Full counting statistics of incoherent Andreev transport

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Abstract. – We study the full counting statistics of heterostructures consisting of normal-metal parts connected to a superconducting terminal. Assuming that coherent superconducting correlations are suppressed in the normal metals, we show, using the Keldysh-Nambu Green's function approach, that the system can be mapped onto a purely normal system with twice the number of elements. For a superconducting beamsplitter with several normal terminals we obtain general results for the counting statistics.

Introduction. – A complete statistical description of a transport process can be obtained from the full counting statistics (FCS) [1]. The FCS of charge transport in mesoscopic conductors has recently attracted a lot of attention [2]. It was shown that scattering between uncorrelated Fermi leads is described by binomial statistics [1, 3], in which charge is transferred in elementary units. Subsequently, the FCS in normal-superconducting systems have been studied and it was shown that Andreev reflection leads to a doubled charge transfer [4–6]. Later on, several approaches to FCS have been put forward. A theory of full counting statistics based on Keldysh-Green's function was developed [7]. This formulation allows a straightforward generalization to superconductors [8, 9], multi-terminal structures [10] and Coulomb blockade systems [11]. As another example, the FCS of charge transport was expressed via the counting statistics [12] of photons emitted from the conductor. Very recently, a classical path-integral approach to FCS was developed [13].

The second moment of the FCS is related to the shot noise [14]. Suppression of the shot noise from the classical Poisson value can result from the Fermi statistics of the electrons [15–17]. Particularly interesting are the correlations between currents at different terminals —the cross-correlations. For uncorrelated Fermi leads they are in general negative [17, 18], which has been experimentally confirmed [19–21]. If the current is injected from a superconducting terminal, the sign of the cross-correlations may become positive [22–27]. If the proximity effect is absent, both negative cross-correlations [6, 28] and positive cross-correlations [29] were theoretically predicted.

The positive correlations are a direct consequence of the charge being transferred in pairs across the normal-superconducting interface [4, 26, 27, 30]. This is the case, independent of the presence or absence of the proximity effect in the normal metal. Since the proximity effect introduces additional pair correlations, it is of special interest to investigate, under general

circumstances, the charge transfer in normal-superconducting systems with suppressed proximity effect. This limit is also generally accessible in experiments on metallic or semiconductor normal-superconducting heterostructures.

Circuit theory of incoherent Andreev transport. – All transport properties of quasiclassical superconductor–normal-metal heterostructures can be calculated from the so-called circuit theory [7, 10, 31–33]. This theory is a discretization of the normal-metal parts into connectors and nodes. The mesh has to be chosen in a way that approximates the real structure to the required precision. To each node (label i) a matrix Green's function $\check{G}_i$ is ascribed. A matrix current $\check{I}_{ij}$ flows through the connector between nodes i and j. Conservation of the matrix current on each node $\sum_j \check{I}_{ij} = 0$ together with the normalization condition $\check{G}_i^2 = 1$ determine the unique solution for the whole circuit. Connections to the external world are represented by terminals (labelled with α) with a fixed matrix Green's function $\check{G}_\alpha$.

Electrons and holes (connected by Andreev reflection at the superconducting terminals) propagate through the structure. In the circuit theory the decoherence due to the finite energy difference $2E$ between electron and hole is described by a *decoherence terminal D*, which is connected to the respective node [32]. The matrix current from the node into the decoherence terminal has the form $\check{I}_{iD} = -i(e^2/\hbar)(E/\delta)[\check{\sigma}_z, \check{G}_i]$, where δ is the spacing. Alternatively the coherence is suppressed by a magnetic flux Φ penetrating the node. The corresponding matrix current reads $\check{I}_{iM} \sim e^2(\Phi/\Phi_0)^2[\check{\sigma}_z\check{G}_i\check{\sigma}_z, \check{G}_i]$. Here $\check{\sigma}_z$ denotes the third Pauli matrix in the Nambu matrix space and Φ_0 is the flux quantum.

The full matrix current conservation on node i then takes the from

$$\sum_j \check{I}_{ij} + \check{I}_{iD} + \check{I}_{iM} = 0 \,. \tag{1}$$

If we assume that the relevant energies are large in comparison to the mean level spacing δ multiplied by the dimensionless conductances of the connectors to the node (or if the magnetic field is large), all off-diagonal elements of $\check{G}_i$ in the Nambu space are suppressed. To show this, we choose a representation of the Green's functions, in which the electron- and hole-Keldysh Green's functions are block-diagonal. For example, a normal terminal (labelled α) has the form

$$\check{G}_\alpha = \begin{pmatrix} \hat{G}_\alpha^{\mathrm{e}}(\chi_\alpha) & 0 \\ 0 & \hat{G}_\alpha^{\mathrm{h}}(\chi_\alpha) \end{pmatrix}. \tag{2}$$

Here electron (hole) Green's function are still matrices in Keldysh space, defined as

$$\hat{G}_\alpha^{\mathrm{e(h)}}(\chi_\alpha) = \pm e^{\pm i\chi_\alpha\hat{\sigma}_z/2} \begin{pmatrix} 1 - 2f_\alpha^{\mathrm{e(h)}} & -2f_\alpha^{\mathrm{e(h)}} \\ -2\left(1 - f_\alpha^{\mathrm{e(h)}}\right) & 2f_\alpha^{\mathrm{e(h)}} - 1 \end{pmatrix} e^{\mp i\chi_\alpha\hat{\sigma}_z/2} \tag{3}$$

with $f_\alpha^{\mathrm{e(h)}}(E) = (1 + \exp[(E \mp eV_\alpha)/k_{\mathrm{B}}T])^{-1}$. The Green's function of the node i is

$$\check{G}_i = \begin{pmatrix} \hat{G}_i^{\mathrm{e}} & \hat{F}_i^1 \\ \hat{F}_i^2 & \hat{G}_i^{\mathrm{h}} \end{pmatrix}. \tag{4}$$

The decoherence matrix current is

$$\check{I}_{iD} \sim \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} \hat{G}_i^{\mathrm{e}} & \hat{F}_i^1 \\ \hat{F}_i^2 & \hat{G}_i^{\mathrm{h}} \end{pmatrix} \right] = 2 \begin{pmatrix} 0 & \hat{F}_i^1 \\ \hat{F}_i^2 & 0 \end{pmatrix}. \tag{5}$$
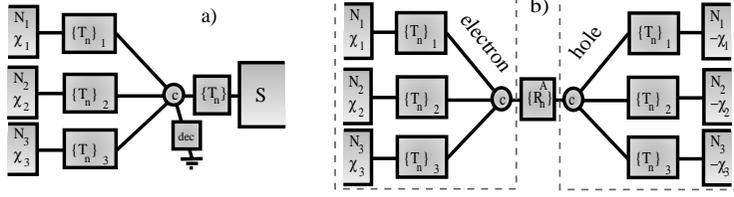
Fig. 1 – Mapping of a beamsplitter circuit. a) A number of normal terminals (here: three) is connected to one node c. The node is connected to a superconducting terminal by a contact with transmission eigenvalues $\{T_n\}$. Strong decoherence of electrons and holes is accounted for by a decoherence terminal connected to the node. This allows to map the circuit on the structure depicted in b). All normal parts of the structure a) are doubled and the superconducting terminal connects these two circuits. The connector between electron and hole circuit has transmission eigenvalues $\{R_n^A\}$, where $R_n^A = T_n^2/(2 - T_n)^2$ is the Andreev reflection probability corresponding to the transmission eigenvalues of the original circuit a).

Thus, if the decoherence current dominates over all other currents, the matrix current conservation equation (1) becomes $\check{I}_{iD} = 0$ and the off-diagonal components of $\check{G}_i$ vanish, *i.e.* $\check{G}_i$ becomes block-diagonal.

The connection between two nodes is described in general by a matrix current

$$\check{I}_{ij} = -\frac{2e^2}{h} \sum_n \frac{T_n[\check{G}_i, \check{G}_j]}{4 + T_n(\{\check{G}_i, \check{G}_j\} - 2)} \,, \tag{6}$$

where $\{T_n\}$ is the ensemble of transmission eigenvalues. The matrix current between two nodes with block-diagonal Green's functions is also block-diagonal. Each block has again the form (6). Thus, in the whole circuit (except at the superconducting terminals) the electron and hole blocks decouple.

At energies well below the superconducting gap, a superconducting terminal at zero chemical potential has the Green's function

$$\check{G}_{\mathrm{S}} = \begin{pmatrix} 0 & \hat{1} \\ \hat{1} & 0 \end{pmatrix}. \tag{7}$$

The matrix current to the superconducting terminal has the form (6), where the Green's function on the normal side has a block-diagonal form. Then, the block-diagonal components of the current can be rewritten, after some algebra, as

$$\check{I}_{\mathrm{S}i} = \begin{pmatrix} \hat{I}_{\mathrm{S}i}^{\mathrm{eh}} & 0 \\ 0 & -\hat{I}_{\mathrm{S}i}^{\mathrm{eh}} \end{pmatrix}, \qquad \hat{I}_{\mathrm{S}i}^{\mathrm{eh}} = -\frac{2e^2}{h} \sum_n \frac{R_n^A[\hat{G}_i^{\mathrm{e}}, \hat{G}_i^{\mathrm{h}}]}{4 + R_n^A(\{\hat{G}_i^{\mathrm{e}}, \hat{G}_i^{\mathrm{h}}\} - 2)} \,. \tag{8}$$

Here we introduced the Andreev reflection probabilities $R_n^A = T_n^2/(2 - T_n)^2$. Thus, the superconductor matrix current constitutes a (2×2)-matrix current between electron and hole Green's functions of the node. Note that the "transmission probabilities" are given by the Andreev reflection probabilities. We conclude that the transmission properties between electron and hole blocks (mirrored at the superconductor) have *exactly* the form of a normal contact with the usual transmission probabilities T_n replaced by R_n^A.

With this result, we are ready to present the mapping rules [34], illustrated in fig. 1:

1) All normal elements, normal terminals and all connectors between normal parts of the circuit are doubled (electron and hole circuit).

2) The superconducting terminals are the only connections between electron and hole circuit. They play the role of normal connectors, in which the normal transmission eigenvalues T_n are replaced by the Andreev reflection probabilities $R_n^A = T_n^2/(2 - T_n)^2$.

3) A counting field $\chi_\alpha^{e(h)} = \pm\chi_\alpha$ is assigned to corresponding electron and hole terminals.

4) The counting statistics is obtained from the solution of the normal circuit.

An important general property of the extended circuit is electron-hole symmetry. In our case, this is formally a consequence of the perfect symmetry of the extended circuit. Mathematically, the symmetry relation reads

$$\hat{\sigma}_x \hat{G}_i^e(E) \hat{\sigma}_x = -\hat{G}_i^h(-E). \tag{9}$$

Below we will use this symmetry relation to obtain some general properties of beamsplitters.

We can draw a general conclusion about the parity of the transferred charge number through the superconductor. Due to the Andreev process this number should be even. Since we have replaced the superconducting circuit by a purely normal circuit, the even parity is not obvious anymore. However, for a two-terminal device the FCS is 2π-periodic in the difference between the counting fields $\chi_e - \chi_h$, which, according to our rules, should be replaced by 2χ. Thus, the CGF is now a π-periodic function of χ. This argument can be generalized to an arbitrary number of terminals, if we are interested only in the charge transfer through the superconductor. The total transferred charge is obtained by taking all counting fields equal. Again, the CGF depends only on differences between counting fields and, therefore, charge transfers between two-electron (or hole) terminals are not counted. With the same argument as given above, it follows that the total CGF is invariant under a shift of all counting fields by π.

Counting statistics of two-terminal devices. – As a check, we first apply the mapping rules to a simple two-terminal contact without internal structure. The results for the cumulant generating function (CGF) of a normal contact with transmission eigenvalues $\{T_n\}$ is [1] ($k_B T = 0$)

$$S_{NN}(\chi_1, \chi_2) = M \sum_n \ln\left[1 + T_n\left(e^{i(\chi_1 - \chi_2)} - 1\right)\right], \tag{10}$$

where we have introduced the number of attempts per channel $M = 2eVt_0/h$. Using the recipe outlined above we replace T_n by R_n^A and $\chi_1 - \chi_2$ by 2χ and find

$$S_{SN}(\chi) = M \sum_n \ln\left[1 + R_n^A\left(e^{i2\chi} - 1\right)\right], \tag{11}$$

which is the result first obtained in ref. [4].

The counting statistics of a diffusive wire between a normal and a superconducting terminal can also be found analytically. Let us write the CGF of a normal diffusive conductor with conductance g_N as [7, 33, 35]

$$S_{NN}(\chi_1, \chi_2) = \frac{g_N V t_0}{4e} \text{acosh}\left(2e^{i(\chi_1 - \chi_2)} - 1\right). \tag{12}$$

Now, according to the mapping rules, we have to replace the diffusive connector by two diffusive wires in series. We can neglect the interface resistance and use that the CGF for a

series of two diffusive conductors is the same as for a single diffusive conductor. The total conductance is halved and we thus find

$$S_{\mathrm{SN}}(\chi) = \frac{g_{\mathrm{N}} V t_0}{8e} \mathrm{acosh}\big(2e^{i2\chi} - 1\big).$$ (13)

This result proves that the FCS of the diffusive SN wire in the incoherent regime is the same as in the coherent regime [33], which has so far only been demonstrated for the first two moments [5, 6, 9].

As another example, we consider a chaotic dot, which is much stronger coupled to the superconductor than to the normal terminal. In this case, the connector between the electron and hole node does not contribute to the FCS (the Green's functions in the electron and the hole dots are the same). We can then consider the equivalent circuit with a single dot, symmetrically coupled to the electron and hole terminals. The Green's functions of electron and hole terminal are (for $|E| < eV$ and $k_{\mathrm{B}}T = 0$)

$$\hat{G}^{\mathrm{e(h)}} = \mp \hat{\sigma}_z - \big(\hat{\sigma}_x \pm i\sigma_y\big)e^{\pm i\chi_{\mathrm{e(h)}}}\,.$$ (14)

For a moment we take independent counting fields in the electron and hole terminal, and replace $\chi_{\mathrm{e(h)}} \to \pm\chi$ only in the end. The solution for the central node is

$$\hat{G}_{\mathrm{c}} = -\frac{1}{Z}\big[\hat{\sigma}_x\big(e^{i\chi_{\mathrm{e}}} + e^{-i\chi_{\mathrm{h}}}\big) + i\hat{\sigma}_y\big(e^{i\chi_{\mathrm{e}}} - e^{-i\chi_{\mathrm{h}}}\big)\big],$$ (15)

where $Z = \sqrt{\exp[i\chi_{\mathrm{e}} - i\chi_{\mathrm{h}}]}$. Using this result, we find the CGF

$$S(\chi_{\mathrm{e}}, \chi_{\mathrm{h}}) = M \sum_n \ln\left[1 + \frac{T_n}{2}\Big(\sqrt{\exp[i\chi_{\mathrm{e}} - i\chi_{\mathrm{h}}]} - 1\Big)\right].$$ (16)

At this stage we can safely take the limit $\chi_{\mathrm{e(h)}} = \pm\chi$ ($\chi \in [-\pi/2, \pi/2]$) and obtain

$$S(\chi_{\mathrm{e}}, \chi_{\mathrm{h}}) = M \sum_n \ln\left[1 + \frac{T_n}{2}\big(e^{i(\chi \bmod \pi)} - 1\big)\right],$$ (17)

which ensures the π-periodicity of the CGF.

We now turn to the general case of the counting statistics of a beamsplitter as depicted in fig. 1. The central node is connected to several normal terminals (labelled with α) and one superconducting terminal. This structure has been studied in different limits in refs. [25–27, 29]. From eq. (8) it is clear that the Andreev conductance $g_{\mathrm{A}} = (4e^2/h)\sum_n R_n^{\mathrm{A}}$ governs the coupling between the electron and hole circuit. In what follows, we focus on the limits of weak and strong coupling of the superconductor to the beamsplitters. These limits are, besides being analytically tractable, of particular interest since they most clearly display the effects of the pair transfer across the normal-superconductor interface.

Counting statistics of a beamsplitter —weakly coupled superconductor. – We assume here that the superconductor is only weakly coupled to the beamlsplitter (see fig. 1) in the sense that $g_{\mathrm{A}} \ll g_{\Sigma} \equiv \sum_\alpha g_\alpha$, where $g_\alpha = (2e^2/h)\sum_n T_n$ is the conductance of the connector to terminal α. All normal terminals are held at the same potential V and zero temperature. Expanding all quantities to first order in $g_{\mathrm{A}}/g_{\Sigma}$, we find (similar to ref. [29]) that the total CGF can be expressed by the CGF of the superconducting contact alone:

$$S = M \sum_n \mathrm{Tr}\ln\left[1 - \frac{R_n^{\mathrm{A}}}{4}\big(\{\hat{G}_{\mathrm{c}}^{\mathrm{e0}}, \hat{\sigma}_x \hat{G}_{\mathrm{c}}^{\mathrm{e0}} \hat{\sigma}_x\} + 2\big)\right].$$ (18)

Here $\hat{G}_{\mathrm{c}}^{\mathrm{e0}}$ is the electron Green's function of the central node in the absence of the superconductor (for $g_{\mathrm{A}} = 0$). All Green's functions are evaluated at $E < V$ and we used eq. (9) to express S in terms of $\hat{G}_{\mathrm{c}}^{\mathrm{e0}}$ only. The Green's function $\hat{G}_{\mathrm{c}}^{\mathrm{e0}}$ can be obtained quite generally. Due to the triangular shape of all $\hat{G}_{\alpha}^{\mathrm{e}}$ (see eq. (14)), also $\hat{G}_{\mathrm{c}}^{\mathrm{e0}}$ has the same form. The matrix current between the central node and terminal α then becomes $\hat{I}_{\alpha} = \frac{g_{\alpha}}{2}[\hat{G}_{\mathrm{c}}^{\mathrm{e0}}, \hat{G}_{\alpha}^{\mathrm{e}}]$. Thus, all normal connectors behave as tunnel contacts. The solution for the central node is [26]

$$\hat{G}_{\mathrm{c}}^{\mathrm{e0}} = \frac{1}{g_{\Sigma}} \sum_{\alpha} g_{\alpha} \hat{G}_{\alpha}^{\mathrm{e}} = \hat{\sigma}_z + (\hat{\sigma}_x + i\hat{\sigma}_y)\Lambda, \tag{19}$$

where $\Lambda = \sum_{\alpha} p_{\alpha} e^{i\chi_{\alpha}}$ with $p_{\alpha} = g_{\alpha}/g_{\Sigma}$. It then follows from eq. (18) that the CGF is

$$S(\{\chi_{\alpha}\}) = M \sum_{n} \ln\left[1 + R_n^{\mathrm{A}}(\Lambda^2 - 1)\right]. \tag{20}$$

Such a CGF leads to a FCS of the form

$$P(N_1, N_2, \ldots) = P_{\mathrm{C}}(2Q)\frac{1}{2}\left(1 + (-1)^{2Q}\right)\frac{(2Q)!}{N_1! N_2! \cdots}p_1^{N_1} p_2^{N_2} \cdots, \tag{21}$$

where $Q = \sum_{\alpha} N_{\alpha}/2$ is the number of transferred Cooper pairs. The total probability distribution is therefore the probability $P_{\mathrm{C}}(2Q)$ that Q Cooper pairs are transferred, and then distributed among the normal terminals with respective probabilities p_{α}. A similar result has previously been obtained for tunnel contacts [26] and a chaotic cavity [29]. Here, this derivation holds for *any* type of connector, *i.e.* point contacts, diffusive wires, single transparency, etc. This has an important implication for the experimentally accessible current cross-correlations: In any (disordered or chaotic) normal conductor weakly coupled to a superconductor, the cross-correlations are positive independent of the details of the normal conductor.

Counting statistics of a beamsplitters —strongly coupled superconductor. – Another important case is when the superconductor is strongly coupled to the beamsplitter, *i.e.* $g_{\mathrm{A}} \gg g_{\Sigma}$. In this limit, the superconducting connector can be removed from the circuit and the system is equivalent to a chaotic dot coupled to the electron and hole terminals. We consider here the case in which the normal terminals are coupled by tunnel contacts. The solution for the central node is then [26]

$$\hat{G}_{\mathrm{c}} = \hat{K}\Big/\sqrt{\hat{K}^2}, \quad \hat{K} = \sum_{\alpha} \frac{g_{\alpha}}{2}(\hat{G}_{\alpha}^{\mathrm{e}} + \hat{G}_{\alpha}^{\mathrm{h}}). \tag{22}$$

The CGF follows straightforwardly,

$$S(\{\chi_{\alpha}\}) = M\left(\mathrm{Tr}\sqrt{\hat{K}^2} - g_{\Sigma}\right) = M\left[\sqrt{\left(\sum_{\alpha} g_{\alpha} e^{i\chi_{\alpha}}\right)^2} - g_{\Sigma}\right]. \tag{23}$$

In the present form the π-periodicity is evident. However, calculating the cumulants explicitly by taking successive derivatives of $S(\{\chi_{\alpha}\})$ with respect to χ_{α}, we can equally well take the square root in eq. (23), giving $S(\{\chi_{\alpha}\}) = M \sum_{\alpha} g_{\alpha}[\exp[i\chi_{\alpha}] - 1] = \sum_{\alpha} S_{\alpha}(\chi_{\alpha})$. From this form of S, no longer evidently π'-periodic, it follows, interestingly, that all cross-correlators vanish. This can be understood from an electron-hole symmetry argument, noting that in the absence of the superconducting connector the fluctuations emitted from the electron and hole contacts for a given α contribute oppositely, *i.e.* cancel out, to the fluctuations at a contact $\beta \neq \alpha$. This results in effectively independent CGFs for the different contacts.

Conclusions. – We have studied the full counting statistics of normal-metal–superconductor heterostructures in the incoherent regime. The original circuit with one superconducting terminal can be mapped on a purely normal circuit consisting of an electron and a hole block. The superconductor plays the role of a normal connector between the electron and the hole block, with the usual transmission probabilities $\{T_n\}$ replaced by the Andreev reflection probabilities $\{R_n^{\mathrm{A}} = T_n^2/(2 - T_n)^2\}$. We have illustrated our approach with several examples.

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