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Three Roles of Empirical Information in Philosophy: Intuitions on Mathematics do Not Come for Free

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Abstract: This work gives a new argument for 'Empirical Philosophy of Mathematical Practice'. It analyses different modalities on how empirical information can influence philosophical endeavours. We evoke the classical dichotomy between "armchair" philosophy and empirical/experimental philosophy, and claim that the latter should in turn be subdivided in three distinct styles: *Apostate speculator*, *Informed analyst*, and *Freeway explorer*. This is a shift of focus from the source of the information towards its use by philosophers. We present several examples from philosophy of mind/science and ethics on one side and a case study from philosophy of mathematics on the other. We argue that empirically informed philosophy of mathematics is different from the rest in a way that encourages a *Freeway explorer* approach, because intuitions about mathematical objects are often unavailable for non-mathematicians (since they are sometimes hard to grasp even for mathematicians). This consideration is supported by a case study in set theory.

Keywords: empirically informed philosophy, philosophy of mathematical practice, empirical philosophy of mind, empirical ethics, large cardinals, study of cultures and practices

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1 Introduction

Contemporary philosophy employs both armchair and empirically informed approaches. In this paper, we look at different possibilities of how empirical work or results can affect philosophy. We will contrast two different clusters of areas of empirical philosophy; namely, on one side, philosophy of mind, philosophy of science and ethics, and on the other, philosophy of mathematics (or philosophy of mathematical practices).

We will identify three different basic types of empirical philosophy and discuss how three examples of empirically informed philosophy fit that classification. Then, we will use the classification to show what makes empirically informed philosophy of mathematics different from other types of empirical philosophy via a case study. Finally, we will argue that this fundamental difference is key to adopting a *wider empiricism* position for philosophy of mathematics. The main reason for this is that empirical investigation cannot directly aim at mathematical content (in contrast to e.g. our cognitive apparatus). Thus, we are bound to reflect based on the practices dealing with the realm of mathematics, i.e. studying actual mathematical practices.

Our argument in a nutshell goes as follows: Philosophical investigations of a domain require some kind of familiarity with it. As we do not have direct access to mathematical objects, we need to study the actual practices of dealing with them.

Before we will build up our argument in more detail, we will propose a new categorization of empirically informed philosophy that focuses on the use of empirical information. In the literature, there are some categorizations discussed. While the armchair versus empirically informed philosophy and the *narrow* versus *wider empiricism* categorizations consider the whole area of philosophy, we aim at establishing workable categories for just the area of empirically informed philosophy. First, we will review one prominent suggestion that categorizes empirically informed philosophy according to the sources of empirical information, and second, we will establish a new categorization that, we argue, is better suited to analyse the use of empirical information in different areas of philosophy. In a third part, we will introduce the differentiation between *narrow* and *wider empiricism* and connect it to our new categorization.

1.1 Sources of Empirical Information

One well-known categorization by Prinz (2008) was further developed by Löwe (2016): it evokes a nice metaphor with tailoring. According to this idea, philosophers have three different options how to get empirical information:

1. **Ready to wear-style**: by looking into the literature for empirical results that are relevant to their endeavour/analogous to going into the mall and taking a suit off the rack.
2. **Customization**: by requesting custom-made results/analogous to going to a tailor who would tailor them a custom-made suit.
3. **Tailoring themselves**: by doing the empirical work themselves/analogous to tailoring the suit themselves.

The first option is close to what is within acceptable boundaries to a lot of people in the armchair community. As the upcoming section on philosophy of mind shows, philosophers are often willing to use empirical information from psychology and neuroscience. We can look at existing scientific literature and decide which results are important to us. If we cannot find something that fits, we can do the work ourselves (third option), which is excluded from the methodology of armchair philosophy. But this makes it necessary to get trained in adequate methodology that may be foreign to philosophers. A more resourceful option would be to divide labour. But here (second option), it remains to find problems and questions that are important for both communities, for an empirically working scientist would need a reason to collect the data that philosophers would be interested in. While their non-philosophically motivated data may be useful to address philosophical questions, the "suit" may not fit the philosopher's size. This idea is very influential in the philosophy of mathematical practice, especially in what Carter (2019) calls in her recent overview of the discipline "empirical philosophy of mathematics". It is a tradition which substantially profits from a series of conferences in Brussels, see f.i. (François and van Bendegem 2007). As Carter observes, this branch of philosophy is closely related to mathematics education, where the processes by which students learn mathematics are studied. Recently, one of the most prestigious journals in that field included an introduction to philosophy of mathematics for mathematics educators, see Hamami and Morris (2020).

1.2 Uses of Empirical Information: A New Categorization

Löwe's distinction addresses the different sources from which philosophers may get empirical information that is relevant to their work. While this opens the door to important practical debates,[1] we focus on the *use* that philosophy makes of empirical information, because this is more relevant for the current dichotomy

1 One promising debate may be to reflect on the question how such interdisciplinary work could be organised in a way that is fruitful for both disciplines (philosophy and empirical sciences).

between *narrow empiricism* and *wider empiricism* (see below). For this aim, we propose two distinctions that capture different ways in which philosophers can use empirical information.

The first distinction discriminates between empirically informed philosophy that answers classical philosophical questions on the one side and empirically informed philosophy that aims to understand the practice of a field on the other. The subject matter is different. A related, partially overlapping distinction can be made between empirically informed philosophy that presupposes an onto-epistemic frame (or features broad philosophical intuitions) and empirically informed philosophy that does not (the frame arises *a posteriori* for the philosopher; it is developed during the empirical scrutiny of practices).

- **Distinction 1:** *solving philosophical issues* or *describing and understanding a field*
- **Distinction 2:** *presupposing/anticipating an onto-epistemic frame (which includes the objects of analysis*) or *accepting the onto-epistemic frame of what is empirically investigated (e.g. information as is provided by scientists or other actors, or scientific practices)*

Together, these distinctions allow us to discriminate between three types of empirically informed philosophy. Empirical philosophy that sets out to solve already formulated philosophical dilemmas necessarily features onto-epistemic frames in advance: the terms of the debate are set, the ontologies are defined, and what would count as evidence for or against a given point is largely predetermined. Therefore, the first side of Distinction 1 (solving philosophical issues) is not compatible with the second side of Distinction 2 (finding the entity of analysis in the empirical investigation), and we obtain the following three types of empirically informed philosophy:

1) **Apostate speculator**: Addressing philosophical questions via empirical information framed by pre-established epistemologies and ontologies/discursive entities
2) **Informed analyst**: Describing, studying or analytically investigating practices via a philosophical epistemic lens that includes pre-established ontologies/discursive entities
3) **Freeway explorer**: Describing, studying or analytically investigating practices mainly drawing from epistemologies and ontologies/discursive entities established in the empirical research

Since any analysis depends on previous theoretical baggage however minimal it is (see f.i. Chang et al. 2011; Pitt and Pitt 2011; Rittberg and van Kerkhove 2019), the freeway explorer will feature onto-epistemic frames as well. But there is a

difference of degree between the informed analyst and the freeway explorer: the latter works with minimal notions and is easily willing to accept the notions and objects of the practitioners that become apparent or emerge in the empirical analysis, while the former usually works with more philosophically sophisticated notions and ontologies, which they tend to cherish and therefore favour over alternatives. The notion of ontologies is understood broadly as including for example genes and sets, but also abstract concepts such as justice or the maximize principle put forward by Maddy (2007). There is an important difference between ontologies in the philosophy of mathematics and in the philosophy of science. The scientific intended domain often is found in the real world, and therefore more readily available than the mathematical intended domain. This is of course not clear cut, because for instance parts of physics are more rooted in the mathematical domain and parts of mathematics can be quite embodied in the real world, as for example in basic arithmetics. This difference is crucial regarding the philosopher's access to the respective intended domains.

Discriminating between these three types is critical because they deserve differential treatment regarding the *narrow empiricism* or *wider empiricism* dichotomy. Furthermore, it helps make visible the particularities of the third type, to which the philosophy of mathematical practice counts, as we will argue. The first two types constitute the bulk of empirically informed philosophy where the onto-epistemic frame is presupposed or anticipated by the philosophers. Hence, we have only few examples of philosophical investigations where the onto-epistemic frames are found in empirical research. Our case study in set theory will provide an example of such a case.

While these three categories are not exhaustive and may overlap, we will next justify that they are useful to understand and characterize empirical philosophy, and to address the distinction between *narrow empiricism* and *wider empiricism*.

1.3 The Effect of Empirical Information: *Narrow* and *Wider Empiricism*

Apart from the use that philosophy can find for empirical information, there is a debate on the *effect* that empirical information should have on philosophical knowledge: what kind of information overrides the other in situations of conflict? There is a distinction between *First Philosophy* and *Second Philosophy* established by Maddy (2007). Because Maddy proposes a very specific approach under the label of 'Second Philosophy' that is applied to philosophy of mathematics (especially set theory), we instead embrace the labels "narrow empiricism" and "wider empiricism", due to Cornelius Benjamin (1939). We use them in the following way:

Narrow empiricism, closely related to armchair philosophy, refers to the position that philosophical intuitions should have primacy over empirical information, while *wider empiricism* defends that, in case of discrepancy between the two, empirical information overrides previous philosophical intuitions. Cornelius Benjamin (1939), while presenting an early notion of empirical philosophy (more encompassing than current conceptions), uses these terms, acknowledges the pros and cons of each position and situates them in a continuum:

> The positivist ends by being simple-minded; the metaphysician, by being muddle-headed. Narrow empiricism, while not positivistic in the extreme form, prefers simple-mindedness to muddle-headedness; wider empiricism, while not allying itself with speculative metaphysics, prefers muddle-headedness to simple-mindedness. We have, therefore, a continuum of positions: strict positivism, narrow empiricism, wider empiricism, speculative metaphysics. The problem of empiricism, it seems to me, is to specify more precisely the nature of this serial arrangement of attitudes. Apart from such considerations, to say that one is an empiricist is like saying that a stick is long or an object is heavy; specifications of how much are imperative (Cornelius Benjamin 1939, p. 520).

We believe that the kind of use made from empirical information is crucial to determine whether a *narrow empiricism* or *wider empiricism* approach is pertinent. As we will see in the upcoming section, the **apostate speculator** or **informed analyst** types admit both approaches in interesting ways. Therefore, deciding between the two might be a task with no straightforward rules. Both extremes seem to have crucial shortcomings: dismissing strong philosophical intuitions in favour of dubious empirical results naively errs on the side of *narrow empiricism*, towards strict positivism, while ignoring strong and systematic empirical evidence in favour of armchair theorizing is succumbing to blind speculation. Ultimately, this may depend on case-by-case local judgement. We agree with Benjamin that there is a continuum which the "strict positivism" versus "speculative metaphysics" (and perhaps *First Philosophy* vs *Second Philosophy*) dichotomy cannot capture. Below we provide examples on how to judge specific cases in the philosophy of mind, science and in ethics.

The **freeway explorer** type, on the other hand, leans towards wider empiricism. The **freeway explorer**, after all, does not start from invasive philosophical intuitions or ontological frameworks in their analysis of practices, and instead takes for granted what the practitioners claim about their subject matter. Without tension between preceding philosophical baggage and empirical content, the **freeway explorer** does not resort to philosophical speculation but collects what they find. For instance, philosophical analysis on an ontological entity cannot precede the entity itself, and the **freeway explorer** works with ontologies elucidated during the empirical investigation. The case study in set theory in Section 3.2.1 will support this claim.

1.4 Our Argument in a Nutshell

There is an ongoing debate about the right methodology for Philosophy of Mathematical Practice(s), see f.i. (Rittberg 2019) or the many programs mentioned by Carter (2019). One can see a parallelism between the development of PMP and certain events in Philosophy of Science. Kuhn (1962) is credited with being a pioneer of such events or at least with popularizing ideas that we can find f.i. as well in Fleck (1935) with his *The Structure of Scientific Revolutions*, endorsing a perspective on the sciences that stress the irrational component of scientific practice. Theory choice or Paradigm Shifts seemed not to fit purely rational proceedings. These developments yielded many important pieces of Philosophy of Science, some with an institutional backbone like the Society of Philosophy of Science in Practice (SPSP). One anonymous reviewer mentioned a non-exhaustive list of authors which belong in this tradition: Bruno Latour, Nancy Nercessian, Joe Rouse, Karen Barad, Hans-Jörg Greiffenhagen, among others. While we are very sympathetic for these more anthropogenic, ethnomethodological, or in our words freeway explorer-ish endeavours, one can investigate whether there is a point where the parallelism between PMP and Philosophy of Science does not apply. Philosophers could argue that whenever they want to talk about the real world the exact practice of those researching the real world is not relevant for them. Social-constructivists could not argue that, but realism is more mainstream. Philosophy of mathematics presents a very different scenario. Here even the realist must see that mathematical entities (again without a strict metaphysical meaning of entities) are not accessible in the sense that the real world is: Mathematical entities remain abstract. Thus the main point of the paper – besides the categorization – is: Philosophers of science, philosophers of mind etc. *can* go for all three approaches, especially for the apostate speculator and informed analyst types. But philosophers of mathematics *need* to go for a freeway explorer type approach as much as possible.

2 Examples from Mainstream Empirically Informed Philosophical Fields

In this section, we describe prominent examples of empirically informed philosophy and show how they fit either the **apostate speculator** or the **informed analyst** types, hardly admitting the **freeway explorer** approach.

2.1 Empirically Informed Philosophy of Mind

We first look at empirical philosophy of mind that we identify mainly with an **apostate speculator** type.

Due to the abstract conceptualization of the mind, its philosophical study has traditionally relied heavily on speculative methods rather than what today would be considered appropriate empirical investigation by modern scientific tenets.[2] A classic example is Descartes' rationalist approach to the mind-brain problem (insofar as the proposition is not founded on empirical findings), and plenty of modern examples can be found in the form of thought experiments (the p-zombie argument (Campbell 1970; David 1996; Kirk 1974a, 1974b), Mary's room (Jackson 1982, 1986), and so forth, which if even some consider empirical in the sense that one's thought is observed, they consist on thought experiments rather than empirical observations of the world). And in fact, some philosophers defend that this is all that philosophy of mind can ever hope to achieve with some degree of reliability (Brandt 1967; Kim 1966). Tibbetts (1973) claims that, rather than an empirical issue, the mind-brain problem is a methodological issue. Bermúdez (2014) acknowledges that empirical information is critical in philosophy of psychology but irrelevant in philosophy of mind. Irvine et al. (2014) sustains that when a thesis in philosophy of mind is vague, general and/or tries to fit rather than predict empirical data, chances are that it cannot be tested empirically. Carreno and Pérez-Escobar (2019) argue that empirical information is useful to tackle practical issues and make clinical decisions based on preliminary notions on the character of the (abnormal) mind and its brain bases but does not settle metaphysical issues.

However, as psychology, cognitive science, and neuroscience have matured as disciplines, more and more philosophers of mind have drawn from empirical information to avail their premises. An extreme supporter of these proceedings is Foss (1987), who argues that the mind-brain problem is merely an empirical issue. As such, positions that entail metaphysical stances regarding the mind-brain problem (like materialism,[3] dualism,[4] emergentism,[5] functionalism,[6] idealism[7] and epiphenomenalism[8]), are open and must be subjected to empirical testing. In

2 This is meant as a generalization, rather than as the claim that there is a unified scientific worldview. For example, most experimental psychologists today would not accept introspection as a source of valid empirical information. Whether a method is speculative or not depends on more-or-less vague notions of empirical appropriateness held by different communities.

3 See Place (1960) and Paul Hellman and Thompson (1976).

4 See Margolis (1978, 1984).

5 See Bunge (1977, 1980), Popper and Eccles (1975), and Roger (1983).

6 See Fodor (1981) and Putnam (1960).

7 See Foss (1987, pp. 521–526).

8 See Jackson (1982).

this line or in more moderate grounds, some philosophers have appealed to and interpreted empirical findings in the sciences to underpin their arguments on the mind-brain problem.

For instance, let us consider eliminative materialism, the position that denies the existence of (most) folk-psychological mental states, such as beliefs and desires. Its proponents insist on paying attention to neuroscientific empirical findings and claim that no neuronal basis of folk-psychological mental states has been or will be found. Therefore, they hold that folk-psychological mental states do not exist in an ontologically strong sense.[9] For instance, Paul and Patricia Churchland dismiss propositional attitudes on these grounds (Churchland 1981, 1989).

Eliminative materialists dismiss introspection as a valid method to gather reliable empirical information, since it is influenced by folk-psychological notions themselves (Churchland 2013). Thus, empirical support of folk-psychological notions derived from introspection is eschewed by eliminative materialists. To defend this attitude, they may appeal to empirical information suggesting that introspection fails to provide appropriate empirical information (see Nisbett and Wilson 1977 for a critique of introspective methods). This is another pathway by which eliminative materialism is informed by empirical results.

However, such empirical support does not straightforwardly lead to the veracity of eliminative materialism, for several reasons. First, there is controversy regarding whether stances on the mind-brain issue are empirical matters at all, as we mentioned before. For example, it could be contended that it contains unfalsifiable premises. Second, as Foss (1985, 1987) notes, eliminative materialism not only appeals to current empirical information, but also to future empirical findings: neither current nor future neuroscience does/will support the existence of desires, beliefs and so forth. Therefore, this position is not only based on empirical information but also on aggressive induction. And third, empirical information can receive heterogeneous treatment and interpretation by different communities: some may consider introspection to be a better source of information than the measurement of neurophysiological parameters, some consider folk-psychological concepts warrant enough predictive power to deserve a more serious treatment (Lahav 1992), etc. But this issue does not undermine the structure we want to stress here. What is relevant is that some philosophers propose solutions to philosophical problems claiming that there is empirical evidence that backs up their accounts. There is a philosophical issue (the character of the interface mind-brain), empirical information that may shed some light

9 There are variations of eliminative materialism that do not tightly conform to this general description. This is a pertinent remark but there is no need to go further into distinct varieties for the sake of our point here.

on the issue (results from psychology, cognitive science, and neuroscience in the mind-brain interface), and a philosophical stance on the issue informed by such empirical information (eliminative materialism). This use of empirical information in philosophy is a clear example of the **apostate speculator** style.

2.2 Empirically Informed Philosophy of Science

Another area of philosophy that draws from empirical information is philosophy of science. There are three ways in which philosophers of science can make use of empirical information:

- Philosophers intending to ascertain the character of scientific theories, explanations, concepts, reasoning and so forth based on empirical data about scientific practices (**apostate speculator**)
- Philosophers intending to ascertain how scientists conceive of and work with scientific theories, explanations, concepts and so forth to check whether they adhere to "good scientific practices" (**informed analyst**, normative)
- Philosophers intending to ascertain how scientists conceive of and work with scientific theories, explanations, concepts and so forth to better understand the field, work out why and how certain theories succeed or fail in their own terms, contribute with clarifications, and so forth (**informed analyst**, descriptive)

Here, we present a concise example to illustrate this distinction. Machery et al. (2016) discusses work in experimental philosophy of science where the uses of concepts by philosophers, scientists and lay people are compared. He discusses empirical work on the scientific notion of "gene" (Stotz and Griffiths 2004; Stotz, Griffiths, and Knight 2004) and concludes that it is quite variable in meaning across scientific communities and different epistemological situations. He also points to empirical research on the notion of "innateness" (Griffiths, Machery, and Linquist 2009). In this case, a formal characterization of innateness in terms of sufficient and necessary features (fixity, typicality, and teleology) is put to the test by contrasting it to the actual conception of innateness by lay people.

In the case of "innateness", the authors propose to debunk previous philosophical views on the concept on the grounds that it lacks empirical validity. For this reason, it qualifies as the **apostate speculator** empirical philosophy. In the case of "gene", it is descriptive work of the **informed analyst** style.

However, it does not qualify as **freeway explorer**. Although there was not a philosophically rigorous notion of the object "gene" to be tested, more minimal or primitive intuitions were displayed (the word itself for example, and phenotypic

effects as essential properties of genes) to at least be able to conduct the analysis of practices. Therefore, the object preceded and conditioned the analysis of practices. For instance, a philosopher may study biological practices *in situ* with the aim to test their conception of "gene" and how it is relevant in their knowledge and techniques. However, if the philosopher found that the scientist call a cup of tea "gene", and spilled tea "gene expression", the philosopher is not likely to take the practices seriously, but instead they would dismiss them: "they are out of their minds", "they are pulling my leg so that I leave their lab".

2.3 Empirically Informed Ethics

The last empirically informed philosophical subdiscipline that we will discuss in this section is empirical ethics. Similar to the aforementioned case of empirically informed philosophy of science, empirical ethics may set up to solve philosophical questions (support or argue against consequentialism, intuitionism, pluralism, deontology …) and, with regard to practices, may be not only descriptive but also normative.

In the case of ethics, the latter point may sound less intuitive: how can information on the values, choices and behaviour of people lead to any insight about what values people ought to have, what choices they ought to make and how they ought to behave? This seems to violate the is-ought dichotomy. Nevertheless, new developments of empirical ethics challenge the strength of this distinction and propose further study of the relations between facts and norms (see for example the introduction of Christen et al. 2014). Such facts are not limited to those of descriptive ethics but also include empirical results from psychology, cognitive science, neuroscience, anthropology, and more (see Greene 2015) for an example. Kauppinen et al. (2014) therefore distinguishes two positions in ethics: "armchair traditionalism" – the view that empirical information is not relevant for the justification of normative theories – and "ethical empiricism" – the view that empirical information can be insightful for ethics.

We will briefly discuss a typical experimental approach in ethics: subjecting people to situations where they must make choices that reveal their moral character. Experimentally studying ethical intuitions of people is useful, first, because it helps to determine who holds the burden of proof (Levy 2009). That is, given two opposing ethical accounts, the one which must find arguments in its favour is the one which is further away from lay intuitions. Second, empirical information may provide direct support for a given ethical account. The usual procedure consists in presenting the subjects with moral dilemmas and predefined options or paths of action. Then, the experimentalist draws conclusions from the choices made by the

subjects: such values are present, such strategies are followed, such imperatives apply, such circumstances are relevant, and so forth. The experimental philosopher abstracts from the choices made by the subjects. Furthermore, the subjects may not have any interpretation or apparent intuition on their moral choices, or, if they have an interpretation, it may be in tension with that of the experimenter. If the subjects are asked to give reasons about their choices, they tend not to reflect the "real reasons" according to the experimenters, or fail to give reasons at all, being "morally dumbfounded"[10] (Haidt 2001). It is argued that the choices made by the subjects are based on a more socially distributed process of reason-giving (Haidt et al. 2008), but for some reason they are still unable to access them. This contrasts with the case study in philosophy of mathematical practice that will be described next, where the intuitions of the mathematician are the ones that matter the most, and therefore they are not questioned.

There are more examples from these and other philosophical disciplines – philosophy of language, aesthetics, philosophy of technology … – which cannot be mentioned here due to space limitations. In the light of the examples discussed, our view is that the **apostate speculator** and the (descriptive or normative) **informed analyst** types are common and arguably fruitful modalities of empirically informed philosophy, perhaps especially in those areas featuring ontologies accessible and/or familiar to philosophers to a certain degree. These modalities feature a broad philosophical (especially ontological) framing of the issue empirically investigated.

Furthermore, we can see how it is not quite clear whether these types of empirical philosophy should adhere to a *narrow* or *wider empiricism* approach. In the case of eliminative materialism, some empirical information is considered to support the position, while other information is either disregarded or presupposed before it comes. So, there is an important degree of speculation. In the example of empirical philosophy of science, we see how (some) philosophers are willing to revise their previous notions, be they minimal (gene) or slightly more elaborated (innateness), relegating them to the results they obtain. In the case of empirical ethics, some philosophers are willing to incorporate the results of the studies into their accounts even if they are strange to them, but on the other hand they are not

10 This may depend on the kind of moral judgment in question (Stanley, Yin, and Walter 2019) or the characteristics of the population sampled instead of being a generalized phenomenon. We argue later that mathematicians (a very specific population) give reasons about mathematics (a radically different domain). There could be a continuum where the more formal a field is, the more subjects tend to give "proper" reasons. But it could also be the case, as we suggest, that experimental ethicists tend not to take the reasons of subjects as seriously as the philosophers of mathematical practice take the reasons of mathematicians, who do not impose such rigid philosophical intuitions and ontological systems on their analyses.

willing to let the interpretation of the subjects of their own behaviour (which is also empirical information) trump theirs.

It must be acknowledged that some ethnographic approaches (Latour 2005; Latour and Woolgar 1979 is a prominent example) may lean more clearly towards the **freeway explorer** type of empirical philosophy. For instance, Latour (2005) puts forward a practical metaphysics which consists in accepting the metaphysical perspectives of actors and the ontologies they use as "real", without the social scientist trying to reformulate them (in other words, actors are not "dumbfounded"). Whether Latour's later stances deviate from this approach, for instance regarding his taking sides in the issue of climate change, and whether his approach is ultimately feasible in science, is a matter of debate (Stamenkovic 2020), and outside the scope of this work.

In any case, these are a few examples among many, and the aim of this paper is not to offer a comprehensive analysis of empirical philosophy in general. The takeaway message is that, in the areas of empirical philosophy discussed here, there does not seem to be an overall rule of thumb to go one route or the other: which route is more fruitful may depend on local, case-by-case judgement on how solid the preceding philosophical intuitions are, and how reliable the empirical data gathered is. We now proceed to argue that the philosophy of mathematical practice is different. We will illustrate this via a case study.

3 Examples from Empirically Informed Philosophy of Mathematics

As we have seen in the case of experimental ethics, it could be argued that it is hard to tackle the propositions and the fundamental ontological structure of the subjects directly with empirical methods. The experimenters tend to project their own conceptual baggage to frame the empirical results and dismiss that of the subjects on the grounds of unreliability.

The case of mathematics tends to be different. According to the standard view of Mathematics, mathematical objects (speaking loosely, not committing us to a metaphysical position that there are such things), themes/questions or propositions are either grounded in an abstract realm (see logical objectivist, semantic realist) or we deal with a constructivist-ish position rooting even such "objects" in humankind. But in the first case (which is often claimed to be the standard position in philosophy) we cannot directly aim at mathematical objects *qua* their abstract status and in the latter case we must investigate the mathematical practice anyway as this practice constitutes their "objects". While this may be the case in ethics too,

the crucial difference is that philosophers of mathematical practice do not start from critical philosophical intuitions as a reference point. Mathematics is so abstract that it is hard for philosophers to have preceding philosophical intuitions about mathematical objects. For instance, non-mathematicians may find it hard to develop intuitions about the set-theoretic axiom V = L, which says that all sets are constructible, while most set theorists share the intuition that this axiom has some nice features but should not be adopted as a new axiom. Therefore, philosophers should not project their own onto-epistemic baggage into mathematics, unlike in the case of ethics, where philosophers can have pre-existing rigid philosophical intuitions.[11]

One may object that assuming a realistic position on mathematics could enable the philosopher to have access to mathematical objects to the same extent as mathematicians. However, due to our limited lifespan, it is unreasonable to believe that a philosopher could get used to advanced mathematical notions (or objects) up to a "native" level, where proper intuitions are acquired such that questions relevant to the mathematical community can be settled. Even if it did, their philosophical baggage may be at odds with mathematical intuitions. Even more, the genesis of such mathematical objects is often carried out in a series of iterated tasks, where successive acts of formalization and mathematization may come with a set of intuitions that occur *in situ* when studying mathematics and which are not strongly codified.[12] Coming back to our example from above, we expect that someone who developed stable intuitions about the axiom V = L, bases their intuition on the set-theoretic knowledge about L including L's specific construction, its feature to allow for only small large cardinals, the existence of a Suslin tree in L etc.[13] In other words, there is a lot of advanced mathematics to understand before developing stable intuitions about a mathematical object.

Regarding the above-mentioned armchair approach, we argue that it is blind in important respects as some parts of mathematical practice cannot be found in the mathematical literature. Examples would be for instance proofs of "folklore" theorems (theorems which most members of the community know but there is no (known) published proof of them) or theorems which are just known to (some of

11 While in principle it could be possible to have an experimental ethics where the philosophical framing is not as rigid, we refer only to recent cases. Being experimental ethics a young discipline, it is possible that it will see deep methodological change over time.

12 See Kant and Sarikaya (2020) for a study of these iterated acts of mathematization, and Carl et al. (2021) and Fisseni et al. (2019) for a study of the idea of learning mathematical objects via familiarizing with prototypical situations.

13 The reader does not have to be familiar with the details on L. Actually, this is exactly what we claim, that only mathematicians are sufficiently familiar with the details on L to have stable intuitions.

the) members of the communities (the latter have been called *ghost theorems*, see f.i. Rittberg, Tanswell, and van Bendegem 2018). Another example from set theory would be the work of Woodin who does not regularly publish his proofs. This together makes it very plausible that dealing with mathematical *practices* which go beyond what is written elucidates crucial aspects of mathematics for an empirically informed philosopher of mathematics.

This need became especially clear after Lakatos' seminal work on philosophy of mathematics, which served as a precursor for the later philosophy of mathematical practice. Lakatos put mathematics at the same level as the rest of science, breaking with the Popperian doctrine that, while scientific theories are fallible and revisable, mathematics is not. Most of Lakatos' work in philosophy of mathematics was compiled in his book *Proof and Refutations* (Lakatos 1976), posthumously published. He famously elaborates a discussion around how Euler's conjecture for polyhedra was preserved even in the advent of counterexamples, meaning that decisions in mathematics are not made attending exclusively to the virtues of platonist forms or some formalism. Hence, it is warranted to empirically study mathematical practices, just like scientific practices, and many would subsequently endorse and further develop this view (see, for instance, Geist, Löwe, and van Kerkhove 2010; Johansen et al. 2016; Kaufman and Lavor 2016; Löwe, Martin, and Pease 2021).

In the next paragraph, we will shortly depict the community of researchers concerned with mathematical practice. We will then look at a debate within philosophy of mathematics, namely the debate about large cardinal axioms as an extension of the axiom system ZFC, and argue that *wider empiricism* in contrast to *narrow empiricism* is much more successful in capturing the relevant aspects surrounding the philosophical issues at stake in this debate.

3.1 What is Empirically Informed Philosophy of Mathematics?

There is a group of areas in philosophy with the form "philosophy of …". When we insert "physics" | "social sciences" etc. we get a subject where, since the Practice Turn, a sizable part of the community is focusing on the practice of the respective academic field. Philosophy of mathematics used to be an exception; it had to wait longer to receive the same treatment as Philosophy of Science. The main focus was on ontological and epistemological questions. A subfield stressed the need to work empirically informed and to focus on the actual practice. From this the discipline "philosophy of mathematical practice" emerged.

Prominent names in this field stressed the need to incorporate the work of mathematicians via case studies.[14] Or in the words of a founding member of the 2009 created *Association for Philosophy of Mathematical Practice*, Paolo Mancosu:

> [A]nyone familiar with contemporary philosophy of mathematics will be aware of the need for new approaches that pay closer attention to mathematical practice (Mancosu 2008, Preface).

This community is constantly growing. Philosophers of mathematics often ignored mathematical practice, and philosophers and sociologists of science/humanities often ignored mathematics. Bettina Heintz for instance wrote about her profession that "Sociology meets mathematics with an idiosyncratic mixture of devotion and disinterest".[15]

The shift to philosophy of mathematical practice leads not necessarily to the abandonment of old questions of philosophy of mathematics. It rather extends the scope of philosophical questions and mainly brings in a new methodology. It is a diverse branch of research. To get a broad overview, based on distinctions made by van Bendegem (2014, p. 221), Jullien et al. (2014), and Mancosu (2008, pp. 3–7), here are some traditions subsumed under the label philosophy of mathematical practice:

A. **The Lakatonean tradition (sometimes labelled as the maverick tradition)**: Here we mainly focus on a historically informed approach. A key view in this tradition is anti-foundational, which stands especially against the importance of mathematical logic and set theory.
B. A **naturalistic** (or *Second Philosophy*) position states that the authority lies with the researchers of the analysed field and not with the philosopher. This may include ontological, methodological, or epistemological dimensions, see especially (Maddy 2007).
C. A **normative** (or *First Philosophy*) approach, which wants to revise mathematics based on philosophy.[16]

These approaches are accompanied by non-philosophical dimensions. Those are:

D. A **sociologically** informed approach and empirical studies of the mathematical community (Greiffenhagen 2008; Heintz 2000; MacKenzie 2001).

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14 See for instance Corfield (2003), Heintz (2000), Löwe and Müller (2010), and Mancosu (2008).

15 (Heintz 2000, p. 9) (Translated by the authors); German original: „Die Soziologie begegnet der Mathematik mit einer eigentümlichen Mischung aus Devotion und Desinteresse." Apparently there are counterexamples, like (Bloor 1976; MacKenzie 2001; Restivo, van Bendegem, and Fischer 1993) and of course herself.

16 The most notable example is possibly the intuitionistic movement.

E. A **mathematics educational** approach, including both philosophical aspects of mathematics education and aspects of the process of learning mathematics (François and van Bendegem 2007; Paul 1991; van Kerkhove 2007).
F. **Ethnomathematical** ideas, taking cultural aspects into account (François and van Kerkhove 2010).
G. Approaches that include **cognitive** and **biological** factors. See f.i. Giaquinto (2007), Mancosu, Jørgensen, and Pedersen (2005), Pease, Guhe, and Smaill (2013), and Schlimm (2008, 2018).

Each of these modalities can be realized via different methodological strategies. For instance, G could be conducted via experiments or by literature review. Thus, this might be armchair philosophy or experimental philosophy.

We think that all of these approaches introduced fruitful investigations into the philosophy of mathematics. Important topics addressed in the philosophy of mathematical practice are the epistemological roles of diagrams (Manders and Mancosu 2008), visualization (Giaquinto 2007), and terminology (Schlimm 2018), the evaluation of 'good' definitions (Tappenden and Mancosu 2008), and the dimensions of proof appraisal (Inglis and Andrew 2015, 2016). Recent work looks into social aspects such as the background of mathematics represented in web blogs (Pease, Aberdein, and Martin 2019) and peer reviewing practices (Andersen 2017). All of them have in common that they reveal aspects of mathematics that are not accessible to the philosopher who does not know about the views of the practitioners.

We will look at one debate that is of interest both to classical philosophers of mathematics and for philosophers of mathematical practice. We will look at the study of so called large cardinal axioms and argue that we need to adapt to a *wider empiricism* approach (understood as described in Section 1) and empirical input is essential, as the preceding philosophical baggage is simply insufficient to onto-epistemically frame the relevant content.

3.2 The Case of Large Cardinals

The questions of independence are well known in philosophical and mathematical literature. On one side there are the Gödelian incompleteness theorems. In 1931, Gödel (1931) showed that every consistent, effectively axiomatized theory containing 'some arithmetic' is incomplete.[17] This means that there is a formula in the language of the theory which cannot be proven and whose negation cannot be

17 "Effectively axiomatizable" stands for something like "*there is* an algorithm that is able to *decide* for any given statement whether it is or is not an axiom". The meaning of "a theory contains

proven either. His second incompleteness theorem gives even an example for such a statement, namely consistency statements of the theory in question (again with the above-mentioned restrictions).

There are approaches which aim to find independent statements that are relevant to mainstream mathematics.[18] A branch of mathematics that is particularly engaged in questions that are independent from its current axiomatic setting is set theory. There are several results independent from ZFC.

A prominent example is the continuum hypothesis (CH): If we denote the cardinality of the natural numbers by $\aleph_0$ and of its power set (i.e., the set of all subsets of the natural numbers, or equally of the continuum) with $2^{\aleph_0}$, then the continuum hypothesis states that $2^{\aleph_0}$ is the second smallest infinite cardinality, or in formula:

$$(CH): 2^{\aleph_0} = \aleph_1$$

This was the first question on Hilbert's list of the most important questions at the beginning of the 20th century. It happens to hold that one can neither prove CH nor its negation within ZFC. So, one might think that ZFC is not a good basis to do set theory and we should rather look for new axioms. Gödel held this position and wanted to justify new axioms (partly) by inductive arguments. He said that

> probably there exist other […] axioms of infinity even disregarding the intrinsic necessity of some new axiom, and even in the case it has no intrinsic necessity at all, a decision about its truth is possible also in another way, namely, inductively, by studying its 'success'.[19]

So, he already introduced a quasi-empirical component, the study of such axioms. But this can be done by respecting the literature and seeing which results were proved.

Large cardinal axioms are interesting in that respect since they are the most acceptable ones in set-theoretic research. They are much used and set theorists have substantial knowledge about them.[20]

A large cardinal axiom states the existence of a specific large cardinal, whose existence is not provable in ZFC. Although their consistency cannot be proven, just one turned out to be inconsistent, namely the "Reinhardt cardinal", whose existence was shown to be inconsistent with the Axiom of Choice (Kunen 1971).

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some arithmetic" can be made very precisely, see f.i. Willard (2001); for the purposes of this work it is sufficient to know that a theory containing Peano arithmetic already contains "some arithmetic".

18 See for instance: (Friedman 1998; Paris and Harrington 1977).

19 [Gödel 1947, p. 182]. He talks about „*probable* decision" in a later version from 1964.

20 Twenty three out of 28 interview partners indicate that they use large cardinal axioms in their research. See below for more information on this interview study.

We saw above that Gödel had some hope that large cardinal axioms may settle the continuum problem. However, due to the theorem of Levy and Solovay (1967), it is known that large cardinal axioms are compatible with both the continuum hypothesis and its negation.[21]

When it comes to the question of how large cardinal axioms can be justified, there are two different claims to differentiate: we either try to argue in favour of the consistency of large cardinal axioms, or in favour of the actual existence of large cardinals. Both claims are important in set-theoretic research. The consistency of large cardinals is used to prove the consistency of other new axioms. The actual existence of large cardinals is often used in the form of their consequences since they settle some independent questions, for example, many questions about the projective subsets of the reals.

One important strategy for the justification of large cardinal axioms proceeds through reflection principles (justification of the existence of large cardinals not only the consistency of the axioms). There is a reflection principle provable in ZFC (which says for each formula that it is equivalent to the existence of a bounded part of the set-theoretic hierarchy satisfying the formula). The strategy is to strengthen this principle and use it to prove the existence of specific large cardinals. Koellner investigated this line of thought in detail relating technical results (which reflection principles imply the existence of which large cardinals?) to philosophical justification (how can a specific reflection principle be justified on the iterative conception of set?). He concludes that

> the Erdös cardinal $\kappa(\omega)$ appears to be an impassable barrier as far as reflection is concerned. This is not a precise statement. But it leads to the following challenge: Formulate a strong reflection principle which is intrinsically justified on the iterative conception of set and which breaks the $\kappa(\omega)$ barrier (Koellner 2009, p. 217).

So, he claims that it seems very hard to justify the existence of larger cardinals than $\kappa(\omega)$ following the strategy of reflection principles ($\kappa(\omega)$ is not very large). Recently, Welch et al. (2019) tried to give a stronger case by means of his 'Global Reflection Principle', which implies the existence of Woodin cardinals (which are much larger than $\kappa(\omega)$).

Another strategy is pursued by Maddy, who collects first arguments in favour of large cardinals in her two articles *Believing the Axioms I, II* (Maddy 1988a, 1988b). In the first article, she considers measurable cardinals and concludes that

21 See also the survey article (Honzik et al. 2018).

> [here] we have, as Gödel predicted, an axiom so rich in extrinsic supports that '… whether or not [it is] intrinsically necessary, [it is] accepted at least in the same sense as any well-established physical theory' (1947/64, p. 477) (Maddy 1988a, p. 508).

Her reasoning works by identifying specific rules of thumb that guide the set theorists' judgments whether some axiom (or consequence of an axiom) is appealing or not. Extrinsic support means that the consequences of the axiom are judged appealing.

In view of the distinction between the consistency of large cardinal axioms and actual existence of large cardinals, Woodin developed an interesting argument, in which he sees the existence of large cardinals as the best explanation for the consistency of large cardinal axioms, which is, in turn, an observation in the set-theoretic community: so far, set theorists did not find an inconsistency in any theory of the form ZFC + Large Cardinal Axiom, see below. Rittberg elaborated on this argument (Rittberg 2020).

3.2.1 Case Study: Large Cardinals are Expensive

Against the backdrop of these philosophical debates about large cardinal axioms, empirical data on the set theorists' views on large cardinal axioms seems desirable – whether or not set theorists find them plausible or acceptable, or what mathematical features they find appealing about large cardinal axioms. In general, empirical results can contribute to philosophy in that they provide new insights into the subject matter, which are hardly available for the empirically uninformed philosopher. We will give in this section a specific such case. In the previous section, we have seen some philosophical ideas on an appropriate solution to the incompleteness problem in set theory regarding large cardinal axioms. The results of a sociological research project about views on new axioms show that those philosophical ideas can fruitfully be supplemented by a more differentiated treatment of new axioms that is found among the practitioners. A phenomenon discovered by empirical research (large cardinals are expensive) is probably philosophically relevant in the context of new axioms. Since this principle seems to play a role in set-theoretic research it should be included in the ontology of the philosopher who wants to describe and understand set-theoretic practice. Therefore, this is an approach of the **freeway explorer** type rather than of the **informed analyst** type.

Let us first consider the idea that the philosophical use of sociological results here is more of a **freeway explorer** type than the previously considered examples. We described above some philosophical research on the justification of large cardinal axioms. With this philosophical debate in mind, set theorists were asked

what they think about new axioms. This includes positive and negative judgments on new axioms, the kind of features that are referred to in such judgments, and candidate reasons in favour or against specific new axioms given by the practitioners. On the one hand, the interviewees offered views that are known arguments in the philosophical debate, but, on the other hand, they also offered views which are new or they presented an argument that was not made as such in the philosophical discourse. The latter is what the interview study was designed for; the study provides us with evidence in favour of new hypotheses which could be introduced into philosophical discourse.

We will see below that a new view on large cardinal axioms appears (*reluctance against using large cardinals while believing that they are consistent*), and therefore, this is of a **freeway explorer** type, where the philosopher did not have in mind this new view as a pre-established object. On the other hand, the philosopher uses some pre-established ontologies, because they look for reasons in favour or against large cardinal axioms. Not every view that comes up in the study is introduced in philosophy, but rather those objects that have the right form that was searched for.

3.2.2 Methodology: Explorative Interviews

We briefly present the methodology of the interview study. The interview study exploring the question "what do set theorists think about independence?" started in 2017. The study was done by the first author. After a brief pilot study with three interviewees (included in the main study), in which different possible interview questions were tested, a questionnaire was fixed. The questions addressed the following topics: the research area, new axioms, naturalness, surprising results, forcing, object and meta level, and the search for new axioms. The questions avoided philosophical vocabulary and focused on the practical aspects of the topics, which were expected to be, if not familiar, at least not foreign to the interviewees.

Further 25 interviews were conducted in 2018 and 2019 in different places: either at set-theoretic research conferences or near the home institution of the interviewees. A set theorist only qualified as an interview partner if they had a permanent research position in mathematics with their research focus on set theory. This makes it probable that they are senior researchers and, therefore, that they have a lot of research experience as well as rather settled views. Further aspects influencing the decision to send an interview invitation were the possibility of a personal meeting (all interviews were conducted by meeting in person), the probability of a positive answer (e.g. if there had been contact between the interviewer and the interviewee before, which applies to six interviewees), and the

goal of broadening the sample set regarding geographic location, research area, and views on independence (to lower risks for biases). Interviews took between 14 and 95 min. Fifteen of the interviewees are affiliated to a European university, 10 to a university in the USA, and three to a university outside of Europe and the USA.[22] All main research areas in set theory are represented: combinatorics, descriptive set theory (including ergodic theory), inner model theory, forcing axioms, large cardinals and forcing, forcing, set-theoretic and general topology, cardinal characteristics, and some additional smaller ones. Regarding their views on independence, 11 of the interviewees hold a more absolutist view and another 11 a more pluralistic view.[23] This data on the sample set suggests that although the study is not representative, a broad cross-section of views is represented.[24]

The interviewees mentioned large cardinal axioms in different places of the interview. Sometimes already at the beginning when they described their research area. But mostly during discussion of new axioms. The relevant interview question that was posed to the interviewees was: "Which axioms, apart from the ZFC axioms, do you use in your work?", and sometimes, the interviewer asked explicitly about large cardinal axioms; either when they were not mentioned, or when they were mentioned and the interviewer wanted to hear more about it. Large cardinal axioms also came in when discussing the other topics (e.g. naturalness, surprising results or object and meta level).

The sociological research questions guiding the interview analysis on large cardinal axioms were: (1) Who uses large cardinal axioms? (2) How do set theorists work with large cardinal axioms? (3) What are their views on large cardinal axioms, and which (mathematical) features of large cardinal axioms underlie these views?

Here, we will focus on the specific results regarding (3), which were coded in the category **[New axioms – Descriptions, views, opinions – Large cardinal axioms].** After a detailed transcription of the interviews, phrases up to an answer to one interview question (a *coding*) were included (or *coded*) in this category if the interviewee described, characterised, or offered a view on large cardinal axioms (if

22 Specific locations are not indicated because of anonymity.

23 Roughly summarized, according to an absolutist view, it makes sense to adopt new axioms while according to a pluralist view, it does not. Independence results are seen as insufficient in an absolutist view but sufficient as acceptable answers in a pluralist view. Regarding the categorization, further six interviewees were not included in one of these two categories, because they did not share a strong view in this respect. This was rather due to indifference than due to indecision.

24 More details on the results of the interview study will be provided in the PhD thesis of the first author.

they said something *about* large cardinal axioms). The codings were summarised according to more specific topics, among them *consistency* and *reluctance*.[25]

3.2.3 Results and Discussion

The views on large cardinals extracted from the interview study turn out to reveal a phenomenon that was not expected. Upon the introduction of large cardinals into set-theoretic research in the first half of the 20th century, people were rather sceptical about their consistency for quite some time. Since large cardinal axioms extend ZFC in terms of consistency strength (they imply the consistency of ZFC), it is impossible to prove their relative consistency from ZFC. Therefore, it was not certain whether those principles can freely be used or rather lead to inconsistencies. Above, we mentioned the Reinhardt cardinal, which is the only example of a large cardinal, where set theorists proved an inconsistency. In the research on all other large cardinals, no inconsistency was found. One result of the interview study confirms that some set theorists explicitly think that there are no worries about the consistency of large cardinals anymore. For example, it was suggested that the worries were dissolved because of the many years of experience with those mathematical objects while always paying careful attention to possible inconsistencies. So, for the practitioners, it seems very unlikely that an inconsistency was overlooked.

Still, there were expressed individual worries about the use of large cardinals when trying to prove a set-theoretic theorem. Some interview partners said that they were very comfortable using large cardinals up to a certain strength but were more uncomfortable with larger large cardinals. Sometimes, even a specific threshold was mentioned, e.g. a supercompact cardinal.[26] The worries are

25 In more detail: After coding, the subsequent analysis steps comprised of (1) paraphrasing the coded passages, (2) sorting out paraphrases that were too detailed, that only contained background information or that focused on another topic than large cardinal axioms, and (3) ordering them by the following topics: linear order and consistency, consistency and inner models, consistency based on large cardinal axioms, equiconsistency, reformulation, Foreman-Magidor-Shelah and Martin-Steel, large cardinals and determinacy, large cardinals and forcing, class-set distinction, large cardinals outside of set theory, consistency, naturalness, partial acceptance, methodological guide, Ultimate-L, reluctance, Woodin cardinals, supercompact cardinals, and models of ZFC. The topics were chosen from the interviews; a new topic was introduced when two interviews contained a coding about the same topic. As a last step, (4) the paraphrases for one topic were generalised and summarised. During this whole process, some details as well as quotes were kept for illustration. This analysis of the interview transcriptions follows the methodology of Mayring's content analysis (Mayring 2015).

26 The reason indicated for this threshold was the discontinuity at the successor of the cardinal of the elementary embedding provided by the existence of a supercompact.

expressed in views stating that some large cardinals are "expensive", that it is "an extra effort" to use them, or that they are "too strong". Importantly, those individual worries were never related to any worries about consistency.

What we can observe here is that some practitioners are hesitant to use certain large cardinals while there is no evidence that they are worried about the consistency of those large cardinals. The question arises why they do not like to use powerful mathematical principles which are available while they do not think that this can lead to inconsistency.

What happens when such a result enters philosophy? The philosophers of set theory are presented with empirical evidence about a new aspect that is probably relevant in the evaluation of new axioms. They also can get an empirical description of this aspect, hypotheses about its nature.

The first insight is that philosophers could have missed something relevant in the context of large cardinals. Let us explore how this could help to develop the framework in which new axioms are evaluated. It is important to note that set theorists are very much used to working with the various new axioms that are available. Besides large cardinal axioms, there are also forcing axioms, determinacy principles, and combinatorial principles. This situation demonstrates that the use of a new axiom in a set-theoretic proof does not presuppose that the practitioner accepts the axiom, but only that they are sufficiently convinced of its consistency. Following this line of thought, one could suspect that accepting the consistency of a new axiom is enough to use it whenever it is useful to prove a theorem. However, this is exactly the claim that our results question. Apparently, practitioners who are convinced of the consistency of large cardinal axioms do not use them whenever they are useful. There opens up a large middle-ground between having accepted the consistency of an axiom and using the axiom generously.

There is one methodological problem that we should consider at this point. When we proceed along these lines, it seems that we must assume that the mathematicians faithfully shared their beliefs with the interviewer, that is, the empirical results do not present us with views that are retold out of other reasons than belief (such as: the supervisor held this view, or the view is a good argument to justify the relevance of their own research etc.). One may argue that this is problematic for integrating results of an interview study into philosophy, however, we do not think so. Unless our philosophical focus lies directly on the beliefs of the people, a coherent, interesting view is still a benefit for philosophical discourse. The important aspect is that the empirical results provide evidence that the discovered phenomenon is probably relevant for the philosophical problem, but philosophers do not take the empirical results as evidence directly for the validity of the statements made by the practitioners. The next step, the evaluation of the empirical results within the philosophical discourse, is a substantial theoretical

step that takes place outside of the empirical context. The argument is not: the practitioners said something and therefore our philosophy must account for it; the argument is rather: if the empirical results are philosophically relevant, then they can both give rise to and constitute an argument in favour of a philosophical account that integrates them and extends them conceptually.

If philosophers want to take into account this empirically discovered reluctance phenomenon, which tasks do they have to fulfil? The first task is certainly to clarify this new aspect. This could be done by looking for a similar philosophical idea that was developed in another context, for example, in connection to other new axioms, or simply by enfolding the details about the new aspect given in the empirical results. This task is important since the empirical results must be embedded in the existing philosophical account, which might differ, for example, in terms of the terminology since the terminology of empirical research is much more oriented towards the natural language of mathematicians. This task could also require further empirical research, see below.

With possibly more than one suggested clarification of the new aspect at hand, the second task is to evaluate whether the new aspect can be added coherently to existing justification accounts for new axioms. The guiding question here is whether the new aspect conflicts with other philosophical claims of the respective account, or whether it weakens a philosophical position which presupposed that such an aspect would be irrelevant (or does not exist).

The philosophical reflection about this new aspect on large cardinals could be guided by the following question: What explains these forms of reluctance other than consistency worries?

Possible explanations partly supported by results of the study are that mathematicians outside of set theory should have access and should accept set-theoretic results and therefore proofs should not assume large cardinal axioms, or that set theorists are unfamiliar with some large cardinals and therefore avoid working with them, or that one understands the subject matter better if one does not use large cardinal axioms that are not necessary which motivates set theorists to use the weakest possible large cardinal axioms in a proof. Probably different explanations have to be given for different cases of reluctance.[27]

We argued in our case study that the empirical results may be philosophically important because they reveal a further aspect relevant to the evaluation of new axiom candidates. We suggest that empirical results of such a sort may be especially fruitful for philosophy of mathematics since it seems very improbable that the revealed aspects would have been found without studying mathematical

27 More details on these ideas and the empirical results concerning large cardinal axioms are given in the draft paper “Large cardinal axioms in practice” by the first author.

practices. The reason is that, in mathematics, many efforts are required to finally develop stable intuitions (perhaps more than in other areas, as suggested by Section 2). Philosophers simply often do not have intuitions on every new axiom candidate since they have few experiences of working with them.

Besides this, mathematics often features "unintuitive intuitions", which are not necessarily complex, like about the axiom of constructibility, V = L, mentioned at the beginning of Section 3. And perhaps due to the character of mathematical reasoning, these intuitions are not as contested by the empirical philosopher as those from biology and ethics. In this sense, empirical philosophy of mathematics features a "thinner lens", which draws it to the **freeway explorer** type, and a *wider empiricism* approach (remember that, as argued before, some ontologies are always presupposed for the empirical analysis, and onto-epistemical intrusiveness is better understood as a continuum). Of note, empirical philosophers are probably more inclined to accept the biologist's ontology than the moral subject's ontology: in the former case, the empirical philosopher will reject it in certain justified cases, while in the latter, it is often the default proceeding. This means, these areas occupy different positions on the aforementioned continuum. One may speculate whether the more formal or abstract a field is, the more seriously the practitioners' reasons are taken, or the less intrusive outsider ontologies are (for example, those of the empirical philosopher). This may however be less relevant for those who endorse a critical realist stance, who usually engage in qualitative research, and are more respectful to local ontologies while studying them, see for instance (Archer et al. 2013; Fletcher 2016).

4 Summary

We have described three different types of empirical information in philosophical work: the **apostate speculator,** the **informed analyst**, and the **freeway explorer**. The first two are common and yield arguably fruitful results in mainstream empirical philosophy. To illustrate these first two types, we described examples of empirical philosophy of mind/science and ethics. We saw how advocating for eliminative materialism is an example of the **apostate speculator**, and how empirical philosophy of science and ethics may follow the **apostate speculator** or the **informed analyst** types. Also, they can take either a *narrow empiricism* or *wider empiricism* approach. Then, we presented a case study from the philosophy of mathematical practice, which, we argue, is close to a **freeway explorer** approach, while keeping aspects of an **informed analyst** approach. This third type is characterized by a lack of rigid philosophical intuitions and onto-epistemic frames preceding the analysis. We provide reasons to believe that empirically

informed philosophy of mathematics may be fundamentally different from mainstream empirically informed philosophy due to "unintuitive intuitions" and the types of objects created in-practice. This difference is key to advocate a *wider empiricism* stance specifically in the philosophy of mathematical practice, and it makes empirical information especially fruitful at many stages, as proceeding speculatively often leaves too many blind spots.

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