The substrate frame (Si, white) is cleaved on the top of the picture. The cleaved edge appears black. We measure on the first stripe from the top. The measurements on the fundamental oop mode are performed with the cavity centered along the stripe. For the measurements on the second oop mode the cavity is positioned closer to the left frame.
The sample is etched from a Norcada (NX5050XS) membrane window by Sebastian Stapfner at the LMU Munich [77]. The stripes are defined with electron beam lithography, the resist was used as an etch mask in the subsequent ICP etch. Details of the fabrication process on stoichiometric Si$_3$N$_4$ sample can be found in Ref. [77]. The resulting stripe under investigation has a width of 30µm, a length of 500µm and a thickness of 30nm. Figure 5.1 shows an optical micrograph of the sample used in this chapter.

5.1 Static spectroscopy

Again, we start with the measurement of the static optomechanical coupling. Compared to the measurements presented in Ch. 4, we split the output of the HV amplifier and feed it to both cavity piezos. In that way, both fibers are scanned simultaneously in opposite directions, keeping the cavity field distribution at the sample constant. The map of the cavity transmission for cavity length scans at different sample positions is shown in Fig. 5.2. The cavity resonance appears as white points in this map. It shows a sinusoidal modulation with the sample position. Compared to the measurements on SiN, the resonant transmission away from the nodes and anti-nodes is enhanced.

![Figure 5.2: Normalized cavity transmission $S_{21}^2$. The cavity transmission is color-coded for cavity scans (along the $\Delta$ axis) for different sample positions. The transmission shows clear resonances (white dots) whose detuning $\Delta$ depends on the sample position in a periodic manner. The periodicity matches $\lambda/2$ of the intracavity field.](image-url)

Overall, the cavity resonance is detuned by around 240GHz, which is considerably less than what we measure for the SiN sample. We calculate frequency pull parameters of up to $|G_\omega|/2\pi = 1$ GHz nm$^{-1}$ (see Fig. 5.3 (a)) and $|G_{\omega_{p}}|/2\pi = 10$ MHz nm$^{-2}$ (Fig. 5.3 (b)). Both frequency pull parameters show good agreement with the simulations. Deviations mostly stem from nonlinearity of the piezo used for the scan of the sample position and from the spline that we use to calculate the derivatives. Due to
the symmetric cavity scan, the frequency pull parameters above and below the node show the same magnitude with opposite signs. Because of that, the maximum values of $G_\omega$ are a bit smaller compared to the asymmetric scan shown in Ch. 4.2.3. Still, we obtain couplings that are only around $40\%$ of the values extracted for the non-stoichiometric sample. Since the thickness of both samples is the same, we expect that the refractive indices of the two materials to differ quite significantly. To reproduce the dispersive coupling of the Si$_3$N$_4$ sample with the simulations, we use a refractive index of $\text{Re}(n) = 1.725$. Reference [115] reports a decrease of the refractive index of SiN grown in PECVD with decreasing Si content with values down to $\text{Re}(n) = 1.6$.

Figure 5.3: Dispersive optomechanical coupling. (a) We extract the dispersive frequency pull parameter $G_\omega = \frac{\partial \omega}{\partial z}$ from the cavity detuning $\Delta$ plotted in Fig. 5.2. The sinusoidal dependence on the sample position of the simulations (black line) is recovered in the experiment. (b) The quadratic dispersive coupling $G_{\omega^2} = \frac{\partial^2 \omega}{\partial z^2}$ follows the simulations.

To determine the dissipative coupling, we again extract the cavity linewidths from the resonant transmission. Figure 5.4 (a) compares the measured linewidths with the simulation. With the sample placed at the node or anti-node of the cavity field, we measure linewidths down to $\kappa/2\pi = 17.5$ MHz, corresponding to a loaded finesse of $\mathcal{F} = 195000$. The reduced optical absorption compared to the Si-rich sample is apparent from the overall reduced cavity linewidth with the sample inserted and from the greatly reduced linewidth with the sample positioned away from the node of the cavity field. Even with the sample positioned at places of maximal dispersive coupling, the cavity finesse reaches values of more than $\mathcal{F} = 30000$. The dips right in the middle between the nodes and the anti-nodes are not reproduced by the simulations with $\text{Im}(n) = 3.5 \cdot 10^{-5}$ and are of unknown origin.

Because $\kappa$ is less dependent on the sample position, the dissipative coupling is reduced compared to the SiN sample with values of $|G_\kappa|/2\pi < 0.2$ MHz nm$^{-1}$ as shown in Fig. 5.4 (b). The experimental values are slightly higher, mainly because they are barely resolved from the scatter of the linewidth. The dissipative coupling is negligible compared to the dispersive coupling.
5. Dynamical backaction on stochiometric Si$_3$N$_4$ membrane stripes

Figure 5.4: Dissipative optomechanical coupling. (a) The cavity linewidth $\kappa$ depends on the sample position and is calculated from the resonant transmission. At the nodes and the anti-nodes, it reaches values of well below $\kappa/2\pi = 20\text{MHz}$. We are unable to reproduce the double-dip structure with our simulations. (b) The dissipative frequency pull parameter $G_\kappa = \partial\kappa/\partial z$ is calculated from the data in panel (a). We are unable to resolve the predicted shape from the simulations because of the scatter and the double-dip structure of the linewidth.

5.2 Mechanical displacement sensing and mode temperature

Before looking into the measurements of dynamical backaction, we consider the mechanical modes of the stripes and discuss mechanical displacement measurements.

The easiest way to measure displacements is with the low finesse 780nm interferometer incorporated through the backside MM fiber. The sensitivity depends on the slope of the fringe profile which can be tuned by moving the sample position with respect to the MM fiber. We increase the light intensity to boost the signal strength. For the optimized configurations, the Brownian motion of the fundamental oop mode of the stripes lateral vibrations rises 30dB above the noise floor. We calibrate the displacement spectral density and fit the data with a Lorentzian. Figure 5.5 (a) shows one measurement from the interferometer and a Lorentzian fit (black line) with a damping constant of $\Gamma/2\pi = 8\text{Hz}$. The noise level of the interferometer (red solid line), the detector (red dashed line) and the spectrum analyzer (red dotted line) are shown for comparison.

Drifts in the mechanical resonance frequency (for example due to thermal fluctuations) during the data acquisition distort the resonance shape especially for small dampings and long data acquisition times. This is apparent in the spectrum plotted in Fig. 5.5 (a). To obtain a better estimate of the mechanical damping, we perform ringdown experiments. To this end, the mechanical motion is driven resonantly via the sample holder piezo. The mechanical amplitude at the drive frequency is measured by demodulating the interferometer signal at the corresponding frequency. A sufficient lowpass filter cutoff frequency provides the demodulated signal with enough temporal resolution to capture the ringdown process when the drive is turned off.

A ringdown trace is displayed in Fig. 5.5 (b) with a corresponding exponential regression. We extract a
5.2 Mechanical displacement sensing and mode temperature

Figure 5.5: Mechanical displacement sensing with the 780 nm interferometer. (a) The Brownian motion of the second oop mode of the stripe clearly rises from the noise floor. Due to frequency drifts during the data acquisition, a Lorentzian regression (black line) fails to accurately estimate the linewidth. The interferometer noise floor (red line), the noise level of the photodetector (red dashed line) and the spectrum analyzer noise floor (red dotted line) are plotted. (b) To perform a mechanical ringdown of the mechanical mode the resonant drive is switched off at the time $t = 0 \text{s}$. An exponential regression of the energy decay (black line) yields a mechanical damping constant of $\Gamma/2\pi = 4.74(6) \text{ Hz}$. Mechanical damping of $\Gamma/2\pi = 4.74(6) \text{ Hz}$ corresponding to a mechanical $Q = 197000$ for the second oop mode at $\Omega_m/2\pi = 932.58 \text{ kHz}$. Similarly, for the fundamental oop mode we obtain a resonance frequency of $\Omega_m/2\pi = 468.2 \text{ kHz}$ and a damping constant of $\Gamma/2\pi = 10.8(2) \text{ Hz}$ yielding $Q = 43000$. This significant increase of the mechanical $Q$-factor for higher order modes is unusual. Identifying the underlying reason is beyond the scope of this thesis.

In the presence of optomechanical cooling, the thermal force driving the mechanical resonator remains constant, but the effective mechanical linewidth is altered. To measure the mode temperature, we extract the effective mechanical damping $\Gamma_{\text{eff}}$ from a fit of the measured mechanical spectra and obtain the effective mode temperature $T_{\text{eff}}$ from the phonon bath temperature $T_{\text{bath}}$ as follows:

$$T_{\text{eff}} = T_{\text{bath}} \frac{\Gamma}{\Gamma_{\text{eff}}}. \quad (5.1)$$

Because of the limited sensitivity of the low finesse interferometer, we can in theory only measure effective mode temperatures down to approximately 10K. However, in our optomechanical measurements the sample position is adjusted to select a certain dispersive coupling. This leads to a sample position where the interferometric sensitivity is reduced considerably. This limits the readout with the 780nm interferometer to even higher mode temperatures.

To overcome this limitation, we use the cavity transmission in a heterodyne scheme as explained in Ch. 3.1.2 to measure the mechanical motion with greatly enhanced sensitivity. As the sensitivity depends on the photon number, the cavity enhances mechanical displacement sensing especially for large pump powers and under strong dynamical backaction.

Finally, the mechanical resonance frequency is altered as soon as the cavity is locked. Fluctuations in
the cavity length affect the detuning. Since this optical spring effect depends on the detuning, those cavity length fluctuations translate to mechanical frequency noise. As the noise is of Gaussian origin, the resulting mechanical spectrum is a convolution of the Lorentzian peak with a Gaussian noise distribution. The resulting distribution is a Voigt profile as commonly occurring in spectroscopy where lines appear Doppler broadened [116]. The Voigt profile \( V(\omega, \sigma, \Gamma) \) is given by
\[
V(\omega, \sigma, \Gamma) = G(\omega, \sigma) \ast L(\omega, \Gamma/2).
\]
(5.2)

\( G(\omega, \sigma) \) denotes the Gaussian distribution with standard deviation \( \sigma \), \( L(\omega, \Gamma/2) \) is the textbook Cauchy-Lorentz distribution with the half-width at half maximum \( \Gamma/2 \). Voigt profiles can also be used to fit peaks that are too narrow to be resolved by the finite bandwidth of the measurement devices [117].

Figure 5.6 (a) displays the Brownian motion resonance of the fundamental oop mode without a cavity mode present. We can fit the measurement both with a Lorentzian function (black line) and with a Voigt profile (red line). Both fits yield low mechanical dampings of \( \Gamma/2\pi = 5.0 \) Hz and \( \Gamma/2\pi = 3.3 \) Hz, respectively. The Voigt profile yields a Gaussian broadening of \( \sigma/2\pi = 0.1 \) Hz probably limited by the resolution bandwidth of the spectrum analyzer. In Fig. 5.6 (b) we lock the cavity with the lock tone blue detuned. As expected, the optical spring effect shifts the mechanical resonance to slightly higher frequencies. The expected narrowing of the effective mechanical linewidth is obscured by broadening of the resonance (Fig. 5.6 (b)). A Lorentzian fit of the central part of the peak (black dotted line) largely overestimates the tails, while a Lorentzian fit of the tails (black solid line) clearly deviates from the resonance shape. A Voigt profile (red line) nicely reproduces the data and yields values \( \Gamma/2\pi = 1.7 \) Hz and \( \sigma/2\pi = 30 \) Hz. The extracted effective mechanical damping is reduced from the natural damping as expected.

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**Figure 5.6:** Mechanical peak broadening under cavity backaction noise measured with the 780nm interferometer. (a) The spectrum of the Brownian motion of the fundamental oop mode can be fitted with both a Lorentzian fit (red line) and a Voigt profile fit (black line). Both regressions yield single-digit mechanical dampings of \( \Gamma/2\pi = 5.0 \) Hz and \( \Gamma/2\pi = 3.3 \) Hz, respectively. The Voigt profile yields a Gaussian broadening of \( \sigma/2\pi = 0.1 \) Hz. (b) Under weak dynamical backaction, the peak is slightly shifted to higher frequencies and appears broadened. Lorentzian fits (black dotted line fits the central peak, black solid line fits the tails) cannot describe the resonance shape. A Voigt profile (red line) fits the data and yields \( \Gamma/2\pi = 1.7 \) Hz and \( \sigma/2\pi = 30 \) Hz.
5.3 Dynamical backaction

To measure dynamical backaction with the Si$_3$N$_4$ stripe, we place the sample close to the node in order to obtain a cavity mode with high finesse. We switch to the homebuilt detector and stabilize the cavity as described in Ch. [4.3]. We use a second Koheras Basik E15 laser for the probe tone to avoid the frequency noise of the Agilent laser that limited the measurements on the Si-rich SiN sample. The second Koheras laser can be tuned thermally close to the cavity resonance. A fast piezo tuning allows to scan the laser wavelength across the resonance. In a second measurement, we fix the probe detuning on the blue side and measure the influence of the probe power on the dynamical backaction cooling performance.

5.3.1 Optical spring effect

![Graph showing the regression to the optical spring theory. The number of probe photons $n_{\text{circ},p}$ as a function of the effective probe detuning $\Delta_{\text{eff},p}$ deviates from the expected Lorentzian lineshape. During the measurement, $\Delta_{\text{eff},p}$ is scanned from positive to negative values. The maximum photon number is obtained for slightly negative detunings because of the scan direction. The contour lines correspond to constant mechanical detunings $\delta\Omega_m = \Omega_m - \Omega_0$ at the corresponding points in the $\Delta_{\text{eff},p} - n_{\text{circ},p}$ phase-space. We fit them to match the measured mechanical resonance frequencies.](image)

We measure the optical spring effect by scanning the probe detuning across the cavity resonance. In this experiment we measure dynamical backaction on the first harmonic oop mode. To optimize coupling to this mode, we displace the cavity along the stripe until the sensitivity of the interferometric read-out of the Brownian motion of the mode is maximized. We assume, that a maximized sensitivity of the interferometer also corresponds to a situation of maximized optomechanical coupling. Again, we calculate the circulating photon number $n_{\text{circ}}$ from the transmitted power. From the resonant transmission, we obtain an effective cavity linewidth of $\kappa/2\pi = 21\text{MHz}$ with the sample placed at a position of...
around $\bar{z} = -20\text{nm}$ and an input power of $P_{in} = 7\mu\text{W}$. While this corresponds to a loaded finesse of around $\mathcal{F} = 165\,000$, we repeat the measurement at different sample positions and reach values of up to $\mathcal{F} = 195\,000$. This value is close to the empty cavity finesse. We measure the mechanical spectra with the 780nm interferometer and extract the mechanical resonance frequency from the PSDs.

Figure 5.7 displays the extracted number of probe photons $n_{\text{circ},p}$ as a function of the effective probe detuning $\Delta_{\text{eff},p}$ (black circles). The response appears bistable. We sweep the detuning from positive to negative values. First, the photon number increases slightly. At around $\Delta_{\text{eff},p} = 70\text{MHz}$, the photon number forms a plateau with a constant low value. As the detuning approaches zero, the system transitions to a solution with high photon number. For negative detunings, the photon number decreases gradually. Here, the response appears Lorentzian as expected. The maximum of the response is slightly shifted to negative detunings, probably because of the scan direction.

As discussed in Ch. 4.3.4, we use both the effective detuning and the photon number to fit the measured mechanical resonance frequency to the purely dispersive optical spring theory. For the Si$_3$N$_4$ sample, we only fit a dispersive coupling because the dissipative coupling is several orders of magnitude weaker. The resulting mechanical detunings $\delta\Omega_{m} = \Omega_{m} - \Omega_{0}$ for different photon numbers and probe detunings is shown in the $\Delta_{\text{eff},p}$-$n_{\text{circ},p}$ phase-space in Fig. 5.7.

Figure 5.8 (a) displays the measured mechanical resonance frequency $\Omega_{m}$ as a function of effective probe detuning $\Delta_{\text{eff},p}$. The regression (black line) with fitted parameters $\kappa/\pi = 62\text{MHz}$ and $g_{0,0}/2\pi = 575\text{Hz}$ nicely describes the softening of the mode for negative detunings. Around the plateau of the photon number, the mechanical mode oscillates close to its natural frequency. We neglect those datapoints in the fit. The small stiffening of the mode above a detuning of $\Delta_{\text{eff},p} \approx 70\text{MHz}$ is reproduced by the theory. The asymmetry of the fit between negative and positive detunings stems from the large difference in photon numbers.

The cavity linewidth that we obtain from the fit in Fig. 5.8 (a) is three times larger than the cavity linewidth calculated from the resonant transmission. We assume that this is due to noise in the detuning from cavity length fluctuations. The replacement of the probe laser compared to the measurement in Ch. 4 leads to a four-fold decrease of the cavity linewidth extracted from the fit, $\kappa/2\pi = 62\text{MHz}$. The obtained single-photon coupling of $g_{0,0}/2\pi = 575\text{Hz}$ corresponds to a frequency pull parameter of $G_{0}/2\pi = 100\text{MHznm}^{-1}$ which is of the order that is expected from the static coupling measurement.

Especially under strong dynamical backaction, the mechanical spectra appear non Lorentzian. The effective mechanical damping cannot be extracted from the Lorentzian fits. Since the sensitivity of the 780nm interferometer is constant during the measurement, we calculate the mechanical linewidth from the height of the spectral peak using Eq. (2.20) evaluated at $\omega = \Omega_{0}$. The obtained linewidths are plotted in Fig. 5.8 (b) together with the theoretical curve (black line) with the parameters from the fit of the optical spring measurement. For the detunings where we measure a plateau of the photon number, we calculate a negative effective mechanical damping (black dashed line). The mechanical mode self-oscillates for those detunings. In the measurements, we observe an effective damping close to zero, limited by the bandwidth of the measurement and the quality of the Voigt profile fits that we use to extract the mechanical linewidth. Compared to this heating for positive detuning, we observe a broadening of the mode for negative detunings. The magnitude of the effective mechanical damping and the peak position match the theoretical curve, but the measurement clearly deviates from the theory.
5.3 Dynamical backaction

(a) The measured mechanical resonance frequencies $\Omega_m$ for varying effective probe detunings $\Delta_{eff,p}$ are fitted to the dispersive optical spring effect using Eq. (2.38) (black line). The regression yields a single-photon coupling of $g_{0\omega}/2\pi = 575\text{Hz}$. The mechanical mode self-oscillates for datapoints between $\Delta_{eff,p}/2\pi = 0\text{MHz}$ and around 70MHz. Those points are neglected in the regression. The asymmetry between positive and negative detunings stems from the bistable cavity response. (b) The extracted effective mechanical dampings $\Gamma_{eff}$ is compared to the theoretical curve (black line) calculated with the parameters from the regression shown in panel (a). The dotted line corresponds to a negative effective mechanical damping.

Figure 5.8: Dynamical backaction measurement and theoretical fit. (a) The measured mechanical resonance frequencies $\Omega_m$ for varying effective probe detunings $\Delta_{eff,p}$ are fitted to the dispersive optical spring effect using Eq. (2.38) (black line). The regression yields a single-photon coupling of $g_{0\omega}/2\pi = 575\text{Hz}$. The mechanical mode self-oscillates for datapoints between $\Delta_{eff,p}/2\pi = 0\text{MHz}$ and around 70MHz. Those points are neglected in the regression. The asymmetry between positive and negative detunings stems from the bistable cavity response. (b) The extracted effective mechanical dampings $\Gamma_{eff}$ is compared to the theoretical curve (black line) calculated with the parameters from the regression shown in panel (a). The dotted line corresponds to a negative effective mechanical damping.

around the tails of the cavity response.

Overall, we measure the optical spring effect and optomechanical cooling and heating of the mode under investigation. Due to the small natural linewidth of the mechanical mode, even small negative optical dampings lead to self-sustained oscillations of the mode. Those self-sustained oscillations cause a bistability in the cavity response, apparent both in the cavity photon number and in the optical spring effect. During the measurement presented above, a slightly red-detuned probe tone leads to an around 25-fold increase of the mechanical linewidth (see Fig. 5.8(b)). According to Eq. (5.1) this corresponds to an effective mode temperature of $T_{eff} \approx 12\text{K}$.

5.3.2 Optomechanical cooling

To confirm our observations, we perform a power sweep for a fixed probe detuning. According to Eqs. (2.38) and (2.40) we expect both the mechanical resonance frequency and the effective mechanical linewidth to increase linearly with the number of cavity photons.

Contrary to the measurements above, we consider the fundamental oop mode in the following. We center the cavity along the stripe to maximize coupling to this mechanical mode. We place the sample at a position of around $\bar{z} = -30\text{nm}$ (below a node of the cavity field) resulting in a cavity linewidth of $\kappa/2\pi = 22.3\text{MHz}$ and stabilize the cavity in a way that the lock tone is blue detuned. The backaction from the lock tone shifts the mechanical resonance by 200Hz to $\Omega_m/2\pi = 468.24\text{kHz}$. The optical damping from the lock tone is estimated to be enough to set the mode into self-oscillation as we discuss later.
Now, we fix a probe detuning of $\Delta_p/\kappa = 0.56$ and we scan the probe power from $P_{in} = 0.2\,\mu W$ to $P_{in} = 18\,\mu W$. That corresponds to a number of intracavity photons due to the probe tone of up to $n_{\text{circ,p}} = 160000$. The photon number due to the lock tone is constant during the measurement: $n_{\text{circ,l}} \approx 90000$. We select a large lock detuning to minimize backaction from the lock tone. At the same time, a large lock photon number ensures a strong feedback signal that boosts the lock quality.

We record the mechanical spectra of both the 780nm interferometer and the $X$ quadrature extracted from the heterodyne signal in transmission. The mechanical PSDs are fitted with Voigt profiles to obtain the mechanical resonances and the effective mechanical dampings. The sensitivity of the heterodyne measurement depends on the light intensity and on the detuning. For values of $n_{\text{circ,p}} < 5 \cdot 10^4$ the transmitted field is dominated by the lock tone and the sensitivity of the heterodyne is almost constant. Because of that, we average over the fit parameters extracted from the 780nm interferometer and from the heterodyne measurement for moderate probe powers. In the range of $n_{\text{circ,p}} \approx 5 \cdot 10^4$ the mode starts to be cooled considerably and the heterodyne signal vanishes. In that range, we can only fit the spectra recorded with the 780nm interferometer. For probe powers with resulting photon numbers of $n_{\text{circ,p}} > 5 \cdot 10^4$ the probe tone boosts the sensitivity of the heterodyne measurement while the signal in the 780nm interferometer vanishes. For those stronger probe powers we use the heterodyne spectra.

Figure 5.9(a) displays the mechanical frequency as a function of probe photon number. From around $n_{\text{circ,p}} = 30000$ the mechanical resonance decreases linearly with the probe photon number. The slope of a linear regression (black line) in this range yields a dispersive single-photon coupling of $g_{0\omega}/2\pi = 395(5)$ Hz. If we consider the effective mechanical damping in Fig. 5.9(b) we observe the expected linear increase of $\Gamma_{\text{eff}}$ with the photon number in the range between $n_{\text{circ,p}} = 30000$ and $n_{\text{circ,p}} = 130000$. Again, we extract the single-photon coupling from a linear regression and we obtain $g_{0\omega}/2\pi = 401(10)$ Hz. Those values appear reasonable compared to the single-photon couplings derived in the detuning sweep measurement discussed above: $g_{0\omega} = 575$ Hz.

We observe, that the mode self-oscillates until the sum of the intrinsic damping of the mode, the optical damping induced by the lock tone, and the optical damping induced by the probe tone reaches a positive value. While the mode self-oscillates, the mechanical resonance frequency remains constant and the observed effective damping is close to zero. The linear regression allows to estimate the optical damping induced by the lock tone to be of the order $\Gamma_{\text{opt,l}}/2\pi = -40$ Hz.

For very strong probe powers, the mechanical damping decreases again. We expect that phase noise imprinted on the light field by cavity length fluctuations starts to heat the mode. Together with the lower mechanical $Q$ factor compared to the mechanical mode discussed above, this limits the attainable minimal mode temperature in this measurement to $T_{\text{eff}} \approx 22$ K as plotted in Fig. 5.9(c).

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*aSince there is no way to precisely know the lock detuning, we estimate the probe detuning from the measured cavity linewidth, the calibrated input power and the power measured in transmission using Eq. (2.4).*
5.4 Discussion

With the switch to stochiometric Si$_3$N$_4$ membrane stripes and the change of the probe laser, we circumvent the limitations of the measurements presented in Ch. 4. Compared to the low-stress SiN membrane, the static spectroscopy yields reduced frequency pull parameters as a result of a reduced refractive index. We measure values of up to $|G_\omega| / 2\pi = 1\ \text{GHz}\text{nm}^{-1}$ corresponding to single-photon couplings of up to $|g_{0\omega}| / 2\pi = 7\ \text{kHz}$. The dissipative coupling is around three orders of magnitude smaller. The reduced overall absorption boosts the loaded cavity finesse up to $\mathcal{F} = 195\,000$, close to the empty cavity value.

We demonstrate dynamical backaction on the fundamental oop flexural mode and on the first overtone. In two parameter sweeps, we scan the probe detuning and the probe power. Under cavity backaction, the

![Figure 5.9: Dynamical backaction cooling. (a) The mechanical resonance frequency $\Omega_m$ decreases linearly with increasing photon number of the probe tone $n_{\text{circ}}$. A linear regression (black line) yield a single-photon coupling of $g_{0,\omega} / 2\pi = 395(5)$ Hz. (b) The effective mechanical damping $\Gamma_{\text{eff}}$ rises linearly with increasing photon number. For very high probe powers the damping decreases again. A regression of the linear part (black line) yields $g_{0,\omega} / 2\pi = 401(10)$ Hz. (c) Effective mode temperature calculated from the data in panel (b).](image)
5. Dynamical backaction on stochiometric Si$_3$N$_4$ membrane stripes

mechanical lineshape is altered. We fit the mechanical spectra to Voigt profiles to extract the effective mechanical damping and the mechanical resonance frequency from the mechanical spectra. Because of the small natural damping of the mechanical modes, the modes are driven into self-sustained oscillations even under weak optomechanical driving. That leads to instabilities and a bistable cavity response. Away from this instability regime, we describe both parameter sweeps with the theory and extract reasonable values. We obtain a single-photon coupling of $|g_{0\omega}|/2\pi = 575$Hz and $|g_{0\omega}|/2\pi = 400$Hz for the detuning sweep and the probe power sweep, respectively. Those values are compatible with the dispersive couplings from the static measurements close to the node of the intracavity field.

Starting from room temperature, we cool the fundamental mechanical mode and the first overtone down to around 22K and 12K, respectively. This value is around 20 times larger than the theoretical limit. We suppose that frequency noise in the system limits the cooling performance. A more detailed study of the noise sources in the system and of the stability of the cavity lock is needed to further improve the cooling performance.

To sum up, we show for the first time the optical spring effect and optical damping of a high $Q$ mechanical mode inside an ultrahigh finesse FFPC operated at room temperature. For those experiments, we operate the system with a single-photon cooperativity of $C_0 = 4g_0^2/\kappa\Gamma \approx 0.01$. For different sample positions, values up to $C_0 = 0.8$ are accessible, which is a competitive value considering the dimensions of the mechanical resonator [14].
Chapter 6

Optomechanical coupling to a free-standing CNT

Following the proposal of Ref. [53], free-standing carbon nanotube mechanical resonators inside a FFPC poses a promising research direction. Because of their outstandingly low mass, they feature huge zero-point fluctuations, giving rise to large single-photon couplings. In a first proof-of-principle experiment, our group demonstrated optical detection of CNT Brownian motion [51]. The experiment was done in a FFPC, but the sample substrate hindered measurements on different positions of the sample inside the cavity mode and lead to a lot of scattering and absorption losses from the substrate.

In the following, we present two different advances towards dynamical backaction measurements on CNTs inside our system. In the first approach, commercial quartz tuning forks are used as mounts for the free-standing CNTs. In the second, more promising approach, SiN membrane stripes serve as holder with CNTs grown between two adjacent stripes.

6.1 Chemical vapor deposition of CNTs

We grow the CNTs in a collaboration with the group of Andreas Hüttel at the University of Regensburg. A thin cobalt (Co) layer serves as catalyst in the chemical vapor deposition (CVD) growth process. Since there is no access to a Co sputter coater in the faculty, we evaporate around 3nm of Co via e-beam evaporation.

The CVD growth itself happens at 950°C in hydrogen and methane atmosphere. The temperature is elevated from the standard recipe to enhance the length of the CNTs grown in the process. Although the main growth happens in the first few seconds, we keep the sample in the CVD for at least 20mins to remove defects and ensure high purity CNTs [41] [118]. During heating and cool-down, an argon atmosphere prevents oxidation of the CNTs.

Figure 6.1(a) shows the growth setup. We mount the sample on a quartz plate (inset) and insert it into the quartz tube. Since the growth is assumed to happen mainly in gas flow direction, we insert the samples
6. Optomechanical coupling to a free-standing CNT

Figure 6.1: Chemical vapor deposition setup at the university of Regensburg used for the CNT growth. (a) The furnace holds a quartz tube that is used as process chamber. The samples are placed on a quartz plate (inset) inside the tube. Process gases enter the chamber on the left, the exhaust is routed to the fume hood. (b) The process gas lines pass a first mass flow controller (MFC Bypass) to set the flow for the flushing of the chamber. Afterwards they can be sent to a second MFC (MFC Growth) to adjust the gas flow for the growth process. Finally, shutoff valves allow to enable and disable every gas line individually.

with the gas flow oriented perpendicular to the stripes lateral extension. The samples are centered over the temperature sensor in the furnace. The growth happens under a fume hood for health safety reasons. The process gases enter the fume hood via the black tube on the left. After flushing through the quartz tube, the exhaust leaves the furnace via the stainless steel tube on top of the image.

The process gases are routed through several mass flow controllers (MFC) and shutoff valves as shown in Fig. 6.1 (b). The first MFCs limit the overall gas flow for flushing the system. The several gas lines are routed either directly to the process chamber or through remotely controlled MFCs for the growth. Finally several shutoff valves allow to disconnect all the gas lines from the process chamber. We list the exact sequence and the relevant gas flows in App. B.

6.2 CNT imaging

To estimate the amount of growth after a CVD run, we inspect the samples in a scanning electron microscope (SEM). Due to different electronic properties compared to the SiN membrane, CNTs appear with a different contrast on the membrane. The free-standing sections of the CNTs are visible mainly in the in-lens detector due to scattering of electrons. We employ low electron currents and low acceleration voltages to minimize contamination of the CNTs with electron beam deposited (EBD) carbon and to enhance interaction of the electron beam with the thin CNTs. We scan along the stripe edges with contrast settings that allow to resolve even tiny contrast variations over the dark background. A magnification of around 30k allows to image even individual SWCNTs close to their clamping point. Individual SWCNTs appear blurred over the free-standing part. Bundles of CNTs can be identified by the branching either close to the stripes or on the membrane substrate as displayed in the SEM micrograph in Fig. 6.2.
6.3 Tuning fork geometry

As discussed before, clean single-walled CNTs (SWCNTs) are usually grown in a CVD process at elevated temperatures. Interfacing those CNTs with superconducting leads, as it is required in the Hüttel group, poses challenges in fabrication because most materials lose their superconducting properties when subject to the high temperatures during CVD growth. Because of that, several groups nowadays use transfer techniques [119–121]. CNTs are grown on a different substrate and are then integrated in complex, large-scale circuits. From a variety of transfer approaches, commercial quartz tuning forks have been developed as a transfer platform in the Hüttel group (see Ref. [122]) and turned out to be a useful geometry for our experiment.

The tuning fork devices in this work have dimensions of $2000 \times 800 \times 150\mu m^3$. The spacing between the tines of the fork is around $140\mu m$, which makes them ideal candidates to enter the cavity as shown in Fig. 6.3. The electrodes on the tuning forks are removed and CNTs are grown on the top surface of the tuning forks with several CNTs spanning the trench between the two tines of every sample. The tuning fork is positioned in a way that the top surface is centered in the cavity as observed with the camera view shown in Fig. 6.3(b) and then the sample is slowly inserted while the cavity response is monitored. The second camera view (Fig. 6.3(c)) allows to estimate the position of the cavity to compare with SEM images.

Figure 6.2: SEM micrograph of a bundle of free-standing CNTs close to the supporting SiN stripe (top). Above the black background, it is clearly visible that the CNT leaving the picture on the bottom is a bundle consisting of several CNTs. Catalyst for the CVD process and CNTs can be seen on the SiN membrane surface.
6. Optomechanical coupling to a free-standing CNT

Figure 6.3: Optical micrographs of the tuning fork geometry. (a) The top view shows the tuning fork (light grey) glued to the sample holder (metallic surface on the right). CNTs are grown between the tines of the tuning fork. Electrode residues (gold-colored) on the right tine help to track the orientation of the tuning fork. (b) We use the camera front view to center the sample surface between the optical fibers and the position of the trench. (c) The camera side view allows to estimate the position of the cavity along the tuning fork as indicated by the yellow arrow. The fiber diameter is 125 µm.

6.3.1 Static coupling

Figure 6.4 shows the resonant cavity transmission of the cavity mode while the CNT passes the cavity mode in lateral direction (y direction as illustrated in Fig. 6.3 (c)) for different sample positions (corresponding to a sweep in z direction in Fig. 6.3 (c)). We calibrate the travel in y direction interferometrically with a mirror that is glued on the positioner stack. With the CNT inside the mode, a displacement in z modulates the cavity transmission with a period of \( \lambda/2 \). The range of interaction of the CNT with the cavity mode in lateral direction is approximately twice the cavity mode diameter (\( w_0 = 5.2 \mu m \)) which corresponds to a convolution of the CNT with the cavity mode profile.
6.3 Tuning fork geometry

Figure 6.4: Resonant transmission for different CNT positions. The resonant cavity transmission is modulated with the sample position showing the expected $\lambda/2$ periodicity. When the CNT is scanned through the mode in $y$ direction, minimum transmission is achieved with the CNT centered in the mode.

To probe the dispersive coupling, we look on the cavity response with the CNT placed centered in the mode (at $\delta y = 0 \mu m$). The two cavity polarization modes at $\Delta/2\pi \approx 0 \text{MHz}$ and at $\Delta/2\pi \approx 100 \text{MHz}$ are visible in the cavity scans as shown in Fig. 6.5. Frequency shifts of the two modes cannot be resolved, only small drifts are observed. Those drifts originate from temperature variations during the measurement. Both modes show a clear modulation of the transmission levels, hinting at a linewidth modulation. The modulation patterns are different for the two polarizations and probably depend on the CNT orientation.

In order to measure the mechanical modes of the CNT, the sample is placed at a position of big dissipative coupling and the cavity is locked with a detuning of around $\Delta = \kappa/2$. We are unable to resolve any mechanical signal over the noise floor of a measurement with the empty cavity. The CNTs used in this geometry are rather long (at least $L = 140 \mu m$). CNTs grown in CVD are expected to have no tensile stress, leading to expected resonance frequencies of only several kHz for the fundamental oop flexural modes of our samples. This comparatively low eigenfrequency range lies inside the lock bandwidth and is obscured by technical noise of the setup.

The optomechanical coupling in our system is mediated by light scattering. For considerable tilts of the CNT with respect to the cavity mode, light is mainly scattered out of the cavity mode. This mechanism leads to additional optical losses depending on the field intensity at the CNT position. Because of that, dissipative coupling dominates in our measurement and we are unable to resolve the dispersive coupling. If we minimize the tilt, we expect the dissipative coupling to decrease. As the light is mostly scattered back into the cavity mode, a dispersive coupling should be observable. Due to the sample geometry, angular alignment of the tuning fork with the cavity is very tricky and the accessible tilt range is very
Figure 6.5: Cavity scans for different sample positions. The CNT is centered in the mode ($\delta y = 0\mu m$). A scan of the sample height results in a modulation of the transmission of the two polarization modes. The periodicity from the intracavity field is clearly visible. No dispersive coupling is resolved in the measurement.

limited. We are unable to improve the angular alignment.

Both limitations are overcome with the membrane stripe geometry presented in the following. With stripe separations of 30µm, expected CNT resonance frequencies of several hundred kHz should be clearly resolved from the noise floor and angular alignment can greatly be improved.

6.4 Membrane stripes geometry

For the second geometry we use SiN membrane stripes with dimensions of $500 \times 30 \times 0.2\mu m^3$. The stripes' separations are 30µm. The sample under investigation was cleaved before the CNT growth. In a quick SEM scan, we observe one single SCWNT between the rear two stripes, placed around 90µm from the left frame. Figure 6.6(a) shows a SEM image of the CNT under investigation. The SiN stripes appear white on the top and bottom of the image, the CNT spans the gap (black background) between the two stripes. A blurring in the middle part originating from the vibrations of the CNT indicates that the CNT under investigation is a single nanotube rather than a bundle of SWCNTs [123].

We glue the sample to a sample holder and install it in the setup. With the front view camera (Fig. 6.6(b)) we can estimate the distance of the cavity from the frame, the tilted side view (Fig. 6.6(c)) resolves the individual stripes and helps to center the cavity mode between the stripes. To start, we align both angles to the cavity by eye.
6.4 Membrane stripes geometry

Figure 6.6: CNT sample with membrane stripe geometry. (a) SEM micrograph of the CNT under investigation. The CNT spans the gap (black background) between the membrane stripes (white on top and bottom). (b) Camera front view of the cavity with the CNT inside the cavity mode. The membrane (contrast step due to illumination) is placed between the two cavity fibers. The triangular shape of the Si frame is visible on the bottom left. The distance of the cavity mode from the frame can be estimated in this view. (c) Tilted side view of the sample with the CNT inside the cavity. The u-shaped frame is inserted in the cavity. Three SiN stripes (blue) are visible over the black sidewall of the rear frame. The fiber diameter is 125µm.

6.4.1 SiN stripe residual coupling

First, we center the cavity mode between the rear two stripes. Even though we expect a cavity mode diameter well below 10µm, we observe a clipping of the cavity mode. Due to the ultrahigh finesse of the cavity ($\mathcal{F} = 205000$ as measured in Ch. 3), we are sensitive to clipping losses even at the tails of the Gaussian distribution of the cavity mode. Depending on the exact sample position, we measure a cavity linewidth of $\kappa/2\pi \approx 20$MHz. We mainly attribute those losses to scattering at the stripe edges. To rule out optomechanical coupling to the SiN stripes at that position, we measure the static coupling. The cavity resonance drifts by around 100MHz during the measurement (Fig. 6.7 (a)) due to thermal
drifts. Fortunately, even if the stripe edges couple dispersively to the cavity mode, the corresponding frequency shifts of the cavity mode are far below the cavity linewidth and therefore below the resolution limit of the setup. We measure a small dissipative coupling with a periodic modulation increase of the cavity linewidth by up to $2\pi \cdot 4\text{MHz}$ as shown in Fig. 6.7(b). Again, we observe that the sample position slightly shifts in $y$ direction during the measurement. As a result, the cavity mode moves closer to one stripe and the overall losses slightly increase. This leads to a slightly linear slope of the cavity linewidth with respect to the sample position inside the mode.

### 6.4.2 Static coupling

In order to find the CNT we step it through the cavity mode in lateral direction ($x$ direction as illustrated in Fig. 6.6(a)). For each step, we record cavity scans for different sample positions. From those measurements, we extract the cavity linewidth and plot it as a function of the sample position in the mode and lateral displacement in Fig. 6.8. With the CNT far from the cavity mode (a large negative number of steps), we observe a modulation of the cavity linewidth with sample position from clipping at the stripe edges as observed before. This modulation background increases with increasing lateral displacement and becomes more apparent for displacements of more than 20 steps. This increase in clipping stems from a misalignment of the stripes with the positioner axes. Therefore, the cavity mode approaches one stripe as the lateral position changes. With the CNT inside the cavity mode, the linewidth increases to around $\kappa/2\pi = 38\text{MHz}$ and depending on the sample position reaches values of up to $\kappa/2\pi = 80\text{MHz}$. The lateral displacement is not calibrated since we have no interferometric access to the corresponding positioner axis. Nevertheless, if we assume a similar travel range as the other horizontal positioner axis, the interaction range is of the same magnitude as in the tuning fork measurement presented above. We laterally center the CNT in the cavity and record another scan of the sample position. For this measurement, the laser line is aligned with the low frequency polarization mode which features the larger finesse. On top of a thermal drift of the cavity resonance frequency by around $40\text{MHz}$, we observe a
Figure 6.8: Cavity linewidth for different CNT positions. With the CNT centered in the cavity mode, the cavity linewidth shows the expected modulation with the sample position. The coupling to the CNT decreases when the sample is displaced in \( x \) direction. Far from the mode, residual modulation of the linewidth stems from clipping on the stripe edges.

Figure 6.9: Static coupling of the CNT to the low frequency cavity mode. (a) The cavity resonance shows a periodicity in the sample position with a double period of around the cavity wavelength. A thermally induced drift of around 40MHz during the measurement is visible. (b) The cavity linewidth is modulated between \( \kappa/2\pi = 38 \text{MHz} \) (CNT placed at a node) and 80MHz (CNT placed at a anti-node).
Figure 6.9 (b) shows the modulation of the cavity linewidth between \( \kappa / 2\pi = 38\text{MHz} \) and 80MHz. Since the loss rates add inversely, residual clipping effects of the SiN stripes are negligible. We calculate a dissipative frequency pull parameter of up to |\( G_\kappa / 2\pi = 150\text{kHznm}^{-1} \)|. The dispersive frequency pull parameter is of the same magnitude |\( G_\omega / 2\pi = 120\text{kHznm}^{-1} \)|. From the expected CNT geometry we estimate \( z_{\text{zpf}} \approx 10\text{fm} \). This corresponds to a single-photon coupling of around \( g_0 \omega / 2\pi \approx 10\text{kHz} \) with a loaded cavity finesse of \( \mathcal{F} = 62000 \) at the position of maximum coupling. Reference [51] estimates a value of \( G_\omega / 2\pi \approx 1\text{MHznm}^{-1} \) for a similar CNT inside a cavity at \( \lambda = 780\text{nm} \). Since \( G_\omega \) scales as \( 1/\lambda^2 \), our value is of the expected magnitude.

The fact that we observe light being scattered out of the cavity hints that the alignment of the CNT with the cavity axis is not perfect. We are able to reduce the cavity linewidth down to around \( \kappa / 2\pi = 30\text{MHz} \) with the CNT centered at a node. Depending on the sample position, the linewidth increases up to 40MHz. The CNT is observed to break during a cavity scan with an input power of \( P_{\text{in}} = 100\mu\text{W} \) and a scan speed of 10.4Hz. The cavity linewidth drops abruptly to \( \kappa / 2\pi = 22\text{MHz} \). Possible reasons might be absorption due to SEM contaminations or static optomechanical effects. Both effect should be boosted by the loaded finesse of almost \( \mathcal{F} = 100000 \). Unfortunately, this hindered the acquisition of a measurement of the dispersive coupling with the improved alignment.

The high frequency polarization mode shows a dispersive frequency pull parameter of the same magnitude. With the sample placed at the node, the linewidth of this mode is around 2MHz bigger compared to the high finesse polarization mode, but at different sample positions it reaches values up to \( \kappa / 2\pi = 110\text{MHz} \).

### 6.4.3 CNT mechanics and hints at dynamical backaction

We measure the mechanical spectra of the membrane stripes with the 780nm interferometer to distinguish between stripe modes and mechanical modes of the CNT under investigation. The Brownian motion spectrum of one stripe is shown in Fig. 6.10 (a). Due to the increased thickness compared to the stripes discussed in previous chapter, the eigenfrequencies are reduced. For the first three oop modes we measure eigenfrequencies of around 125kHz, 250kHz and 375kHz. We extract quality factors in the range of \( Q \approx 50000 - 100000 \). We assume that the \( Q \) factors are reduced due to the catalyst coating and the annealing during the CVD process.

In the next measurement, we place the sample at the position of maximum coupling. At that position, the linewidth of the loaded cavity of \( \kappa / 2\pi = 35\text{MHz} \) corresponds to a finesse of \( \mathcal{F} = 98000 \). We switch to the homebuilt detector and lock the cavity on the \( X \) quadrature extracted from the cavity transmission. In order to obtain a sufficient signal in transmission, we increase the lock tone intensity to \( P_{\text{in}} = 20\mu\text{W} \). We vary the lock detuning with an offset applied to the error signal and record cavity transmission spectra at a number of lock detunings. Since we have no access to the exact length of the locked cavity, we are unable to extract the value of the lock detuning in this one-tone measurement. We repeat the measurement without the CNT in the cavity mode.

Figure 6.10 (b) displays spectra acquired from the cavity transmission for a blue detuned lock with (blue line) and without (black line) the CNT in the cavity mode. We observe a variety of technical noise and substrate modes up to around 150kHz. With the CNT in the cavity, one mode at around 190kHz clearly rises above the noise background. We vary the uncalibrated lock detuning and record several
6.4 Membrane stripes geometry

Figure 6.10: Mechanical spectra of the SiN stripes and the CNT. (a) Brownian motion spectrum of one membrane stripe measured with the 780nm interferometer. The first three oop modes have eigenfrequencies of around 125kHz, 250kHz and 375kHz (marked with arrows). (b) Mechanical spectra measured with the locked cavity. The black line shows the noise background acquired without the CNT, the blue line is measured with similar lock conditions but with the CNT inside the cavity mode. One peak at around 190kHz clearly rises above the noise floor and is attributed to the CNT. (c) Mechanical spectra for different lock detunings. The black line corresponds to the noise background. The red line and the blue line are recorded with a lock detuned to the red and blue respectively. We fit the peaks with a Voigt profile and obtain $\Omega_m/2\pi = 168.7$kHz and $\Gamma_{\text{eff}}/2\pi = 220$Hz (red peak) and $\Omega_m/2\pi = 189.2$kHz and $\Gamma_{\text{eff}}/2\pi = 91$Hz (blue peak). In both cases, the noise broadening of the peak is around $\sigma/2\pi = 250$Hz. The peak at around 194kHz is present even without the CNT in the mode and is attributed to a non-mechanical origin.

spectra on both sides of the resonance. We compare two spectra with similar cavity transmission but one with the lock detuned to the red (red line in Fig. 6.10 (c)) and one with the lock detuned to the blue (blue line). Voigt profile fits to the spectra yield $\Omega_m/2\pi = 168.7$kHz and $\Gamma_{\text{eff}}/2\pi = 220$Hz (red peak) and $\Omega_m/2\pi = 189.2$kHz and $\Gamma_{\text{eff}}/2\pi = 91$Hz (blue peak). In both cases, the noise broadening of the peak is around $\sigma/2\pi = 250$Hz. The tuning behavior of the mechanical resonance frequency we observe qualitatively matches with what we calculate from the optical spring effect with a estimated single-photon
coupling of $g_0\omega/2\pi = g_0\kappa/2\pi = 2\text{kHz}$, a cavity linewidth of $\kappa/2\pi = 35\text{MHz}$ and the photon number calculated from the input power as plotted in Fig. 6.11. The dots corresponds to situations where the lock tone is red or blue detuned. We estimate the natural mechanical quality factor to be below $Q = 1000$, which is the expected magnitude of CNTs at room temperature [41, 124, 125].

6.5 Discussion

We present two different sample geometries that allow to introduce free-standing CNTs into the cavity. The tuning fork geometry greatly reduces fabrication complexity. We grow CNTs on the top surface spanning the gap between the two tines and introduce the CNT into the cavity. We observe an optomechanical coupling between the CNT and the cavity mode with the expected periodicity in the standing wave pattern, but the coupling is purely dissipative in nature. Due to misalignment of the CNT with the cavity, we are unable to resolve any dispersive coupling. The tuning fork geometry limits the angular alignment of the CNT to the cavity axis. Because of the large separation between the tines, the CNTs are long with expected mechanical eigenfrequencies in the range of the cavity feedback.

The second, more promising geometry relies on CNTs grown between adjacent SiN stripes. Static spectroscopy resolves both a dispersive and a dissipative optomechanical coupling of a CNT to an optical cavity mode for the first time. Recent cavity optomechanical experiments with CNTs rely on dissipative couplings [55] or photothermal force gradients [54]. We measure a dispersive frequency pull parameter of around $|G_0|/2\pi = 120\text{kHz}\text{nm}^{-1}$ in accordance with the values expected from cavity perturbation theory as estimated in Ref. [51]. With the CNT at a position of largest dispersive coupling, this corresponds to a competitive single-photon coupling of $g_0\omega/2\pi = 10\text{kHz}$ and a loaded cavity finesse of almost $\mathcal{F} = 100000$. 

Figure 6.11: Optical spring effect (a) and optical damping (b) on a vibrational mode of the CNT. The theory curves use simulation parameters: $g_0\omega/2\pi = g_0\kappa/2\pi = 2\text{kHz}$, $\kappa/2\pi = 35\text{MHz}$ and $P_{in} = 10\mu\text{W}$. Colored dots corresponds to experimental situations with equal detuning to the red and to the blue. Note that the mechanical resonance frequency and the mechanical damping are chosen arbitrarily in order to qualitatively describe the experimental observations.
We observe peaks in the locked cavity transmission that attribute to vibrational modes of the CNT. Depending on the lock detuning, one peak can be tuned in the range of $\Omega_m/2\pi = 170\,\text{kHz}$ to $190\,\text{kHz}$. In the same time, we obtain (mechanical) linewidths between $\Gamma_{\text{eff}}/2\pi = 220\,\text{Hz}$ and $90\,\text{Hz}$. Those finding quantitatively match the optical spring effect and the optomechanical damping theory with the extracted parameters of our system and therefore hint that the observed mode is of mechanical origin. To further verify those findings, a more systematic approach is needed. Especially two-tone measurements of the optical spring effect and optomechanical damping as presented in Ch. 4 and in Ch. 5 are in reach.
6. Optomechanical coupling to a free-standing CNT
Chapter 7

Photothermal effects on SiN membranes inside a 780 nm FFPC

This chapter presents measurements that were conducted at an earlier stage of the experiment before switching from the 780 nm to the 1550 nm FFPC technology. Residual absorption in SiN in a $\lambda = 780$ nm cavity pose limitation on the sample geometry (especially the sample thickness). On top of that, arising photothermal effects in combination with huge optomechanical coupling strengths hinder the linear operation of the system. For that reason, we only discuss static optomechanical measurements in the following.

The measurements in this chapter were performed on the same sample (SiN, 30 nm thickness) as discussed in Ch. 4 and shown in Fig. 4.2. The cavity consists of a SM800-125CB input SM fiber and a GI50-125CB output MM fiber (both from Oxford Electronics). The input mirror on the SM fiber has a measured curvature of 208 µm, a MM fiber from the same batch as the one used for the cavity has a measured curvature of 133 µm. The coatings were done by ATF Inc. in 2011 with a design transmission of 10 ppm for each fiber.

The cavity characterization is similar to the procedure described in Ch. 3. We measure the FSR of the cavity with a broadband SLED illumination (Exalos EXS7505-8411). An optical spectrometer (Ocean Optics USB4000) measures the transmitted modes. We obtain a value of $\omega_{fsr}/2\pi = 4.96$ THz, corresponding to an effective mirror separation of 30.22 µm. An empty cavity linewidth of $\kappa/2\pi = 36$ MHz yields a finesse of $\mathcal{F} = 138,000$.

7.1 Static coupling

For the 780 nm cavity, we observe that absorption in SiN complicates the measurement procedures. The alignment procedure is very tricky because even for a small misalignment, the cavity mode vanishes when we insert the stripe in the cavity. We try a variety of samples with different geometries and find that for thicker samples, we are unable to recover the cavity mode with the sample fully inserted. Measurements
Figure 7.1: Dispersive coupling of the cavity with the SiN membrane stripe inserted. The periodicity of the intracavity field is clearly visible.

with the sample partially inserted yield couplings, but these are weak and very sensitive to position drifts. Also for these positions, scattering at the stripe edges limits the resulting cavity finesse.

For the sample that we discuss in the following, we recover the cavity mode quickly. Adjustments of the two goniometer angles allow to gradually increase the resonant transmission. Once the sample is aligned, we measure the static optomechanical coupling similar as described above. We use the external cavity diode laser (ECDL) at 780 nm to measure the cavity. A 1310 nm SLED (Thorlabs S5FC1018S) serves as laser for the low finesse interferometer. The low coherence length of the SLED of several tens of µm (the specified FWHM bandwidth of $15 – 20$ nm corresponds to a coherence length of $30 – 40$ µm) allows to measure the sample position and sample vibrations while it eliminates cavity effects in the fibers. Again, we use an asymmetric scan of the cavity (one fiber is fixed, while we scan the other one). Figure 7.2 shows the cavity response for different sample positions.

The cavity resonance frequency is modulated by the sample with a periodicity in the intracavity field as shown previously in the 1550 nm setup. The period matches quite well the expected period of $\lambda/2$. Below the node, the cavity response shows a nonlinear shape (white triangular areas). We discuss linecuts in the nonlinear regime in Ch. 7.2. This nonlinear effect greatly scales with the decreasing cavity linewidth when the sample approaches the node. At the node the dispersive coupling vanishes and the cavity response recovers its Lorentzian lineshape. Above the node, the cavity response looks linear.

Overall, the cavity resonance is shifted by up to 1.1 THz. We observe dispersive frequency pull parameters of up to $G_\omega/2\pi = 13.8$ GHz nm$^{-1}$ (Fig. 7.1(a)) and $G_\omega^2/2\pi = 560$ MHz nm$^{-2}$ (Fig. 7.1(b)). We are able to verify the magnitudes of all those quantities in a numerical simulation. Compared to the measurements in Ch. 4 the laser wavelength is reduced by approximately 1/2, leading to a doubling of
7.2 Nonlinear cavity response

With the sample placed slightly below the node, the cavity response forms a nonlinear shape that spans a broad frequency region of up to 200GHz. Figure 7.3 shows cavity scans with the sample placed 9nm (black line), 50nm (blue line) and 80nm (red line) below the node. We scan the cavity in increasing detuning direction. All curves are normalized by the same factor. Since the input couplings are unknown for this cavity, the response is shown in arbitrary units. The response clearly deviates from the static bistability shape as discussed above. We observe two bifurcation points, hinting at a higher order non-linearity or a second non-linear mechanism. Since we do not observe this behavior in the measurements in the 1550nm cavity, we expect photothermal effects to play a role here. Localized heating of the membrane stripe might lead to distortion or buckling of the resonator, which would in turn influence the cavity response. For sample positions closer to the mode, the cavity finesse increases. An increased cavity finesse boosts the circulating power and thus enhances those photothermal mechanisms. The dispersive coupling shifts the resonances to higher detunings. Sideband residues appear in the response curves.

Figure 7.2: Dispersive coupling. (a) Dispersive frequency pull parameter $G = \frac{\partial \omega_{\text{cav}}}{\partial z}$ extracted from the cavity detuning measurement. (b) Quadratic dispersive coupling.

the cavity resonance frequency $\omega_{\text{cav}}(0)$. Additionally, the spatial variation of the cavity field is twice as fast compared to a 1550nm standing wave. According to Eq. (2.28) we expect an increase of $G_{\omega}$ by a factor of 4 and of $G_{\omega^2}$ by a factor of 8 compared to the 1550nm cavity. On top of the difference in cavity wavelength, the two cavities have different lengths. This length difference explains the fact, that the couplings reported here exceed the couplings to the 1550nm cavity by more than this wavelength factor.
7. Photothermal effects on SiN membranes inside a 780nm FFPC

![Graph]

**Figure 7.3:** Non-linear cavity response measured with the sample place 9nm (black line), 50nm (blue line) and 80nm (red line) below a cavity node for a strong input power. The small ripple close to the left bifurcation point stems from a modulated sideband with a spacing of 5GHz.

### 7.3 Discussion

We present static spectroscopy measurements of a 30nm thick SiN membrane stripe in a 780nm FFPC. The cavity features a mirror separation of $L = 30.22 \, \mu m$ and the empty cavity finesse is $\mathcal{F} = 138000$. We extract dispersive frequency pull parameters of up to $|G_\omega| / 2\pi = 14\text{GHz nm}^{-1}$. Compared to the values obtained in the 1550nm cavity in Ch. 4, this corresponds to more than a four-fold increase. We explain this increased frequency pull parameter by the reduced wavelength and the smaller cavity length.

For sample positions that result in a large dispersive coupling and large circulating photon numbers as a result of small effective cavity linewidths, we observe a non-linear cavity response with two bifurcation points. We expect that an optomechanical static bistability in combination with photothermal forces leads to the observed shape. In order to understand the mechanisms involved here in more detail, the influence e.g. of the scan direction or the input power on the measurement need to be studied. Additionally, the interferometer can in principle resolve a static deflection of the stripe during the cavity scan. Resolving this "latching" [126] allows to compare the measurement to theoretical predictions. Measuring the equilibrium position of the stripe during the scans, would give access to a second observable of this coupled system. Overall, the experimental challenges arising due to the photothermal effects motivated the switch to the 1550nm FFPC technology.
Conclusion and outlook

This thesis presents a fiber-based microcavity setup that poses a powerful platform for cavity optomechanical experiments at room temperature. We demonstrate dynamical backaction in a MIM type configuration with free-standing SiN membrane stripes inserted into the optical cavity mode. Additionally, we present an important step towards the experimental realization of cavity nano-optomechanics with nanometer sized scattering objects dispersively coupled to the cavity mode as proposed in Ref. [53]. We discuss theoretical frameworks to describe the optical cavity mode and the mechanical resonators under investigation in Ch. 2. Different optomechanical models are reviewed and the influence of dissipative effects are included.

A main achievement of this work is the installation and optimization of the experimental setup. The resulting optical and electrical configurations are described in Ch. 3. We quickly summarize the cavity fiber shooting, coating and the cavity gluing. We characterize the cavity and extract all the parameters that we need for our optomechanics experiments. The empty cavity finesse of $F = 203,000$ is compatible with values reported in literature for recent FFPC while the cavity is as short as 43.8 $\mu$m and features a mode volume of around $250\lambda^3$.

With the proof-of-principle experiments described in Ch. 4 we demonstrate the optical spring effect on the fundamental out-of-plane flexural mode of a SiN membrane stripe. All measurements are supported by theoretical models, allowing to quantitatively extract the parameters of the system. Enabled by the rigid cavity gluing scheme and the improved electrical setup, we are able to operate the loaded cavity with a finesse up to $\mathcal{F} = 126,000$. We report an optomechanical coupling strength of $|g_{0\omega}|/2\pi = 2.5$ kHz while values up to $|g_{0\omega}|/2\pi = 11$ kHz are accessible at a different position of the membrane stripe in the cavity mode.

Absorption losses in the cavity decreases with the use of high-stress Si$_3$N$_4$ as presented in Ch. 5. Thus, we boost the loaded cavity finesse up to $\mathcal{F} = 195,000$, which is only slightly lower than the empty cavity finesse. To the best of our knowledge, this is the highest finesse reported in a loaded FFPC system to date. We demonstrate the optical spring effect as well as optomechanical damping on two oop flexural modes of the membrane stripes. In a detuning sweep we observe dynamical multistabilities.
8. Conclusion and outlook

as the effective mechanical linewidth reaches zero. From the measurement we extract a single-photon coupling of \( |g_{0\omega}| / 2\pi = 575 \text{ Hz} \). In a second experiment we sweep the probe power and reproduce the measurements with theory. We obtain a single-photon coupling of \( |g_{0\omega}| / 2\pi = 400 \text{ Hz} \). Using dynamical backaction, we are able to cool the effective mode temperature from room temperature down to around 12 K.

In Ch. 6 we use a tuning fork geometry to measure a static dissipative coupling of the FFPC mode to a free-standing CNT. With an improved sample geometry, we are able to improve the angular alignment of the CNT with the cavity mode. In a static spectroscopy measurement, we measure a dispersive coupling of around \( G_\omega / 2\pi = 120 \text{ kHz nm}^{-1} \). With estimated zero-point fluctuations of several tens of pm, this yields an estimated single-photon coupling of order \( |g_{0\omega}| / 2\pi = 10 \text{ kHz} \) while the finesse is as large as \( \mathcal{F} = 95000 \). We measure a peak in the output spectrum of the cavity that we attribute to a flexural vibrational mode of the CNT. If we detune the laser frequency with respect to the cavity resonance, we observe the peak to shift and broaden qualitatively in accordance with optomechanical theory.

One main limitation of the setup is mechanical noise in the system. Especially for samples with large frequency pull parameters small position fluctuations translate to large frequency noise and hinders the active cavity stabilization. In a replicated setup we replaced the nanopositioners tower with a more rigid nanopositioning system. Measurements on the mechanical stability of the sample position are pending. Additionally, we observe a broadened linewidth of the cavity in the two-tone experiments. This is apparent from the linewidth obtained from the fit to the optical spring theory. We assume that there is room for improvement in the quality of the cavity stabilization. We designed a HV amplifier that shows an improved noise performance compared to the HV amplifier that is used in this work (see Ap. E.2). A switch to a digital proportional integral derivative (PID) controller allows to implement an infinite impulse response (IIR) filter. Combining those two improvements, one can measure the transfer function of the system consisting of the HV amplifier, the cavity piezos and the fiber mirrors acting as mechanical cantilevers and correct the PID output accordingly. A convenient way to achieve this is the open-source software package PyRPL [127] on the RedPitaya STEMLab boards.

Under detuning sweeps we observe asymmetric lineshapes of the quadratures spectra obtained with the heterodyne measurement. A theoretical model describing the signal levels and quadratures of the light leaving the cavity output ports including (thermal) noise in the system would help to better understand noise sources in the system. This is important to predict the limit to backaction cooling in our system or to estimate the displacement sensitivity of the cavity.

While the first measurement on CNTs are very promising, a more systematic approach is needed. We assume that the dispersive coupling can be enhanced and the dissipative coupling decreased by better alignment of the CNT to the cavity axis. Additionally, a two- to four-fold increase of the single-photon coupling is feasibly by the use of shorter cavities. Together with an increased cavity loaded finesse, the strong coupling regime \( (g_\omega \gg \kappa) \) might be in reach. Systematic two-tone measurements as performed on the stochiometric Si\(_3\)N\(_4\) stripes are the consequent next step to verify the single-photon coupling and to demonstrate the mechanical nature of the observed spectral peaks. This would pose a proof-of-principle demonstration of the proposal in Ref. [53].
Due to the versatility of the presented setup other interesting nanoscale mechanical structures such as nanowires, nanorods or two-dimensional materials like boron nitride monolayers can be studied. By additionally exploiting optical dipole transitions in the said materials, the realization of hybrid optomechanical systems seems to be in reach.
8. Conclusion and outlook
# List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Separation of the cavity mirrors</td>
</tr>
<tr>
<td>$R =</td>
<td>r</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Light wavelength</td>
</tr>
<tr>
<td>$k = 2\pi/\lambda$</td>
<td>Light wave vector</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light in vacuum</td>
</tr>
<tr>
<td>$\omega = 2\pi c/\lambda$</td>
<td>Light frequency</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Laser detuning</td>
</tr>
<tr>
<td>$\omega_{\text{FSR}}$</td>
<td>Free spectral range</td>
</tr>
<tr>
<td>$\kappa = \kappa_0 + \kappa_e + \kappa_2$</td>
<td>Optical damping from internal losses and external couplings</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>Cavity finesse</td>
</tr>
<tr>
<td>$E$</td>
<td>Electro-magnetic field</td>
</tr>
<tr>
<td>$P$</td>
<td>Light power</td>
</tr>
<tr>
<td>$n_{\text{circ}}$</td>
<td>Number of intracavity photons</td>
</tr>
<tr>
<td>$R$</td>
<td>Mirror curvature</td>
</tr>
<tr>
<td>$g$</td>
<td>Cavity stability parameter</td>
</tr>
<tr>
<td>$w$</td>
<td>Mode radius</td>
</tr>
<tr>
<td>$z_R$</td>
<td>Rayleigh range</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Gouy phase</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Input coupling efficiency</td>
</tr>
<tr>
<td>$n$</td>
<td>Refractive index</td>
</tr>
</tbody>
</table>

*Table A.1: List of symbols used to describe the optical subsystem.*
## A. List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_m )</td>
<td>Mechanical eigenfrequency</td>
</tr>
<tr>
<td>( k_{\text{eff}} )</td>
<td>Effective spring constant</td>
</tr>
<tr>
<td>( m_{\text{eff}} )</td>
<td>Effective mass</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Mechanical damping</td>
</tr>
<tr>
<td>( F )</td>
<td>Force</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>Mechanical displacement</td>
</tr>
<tr>
<td>( S )</td>
<td>Spectral density</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Susceptibility</td>
</tr>
<tr>
<td>( k_B )</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Calibration factor of the displacement read-out</td>
</tr>
<tr>
<td>( Q )</td>
<td>Mechanical quality factor</td>
</tr>
<tr>
<td>( L_{\text{beam}}, w_{\text{beam}}, t_{\text{beam}} )</td>
<td>Length, width and thickness of the mechanical resonator</td>
</tr>
<tr>
<td>( A )</td>
<td>Resonator cross section</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Mass density</td>
</tr>
<tr>
<td>( E )</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Tensile pre-stress</td>
</tr>
<tr>
<td>( I )</td>
<td>Area moment of inertia</td>
</tr>
</tbody>
</table>

Table A.2: List of symbols used for the mechanical subsystem.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Light field amplitude</td>
</tr>
<tr>
<td>( z_m = \bar{z} + \delta z )</td>
<td>Position of the mechanical resonator</td>
</tr>
<tr>
<td>( G_{\omega} )</td>
<td>Frequency pull parameter</td>
</tr>
<tr>
<td>( G_\chi )</td>
<td>Dissipative coupling parameter</td>
</tr>
<tr>
<td>( g_{0\omega}, g_{0\kappa} )</td>
<td>Dispersive, dissipative single-photon coupling</td>
</tr>
<tr>
<td>( g_{\omega}, g_{\kappa} )</td>
<td>Effective coupling</td>
</tr>
<tr>
<td>( \varepsilon_0, \varepsilon_r )</td>
<td>Permittivity, dielectric constant</td>
</tr>
<tr>
<td>( \Delta_{\text{eff}} )</td>
<td>Effective detuning due to the shifted equilibrium position</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>Optomechanical self-energy</td>
</tr>
<tr>
<td>( \zeta_{\text{opt}} )</td>
<td>Zero-point fluctuation</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Single-photon cooperativity</td>
</tr>
<tr>
<td>( \delta (\Omega)^2 )</td>
<td>Mechanical frequency shift</td>
</tr>
<tr>
<td>( \Gamma_{\text{opt}} )</td>
<td>Optically induced damping</td>
</tr>
<tr>
<td>( \Gamma_{\text{eff}} )</td>
<td>Effective mechanical damping</td>
</tr>
<tr>
<td>( \mathcal{H} )</td>
<td>Hamiltonian</td>
</tr>
<tr>
<td>( N )</td>
<td>Phonon number</td>
</tr>
</tbody>
</table>

Table A.3: List of symbols used to describe optomechanical system.
Fabrication parameter

Membrane handling

<table>
<thead>
<tr>
<th>Process step</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-cleaning</td>
<td>Not required The membrane windows are shipped clean with minimized particle contamination and rendered organic free.</td>
</tr>
<tr>
<td>Handling</td>
<td>Use sharp and clean tweezers (Ideal-tek 5SVR.SA) Handle by the edge Always keep membrane surface face up, The etched pit on the backside is visible by eye</td>
</tr>
<tr>
<td>Storage</td>
<td>During fabrication dust-free, anti-static sample boxes</td>
</tr>
<tr>
<td></td>
<td>During transport gel boxes</td>
</tr>
<tr>
<td></td>
<td>Long term sealed with plastic wrap in desiccator with dry N₂ atmosphere</td>
</tr>
</tbody>
</table>

**Table B.1:** Sample preparation and membrane handling.
### Optical lithography

<table>
<thead>
<tr>
<th>Process step</th>
<th>Device</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desorption of H\textsubscript{2}O</td>
<td>Hot-plate</td>
<td>10 min @ 110°C</td>
</tr>
<tr>
<td>Adhesion promotion</td>
<td>Hot-plate</td>
<td>80 µl HDMS for two samples</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Heat HDMS on clean-room wipe in petri dish @ 100°C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Place sample upside-down over clean-room wipe for 10 – 15 s</td>
</tr>
<tr>
<td>Resist coating</td>
<td>Spin-coater</td>
<td>AZ MiR 701</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ramp up: 3 s @ 1000 rpm, acceleration 1000 rpm/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soak: 40 s @ 3000 rpm, acceleration 2500 rpm/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ramp down: 2 s @ 100 rpm, acceleration –1000 rpm/s</td>
</tr>
<tr>
<td>Soft-bake</td>
<td>Hot-plate</td>
<td>70 s @ 90°C</td>
</tr>
<tr>
<td>Lithography</td>
<td>Smart Print</td>
<td>Exposure time 2.0 s</td>
</tr>
<tr>
<td>Development</td>
<td></td>
<td>AZ 726 MIF</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60 s at room temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 s rinse in DI water</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 s rinse in hot DI water @ 80°C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Blow dry with \textsubscript{N} \textsubscript{2}</td>
</tr>
</tbody>
</table>

**Table B.2:** Optical lithography to define the membrane stripes.

### Reactive ion etch

<table>
<thead>
<tr>
<th>Process step</th>
<th>Device</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etch mask evaporation</td>
<td>AJA Orion 8</td>
<td>E-beam evaporation of Aluminium in UHV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 nm @ 0.5 Å/s</td>
</tr>
<tr>
<td>Lift-off</td>
<td></td>
<td>Flush in acetone for several minutes</td>
</tr>
<tr>
<td>Anisotropic RIE</td>
<td>Oxford Plasmalab 100</td>
<td>Etch time: 6 per 10nm membrane thickness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>350 W ICP power</td>
</tr>
<tr>
<td></td>
<td></td>
<td>65 W RF power</td>
</tr>
<tr>
<td></td>
<td></td>
<td>280 V bias voltage</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 sccm / 4 sccm flow rates SF\textsubscript{6} / Ar</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 °C table temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 mTorr pressure</td>
</tr>
<tr>
<td>Etch mask removal</td>
<td></td>
<td>5 min in NaOH 0.5 mol/l</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rinse twice 20 s in DI water</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Blow dry with \textsubscript{N} \textsubscript{2}</td>
</tr>
</tbody>
</table>

**Table B.3:** Reactive ion etch.
CVD of long ultra-clean CNTs

<table>
<thead>
<tr>
<th>Process step</th>
<th>Device</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catalyst evaporation</td>
<td>E-beam evaporation of Co in UHV</td>
<td>3 nm @ 0.1 Å/s</td>
</tr>
<tr>
<td>Flush</td>
<td></td>
<td>Flush with H₂, Ar and HC₄ for at least two minutes</td>
</tr>
<tr>
<td>Heat</td>
<td>Thermo Lindberg/Blue M</td>
<td>Heat to 950°C in Ar atmosphere (14 sccm)</td>
</tr>
<tr>
<td>Growth</td>
<td>Thermo Lindberg/Blue M</td>
<td>20 min in H₂ (20 sccm) and CH₄ (10 sccm) atmosphere</td>
</tr>
<tr>
<td>Cool down</td>
<td>Thermo Lindberg/Blue M</td>
<td>Cool down to 300°C in Ar and H₂ atmosphere</td>
</tr>
<tr>
<td></td>
<td></td>
<td>below 300°C in Ar atmosphere</td>
</tr>
<tr>
<td></td>
<td></td>
<td>below 180°C in ambient atmosphere</td>
</tr>
</tbody>
</table>

Table B.4: Chemical vapor deposition process.
B. Fabrication parameter
Gluing v-groove cavities

The procedure to glue our FFPCs with the help of v-groove chips is explained in great detail in Ref. [77]. In the following a few additional details on the gluing process are given.

Piezo cables

We use miniature coaxial cables (Axon PCX38K10AK) for the cavity piezos. We prepare a 60 cm long cable for the SM fiber piezo and a 50 cm long cable for the MM fiber piezo. The settings for the stripper tool are: 0.3 mm for the jacket and 0.1 mm for the conductor. For the connections to the setup we use IC pins. We solder the ground to the cable shield, the high voltage is sent through the inner conductor. We isolate the cable at the IC pins with stycast and mark the polarity of the connector.

Cavity mount

We start with a quartz plate with dimensions 4 × 3 × 25 mm. Note that the plates are cut from a 4 mm thick quartz piece to obtain smooth surfaces on top and on the bottom to ensure that the cavity is well aligned to the mount.

The cavity piezos are PIC 255 shear piezo elements from PI Ceramics with dimensions 5 × 5 × 1 mm$^3$. A groove on the piezo surface marks the polarization direction. We place the piezos with opposing travel directions (grooves pointing outwards) and align them using the Aluminum lever. The gap between the two piezos should be around 2 mm. Now we center the quartz plate over the piezos and glue it with the fast curing UV glue (Dymax OP 4-20632). We cure the UV glue with a LED (LedEngin Inc LZ4-44UV00-0000) driven by a constant current LED driver (Mean Well LPC-20-700). At 700 mA the LED outputs 2 W at a center wavelength of 365 nm. The UV LED is displayed in Fig. C.1(a). For permanent joints, we use exposure times of at least 30 s to cure the glue.

We solder the piezo cables to the piezo elements. Ground is connected to the top electrodes to shield the sample and the cavity from the high voltage. A drop of glue serves as strain reliefs for the piezo cables.
Finally, we glue a glass sheet on top of the SM piezo. The glass sheet has an approximate dimension of $3 \times 4 \times 0.15$ mm and enables angular alignment of the second fiber by imposing a height difference between the two mounting points. The finished mount with the piezos and the cables is shown in Fig. C.1(b).

**Figure C.1:** (a) Photograph of the UV LED mounted on a black heat sink. (b) Photograph of the finished cavity mount with soldered piezo cables.

**V-grooves**

The V-groove chips are commercial Si v-grooves with 16 channels, 250µm center-to-center spacing and dimensions $5.25 \times 12 \times 0.525$ mm from Mizur Technology. We remove the strain relief area and cleave the remaining area in four parts. Under gentle pressure with a sharp blade, the chip breaks along the grooves. We make sure, that the fiber is mounted in the second undamaged groove from the edge of the chip. Figure C.2 shows a micrograph of a v-groove that is being aligned and glued to the SM fiber piezo.

**Fiber rotation**

Cleaving the fibers results in small angles of the cleaved surfaces relative to the fiber core. In addition with imperfect centering in the shooting process, this results in a degeneration of the two polarization modes of the cavity. The frequency splitting of those two polarization modes depends on the relative angle of the two mirrors with respect to each other [128]. Therefore, rotating the fibers allows to align the two mirror surfaces and to tune the frequency splitting. To implement this in the current gluing process, it is convenient to first glue the SM fiber. Afterwards, the rotation of the MM fiber is aligned before it is glued to the second v-groove chip. This adds another cavity alignment step and only allows coarse alignment of the fiber rotation. There might be a better way to implement the fiber rotation alignment in our gluing process.

**Glue selection**

We glue the second fiber with a slow curing epoxy to be able to realign the cavity while the glue shrinks during hardening. For lab temperatures well above $20^\circ$C we made very good experience
with Epo-Tek 301/B4 two-component optical adhesive. It is vacuum compatible and shows very little shrinkage during the curing process. Additionally, the resist cures during 24h allowing to gradually compensate for small drift. Thus we achieve very good alignments of the final cavities. Due to too low temperatures in our lab, we observed curing times of up to 72h.

For that reason, we switched to using Stycast 2850 FT with catalyst 24LV for the slow curing joints. Curing times were greatly reduced, with the main shrinkage happening around 6 – 10h in the curing process. The resulting epoxy quality critically depends on the the mixing ratio between the two components.
C. Gluing v-groove cavities
Appendix D

External cavity diode laser

The 780nm laser used in the setup is a homebuilt external diode cavity laser (ECDL) in Littrow configuration as described in Ref. [105]. This laser is a cheap and easy platform the be used in different configurations: The wavelength and the output power can be chosen by the choice of the laser diode (LD). A feedback piezo allows to tune the frequency by some tens of GHz and a bias-tee allows for direct modulation of the diode current and can be used e.g. for sideband modulation.

In the following the setup, calibration and characterization of the laser is described in detail. Since all the drawings are available, most of the parts can be manufactured in the workshops of the university.

D.1 Mechanical design

The mechanical drawing on the following page gives an overview of the mechanical design. The base block is made from aluminium. On the base block a peltier is mounted to allow for temperature control of the laser. On top of the peltier, there is laser base plate for the LD and the grating. The grating holder can be bent open with a piezo actuator and a micrometer screw. With another screw it is fixed to the laser base plate and allows coarse adjustment of the feedback angle. The LD is mounted inside a Thorlabs SM05 tube. A collimation lens is mounted in front of the diode to collimate the laser beam. The tube is fixed in the collimator holder.

The whole laser housing is closed with acrylic glass to avoid air currents and the prevent dust from entering the laser.

D.2 Electrical design

The laser head features a small PCB that operates the laser. A schematic of the individual branches of the laserhead PCB is displayed in Fig. D.2. It features a modulation input and two D-sub inputs to control the diode current and the diode temperature. A security circuit allows to switch off the laser head in a controlled way.
D.3 Laser head components

In the following, we summarize all the components that are needed for the laser head. Specs are given as far as they are available.

**Peltier** We use a Laird Technologies Peltier element (CP1.4-127-06L-RTV51.4W) (40 × 40mm) from RS-Components. It provides sufficient power even at lower voltages and low currents. The maximum output voltage of the controller operational amplifiers is limited to around 13.5 V.

**Piezo Actuator** The piezo actuator for the grating scan is ordered from Piezomechanik GmbH, the part number is PST150/4/7.

**Grating** The grating is a holographic sinus grating from Edmund Optics. It features 1800 grooves/mm and is designed for VIS wavelengths.

**Laser Diode** We use TO can laser diodes. We use laser diodes at 785 nm and tuned them down in wavelength to match the Rb transition. To enhance the lifetime of the diode, we use it close to operating current. We use either L785P090 diodes from Thorlabs or QL78M6SA from Roithner (cheaper). Since AR coated diodes are very expensive, we remove the diode can (with Thorlabs WR1 can opener) to avoid reflections from the can window. This greatly improves the feedback alignment and should improve the feedback quality.

**Collimation Lens** CM230TM-B from Thorlabs.
D.4 Installation

Now we discuss the assembly of the laser, the installation of the laser diode, the alignment of the grating and finally the wavelength tuning of the laser is described.

**Figure D.2:** Schematics of the laserhead. Top: The AC branch consists of a bias-tee that allows to modulate the diode current. The DC branch allows to operate the laser diode with a constant current. A voltage sense allows to measure the current flowing through the laser diode. A thermistor measures the diode temperature and a peltier allows to control the temperature.
D.4.1 Installing the laser diode

All the parts of the laserhead are assembled as shown in the drawings. Permanent connections are glued with epoxy. The Peltier and the thermistor are soldered to the laser head PCB. The Laser Controller is powered with a ±15V power supply (1 A max current minimum!). It is connected to the Arduino and plugged to the laser head.

To check the polarity of the pin on the laserhead PCB, we select a small pumping current with the controller and measure the voltages on the PCB pins. The LD Anode/Cathode should measure the maximum voltage set by the controller (around 2.8 V). The sign can be adjusted with the jumpers on the PCB board (and the pos/neg switch on the Toptica controller). Some laser diodes feature an integrated photodiode to directly measure the diode output power. This can be used to stabilize the light power. This integrated photodiode cannot be used with the homebuilt controller!

Once we are sure the polarity matches the diode, we solder the LD to the laser head PCB. We have to make sure to mount the diode inside the tube first before soldering it. Now we power the diode until we see the output laser beam. We remove the grating and make sure the beam leaves the laser head box. We adjust the collimation lens to collimate the laser beam. Afterwards we reinstall the grating.

D.4.2 Feedback alignment

To align the feedback, the zeroth order of the grating is fed back to the diode while the first order is used as output beam. Two angles can be aligned with the two screws on the grating mount. First the piezo is scanned with a sine wave with ±10 V at a slow frequency. We use a scanning frequency of 1 Hz to be able to observe a flashing by eye. The coarse alignment of the grating is done before fixing the grating to the mount. With a detector card the feedback is centered onto the laser diode. Once the coarse alignment is done and the grating is screwed, the pump current is reduced close to the threshold. Now the fine alignment can be done with the mm screws. A good alignment can be seen by a “flashing” of the output beam. The flashing is maximized, then the current is reduced and the procedure is repeated.

For the final alignment, the laser threshold is clearly reduced and the output power is reduced while the linewidth is drastically improved. Exemplary data is shown in Fig. D.3(a). Note that the mm screw for the wavelength tuning has to be tight enough to fix the tuning piezo, otherwise the piezo will oscillate between the screw and the backside of the grating. On the other hand the screw should not be too tight (maximum preload on the piezo should not be exceeded).

D.4.3 Absorption spectroscopy on Rubidium

We lock the laser wavelength to a Rubidium (Rb) transition in a Doppler-free absorption spectroscopy scheme. A bias-tee in the laser head allows to frequency modulate the diode current. A small portion of the laser beam is split off and passes a Rb vapour cell in a Michelson type configuration. We align the incident and the reflected beam to overlap inside the vapour cell. We measure the light that passes the Rb cell twice on a fast photo-detector and demodulate the signal in a homebuilt lock-in amplifier. The modulation frequency is around 100 MHz. A slow photodetector records the absorption spectrum, while we scan the laser wavelength with the grating piezo around a transition in Rb. The resulting absorption
Figure D.3: (a) Laser output beam power for varying pump currents. Measured values are marked with circles. A linear fit allows to extract the laser threshold. The black line shows the bare bare laser diode, the blue line corresponds to the laser with the grating aligned. (b) Absorption spectroscopy of Rb. Under a sweep of the grating piezo, several absorption lines are visible (black line). The demodulated signal is plotted in blue.

spectrum is shown in Fig. D.3 (b) together with the lock-in output. We use the lock-in signal as a feedback to stabilize the laser wavelength.

D.5 Laser Controller

In the course of this work, we use a commercial controller (Toptica DC 110). To replicate the laser, another controller was designed by Alex with support of the electronic service center. The schematic of this controller is presented in Fig. D.4.

The controller is operated by an Arduino Due. The arduino enables the outputs with digital HIGH/LOW signals (3.3V) on the corresponding E/S pins. A DAC signal controls the voltage send to the diode and the peltier respectively. The temperature is read from a thermistor, the diode current can be extracted from two voltage measurements. Both DAC outputs are offset and amplified to the desired voltage range on the controller PCB. The controller is powered with a ±5 V and ±15 V DC power supply and draws below 1 A for a 100 mA laser diode.

The Arduino program implements two PIDs for the temperature and for the diode current. The laser controller can be remote controlled via serial connection. The code of the Arduino program is presented in Secs. D.5 and D.5.

Both the laser head electronics and the laser controller can be ordered from the electronic service center. The come populated and tested. The wiring is done with two non-twisted D-Sub 9 cables (female-female for TEC, male-male for LD). Pay attention to right polarity of the cables!
Figure D.4: Schematics of the laser controller.
Arduino Program for the Current Control

/* *************************************************/
/* Control the current sent to the laser diode
/* Current is calculated from voltage difference at Pins
/* A3 and A2. A voltage feedback is sent with DAC1
/* *************************************************/

// Define Variables we’ll be connecting to
float I1, I2, I3, I4, Setpoint = 0.07;
float e2 = 0, e1 = 0, e0 = 0, u2 = 0, u1 = 0, u0 = 0;
float Kp = 1; // proportional gain
float Ki = 10; // integral gain
float N = 20; // filter coefficients
float Ts = 0.1; // This must match actual sampling time
float ku1, ku2, ke0, kel, ke2;

void setup()
{
    // initialize serial:
    Serial.begin(9600);
    Serial.println("Laser Diode Current Control");
    Serial.print("Diode Current \[mA\]: ");
    Serial.println(1000*Setpoint, 0);
    Serial.println("To change setpoint send via serial!");
    // use full DAC resolution
    analogReadResolution(12);
    analogWriteResolution(12);
    // Set laser head Cathode to 0V
    analogWrite(DAC1,2047);
    // Enable laser
    pinMode(48, OUTPUT);
    pinMode(50, OUTPUT);
    digitalWrite(48, LOW); // Laser current enabled
digitalWrite(50, HIGH); // Control LED enabled
    // Set PID gain and calculate parameters needed
    // https://www.scilab.org/discrete-time-pid-
    // controller-implementation
    ku1 = -(2 + N*Ts) / (1+N*Ts);
    ku2 = 2/(1+N*Ts);
    ke0 = (Kp*(1+N*Ts) + Ki*Ts*(1+N*Ts)) / (1+N*Ts);
    kel = -(Kp*(2+N*Ts) + Ki*Ts) / (1+N*Ts);
    ke2 = (Kp) / (1+N*Ts);
}
void loop()
{
    if (Serial.available() > 0) {
        Setpoint = (float)Serial.read();
        Serial.print("Diode Current [mA]: ");
        Serial.println(1000*Setpoint, 0);
    }
    e2=e1; e1=e0; u2=u1; u1=u0; // update variables
    // read plant output
    I1 = (float)(analogRead(A2)) - float(analogRead(A3))*3.3/4095;
    I2 = (float)(analogRead(A2)) - float(analogRead(A3))*3.3/4095;
    I3 = (float)(analogRead(A2)) - float(analogRead(A3))*3.3/4095;
    I4 = (float)(analogRead(A2)) - float(analogRead(A3))*3.3/4095;
    e0 = Setpoint - (I1+I2+I3+I4)/4; // compute new error
    u0 = -ku1*u1 - ku2*u2 + ke0*e0 + ke1*e1 + ke2*e2 + 2047;
    if (u0 > 4095) u0 = 4095; // limit to DAC or PWM range
    if (u0 < 0) u0 = 0;
    analogWrite(DAC1,u0);
    delay(1000);
}

Arduino Program for the Temperature Control

/*********************************************************
* Control the thermoelectric cooling of the laser diode
* Reading A1 to control analog DAC 0
***********************************************************/

// Define Variables we’ll be connecting to
float TAct, TSet = 23.0;
float e2 = 0, e1 = 0, e0 = 0, u2 = 0, u1 = 0, u0 = 0;
float Kp = 20; // proportional gain
float Ki = 1; // integral gain
float Kd = 1; // derivative gain
float N = 1; // filter coefficients
float Ts = 0.01; // This must match actual sampling time
float ku1, ku2, ke0, ke1, ke2;

void setup()
{
    // initialize serial:
    Serial.begin(115200);
    Serial.println("Laser Diode Temperature Control");
    Serial.print("Diode Temperature [°C]: ");
}
Serial.println(TSet, 1);
// use full DAC resolution
analogReadResolution(12);
analogWriteResolution(12);
// Set laser head Cathode to 0V
analogWrite(DAC1, 0); delay(100);
// Set Peltier voltage to 0V
analogWrite(DAC0, 2047);
// Enable laser
pinMode(48, OUTPUT);
pinMode(50, OUTPUT);
// pinMode(52, OUTPUT);
digitalWrite(48, LOW); // Laser current enabled
digitalWrite(50, LOW); // Peltier enabled
// digitalWrite(52, HIGH); // Control LED enabled
// Set PID gain and calculate parameters needed
// https://www.scilab.org/discrete-time-pid-controller-implementation
ku1 = (2 + N*Ts) / (1 + N*Ts);
ku2 = 1 / (1 + N*Ts);
ke0 = (Kp*(1+N*Ts) + Ki*Ts*(1+N*Ts) + Kd*N) / (1+N*Ts);
ke1 = (Kp*(2+N*Ts) + Ki*Ts + 2*Kd*N) / (1+N*Ts);
ke2 = (Kp+Kd*N) / (1+N*Ts);
e0 = -TSet + readTemperature();
e1 = -TSet + readTemperature();
}

void loop()
{
e2=e1; e1=e0; u2=u1; u1=u0; // update variables
// read plant output
TAct = readTemperature();
e0 = -TSet + TAct; // compute new error
u0 = ku1*u1 - ku2*u2 + ke0*e0 - ke1*e1 + ke2*e2; // eq (12)
if (u0 > 2047) u0 = 2047; // limit DAC range
if (u0 < -2047) u0 = -2047;
analogWrite(DAC0, u0+2047);
analogWrite(DAC0, 0);
analogWrite(DAC1, TAct*1000);
// Serial.println(TAct);
// Serial.println(e0);
// Serial.println(u0);
// Serial.println(u0);
// delay(1000);
}
```c
float readTemperature() {
    int bitNTC = 0; // measured voltage
    int beta = 3988; // NTC Beta from datasheet (25°C)
    float rNTC = 0; // calculated NTC resistance
    float T = 0; // calculated temperature [°C]
    bitNTC = (analogRead(A1)+analogRead(A1)+analogRead(A1)+analogRead(A1))/4;
    rNTC = 10000*(((double)bitNTC/4098)/((1-((double)bitNTC/4098)));
    T = 1/(1/298.15)+((double)1/beta)*log((double)rNTC/10000)-273.15;
    return T;
}
```
Homebuilt electronics

E.1 Photodetectors

For the measurements we need photodetectors that are sufficient fast to measure even higher order nanomechanical modes around up to tens of MHz as well as sideband for PDH error signal generation. On the other hand a high gain is desirable to be able to operate the system with lower photon numbers. Additionally the added noise should be minimal. Since Stefan Manus (HF technician from the Kothaus chair at LMU) designed and fabricated photodetectors that meet all the requirements mentioned before, we reverse engineered and adapted those detectors to work in the NIR regime.

The detectors provide two outputs. A DC coupled monitor output features a bandwidth of around 1MHz and variable gain from 1000V/W up to around 1000000V/W. The fast AC coupled output has a bandwidth of 30MHz and a gain of around 300000V/W. We use a Hamamatsu photodiode with a ball lens to enable efficient fiber coupling. For the VIS range photodetectors we use a S5973-01 Si based photodiode, for the NIR photodetectors we use G6854-01 InGaAs diode. The diode is biased in reverse. The absorption of photons will give rise to a photocurrent which is converted into a voltage in the transimpedance amplification stage that is shared between the two output lines. The feedback capacitor has to be carefully matched to the parasitic capacitance of the diode and the PCB. This is done with a variable capacitor (C5 in the schematic on the next page). After the first stage the signal is split.

The AC path is capacitively coupled with C4. In the second amplification stage the offset can be adjusted with the potentiometer R15. The following LC circuit acts as a Butterworth lowpass filter of fith order with values tuned to obtain a 3dB frequency of 50Mhz. The impedances are 100Ohms for sake of noise performance. The last stage amplifies the signal to use the full voltage range of ±5V.

The DC path consists of three amplification stages. The first stage sets the offset (R35) and allows to tune the gain over several orders of magnitude. Again the two subsequent stages amplify the output signal to the voltage range of ±15V.

Since the photodetectors don’t have a lot of voltage regulation on board, the supply voltages have to be rather clean. Therefore it is convinient to use the power supplies with the three pin Fischer connectors.
The detectors come in anodized boxes. When mounting one has to make sure the cases are galvanically separated from the table ground. The photodiodes are centered under the SM05 hole on the front side of the detectors. Thorlabs FC/APC adaptors can be screwed into the SM05 threading to allow for efficient fiber coupling. To optimize the coupling, a fiber is connected. Then the box is opened, the screws that hold the PCB are loosened and the PCB position is adjusted to maximize the detector signal. The electrical schematic is attached in the following Fig. E.1.

### E.2 HV amplifier

We designed a ultra-stable high voltage amplifier inspired by the design in Ref. [129]. The schematics is shown in Fig. E.2. We reduce noise by separating the DC HV offset from faster components, e.g. cavity length scans or cavity feedback. The DC offset input is amplified in a ultra-low noise precision amplifier AD8429ARZ. A subsequent lowpass filter stage with corner frequency of 1.6Hz stabilizes the output. In a second amplification stage, it is combined with the modulation signal. It beats the noise performance of the HV amplifier used in this work but its output is limited to 0 – 250 V. Unfortunately, this is not enough to bring the cavity to resonance with the 1550nm laser lines. If one carefully controls the cavity length during the gluing process, this improved HV amplifier can be used in the experiments. This should greatly improve the quality of the cavity lock.
Figure E.1: Schematics of the homebuilt photodetector.
Figure E.2: Schematics of the high voltage amplifier.
Appendix F

Notation dissipative optomechanics

Here we show that our notation is equivalent to the notation in Ref. [88]. We write the effective mechanical resonance frequency as $\Omega_{m,\text{eff}} = \sqrt{\Omega_0^2 + \delta(\Omega^2)}$ with

$$\delta(\Omega^2) = 2\Omega_0 g^2 \left[ \frac{\Omega_0 - \Delta}{(\Omega_0 + \Delta)^2 + \left(\frac{\kappa}{2}\right)^2} - \frac{\Omega_0 - \Delta}{(\Omega_0 - \Delta)^2 + \left(\frac{\kappa}{2}\right)^2} \right]$$

$$- \Omega_0 g g_{\text{eff}} \kappa \left[ \frac{1}{(\Omega_0 + \Delta)^2 + \left(\frac{\kappa}{2}\right)^2} + \frac{1}{(\Omega_0 - \Delta)^2 + \left(\frac{\kappa}{2}\right)^2} \right]$$

$$= \delta(\Omega^2)_{\text{dispersive}} + \delta(\Omega^2)_{\text{dissipative}}.$$
We show that the two terms match the formula given in the paper. For the dispersive $\delta (\Omega^2)_{\text{dispersive}}$ term we obtain:

$$\delta (\Omega^2)_{\text{dispersive}} = 2\Omega_0 g^2 \frac{(\Omega_0 + \Delta)[(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2] - (\Omega_0 + \Delta)[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= 2\Omega_0 g^2 \frac{\Omega_0^2 - 2\Omega_0 \Delta + \Delta^2 + \left(\frac{\omega}{2}\right)^2 - \Omega_0^2 - 2\Omega_0 \Delta - \Delta^2 - \left(\frac{\omega}{2}\right)^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$+ \frac{\Delta \Omega_0^2 - 2\Omega_0 \Delta + \Delta^2 + \left(\frac{\omega}{2}\right)^2 + \Omega_0^2 + 2\Omega_0 \Delta + \Delta^2 + \left(\frac{\omega}{2}\right)^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= 2\Omega_0 g^2 \frac{-4\Omega_0^2 \Delta + 2\Delta \Omega_0^2 + 2\Delta^2 + 2\Delta \left(\frac{\omega}{2}\right)^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= 2\Omega_0 g^2 \frac{2\Delta \left(\frac{\omega}{2}\right)^2 - \Omega_0^2 + \Delta^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= \Omega_0 (2g)^2 \frac{\Delta \left(\frac{\omega}{2}\right)^2 - \Omega_0^2 + \Delta^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}.$$

With $G \equiv -2g$ and $\kappa T(q) \equiv \frac{\omega}{2}$, evaluated at $\omega = \Omega_0$ this matches the formula given in \cite{88}. Note that the detuning is defined with opposite sign in the paper. For the dissipative term $\delta (\Omega^2)_{\text{dissipative}}$ we arrive at:

$$\delta (\Omega^2)_{\text{dissipative}} = -\Omega_0 g g \kappa \frac{[(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2] + [(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= -\Omega_0 g g \kappa \frac{\Omega_0^2 - 2\Omega_0 \Delta + \Delta^2 + \left(\frac{\omega}{2}\right)^2 + \Omega_0^2 + 2\Omega_0 \Delta + \Delta^2 + \left(\frac{\omega}{2}\right)^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= -\Omega_0 g g \kappa \frac{2\Omega_0^2 + \Delta^2 + \left(\frac{\omega}{2}\right)^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}$$

$$= -\Omega_0 (2g)(2g \kappa) \frac{\kappa}{2} \frac{\left(\frac{\omega}{2}\right)^2 + \Omega_0^2 + \Delta^2}{[(\Omega_0 + \Delta)^2 + \left(\frac{\omega}{2}\right)^2][(\Omega_0 - \Delta)^2 + \left(\frac{\omega}{2}\right)^2]}.$$

Here in the paper they use $\Gamma \equiv 2g \kappa$. Again this part matches our formula.
Similarly the optical damping is expanded by a dissipative term

\[ \Gamma_{\text{opt}} = \Gamma_{\text{opt, dispersive}} + \Gamma_{\text{opt, dissipative}} \]

\[ = g^2 \kappa \left[ \frac{1}{(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2} - \frac{1}{(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2} \right] 
- 2gg_k \left[ \frac{\Omega_0 - \Delta}{(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2} + \frac{\Omega_0 + \Delta}{(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2} \right]. \]

For the dispersive term:

\[ \Gamma_{\text{opt, dispersive}} \]

\[ = g^2 \kappa \left[ \frac{[\Omega_0 - \Delta]^2 + \left( \frac{\Omega}{2} \right)^2 - [\Omega_0 + \Delta]^2 + \left( \frac{\Omega}{2} \right)^2}{[(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2] \cdot [(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2]} \right] 
- 2gg \kappa \left[ \frac{\Omega_0 - \Delta}{[(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2] \cdot [(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2]} \right] 
+ \frac{\Omega_0}{[(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2] \cdot [(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2]} \right] 
- 2gg_k \Omega_0 \left[ \frac{2\Delta + 2\Delta^2 + 2 \left( \frac{\Omega}{2} \right)^2 - 4\Delta^2}{[(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2] \cdot [(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2]} \right] 
- \frac{[\Omega_0 + \Delta]^2 + \left( \frac{\Omega}{2} \right)^2 - \Delta^2}{[(\Omega_0 + \Delta)^2 + \left( \frac{\Omega}{2} \right)^2] \cdot [(\Omega_0 - \Delta)^2 + \left( \frac{\Omega}{2} \right)^2]} \right]. \]
List of publications


Vielen Dank

Hinfln. Aufstehen. Kroner. Weitergehen. [sic]

—P. B.

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Bibliography


