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## Price Dispersion and the Costs of Inflation

The new Keynesian literature typically makes the assumption that firms always have to satisfy demand, which is at odds with profit-maximizing behavior under Calvo pricing when long-run inflation is positive. Our model, which relaxes this assumption, predicts that inflation causes a substantially smaller loss in effective aggregate productivity compared to a benchmark model without the possibility of rationing. Moreover, under positive inflation, firms choose smaller markups over marginal costs in our model than in the benchmark model. As a result, our analysis suggests that the standard new Keynesian model may exaggerate the welfare costs of inflation.

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WHAT MAKES INFLATION COSTLY? How large are the social costs of inflation? These core questions in monetary economics are not only interesting in their own right. They also have important implications for monetary policy-making as the answers to them determine the level of the inflation target that central banks should try to achieve.

The mainstream new Keynesian model implies that inflation is costly because, in combination with staggered price-setting, it leads to distortions in relative prices and thereby to an inefficient allocation of economic resources (see, e.g., Woodford 2003). Our paper shows that this mechanism is substantially weaker once one relaxes the implicit assumption typically made in new Keynesian models that firms always satisfy demand. Although this behavior is optimal when inflation is zero and prices are set above marginal costs, profit-maximizing firms would not satisfy demand completely after sufficiently long spells of fixed nominal prices when inflation is positive.

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Despite the fact that the fraction of firms that profit from rationing demand is typically small, we show that the aggregate consequences may be nonnegligible. In our benchmark new Keynesian model without the possibility to ration demand, even a moderate increase in the inflation rate from 0% to 5% causes inefficient price dispersion that results in a decrease in effective productivity by 5.8%. By contrast, in our main model, where firms choose their output optimally in every period, an inflation rate of 5% rather than 0% induces an effective productivity drop by a mere 0.7%.

The intuition is straightforward. In the standard new Keynesian model under positive inflation, too many resources are allocated to firms that have not adjusted their prices for long periods of time and therefore have low relative prices. If these firms have the possibility to constrain production to some extent, this will alleviate the inefficiencies caused by inflation.

There is also another effect that leads to lower social costs of inflation when firms have the possibility to ration demand. Without this possibility, it is risky for firms to choose small markups when inflation is positive, as the corresponding nominal price may be fixed for a considerable time period, which would result in a very low relative price and possibly high losses. The possibility to ration demand alleviates this risk and thus induces firms to choose smaller markups, which leads to more moderate distortions stemming from monopolistic competition.

To obtain these findings, we compute the stationary equilibria of a new Keynesian model with Calvo (1983) pricing where firms can ration demand but which is completely standard otherwise. We utilize this framework to analyze the long-term relationship between inflation and other economic variables. More specifically, we derive the following results. First, we provide a formal proof of our previous claim about the consequences of inflation for effective aggregate productivity. More precisely, we show analytically that, under positive inflation, price dispersion, which is inversely related to effective aggregate productivity, is always lower in our model with demand rationing compared to the standard new Keynesian model where firms always have to satisfy demand.

Second, we prove that our model has an equilibrium for arbitrary levels of inflation. This contrasts with the well-known finding that the standard new Keynesian model with Calvo pricing does not have an equilibrium for sufficiently high inflation rates (see, for example, Ascari and Sbordone 2014).<sup>1</sup> For high inflation rates, the inefficiencies due to distorted relative prices are so severe in the standard new Keynesian model such that no positive quantity of aggregate output can be produced. By contrast, the harmful effect of inflation on effective productivity is significantly smaller in our model, which explains why an equilibrium exists for arbitrarily high inflation rates.

Third, we derive the conditions under which firms ration demand. For severe deflation, firms ration demand for some time after re-setting their prices. For moderate deflation, firms never ration demand. For positive inflation, rationing occurs for firms

1. This would hold under Rotemberg (1982) pricing as well.

that have not adjusted their prices for some time, as their relative prices are comparably low and the demands for their goods are high.

Fourth, we prove our previous claim about the size of markups and rationing. More precisely, we show that, when inflation is positive, firms always choose smaller markups over marginal costs in our main model compared to the benchmark new Keynesian model without rationing.

Fifth, together with the finding that the effective productivity loss caused by inflation is considerably smaller in our main model, this effect entails that the standard new Keynesian model exaggerates the welfare costs of inflation. For example, the standard model predicts that 5% inflation leads to a 5.7% consumption-equivalent welfare loss over and against zero inflation. In our main model, the respective loss is only 0.7%. For larger inflation rates, the difference between the welfare costs of inflation is even stronger.

To the best of our knowledge, this paper is the first to study the possibility of rationing demand in a new Keynesian model with Calvo pricing. Extending Danziger (1988), Danziger (2001) considers the possibility of rationing in a menu-cost model. His main point is to highlight the importance of quantity adjustment costs for the welfare effects of inflation. Quantity adjustment costs eliminate the socially wasteful differences in output across firms, which represent the standard source of welfare costs in new Keynesian models with staggered price setting.<sup>2</sup> In our paper, rationing does not lead to an elimination but to a considerable reduction of these differences in output. In contrast with our model, Danziger's framework involves that, under positive inflation, a fraction of output is discarded as a consequence of insufficient demand at the beginning of a price spell. In a more traditional Keynesian framework, rationing has been considered by Barro and Grossman (1971), who lay out a static disequilibrium model with exogenous prices in which they carefully distinguish between situations with excess demand and excess supply on goods and labor markets.

Although there are only few papers that focus on the role of rationing for the relationship between inflation and welfare, the general literature on the costs of inflation and the socially desirable level of inflation in the long term is very large (see Schmitt-Grohé and Uribe 2010, Diercks 2017, for surveys). The traditional view about the costs of inflation stresses that higher inflation is associated with higher nominal interest rates and thereby larger opportunity costs of holding money (see, e.g., Fischer 1981, Lucas 1981). As a consequence, the Friedman rule, that is, permanent deflation that eliminates the opportunity costs of holding money, is optimal. By contrast, zero inflation is typically welfare-maximizing in the standard new Keynesian model in the limiting case where real money balances are zero, as it alleviates the distortions in relative prices under staggered price setting. In a model

2. As in our paper, the impact of inflation on markups over marginal costs is another important factor influencing the welfare costs of inflation.

where both channels are at work, Khan, King, and Wolman (2003) find optimal rates of inflation that are slightly negative.<sup>3</sup>

Recently, the standard mechanism for the costs of inflation in the new Keynesian model has come under fire by Nakamura et al. (2018). With the help of a new data set on prices that also covers the Great Inflation of the 1970s, they argue that empirical support for a link between price dispersion and inflation rates is wanting. Our paper is complementary to theirs, as we provide a theoretical argument for why the standard new Keynesian model may exaggerate the costs of inflation.

In the aftermath of the 2007/2008 global financial crisis in particular, the literature has examined to which degree the so-called zero lower bound on interest rates and the possibility of a lower natural real rate of interest provide a rationale for higher inflation targets. This question is addressed by Reifschneider and Williams (2000), Coibion, Gorodnichenko, and Wieland (2012), and Andrade et al. (2019), among others. Because our paper concentrates on analyzing the steady state for different inflation rates and does not study shocks that can temporarily push interest rates to zero, we abstract from the zero lower bound for most of our analysis.

Although much of the new Keynesian literature considers models that are log-linearized around a zero-inflation steady state, there are several authors who allow for positive trend inflation (Ascari 2004, Hornstein and Wolman 2005, Yun 2005, Amano, Ambler, and Rebei 2007, Ascari and Ropele 2007, Ireland 2007, Kiley 2007, Levin and Yun 2007, Cogley and Sbordone 2008, Coibion and Gorodnichenko 2011, Kurozumi 2016, Ascari, Phaneuf, and Sims 2018). This literature, which is surveyed in Ascari and Sbordone (2014), examines, among other things, how positive trend inflation affects equilibrium determinacy as well as the equilibrium dynamics in response to shocks.<sup>4</sup> None of these papers examine how the welfare costs of inflation depend on the assumption that firms always have to satisfy demand.

Following many authors, this paper focuses on Calvo price-setting. In the literature, other time-dependent approaches to price setting are considered. In particular, it is well known that Taylor (1980) pricing or truncated Calvo pricing (see Amano, Ambler, and Rebei 2007) lead to lower costs of inflation compared to Calvo's approach. Similarly to our paper, these approaches avoid the sizable distortions that occur because some firms have not updated their prices for a long time. Compared to these other types of time-dependent pricing, the standard Calvo approach has the advantage of implying a constant hazard function, which is more in line with the microeconomic evidence on price setting (see Klenow and Kryvtsov 2008).<sup>5</sup>

3. Adam and Weber (2019) and Lepetit (2017) show that different forms of heterogeneity may entail a positive optimal rate of inflation. Blanco (2021) demonstrates that a positive inflation target can be optimal in a model with menu costs, idiosyncratic shocks to firms' productivities, and the zero lower bound.

4. Yun (2005) analyzes how optimal policy depends on the initial degree of price dispersion.

5. Both Taylor and truncated-Calvo pricing would imply a spike in the hazard function once the maximum price spell is reached.

A part of the literature that uses models with Calvo pricing considers an endogenous frequency of price adjustment.<sup>6</sup> For example, Levin and Yun (2007) assume that firms can choose the mean duration of the price spell when they adjust their prices. This assumption provides a complementary mechanism that results in costs of inflation that are smaller than in the standard framework with exogenous frequency of price adjustment. In future work, it would be interesting to combine the two approaches, which would plausibly lead to even smaller costs of inflation.

Another active strand of the literature examines state-dependent pricing. These papers typically find that menu costs imply lower costs of inflation compared to Calvo pricing (see, e.g., Burstein and Hellwig 2008). Like rationing, the menu-cost approach avoids the large distortions under Calvo that arise because some firms have not adjusted their prices for a long time. Our approach has the advantage of being relatively easy to implement. For stationary equilibria, this is demonstrated in the present paper. For equilibria with aggregate shocks, Calvo price setting with rationing is more complex than the standard approach without rationing, as the entire distribution of relative prices becomes a state variable. However, such models could be solved with by-now-standard numerical techniques (see, e.g., Krusell and Smith 1998).

In future work, it would be interesting to integrate rationing into medium-scale models of inflation. In particular, rationing could be introduced into models of staggered wage setting. For example, Ascari, Phaneuf, and Sims (2018) show that inflation increases the wage markups chosen by households as well as wage dispersion. Plausibly, both effects would be weakened if households were allowed to ration their hours worked.

This paper is organized as follows. In Section 1, we present a benchmark new Keynesian model and our main model, in which we allow firms to ration demand. Section 2 solves the benchmark model and identifies the situations in which rationing would be profitable to firms. In the subsequent Section 3, we solve our main model and determine several properties of the equilibrium analytically. Our main results regarding price dispersion and the costs of inflation are presented in Section 4. In order to assess the robustness of our results, we consider several variants of our model in Section 5. Section 6 concludes.

## 1. MODEL

Our model is based on a textbook macroeconomic model (see Woodford 2003) with price setting à la Calvo (1983) and positive trend inflation. We will analyze two variants of this model. In Section 2, we will examine our benchmark model, that is, the standard case where each firm always satisfies the demand for its goods. In Section 3, we examine our main model, where firms can supply smaller quantities than demanded if that is profitable to them. In line with our objective to examine the costs of inflation in the long run, we consider stationary equilibria of both economies for different inflation rates.

6. An early paper that considers endogenous price stickiness is Ball, Mankiw, and Romer (1988).

Time is continuous and denoted by  $t = [0, \infty)$ .<sup>7</sup> The model is populated by households, a representative, perfectly competitive final-goods producer, monopolistically competitive intermediate-goods producers, and a central bank. We provide details about each of these groups in turn.

### 1.1 Households

There is a continuum of households that own identical shares of all firms and receive firms' profits as dividends. Each household has the following instantaneous utility function

$$u(c(t), n(t)) = \frac{c(t)^{1-\sigma} - 1}{1-\sigma} - \Psi \frac{n(t)^{1+\varphi}}{1+\varphi}, \quad (1)$$

where  $\sigma$ ,  $\varphi$ , and  $\Psi$  are positive parameters,  $n(t)$  stands for the household's supply of labor, and  $c(t)$  is consumption of a final good.

Utility in future periods is discounted at a positive rate  $\rho$ . In all periods  $t$ , the nominal budget constraint is

$$P(t)c(t) + \dot{B}(t) = i(t)B(t) + P(t)w(t)n(t) + P(t)T(t), \quad (2)$$

where  $P(t)$  denotes the price of the final good,  $B(t)$  bond holdings,  $i(t)$  the nominal interest rate,  $w(t)$  the economy-wide real wage, and  $T(t)$  a real transfer, which includes the profits of firms and the government's seigniorage revenues. Throughout the paper, a dot on top of a variable stands for the derivative with respect to time. Bonds are in zero net supply.

As is well-known, the household's optimization problem leads to the following consumption Euler equation and condition for the optimal supply of labor:

$$\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}(i(t) - \pi(t) - \rho) \quad (3)$$

$$\Psi n(t)^\varphi = w(t)(c(t))^{-\sigma}, \quad (4)$$

where  $\pi(t) := \dot{P}(t)/P(t)$  is the inflation rate.

7. The assumption of continuous time is mathematically convenient in our main model. As we will see, there is typically a specific price spell for which firms start rationing demand (or stop rationing demand). For continuous time, this price spell is a differentiable function of the initial price chosen by an intermediate firm. As a consequence, the solution to the firm's profit maximization problem can be obtained by solving a standard first-order condition.

### 1.2 Final-Goods Producers

Perfectly competitive final-goods producers assemble the different intermediate goods  $y_j(t)$  ( $j \in [0, 1]$ ) to produce a final good according to the production function

$$y(t) = \left[ \int_0^1 (y_j(t))^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad (5)$$

where  $\theta$  ( $\theta > 1$ ) stands for the elasticity of substitution between the differentiated goods. The resulting demand for intermediate good  $j$  is

$$y_j^d(t) = \left( \frac{P_j(t)}{P(t)} \right)^{-\theta} y(t). \quad (6)$$

In the benchmark model, intermediate-goods producers always satisfy the demand for their goods. In our main model, these firms may ration demand if that is profitable.

### 1.3 Intermediate-Goods Firms

There is a continuum of monopolistically competitive intermediate-goods producers, indexed by  $j \in [0, 1]$ . The production function of firm  $j$  is of the form

$$y_j(t) = A(n_j(t))^\gamma, \quad (7)$$

where  $A > 0$ ,  $\gamma \in (0, 1)$ , and  $n_j(t)$  is the labor input of firm  $j$  at time  $t$ . As  $\gamma < 1$ , there are decreasing returns to scale.<sup>8</sup> One possible interpretation of this case is that output is given by a standard Cobb–Douglas production function with fixed capital.<sup>9</sup>

In line with Calvo (1983), intermediate-goods firms can only adjust their prices if they receive a shock, which arrives with constant rate  $\delta$ . Thus we do not consider indexation or rule-of-thumb pricing for firms that are not able to re-optimize their prices. These assumptions are sometimes made in the literature (see, for example, Yun 1996), but are incompatible with the microeconomic evidence that prices are fixed for considerable periods (Klenow and Kryvtsov 2008, Nakamura and Steinsson 2008).

As has been mentioned before, we distinguish between two cases. First, as is standard in the new Keynesian literature, we assume in our benchmark model that firms always have to satisfy demand at the current price even if this entails losses in this

8. Decreasing returns to scale are frequently used in the literature. In our case, they have the convenient implication that firms that choose to ration demand still supply positive quantities of intermediate goods. For constant returns to scale, firms would stop production completely when relative prices drop below marginal costs. This case is considered in Section 5.1.

9. It would be easy to include capital into our model and to examine the consequences of different inflation rates for the capital stock in the long run. Here we follow the large part of the literature that disregards capital accumulation (see the discussion on p. 724 in Ascari and Sbordone 2014). At any rate, our main findings are plausible to hold in a model with endogenous capital as well.

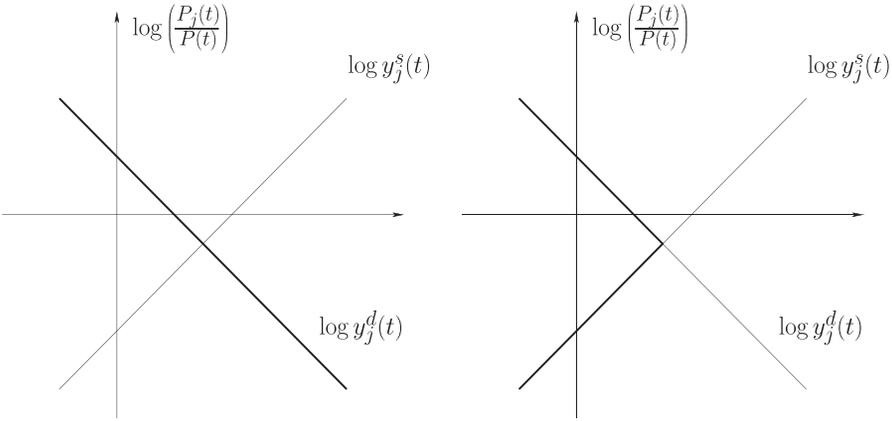


Fig 1. Log Output (Thick Black Line) for Different Levels of a Firm's Log Relative Price in the Benchmark Case, where Output is Always Determined by Demand (left panel) and the Main Model, where Output is given by Demand or Supply, whichever is Smaller (Right Panel).

period. Second, in our main model, we adopt the assumption that firms are able to ration demand at the current price. It is straightforward to derive that, in the case where firm  $j$  could choose its output freely, its optimal output, for given price  $P_j(t)$ , would be

$$y_j^s(t) = \left( \gamma \frac{A^{\frac{1}{\gamma}} P_j(t)}{w(t) P(t)} \right)^{\frac{\gamma}{1-\gamma}}. \tag{8}$$

Figure 1 illustrates schematically how output is determined in the two model variants. In the benchmark model, output is always given by demand (see the left panel). Although this is a perfectly reasonable assumption under zero inflation rates because monopolistically competitive firms set prices above the market-clearing level, this assumption is more problematic under positive inflation rates and rigid prices. If a price  $P_j(t)$  has not been adjusted for some time, the relative price  $P_j(t)/P(t)$  may drop below the market-clearing level. In this case,  $y_j^s(t) < y_j^d(t)$ . As a consequence, the main model (illustrated in the right panel) involves that output is supply-determined in this case.

In our main model, firm  $j$ 's optimal choice in a period  $t$  in which it can change its price is given by the solution to the following maximization problem:

$$\begin{aligned} \max_{P_j(t)} \mathbb{E}_t \left[ \int_t^\infty \frac{\lambda(\tau)}{\lambda(t)} e^{-(\rho+\delta)(\tau-t)} \left( \frac{P_j(t)}{P(\tau)} y_j(\tau) - w_\tau \left( \frac{y_j(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) d\tau \right] \\ \text{s.t. } y_j(\tau) = \min \{ y_j^d(\tau), y_j^s(\tau) \}, \quad \forall \tau \geq t, \end{aligned} \tag{9}$$

where  $\lambda(t) = c(t)^{-\sigma}$ . The optimization problem in the benchmark case without the possibility of rationing is identical, except that the constraint is  $y_j(\tau) = y_j^d(\tau), \forall \tau \geq t$ , in this case.

#### 1.4 Monetary Policy

We close the model by assuming that the central bank follows a simple Taylor rule:

$$i(t) = \rho + \pi^* + \phi(\pi(t) - \pi^*), \quad (10)$$

where  $\phi > 1$  and  $\pi^*$  is the central bank's inflation target.

## 2. BENCHMARK MODEL

Before exploring the possibility that firms ration demand, it will be instructive to solve the benchmark case where intermediate-goods firms always have to satisfy demand.<sup>10</sup> In a stationary equilibrium, equations (3) and (4) simplify to

$$i_B = \pi^* + \rho, \quad (11)$$

$$\Psi(n_B)^\varphi = w_B(y_B)^{-\sigma}, \quad (12)$$

where we have introduced the subscript  $B$  to denote equilibrium values for the benchmark economy and have used  $c_B = y_B$ .

Equation (5) can be rewritten as

$$y_B = \left[ \delta \int_0^\infty e^{-\delta\tau} (q_B^{-\theta} e^{\theta\pi^*\tau} y_B)^{\frac{\theta-1}{\theta}} d\tau \right]^{\frac{\theta}{\theta-1}}, \quad (13)$$

where  $q_B$  is a firm's reset price in relation to the price index  $P(t)$ , that is,  $q_B = P_j(t)/P(t)$  at the time when it adjusts its price. Computing the integral in (13) and re-arranging yields<sup>11</sup>

$$q_B = \left( \frac{\delta}{\delta - (\theta - 1)\pi^*} \right)^{\frac{1}{\theta-1}}. \quad (14)$$

10. A similar analysis for nonzero steady-state inflation can be found in, for example, Ascari and Sbordone (2014).

11. According to the terminology introduced by King and Wolman (1996) and used in Ascari and Sbordone (2014),  $q_B$  corresponds to the inverse of the price-adjustment gap.

We note that  $q_B$  satisfies  $q_B = 1$  for  $\pi^* = 0$  and increases with inflation. The fact that  $q_B > 1$  for  $\pi^* > 0$  has the straightforward interpretation that, under positive inflation, newly adjusted prices are higher than prices that were selected in the past. It is also noteworthy that we have to impose the restriction  $\pi^* < \delta/(\theta - 1)$ .<sup>12</sup> Otherwise the integral in (13) cannot be computed and no solution for  $q_B$  exists.

The production function for the final good, (5), can be combined with the demand for intermediate goods, (6), to obtain the following well-known equation for the price level:

$$P(t) = \left[ \int_0^1 (P_j(t))^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (15)$$

It may be worth stressing that this equation will not hold automatically in the main model with possible rationing, as an intermediate firm's output is not necessarily given by the respective demand, (6).

The labor-market clearing condition,  $n_B = \int_0^1 n_j(t) dj$  can also be stated as  $n_B = \int_0^1 (y_j(t)/A)^{1/\gamma} dj$ . Using (6),  $P_j(t)/P(t) = q_B e^{-\pi^* \tau}$  for the relative price of a firm  $j$  that adjusted its price  $\tau$  periods ago, and  $\delta e^{-\delta \tau}$  for the density function for the ages of prices, we obtain

$$n_B = \frac{\delta}{\delta - \frac{\pi^* \theta}{\gamma}} \left( \frac{y_B}{A} \right)^{\frac{1}{\gamma}} (q_B)^{-\frac{\theta}{\gamma}}. \quad (16)$$

The integral that we evaluated to compute  $n_B$  only exists if  $\pi^* < (\gamma \delta)/\theta$ .

Equations (14) and (16) can be combined to yield

$$y_B = A/s_B \cdot (n_B)^\gamma, \quad (17)$$

where

$$s_B := \left( \frac{\delta}{\delta - \frac{\pi^* \theta}{\gamma}} \right)^\gamma \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\frac{\theta}{\theta-1}}. \quad (18)$$

In the literature,  $s_B$  is viewed as a measure of price dispersion.<sup>13</sup> In line with (17), we can interpret  $A/s_B$  as a measure of effective aggregate productivity.

As is well known,  $s_B$  attains its global minimum of 1 at  $\pi^* = 0$  (Schmitt-Grohé and Uribe 2007).<sup>14</sup> Analogously, effective productivity  $A/s_B$  reaches its global maxi-

12. It is well known that inflation must not be too high for the new Keynesian model with Calvo (1983) pricing to have an equilibrium. The respective conditions for the discrete-time case are stated in, for example, Ascari and Sbordone (2014).

13. See, for example, the survey by Ascari and Sbordone (2014).

14. In the limit where prices are perfectly flexible, that is, for  $\delta \rightarrow \infty$ , price dispersion goes to one as well.

imum of  $A$  at  $\pi^* = 0$ . For  $\pi^* < 0$ ,  $A/s_B$  increases monotonically with  $\pi^*$ . It decreases with  $\pi^*$  for positive inflation rates. Importantly, as  $\pi^*$  approaches  $(\gamma\delta)/\theta$ ,  $s_B^*$  goes to infinity and effective productivity approaches zero accordingly. In this case, even a very high quantity of labor can produce only minimal aggregate output. For inflation rates equal to or higher than  $(\gamma\delta)/\theta$ , no equilibrium of the standard new Keynesian model exists.

An intermediate firm  $j$  that can adjust its price in period  $t$  will have the following profits in period  $\tau \geq t$  as a function of the relative price chosen in  $t$ ,  $q_j(t) = P_j(t)/P(t)$ , provided that it will not have been able to change its price in the meantime:

$$\Pi_j^t(\tau) := \frac{e^{\pi^*(\theta-1)(\tau-t)}}{q_j(t)^{\theta-1}} y_B - \left( \frac{e^{\pi^*\theta(\tau-t)} y_B}{A q_j(t)^\theta} \right)^{\frac{1}{\gamma}} w_B,$$

where we have used that  $y_j(\tau) = y_j^d(\tau)$  and  $P(\tau) = P(t)e^{\pi^*(\tau-t)}$ . The optimal value of  $q_j(t)$  solves the optimization problem

$$\max_{q_j(t)} \int_t^\infty e^{-(\rho+\delta)(\tau-t)} \Pi_j^t(\tau) d\tau.$$

We observe that we have to require  $\pi^* < (\gamma(\rho + \delta))/\theta$  for the integral to exist. Then the equilibrium value of  $q_j(t)$ ,  $q_B$ , has to satisfy

$$q_B = \left( \frac{\theta}{\theta - 1} \frac{\rho + \delta - \pi^*(\theta - 1)}{\gamma(\rho + \delta) - \pi^*\theta} \frac{(y_B)^{\frac{1-\gamma}{\gamma}}}{A^{\frac{1}{\gamma}}} w_B \right)^{\frac{\gamma}{\gamma+(1-\gamma)\theta}}. \tag{19}$$

We summarize our results in the following proposition:

**PROPOSITION 1.** *For a given level of  $\pi^*$  that satisfies  $\pi^* < (\gamma\delta)/\theta$ , the equilibrium of the benchmark economy is given by the values of  $i_B$ ,  $w_B$ ,  $q_B$ ,  $n_B$ ,  $s_B$ , and  $y_B$  that satisfy (11), (12), (14), and (17)-(19). For  $\pi^* \geq (\gamma\delta)/\theta$ , no equilibrium exists.*

We observe that the condition mentioned in the proposition,  $\pi^* < (\gamma\delta)/\theta$ , ensures that the other parameter restrictions stated in this section,  $\pi^* < (\gamma(\rho + \delta))/\theta$  and  $\pi^* < \delta/(\theta - 1)$ , hold automatically. Although no equilibrium of the standard new Keynesian model exists for sufficiently high inflation rates,  $\pi^* \geq (\gamma\delta)/\theta$ , the model is compatible with arbitrarily low negative rates inflation.

In the next proposition, we identify the situations in which there are incentives for firms to ration demand, that is, in which firms would profit from lowering output if this were possible.

**PROPOSITION 2.** *Consider a fixed level of  $\pi^*$  with  $\pi^* < (\gamma\delta)/\theta$ . In the equilibrium of the benchmark economy, the incentives to ration demand are as follows:*

- (1) For severe deflation,  $\pi^* < -\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$ , each firm would benefit from rationing demand in each period where it adjusts its price and in subsequent periods where the price spell satisfies  $\tau < \hat{\tau}_B$  with

$$\hat{\tau}_B := \gamma \frac{\ln \left( \gamma \frac{\theta}{\theta-1} \frac{(\rho+\delta)-\pi^*(\theta-1)}{\gamma(\rho+\delta)-\pi^*\theta} \right)}{\pi^*(\gamma + (1 - \gamma)\theta)}. \quad (20)$$

When a time period of at least  $\hat{\tau}_B$  has elapsed since the last adjustment, rationing would no longer be desirable.

- (2) For moderate deflation,  $\pi^* \in [-\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1)), 0]$ , firms would never benefit from rationing demand.
- (3) For positive inflation,  $\pi^* \in (0, (\gamma\delta)/\theta)$ , firms would benefit from rationing demand after not having adjusted their prices for more than  $\hat{\tau}_B$  periods. They would not benefit from rationing demand before.

The intuition for the proposition, which is proved in Appendix A, is straightforward. For positive inflation, the relative price of a firm decreases over time. At some point, the price will be so low and demand so high such that it would be desirable to supply fewer goods than demanded. For moderate deflation, the relative price increases during a spell of a constant nominal price. Hence the relative price never drops below the point where it would be optimal to ration demand. For severe deflation, each firm chooses a price that, in real terms, is so low initially such that it would be desirable not to satisfy the entire demand at the beginning. As time passes by, the relative price increases under deflation and demand drops by sufficiently much so that rationing demand is no longer attractive.

It may be noteworthy that, if we imposed a zero lower bound on nominal interest rates, a steady state would only exist for  $\pi^* \geq -\rho$ . This implies that the case with severe deflation could not arise under the zero lower bound if  $-\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1)) < -\rho$ . Nevertheless the case of severe deflation would still be relevant in cashless economies, which are plausible to be increasingly relevant in the future. In Scandinavian countries, for example, cash is rarely used and its significance is waning further.

### 3. MAIN MODEL WITH POSSIBLE DEMAND RATIONING

Having seen that there are situations where firms would benefit from rationing demand in the benchmark model, we now turn to our main model, where firms actually have the possibility to do so. We note that the possibility to ration demand will lead to equilibria that differ in a nontrivial way from those identified in the previous section, as this possibility will affect firms' price-setting in general. For example, under positive inflation, one might expect that firms choose lower prices for given marginal costs compared to the benchmark model because they take into account that low relative

prices are less harmful after long spells of constant prices when rationing is possible. Later we will show that this is indeed the case.

It will be useful to examine the three regions for the inflation rate specified in Proposition 2 separately. We begin our analysis with the case of severe deflation.

**PROPOSITION 3.** *For severe deflation, that is,  $\pi^* < -\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$ , a unique equilibrium exists in our main model. Each firm rations demand in every period where it adjusts its price and in all subsequent periods until a time period of  $\hat{\tau}_D$ , which is specified in Appendix B, has elapsed since it last changed its price. If a time period of  $\hat{\tau}_D$  has elapsed since the last adjustment and in all subsequent periods until the next price adjustment, firms satisfy demand completely.*

The proof is given in Appendix B. Although markups over marginal costs are different than in the equilibrium of the benchmark economy with severe deflation, we note that the intuition for when rationing is profitable is identical. When deflation is severe, firms choose very low prices initially, as they anticipate their prices to increase over time in relative terms. This makes it optimal to ration demand for newly adjusted prices. If prices have not been changed for some time, they become sufficiently high relative to the other prices such that rationing ceases to be profitable.

Having explored the case of severe deflation, we now turn to moderate deflation:

**PROPOSITION 4.** *For moderate deflation, that is,  $\pi^* \in [-\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1)), 0]$ , a unique equilibrium exists in our main model. In this equilibrium, firms never ration demand. The equilibrium is identical to the one for the benchmark economy.*

Recall that Proposition 2 has shown that there are no incentives for firms to ration demand in the benchmark scenario under moderate deflation. Thus it appears plausible that the equilibrium of the benchmark economy also constitutes an equilibrium in our main model. This is shown formally in Appendix C.

It remains to analyze the case of positive inflation. In Appendix D, we prove

**PROPOSITION 5.** *For  $\pi^* > 0$ , a unique equilibrium exists in our main model. Firms do not ration demand for  $\hat{\tau}_R$  periods after adjusting their prices, where  $\hat{\tau}_R$  is specified in Appendix D. Afterward they start rationing demand.*

We would like to mention that, like in the case with severe deflation considered in Proposition 3, the equilibrium of our main model with positive inflation involves markups that are different from those in the benchmark case. As firms know that they will have the possibility to ration demand in the future, they choose lower markups over marginal costs. This effect and the resulting consequences for the welfare costs of inflation will be examined in more detail in the next section.

## 4. THE COSTS OF INFLATION

### 4.1 Theoretical Results

Before stating our main results regarding the costs of inflation in the main model compared to the benchmark model, we analyze how the possibility to ration demand affects the relative prices of firms. In Appendix E, we prove

**LEMMA 1.** *Consider the range of positive inflation rates for which an equilibrium of the benchmark economy exists, that is,  $\pi^* \in (0, (\gamma\delta)/\theta)$ . In the model with the possibility to ration demand, the initial relative price set by firms when they are able to adjust their prices,  $q_R$ , is always lower than  $q_B$ , the respective value in the benchmark case without the possibility to ration demand.*

Here and henceforth the subscript  $R$  stands for the main model where rationing is possible.

The lemma can be interpreted in the following way. As has been mentioned in our discussions of the benchmark model, the relative price chosen by a firm when it adjusts its price,  $q_B$ , is strictly larger than one under positive inflation, and it increases with inflation. This is simply a consequence of the fact that, under positive inflation, newly chosen prices are higher than prices that were chosen in the past. It is obvious that this pattern also holds in our main model with rationing. However, it may appear surprising at first why  $q_B > q_R$  holds for a given positive level of inflation, which is implied by Lemma 1. This finding can be explained by noting that, in our main model, prices that have not been adjusted for a long time and where rationing is profitable as a result receive a smaller weight in the price index.

The effect that demand rationing reduces the weights on very low prices is also responsible for a comparably small price dispersion  $s$  in our main model. This is demonstrated by the following proposition, which is proved in Appendix F.

**PROPOSITION 6.** *Suppose  $\pi^* \in (0, (\gamma\delta)/\theta)$ . In our main model with the possibility to ration demand, price dispersion  $s$  is always lower than in the benchmark case without the possibility to ration demand.*

As a consequence, effective productivity  $A/s$  is always higher in the main model than in the benchmark model under identical, positive inflation rates. In this sense, the benchmark model overstates the costs of inflation that arise from price dispersion.

In the next step, we compare the firms' markups across the two models. In Appendix G, we show

**PROPOSITION 7.** *Consider the range of positive inflation rates for which an equilibrium of the benchmark economy exists, that is,  $\pi^* \in (0, (\gamma\delta)/\theta)$ . When adjusting their prices, firms always choose a smaller markup over marginal costs in the model with the possibility to ration demand than in the benchmark case without this possibility. For  $\pi^* \rightarrow 0$ , the difference in markups converges to zero.*

Thus, for positive inflation rates, markups are lower in our main model than in the benchmark case. Markups are comparably high in the benchmark model, as high

markups are a precautionary measure against long price spells, which imply low relative prices and potentially high losses. In the main model, long price spells and the resulting low relative prices are less costly because firms can avoid to sell the large quantities demanded at these low prices.

Proposition 7 identifies a second reason why the assumption that firms cannot supply fewer goods than demanded tends to lead to higher costs of inflation: The possibility to ration demand entails lower markups over marginal costs and thereby reduces the inefficiencies resulting from imperfect competition.

#### 4.2 *Quantitative Results*

Our theoretical analysis has shown that the possibility to ration demand always leads to more moderate effects of inflation on price dispersion and effective aggregate productivity compared to the benchmark model where rationing is not possible. Moreover, the possibility to ration demand leads to smaller markups over marginal costs under positive inflation rates. Both effects tend to entail smaller welfare costs of inflation in our main model. In order to be able to assess the plausible magnitudes of these effects, we have to assign numerical values to the parameters of our model.

We normalize aggregate productivity to  $A = 1$ . Under the interpretation that the production function (7) is of the Cobb–Douglas form with constant capital, we can adopt  $\gamma = 2/3$ , which is a value that is often found in the literature.  $\rho$  is approximately equal to the annual real interest rate, which leads us to choose  $\rho = 0.03$ . The value of  $\theta$  is crucial for our quantitative results, as it determines the initial markup when firms set their prices. Using two different estimation techniques, Basu and Fernald (2002) find values of 13% and of as low as 5%. To stack the deck against large effects of rationing, we focus on the larger value, which implies a value of  $\theta = 1.13/(1.13 - 1) \approx 8.7$ .<sup>15</sup> We set the inverse of the intertemporal elasticity of substitution to  $\sigma = 1$  and the inverse of the Frisch elasticity of labor supply to the same value, that is,  $\varphi = 1$ . Without loss of generality, we choose  $\Psi = 1$  for the weight on the disutility of labor in the utility function. The Calvo parameter  $\delta$  determines the length of price spells,  $1/\delta$ . Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) find that prices typically remain unchanged for roughly three quarters, which leads to the choice  $\delta = 4/3$ .<sup>16</sup>

Before comparing the solutions for our two model variants, it may be interesting to compute  $(\gamma\delta)/\theta$ , the level of inflation, where an equilibrium of the benchmark economy ceases to exist. For our parameter choices,  $e^{(\gamma\delta)/\theta} \approx 1.108$ . As a consequence, the benchmark model makes the prediction that effective productivity reaches zero

15. We note that this value of  $\theta$  implies that, for positive inflation rates, the markup of firms is even higher than 13%. For example, for an inflation rate of 2%, the benchmark model implies an initial markup of over 20%.

16. Table 1 in Nakamura and Steinsson (2013) reports values of the mean duration of regular prices in the range from 8.6 to 11.7 months. Choosing a duration at the lower end of this range tends to lead to an underestimation of the effects of rationing.

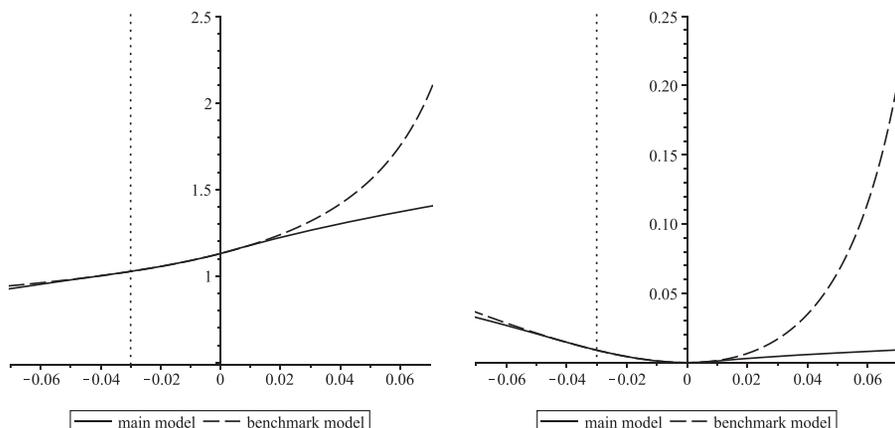


Fig 2. The Markup Over Marginal Costs Chosen by Firms that Readjust their Prices (Left Panel) and Price Dispersion (Measured as a Deviation from One, Right Panel) as Functions of Inflation  $\pi^*$ .

NOTE: The vertical dotted lines highlight the minimum inflation rate that could arise in a steady state if we imposed a zero lower bound on nominal interest rates.

at an annual inflation rate of 10.8%. This is clearly implausible as inflation rates of similar magnitude prevailed, for example, in the United States during the 1970s. As a next preliminary step, we evaluate the critical level of inflation that separates the moderate deflation scenario and the severe deflation scenario in Propositions 2–4. Evaluating  $-\gamma/(1-\gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$  yields that rationing does not occur for negative inflation rates above  $-4.0\%$ . For lower inflation rates and positive inflation rates, firms find it sometimes desirable to ration demand. One might also ask how strongly firms ration demand in our main model. Our calculations imply that, for an inflation rate of 2%, the output of intermediate goods is approximately 1.1% lower than demand on average.

To illustrate how inflation affects price-setting, we plot the initial markup over marginal costs that is selected by firms that re-adjust their prices as a function of the inflation rate for the benchmark case (dashed line) and the main model (solid line) in the left panel of Figure 2. The figure illustrates the finding of Proposition 7 that, for positive inflation, markups are smaller in the main model than in the benchmark version. In addition, it reveals that this effect is nonnegligible quantitatively even for comparably low inflation rates of a few percent. According to the figure, markups in the main model are smaller than in the benchmark case for severe deflation as well. Moreover, the figure confirms that the initial markup of adjusting firms increases with inflation in both model variants.<sup>17</sup> The dotted vertical lines in both panels represent the lowest possible steady-state inflation rate under the zero lower bound, that is,  $-\rho$ .

17. The fact that the initial markup increases with inflation does not imply that, for all given durations of price spells, markups increase with inflation.

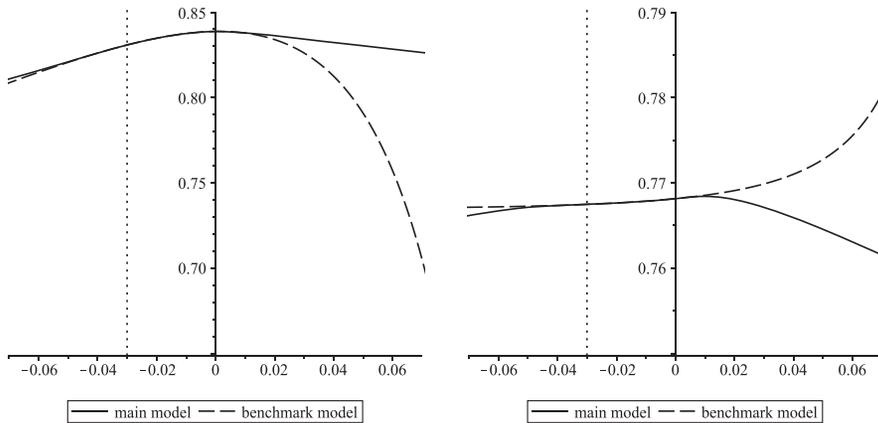


Fig 3. Final-Good Output (Left Panel) and Employment (Right Panel) as Functions of Inflation  $\pi^*$ .

NOTE: The vertical dotted lines highlight the minimum inflation rate that could arise in a steady state if we imposed a zero lower bound on nominal interest rates.

Thus the case with severe deflation, where firms that adjust their prices ration demand initially, cannot occur for the calibration under consideration if we impose the zero lower bound on nominal interest rates.

The panel on the right-hand side of Figure 2 shows price dispersion  $s$  for different inflation rates  $\pi^*$ . In both model variants, price dispersion is minimal at zero inflation. The figure illustrates the claim of Proposition 6 that, under positive inflation rates,  $s$  is larger in the benchmark case than in the main model. The figure also shows that the effect is quantitatively large. A closer look at our simulation results reveals, for example, that an annual inflation rate of 5% leads to an increase of price dispersion by 6.1% compared to the zero-inflation scenario. By contrast, our main model predicts a moderate increase of 0.7% in this case. At this point, it may be worth remembering that increases in price dispersion directly translate into decreases in effective productivity,  $A/s$ . Thus the benchmark model predicts that effective productivity decreases by 5.8% percent when inflation is at 5% rather than at zero. Our main model with rationing predicts that an inflation rate of 5% leads to a drop in effective productivity by only 0.7%.

The differences in effective productivities and markups across the two models also lead to differences in output and employment. These differences are shown in Figure 3. The left-hand panel, which shows output for the two models, demonstrates that the output loss caused by inflation is considerably more moderate in our main model than in the benchmark scenario. The right-hand panel reveals that both models predict a very different response of employment. In the benchmark model, higher inflation rates lead to sizable increases in employment, whereas the main model predicts decreases in employment in response to higher inflation rates above 1%.

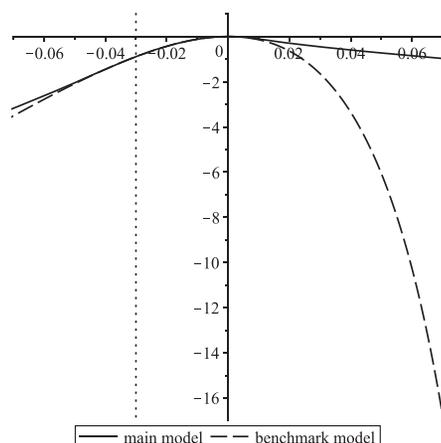


Fig 4. Consumption-equivalent Welfare, Measured in Percent, as a Function of  $\pi^*$ .

NOTE: The vertical dotted line highlights the minimum inflation rate that could arise in the steady state if we imposed a zero lower bound on nominal interest rates.

We conclude this section with a quantitative analysis of the effects of inflation on welfare. For this purpose, Figure 4 plots the relative change in consumption in the zero-inflation steady state that would make the household indifferent to the steady state with an inflation rate of  $\pi^*$  in the benchmark model (dashed line) and the main model (solid line). The figure confirms that the welfare costs of inflation are substantially higher in the benchmark model. For example, the benchmark model predicts that an inflation rate of 5% leads to a consumption-equivalent welfare loss of 5.7% compared to the steady state with zero inflation. In our main model, the welfare loss is only 0.7% in this case. It is clear that the differences in welfare losses would be even higher for larger inflation rates.

It may be instructive to compare the value of 0.7% to estimates of welfare losses obtained by the traditional approach that considers the area under the money demand function (see, e.g., Fischer 1981, Lucas 1981). The traditional approach tends to yield even smaller losses caused by moderate inflation. For example, Lucas (1981) estimates the welfare costs of 5% inflation to be equivalent to a 0.1% drop in consumption.

Interestingly, in the benchmark model the costs of inflation and deflation are highly asymmetric. For example, a positive inflation rate of 3% involves welfare losses of 1.6% and thus is more costly than a negative inflation rate of -3%, which entails welfare losses of 0.9%. For our main model, the pattern is reversed qualitatively: A positive rate of inflation is always socially more desirable than a negative inflation rate of the same magnitude. For example, an inflation rate of 3% corresponds to a 0.4% drop in consumption, while a negative inflation rate of -3% is equivalent to a decrease of consumption by 0.9%.

## 5. ROBUSTNESS

In this section, we evaluate the robustness of our results by considering three different variants of our model. First, we analyze constant returns scale. Second, we examine how the introduction of a fixed cost of production affects our findings. Third, we discuss our implicit assumption that rationing demand involves no costs.

### 5.1 Constant Returns to Scale

The welfare losses caused by inflation in our main model are affected by the differences in marginal costs across firms. Under constant returns to scale, these differences would disappear and thus the social costs of inflation are plausible to be lower. Hence this section assesses the extent to which our results change under constant returns to scale.

By setting  $\gamma = 1$ , it is straightforward to extend our previous results to the case of constant returns to scale. First, the benchmark economy only has an equilibrium for  $\pi^* < \delta/\theta$ . Second, firms never have an incentive to ration demand under deflation, irrespective of the level of  $\pi^*$ . For positive inflation, firms find it profitable to ration demand by stopping production completely after they have not adjusted their prices for  $\hat{\tau}_{CRTS}$  periods, where  $\hat{\tau}_{CRTS} > 0$ . Third, for positive inflation, price dispersion and the initial markup are always higher in the benchmark model than in the main model. Under deflation, both models lead to identical outcomes.

Having seen that the qualitative results under constant returns to scale are similar to the results obtained before, we now turn to the magnitude of the effects in the scenario with constant returns to scale. Figure 5 shows the consequences of inflation for price dispersion and the welfare costs of inflation as functions of  $\pi^*$  for the model with rationing and the benchmark model. Although the effects of rationing are smaller compared to the case of decreasing returns to scale, they are nonetheless nonnegligible, in particular for inflation rates higher than 7%.

### 5.2 Fixed Cost of Production

Introducing a fixed cost of production (see, e.g., Smets and Wouters 2003, Christiano, Eichenbaum, and Evans 2005) into our model is plausible to make rationing more prevalent. Thus one might conjecture that, in the presence of a fixed cost, the possibility that firms ration demand alleviates the social costs of inflation even more strongly than in our basic model. This conjecture is examined more closely in the following.

For this purpose, we replace the production function (7) by

$$y_j(t) = \begin{cases} A(n_j(t))^\gamma - \Phi & \text{if } A(n_j(t))^\gamma - \Phi \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (21)$$

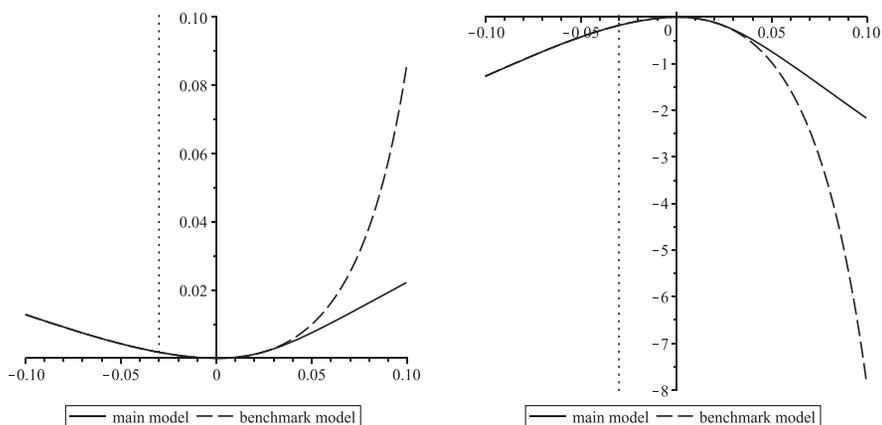


Fig 5. The Case of Constant Returns to Scale: Price Dispersion (Measured as a Deviation from One, Left Panel) and the Consumption-equivalent Welfare Losses (Right Panel) as Functions of Inflation  $\pi^*$ .

NOTE: The vertical dotted lines highlight the minimum inflation rate that could arise in a steady state if we imposed a zero lower bound on nominal interest rates.

where  $\Phi$  ( $\Phi > 0$ ) stands for the fixed cost. It is straightforward to derive that, in such a variant of our model, three regions for a firm's relative price can be distinguished. First, the price may be so high such that the firm always satisfies the demand for its goods. Second, the price may be intermediate such that the firm finds it optimal to ration demand, while continuing to produce a positive level of output. Third, the price may be so low such that the firm stops operating completely.

As it is not possible to solve this model variant analytically, we proceed by analyzing it numerically, where the algorithm is described in Appendix H. We set  $\Phi = 0.29$  and otherwise choose the same calibration as before. The value of  $\Phi$  implies that the fixed cost corresponds to roughly 45% of output for a zero-inflation steady state, which is the mean of the respective prior distribution in Smets and Wouters (2003).<sup>18</sup>

According to Figure 6, rationing is comparably common in the model with fixed cost. The fraction of firms that ration demand increases with inflation, as higher inflation rates entail larger fractions of firms with low relative prices. For inflation rates of 5%, approximately 17% of all firms produce a positive level of output that is lower than demand. For rather high inflation rates of around 7%, approximately 1% of firms have real prices that are so low such that they do not produce at all.

However, the prevalence of rationing does not imply particularly low social costs of inflation as rationing is a two-edged sword in the presence of a fixed cost. On the one hand, it avoids the high output levels of firms that have not adjusted their prices for some time. This effect is also present in the absence of a fixed cost. On the other hand,

18. In the literature,  $\Phi$  is often chosen such that steady-state profits are zero. This approach cannot be applied here, as profits depend on the inflation rate.

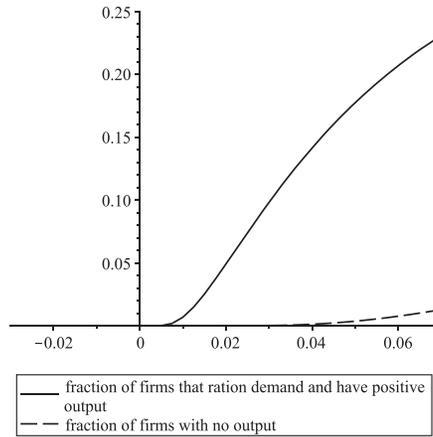


Fig 6. The Case of a Fixed Cost of Production: the Shares of Firms that Ration Demand as Functions of Inflation  $\pi^*$ .

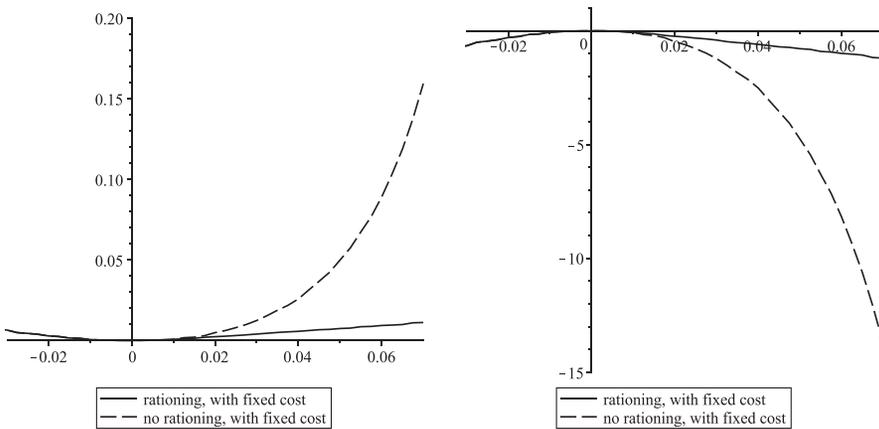


Fig 7. The Case of Fixed Costs of Production: Price Dispersion (Measured as a Deviation from One, Left Panel) and the Consumption-equivalent Welfare Losses (Right Panel) as Functions of Inflation  $\pi^*$ .

for sufficiently high inflation rates, there are some firms that discontinue production entirely. The resulting reduction in the number of intermediate-product varieties has the potential to lower welfare.

Moreover, the introduction of a fixed cost makes marginal costs more similar across firms, as can be verified easily. This effect tends to lower the social costs of inflation both in the absence of rationing and in the case where rationing is possible. Overall, Figure 7 shows that the possibility of rationing has effects on welfare and price dispersion, that is,  $(An^\gamma - \Phi)/y$ , that are similar in magnitudes to the ones in our main model.

### 5.3 *Costs of Rationing*

Standard new Keynesian models, which exclude the possibility of demand rationing, can be interpreted as assuming that rationing demand would involve very large costs, e.g. permanent exclusion from markets. From this perspective, the present paper takes the opposite view that rationing involves no costs at all. One might argue that some intermediate case with finite, yet nonnegligible costs of rationing is plausible. Under this interpretation, the standard new Keynesian model establishes an upper bound on the social costs of inflation, whereas the present paper establishes a lower bound.

Are the true social costs of inflation more likely to be closer to the lower bound identified in this paper or the upper bound identified by the standard approach? To answer this question, it may be helpful to compare the frequency of rationing in our model and its empirical counterpart. If the frequency of rationing in the data were much lower than in our model, this would be an indication that the costs of rationing are nonnegligible in practice.

For inflation rates of 2%, the main model can be shown to imply that 1.1% of the demand for goods is not satisfied. For the scenario with fixed costs considered in Section 5.2, which entails substantially more frequent rationing, we obtain a value of approximately 4.9%. Even the latter value is not implausibly large, as Bils (2016) reports average stockout rates of 4.6% or 7.8%, depending on the method used. Although a more thorough analysis would also take into account additional factors that could drive stockouts, the fact that our model does not predict unreasonably high levels of stockouts can be seen as an indication that its predictions about welfare may not be wide of the mark.

## 6. CONCLUSIONS

This paper has pointed out that the large costs of inflation under Calvo pricing rely on the implicit assumption that firms always have to satisfy the demands for their goods, even in cases where doing so entails negative profits. Relaxing this assumption leads to substantially more moderate costs of inflation and empirically more plausible relationships between inflation and other economic variables like employment and output.

Although this paper has focused on stationary equilibria, it would be interesting to study the dynamics of our economy in response to shocks in future work. In such an extension to our framework, one could assess how the results in the literature about the impact of trend inflation on the short-term dynamics of the new Keynesian model and equilibrium determinacy would be affected by the assumption that firms can choose the quantities they produce optimally. As explained in Ascari and Sbordone (2014), trend inflation makes price setters effectively more forward-looking under Calvo pricing by increasing the importance of future marginal costs for price-setting. This effect tends to increase the parameter region for the Taylor-rule coefficients that result in

indeterminacy. It appears plausible that rationing would mitigate this effect, as it reduces the influence of future marginal costs on price-setting.

Finally, it might be interesting to speculate how the mechanism outlined in this paper would affect the optimal level of the inflation target when shocks may push interest rates toward the zero lower bound from time to time. As our mechanism is plausible to reduce the costs of inflation also in a richer model that features the zero lower bound on nominal interest rates, it might provide an argument in favor of higher inflation targets.

However, caution may be advisable with respect to raising inflation targets as there may be other costs of inflation that are not captured by the standard new Keynesian model and the model in this paper. At any rate, this paper supports the point made by Nakamura et al. (2018) that the specific mechanism that is responsible for the costs of inflation in new Keynesian models may be less relevant than previously thought.

## APPENDIX A: PROOF OF PROPOSITION 2

Let us assume, without loss of generality, that a particular firm  $j$  has adjusted its price in period 0. Let  $\hat{\tau}$  be the duration of the price spell where the firm is indifferent between rationing demand or not. With the help of  $q$ , which is the initial relative price chosen by a firm when it adjust its price, we can state the following condition for  $\hat{\tau}$

$$\left( \gamma \frac{A^\frac{1}{\gamma}}{w} q e^{-\pi^* \hat{\tau}} \right)^{\frac{\gamma}{1-\gamma}} = q^{-\theta} e^{\pi^* \theta \hat{\tau}} y, \quad (\text{A1})$$

where we have used  $y_j^s(\hat{\tau}) = y_j^d(\hat{\tau})$ , (6), (8), as well as the observation that,  $\hat{\tau}$  periods after the relative price of firm  $j$  has been set to  $q$ , it is  $e^{-\pi^* \hat{\tau}} q$ . Solving (A1) for  $\hat{\tau}$  yields

$$\hat{\tau} = \frac{\log(q)}{\pi^*} + \frac{\log\left(\left(\frac{\gamma}{w}\right)^\frac{\gamma}{1-\gamma} \frac{A}{y^{1-\gamma}}\right)}{\kappa \pi^*}, \quad (\text{A2})$$

where

$$\kappa := \gamma + (1 - \gamma)\theta. \quad (\text{A3})$$

Using (19) to substitute for  $q$  in (A2), we obtain the expression for  $\hat{\tau}_B$  stated in the proposition, where we have added the subscript  $B$  to denote the benchmark solution. It is obvious from this expression that  $\hat{\tau}_B$  is strictly positive for  $\pi^* > 0$ . It is also clear that, for positive inflation rates, the relative price of the intermediate firm's good,  $q e^{-\pi^* \tau}$ , decreases over the duration of the price spell. For positive inflation rates, we therefore conclude that the price is sufficiently high for  $\tau < \hat{\tau}_B$  such that  $y_j^s(\tau) > y_j^d(\tau)$  and that it is so low for  $\tau > \hat{\tau}_B$  such that  $y_j^s(\tau) < y_j^d(\tau)$ . This implies that rationing is desirable if and only if  $\tau > \hat{\tau}_B$ .

For  $\pi^* \in [-\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1)), 0]$ , no solution for  $\hat{\tau}_B$  exists in Equation (20), and  $y_j^s(\tau) \geq y_j^d(\tau)$  holds for all  $\tau \geq 0$ . Hence rationing would never

be profitable for intermediate firms. For  $\pi^* < -\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$ ,  $\hat{\tau}_B$ , given by (20), is strictly positive. Moreover, we note that the relative price of the firm's good increases over time under deflation. As a consequence,  $y_j^s(\tau) < y_j^d(\tau)$  for  $\tau < \hat{\tau}_B$  and  $y_j^s(\tau) > y_j^d(\tau)$  for  $\tau > \hat{\tau}_B$ . Thus rationing would be profitable initially after an intermediate firm has adjusted its price. After  $\hat{\tau}_B$  periods, rationing demand would cease to be attractive.

APPENDIX B: PROOF OF PROPOSITION 3

As has been shown in Proposition 2, an equilibrium for  $\pi^* < -\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$  would necessarily involve that intermediate firms sometimes ration demand. Assume, again without loss of generality, that firm  $j$  adjusts its price in period 0. Then it has to choose its relative initial price  $q$  to maximize its profits over the duration of the price spell,

$$\int_0^{\hat{\tau}_D} \left( qe^{-\tau\pi^*} y_j^s(\tau) - w \left( \frac{y_j^s(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau + \int_{\hat{\tau}_D}^{\infty} \left( qe^{-\tau\pi^*} y_j^d(\tau) - w \left( \frac{y_j^d(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau, \tag{B1}$$

where  $\hat{\tau}_D$  is given by (A2), the relative price in period  $\tau$  is  $qe^{-\tau\pi^*}$ , and we have taken into account that the intermediate firm's output is determined by (8) until period  $\hat{\tau}_D$  and by (6) afterward. It is very tedious but straightforward to show that the first-order condition can be rewritten as<sup>19</sup>

$$\xi^{\frac{1}{1-\gamma} + \frac{\delta+\rho}{\pi^*}} = 1 - \frac{((\rho + \delta) + \theta(\theta - 1)\pi^* \frac{1-\gamma}{\gamma})(\rho + \delta + \frac{\pi^*}{1-\gamma})}{(\rho + \delta - \frac{\pi^*\theta}{\gamma})(\rho + \delta - (\theta - 1)\pi^*)}, \tag{B2}$$

where, with the help of  $\kappa$  (see (A3)), we have introduced

$$\xi := q \left( \left( \frac{\gamma}{w} \right)^\gamma \frac{A}{y^{1-\gamma}} \right)^{\frac{1}{\kappa}}. \tag{B3}$$

As a next step, we analyze the fraction on the right-hand side of (B2) more closely. As  $\pi^* < 0$ , the denominator is strictly positive. The first term in the numerator,  $(\rho + \delta) + \theta(\theta - 1) \cdot \pi^* \cdot (1 - \gamma)/\gamma$ , is strictly negative for the values of  $\pi^*$  under consideration. For the remaining term in the numerator,  $\rho + \delta + \pi^*/(1 - \gamma)$ , two cases need to be distinguished. First, it may be strictly positive. In this case, the right-hand side of (B2) is strictly larger than 1. As the exponent of  $\xi$  on the left-hand side is strictly negative for  $\rho + \delta + \pi^*/(1 - \gamma) > 0$ , we can conclude that, for

19. A detailed derivation is available on request.

$\rho + \delta + \pi^*/(1 - \gamma) > 0$ , (B2) has a unique solution for  $\xi$ , which is positive and smaller than 1. Second,  $\rho + \delta + \pi^*/(1 - \gamma) < 0$  may hold, which implies that the right-hand side of (B2) is strictly smaller than 1.<sup>20</sup> It is easy to see that the right-hand side of (B2) is also strictly positive. As the exponent of  $\xi$  on the left-hand side is positive in the constellation under consideration, we can conclude again that a unique solution for  $\xi$  exists and that this solution is positive and smaller than 1.

We note that  $\xi$  and  $\hat{\tau}_D$  are linked via

$$\hat{\tau}_D = \log(\xi)/\pi^*. \tag{B4}$$

As  $\pi^* < 0$  and  $\xi < 1$ , we obtain that  $\hat{\tau}_D$  is strictly positive.

Having analyzed the first-order condition for an intermediate firm’s profit-maximization problem, we now turn to the condition that links aggregate output and the output levels of intermediate goods, that is, equation (5). As in our case, the intermediate output of a firm that adjusted its price fewer than  $\hat{\tau}_D$  periods ago is given by  $y_j^s(\tau)$  (see (8)) and by  $y_j^d(\tau)$  (see (6)) otherwise, (5) can be stated as

$$y^{\frac{\theta-1}{\theta}} = \delta \left( \gamma \frac{A^{\frac{1}{\gamma}}}{w} q \right)^{\frac{\gamma}{1-\gamma} \frac{\theta-1}{\theta}} \int_0^{\hat{\tau}_D} e^{-\left(\delta + \frac{\gamma}{1-\gamma} \frac{\theta-1}{\theta} \pi^*\right) \tau} d\tau + \frac{\delta}{q^{\theta-1}} y^{\frac{\theta-1}{\theta}} \int_{\hat{\tau}_D}^{\infty} e^{-(\delta - \pi^*(\theta-1))\tau} d\tau. \tag{B5}$$

Using (A2), (A3), as well as (B3) and re-arranging yields

$$q = \left[ \left( \frac{\delta}{\delta(1-\gamma)\theta + \pi^*\gamma(\theta-1)} \right) \left( (1-\gamma)\theta \xi^{\frac{\kappa}{1-\gamma} \frac{\theta-1}{\theta}} + \frac{\pi^*(\theta-1)\kappa \xi^{\theta-1-\frac{\delta}{\pi^*}}}{(\delta - \pi^*(\theta-1))} \right) \right]^{\frac{1}{\theta-1}}. \tag{B6}$$

With the help of  $0 < \xi < 1$ , (A3), and  $\pi^* < -\gamma/(1 - \gamma) \cdot (\rho + \delta)/(\theta(\theta - 1))$ , it is possible to show that the term in parentheses on the right-hand side is strictly positive. Hence (B6) always defines a unique value of  $q$ .

Having examined the intermediate-goods firm’s first-order condition and the final-goods producers’ production function, it remains to analyze the labor-market clearing condition  $n = \int_0^1 n_j(t) dj = \int_0^1 (y_j(t)/A)^{\frac{1}{\gamma}} dj$ . Using that  $y_j(\tau) = y_j^s(\tau)$  for firms with price spells  $\tau < \hat{\tau}_D$  and  $y_j(\tau) = y_j^d(\tau)$  for  $\tau > \hat{\tau}_D$ , where  $y_j^d(\tau)$  and  $y_j^s(\tau)$  are given in (6) and (8), we obtain

$$n = \frac{\delta}{A^{\frac{1}{\gamma}}} \left( \gamma \frac{A^{\frac{1}{\gamma}}}{w} q \right)^{\frac{1}{1-\gamma}} \int_0^{\hat{\tau}_D} e^{-\left(\delta + \frac{\pi^*}{1-\gamma}\right) \tau} d\tau + \frac{\delta}{A^{\frac{1}{\gamma}} q^{\frac{\theta}{\gamma}}} y^{\frac{1}{\gamma}} \int_{\hat{\tau}_D}^{\infty} e^{-\left(\delta - \frac{\pi^*\theta}{\gamma}\right) \tau} d\tau. \tag{B7}$$

20. For the sake of brevity, we omit the analysis of the knife-edge case where  $\rho + \delta + \pi^*/(1 - \gamma) = 0$ , as it is straightforward and does not deliver additional insights.

After several steps, this expression can be shown to be equivalent to

$$n = \frac{1}{A^{\frac{1}{\gamma}}} \frac{\delta y^{\frac{1}{\gamma}}}{q^{\frac{\theta}{\gamma}}} \frac{\xi^{\frac{\kappa}{\gamma(1-\gamma)}}}{\delta + \frac{\pi^*}{1-\gamma}} \left( 1 + \frac{\xi^{-\frac{\delta}{\pi^*} - \frac{1}{1-\gamma}} \kappa \pi^*}{(1-\gamma)(\gamma\delta - \pi^*\theta)} \right). \tag{B8}$$

The measure of price dispersion,  $s = (An^\gamma)/y$  can thus be written as

$$s = \left[ \left( \frac{\delta}{\delta + \frac{\pi^*}{1-\gamma}} \right) \left( 1 + \frac{\xi^{-\frac{\delta}{\pi^*} - \frac{1}{1-\gamma}} \kappa \pi^*}{(1-\gamma)(\gamma\delta - \pi^*\theta)} \right) \right]^\gamma \frac{\xi^{\frac{\kappa}{1-\gamma}}}{q^\theta}. \tag{B9}$$

It is immediate to verify that the expression in brackets is always positive. Hence, for given positive values of  $\xi$  and  $q$ , a unique positive value of  $s$  exists.

The uniqueness of the equilibrium follows from our previous finding that (B2), (B6), and (B9) admit unique positive solutions for  $\xi$ ,  $q$ , and  $s$ . Together with  $s = (An^\gamma)/y$  and (B3), (11) and (12) can be used to determine the equilibrium values of  $i$ ,  $n$ ,  $y$ , and  $w$ . The value of  $\hat{\tau}_D$  can be easily derived with the help of (B4).

APPENDIX C: PROOF OF PROPOSITION 4

As shown in Proposition 2, provided that  $q$  is set as in the benchmark equilibrium, rationing demand would always make firms worse off. It remains to verify that firms would not benefit from setting  $q$  to a value sufficiently low so that rationing is optimal for some time. However, our analysis in Appendix B has shown implicitly that this is not the case for  $\pi^* \in [-\gamma/(1-\gamma) \cdot (\rho + \delta)/(\theta(\theta - 1)), 0]$ .

APPENDIX D: PROOF OF PROPOSITION 5

We begin our analysis of the economy with possible rationing of demand for  $\pi^* > 0$  by looking at the profit-maximization problem of an intermediate firm. As before, we assume without loss of generality that the firm adjusts its price in period 0. Let us use  $q$  again to denote the relative price that the firm chooses in period 0. Because the relative price of the firm,  $qe^{-\pi^*\tau}$ , will fall over the duration of a price spell, it is clear that there will be a price duration,  $\hat{\tau}_R$ , after which rationing demand is strictly profitable.  $\hat{\tau}_R$  is strictly positive, as selecting a value of  $q$  that is so low such that rationing demand is always desirable cannot be profitable.<sup>21</sup>

21. Note that per-period profits, under the assumption that  $y_j^d(t) < y_j^s(t)$ , decrease with the relative price in the respective period.

The firm's discounted profits over the duration of the price spell that starts in period 0 are

$$\int_0^{\hat{\tau}_R} \left( qe^{-\tau\pi^*} y_j^d(\tau) - w \left( \frac{y_j^d(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau + \int_{\hat{\tau}_R}^{\infty} \left( qe^{-\tau\pi^*} y_j^s(\tau) - w \left( \frac{y_j^s(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau, \quad (D1)$$

where  $\hat{\tau}_R$  is given by (A2). Additionally, we have taken into account that the relative price in period  $\tau$  is  $qe^{-\tau\pi^*}$  and that output is determined by  $y_j^d(\tau)$  until period  $\hat{\tau}_R$  and by  $y_j^s(\tau)$  afterward. Utilizing (6), (8), and (B4), the resulting first-order condition can be simplified to

$$f(\xi) := -(\theta - 1) \frac{\xi^{\frac{\theta+\delta}{\pi^*} - (\theta-1)} - 1}{\rho + \delta - (\theta - 1)\pi^*} + \theta \frac{\xi^{\frac{\theta+\delta}{\pi^*} - \frac{\theta}{\gamma}} - 1}{\rho + \delta - \frac{\pi^*\theta}{\gamma}} + \frac{1}{\rho + \delta + \frac{\pi^*}{1-\gamma}} = 0. \quad (D2)$$

We note that  $\hat{\tau}_R \geq 0$  requires  $\xi \geq 1$ , as  $\hat{\tau}_R = \log(\xi)/\pi^*$ . In order to establish that (D2) has a unique solution for  $\xi$  with  $\xi \geq 1$ , we highlight several properties of  $f(\xi)$ . First,  $f(\xi)$  is a continuously differentiable function. Second,  $f(1) = 1/(\rho + \delta + \pi^*/(1 - \gamma)) > 0$  holds. Third, we note that  $f'(1) = 1/\pi^*$ . Fourth,  $f'(\xi) = 0$  has a unique solution for  $\xi > 1$ . It is given by  $\xi = (\theta/(\theta - 1))^{\gamma/\kappa}$ . Fifth,  $\lim_{\xi \rightarrow \infty} f(\xi) = -\infty$ . Taken together, these properties imply that  $f(\xi) = 0$  has a unique solution for  $\xi$ , which satisfies  $\xi > (\theta/(\theta - 1))^{\gamma/\kappa} > 1$ .

As a next step, we evaluate the condition that relates aggregate output and intermediate inputs, that is, (5). For positive inflation rates, the intermediate output of a firm that adjusted its price fewer than  $\hat{\tau}_R$  periods ago is given by  $y_j^d(\tau)$  (see (6)) and by  $y_j^s(\tau)$  (see (8)) otherwise. Consequently, (5) can be written as

$$y^{\frac{\theta-1}{\theta}} = \frac{\delta}{q^{\theta-1}} y^{\frac{\theta-1}{\theta}} \int_0^{\hat{\tau}_R} e^{-(\delta-\pi^*(\theta-1))\tau} d\tau + \delta \left( \gamma \frac{A^{\frac{1}{\gamma}}}{w} q \right)^{\frac{\gamma}{1-\gamma} \frac{\theta-1}{\theta}} \int_{\hat{\tau}_R}^{\infty} e^{-\left(\delta + \frac{\gamma}{1-\gamma} \frac{\theta-1}{\theta} \pi^*\right)\tau} d\tau. \quad (D3)$$

Using (A2), (A3), and (B3) and re-arranging yields

$$q = \left[ \frac{\delta}{\delta - (\theta - 1)\pi^*} \left( 1 - \frac{\pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} \right) \right]^{\frac{1}{\theta-1}}. \quad (D4)$$

For  $\xi > 1$  and  $\pi^* < \frac{\delta}{\theta-1}$ , one can easily demonstrate

$$\frac{\pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} < 1. \quad (D5)$$

Because of (D5) and  $\delta/(\delta - (\theta - 1)\pi^*) > 0$ , (D2) involves a real solution for  $q$  for any given level of  $\xi$  with  $\xi > 1$ .

The labor-market clearing condition,  $n = \int_0^1 n_j(t) dj = \int_0^1 (y_j(t)/A)^{\frac{1}{\gamma}} dj$ , can be shown to be equivalent to

$$s = q^{-\theta} \left[ \frac{\delta}{\delta - \frac{\pi^* \theta}{\gamma}} \left( 1 - \frac{\pi^* \kappa \xi^{-\frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}}{\gamma(\delta(1-\gamma) + \pi^*)} \right) \right]^\gamma, \tag{D6}$$

where  $s = (An^\gamma)/y$ .

The uniqueness of the equilibrium follows from the fact that (D2), (D4), (D6) admit unique positive solutions for  $\xi$ ,  $q$ , and  $s$ . Together with  $s = (An^\gamma)/y$  and  $\xi = q[(\gamma/w)^\gamma \cdot A/y^{1-\gamma}]^{1/\kappa}$ , (11) and (12) can be used to determine the equilibrium values of  $i$ ,  $n$ ,  $y$ , and  $w$ . The value of  $\hat{r}_R$  follows from (B4). As  $\xi > 1$ , this value is strictly positive.

APPENDIX E: PROOF OF LEMMA 1

To show the lemma, we have to compare  $q$  in the benchmark case, which is given by (14), with the respective value for the model with the possibility of rationing, that is, (D4). For  $\pi^* > 0$ , it is clear that

$$\left( 1 - \frac{\pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1-\gamma)\theta + \pi^*\gamma(\theta - 1)} \right)^{\frac{1}{\theta-1}} < 1, \tag{E1}$$

which proves the statement of the lemma.

APPENDIX F: PROOF OF PROPOSITION 6

To establish that  $s$  is smaller in the economy with the possibility of rationing than in the benchmark case, we combine (D4) and (D6) to yield the following expression for  $s$  in the main model with positive inflation:

$$s_R = \left( \frac{\delta}{\delta - \frac{\pi^* \theta}{\gamma}} \right)^\gamma \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\frac{\theta}{\theta-1}} \frac{\left( 1 - \frac{\pi^* \kappa \xi^{-\frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}}{\gamma(\delta(1-\gamma) + \pi^*)} \right)^\gamma}{\left( 1 - \frac{\pi^*(\theta-1)\kappa \xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1-\gamma)\theta + \pi^*\gamma(\theta-1)} \right)^{\frac{\theta}{\theta-1}}} \tag{F1}$$

For the benchmark case, it has been shown before that  $s$  amounts to

$$s_B = \left( \frac{\delta}{\delta - \frac{\pi^* \theta}{\gamma}} \right)^\gamma \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\frac{\theta}{\theta-1}}. \tag{F2}$$

Proving the proposition therefore amounts to establishing that

$$\left( 1 - \frac{\pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1-\gamma)\theta + \pi^*\gamma(\theta - 1)} \right)^{\frac{\theta}{\theta-1}} > \left( 1 - \frac{\pi^* \kappa \xi^{-\frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}}{\gamma(\delta(1-\gamma) + \pi^*)} \right)^\gamma \tag{F3}$$

or, equivalently, that

$$\left(1 - \frac{\pi^*(\theta - 1)\kappa\xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)}\right)^{\frac{\theta}{\theta - 1} \frac{1}{\gamma}} - 1 + \frac{\pi^*\kappa\xi^{-\frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}}{\gamma(\delta(1 - \gamma) + \pi^*)} > 0. \quad (\text{F4})$$

As  $\theta/(\theta - 1) \cdot 1/\gamma > 1$  and (D5), Bernoulli's inequality implies for the first term in (F4) that

$$\left(1 - \frac{\pi^*(\theta - 1)\kappa\xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)}\right)^{\frac{\theta}{\theta - 1} \frac{1}{\gamma}} > 1 - \frac{\theta}{\theta - 1} \frac{1}{\gamma} \frac{\pi^*(\theta - 1)\kappa\xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)}. \quad (\text{F5})$$

Hence to prove (F4), it is sufficient to show

$$-\frac{\theta}{\theta - 1} \frac{1}{\gamma} \frac{\pi^*(\theta - 1)\kappa\xi^{-\frac{\delta}{\pi^*} + \theta - 1}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} + \frac{\pi^*\kappa\xi^{-\frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}}{\gamma(\delta(1 - \gamma) + \pi^*)} > 0. \quad (\text{F6})$$

Equation (F6) holds if

$$-\frac{\theta\xi^{\theta - 1 - \frac{\theta}{\gamma}}}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} + \frac{1}{\delta(1 - \gamma) + \pi^*} > 0. \quad (\text{F7})$$

As the exponent of  $\xi$  satisfies  $\theta - 1 - \theta/\gamma < 0$ , the left-hand side of (F7) is an increasing function of  $\xi$ . According to the proof of Proposition 5,  $\xi > (\theta/(\theta - 1))^{\gamma/\kappa}$ , where  $\kappa = \gamma + (1 - \gamma)\theta$ . As a consequence, it suffices to show that the left-hand side of (F7), evaluated at  $\xi = (\theta/(\theta - 1))^{\gamma/\kappa}$ , is strictly positive. This gives us the following sufficient condition

$$-\frac{\theta - 1}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} + \frac{1}{\delta(1 - \gamma) + \pi^*} > 0, \quad (\text{F8})$$

which can be re-arranged as

$$(\theta - 1)\pi^* < \delta. \quad (\text{F9})$$

As (F9) holds for  $\pi^* < (\gamma\delta)/\theta$ , we have shown the claim of the proposition.

## APPENDIX G: PROOF OF PROPOSITION 7

We can define firm  $j$ 's marginal cost of production in period  $t$  as

$$MC_j(t) := w(t) \frac{dn_j(t)}{dy_j(t)}. \quad (\text{G1})$$

In the period where the price is set, we can use the equation for demand (6) and firm  $j$ 's production function (7) to write the marginal cost as

$$MC = \frac{w (q^{-\theta} y)^{\frac{1-\gamma}{\gamma}}}{\gamma A^{\frac{1}{\gamma}}}. \quad (\text{G2})$$

With the help of (G2), we can reformulate (19) as

$$q_B = \frac{\theta}{\theta - 1} \frac{\gamma(\rho + \delta - \pi^*(\theta - 1))}{\gamma(\rho + \delta) - \pi^*\theta} MC. \quad (\text{G3})$$

In the special case where  $\pi^* = 0$ , we obtain  $q_B = \theta/(\theta - 1) \cdot MC$ , which is the standard result that firms choose a markup of  $\theta/(\theta - 1)$  over marginal costs.

Using (B3), and (G2), we obtain that the optimal price is given by  $q_R = \xi^{\kappa/\gamma} MC$  in the main model, where  $\xi$  is the solution to  $f(\xi) = 0$  (see (D2)). The claim of the proposition,  $q_B > q_R$  for fixed  $MC$ , is thus equivalent to

$$x > \xi, \quad (\text{G4})$$

where

$$x := \left( \gamma \frac{\theta}{\theta - 1} \frac{\rho + \delta - \pi^*(\theta - 1)}{\gamma(\rho + \delta) - \pi^*\theta} \right)^{\frac{\gamma}{\kappa}}. \quad (\text{G5})$$

As  $f(\xi)$  decreases monotonically for values of  $\xi$  larger than the one implied by  $f(\xi) = 0$ , it is sufficient to show

$$f(x) < 0. \quad (\text{G6})$$

It is tedious but straightforward to derive

$$f(x) = - \frac{\pi^*(\rho + \delta)\kappa^2}{(\rho + \delta - \pi^*(\theta - 1))(\gamma(\rho + \delta) - \pi^*\theta)(\pi^* + (1 - \gamma)(\rho + \delta))}. \quad (\text{G7})$$

This expression is strictly negative for  $\pi^* \in (0, (\gamma\delta)/\theta)$  and converges to zero for  $\pi^* \rightarrow 0$ , which proves the claims of the proposition.

## APPENDIX H: ALGORITHM FOR THE MODEL WITH FIXED COST

For each given level of steady-state inflation, we adopt the following iteration procedure. We start with a guess for aggregate output and wages. For given levels of inflation, aggregate output, and wages, we compute the optimal relative price of a firm that is allowed to reset its price, where, for each point in time in the future, the firm has to take into account which option will be optimal: (i) satisfying demand, (ii) rationing demand but producing a positive quantity, or (iii) not producing at all. As a next step, we compute aggregate output (with the help of (5)) and employment (with

the help of  $n = \int_0^1 n_j(t) dj$ ) under the assumption that all firms choose the optimal relative price whenever they adjust their prices. We then use the labor-supply condition (12) to calculate the wage implied by these values of employment and output. We use a weighted average of the previous guesses and the new values for the wage and aggregate output as starting points for the next iteration. Once the change in output and employment in an iteration is smaller than a prespecified value, we stop.

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