A Macroeconomic Analysis of Tax Evasion and Informality

Dissertation

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Summary

Informality, which is defined as unofficial and unrecorded production with a purpose of tax and social security avoidance, is a widespread phenomenon in developing as well as in developed economies (Schneider et al. (2010)). The informal sector offers an alternative untaxed occupational sector and its existence may lead to unintended consequences of government interventions. This thesis evaluates the macroeconomic consequences of tax evasion and informality employing the quantitative dynamic stochastic general equilibrium models with heterogeneous agents.

In Chapter 1, which is joint work with A. Di Nola, G. Kocharkov and A. Scholl we study the effects of tax evasion on occupational choice, efficiency and wealth inequality in the United States. Tax evasion in the U.S. is substantial and concentrated among the self-employed businesses. While only 1% of wages and salaries are not reported, this figure rises to 57% for the self-employed income (Johns and Slemrod 2010). Self-employed businesses constitute an important component of the U.S. economy. Hence, the fact that self-employed substantially evade taxes may have important aggregate and distributional implications. To study the aggregate consequences of tax evasion, we develop a dynamic general equilibrium model with incomplete markets and occupational choice in which self-employed business owners may hide a share of their business income but face the risk of being detected by the tax authorities. The model replicates important quantitative features of the U.S. economy in terms of the distribution of income and wealth, self-employment, and tax evasion. We show that tax evasion in the self-employment sector has a non-trivial quantitative impact on aggregate outcomes and welfare. We quantify three important channels through which tax evasion affects the overall economy. The subsidy channel emphasizes that tax evasion acts like a subsidy and stimulates asset accumulation. The selection channel highlights that the opportunity to evade taxes induces less talented agents to run self-employed businesses and depresses the average productivity in the self-employment sector. The detection channel reflects the fact that self-employed business owners have incentives to keep their businesses small to stay under the radar of the tax authorities. Tax evasion generates positive welfare effects in the aggregate because it subsidizes self-employed business owners. However, welfare is affected heterogeneously along the dimensions of occupation and wealth. While tax evasion reduces the welfare
of poor workers, it is particularly beneficial for poor self-employed agents. In our setup, implementing a perfect tax enforcement technology and using the additional revenues for a targeted tax reduction for poor self-employed agents leads to an increase in aggregate welfare and to a more productive economy. Our analysis has important implications for tax enforcement and tax policy: macroeconomic models which abstract from tax evasion might deliver biased policy recommendations.

In Chapter 2, I evaluate alternative pension system reforms in the presence of informality and analyze the mechanisms through which the informal sector and the pension system interact. I focus on Brazil which operates a pay-as-you-go pension system. The current system design allows to retire early with a generous pension benefit replacing more than 70% of the previous earnings (OECD (2016)). The World Bank (2017) reports that the old-age dependency ratio will triple in the next 35 years raising concerns about the pension system sustainability. Hence, the government of Brazil is currently designing a pension system reform. The phenomenon which should not be overlooked in this context is the existence of a sizable informal sector in Brazil (Schneider et al. (2010)). Around 40% of the entire labor force is employed informally (Meghir et al. (2015)). Informal workers do not contribute to the pension system but are entitled to a minimum pension benefit upon retirement. Additionally, the pension system reforms may potentially create incentives to exit the social security system by working informally leading to unforeseen consequences. To evaluate alternative pension reforms, I develop an overlapping-generation life-cycle model with incomplete markets, in which heterogeneous agents choose whether to work in the formal or informal sector. Working informally reduces the current tax burden but comes at a cost of lower pension benefits in the future. I calibrate my model to Brazil and the model matches well the characteristics of the informal sector and the pension system. I find that a demographic change, projected by the Brazilian Statistical Office (IBGE), leads to a significant reallocation of labor from the informal to the formal sector which increases tax revenues without direct government interventions. This mitigates a negative effect on the fiscal balance of an increased dependency ratio. Next, I evaluate four alternative pension system reforms which are designed to cover the additional deficit of around 3pp induced by the demographic change: (1) an increase of the individual payroll tax from 11% to 18.9%; (2) a reduction of the pension benefits by 16%; (3) a rise of the required years of contribution from 15 to 22 years; (4) a retirement age increase from 60 to 65 years. Among the considered reforms, increasing the minimum required years of contribution by 7 years minimizes informality and inequality. Raising the retirement age by 5 years maximizes welfare and minimizes the pension expenditure to GDP ratio. Finally, I study the role of informality by assessing the same policy instruments in an economy without the informal sector. The presence of informality requires stricter pension reforms in response to the demographic change. Apart from the payroll tax hike, the reforms lead to a higher long-run
welfare gain in the absence of informality. The countries with high informal sectors should implement the pension system reforms with a simultaneous reduction of informality.

In Chapter 3, I analyze the aggregate and distributional long-run effects of education policies in the presence of a large informal sector. Empirical studies show that low-educated individuals opt for informal employment (Araujo et al. (2013), Lopez Garcia (2015), Fairris and Jonasson (2016), Meghir et al. (2015)). Education subsidies make education more affordable and increase the skill level of population raising formal sector participation. On the other hand, education subsidies have to be financed either with debt accumulation or additional taxation. Raising taxes to finance the education subsidy may encourage individuals to enter the informal sector. In this chapter, I study the interrelation between education policies, taxation and income distribution in the presence of an informal sector. I develop an overlapping-generation life-cycle model with heterogeneous agents and incomplete markets, where agents make educational and occupational choices. Education is costly but ensures a wage premium in the future. A part of the education costs is subsidized by the government. Individuals choose to operate either in the formal or in the informal sector taking into account that the education premium is higher in the former. At the same time, formal workers face progressive labor taxation whereas informal workers do not pay income taxes. I calibrate my model to Brazil and assess the effects of education subsidies on the overall economy. The increase in the education subsidy improves educational attainment, expands formalization and substantially increases tax revenues. Raising the education subsidy from the current 26% to 80% reduces the informal sector participation by around 10pp and increases income tax revenues by 6pp. Cheaper education allows poor individuals to accumulate human capital and enter the formal sector which reduces inequality in the long-run. The education subsidy which covers 80% of the education costs maximizes welfare and the overall welfare gain is around 10% measured in a consumption equivalent variation. Interestingly, due to a significant positive effect on the tax base, the education subsidy is self-financing in the long run.

References


Zusammenfassung

Informalität, die als inoffizielle und nicht registrierte Produktion mit dem Ziel der Steuervermeidung definiert wird, ist sowohl in Entwicklungs- als auch in Industrieländern ein weit verbreitetes Phänomen (Schneider et al. (2010)). Der informelle Sektor bietet einen alternativen unversteuerten Berufssektor, und seine Existenz kann zu unbeabsichtigten Folgen staatlicher Interventionen führen. Diese Arbeit bewertet die makroökonomischen Konsequenzen von Steuerhinterziehung und Informalität im Rahmen von quantitativen heterogene Agenten allgemeinen Gleichgewichtsmodellen.


dungszuschuss, der 80% der Bildungskosten abdeckt, maximiert das Wohlbefinden und der allgemeine Wohlfahrtsgewinn liegt bei etwa 10%, gemessen in einer konsumäquivalenten Variation. Interessanterweise finanziert sich der Bildungszuschuss aufgrund eines deutlich positiven Effekts auf die Steuerbemessungsgrundlage langfristig selbst.

References


Chapter 1

The Aggregate Consequences of Tax Evasion
Chapter 1. The Aggregate Consequences of Tax Evasion

1.1 Introduction

The evasion of individual income taxes is substantial in the United States. The Internal Revenue Service (IRS) estimates that the lost federal tax revenue due to underreported individual income is $197 billion in 2001, which is 18% of the actual individual income tax liability (U.S. Department of the Treasury 2006).\(^1\) Tax evasion is concentrated among the self-employed businesses. While only 1% of wages and salaries are not reported, this figure rises to 57% of self-employed income (Johns and Slemrod 2010). Self-employed businesses constitute an important component of the U.S. economy. They account for 39% of the assets and 21% of the income in the economy.\(^2\)

What are the aggregate consequences of tax evasion in the self-employment sector in the U.S.? Does evading taxes by self-employed businesses matter for aggregate outcomes, inequality, and welfare? What are the channels through which such effects operate? What are the implications for tax enforcement and tax policy?

To answer these questions we develop a dynamic general equilibrium model with incomplete markets and occupational choice and analyze how imperfect tax enforcement affects aggregate outcomes, the distribution of wealth, and welfare. In our model environment, infinitely lived agents face idiosyncratic and persistent shocks to their labor productivity and their talent of running a business. They pay progressive income taxes and choose between being a worker or running a self-employed business each period. Workers supply inelastically their effective time endowment to corporate firms, which operate with a constant returns to scale technology. These firms use competitively labor and capital and produce a consumption good. Workers cannot evade taxes and make consumption and saving decisions. Self-employed business owners use a decreasing returns to scale technology in capital to produce the consumption good. They may hide a share of their business income but are confronted with the probability of being detected by the tax authorities and punished by paying the evaded taxes and a proportional fine. Self-employed business owners optimally determine the size of their firms by choosing capital, taking into account that detection becomes more likely as their firm grows. In doing so, they face borrowing constraints proportional to the amount of their savings.

We calibrate the model to the U.S. economy at the start of the 2000s. First, we use the Panel Survey of Income Dynamics (PSID) data to estimate the labor productivity process for workers and impose parametric functions for the progressive income taxes paid by workers and self-employed business owners. Second, we set the parameter values related

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\(^1\)For the remainder of the paper we refer to the underreporting of individual income as tax evasion. This is a violation of the tax code and differs from tax avoidance practices, which reduce tax liability via legal means.

\(^2\)These numbers are derived from the Panel Study of Income Dynamics (PSID). For more details on the data work, see Appendix 1.A.
to production, the talent of running a self-employed business, and tax evasion by matching a selected number of data targets via a method-of-moments estimation. In particular, we target the capital-output ratio and the interest rate of the U.S. economy, the share of self-employed business owners, their assets and income, and the annual exit rate from self-employment. Importantly, the parameters related to tax evasion are set to match the average misreporting rate of income as well as the cross-sectional misreporting rates conditional on the level of income.

The model replicates the empirical distributions of income and wealth even though they are not explicitly targeted. Another non-targeted dimension, which the model successfully matches, is the size distribution of self-employed businesses. The overall excellent fit of the model with respect to this broad set of empirical facts for the U.S. economy gives us confidence to use the model for a quantitative analysis.

In our quantitative analysis, we study the impact of tax evasion by comparing our benchmark economy with a counterfactual economy in which taxes are perfectly enforced. The optimal decision rules highlight three important channels through which tax evasion affects aggregate outcomes. (i) The subsidy channel: tax evasion acts like a subsidy and stimulates asset accumulation, allowing higher investment in business capital. (ii) The selection channel: The opportunity to evade taxes induces less talented agents to run self-employed businesses. (iii) The detection channel: self-employed business owners have incentives to keep their businesses small to stay under the radar of the tax authorities and to reduce the probability of being audited.

The quantitative analysis of the stationary equilibrium suggests that tax evasion by self-employed businesses matters for aggregate outcomes and inequality. The opportunity to evade taxes increases the number of self-employed businesses but reduces the average productivity of the self-employment sector. Moreover, tax evasion increases the share of small businesses, which is crucial for replicating the empirical self-employed firm size distribution. Furthermore, the economy with tax evasion is characterized by higher aggregate savings and larger aggregate output than the counterfactual economy with perfect tax enforcement. The increase in the aggregate capital stock lowers the interest rate and raises the wage generating a lower wealth inequality. A quantitative decomposition of the aggregate effects of tax evasion reveals that the subsidy channel as well as the selection channel are crucial while the detection channel is quantitatively less important.

Next, we study the welfare implications of tax evasion. To this end, we calculate the welfare effects of eliminating tax evasion by adopting a perfect tax enforcement technology. Our analysis suggests that the elimination of tax evasion decreases aggregate output in the economy, leading to an aggregate welfare loss of about 4%, measured in consumption equivalence units. This is not a trivial result since tax evasion distorts the self-employed firm size and induces less able agents to self-select into entrepreneurship. Perfect tax
enforcement raises tax revenues by around 1.6% of GDP, which roughly corresponds to
the empirical estimate of the U.S. tax gap of 2% of GDP (U.S. Department of the Treasury
(2009)). If these additional tax revenues are redistributed to the households via lump-sum
transfers or tax cuts, the aggregate welfare loss of perfect tax enforcement is substantially
reduced to about 1.7% and 1.2%, respectively. Importantly, there is a large degree of
heterogeneity: perfect tax enforcement is associated with sizable welfare gains for the
workers while self-employed business owners face substantial welfare losses. This pattern
is emphasized for poor workers and poor self-employed. If the additional tax revenues
are used for tax cuts targeted to the poor self-employed business owners, then perfect tax
enforcement generates higher aggregate productivity and an overall aggregate welfare gain
of 0.89%.

Finally, we study the implications of our analysis for tax enforcement and tax policy.
First, we vary the fine that detected evaders have to pay to the tax authorities. It turns
out that within a reasonable range of the penalty, tax revenues follow a Laffer curve in
the size of the fine. This hump-shaped pattern is generated by two opposing forces. On
the one hand, a higher fine allows the government to collect more revenues. On the other
hand, a higher fine makes misreporting more risky and reduces the share of self-employed
businesses. This, in turn, reduces aggregate output such that the lower tax base decreases
tax revenues. Our quantitative findings suggest that a fine of around 65% maximizes tax
revenues, which is 10pp lower than the existing civil fraud penalty of 75% on missing taxes
in the U.S.

Second, we vary the average tax burden on workers and self-employed businesses by
scaling their respective non-linear tax functions. Our results show that the tax revenues
collected from self-employed businesses follow a Laffer curve. While tax hikes increase
revenues directly, they induce more (but less productive) agents to become self-employed
and to escape taxation by evading. Lower productivity and higher distortionary taxes
decrease output, which, in turn, reduces the tax base and adversely affects the tax revenues
coming from self-employed businesses. The tax revenues collected from workers increase
strongly if taxes are raised, suggesting that the elasticity of self-employed taxable income
is much higher than the elasticity of taxable labor income. Thus, explicitly modeling tax
evasion has direct quantitative implications for the assessment of tax policy.

The rest of the paper is organized as follows. The next subsection discusses the related
literature. In Section 1.2, we provide further details on the technology of tax evasion in the
U.S. Section 1.3 presents the model. Section 1.4 explains the calibration procedure and
shows the model fit. In Section 1.5, we present and discuss how tax evasion affects aggre-
gate outcomes, inequality, and welfare. Section 1.6 studies the impact of tax enforcement
and tax policy on tax revenues. The last section concludes.
1.1.1 Related Literature

The economic theory of the technology and practices of tax evasion was initiated by the seminal works of Allingham and Sandmo (1972) and Yitzhaki (1974). They present a stylized model of tax evasion by a risk-averse agent who faces the probability of getting caught and penalized by the tax authorities. The theoretical analysis shows that it depends on income and risk aversion how much individuals evade. Andreoni (1992) extends this framework to a two-period model with income uncertainty and borrowing constraints. Other notable extensions of the static theory are presented by Yitzhaki (1974) and Pencavel (1979) who allow for a more general penalization structure and introduce labor supply choice, respectively. For a detailed summary of the literature, see Andreoni et al. (1998), Slemrod and Yitzhaki (2002), and Slemrod (2007). We take this classic modeling approach to tax evasion and incorporate it in a modern heterogeneous agent macroeconomic model of income and wealth inequality.

The macroeconomic literature on the aggregate effects of tax evasion is scarce. Our paper is related to Maffezzoli (2011) who looks at the distributional effects of income tax evasion in a heterogeneous agent framework with incomplete markets. His model, similarly to ours, successfully replicates the cross-sectional pattern of tax evasion which increases in true individual income levels. The results point out that moving from a progressive taxation to a proportional tax rate reduces the amount of evaded taxes and raises government revenues. In contrast to his model, our framework explicitly accounts for the role of self-employed businesses in tax evasion. This allows us to quantitatively document the consequences of tax evasion for capital accumulation and aggregate productivity.

In a related contribution, Fernandez-Bastidas (2018) studies the role of tax evasion in a heterogeneous agent model with occupational choice. The main quantitative experiment differs from ours and evaluates a tax reform in which the existing progressive tax code is replaced by a proportional income tax. The results point out that modeling tax evasion matters when assessing the welfare consequences of the reform.

Our work builds on existing quantitative macroeconomic models with heterogeneous agents and incomplete markets in which entrepreneurs face borrowing constraints. The seminal works of Quadrini (2000) and Cagetti and De Nardi (2006) paved the way for generating adequate distributions of wealth in macroeconomic environments due to the savings behavior of entrepreneurs. Kitao (2008) explores the productive and welfare effects of capital taxation in a similar framework and shows that these effects depend on whether entrepreneurial or non-entrepreneurial capital is taxed. We complement these works by introducing the possibility of tax evasion for self-employed businesses and by exploring its role for aggregate economic outcomes and welfare.

We contribute to the macroeconomic literature of occupational choice and informality
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featuring two-sector models with formal and informal production. Amaral and Quintin (2006) focus on how informality arises from the incomplete enforcement of taxes, whereas Antunes and Cavalcanti (2007) study informality-related distortions such as financial constraints and entry regulations. Ordonez (2014) develops a general equilibrium model with heterogeneous firms to assess the quantitative effect of incomplete tax enforcement on aggregate output and productivity for the case of Mexico. He studies how the informal sector affects the allocation of production factors and occupational choices and finds an inverted-U relationship between the size of the informal sector and output. We extend these models to a dynamic environment with household heterogeneity, realistic self-employed tax evasion patterns and accurate personal income tax schedules. This allows us to conduct an elaborate quantitative analysis on the role of tax evasion for the U.S. economy.

Imperfect tax enforcement and informality have been recently included in the optimal taxation literature based on the Diamond-Mirlees paradigm. A prominent example is Piketty et al. (2014) who extend the optimal top tax rate formula of Saez (2001) to include a tax avoidance response. More related to our work, Doligalski and Rojas (2018) take a Mirrlees approach of optimal taxation to an economy with formal and informal production. They show that the sufficient statistics formula of optimal taxes changes in the presence of informality. In a quantitative application to the Columbian economy, optimal marginal taxes at the bottom of the income distribution are much higher if informality is not taken into account by the planner. The rationale for this result is that at these low income levels workers might be tempted to join the informal economy. In our model, a similar mechanism is at work, but the implications extend to the process of asset accumulation.

1.2 Tax Evasion in the United States

The Internal Revenue Code contains three primary obligations on taxpayers: (i) to file timely returns, (ii) to report accurately on those returns, and (iii) to pay the required tax voluntarily and on time. Thus, non-compliance takes three forms: (i) underreporting (not reporting full liability on a timely-filed return), (ii) underpayment (not paying the full amount of tax reported on a timely-filed return), and, (iii) non-filing (not filing the required returns on time). Given the scope of this paper, we concentrate our attention to the first component of non-compliance, namely, underreporting.

Individual income tax evasion and its distribution. The underreporting tax gap is defined as the amount of tax liability, which is not reported voluntarily by taxpayers who file tax returns on time. The IRS estimates that in 2001 underreporting of individual income tax returns was $315 billion, or 23% of the total tax liability. The IRS considers taxpayers who file returns but underreport their income as nonfilers. The IRS estimates that there are 20 million nonfilers, who owe $150 billion in taxes. The IRS also estimates that 10 million taxpayers who file returns do not pay the full amount of tax due. The underpayment tax gap is estimated to be $50 billion. In conclusion, the underreporting tax gap is much larger than the underpayment tax gap, which is much larger than the non-filing tax gap.

3In a related contribution, Kuehn (2014) concentrates on the role of taxes and governance quality for differences in informality across developed countries.
income led to a tax gap of $197 billion (U.S. Department of the Treasury 2009). This amounts to around 2% of the U.S. GDP in this year.\footnote{The estimate is based on the data collected through the National Research Program (NRP) Individual Income Tax Reporting Compliance Study for the 2001 tax year. The NRP analyzes approximately 46,000 randomly selected individual income tax returns. The estimated underreporting gap excludes unpaid taxes due to purely illegal activities.}

Only 1% of the wages and salaries and 4% of taxable interest and dividends are misreported to the IRS. In contrast, 57% of self-employment business income is not reported. Kleven et al. (2011) support this stylized fact using audit data for Denmark. They document that tax evasion is basically zero for income subject to third party reporting (such as wages and salaries) but is substantial for self-reported income.\footnote{The fact that business income is subject to much more underreporting than other sources of income is also evident in household survey data, as shown by Hurst et al. (2014).} Johns and Slemrod (2010) analyze the micro data from the NRP in order to assess the distribution of tax non-compliance for the fiscal year of 2001. In their analysis, tax payers are grouped according to percentiles of their true income, i.e., the gross income they should have reported if not evading. According to their calculations, 11% of true income is misreported to the IRS. However, the misreporting rate varies with income levels. True income levels in the first decile of income are not misreported at all. Income in all other deciles below the median are misreported at a steady rate of around 5%. Around 7-8% of income in the four deciles above the median are hidden. Finally, tax evasion is highest in the top decile, where more than 15% of income is misreported.

**Detecting and punishing tax evasion.** The IRS had around 13,000 revenue and tax agents in 2002 whose main responsibility is detecting tax evasion (Dubin 2018). The individual income tax examination coverage, i.e., the audit rate was 1.27\% in 1997. In the following years, the audit rate declined and fell below 1\% (TIGTA 2002). The aggregate numbers mask, however, a large variation by type and size of reported income, as documented in Slemrod and Gillitzer (2014) and the U.S. Department of the Treasury (2011). For individuals the probability of auditing is generally rising in reported income: individuals tax returns with reported income between $25,000 and $50,000 had a 0.73\% probability of being audited. The probability raises to 29.93\% for tax returns with reported income over $10 million. Auditing rates depend considerably on the type of income declared. Individuals with business income (which is subject to more evasion) face a higher audit probability than those not reporting business income (2.1\% and 0.6\%, respectively). Likewise, for corporations the audit rate dramatically rises with the amount of total assets.\footnote{We report more data on the relationship between auditing and income levels/size of business activity in Appendix 1.A.5.}

Legally, it is very demanding to prove that a taxpayer knowingly committed a fraudulent act when evading taxes. Therefore, the IRS performs very few criminal investigations.
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and more often pursues civil charges for evasion. Accuracy-related penalties vary between 20-40% of the missing taxes, while the civil fraud penalty is fixed at 75% (U.S. Department of the Treasury 2016).

1.3 The Model

The model builds on the seminal contributions of Quadrini (2000) and Cagetti and De Nardi (2006) who introduce entrepreneurs in macroeconomic models of wealth inequality but it differs from them in three key aspects. First, we introduce income tax evasion following the classic papers by Allingham and Sandmo (1972) and Yitzhaki (1974). Second, we allow for non-linear taxes, which describe the existing tax code more accurately. Third, in the light of the empirical facts on tax evasion, we concentrate our attention on the self-employed businesses and not on the general category of entrepreneurs.

Our model economy includes households, firms, and the government. Households are infinitely lived. Each time period corresponds to one year. Each period, households receive a pair of idiosyncratic realizations of their working ability and their ability of running a business. Based on these realizations and their stock of savings, they decide whether to form a self-employed business or to supply their work to a labor market. As in Aiyagari (1994), asset markets are incomplete, i.e., households cannot insure against shocks to working or business ability. In addition, there is another source of market imperfection: borrowing of self-employed businesses is subject to collateral constraints.

1.3.1 Preferences and Endowments

Households maximize the expected sum of discounted utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$ is the time discount factor and $c_t$ is consumption in period $t$. The utility function $u$ is defined as $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. For simplicity, we assume that labor is inelastically supplied. Each household is endowed with a working ability $\varepsilon \in E$ and a business ability $\theta \in \Theta$, where $\varepsilon$ and $\theta$ are drawn from a finite-state Markov process with transition probability $F(\varepsilon', \theta' | \varepsilon, \theta)$.

1.3.2 Technology

The economy consists of two sectors of production. The single consumption good is produced either in self-employed businesses each of which is run by a household or in a large
corporate sector with a representative firm. Actors in both sectors are price takers.

Following Cagetti and De Nardi (2006), self-employed businesses combine their ability \( \theta \) and capital \( k \) according to a production function,

\[
f(k) = \theta k^v,
\]

where \( 0 < v < 1 \). The production function exhibits decreasing returns to scale capturing the *span of control* idea introduced by Lucas (1978): self-employed business skills gradually deteriorate as the size of the firm increases. The self-employed can save at a risk-free rate \( r \) and use their wealth to finance capital used in the project. In addition to using their own assets, they are allowed to borrow from a financial intermediary at a rate \( r \).\(^7\) This borrowing is limited up to a constant share of the assets, which self-employed businesses can pledge as collateral, \( k \leq \lambda a \), where \( \lambda \geq 1 \). The two polar cases of \( \lambda = 1 \) and \( \lambda = \infty \) capture the two extremes of financial autarky and perfect credit markets, respectively.

The corporate firm operates according to a constant returns to scale technology,

\[
F(K_C, N_C) = K_C^\alpha N_C^{1-\alpha},
\]

where \( 0 < \alpha < 1 \). The corporate firm rents capital from households at rate \( r \) and labor services from workers paying a wage \( w \). Capital in both sectors depreciates at a rate \( \delta \in (0,1) \). Profit maximization in the corporate sector implies that input prices are set according to their marginal products,

\[
r = \alpha K_C^{\alpha - 1} N_C^{1-\alpha} - \delta \tag{1.1}
\]

and

\[
w = (1 - \alpha) K_C^\alpha N_C^{-\alpha}. \tag{1.2}
\]

### 1.3.3 Government and Taxation

The government raises tax revenues to finance exogenously given public spending \( G \). Both workers and self-employed are subject to a non-linear personal income tax \( T_i(\cdot) \) meant to approximate the actual tax code for the U.S. by capturing not only the statutory tax rates but also deductions, exemptions, and tax credits. We allow the tax schedules to be different for workers and self-employed. In particular, following Gouveia and Strauss (1999), we assume that each agent of type \( i = \{W, E\} \), where \( W \) stands for worker and \( E \)

\(^7\)The financial intermediary collects deposits from households and lends the proceeds to the corporate firm or the self-employed businesses. The ability of the self-employed is not observed by the intermediary, and, therefore, borrowing contracts cannot be conditioned on it.
stands for self-employed, has to pay tax liabilities given by the following tax function:⁸

\[
T^i (y) = a_0^i \left[ y - \left( y^{-a_1^i} + a_2^i \right)^{-1/a_1^i} \right].
\]

(1.3)

Note that for \( a_1 > 0 \) we have a progressive tax system since the average tax rate,

\[
\frac{T^i (y)}{y} = a_0^i \left[ 1 - \left( 1 + a_1^i y^{-a_2^i} \right)^{-1/a_1^i} \right],
\]

is increasing with income \( y \).⁹

The crucial element of our modeling exercise is the introduction of imperfect tax enforcement. Whereas workers cannot evade taxes, self-employed agents may hide a share \( \phi \in [0, 1] \) of their business income.¹⁰ The government, knowing that the self-employed evade taxes, can monitor through audits and perfectly verify the individual tax returns. Let \( p(k) \), with \( p'(k) > 0 \), be the probability that a self-employed tax return is subject to monitoring. If the self-employed agent is audited and underreporting is detected, a fine \( s > 1 \) proportional to the amount of the underreported taxes is issued.¹¹ In essence, the self-employed needs to pay back the hidden taxes and an additional proportional penalty. For simplicity, we assume that the auditing efforts of the tax authorities are costless.

Our key assumption here is that the probability of being audited depends positively on the size of the business, \( p'(k) > 0 \), capturing the idea that larger firms are more visible to the tax authorities. This modeling strategy is in line with the empirical evidence: Slemrod and Gillitzer (2014) document that in the U.S. the probability of auditing generally rises with income levels.¹² Furthermore, Lewis (2005) and Ordonez (2014) report that government agencies target larger establishments when it comes to audits.

### 1.3.4 Household Problem

**Timing of events.** The sequence of events in this economy unfolds as follows. At the beginning of each period, the idiosyncratic shocks \( \varepsilon \) and \( \theta \) for working ability and business ability are realized. After observing these shocks, and conditional on the value of assets \( a \)

⁸Guner et al. (2014) show that this tax function is very flexible and provides a good approximation of the effective U.S. tax schedule. This functional form has been used extensively in the quantitative macroeconomic and public finance literature. Notable examples are Conesa and Krueger (2006), Kitao (2008) and Cagetti and De Nardi (2009).

⁹In addition, the degree of tax progressivity is increasing with \( a_1 \). If \( a_1 \to 0 \), then \( T^i (y) \to a_0 y \), i.e. taxes are proportional.

¹⁰We assume that interest income generated by savings cannot be underreported, for both workers and self-employed.

¹¹In the seminal work of Allingham and Sandmo (1972), the fine paid upon detection of tax fraud is proportional to the evaded income. However, the administrative penalty for evading taxes in the U.S. is proportional to the amount of unpaid taxes.

¹²See Appendix 1.A.5 for more evidence on this.
inherited from the previous period, an individual chooses whether to be a worker or a self-employed for the current period. Workers make optimal decisions regarding consumption and savings and pay income taxes to the government. On the other hand, self-employed decide how much to invest (i.e., they choose \( k \), taking the collateral constraint into account) and how much to evade (i.e., they choose \( \phi \)). After business decisions are taken, detection and auditing by the government takes place. After observing if they are detected or not, self-employed agents make consumption and savings decisions. Note that the optimal consumption and saving choice of the self-employed is contingent on detection.

The optimization problem of an agent can be recursively formulated, with the individual states being the assets level \( a \) and the current abilities \( \varepsilon \) and \( \theta \). Let \( V^W \) and \( V^E \) denote the values of being a worker or a self-employed, respectively. The beginning-of-the-period value function is given by:

\[
V(a, \varepsilon, \theta) = \max \left\{ V^W(a, \varepsilon, \theta), V^E(a, \varepsilon, \theta) \right\}.
\]

Let \( o(a, \varepsilon, \theta) \) denote the occupational choice associated with problem (1.4):

\[
o(a, \varepsilon, \theta) = \begin{cases} 
1, & \text{if } V^E(a, \varepsilon, \theta) \geq V^W(a, \varepsilon, \theta) \\
0, & \text{otherwise.}
\end{cases}
\]

**Workers.** The worker’s problem can be written as:

\[
V^W(a, \varepsilon, \theta) = \max_{c, a'} \left\{ u(c) + \beta E\left[ V(a', \varepsilon', \theta') | \varepsilon, \theta \right] \right\}
\]

subject to

\[
y_W = w \varepsilon + ra, \quad (1.6)
\]

\[
c + a' \leq y_W + a - T^W(y_W), \quad (1.7)
\]

\[
a' \geq 0. \quad (1.8)
\]

\( T^W(\cdot) \) is the non-linear tax schedule defined in Section 1.3.3. Each worker supplies her total amount of time to the corporate sector as employed labor, earning a wage \( w \) for each productivity unit \( \varepsilon \). Equation (1.6) represents the worker’s taxable income, which consists of labor income \( w \varepsilon \) and income from financial assets \( ra \). Equation (1.7) states that all available resources net of taxes are split between consumption and savings. The last constraint summarizes the assumption that workers cannot borrow.\(^{13}\) In line with the data, employed workers cannot misreport their true income to the tax authority.

\(^{13}\)More generally, equation (1.8) can be replaced by \( a' \geq -a \) where \( a \geq 0 \) is an *ad hoc* borrowing limit.
Self-employed. The decisions of a self-employed agent production and tax evasion amount to choosing the operational capital \( k \) and the share of business income \( \phi \), which is not reported to the tax authorities. In doing so, the agent takes into account the probability of an audit by the tax authorities, which is conditional on the amount of capital invested in the business. The beginning-of-the-period value function is given by

\[
V^E(a, \varepsilon, \theta) = \max_{k,\phi \in [0,1]} \{ p(k) V^E_d(a, \varepsilon, \theta, k, \phi) + (1 - p(k)) V^E_n(a, \varepsilon, \theta, k, \phi) \} \tag{1.9}
\]

subject to

\[
0 \leq k \leq \lambda a, \tag{1.10}
\]

where (1.10) is the collateral constraint. The value function for the case of detection is given by

\[
V^E_d(a, \varepsilon, \theta, k, \phi) = \max_{c,a'} \{ u(c) + \beta E[V(a', \varepsilon', \theta') | \varepsilon, \theta] \} \tag{1.11}
\]

subject to

\[
\pi = \theta k^v - (\delta + r)k, \tag{1.12}
\]

\[
y_E = \pi + ra, \tag{1.13}
\]

\[
c + a' \leq y_E + a - T^E((1 - \phi) \pi + ra) - s[T^E(\pi + ra) - T^E((1 - \phi) \pi + ra)], \tag{1.14}
\]

\[
a' \geq 0. \tag{1.15}
\]

Equations (1.12) and (1.13) define respectively the profits from business activity and taxable income, which includes both capital profits \( \pi \) and financial income from savings \( ra \). The budget constraint is given by equation (1.14) and states that available resources, net of taxes and fines, are allocated between consumption and savings. Since the self-employed is audited, she has to pay a fine \( s > 1 \) proportional to the amount of the underreported taxes (1.14). Notice that self-employed agents may hide a fraction \( \phi \) of their business income \( \pi \) but they report truthfully their interest income \( ra \).

The value function for the case of non-detection is defined as

\[
V^E_n(a, \varepsilon, \theta, k, \phi) = \max_{c,a'} \{ u(c) + \beta E[V(a', \varepsilon', \theta') | \varepsilon, \theta] \} \tag{1.16}
\]

subject to

\[
\pi = \theta k^v - (\delta + r)k, \tag{1.17}
\]

\[
y_E = \pi + ra,
\]

\[
c + a' \leq y_E + a - T^E((1 - \phi) \pi + ra), \tag{1.17}
\]

\[
a' \geq 0.
\]
The optimization problem in (1.16) is very similar to (1.11) with the only difference coming from the flow budget constraint, which now does not show any tax evasion penalties.

For future reference, let us summarize the policy functions associated with the above problems. After solving the worker’s maximization problem (1.5), we get the policy function for asset holdings \( a' \) if the agent is a worker, \( g^W(a, \epsilon, \theta) \). The solutions to the maximization problems of the self-employed (1.11) and (1.16) imply the policy function for asset holdings if the agent is self-employed and detected, \( g^E_d(a, \epsilon, \theta) \), and if the agent is self-employed and not detected, \( g^E_n(a, \epsilon, \theta) \). \( k(a, \epsilon, \theta) \) refers to the policy function for business capital and \( \phi(a, \epsilon, \theta) \in [0, 1] \) is the policy function for tax evasion.

### 1.3.5 Equilibrium

The stationary equilibrium in the economy is characterized by a stationary distribution of agents over assets and ability realizations when the optimal behavior of agents and firms is taken into account. First, define the functions

\[
1^W(a', a, \epsilon, \theta) = \begin{cases} 
1, & \text{if } g^W(a, \epsilon, \theta) = a' \\
0, & \text{otherwise}
\end{cases}
\]

\[
1^E_d(a', a, \epsilon, \theta) = \begin{cases} 
1, & \text{if } g^E_d(a, \epsilon, \theta) = a' \\
0, & \text{otherwise}
\end{cases}
\]

\[
1^E_n(a', a, \epsilon, \theta) = \begin{cases} 
1, & \text{if } g^E_n(a, \epsilon, \theta) = a' \\
0, & \text{otherwise}
\end{cases}
\]

These functions take the value of one if the current realizations of the state variables \( \{a, \epsilon, \theta\} \) are associated with a future realization of the asset position \( a' \) according to the policy functions for workers and self-employed. Second, redefine the probability of detection as a function of the state variables using the policy function for business capital, \( p_k(a, \epsilon, \theta) \equiv p(k(a, \epsilon, \theta)) \). Then, the stationary distribution \( \mu \) is defined as

\[
\mu(a', \epsilon', \theta') = \int \left[ (1 - o(a, \epsilon, \theta)) 1^W(a', a, \epsilon, \theta) \\
+ o(a, \epsilon, \theta) p_k(a, \epsilon, \theta) 1^E_d(a', a, \epsilon, \theta) \\
+ o(a, \epsilon, \theta) (1 - p_k(a, \epsilon, \theta)) 1^E_n(a', a, \epsilon, \theta) \right] F(\epsilon', \theta' | \epsilon, \theta) d\mu(a, \epsilon, \theta).
\]

The first row of equation (1.18) counts those agents who decide to be workers and reach future states \( \{a', \epsilon', \theta'\} \) given that they are at current states \( \{a, \epsilon, \theta\} \). The second and
third lines represent the flow of self-employed who transit between current states \(\{a, \epsilon, \theta\}\) and future states \(\{a', \epsilon', \theta'\}\) depending on whether they are detected or not by the tax authorities.

In a competitive stationary equilibrium workers, self-employed businesses, and the corporate firm solve their maximization problems, all markets clear, and the distribution over the state variables that govern the behavior of households is stationary over time. Let the vector \(x = (a, \epsilon, \theta)\) contain the state variables, which summarize all the information necessary to solve the household problems in the economy. Specifically, a stationary competitive equilibrium consists of value functions \(V(x), V^W(x), \) and \(V^E(x)\), policy functions for the household \(o(x), g^W(x), g^E(x), g^E_{nd}(x), k(x), \) and \(\phi(x)\), input prices \(r \) and \(w\), government spending \(G\), income tax functions \(T^W(\cdot)\) and \(T^E(\cdot)\), and a probability distribution \(\mu(x)\) such that:

1. Given prices \(\{r, w\}\) and tax functions \(\{T^W(\cdot), T^E(\cdot)\}\), the value functions \(\{V(x), V^W(x), V^E(x)\}\) and the policy functions \(\{g^W(x), g^E(x), g^E_{nd}(x), k(x), \phi(x)\}\) solve problems (1.4), (1.5), (1.9), (1.11) and (1.16).

2. Prices \(\{r, w\}\) satisfy the optimization conditions of corporate firms, (1.1) and (1.2).

3. The government budget constraint is satisfied:
\[
G = \int \left[ (1 - o(x)) T^W(y^W(x)) + o(x) T^E((1 - \phi(x)) \pi(x) + ra) + o(x) sp(k(x)) [T^E(\pi(x) + ra) - T^E((1 - \phi(x)) \pi(x) + ra)] \right] d\mu(x). \tag{1.19}
\]

4. The capital and labor markets clear. Capital demand (by corporate sector and by self-employed businesses) is equal to capital supply:
\[
K_C + \int o(x) k(x) d\mu(x) = \int ad\mu(x).
\]

Labor demand is equal to labor supply:
\[
N_C = \int (1 - o(x)) \epsilon d\mu(x).
\]

By Walras’ law the goods market clearing condition holds in equilibrium as well. Total output can be defined as the sum of aggregate production in the self-employed sector and in the corporate sector:
\[
Y = \int o(x) \theta k(x)^\nu d\mu(x) + K_C^\alpha N_C^{1-\alpha}.
\]
5. The distribution $\mu(x)$ is stationary as implied by (1.18).

### 1.4 Fitting the Model to the Data

We choose the parameters in our model in order to replicate important quantitative features of the U.S. economy. In particular, the focus is on matching (i) the share of self-employed households and their income and assets, and, (ii) the overall misreporting rate and the misreporting rates across quintiles of income.

We use the PSID for the years 1990-2003 to estimate the data moments related to (i).\(^{14}\) In addition, we use wealth supplements, which are available for the years 1994, 1999, 2001 and 2003. The data targets related to (ii) are taken from Johns and Slemrod (2010). For more details on our data work, we refer the reader to Appendix 1.A.

The rest of this section is organized as follows. First, we present parameters that are fixed outside the model. Then, we discuss internally calibrated parameters, which are set so that the model matches the selected data targets. Finally, we report the model fit along several dimensions of the targeted and non-targeted data.

#### 1.4.1 Externally Calibrated Parameters

**Personal income tax.** As explained in Section 1.3.3, we specify the income tax functions separately for workers and self-employed, using the functional form of Gouveia and Strauss (1999),

$$T^i(y) = a^i_0 \left[ y - \left( y - a^i_1 + a^i_2 \right)^{-1/a^i_1} \right],$$

where $i = \{W, E\}$. The parameters $\{a^i_0, a^i_1, a^i_2\}$ are taken from Cagetti and De Nardi (2009) who estimate this functional form on federal taxes levied on the household pre-government income.\(^{15}\)

**Working ability process.** We estimate a stochastic process for working ability following two steps, as it is standard in the literature (see, e.g., Guvenen (2009) and Heathcote

---

\(^{14}\)Another popular choice for estimating moments of the income and wealth distribution is the Survey of Consumer Finance (SCF). While the SCF is very well-designed for analyzing the top of the wealth distribution (it oversamples very rich households), it lacks a panel dimension, which is needed to compute the exit rate from entrepreneurship and to estimate the labor income process. Moreover, we borrow estimates for the tax function parameters (for workers and self-employed) from Cagetti and De Nardi (2009), who use PSID data. A number of papers in the macroeconomics literature on inequality, like Quadrini (2000), De Nardi et al. (2018) or the aforementioned Cagetti and De Nardi (2009), use PSID data.

\(^{15}\)The parameters $a^W_2$ and $a^E_2$ are re-scaled in order to balance the government budget in equilibrium. We re-scale both $a^W_2$ and $a^E_2$ by a constant factor $\chi$, $a^i_2 = a^i_2 / \chi$ for $i = \{W, E\}$. As explained in Section 1.4.2, the parameter $\chi$ is set to match a ratio of total income taxes to GDP of 15.2% as in Maffezzoli (2011).
et al. (2017)). First, we regress labor earnings on observable household characteristics such as education and experience in order to obtain a measure of labor income residuals $\varepsilon_t$. Second, we model the residuals as a first order auto-regressive process:

$$\log \varepsilon_{t+1} = \rho_\varepsilon \log \varepsilon_{t} + \eta_{t+1},$$

(1.21)

where $\eta_{t+1} \sim N(0, \sigma_\varepsilon^2)$. We estimate this process for workers and obtain the persistence parameter $\rho_\varepsilon = 0.89$ and the dispersion parameter $\sigma_\varepsilon = 0.21$. We approximate the stochastic process in (1.21) by a discrete Markov chain following the procedure described in Tauchen and Hussey (1991). More details can be found in Appendix 3.A.2.

**Further parameters.** We fix the coefficient of relative risk aversion $\sigma$ to 2 which is standard in the macroeconomic literature. The parameter $\alpha$ represents the corporate capital share and is set to 0.38, which is the average corporate capital share for the period 1990-2007 (Karabarbounis and Neiman 2014). The choice for the parameter in the collateral constraint (1.10) is more delicate. When the borrowing constraint is binding, $\lambda = \frac{k}{\alpha}$. Therefore, $\lambda$ controls the maximum amount of leverage in the self-employed sector. Since we cannot observe the share of business capital financed with external sources in our data, we set $\lambda$ to 1.2 as in Diaz-Gimenez et al. (1992). Finally, the value of the tax evasion fine $s$ is set to the to the existing penalty for civil fraud of 75% (U.S. Department of the Treasury (2016)). All externally set parameter values are reported in Table 1.1.

### 1.4.2 Internally Calibrated Parameters

Business ability is assumed to follow a first order auto-regressive process:

$$\log \theta_{t+1} = \mu_\theta + \rho_\theta \log \theta_{t} + v_{\theta, t+1},$$

(1.22)

where $v_{\theta, t} \sim N(0, \sigma_\theta^2)$. The probability of tax fraud detection is a logistic function of business capital. In particular, we assume that

$$p(k) = \frac{1}{1 + p_1 \exp(-p_2 k)},$$

(1.23)

16In a series of robustness exercises, we vary the level of $\lambda$ from 1.1 to 1.8, while keeping the other parameters at the benchmark values. Most model moments are affected marginally. However, the share of income (assets) of the self-employed goes from 21.50% (42%) to 30% (45%). For more details, see Appendix 1.B.3.
Table 1.1: Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Preferences</td>
<td>2</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Corp. capital share</td>
<td>0.38</td>
<td>Karabarbounis and Neiman (2014)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Leverage ratio</td>
<td>1.2</td>
<td>Diaz-Gimenez et al. (1992)</td>
</tr>
<tr>
<td>$s$</td>
<td>Tax evasion fine</td>
<td>1.75</td>
<td>U.S. Department of the Treasury (2016)</td>
</tr>
</tbody>
</table>

**Working ability**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\varepsilon}$</td>
<td>Persistence</td>
<td>0.89</td>
<td>Micro data - PSID</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon}$</td>
<td>Standard deviation</td>
<td>0.21</td>
<td>Micro data - PSID</td>
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</tbody>
</table>

**Tax functions**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{0W}$</td>
<td>Workers</td>
<td>0.32</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
<tr>
<td>$a_{1W}$</td>
<td>Workers</td>
<td>0.76</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
<tr>
<td>$a_{2W}$</td>
<td>Workers</td>
<td>0.22</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
<tr>
<td>$a_{0E}$</td>
<td>Self-employed</td>
<td>0.26</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
<tr>
<td>$a_{1E}$</td>
<td>Self-employed</td>
<td>1.40</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
<tr>
<td>$a_{2E}$</td>
<td>Self-employed</td>
<td>0.44</td>
<td>Cagetti and De Nardi (2009)</td>
</tr>
</tbody>
</table>

with $p_1 > 0$ and $p_2 > 0$. As argued before, we assume that the probability of being audited increases with the size of a business unit, following the empirical evidence reported in Lewis (2005), Slemrod and Gillitzer (2014), and Ordonez (2014). Figure 1.1 shows the function $p(k)$ evaluated at the estimated parameters.

In a robustness analysis, we have experimented with a non-increasing functional form, namely, a constant $p(k)$. The results presented in Appendix 1.B.2 show that the logistic functional form achieves a better model fit in terms of tax evasion patterns. Importantly, the increasing functional form helps the model generate a size distribution of self-employed businesses that is in line with the data.

One might object that tax authorities target their audits towards firms with higher profitability such that the probability of auditing depends on business income rather than capital. In our model, however, true business income is not observable by the tax authority. Therefore, we assume the audit probability to be a function of capital and emphasize that in our model business capital and business income are highly correlated (0.81). Moreover, in Figure B.3a in Appendix 1.B.2, we report the implied audit probability as function of business income generated by our benchmark economy. The pattern is very similar to the one shown in Figure 1.1. Figure B.3b in Appendix 1.B.2 shows that the average probability of detection is increasing in business income, which is in line with the empirical evidence reported in Slemrod and Gillitzer (2014).

We need to assign values to the following parameters: the household discount factor

\[ p_1 \] and \[ p_2 \]. We choose the logistic function for its flexibility. The parameter $p_1$ affects the vertical intercept of the function, $p(0) = 1/(1 + p_1)$. The parameter $p_2$ affects the inflexion point of the function. A higher $p_2$ shifts the inflexion to the left.
Figure 1.1: Probability of Auditing

Notes: Capital and income are measured in model units.

\( \beta \), capital depreciation \( \delta \), the span of control for self-employed businesses \( v \), the three parameters for the business ability process \( (\rho_\theta, \sigma_\theta, \mu_\theta) \), and the two parameters for the audit probability, \( p_1 > 0 \) and \( p_2 > 0 \). Additionally, we need to pin down the scaling factor \( \chi \) for the parameters controlling government revenue level in the tax functions (1.20).

A number of data targets are considered, which are sensitive to variations in the parameters. It is well-understood that all the model parameters affect all the targets but we can nonetheless outline which data moment is most informative about a certain parameter. The interest rate and the capital-output ratio identify the discount factor \( \beta \) and the capital depreciation \( \delta \). The parameter \( v \) controls the share of income of self-employed agents. The persistence \( \rho_\theta \) in the stochastic process for the business ability is identified mainly by the annual exit rate from self-employment: a higher persistence of the ability process implies that self-employed change their occupation less frequently. The standard deviation \( \sigma_\theta \) crucially affects the strength of the precautionary saving motive by self-employed agents, and thus, the share of assets owned by them. The parameter \( \mu_\theta \), which relates to the unconditional mean of (1.22), determines the share of self-employed agents in the population.

The parameters \( p_1 \) and \( p_2 \) of the probability function \( p(k) \) are set to match the relationship between tax evasion and income. More precisely, we target the taxable income misreporting rate over quintiles of true household income, which are reported by Johns and Slemrod (2010). The overall taxable income misreporting rate in the U.S. economy is matched, too. Finally, we need to determine the value of the scaling factor \( \chi \) which adjusts the parameters \( a_2^W \) and \( a_2^E \) of the tax functions, so that that income tax revenue is an appropriate fraction of GDP.

To summarize, we set the 9 parameters \( \Theta = \{ \beta, \delta, v, \rho_\theta, \sigma_\theta, \mu_\theta, p_1, p_2, \chi \} \) by matching
the following data targets:

1. *Share of self-employed, shares of total income and assets in possession of self-employed and their annual exit rate. These targets are derived from the PSID.* [4 targets].

2. *Capital-output ratio (NIPA) and an interest rate of 4%* [2 targets].

3. *Overall taxable income misreporting rate and taxable income misreporting rates in each quintile of income (Johns and Slemrod 2010)* [6 targets].

4. *Tax revenue to GDP of 15.2% (Maffezzoli 2011)* [1 target].

In doing so, we use an overidentified method of moments approach. We minimize the squared difference between the 13 model moments and their counterparts in the U.S. data. We compute the difference of the model moments $\hat{m}_i(\Theta)$ from the data moments $m_i$ as $d_i(\Theta) = m_i - \hat{m}_i(\Theta)$. Let $D(\Theta) = (d_1(\Theta), ..., d_{13}(\Theta))$ be the vector of differences between the model moments and the data moments. Then, the minimization problem is given by

$$\hat{\Theta} = \min_{\Theta} D(\Theta)^{\prime}W D(\Theta),$$

where $W$ is a diagonal weighting matrix.

### Table 1.2: Internally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td>Discount factor</td>
<td>0.935 4% interest rate</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
<td>0.11</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Span of control</td>
<td>0.62</td>
<td>Share of income, self-employed</td>
</tr>
<tr>
<td><strong>Self-employed ability</strong></td>
<td>Persistence</td>
<td>0.935 Exit rate, self-employed</td>
<td></td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>Standard deviation</td>
<td>0.77</td>
<td>Share of assets, self-employed</td>
</tr>
<tr>
<td>$\mu_\theta$</td>
<td>Unconditional mean</td>
<td>-1.29</td>
<td>Share, self-employed</td>
</tr>
<tr>
<td><strong>Tax evasion detection</strong></td>
<td>Parameter of $p(k)$</td>
<td>1,500</td>
<td>Tax evasion by income (quintiles)</td>
</tr>
<tr>
<td>$p_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tax functions rescale</strong></td>
<td>Rescaling parameter</td>
<td>1.4</td>
<td>Tax revenues as share of GDP</td>
</tr>
<tr>
<td>$\chi$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The recovered values for the internally set parameters are presented in Table 1.2. Our calibration delivers a value of 0.62 for the span-of-control parameter. Note that this value is lower than the usual ones used in models with an entrepreneurial sector. The reason for this is that, unlike in other papers, our entrepreneurs are self-employed. Therefore, their average size and productive efficiency are much lower.

1.4.3 Model Fit

In this section, we compare the outcomes generated by the model with the corresponding statistics for the U.S. economy, both targeted and non-targeted. A good fit of the model along dimensions that are not explicitly targeted in the parameterization process reinforces our confidence in the validity of our approach when it comes to the counterfactual analysis.

Table 1.3 shows the model fit in terms of the first set of targeted moments of the U.S. data. The interest rate and the capital-output ratio are matched very closely. The model generates all basic targets on the share of self-employed, their income, assets, and exit rate as in the data. The average misreporting rate for taxable income matches the empirical value. Finally, tax revenues from income taxation are matched as a part of the budget balancing condition for the government.

The other targeted moments relate to the patterns of misreporting of taxable income by quintiles of true household income. Figure 1.2 reports the data facts and the model outcomes of misreporting by income level. The model is able to match the increasing pattern of tax evasion with total income (Figure 1.2a). For lower deciles of true household income the share of workers is higher and workers do not evade. For higher deciles of total income there are more self-employed who can potentially evade (Figure 1.2b). The overall effect is, however, non-trivial because richer self-employed agents tend to evade less due to the probability of auditing, which rises in the size of the business as shown in Figure 1.2c.

We report selected moments of the distribution of wealth and income in Table 1.4. Even though we do not target the Gini coefficient, the mean-to-median ratio, and the other measures of wealth and income concentration, the model fits the data very closely in all these dimensions. The model replicates reasonably well both the bottom and the top of the wealth and income distributions, even though it slightly undershoots the concentration of wealth in the top 1%. Figure 1.3a shows the average of self-employed net wealth for

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19 Guvenen et al. (2017) have shown that in order to match the Pareto tail of the U.S. wealth distribution the span-of control parameter should be 0.8 or above. Cagetti and De Nardi (2006) calibrate this parameter to 0.88, while Buera et al. (2011) use 0.79.

20 In Table A.1 in Appendix 1.A.1, we provide descriptive statistics for self-employed households and business owners.

21 It is well known that a standard Bewley model falls short of replicating the high degree of wealth concentration observed in the U.S. data. The inclusion of entrepreneurship as in the present framework improves significantly the ability of the model to fit the data along this dimension.
Figure 1.2: Self-Employment and Tax Evasion by Income

(a) Tax Evasion by Total Income

(b) Share of Self-Employed by Income

(c) Tax Evasion by Self-Employed Income

Notes: U.S. data on the percentage misreporting rate by true income groups are taken from Johns and Slemrod (2010), Table 2.
### Table 1.3: Basic Model Targets

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate (%)</td>
<td>4.00</td>
<td>3.97</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>2.65</td>
<td>2.62</td>
</tr>
<tr>
<td>Share of self-employed (%)</td>
<td>14.70</td>
<td>14.65</td>
</tr>
<tr>
<td>Share of assets, self-employed (%)</td>
<td>39.11</td>
<td>42.72</td>
</tr>
<tr>
<td>Share of income, self-employed (%)</td>
<td>21.04</td>
<td>23.76</td>
</tr>
<tr>
<td>Exit rate, self-employed (%)</td>
<td>15.73</td>
<td>15.90</td>
</tr>
<tr>
<td>Misreporting rate (%)</td>
<td>11.00</td>
<td>10.33</td>
</tr>
<tr>
<td>Tax revenues/GDP (%)</td>
<td>15.20</td>
<td>14.96</td>
</tr>
</tbody>
</table>

*Notes:* The table shows the model targets and the corresponding data targets based on U.S. PSID data for the years 1990-2003. The misreporting rate is taken from Johns and Slemrod (2010).

different quintiles of wealth, while Figure 1.3b reports the model fit in terms of business capital distribution. The fit of the model in terms of quintiles of net wealth and firm size is quite good. In particular, the model is able to reproduce the fact that around 70% of firms are concentrated in the first bin of the size distribution (with capital of less than $522,000).

### Table 1.4: Wealth and Income Distribution

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>Mean/Median</th>
<th>Bottom 40</th>
<th>Top 20</th>
<th>Top 10</th>
<th>Top 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>73.5</td>
<td>2.90</td>
<td>3.26</td>
<td>76.38</td>
<td>63.32</td>
<td>21.53</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>71.1</td>
<td>3.10</td>
<td>2.71</td>
<td>75.64</td>
<td>60.56</td>
<td>26.53</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>36.6</td>
<td>1.34</td>
<td>19.84</td>
<td>45.03</td>
<td>31.71</td>
<td>10.69</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>35.2</td>
<td>1.23</td>
<td>19.32</td>
<td>42.77</td>
<td>28.27</td>
<td>7.60</td>
</tr>
</tbody>
</table>

*Notes:* U.S. income data are from the PSID for the years 1993-2003. Data on wealth are from the PSID wealth supplements for the period 1994 and 1999-2003.

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22Self-employed firm size and net wealth are strictly related due to the collateral constraint $k \leq \lambda a$. 
1.5 The Aggregate Effects of Tax Evasion

To highlight the aggregate effects of tax evasion, we provide a comparison between our benchmark economy and a counterfactual economy in which taxes are perfectly enforced. Think of this economy as an economy in which the penalty on misreporting is so high that tax evasion does not occur. In a first step, to understand the mechanisms, we study how tax evasion affects the optimal decision rules. In a second step, we analyze the impact of tax evasion on aggregate outcomes. Finally, we discuss the welfare implications of tax evasion.

1.5.1 Understanding the Mechanisms

In this section, we analyze the economic mechanisms of tax evasion and discuss the policy functions displayed in Figure 1.4.

In our two-sector model with incomplete credit markets, the agents’ occupational choice, depicted in Figure 1.4a, depends both on business ability $\theta$ and wealth $a$ (given average working ability). For a given level of business ability $\theta$, agents become self-employed as long as they hold sufficient wealth. Poor talented agents who receive a high realization of business ability are credit-constrained so that they are not able to generate sufficient business income. There exists a wealth threshold $a^*(\varepsilon, \theta)$, (weakly) decreasing with business ability $\theta$, such that agents with $a < a^*(\varepsilon, \theta)$ become workers and those with $a \geq a^*(\varepsilon, \theta)$ become self-employed.

Notes: U.S. data on net worth are from the PSID for the years 1993–2003. Panel (a) shows average net worth for each quintile, normalized by average net worth in the population. In panel (b), firm size is measured in terms of capital.
The solid line in Figure 1.4a represents the wealth threshold for running a self-employed business as a function of business ability $\theta$ (given average working ability) in the benchmark economy, while the dashed line refers to the same threshold rule in the counterfactual economy with perfect tax enforcement. Tax evasion distorts the occupational choice at the margin because it makes self-employment more attractive. The opportunity to evade taxes raises the share of self-employed in the economy because a group of relatively less able agents (those between the solid and the dotted line in Figure 1.4a) find it profitable to run self-employed businesses. This suggests that on average the business ability of a self-employed agent in the economy with tax evasion is lower than in the counterfactual economy with perfect tax enforcement. This mechanism, through which tax evasion affects occupational choice and therefore the aggregates in the economy, is dubbed the selection channel.

In Figure 1.4b, we show the policy function for savings of the self-employed as a function of asset holdings (given average working and business ability). The blue solid line refers to the benchmark economy while the dashed red line refers to the counterfactual economy in which taxes are perfectly enforced. Tax evasion reduces the tax burden of self-employed agents and acts as a subsidy that facilitates higher savings. We refer to this as the subsidy channel.

Figure 1.4c shows the optimal decision rule for business capital for the self-employed as a function of asset holdings. The productive abilities $\varepsilon$ and $\theta$ are fixed at their average values. The blue solid line shows the decision rule for capital in the benchmark economy and the dashed red line refers to the counterfactual economy in which tax evasion is absent. For high asset levels the collateral constraint $k \leq \lambda a$ is not binding such that the optimal choice for capital is independent of the asset level. For lower values of assets, instead, the financing constraint binds and the self-employed are not able to run their projects at the optimal scale: in such a case their optimal capital choice does depend on wealth.

Interestingly, tax evasion creates a distortion in capital accumulation at low and medium values of assets. Indeed, the presence of a kink in $k(a, \varepsilon, \theta)$ (blue solid line) shows that wealth-constrained self-employed agents choose a sub-optimal level of capital in order to avoid a sharp increase in the probability of being audited. The intuition goes as follows. For low values of capital, Figure 1.1 shows that $p(k)$ is flat and small so that there is no distortion in capital accumulation: the optimal choice for $k(a, \varepsilon, \theta)$ is increasing. Then, as $k$ approaches the inflexion point in $p(k)$, agents keep $k(a, \varepsilon, \theta)$ flat to avoid a sharp increase in the probability of detection. For larger $k$, however, they stop evading ($\phi = 0$) and can thus freely increase their choice of capital, until the first best is reached. Under perfect tax enforcement, the kink in $k(a, \varepsilon, \theta)$ disappears (red dotted line). We refer to this effect of tax evasion as the detection channel.

Figure 1.4c also shows that in the presence of tax evasion wealthy self-employed busi-
nesses with non-binding collateral constraints utilize more capital in production than in the counterfactual economy with perfect tax enforcement. This finding can be explained by a general equilibrium effect. Due to the subsidy effect of tax evasion, self-employed agents accumulate more capital in the aggregate, which decreases the equilibrium interest rate. In turn, the lower interest rate raises the first best level of capital.

Figure 1.4d displays the misreporting rate of a self-employed agent as a function of assets for different values of her business ability (given average working ability). Clearly, less talented agents misreport higher shares of their income as they are financially more constrained. Moreover, because of their small business size they face a lower probability of getting detected by the tax authorities inducing them to evade more.
Figure 1.4: Policy Functions

(a) Occupational Choice

(b) Savings of Self-Employed

(c) Business Capital

(d) Misreporting Rate

Notes: In panel (a), the solid (dotted) line demarcates the occupational choice \( o(a, \varepsilon, \theta) \) in the benchmark economy with tax evasion (in the counterfactual economy with perfect tax enforcement). Agents with low wealth and/or low business ability become workers (southwest of the demarcation lines). Panel (b) reports the policy functions for next-period assets of self-employed agents \( a' = g^E(a, \varepsilon, \theta) \) in the benchmark economy with tax evasion and in the counterfactual economy with perfect tax enforcement. Panel (c) shows the policy function for business capital \( k(a, \varepsilon, \theta) \) in the benchmark economy with tax evasion and in the counterfactual economy with perfect tax enforcement. The vertical line demarcates the regions in which self-employed evade \( (\phi > 0) \) from regions in which they do not \( (\phi = 0) \). Panel (d) displays the misreporting rate \( \phi(a, \varepsilon, \theta) \) in our benchmark economy with tax evasion for different business abilities \( \theta \). In panels (a) and (d), working ability is fixed to the average value. In panels (b) and (c), working ability \( \varepsilon \) and business ability \( \theta \) are fixed to their average values.
1.5.2 Tax Evasion and Aggregate Outcomes

Table 1.5 presents selected aggregate statistics for the benchmark economy and the counterfactual economy in which taxes are perfectly enforced.

In our benchmark economy the share of self-employed agents is about 4pp larger than in the economy with perfect tax enforcement. At the same time, the average business ability $E(\theta|E)$ is lower highlighting the selection channel: the opportunity to evade taxes induces less talented agents to run self-employed businesses.

There are several opposing forces affecting capital in the self-employment sector. On the one hand, the subsidy channel stimulates asset accumulation and allows higher investment in business capital. On the other hand, the detection channel provides incentives to keep self-employed businesses small to stay under the radar of the tax authorities and to reduce the chances of being audited. In addition, the selection channel lowers the average productive capacity of the self-employed businesses, and thus, their average size. Our quantitative findings suggest that the business capital decision of a self-employed business owner is critically affected by the detection and the selection channels: in the economy with tax evasion the mean value of business capital of a self-employed agent, $E(k|E)$, is lower than in the counterfactual economy with perfect tax enforcement.

In the aggregate, however, business capital $K^E$ in the self-employment sector increases when tax evasion is allowed due to the higher share of self-employed businesses in the economy. As a consequence, tax evasion raises the aggregate output of the self-employed sector. The impact of tax evasion on the firm size distribution is shown in Figure 1.5. Due to the detection channel of tax evasion there are more firms in the smallest bin (from $0 to $522,000). Note that our benchmark economy with tax evasion provides a better description of the empirical firm size distribution than our counterfactual economy in which taxes are perfectly enforced.

Since the opportunity to evade taxes increases the share of self-employed, fewer households become workers and aggregate labor $N^C$ in the corporate sector decreases. The increase in labor productivity is reflected in a higher real wage $w$. The benchmark economy with tax evasion is characterized by a larger aggregate capital stock than the counterfactual economy such that the interest rate $r$ decreases. Both the higher wage and the lower interest rate contribute to a lower wealth inequality measured by the Gini coefficient of the household wealth distribution. In addition, wealth is less concentrated because tax evasion allows poor self-employed agents to save more.

In our quantitative application of the theoretical model to the U.S. economy, tax evasion reduces tax revenues by 1.6pp of GDP. This figure is close to the empirical estimate of the U.S. tax gap of 2% of GDP (U.S. Department of the Treasury 2009).

In our discussion so far, we highlighted the three channels through which tax evasion
### Table 1.5: Aggregate Effects of Tax Evasion

<table>
<thead>
<tr>
<th>Sector of self-employment</th>
<th>Tax Evasion (Benchmark)</th>
<th>Perfect Tax Enforcement</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of self-employed (%)</td>
<td>14.65</td>
<td>10.51</td>
<td>+4.14</td>
</tr>
<tr>
<td>$E(\theta</td>
<td>E)$</td>
<td>0.93</td>
<td>1.02</td>
</tr>
<tr>
<td>$E(k</td>
<td>E)$</td>
<td>12.86</td>
<td>14.65</td>
</tr>
<tr>
<td>$K^E$</td>
<td>1.88</td>
<td>1.54</td>
<td>+18.30</td>
</tr>
<tr>
<td>$Y^E$</td>
<td>0.68</td>
<td>0.56</td>
<td>+17.90</td>
</tr>
<tr>
<td>Corporate sector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^C$</td>
<td>3.84</td>
<td>3.82</td>
<td>+0.53</td>
</tr>
<tr>
<td>$N^C$</td>
<td>0.85</td>
<td>0.89</td>
<td>-4.34</td>
</tr>
<tr>
<td>$Y^C$</td>
<td>1.51</td>
<td>1.54</td>
<td>-2.46</td>
</tr>
<tr>
<td>Prices</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$ (%)</td>
<td>3.97</td>
<td>4.34</td>
<td>-0.37</td>
</tr>
<tr>
<td>$w$</td>
<td>1.10</td>
<td>1.08</td>
<td>+1.48</td>
</tr>
<tr>
<td>Tax revenues</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T/Y$ (%)</td>
<td>14.96</td>
<td>16.61</td>
<td>-1.65</td>
</tr>
<tr>
<td>Wealth inequality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini</td>
<td>73.50</td>
<td>75.24</td>
<td>-1.74</td>
</tr>
</tbody>
</table>

**Notes:** The table compares macroeconomic aggregates of the benchmark economy with tax evasion with those of the counterfactual economy with perfect tax enforcement. The last column reports the percentage changes in the macroeconomic aggregates when moving from the counterfactual economy with perfect tax enforcement to the benchmark economy with tax evasion. $E(\theta|E)$ and $E(k|E)$ denote the mean value of business ability and business capital of a self-employed agent. $K^E$ and $Y^E$ refers to aggregate capital and aggregate output in the self-employment sector. $K^C$, $N^C$, and $Y^C$ denote capital, labor, and output in the corporate sector, respectively. Tax revenues $T/Y$ are given as percentage share of total output. $w$ denotes the real wage rate, while $r$ is the real interest rate in percent.
Chapter 1. The Aggregate Consequences of Tax Evasion

Figure 1.5: Tax Evasion and the Distribution of Self-Employed Businesses

Notes: U.S. data on the size of self-employed businesses are from the PSID for the years 1993-2003. Panel (a) and (b) provide a comparison of the data with the benchmark model with tax evasion and the counterfactual economy with perfect tax enforcement.

The opportunity to evade taxes induces more but less talented agents to become self-employed, which increases the misreporting rate and reduces the mean value of business capital of a self-employed agent. In general equilibrium, the wage increases while the interest rate decreases, such that the impact of tax evasion on aggregate outcomes is mitigated.

In the next step, we decompose the partial equilibrium effects to deduce the strength...
of the subsidy, selection and detection channels. To this end, we run a series of additional counterfactual experiments. Let \( \hat{o}(x) \) and \( \hat{k}(x) \) denote the policy functions for the occupational choice and for business capital in the economy with perfect tax enforcement, respectively. To isolate the effect of the subsidy channel of tax evasion, in column (2), we impose exogenously the policy functions \( o(x) = \hat{o}(x) \) and \( k(x) = \hat{k}(x) \) in the partial equilibrium economy with tax evasion. Thus, we allow for tax evasion but the decisions on occupational choice and business capital are fixed such that the selection and detection channels are shut down. Now tax evasion affects the outcomes of the economy only through the savings behavior of agents. Next, in column (3), we fix only the occupational choice \( o(x) = \hat{o}(x) \) in the partial equilibrium economy with tax evasion and shut down the selection channel. Finally, in column (4), we fix only the choice of business capital \( k(x) = \hat{k}(x) \) to eliminate the detection channel.

Let us first focus on the quantitative importance of the subsidy channel. Our findings in column (2) show that in the absence of the detection and selection channels the opportunity to evade taxes increases the average business capital of a self-employed agent by 7.1% (from 14.65 to 15.69). Misreporting income allows self-employed business owners to pay less taxes and to accumulate more savings and, in turn, to invest more in their business capital. Column (3) shows that the detection channel has a mitigating impact on average business capital because the probability of being audited by the tax authorities induces self-employed businesses to stay small. However, quantitatively this detection channel is less important than the subsidy channel.

A comparison of column (4) with column (2) reveals that the selection channel is of great quantitative importance. In the economy in which both the subsidy and selection channels are in place, average capital of self-employed businesses is reduced by 16% compared to the economy in which only the subsidy channel is present (\( E(k|E) \) drops from 15.69 to 13.23). At the same time, the share of self-employment raises by around 3.7pp (from 11.22 to 14.93), which is close to the total increase in self-employment due to tax evasion shown in column (5). The rise in the number of self-employed implies a substantial increase in aggregate capital in the self-employment sector: \( K^E \) goes up by 12.5% from column (2) to column (4). Moreover, the selection of more but less talented self-employed agents generates a lot of the observed tax noncompliance as evident by the high misreporting rate reported in column (4).
Table 1.6: Decomposition of Aggregate Effects

<table>
<thead>
<tr>
<th>Tax Evasion Economies</th>
<th>Perfect Tax Enforcement</th>
<th>Counterfactual Experiments</th>
<th>Benchmark General Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Setup</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed prices from (1)</td>
<td>-</td>
<td>$r, w$</td>
<td>$r, w$</td>
</tr>
<tr>
<td>Fixed decisions from (1)</td>
<td>-</td>
<td>$o(x), k(x)$</td>
<td>$o(x)$</td>
</tr>
<tr>
<td>Endogenous decisions</td>
<td>all</td>
<td>$a'(x), \phi(x)$</td>
<td>$a'(x), k(x), \phi(x)$</td>
</tr>
<tr>
<td>Operational channels</td>
<td>-</td>
<td>Subsidy</td>
<td>Subsidy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+Detection</td>
<td>+Selection</td>
</tr>
<tr>
<td>Outcomes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of self-employed (%)</td>
<td>10.51</td>
<td>11.22</td>
<td>11.29</td>
</tr>
<tr>
<td>$E(k</td>
<td>E)$</td>
<td>14.65</td>
<td>15.69</td>
</tr>
<tr>
<td>$K^E$</td>
<td>1.54</td>
<td>1.76</td>
<td>1.70</td>
</tr>
<tr>
<td>$K^C$</td>
<td>3.82</td>
<td>4.23</td>
<td>4.31</td>
</tr>
<tr>
<td>Misreporting rate (%)</td>
<td>0</td>
<td>7.77</td>
<td>8.63</td>
</tr>
</tbody>
</table>

Notes: The table provides macroeconomic outcomes for a series of counterfactual experiments in which prices and policy functions are fixed at the perfect tax enforcement economy. $E(k|E)$ denotes the mean business capital of a self-employed agent. $K^E$ and $K^C$ denote aggregate capital in the self-employment sector and corporate sector, respectively.
1.5.3 Tax Evasion and Welfare

In this section, we evaluate how tax evasion affects average welfare. The welfare effects of eliminating tax evasion are in terms of consumption equivalent variations, i.e., we calculate the consumption gain or loss of moving from the benchmark economy with tax evasion to an economy in which taxes are perfectly enforced.\footnote{Appendix 1.B.1 provides technical details.} Thereby, we compare the stationary equilibria and abstract from transitional dynamics. Table 1.7 summarizes the results.

Table 1.7: Tax Evasion and Aggregate Welfare

<table>
<thead>
<tr>
<th></th>
<th>Tax Evasion Benchmark</th>
<th>Perfect Tax Enforcement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Share of self-employed (%)</td>
<td>14.65</td>
<td>10.51</td>
<td>10.45</td>
</tr>
<tr>
<td>( Y ) (%)</td>
<td>2.18</td>
<td>2.10</td>
<td>2.10</td>
</tr>
<tr>
<td>( r ) (%)</td>
<td>3.97</td>
<td>4.34</td>
<td>4.40</td>
</tr>
<tr>
<td>( w )</td>
<td>1.10</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>CEV (%)</td>
<td>N.A.</td>
<td>-4.09</td>
<td>-1.72</td>
</tr>
</tbody>
</table>

Notes: The table summarizes selected statistics for the benchmark economy with tax evasion and four counterfactual economies in which taxes are perfectly enforced. CEV (consumption equivalent units) shows the percentage change in consumption needed to make a household indifferent between being born in the benchmark economy (column (1)) and being born in any of the four counterfactual economies with perfect tax enforcement (columns (2) to (5)). Column (2) is the counterfactual economy without fiscal neutrality in which additional tax revenues are not redistributed. In column (3) the government balances the budget with lump-sum transfers to all households. In column (4) the government balances the budget by implementing tax cuts for all households while in column (5) the tax cut is implemented for self-employed agents only. \( Y \) refers to total aggregate output. \( r \) is the interest rate in percent while \( w \) refers to the wage rate.

Since the elimination of tax evasion increases tax revenues, we distinguish several fiscal policy scenarios. In a first step, in column (2) of Table 1.7, we assume that the additional tax revenues are not redistributed to the agents in the economy. In a second step, we consider fiscal scenarios under fiscal neutrality, i.e., we assume that tax policies are adjusted such that the same level of tax revenues is achieved as in the benchmark economy. Specifically, in column (3), we report the welfare results if the additional tax revenues are redistributed via lump-sum transfers to all households. In column (4), the additional tax revenues are redistributed by tax cuts for all households. To this end, the tax level is decreased by re-scaling proportionately down the terms \( a_W^1, a_E^2 \) in the non-linear tax functions (1.3). Finally, in column (5), fiscal neutrality is imposed by a tax cut for the self-employed only.

Eliminating tax evasion without imposing fiscal neutrality has a negative effect on welfare (column (2)). This is not surprising since in this case aggregate capital and output...
fall. If the additional taxes are rebated via lump-sum transfers to all households, the welfare loss from perfect tax enforcement is much smaller, dropping from 4% to 1.7% (column (3)). If redistribution is accomplished by slashing the level of non-linear taxes (column (4)), the welfare loss is even smaller than in the previous case. If the tax cut is implemented for self-employed agents only (column (5)), welfare is roughly in line with the benchmark economy, but aggregate output is 1.2% higher. The reason is that the additional tax revenues allow the government to reduce the distortionary effect of taxation, which increases aggregate production. This positive effect is enhanced if the tax cut is targeted to the self-employed sector.

To deepen our understanding of the welfare results, we decompose them to study whether workers and self-employed agents are affected differently by tax evasion. Figure 1.6 shows the average welfare changes for workers and self-employed agents along the wealth distribution for the four fiscal policy scenarios. In order to compute these values, first, we identify the decile of the overall wealth distribution that each household type (in terms of asset holdings and abilities) belongs to in the benchmark economy. Second, we identify the occupational choice made by different household types in the benchmark economy. Finally, we compute the average welfare effects in each counterfactual scenario for households occupying each of the original benchmark economy deciles, while distinguishing between workers and self-employed according to their benchmark choice. However, the distribution over household types used in the welfare effects computation comes from the counterfactual economy. Defining tight household categorization according to the abilities and choices made in the benchmark economy allows us to derive the heterogeneous welfare effects of moving to the new economy for the same agents while taking into account that the weights of different household types in the new economy’s distribution change.24

Figure 1.6a displays the welfare results if the additional tax revenues are not redistributed back. The elimination of tax evasion leads to welfare losses for both self-employed agents and workers, except for those in the last decile of wealth. Self-employed agents incur larger losses since perfect tax enforcement has a direct negative impact on them, whereas workers are hurt indirectly from the future option value of becoming self-employed. In addition, the wage is reduced as seen in Table 1.7.

The welfare losses of self-employed business owners decrease in the relative wealth position. Recall that poor self-employed business owners have a higher misreporting rate (Figure 1.2c). Therefore, they lose more if taxes are perfectly enforced. Workers and self-employed in the top decile of wealth encounter welfare gains from the elimination of tax evasion because they benefit from higher interest rates (Table 1.7).

24Our approach to calculating heterogeneous welfare effects in the presence of occupational choice resembles the welfare analysis of tax policies in Brüggemann (2017). Further details are described in Appendix 1.B.1.
Figure 1.6: Tax Evasion and Welfare Across Wealth and Occupation

(a) No Redistribution

(b) Lump-Sum Redistribution

(c) Tax Cuts for Self-Employed and Workers

(d) Tax Cuts for Self-Employed

Notes: This figure shows the percentage change in consumption needed to make a household (worker or self-employed) indifferent between being born in the benchmark economy and being born in any of the four counterfactual economies in which taxes are perfectly enforced. A formal definition of the certainty equivalents among different groups is given in Appendix 1.B.1. Panel (a) is the counterfactual economy without fiscal neutrality in which additional tax revenues are not redistributed. In panel (b), the government balances the budget with lump-sum transfers to all households. In panel (c), the government balances the budget by implementing tax cuts for all households while in panel (d) the tax cut is implemented for self-employed agents only. Note that there are no self-employed agents in the first decile of the wealth distribution.
The overall welfare effect of eliminating tax evasion is negative if the additional tax revenues are redistributed via lump-sum transfers to all households (Table 1.7). However, Figure 1.6b reveals that the welfare changes are heterogeneous along the dimensions of occupation and wealth. Importantly, workers gain while the majority of self-employed business owners lose from the elimination of tax evasion accompanied by lump-sum transfers. Note that poor workers have larger welfare gains because they benefit more from lump-sum transfers. In contrast, if taxes are rebated via tax cuts for all, the welfare gains of workers vary less across deciles of wealth (see Figure 1.6c).

Figure 1.6d displays the average welfare changes for workers and self-employed business owners if the tax cut is implemented for the self-employed only. Clearly, the welfare losses of self-employed agents are much smaller than in the previous scenarios. Although there is no tax cut for the workers, they still have sizable welfare gains because they benefit from a higher wage rate.

To sum up, our welfare results point out that under fiscal neutrality moving to an economy where taxes are perfectly enforced makes workers better off while self-employed, and in particular the poor ones, lose. This observation motivates the last fiscal policy scenario presented in this section: we impose fiscal neutrality by cutting taxes, but only for a sub-group of the self-employed agents who might be particularly affected by perfect enforcement.

In Table 1.8 we target self-employed agents with asset holdings below a given threshold measured as percentile of the benchmark wealth distribution. Our results reveal that targeting self-employed agents with assets holdings below the 60th percentile leads to an average welfare gain of 0.89% for all households. Interestingly, when tax cuts are implemented only for the poorest self-employed with wealth levels below the 50th percentile, eliminating tax evasion generates an overall welfare loss. The intuition behind this finding is that these poor self-employed business owners are likely to have a lower business ability as well. Therefore, subsidizing them via tax reductions has adverse welfare effects.
Table 1.8: Tax Evasion and Aggregate Welfare: Tax Cuts for Self-Employed

<table>
<thead>
<tr>
<th>Threshold as Percentage of the Wealth Distribution</th>
<th>p10</th>
<th>p20</th>
<th>p30</th>
<th>p40</th>
<th>p50</th>
<th>p60</th>
<th>p70</th>
<th>p80</th>
<th>p90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2.14</td>
<td>2.14</td>
<td>2.14</td>
<td>2.16</td>
<td>2.17</td>
<td>2.22</td>
<td>2.20</td>
<td>2.23</td>
<td>2.22</td>
</tr>
<tr>
<td>r (%)</td>
<td>4.13</td>
<td>4.14</td>
<td>4.14</td>
<td>4.12</td>
<td>4.05</td>
<td>3.92</td>
<td>3.96</td>
<td>3.85</td>
<td>3.84</td>
</tr>
<tr>
<td>w</td>
<td>1.09</td>
<td>1.09</td>
<td>1.09</td>
<td>1.09</td>
<td>1.09</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>CEV (%)</td>
<td>-1.33</td>
<td>-1.09</td>
<td>-1.03</td>
<td>-0.82</td>
<td>-0.3</td>
<td>0.89</td>
<td>0.38</td>
<td>0.78</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Notes: The table shows selected statistics for counterfactual economies in which taxes are perfectly enforced. CEV (consumption equivalent units) shows the percentage change in consumption needed to make a household indifferent between being born in the benchmark economy with tax evasion (column (1) of Table 1.7) and being born in a counterfactual economy with perfect tax enforcement in which the government balances the budget by reducing tax rates only for self-employed agents with asset holdings below a given threshold (10th, 20th, ..., 90th percentiles of the self-employed wealth distribution). Y refers to total aggregate output. r is the interest rate in percent while w refers to the wage rate.

1.6 Tax Evasion and Laffer Curves

In this section, we study the impact of tax evasion on tax revenues and perform two types of exercises. First, we focus on the fine that detected evaders have to pay to the tax authorities. We consider the penalty as a policy variable and study the impact of tax enforcement on tax revenues and aggregate outcomes. Second, we analyze how shifts in the tax scheme affect tax revenues and aggregate outcomes in the presence of tax evasion.

1.6.1 The Fine on Tax Evasion

According to the OECD (2011), in developed countries the penalties for tax evasion vary between 10% and 30% for minor offenses and between 40% and 100% for frauds. To study the impact of the size of the penalty on aggregate outcomes and tax revenues, we use our benchmark model and vary the fine $s$ within a range between 20% and 100% of the tax evaded. In Figure 1.7, we display how the share of self-employed business owners, the average productivity in the self-employment sector, aggregate capital and output as well as tax revenues change in response to the size of the penalty.

Figure 1.7c shows that the share of self-employed businesses decreases as the level of the fine rises. This goes hand in hand with an increase in the average productivity of the self-employment sector. The reason is intuitive: if tax evasion is punished with a higher fine, agents with a lower business ability leave the sector of self-employment because misreporting becomes too risky for them. The smaller size of the self-employment sector, however, decreases aggregate capital and output in the economy, as shown in Figure 1.7d.

Figure 1.7b suggests that total tax revenues follow a hump-shaped pattern in the fine $s$. This behavior resembles a Laffer curve and is driven by the tax revenues collected from the
self-employed business owners as the tax revenues collected from the workers hardly change. The hump-shape is generated by two opposing forces. First, an increase in the penalty for tax evasion incentivizes self-employed business owners to report business income more truthfully: the income gap shrinks as shown in Figure 1.7a and the government can collect higher revenues. Second, an increase in $s$ decreases the share of self-employed businesses as well as aggregate output and capital (see Figures 1.7c and 1.7d). The drop in aggregate output reduces the tax base and lowers tax revenues. Within the considered range, we find that a fine of around 65% of the tax evaded maximizes tax revenues. This revenue-maximizing fine is 10pp lower than the existing civil fraud penalty of 75% on evaded taxes in the U.S.

Note that for penalties larger than 200% tax revenues start to increase as the fine $s$ is raised. Very large penalties eliminate tax evasion and the outcomes correspond to the ones of our counterfactual economy in which taxes are perfectly enforced. However, we argue that penalties of this size are politically not feasible.

1.6.2 The Tax Scheme

In this section, we analyze the impact of the tax scheme on aggregate outcomes and tax revenues in the presence of tax evasion. In particular, we are interested in how shifts in the tax scheme affect the size of the self-employment sector and the misreporting rate. To shift the tax scheme, we increase the tax parameters $a_0^W$ and $a_0^E$ proportionally, see the tax function (1.20), such that both workers and self-employed business owners face higher taxes. We display our findings in Figure 1.8.

Figure 1.8b displays total tax revenues and tax revenues collected from workers. Both types of tax revenues increase as the tax rates for workers and self-employed are shifted proportionally. Interestingly, the tax revenues collected from self-employed businesses follow a hump-shaped pattern. Furthermore, the tax revenues coming from workers increase much stronger than the tax revenues from business owners suggesting that the elasticity of taxable income for self-employed businesses is high.

To understand the pattern of tax revenues, we display the income gap in Figure 1.8a, the share and productivity of self-employed businesses in Figure 1.8c, and total aggregate capital and output in Figure 1.8d. The Laffer-type behavior of tax revenues collected from self-employed business owners is driven by opposing forces. On the one hand, tax hikes increase tax revenues directly. On the other hand, higher taxes induce more agents to become self-employed such that the average business ability in the self-employment sector decreases. Lower productivity and higher distortionary taxes decrease aggregate capital and output in the economy. This in turn, reduces the tax base and adversely affects the tax revenues collected from self-employed businesses who now evade more. Note that
those agents who remain being workers cannot escape the higher taxation by working less because labor supply is fixed in this economy. Therefore, tax revenues collected from workers increase if the tax scheme is shifted upwards.

We perform the same experiment for our counterfactual economy in which taxes are perfectly enforceable. We find that as the tax rate increases, the share of self-employed businesses decreases in the economy with perfect tax enforcement while the opposite is true in the economy with tax evasion. The reason is quite intuitive: with tax evasion, self-employed agents can protect themselves against increases in taxes by evading more.

Figure 1.7: The Impact of the Fine

Notes: This figure varies the fine on tax evasion $s$ within a range between 1.2 and 2.2 and report selected variables. The black dotted line indicates the benchmark economy with $s = 1.75$. The percentage income gap shown in panel (a) refers to the share of underreported income for the whole population. The other variables are normalized to the benchmark economy.
Figure 1.8: The Impact of the Tax Scheme

(a) Income Gap

(b) Tax Revenues

(c) Self-Employed: Share and Productivity

(d) Capital and Output (total)

Notes: This figure shifts the tax scheme and reports selected variables. The black dotted line indicates the benchmark economy with tax rates $a_W^0 = 0.32$ and $a_E^0 = 0.26$. $a_W^0$ and $a_E^0$ are rescaled proportionally. The percentage income gap shown in panel (a) refers to the share of underreported income for the whole population. The other variables are normalized to the benchmark economy.
1.7 Conclusions

The evasion of individual income taxes in the U.S. is substantial and concentrated among the self-employed businesses. To study the aggregate consequences of tax evasion we develop a dynamic general equilibrium model with incomplete markets and occupational choice in which self-employed business owners may hide a share of their business income but face the risk of being detected by the tax authorities. The model replicates important quantitative features of the U.S. economy in terms of the distribution of income and wealth, self-employment, and tax evasion.

We show that tax evasion in the self-employment sector has a non-trivial quantitative impact on aggregate outcomes and welfare. We quantify three important channels through which tax evasion affects the overall economy. The subsidy channel emphasizes that tax evasion acts like a subsidy and stimulates asset accumulation. The selection channel highlights that the opportunity to evade taxes induces less talented agents to run self-employed businesses and depresses the average productivity in the self-employment sector. The detection channel is important to replicate the empirical firm size distribution because self-employed business owners have incentives to keep their businesses small to stay under radar of the tax authorities.

Tax evasion generates positive welfare effects in the aggregate because it subsidizes self-employed business owners. However, welfare is affected heterogeneously along the dimensions of occupation and wealth. While tax evasion reduces the welfare of poor workers, it is particularly beneficial for poor self-employed agents. In our setup, implementing a perfect tax enforcement technology and using the additional revenues for a targeted tax reduction for poor self-employed agents leads to an increase in aggregate welfare and to a more productive economy.

Our analysis has important implications for tax enforcement and tax policy: macroeconomic models which abstract from tax evasion might deliver biased policy recommendations.
References


Appendix 1.A  Data

1.A.1  Data Description

In this section, we provide details regarding data, sample selection, and definitions used in this paper.

For our calibration, we estimate the moments from the Panel Study of Income Dynamics (PSID). We use a sample from 1990-2003 to estimate most of the relevant moments related to income. For wealth targets we link the main dataset to the Wealth Supplement File for the years 1994, 1999, 2001 and 2003. The questions in the survey refer to the previous calendar year.

Sample selection. We create our sample including variables related to the characteristics of the households and occupation and merge it with Sample A of Heathcote et al. (2010), which contains information on household tax liabilities. Heathcote et al. (2010) apply basic data cleaning by dropping records if: (a) there is no information on age for either the head or spouse, (b) either the head or spouse has positive labor income but zero annual hours, or (c) either the head or spouse has an hourly wage less than half of the corresponding federal minimum wage in that year. In addition, we select all households where the head of the household is male, has age in the 25-65 range and has worked at least 260 hours during the year.

Definition of self-employed. Traditionally, the entrepreneurial literature distinguishes between two definitions of entrepreneurs, see, e.g., Quadrini (2000). According to the first definition, entrepreneurs are families that own a business or have a financial interest in some business enterprise. This definition is based on the PSID variable “Whether Business” which is based on the following interview question: “Did you (Head) or anyone else in the family own a business at any time during the previous year or have a financial interest in any business enterprise?” If the answer is positive this household is recorded as an entrepreneur and if negative this household is thought of as ‘a worker’. According to the second definition, entrepreneurs are families in which the head is self-employed in his or her main job and the interview question is: “In your main job, are you (head) self-employed or do you work for someone else”. Unlike the previous survey question, which allows only a binary answer (yes/no), this one specifies the occupation of the head and allows to identify a household directly as: a self-employed, an employee, both a self-employed and an employee, or an unemployed.

In our study, we opt for the second definition. First, this definition is more consistent with the data on tax evasion since underreported self-employed business income refers to

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25Although wealth data are also available for 2005 and 2007, we do not extend the analysis to these years since other variables needed are missing.
Table A.1: Summary Statistics for Alternative Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Self-Employed</th>
<th>Business Owners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of entrepreneurs (%)</td>
<td>14.70</td>
<td>20.11</td>
</tr>
<tr>
<td>Share of income, entrepreneurs (%)</td>
<td>21.04</td>
<td>27.98</td>
</tr>
<tr>
<td>Share of assets, entrepreneurs (%)</td>
<td>39.11</td>
<td>46.15</td>
</tr>
<tr>
<td>Ratio of median assets (E/W)</td>
<td>4.02</td>
<td>3.65</td>
</tr>
<tr>
<td>Exit rate, entrepreneurs (%)</td>
<td>15.73</td>
<td>24.43</td>
</tr>
<tr>
<td>Number of observations</td>
<td>22647</td>
<td>22704</td>
</tr>
</tbody>
</table>

*Notes: Summary statistics are derived from the PSID for the years 1990-2003.*

those who are self-employed (see Johns and Slemrod (2010)). Second, the answer to the first question can be positive if the household has “a financial interest in any business enterprise” and it would not reflect the occupation of the household, which we have in mind in the model. Moreover, the second survey question gives more information on the occupation of the head of the household and allows to clearly distinguish between self-employed and workers.

Based on the second survey question, we define an entrepreneurial household as a household where the head is self-employed, a ‘worker’ household where the head is an employee or ‘both a self-employed and an employee’.26 We drop those who answered they are ‘unemployed’ from the sample. As the result, there are 14.70% of self-employed households in our sample. Some important summary statistics for the alternative definitions of entrepreneurs are presented in Table A.1.

### 1.A.2 Estimating Labor Income Process

In our income process estimation, we follow closely the procedure described by Heathcote et al. (2010). Since our model unit is a household, we focus on household labor income. We concentrate on the residual dispersion for logarithm labor household income residuals obtained from a standard Mincerian regression:

$$\lninc_{i,t} = \alpha_0 + \beta_0 \text{educ}_{i,t} + \beta_1 \text{potexp}_{i,t} + \beta_2 \text{potexp}_{i,t}^2 + \epsilon_{i,t},$$

where $i$ is a household index and $t$ is time. The variable $\lninc$ is the logarithm of household labor income, $\text{educ}$ refers to years of education and $\text{potexp}$ represents years of potential experience.

Potential experience is calculated as the difference between the age and years of education less 6, i.e., $\text{potexp} = \text{age} - \text{educ} - 6$, where 6 is the typical age for entering the

---

26There are 0.7% of such households, hence, either dropping those households or including them to either of the group does not change the main moments.
elementary school. Hence, someone who is 40 years old, with 16 years of education can potentially have 18 years of working experience.

We assume that the error term follows a first order Markov process of the form:

$$\log \varepsilon_{t+1} = \rho \varepsilon_t + \eta_{t+1},$$

(1.25)

where $\eta_{t+1} \sim N(0, \sigma^2_\varepsilon)$. We estimate this process for workers and obtain a persistence parameter $\rho = 0.89$ whereas the dispersion parameter is $\sigma = 0.21$.

### 1.A.3 Estimating Entry and Exit Rates

The exit rates are calculated as follows. First, we sort individuals by their identification number and consider two consecutive years. Then, we calculate how many individuals remained workers from one year to another and divide by the initial number of workers. This gives us the share of people who stayed workers. In the same way, we calculate the share of those who stayed self-employed. Exit rates are calculated as one minus the share of those who stayed a worker/self-employed. Finally, we calculate a weighted sum of year-by-year exit rates to get an average number we use for calibration. We get that on average, per year, around 15.73% of those who were self-employed exited self-employment. This number is comparable with the exit rate of 13.6% reported by Quadrini (2000). Table A.2 shows the year-by-year exit rates for workers and self-employed.

<table>
<thead>
<tr>
<th>Year</th>
<th>% Stayed Workers</th>
<th>Number of Workers</th>
<th>Exit Rate Workers (%)</th>
<th>% Stayed Self-Employed</th>
<th>Number of Self-Employed</th>
<th>Exit Rate Self-Employed (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>96.82</td>
<td>1,572</td>
<td>3.18</td>
<td>88.85</td>
<td>278</td>
<td>11.15</td>
</tr>
<tr>
<td>1991</td>
<td>96.32</td>
<td>1,522</td>
<td>3.68</td>
<td>89.93</td>
<td>278</td>
<td>10.07</td>
</tr>
<tr>
<td>1992</td>
<td>96.07</td>
<td>1,424</td>
<td>3.93</td>
<td>79.57</td>
<td>235</td>
<td>20.43</td>
</tr>
<tr>
<td>1994</td>
<td>96.84</td>
<td>1,709</td>
<td>3.16</td>
<td>85.00</td>
<td>260</td>
<td>15.00</td>
</tr>
<tr>
<td>1995</td>
<td>97.96</td>
<td>1,715</td>
<td>2.04</td>
<td>82.14</td>
<td>252</td>
<td>17.86</td>
</tr>
<tr>
<td>1996</td>
<td>97.45</td>
<td>1,728</td>
<td>2.55</td>
<td>84.65</td>
<td>241</td>
<td>15.35</td>
</tr>
<tr>
<td>1997</td>
<td>96.33</td>
<td>1,609</td>
<td>3.67</td>
<td>83.11</td>
<td>219</td>
<td>16.89</td>
</tr>
<tr>
<td>1999</td>
<td>96.07</td>
<td>1,704</td>
<td>3.93</td>
<td>82.45</td>
<td>245</td>
<td>17.55</td>
</tr>
<tr>
<td>2001</td>
<td>95.93</td>
<td>1,844</td>
<td>4.07</td>
<td>80.80</td>
<td>224</td>
<td>19.20</td>
</tr>
</tbody>
</table>

**Exit Rate**

15.73

*Notes:* Summary statistics are derived from the PSID for the years 1990-2003. The year 1993 is excluded because of missing information on the occupation.

### 1.A.4 Estimating Income and Wealth Inequality

In this section, we report additional statistics which we do not include in the main text. We base our estimates of the asset distribution on the PSID variable ‘Wealth’, which includes:
Table A.3: Wealth Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Gini</th>
<th>Mean/Median</th>
<th>Bottom 40</th>
<th>Top 20</th>
<th>Top 10</th>
<th>Top 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>71.1</td>
<td>3.10</td>
<td>2.71</td>
<td>75.64</td>
<td>60.56</td>
<td>26.53</td>
</tr>
<tr>
<td>Self-Employed</td>
<td>68.4</td>
<td>2.61</td>
<td>4.35</td>
<td>72.17</td>
<td>57.23</td>
<td>21.13</td>
</tr>
<tr>
<td>Workers</td>
<td>67.0</td>
<td>2.61</td>
<td>3.17</td>
<td>71.86</td>
<td>55.21</td>
<td>21.14</td>
</tr>
</tbody>
</table>

Notes: Summary statistics are derived from the PSID for the years 1990-2003.

a) net market value of a business, (b) money in checking and savings accounts, money market funds certificates or deposit, government savings bonds or treasury bills, (c) other real estate than your main home, (d) shares of stock in publicly held corporations, mutual funds, etc., (e) value of home equity. In the data, we recode negative asset positions with zeros to zeros to stay consistent with the model. Some important summary statistics on the asset distribution in the data are summarized in Table A.3.

1.A.5 Data on Auditing

Table A.4 is based on Slemrod and Gillitzer (2014) and U.S. Department of the Treasury (2011) and reports auditing rates by type and size of reported income, relative to the fiscal year 2011. On average only 1.11% of individual tax returns are audited but this percentage changes quite substantially across income levels. Generally the probability of being audited by the Internal Revenue Service is rising with income, increasing from less than 1% to almost 30% for tax returns above $10 million. Individuals who include business income in their returns are significantly more likely to be audited. Small corporations, with less than $10 million in total assets, are audited with only 1% probability, whereas larger corporations, with more $10 million in total assets, have an audit rate at 17.6%.

The picture varies quite a lot also when it comes to the fine and punishment. The IRS imposes a relatively low penalty of 20% on underpayment that lacks economic substance, whereas it penalizes at a higher 75% underpayment due to fraud. The U.S. tax code follows on this respect most of existing legislatures around developed countries. According to OECD (2011), penalties for minor offenses are set around 10-30% of evaded tax liability, while more serious offenses are penalized at 40-100% of the tax evaded.
Table A.4: U.S. Auditing Rates by Type and Size of Tax Return

<table>
<thead>
<tr>
<th>Type of return</th>
<th>Percent covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Income Tax</td>
<td>1.11</td>
</tr>
<tr>
<td>No adjusted gross income</td>
<td>3.42</td>
</tr>
<tr>
<td>[1, 25000)</td>
<td>1.22</td>
</tr>
<tr>
<td>[25000, 50000)</td>
<td>0.73</td>
</tr>
<tr>
<td>[50000, 75000)</td>
<td>0.83</td>
</tr>
<tr>
<td>[75000, 100000)</td>
<td>0.82</td>
</tr>
<tr>
<td>[100000, 200000)</td>
<td>1.00</td>
</tr>
<tr>
<td>[200000, 500000)</td>
<td>2.66</td>
</tr>
<tr>
<td>[500000, 1m)</td>
<td>5.38</td>
</tr>
<tr>
<td>[1m, 5m)</td>
<td>11.80</td>
</tr>
<tr>
<td>[5m, 10m)</td>
<td>20.75</td>
</tr>
<tr>
<td>&gt; 10m</td>
<td>29.93</td>
</tr>
<tr>
<td>Corporate income tax</td>
<td>1.5</td>
</tr>
<tr>
<td>Small firms (&lt;$10m in assets)</td>
<td>1.0</td>
</tr>
<tr>
<td>Large firms (&gt;=$10m in assets)</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Notes: The numbers are based on the fiscal year 2011 and are taken from Slemrod and Gillitzer (2014) and U.S. Department of the Treasury (2011).

Appendix 1.B Model

1.B.1 Welfare Analysis: Details

Let us define the state space as \( x = (a, \epsilon, \theta) \). The distribution in the benchmark economy is given by \( \mu^B (a, \epsilon, \theta) \), while in the counterfactual economy is \( \mu^C (a, \epsilon, \theta) \). The corresponding decision rules for the occupational choice are \( o^B (a, \epsilon, \theta) \) and \( o^C (a, \epsilon, \theta) \). Then, the welfare criterion in the benchmark economy is given by

\[
EV^B = \int [o^B (a, \epsilon, \theta) V^{B,E} (a, \epsilon, \theta) + (1 - o^B (a, \epsilon, \theta)) V^{B,W} (a, \epsilon, \theta)] d\mu^B (a, \epsilon, \theta),
\]

whereas welfare criterion in a given counterfactual economy \( C \) is

\[
EV^C = \int [o^C (a, \epsilon, \theta) V^{C,E} (a, \epsilon, \theta) + (1 - o^C (a, \epsilon, \theta)) V^{C,W} (a, \epsilon, \theta)] d\mu^C (a, \epsilon, \theta).
\]

To compare welfare changes between benchmark and counterfactual, we compute the consumption equivalent variation (CEV) needed to make a household indifferent between the two economies. Specifically, we calculate

\[
CEV = (EV^C / EV^B)^{1/(1-\sigma)} - 1,
\]

where \( EV^C \) and \( EV^B \) are defined above. We can further compute the same number based on the decile of household net wealth.\(^{27}\) Therefore, we derive a function which places each

\(^{27}\)Note that it is difficult to consider gross income because of the presence of tax evasion and its pun-
household in a particular decile of household net wealth in the benchmark economy. The function, say \( D(o^B(a, \varepsilon, \theta), a, \varepsilon, \theta, \mu^B) \), depends on the asset position, the abilities \( \varepsilon \) and \( \theta \), the occupational choice, and the distribution over the state variables. It takes discrete values from 1 to 10. A household type is defined as \((i, j)\), where

\[
i = D(o^B(a, \varepsilon, \theta), a, \varepsilon, \theta, \mu^B)
\]

and

\[
j = o^B(a, \varepsilon, \theta).
\]

We can summarize the types in a series of indicator functions,

\[
1_{i,j}(a, \varepsilon, \theta) = \begin{cases} 
1 & \text{if } D(o^B(a, \varepsilon, \theta), a, \varepsilon, \theta, \mu^B) = i \text{ and } o^B(a, \varepsilon, \theta) = j \\
0 & \text{otherwise}.
\end{cases}
\]

We compute 20 numbers,

\[
EV_{i,j}^B = \frac{\int 1_{i,j}(a, \varepsilon, \theta)|o^B(a, \varepsilon, \theta)V^{R,E}(a, \varepsilon, \theta) + (1 - o^B(a, \varepsilon, \theta))V^{R,W}(a, \varepsilon, \theta)|d\mu^B(a, \varepsilon, \theta)}{\int 1_{i,j}(a, \varepsilon, \theta)d\mu^B(a, \varepsilon, \theta)}.
\]

Then, we can go to the counterfactual economy. We keep the definition of the household type from the benchmark economy and compute the new welfare numbers using the new distribution, \( \mu^C \),

\[
EV_{i,j}^C = \frac{\int 1_{i,j}(a, \varepsilon, \theta)|o^C(a, \varepsilon, \theta)V^{C,E}(a, \varepsilon, \theta) + (1 - o^C(a, \varepsilon, \theta))V^{C,W}(a, \varepsilon, \theta)|d\mu^C(a, \varepsilon, \theta)}{\int 1_{i,j}(a, \varepsilon, \theta)d\mu^C(a, \varepsilon, \theta)}.
\]

Finally, in Figure 1.6 in the main text we plot

\[
CEV_{i,j} = (EV_{i,j}^C/EV_{i,j}^B)^{1/(1-\sigma)} - 1,
\]

where \( j = 0,1 \) refers to the occupation (worker or self-employed) and \( i = 1, \ldots, 10 \) to decile of wealth.

1.B.2 The Probability of Auditing: Robustness

In this section, we assess the impact of the functional form of the audit probability \( p(k) \) on the model-generated misreporting rate and the size distribution of self-employed businesses. To this end, we compare the benchmark economy with the logistic probability function \( p(k) \) to an economy in which the audit probability is constant, \( p(k) = p \). \( p \) is set
to the average audit probability in the benchmark economy. All other parameters are kept unchanged.

In Figure B.1, we report the tax evasion targets (in terms of misreported percentage) over quintiles of total income. Since the parameters of $p(k)$ are identified by matching the tax evasion targets, clearly, the logistic probability function $p(k)$ delivers a better fit than the constant audit probability.

Figure B.1: Tax Evasion by Income

Notes: U.S. data on the percentage misreporting rate by true income groups are taken from Johns and Slemrod (2010), Table 2.

in Figure B.2, we report the distributions of self-employed business capital for the two specifications. The economy with a constant audit probability delivers a distribution in which more firms choose higher capital levels. In particular, the economy with a constant $p$ underestimates the share of self-employed businesses in the first bin of the size distribution. Moreover, the economy with a constant audit probability substantially overstates the percentage of self-employed businesses in the second bin.
Figure B.2: Size of Self-Employed Businesses and the Probability of Auditing

Notes: Firm size is measured in terms of capital. U.S. data are from the PSID for the years 1993-2003.

Figure B.3: Probability of Auditing

Notes: Panel (a) measures business income in model units.

One might argue that the probability of being detected depends on business income rather than on business capital. Therefore, in Figure B.3a, we report the audit probability as a function of business income $\pi$ for different values of the entrepreneurial ability $\theta$, as implied by our benchmark economy with a logistic $p(k)$. Figure B.3b shows the average probability of detection for different quintiles of business income. Evidently, our benchmark model implies a probability of being audited that is increasing in business income, which is in line with the empirical evidence reported in Slemrod and Gillitzer (2014).
1.B.3 Collateral Constraint: Robustness

In this section, we provide a sensitivity analysis with respect to the parameter $\lambda$, which measures the tightness of the collateral constraint. We vary $\lambda$ between 1.1 and 1.8 (the benchmark value being $\lambda = 1.2$) and report the model targets and the main model outcomes in Table B.5 and Table B.6, respectively.

As $\lambda$ increases, the collateral constraint for self-employed becomes looser, thus allowing them to borrow and invest more. Accordingly, the average size of business capital and the capital-output ratio increase. At the same time, the share of self-employed income increases from 21.50% to 30%, and the share of assets rises from 42% to 45%. The misreporting rate is affected by two effects. On the one hand, a higher value of $\lambda$ relaxes the borrowing constraint and decreases the incentives to evade taxes. On the other hand, less talented agents become self-employed who evade more. Therefore, the overall misreporting rate follows a non-trivial pattern in $\lambda$.

Table B.5: Basic Model Targets: Varying $\lambda$

<table>
<thead>
<tr>
<th>Moments</th>
<th>Benchmark</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interests rate (%)</td>
<td></td>
<td>3.99</td>
<td>3.97</td>
<td>3.92</td>
<td>3.87</td>
<td>3.86</td>
<td>3.81</td>
<td>3.84</td>
<td>3.88</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td></td>
<td>2.58</td>
<td>2.62</td>
<td>2.64</td>
<td>2.68</td>
<td>2.70</td>
<td>2.74</td>
<td>2.75</td>
<td>2.78</td>
</tr>
<tr>
<td>Share of self-employed (%)</td>
<td></td>
<td>14.07</td>
<td>14.65</td>
<td>15.01</td>
<td>15.36</td>
<td>15.82</td>
<td>16.05</td>
<td>16.30</td>
<td>16.59</td>
</tr>
<tr>
<td>Share of assets, self-employed (%)</td>
<td></td>
<td>42.08</td>
<td>42.72</td>
<td>43.39</td>
<td>43.91</td>
<td>44.36</td>
<td>44.83</td>
<td>45.01</td>
<td>45.11</td>
</tr>
<tr>
<td>Share of income, self-employed (%)</td>
<td></td>
<td>21.50</td>
<td>23.76</td>
<td>24.67</td>
<td>25.87</td>
<td>27.47</td>
<td>28.19</td>
<td>28.86</td>
<td>30.48</td>
</tr>
<tr>
<td>Exit rate, self-employed (%)</td>
<td></td>
<td>15.95</td>
<td>15.90</td>
<td>15.79</td>
<td>15.80</td>
<td>15.73</td>
<td>15.71</td>
<td>15.69</td>
<td>15.71</td>
</tr>
</tbody>
</table>
Chapter 1. The Aggregate Consequences of Tax Evasion

Table B.6: Aggregate Moments: Varying $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Benchmark</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sector of self-employment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of self-employed (%)</td>
<td></td>
<td>14.07</td>
<td>14.65</td>
<td>15.01</td>
<td>15.36</td>
<td>15.82</td>
<td>16.05</td>
<td>16.30</td>
<td>16.59</td>
</tr>
<tr>
<td>$E(\theta</td>
<td>E)$</td>
<td></td>
<td>0.94</td>
<td>0.93</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>$E(k</td>
<td>E)$</td>
<td></td>
<td>11.81</td>
<td>12.86</td>
<td>13.68</td>
<td>14.55</td>
<td>15.34</td>
<td>16.18</td>
<td>16.76</td>
</tr>
<tr>
<td>$K^E$</td>
<td></td>
<td>1.66</td>
<td>1.88</td>
<td>2.05</td>
<td>2.23</td>
<td>2.43</td>
<td>2.60</td>
<td>2.73</td>
<td>2.92</td>
</tr>
<tr>
<td>$Y^E$</td>
<td></td>
<td>0.62</td>
<td>0.68</td>
<td>0.72</td>
<td>0.77</td>
<td>0.81</td>
<td>0.85</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>Corporate sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^C$</td>
<td></td>
<td>3.83</td>
<td>3.84</td>
<td>3.82</td>
<td>3.83</td>
<td>3.81</td>
<td>3.80</td>
<td>3.80</td>
<td>3.80</td>
</tr>
<tr>
<td>$N^C$</td>
<td></td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.84</td>
<td>0.84</td>
<td>0.84</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td>$Y^C$</td>
<td></td>
<td>1.51</td>
<td>1.51</td>
<td>1.50</td>
<td>1.50</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
<td>1.48</td>
</tr>
<tr>
<td><strong>Prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$ (%)</td>
<td></td>
<td>3.99</td>
<td>3.97</td>
<td>3.92</td>
<td>3.87</td>
<td>3.86</td>
<td>3.81</td>
<td>3.84</td>
<td>3.88</td>
</tr>
<tr>
<td>$w$</td>
<td></td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
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<tr>
<td><strong>Tax revenues</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Wealth inequality</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini</td>
<td></td>
<td>73.24</td>
<td>73.50</td>
<td>74.02</td>
<td>74.42</td>
<td>74.65</td>
<td>75.16</td>
<td>75.24</td>
<td>75.30</td>
</tr>
</tbody>
</table>

Notes: The table outlines how the macroeconomic aggregates of the economy change for different values of $\lambda$. $E(\theta|E)$ and $E(k|E)$ denote the mean value of business ability and business capital of a self-employed agent. $K^E$ and $Y^E$ refer to aggregate capital and aggregate output in the self-employment sector. $K^C$, $N^C$, and $Y^C$ denote capital, labor, and output in the corporate sector, respectively. Tax revenues $T/Y$ are given as percentage share of total output. $w$ denotes the real wage rate, while $r$ is the real interest rate in percent.
Chapter 2

Pension System Reforms in the Presence of Informality
2.1 Introduction

Aging populations worldwide raise concerns about the sustainability of public social security systems and call for pension reforms.\(^1\) In emerging economies, policymakers face the additional challenge of sizable informal sectors (Schneider et al. (2010)). The informal sector is defined as unofficial production which remains unrecorded with a purpose of tax and social security avoidance. Informal activities interact with the pension system in a direct and an indirect way. By its nature, informal workers do not contribute to the pension system which directly implies lower pension contributions and tax revenues. At the same time, the pension system design can itself create incentives to exit the social security system by working informally leading to unintended consequences of government interventions.

The aim of the paper is to evaluate alternative pension system reforms in the presence of informality and to analyze the mechanisms through which the informal sector and the pension system interact. To this end, I develop an overlapping-generation life-cycle (OLG) model with incomplete markets, in which heterogeneous agents choose whether to work in the formal or informal sector. I employ the model to explore the impact of different pension system reforms on the size of the informal sector, fiscal revenues, inequality, and welfare.

As a case study, I focus on Brazil. Brazil operates a pay-as-you-go pension system, which implies that the working population pays pension benefits to the retirees.\(^2\) The World Bank (2017) reports that despite a currently low dependency ratio\(^3\) in Brazil, the public spending on pensions is high, amounting to around 11% of GDP. Due to a drastic demographic change, it is projected that this number will rise to 26% of GDP in the next 35 years making the system unsustainable (IMF (2016)). The government of Brazil is currently undertaking a pension system reform in order to maintain the financial sustainability of the system. Crucially, Brazil has a large informal sector which covers around 40% of the entire workforce (Meghir et al. (2015)).

This paper develops a quantitative OLG model in the spirit of Auerbach and Kotlikoff (1987), enriched by an uninsurable idiosyncratic earnings risk as in Huggett (1996). I extend the standard model by introducing two sectors of production: a formal and an informal sector. In each period, individuals endogenously choose the sector of occupation as well as the amount of hours worked, savings and consumption. In the model setup, pension benefits depend on an individual earning index which is the result of the

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\(^1\)For example, Chile addressed the unsustainability of its pay-as-you-go pension system by reforming to a system of individual retirement accounts. See also OECD (2016), ‘Recent Pension Reforms’ for more examples.

\(^2\)A detailed description of the current pension system in Brazil is provided in Section 2.

\(^3\)The ratio of retirees to the working-age population.
occupational choice during the working period and average earnings. Working informally implies that an individual does not contribute to the social security system which lowers the current tax burden but curtails pension benefits in the future.

I calibrate the model to Brazil using the Brazilian survey data Pesquisa Mansal de Emprego (PME) which contains detailed information about the individual characteristics, labor income and hours of work for both, workers in the formal and informal sectors. For the details of the pension system I rely on the OECD (2016) report “Pensions at a Glance”. My model matches well the size of the informal sector in terms of GDP and labor share, the income difference between the two sectors, the replacement rate, the pension expenditures to GDP ratio and a range of other macroeconomic indicators.

I use the model to conduct a series of experiments. In the first step, I evaluate the long-run consequences of the projected demographic change in Brazil assuming that no reform takes place. For this experiment, I multiply the survival rates of the benchmark economy by a factor of \( \Delta \) to reach the new dependency ratio of 36% as projected by the Brazilian Institute of Statistics (IBGE). As a consequence, the pension system deficit rises by 2.7pp and informality drops by around 6pp. Due to a higher life-expectancy, individuals care more about their retirement income. Working formally offers higher pension benefits in the future, hence, the formal sector becomes more attractive and the share of informal workers decreases. A substantial reallocation of labor from the informal to the formal sector mitigates the negative effect of an increased dependency ratio due to the enlarged tax base.

Next, I use my theoretical framework to quantitatively evaluate a series of policy reforms which close the budget gap caused by the demographic change. First, I assess how much adjustment is needed for each policy instrument to cover the induced fiscal deficit. Afterwards, I compare the aggregate consequences of these measures. I consider (1) an increase of the individual payroll tax from current 11% to 18.9%; (2) a reduction of the pension benefits by 16%; (3) a rise of the required years of contribution from 15 to 22 years; (4) a retirement age increase from 60 to 65 years.

I find that an increase in the individual payroll tax creates strong incentives to exit the formal sector and operate informally. The share of informal workers increases by 17pp. This reform distorts labor supply and capital accumulation and results in the largest welfare loss compared to the alternatives.\(^4\) Reducing the pension benefits has a small effect on the reallocation of labor since the whole pension scheme, including the minimum pension benefit, is scaled down. Raising the required years of contribution results in the lowest share of informal workers of around 26%. In this scenario, individuals spend more years in the formal sector to qualify for a decent pension benefit. Raising the retirement

\(^4\)Welfare is measured by the consumption equivalent variation and compares ex-ante expected lifetime utility of a newborn in a stationary equilibrium before and after the reform.
age shortens retirement and extends the working period for the individuals. Keeping the pension requirements unchanged, agents prefer to work the additional years in the tax-free informal sector and the share of informal labor rises. Individuals can afford to consume more due to higher income and lower retirement savings generating a positive welfare effect of 3% measured in consumption equivalent variation. The last reform leads to the highest long-run welfare gain compared to the alternatives.

Additionally, I evaluate the reform proposed by the Brazilian government which combines the increase in both: the retirement age to 65 years and required years of contribution to 25 years.\(^5\) Combining the two measures results in a synergistic effect. The government proposal does not only compensate for the budget gap created by the demographic change but additionally reduces the fiscal deficit. It reduces the pension deficit by around 6\(pp\) and the pension expenditure to GDP ratio by around 7\(pp\). GDP increases by around 21\% and the welfare gain amounts to 3.2\% measured consumption equivalent variation in the long-run.

Finally, I study the role of informality by considering the pension system reforms in an economy where the informal sector is absent. As a result of the demographic change, the pension system deficit rises by around 3\(pp\). Comparing the benchmark set-up with the counter-factual economy reveals that the government has to undertake more severe policy adjustments in the presence of informality. The payroll tax rate has to increase by 7.9\(pp\) with informality as opposed to 2.5\(pp\) without the informal sector. Increasing the payroll tax rate raises the incentives to operate informally. This lowers the tax base and requires an even higher increase in the payroll tax to cover the additional fiscal deficit. Similarly, the pension benefits have to decrease by 16\% as opposed to 12\%, the required years of contribution have to increase by 7 in contrast to 5 years, and the retirement age has to rise by 5 instead of 2.5 years in the presence of informality.

All the considered reforms, apart from the payroll tax hike, lead to a higher long-run welfare gain in the absence of informality. For example, the increase in the retirement age results in a 5\% instead of a 3\% welfare gain. In the case of the payroll tax hike, the opportunity to evade taxes mitigates the tax distortion in the economy. In the presence of informality, the payroll tax hike leads to a 1.4\% welfare loss in contrast to a 2.7\% welfare loss without informality. The government proposal results in higher output, lower pension expenditures, lower inequality and a higher welfare gain of 3.3\% in the absence of informality. Informality does not change the ranking of the reforms in relative terms such that increasing the retirement age remains the best option in terms of long-run welfare. However, the results of the policy measures differ significantly in the quantitative respect. The policy implication of this analysis is that the reforms should be implemented along with the reduction of the informal sector.

\(^5\)More details on the government proposal are described in the next section.
This project builds on two strands of literature. First, it is related to the social security literature developed by Auerbach and Kotlikoff (1987), Imrohoroglu et al. (1995), Conesa and Krueger (1999) and De Nardi et al. (2001), among others. Similarly to this paper, Kitao (2014) studies the effects of the demographic change and alternative pension reforms in the United States. However, all of the studies mentioned above abstract from informality since the analysis is conducted for developed economies where the size of the informal sector is small.

The second strand of literature focuses on the role of the informal sector but abstracts from the questions related to the social security system. My study is related to the work of Amaral and Quintin (2006), Antunes and Cavalcanti (2007), Ihrig and Moe (2004), Busato and Chiarini (2013), D’Erasmo and Moscoso Boedo (2012) and Prado (2011), among others, in that I model a two-sector economy. All these studies consider a formal and an informal sector but they impose different costs of informality. Amaral and Quintin (2006), Antunes and Cavalcanti (2007) and Ihrig and Moe (2004) focus on the lack of access to external finance for informal individuals. D’Erasmo and Moscoso Boedo (2012) assume different production technologies available for formal and informal firms such that informal firms have access only to a low-productivity process. Busato and Chiarini (2013) and Prado (2011) consider a penalty for tax evasion. In this paper, the cost of informality is assumed to be increasing in the agent’s ability to ensure a self-selection of high-ability individuals to the formal and low-ability individuals to the informal sector. Moreover, I emphasize the difference between the old-age insurance for formal and informal agents which is the core difference to the studies mentioned above.

This paper contributes to a rather small strand of literature that combines the questions of social security and informality. Joubert (2015) analyzes the role of mandatory pension contributions in Chile in the presence of an informal sector in a partial equilibrium set-up. He finds that mandatory pension contributions encourage informality. Similarly, McKiernan (2018) focuses on Chile and quantifies the welfare effects of the transition from the pay-as-you-go system to individual retirement accounts. She finds that the transition has substantial positive long-run welfare consequences with a mild welfare loss of transitional generations. In a related framework, Jung and Tran (2012) study the effects of government transfers to informal elderly in Brazil. They find that providing retirement benefits to informal sector workers induces efficiency losses and increases welfare. To distinguish between formal and informal retirees, the authors assume that individuals do not change their occupation during their lives. In contrast, I keep track of the individual occupational choice which defines the future pension benefit. Although the studies above analyze the interaction of social security and informality, they do not evaluate alternative pension

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6Other important contributions are, e.g, French (2005), Alonso-Ortiz (2014), Fehr et al. (2013), Fehr et al. (2008), Díaz-Gimenez and Díaz-Saavedra (2009).
system reforms in Brazil, which is the focus of my paper.

The rest of the paper is organized as follows. In the next section, I present stylized facts about the Brazilian pension system. In Section 3, I discuss the model. Section 4 describes the calibration. Finally, I present the results and draw conclusions.

2.2 The Pension System in Brazil

Brazil operates a pay-as-you-go pension system which implies that the current working population finances current retirees.

The first defining feature of a current pension system in Brazil is its generosity. The replacement rates in Brazil are considerably higher than the OECD average and range from 70 to 100% (OECD (2016)). Figure 2.1 illustrates the net replacement rate for a full-career worker having entered the labor market in 2014. An average-wage male worker entering the labor market at the age of 20 with a full career gets a full pension paying 80% of pre-retirement net earnings, at age of 55. The poor workers are entitled to a pension benefit which replaces more than 100% of their pre-retirement net income.

Second, the Brazilian pension system is characterized by a low average retirement age. On average, individuals start to collect pensions when they are 58 years old in contrast to the OECD average of 64 years (World Bank (2017)). There are two alternative

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7The OECD average is below 60% for the gross replacement rate.
8Defined as workers who receive less than 50% of the average wage.
eligibility criteria for a pension. Based on the years of contribution, with 35 (30) years of contributions men (women) can retire at any age. Based on the age, with at least 15 years of contribution, the individual can retire at age 65 (men) and 60 (women).\textsuperscript{9} Importantly, contributing only 15 years to the system guarantees a minimum pension which is equivalent to the minimum wage of those working today.\textsuperscript{10} For comparison, in the OECD countries on average, an individual has to contribute 26 years in order to receive a minimum pension which is not tied to the minimum wage. In Brazil, even if no contributions were made, after reaching the retirement age, pension-like assistance benefits are available. The benefit is equal to the minimum wage pension which in turn is equivalent to the minimum wage.

From the contribution side, the payroll tax is around 11% for the employees and 20% for the employers.

Currently, the Brazilian system runs a deficit of 3.7% of GDP (World Bank (2017)). The Brazilian Institute of Statistics (IBGE) projects that the old age dependency ratio will increase from 11% today to 36% by 2050, creating a substantial financial pressure.

Due to concerns about the sustainability of the Brazilian pension system, the Brazilian government has proposed a reform which includes several important changes. First, it is suggested to adopt a minimum retirement age to 65 years irrespective of the length of the contribution period. Moreover, the Brazilian government plans to raise the minimum contribution requirement from 15 to 25 years. Additionally, the system requirements for women have to gradually converge to those for men over the next 20 years and the minimum pension should be independent of the minimum wage in the long run.

2.3 The Model

I build a dynamic general equilibrium overlapping generations model in the spirit of Auerbach and Kotlikoff (1987) with income and lifespan uncertainty as in Huggett (1996). Individuals may work in either a formal or an informal sector and may switch between them during their lives. Formal workers must pay an income and social security tax. Informal workers, instead, do not pay taxes but bare a cost associated with informal employment. Individuals retire when they reach the retirement age. Their pension benefit depends on two things, first, the years of contribution to the system, second, on their earning index which captures the individual’s average income over the working period. Importantly, when individuals operate informally, the years of contribution are not counted which lowers their future pension benefit. Additionally, there are formal and informal firms which hire formal and informal labor, respectively. The government collects all the tax revenues to finance its expenditures which include government consumption and pension payouts.

\textsuperscript{9}60 and 55 respectively for rural workers.
\textsuperscript{10}The minimum wage is around 40% of the average wage.
2.3.1 Households

Demographics:

The economy is populated by overlapping generations of individuals. Individuals enter the economy at age \( j_0 \) and face lifespan uncertainty. The conditional probability of survival from age \( j \) to age \( j+1 \) is denoted as \( \psi_j \). The maximum possible age is \( j = J \), with \( \psi_J = 0 \). The size of a new cohort grows at a constant rate \( n \). Bequests are assumed to be collected and distributed as a lump-sum transfer to the entire population. Individuals enter the economy with no assets except for the lump-sum bequests \( b \).

Preferences and Endowments:

Households differ with respect to their age \( j \), asset position \( a \), labor productivity \( \epsilon \), earning index \( e \) and years of contribution \( h \). Given these characteristics, in each period, individuals decide whether to become a formal or an informal worker. Asset markets are incomplete, that is, households cannot insure against shocks to labor productivity \( \epsilon \). Moreover, individuals are borrowing constrained \( a' \geq 0 \).

Households maximize the expected sum of discounted utility given by:

\[
E_{j_0} \sum_{j=j_0}^J \beta^{j-j_0} \psi_j u(c_j, 1-l_j),
\]

where \( \beta \in (0,1) \) is the time discount factor, \( c_j \) is consumption, and \( 1-l_j \) is leisure at the age of \( j \). The total time endowment is normalized to one.

Each individual can allocate one unit of disposable time to leisure or market work. Individuals ability \( \epsilon \) consists of two components, \( \epsilon = \varpi_j \eta \). \( \varpi_j \) depends on the age and reflects the life-cycle component of the ability process.\( \eta \) denotes an idiosyncratic labor productivity and is drawn from a finite-state Markov process with transition probability given by \( F(\eta' | \eta) \).

In every period, the individual decides whether to work formally or informally. The maximization problem of the working population is given:

\[
V(j, a, \epsilon, e, h) = \max \{ V^F(j, a, \epsilon, e, h), V^I(j, a, \epsilon, e, h) \},
\]

where \( V^F(.) \) and \( V^I(.) \) are the value functions associated with becoming a formal and an informal worker, respectively.

\( ^{11} \)The estimation procedure of the life-cycle component is discussed in the Calibration Section.
Formal Workers:

In the formal sector, individuals receive income $y^F$ which depends on the individual ability $\epsilon$, the market formal wage $w^F$ and labor supply $l$. In addition to a consumption tax $\tau_c$ and a capital tax $\tau_k$, formal workers pay an income tax $\tau_l$ and a social security contribution tax $\tau_p$. The social security contribution tax is levied on the labor income but only up to a particular cap. This income ceiling is captured by the variable $\bar{y}$. If a worker is employed formally she contributes to the social security system and the government counts the years of contribution $h$ and updates the earning index $e$. The earning index captures the average income during the contribution years. The exact specification of $f(e, h, y^F)$ is discussed in the Appendix and approximates the earning index calculation in Brazil.

Individuals choose consumption, labor supply and savings in order to maximize their expected lifetime utility subject to their budget constraint.

\[
V^F(j, a, \epsilon, e, h) = \max_{c, l, a'} \left( u(c, l) + \beta \psi_{j} E \left[ V(j + 1, a', \epsilon', h', e') | \epsilon \right] \right) \tag{2.3}
\]

subject to

\[
y^F = \epsilon w^F l,
\]
\[
(1 + \tau_c)c + a' = (1 + r(1 - \tau_k))(a + b) + (1 - \tau_l)y^F - \tau_p \min \{ y^F, \bar{y} \}, \tag{2.4}
\]
\[
h' = h + 1,
\]
\[
e' = f(e, h, y^F).
\]

Informal Workers:

Informal workers do not pay the labor and social security tax, in contrast to the formal employees. Their income $y^I$ depends on the market informal wage $w^I$, their individual productivity and labor supply. Importantly, informal workers bear a utility cost of informality $c(\epsilon)$ which is increasing in their ability level, $c'(\epsilon) > 0$. The intuition behind this assumption is that individuals with a higher working ability face higher opportunity costs of operating informally.\textsuperscript{12} Technically, this assumption generates a self-selection of highly productive individuals into the formal sector and low-productive individuals into the informal sector which is in accordance with the empirical evidence (Galiani and Weinschelbaum (2012), Gasparini and Tornarolli (2009), Amaral and Quintin (2006), Meghir et al. (2015), among others). Workers who work informally do not contribute to the social security system and their earning history is not updated.

\textsuperscript{12}This cost can also be interpreted as the fact that more educated individuals assign a higher relative importance to non-wage benefits, particularly in formal jobs. Moreover, there is empirical evidence that low ability workers are more productive in the formal sector, while low ability workers are more productive in the informal sector. See Lopez García (2015) for more details.
\[ V^I(j, a, \epsilon, e, h) = \max_{c, l, a'} (u(c, l) - c(\epsilon) + \beta \psi_j \mathbb{E}[V(j + 1, a', \epsilon', h', e')|\epsilon]), \]  

subject to
\[ y^I = \epsilon w^I l, \]
\[ (1 + \tau_c)c + a' = (1 + r(1 - \tau_k))(a + b) + y^I, \]
\[ h' = h, \]
\[ e' = e. \]  

**Retirees:**

After reaching the retirement age of \( J_R \) individuals stop working and receive the pension benefit \( p(e, h) \). The pension depends on the individual earning history index \( e \), which is calculated as the average income during the contribution years. Importantly, if an individual contributed less than \( \bar{H} \) years to the system, she gets the minimum pension \( \bar{p} \) upon retirement. The maximum pension is capped from above, denoted by \( \bar{p} \), and is fixed by Brazilian law. Hence, the pension benefit is calculated as follows:

\[ p = \begin{cases} 
\bar{p} & \text{if } h < \bar{H} \\
 p(e, h) & \text{if } h \geq \bar{H} \\
\bar{p} & \text{if } e > \bar{e}.
\end{cases} \]

Retirees choose their consumption and saving in order to maximize:

\[ V^R(j, a, e, h) = \max_{c, a'} (u(c, 0) + \beta \psi_j \mathbb{E}[V^R(j + 1, a', e, h)]) \]  

subject to
\[ (1 + \tau_c)c + a' = (1 + r(1 - \tau_k))(a + b) + p. \]  

Since retirement excludes the possibility to work, the earning index is not updated after retirement.

**2.3.2 Firms**

**Formal firms:**

Formal firms hire formal labor \( L_F \) and rent capital \( K \) in order to produce. A representative formal firm operates under a Cobb-Douglas production function with a technological parameter \( A^F \). Formal firms hire labor at the wage \( w^F \) and borrow capital at the rate \( r \). Capital depreciates at the rate \( \delta \).
Formal firms maximize their profits by solving the problem of the form:

$$\pi^F = \max_{K,L^F} \left( A^F K^\alpha L^F_1^{1-\alpha} - w^F L^F - (r + \delta)K \right), \quad (2.9)$$

where $\alpha \in (0,1)$ is the share of capital in the production function.

**Informal firms:**

Informal firms hire only informal labor $L^I$ in order to produce. La Porta and Shleifer (2014) document that informal firms are much more labor intensive than the formal ones. I follow Ihrig and Moe (2004), Busato and Chiarini (2004) and assume that informal firms do not use capital and produce with linear production function in labor and a productivity parameter $A^I$.

Informal firms hire informal labor in order to maximize their profits:

$$\pi^I = \max_{L^I} \left( A^I L^I - w^I L^I \right). \quad (2.10)$$

Both formal and informal firms produce identical goods.

**2.3.3 Government**

The government collects revenues from the consumption and capital tax paid by all individuals. Additionally, it receives payroll and income tax from formal workers. The revenues are spent on government consumption $G$ and pensions.\(^{13}\)

**2.3.4 Timing of Events**

The sequence of events in this economy unfolds as follows. Individuals enter the economy at the age of $j = j_0$ with no assets except of the accidental bequests $a_0 = b$. Additionally, at the beginning of their active lives, individuals draw an idiosyncratic ability component from a Normal distribution $\eta_0 \in N(0,\sigma_0)$. In each period, individuals make their formal-informal sector choice, $o \in \{F,I\}$. At the age of $j = J_R$ individuals retire and receive their pension which depends on the earning and contribution history of an individual. At the age of $j = J$ individuals die with certainty.

\(^{13}\)Note that government consumption is assumed to be wasteful.
2.3.5 Stationary Competitive Equilibrium

For a given set of exogenous demographic parameters \(\{n, \psi_j\}\) and a given stream of government consumption \(\{G\}\), a competitive equilibrium is characterized by sequences of household policies and value functions \(\{c, l, l', o, V, V^F, V^I, V^R\}\) for each state \(x \in (j, a, \epsilon, e, h)\), production plans \(\{Y, K, L_F, L_I\}\), sequences of tax and social security policies \(\{\tau_c, \tau_k, \tau_l, \tau_p, \rho\}\), sequences of prices \(\{w^F, w^I, r\}\), sequences of transfers of accidental bequests \(\{b\}\) as well as a distribution of individuals \(\mu\) that satisfy the following conditions:

1. Taking prices as well as tax and social security policy parameters as given, individuals choose their consumption, labor input and savings in order to maximize their expected discounted utility specified by equations (2), (3), (5) and (7).

2. Factor prices are determined competitively:

\[
 r = \alpha A^F K^{\alpha-1} L_F^{1-\alpha} - \delta, \\
 w^F = (1 - \alpha) A^F K^\alpha L_F^{-\alpha}, \\
 w^I = A^I.
\]

3. The lump-sum bequest transfer is equal to the amount of assets left by the deceased:

\[
 b = \int_x a'(x)(1 - \psi_j) d\mu(x).
\]

4. The labor and capital as well as the goods markets clear:

\[
 L_F = \int_x 1^F \ell(x) \ d\mu(x), \\
 L_I = \int_x 1^I \ell(x) \ d\mu(x), \\
 K = \int_x a'(x) \ d\mu(x), \\
 Y = \int_x c(x) d\mu + \tilde{I},
\]
where $1^F$ and $1^I$ are the indicator functions for the formal and the informal occupation, respectively. $Y$ denotes total production in the economy comprising production in both sectors and $\bar{I}$ is the total investment.

5. The government budget constraint is balanced:

$$G + \int_x 1^R p(x) \mu(x) = \int_x (\tau_p r(a(x) + b) + \tau_c c(x)) \mu(x) + \int_x 1^F (\tau_k y^F(x) + \tau_p \min \{y^F(x), \bar{y}\}) \mu(x),$$

where $1^R$ is an indicator function for retirees.

6. The distribution of individuals across states is stationary:

$$\mu(x) = R_{\mu} [\mu(x)].$$

2.4 Calibration

My main source of data consists of a panel of individuals of working age, sampled by the labor force survey of Brazil, Pesquisa Mensal de Emprego (PME). PME is designed and conducted by the National Statistics Bureau and follows individuals of the six main metropolitan regions of Brazil. The sample period starts in January 2010 and goes until December 2015.\textsuperscript{14}

For the purpose of this paper, I focus on workers of age 20-60 who are household heads. The survey provides information that allows me to classify employees as either formal or informal. Individuals are asked if their labor card is signed by their employer. If it is not, they are not registered and are not entitled to labor rights and benefits. I define an individual to be working informally if she is an employee or a domestic worker without a signed labor card.\textsuperscript{15} Additionally, I consider self-employed as informal workers due to the fact that around 95% of self-employed in Brazil do not contribute to the social security system (Henley et al. (2009)). This definition of informality follows the literature and is discussed in a greater detail in Cardoso (2016) with a list of papers following this approach. I do not consider unpaid workers, unemployed or individuals who did not report their occupation.\textsuperscript{16} The summary statistics of my sample are presented in the Appendix.

The rest of this section is organized as follows: First, I report parameters which are

\textsuperscript{14}I abstract from earlier periods to better capture the characteristics of the current Brazilian economy.

\textsuperscript{15}Domestic workers in Brazil report to receive income and have to sign a working contract similarly to employees (see Ben Yahmed (2018)).

\textsuperscript{16}Since I focus on household heads, who are by definition the most financially responsible unit of the household, they are unlikely to be considered in one of the category listed above.
fixed outside the model, then I discuss internally calibrated parameters which are set to match selected data targets. Finally, I present the model fit along several dimensions of the data and discuss the model performance.

2.4.1 Externally Fixed Parameters

The demographic structure of the economy is set externally. The population growth \( n \), is fixed to 1.8% which is the number used by Jung and Tran (2012) and describes the annual population growth in Brazil. Since for the computational reasons, one period in the model is equivalent to five years the number for a model period is \((1 + n)^5 - 1 = 0.093\) or 9.3%.

The conditional surviving probabilities, \( \{ \psi_j \}_{j=0}^J \), are taken from the life-tables provided by the World Health Organization.\(^{17}\) In Figure B.2 the red solid line shows the conditional surviving rates along age. Individuals enter the economy at the age of \( j_0 = 20 \). I set the maximum age in the economy to 14 periods which is equivalent to 90 years.\(^{18}\) The age at which individuals retire is heterogeneous in Brazil. Since I abstract from endogenous retirement, I set the retirement age to 60 years which is close to the average retirement age in Brazil.\(^{19}\)

The utility function \( u \) is defined as
\[
u(1 - l) = \left( \frac{e^{(1-l)^{1-\nu}}}{1-\sigma} \right)^{1-\sigma},
\]
where \( \nu \) denotes the intratemporal elasticity of substitution between consumption and leisure and \( \sigma > 0 \) is the relative risk aversion coefficient.\(^{20}\) I fix the coefficient of relative risk aversion \( \sigma \) to 2 which is standard in the macroeconomic literature. In order to estimate the age-efficiency profile \( \varpi \), and the stochastic component \( \eta \), I do the following: First, I regress hourly labor earnings\(^{21}\) on observable individual characteristics such as education, race and age.\(^{22}\) I use the estimated coefficients on the age polynomials to set the age-efficiency profile, \( \varpi \). Next, I model the stochastic part as a first order auto-regressive process: \( \log \eta_{t+1} = \rho \log \eta_t + \zeta_{t+1} \), where \( \zeta_{t+1} \sim N(0, \sigma^2_\eta) \). I estimate this process and obtain a persistence parameter on annual basis \( \rho = 0.776 \) which translates to 0.282 for a model period of 5 years. The dispersion parameter is \( \sigma_\eta = 0.382 \) which is equivalent to 0.553 for a five-year span. The details on the estimation and translation from the annual to a five-year period are provided in the Appendix.

The parameter \( \alpha \) represents the capital share in the formal sector production and is set to 0.4, which is the value used by Jung and Tran (2012). The total factor productivity

\(^{17}\)I use a calculated weighted average for males and females over the period of 2010-2015.
\(^{18}\)The surviving probabilities after age 85 are not reported in the life-tables of Brazil.
\(^{19}\)On average individuals retire when they are 58 (World Bank (2017)).
\(^{20}\)The advantage of this utility function specification is that it is consistent with the balanced growth path.
\(^{21}\)Note, that I use the hourly wage since I have endogenous labor supply in the model.
\(^{22}\)Note, this is done for the whole population and not separately for formal and informal workers. The reason is that the efficiency profile as well as the idiosyncratic shocks are unconditional on the occupation.
of the formal sector, $A^F$, is normalized to unity.

I fix the parameters for the consumption and capital tax to 23% and 22%, respectively, as reported in Jung and Tran (2012). The income tax $\tau_l$ combines 15% reported by Jung and Tran (2012) and the social security tax rate of 20% which is not capped, reported by the OECD (2016).\textsuperscript{23} The payroll tax $\tau_p$ is set to 11% (OECD (2016)). The cap for taxable income $\bar{y}$ is roughly twice the mean income in the economy based on calculations that the ceiling is 4390BRL and the mean income is around 2180BRL in Brazil (OECD (2016)). The minimum number for years of contribution is set to 3 periods, which is 15 years, and is in accordance with the Brazilian law. All externally set parameters are reported on annual basis in Table 3.2.

Table 2.1: Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>population growth</td>
<td>1.8%</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>${\psi^j}_{j=0}^J$</td>
<td>surviving probabilities</td>
<td></td>
<td>Figure B.2 WHO</td>
</tr>
<tr>
<td>$J$</td>
<td>maximum age</td>
<td>14</td>
<td>90 years</td>
</tr>
<tr>
<td>$R$</td>
<td>retirement age</td>
<td>8</td>
<td>60 years</td>
</tr>
<tr>
<td><strong>Preference and ability process</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>risk aversion</td>
<td>2</td>
<td>standard value</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>persistence</td>
<td>0.78</td>
<td>micro data (PME)</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>standard deviation</td>
<td>0.38</td>
<td>micro data (PME)</td>
</tr>
<tr>
<td><strong>Technology and production</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.4</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>$A^F$</td>
<td>TFP, formal sector</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>consumption tax</td>
<td>23%</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>capital tax</td>
<td>22%</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>income tax</td>
<td>35%</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>payroll tax</td>
<td>11%</td>
<td>OECD</td>
</tr>
<tr>
<td>$\bar{H}$</td>
<td>years of contribution</td>
<td>3</td>
<td>OECD</td>
</tr>
</tbody>
</table>

2.4.2 Internally Calibrated Parameters

After calibrating the external parameters, there are 7 parameters that are calibrated internally: the discount factor $\beta$, depreciation rate $\delta$, weight on leisure in the utility function $1 - \nu$, TFP in the informal sector $A^I$, two parameters in the cost function of informality $c_1, c_2$, and the minimum pension $p$. I calibrate those parameters to match 7 selected moments from the data listed in Table 3.3.

\textsuperscript{23}The payroll tax of 20% refers to the tax rate paid by firms. In the model, the tax burden of firms will be directly reflected in the worker’s wage.
It is well understood that all the model parameters affect all the targets, but we can nonetheless outline which data moment is most informative about a specific parameter. The interest rate and the capital-output ratio identify the discount factor $\beta$ and the depreciation rate $\delta$. The weight on leisure $1 - \nu$ is pinned down to match the average labor supply in Brazil, measured as the percentage of time endowment devoted to market work.\(^{24}\) The productivity of the informal sector $A^I$ and the two parameters in the cost function of informality $c_1$ and $c_2$ jointly affect the size of the informal sector in terms of GDP, the share of informal workers in the economy and the income ratio between formal and informal workers.

Finally, I determine the minimum pension in the economy, $p$, so that the implied replacement rate in the model and in the data are the same.\(^{25}\) Table 3.3 summarizes the recovered values for the internal parameters. The presented values are annualized.

Table 2.2: Internally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.955</td>
<td>interest rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.05</td>
<td>capital-output ratio</td>
</tr>
<tr>
<td>$\nu$</td>
<td>weight on leisure</td>
<td>0.4</td>
<td>labor supply</td>
</tr>
<tr>
<td>$A^I$</td>
<td>TFP, informal sector</td>
<td>0.685</td>
<td>size of informal sector</td>
</tr>
<tr>
<td>$c_1$</td>
<td>cost parameter</td>
<td>0.25</td>
<td>share of informal workers</td>
</tr>
<tr>
<td>$c_2$</td>
<td>cost parameter</td>
<td>0.6</td>
<td>average income ratio, formal-informal</td>
</tr>
<tr>
<td>$p$</td>
<td>minimum pension</td>
<td>0.05</td>
<td>replacement rate</td>
</tr>
</tbody>
</table>

2.4.3 Model Fit

Before analyzing the mechanism of the model and discussing the results, I present how the model matches the data and discuss some limitations.

The model fit is presented in Table 3.4. The annual interest rate in the model is close to the annual real interest rate in the data. The latter is taken from the Federal Reserve Bank of St. Louis.\(^{26}\) The capital-output ratio as well as the labor supply in the model and in the data are also fairly close. The data target for capital-output ratio is taken from the empirical study of Filho (2002). Note, that labor supply is calculated as the percentage of time endowment devoted to market work. The data target was calculated using PME for a pooled sample of formal and informal workers.

---

\(^{24}\) Time endowment is defined to be 120 hours per week which is the maximum possible value in the survey.

\(^{25}\) The maximum pension level is set to be 6 times the minimum wage as it is defined by Brazilian law.

\(^{26}\) Note, that I take the net interest rate and adjust for the inflation over the period of 2010-2015. This time range is chosen to stay consistent with the period I use in PME.
Chapter 2. Pension System Reforms in the Presence of Informality

The size of the informal sector measured as a fraction of GDP lies within the estimated values for Brazil. Filho (2012) estimates the size of informality to be around 20% of GDP. Medina and Friedrich (2018) report the size of the informal sector to be between 24.2% and 37.6% depending on the estimation approach. I calculate the share of informal workers from PME using the definition of informality described above. The recovered number of 34.98% is in line with other recent studies (Henley et al. (2009), Cardoso (2016), Meghir et al. (2015), among others).

The model generates a slightly higher gross income ratio between formal and informal workers compared to the data. The reason is that in the model there is a sorting of high-ability individuals to the formal sector and low-ability individuals to the informal. Empirical evidence suggests that on average informal workers are less educated and informal work is less ability-demanding (Amaral and Quintin (2006)). However, there is no clear cut-off in the data, whereas it is present in the model, and the exact mechanism is discussed below.

Importantly, the implied replacement ratio, which is calculated as the ratio between the mean pension to the mean wage, is matched. The model generates a pension-expenditure to GDP ratio of 13.55% and a pension system deficit of 4.07% of GDP which are both close to the data counterparts of 11.3% and 3.7% reported by the IMF and the World Bank.\footnote{Similarly to Kitao (2014) I do not clear the pension system budget separately from the aggregate government budget.}

Note that these two numbers describing the pension system in Brazil are not explicitly targeted.

Table 2.3: Model Fit

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>4.78%</td>
<td>4.72%</td>
<td>FRED</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>3</td>
<td>2.89</td>
<td>Filho (2002)</td>
</tr>
<tr>
<td>Labor supply</td>
<td>34.98%</td>
<td>35.19%</td>
<td>PME</td>
</tr>
<tr>
<td>Size of informal sector</td>
<td>20-35%</td>
<td>29.81%</td>
<td>Medina and Friedrich (2018), Filho (2012)</td>
</tr>
<tr>
<td>Share of informal workers</td>
<td>35.66%</td>
<td>35.49%</td>
<td>PME</td>
</tr>
<tr>
<td>Average income ratio, F/I</td>
<td>1.61</td>
<td>1.77</td>
<td>PME</td>
</tr>
<tr>
<td>Replacement rate</td>
<td>70%</td>
<td>67.36%</td>
<td>OECD (2016)</td>
</tr>
<tr>
<td>Pension deficit</td>
<td>3.7%</td>
<td>4.07%</td>
<td>World Bank (2017)</td>
</tr>
<tr>
<td>Pen. exp. to GDP</td>
<td>11.3%</td>
<td>13.55%</td>
<td>IMF (2016)</td>
</tr>
<tr>
<td>Gini, income</td>
<td>0.466</td>
<td>0.487</td>
<td>PME</td>
</tr>
</tbody>
</table>

An important limitation of the model is its inability to replicate the pattern of informality along the age dimension as shown in Figure B.5 in the Appendix. In the model, the
share of informal workers decreases with age. Since formal workers have to pay taxes in the present period but the pension benefits are expected in the future, there is an incentive to shift the contribution years to the future. However, in the data for Brazil the pattern is reversed. A possible explanation for the empirical evidence is that there is a trend component in the data which cannot be accounted for in the steady state of the model. In the last decades, informality in Brazil has decreased (Cardoso (2016), Fairris and Jonasson (2016), Medina and Friedrich (2018)). Informal workers in their late fifties started to work when the labor conditions were different from those now. Since the transition rates for the older population from informality to formality are low, the life-cycle pattern in the data most likely reflects the long-term trend of informality and less so the stationary occupational choice. It is worth mentioning that the recent study by ILO (2018) documents that on the world’s level the participation in the informal sector is high for the young and for the old. Amaral and Quintin (2006), Maloney (1999) and Maloney (2004) note that informal workers in developing countries are on average younger which is in line with the model predictions.

2.5 Results

In this section, I first discuss the occupational choice and the role of the earning history in the model. Then, I present the aggregate outcomes. Particularly, I explain the life-cycle patterns of consumption, labor supply and aggregate savings for the whole population, as well as, for the subgroups of formal and informal workers. Next, I present the consequences of the demographic change by increasing the benchmark surviving probabilities by a factor of $\Delta$ to reach the projected dependency ratio for the year 2050. After changing the demographic structure, I compare the new economy to the benchmark. Using the new economy which I label as ‘No Reform Economy’, I run four alternative pension reforms which cover the fiscal deficit caused by the demographic change and compare them in terms of aggregate outcomes and welfare. I also evaluate the reform proposed by the Brazilian government. Finally, I assess the role of informality by comparing the outcomes of alternative reforms in a counter-factual economy where the informal sector is absent.\(^{28}\)

2.5.1 Occupational Choice

Figure 2.2 panel a) shows the occupational choice as a function of ability and assets in the benchmark economy. Age, earning index and the years of contribution are fixed to the average value on the state space. The blue shaded area refers to the informal occupation, whereas the yellow shaded area to the formal. For a fixed value of assets individuals

\(^{28}\)For this experiment I do not recalibrate the model.
with a high ability choose to be formal whereas individuals with a low ability prefer to be informal. This is due to the assumption that being informal implies a cost that is increasing in ability as discussed in Section 3.\footnote{The function of the cost of informality and the underlying distribution is shown in Figure B.4.}

For a fixed ability level, individuals with higher assets choose to operate formally. Individuals with higher assets prefer to work fewer hours due to the income effect. Since the net wage in the formal sector is lower than in the informal sector, working less is more attractive in the formal sector due to a lower price of leisure. Note that there is an endogenous threshold level of ability $\bar{\epsilon}$ above which individuals choose to become formal. The threshold is endogenous since it depends on the relative wages in the formal and informal sectors which is a general equilibrium outcome.

Figure 2.2 panel b) depicts the occupational choice as a function of the earning index and assets. As before, the rest of the state space variables (age, ability and years of contribution) are fixed to the average value on the state space. Everything else equal, an individual with a higher earning index chooses to be formal to count this working period and receive a higher pension in the future.

![Figure 2.2: Occupational Policy](image)

(a) Assets - Ability  
(b) Assets - Earning Index

**Notes:** The left panel shows the occupational choice as a function of ability and assets in the benchmark economy. The right panel shows the occupational choice as a function of the earning index and assets. The blue (yellow) shaded area defines the informal (formal) occupation. For both figures the rest of the state-space variables are fixed to their average values.

### 2.5.2 Aggregate Outcomes

Figure 3.2 summarizes the average life-cycle profiles of consumption, labor supply in efficiency units and asset accumulation for the whole population. The outcome is standard
for an OLG model. Since the time discount factor of the household is smaller than the interest rate in the economy, the household favors an increasing consumption path over the life cycle. The labor supply follows a hump shape for two reasons. On the one hand, the labor ability has an efficiency component which rises with age (Figure A.1) and, therefore, individuals increase their labor supply in the initial stage of their economically active life. On the other hand, since individuals demand an expanding consumption path, they also increase their leisure consumption. From age 25 onwards, the second effect dominates and the hours worked decrease with age.

The combination of an increasing consumption path and a hump-shaped labor-related income profile induces households to save a substantial amount. These savings are meant to finance consumption in the retirement period.

Figure B.6 compares total and mean asset holdings of formal and informal workers. Panel (a) reveals that in aggregate terms informal workers hold almost no assets compared to formal workers. The reason behind this result is two-fold. First, the analysis of the occupational choice revealed (Figure 2.2a) that a low asset position induces individuals to work informally. Second, since informal workers are on average less able they receive lower income and save less. As a result, informal individuals tend to consume most of their income. Panel (b) shows that even after taking into account that there are fewer informal workers in the economy, informal workers’ asset accumulation is on average much lower compared to the formal.

\[^{30}\text{Note, that I can distinguish the sector of occupation only during the working period.}\]
Chapter 2. Pension System Reforms in the Presence of Informality

Figure 2.4: Asset Holdings Comparison

Notes: The left panel compares total asset holdings of formal and informal workers over the life-cycle whereas the right panel compares the respective means, measured in model units.

Figure 2.5 presents total and mean consumption of formal and informal workers over the life cycle. Both total and average consumption in the informal sector are lower compared to the formal except for the first period. In the first period, the model generates a large share of informal workers such that total consumption in the informal sector is larger than in the formal sector. The informal workers are on average poorer and can afford less consumption.

Next, Figure 2.6 compares the total and average labor supply measured in efficiency units in the formal and informal sectors. Informal workers supply on average less efficient labor compared to formal workers due to a lower productivity. However, as shown in Figure 2.7a informal workers work on average more hours. Since the informal market wage is larger than the net formal market wage for a given ability level, the price of leisure is smaller for the formal workers. Hence, they enjoy more leisure than informal workers.

Figure 2.7 compares hours of work by the sector of occupation in the model and in the data. Hours of work by formal employees decrease over the life-cycle both in the model and in the data, however the drop is less pronounced in the latter. The average hours of work in the informal sector remain relatively stable in the model whereas it has a reversed U-shape in the data. A possible explanation is that the young and the old are informally employed to make use of flexible hours of work and operate only as part-time workers. Policy functions with respect to savings, consumption and labor supply are reported in

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31 In the model the decrease is much sharper due to the mathematical properties of the model. 32 For example, the young could work part time in the informal sector and attend a college at the same time. I do not model neither schooling decisions nor early retirement explicitly.
Figure 2.5: Consumption Comparison

Notes: The left panel compares total consumption of formal and informal workers over the life-cycle whereas the right panel compares the respective means, measured in model units.

Finally, Table 3.6 reports the inequality measures for the gross income for the total working population as well as separately for the formal and informal employees. The model provides a decent match on the aggregate level. However, it overshoots the inequality in the formal sector and understates the inequality in the informal sector. This finding indicates that the model exaggerates the self-selection mechanism into informality and there are a lot of equally poor informal workers. On the contrary, the income of the informal employees in the data is more dispersed.

Table 2.4: Inequality Statistics, Pre-government Income

<table>
<thead>
<tr>
<th>Moments</th>
<th>Gini</th>
<th>Mean/Median</th>
<th>Bottom 40</th>
<th>Top 20</th>
<th>Top 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.487</td>
<td>1.479</td>
<td>10.857</td>
<td>52.233</td>
<td>34.119</td>
</tr>
<tr>
<td>Data</td>
<td>0.466</td>
<td>1.673</td>
<td>15.304</td>
<td>54.441</td>
<td>39.063</td>
</tr>
<tr>
<td><strong>Formal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.512</td>
<td>1.393</td>
<td>8.198</td>
<td>52.551</td>
<td>32.779</td>
</tr>
<tr>
<td>Data</td>
<td>0.455</td>
<td>1.676</td>
<td>15.664</td>
<td>53.653</td>
<td>38.248</td>
</tr>
<tr>
<td><strong>Informal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.297</td>
<td>1.023</td>
<td>20.480</td>
<td>37.179</td>
<td>21.205</td>
</tr>
<tr>
<td>Data</td>
<td>0.476</td>
<td>1.673</td>
<td>15.328</td>
<td>54.973</td>
<td>40.771</td>
</tr>
</tbody>
</table>

Notes: This table reports inequality statistics for the pre-government gross income for the total population as well as for the formal and informal workers separately. Data source is PME from 2010 to 2015.
Figure 2.6: Labor Supply Comparison

Notes: The left panel compares total labor supply in efficiency units of formal and informal workers over the life-cycle. The right panel compares the respective means.

Figure 2.7: Hours Worked Comparison

Notes: The figures compare the hours worked as a % of time endowment for formal and informal workers in the model and in the data. Data source for the right panel is PME from 2010 to 2015.
2.5.3 Demographic Change

In this subsection, I analyze the aggregate impact of the projected demographic change. The Brazilian Institute of Statistics (IBGE) states that the old-age dependency ratio, which is calculated as the ratio of retirees to the working population, will reach 36% by 2050 in Brazil. For this experiment, I multiply the survival rates of the benchmark economy by a factor of $\Delta$ to reach the new dependency ratio. The upward shift of surviving probabilities is shown in Figure B.2. Due to the demographic change the life expectancy at the age of 60 rises from 19.6 to 26.2 years.

Table 2.5 summarizes the main results. In the new economy, labeled as a No-Reform scenario, individuals live longer such that the retirement income becomes more important. Working in the formal sector increases future pensions, hence, in the No-Reform economy the formal sector becomes more attractive and the share of informal workers drops. In order to insure against low income during the retirement period, individuals accumulate more assets during their working life. Higher capital accumulation decreases the interest rate and increases the formal wage. This makes the formal sector even more attractive for the individuals. Since capital and labor flows into the formal sector increase, formal production increases as well. At the same time, informal production substantially decreases which leads to an overall output decline.

In spite of higher pension expenditures due to a larger number of retirees, the increasing size of the formal sector raises pension revenues. The expenditure-to-GDP ratio increases by around 4.9pp and the pension system deficit rises by 2.7pp. The projected demographic change leads to the reallocation of labor from the informal to the formal sector thus mitigating a negative effect of an increased dependency ratio on the pension system deficit.

2.5.4 Pension System Reforms

In this subsection, I analyze the effects of four alternative pension system reforms. The demographic change discussed above resulted in an amplification of the fiscal deficit by 2.7pp compared to the benchmark economy. I assess four options which compensate the additional deficit caused by the demographic change. I discuss the effects of an increase in the payroll tax, a reduction of the pension benefits, a hike in the minimum required years of contribution and an increase in the retirement age. The combination of the last two measures are proposed by the Brazilian government. I evaluate the consequences of this proposal later in the subsection.

Table 2.6 summarizes the main findings. In order to cover the additional pension system deficit the government has to increase the payroll tax from the current 11% to 18.9%. This

---

33 The World Bank projects those numbers to be around 15pp and 7pp, respectively.
### Table 2.5: Aggregate Results of the Demographic Change

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark</th>
<th>Demographic change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of inf. labor</td>
<td>35.49%</td>
<td>28.93%</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>30.44</td>
<td>29.61</td>
</tr>
<tr>
<td>$Y^{F*}$</td>
<td>21.37</td>
<td>23.15</td>
</tr>
<tr>
<td>$Y^{I*}$</td>
<td>9.07</td>
<td>6.45</td>
</tr>
<tr>
<td>$K$</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>$L^{F*}$</td>
<td>24.30</td>
<td>26.11</td>
</tr>
<tr>
<td>$L^{I*}$</td>
<td>13.25</td>
<td>9.42</td>
</tr>
<tr>
<td>$C^*$</td>
<td>23.12</td>
<td>23.40</td>
</tr>
<tr>
<td>$w^F/w^I$</td>
<td>1.77</td>
<td>1.79</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>4.72%</td>
<td>4.59%</td>
</tr>
<tr>
<td>Deficit</td>
<td>4.07%</td>
<td>6.76%</td>
</tr>
<tr>
<td>Pen. exp. to GDP</td>
<td>13.55%</td>
<td>18.45%</td>
</tr>
<tr>
<td>Pen. exp. change</td>
<td>-</td>
<td>29.40%</td>
</tr>
<tr>
<td>Pen. rev. change</td>
<td>-</td>
<td>17.99%</td>
</tr>
</tbody>
</table>

Notes: The table reports the results of a demographic change with no government interventions. The increase in dependency ratio raises the pension expenditures by 29.4%. However, also the pension revenues enlarge due to an increase in the tax base. The expenditure-to-GDP ratio rises by around 4.9pp and the pension system deficit by 2.7pp.

* denotes multiples of 100.

### Table 2.6: Pension Experiments

<table>
<thead>
<tr>
<th>Variable</th>
<th>No Reform</th>
<th>Reform 1 $\tau_p = 18.9%'$</th>
<th>Reform 2 $\tau_p = 16%'$</th>
<th>Reform 3 $H = 22$</th>
<th>Reform 4 $JR = 65$</th>
<th>Proposal $JR = 65 &amp; H = 25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of inf. labor</td>
<td>28.93%</td>
<td>45.51%</td>
<td>27.43%</td>
<td>26.67%</td>
<td>31.92%</td>
<td>29.22%</td>
</tr>
<tr>
<td>$E(\epsilon_{formal})$</td>
<td>1.3</td>
<td>1.31</td>
<td>1.29</td>
<td>1.29</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>29.61</td>
<td>29.81</td>
<td>30.12</td>
<td>30.41</td>
<td>34.94</td>
<td>35.95</td>
</tr>
<tr>
<td>$Y^{F*}$</td>
<td>23.15</td>
<td>24.34</td>
<td>24.90</td>
<td>26.62</td>
<td>28.76</td>
<td></td>
</tr>
<tr>
<td>$Y^{I*}$</td>
<td>6.45</td>
<td>5.78</td>
<td>5.51</td>
<td>8.31</td>
<td>7.20</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>0.97</td>
<td>0.89</td>
<td>1.02</td>
<td>1.04</td>
<td>1.08</td>
<td>1.18</td>
</tr>
<tr>
<td>$L^{F*}$</td>
<td>26.11</td>
<td>27.44</td>
<td>28.01</td>
<td>30.56</td>
<td>32.88</td>
<td></td>
</tr>
<tr>
<td>$L^{I*}$</td>
<td>9.42</td>
<td>8.44</td>
<td>8.05</td>
<td>12.14</td>
<td>10.50</td>
<td></td>
</tr>
<tr>
<td>$C^*$</td>
<td>23.40</td>
<td>22.82</td>
<td>22.92</td>
<td>26.56</td>
<td>26.23</td>
<td></td>
</tr>
<tr>
<td>$w^F/w^I$</td>
<td>1.79</td>
<td>2.02</td>
<td>1.79</td>
<td>1.79</td>
<td>1.73</td>
<td>1.75</td>
</tr>
<tr>
<td>$\tau_v$</td>
<td>4.59%</td>
<td>3.62%</td>
<td>4.57%</td>
<td>4.57%</td>
<td>4.88%</td>
<td>4.79%</td>
</tr>
<tr>
<td>Deficit</td>
<td>6.76%</td>
<td>4.07%</td>
<td>1.07%</td>
<td>1.07%</td>
<td>1.07%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Pen. exp. to GDP</td>
<td>18.45%</td>
<td>17.15%</td>
<td>14.64%</td>
<td>14.66%</td>
<td>13.51%</td>
<td>10.92%</td>
</tr>
<tr>
<td>Pen. exp. change</td>
<td>-</td>
<td>-4.10%</td>
<td>-16.57%</td>
<td>-15.97%</td>
<td>-11.92%</td>
<td>-27.00%</td>
</tr>
<tr>
<td>Pen. rev. change</td>
<td>-</td>
<td>15.84%</td>
<td>-8.50%</td>
<td>-4.38%</td>
<td>-0.29%</td>
<td>12.02%</td>
</tr>
<tr>
<td>Gini</td>
<td>48.81</td>
<td>51.94</td>
<td>48.68</td>
<td>48.49</td>
<td>49.73</td>
<td>49.36</td>
</tr>
<tr>
<td>Welfare change (cev)</td>
<td>-</td>
<td>-1.39%</td>
<td>-0.16%</td>
<td>0.02%</td>
<td>3.06%</td>
<td>3.16%</td>
</tr>
</tbody>
</table>

Notes: The table compares the outcomes of four alternative reforms covering the additional pension system deficit induced by the projected demographic change. Increasing the minimum required years of contribution from current 15 to 22 years minimizes informality and inequality. Raising the retirement age by 5 years leads to the best outcome in terms of welfare and the pension expenditure to GDP ratio. Increasing the payroll tax rate leads to the worst outcome in terms of welfare, inequality and pension expenditures.

* denotes multiples of 100.
reform has a significant impact on the share of informal workers. A higher tax burden creates a strong incentive to exit the social security system generating a reallocation of labor from the formal to the informal sector. Hence, informal output rises whereas formal output drops. Overall GDP, which accounts for output in both sectors, slightly rises. The shrinkage of the formal sector lowers capital demand which substantially cuts down the interest rate. Moreover, the decline in the formal labor supply increases the relative wage in the formal sector. The adjustment of prices in the general equilibrium plays a mitigating role, since the tax hike is partially compensated by a higher gross formal wage. The reform affects the pension system balance in two ways. First, the increase in taxes directly expands the pension system revenues, even though the tax base shrinks. Second, since informal workers claim lower pensions, the increase in informality lowers the expenditure side of the system as well. All in all, the pension expenditures to GDP ratio drops but does not reach the before-demographic-change value. More informality is associated with lower consumption, higher inequality and a welfare loss of around 1.4% measured in consumption equivalent variation.\(^{34}\)

Column 4 of Table 2.6 shows the results of a reduction of the generosity of pension benefits. In order to compensate the deficit they have to decrease by 16%. Since this reform implies a shift of pensions downwards for everyone it has little effect on the occupation choice. Lower pension benefits motivate individuals to accumulate more assets to self-insure against the low income during the retirement period. The increase in capital supply decreases the interest rate. Pension expenditure to GDP ratio declines because pension benefits reduce and output increases. In terms of welfare, individuals lose from the reform but less compared to the first option. The slight decrease in informality has also a favorable effect on inequality.

Column 5 of Table 2.6 illustrates the consequences of the increase in the required years of contribution by 7 years, from 15 to 22 years (Reform 3).\(^{35}\) Since the entitlement requirements become stricter, individuals have to spend more time working formally to qualify for a decent pension benefit. The share of informal workers declines. The increase in the formal production dominates the decrease in the informal production and the aggregate output rises. Capital supply boosts in the economy, leading to a drop in the interest rate. Even though there are more formal workers, there are fewer individuals who meet the contribution requirement (there are more formal workers but more individuals receiving minimum pension), hence pension expenditures decrease by 15.97%. Combined with the increased output, the pension expenditure to GDP ratio drops. This reform is welfare neutral in the long run. At the same time it leads to the lowest size of the informal sector.

\(^{34}\)The welfare criterion I employ is ex-ante expected lifetime utility of a newborn in a stationary equilibrium.

\(^{35}\)Note, that due to the fact that one model period is 5 years I use linear interpolation to evaluate the required adjustment.
and lowest inequality measured by the Gini coefficient.

The last reform suggests to raise the retirement age from 60 to 65 years (Reform 4). Pensions become less important since individuals are forced to work more years and the pension period becomes shorter. It results in an increase in the share of informal workers. Individuals supply their labor for 5 more years and labor supply in both sectors rises. This leads to an expansion of the aggregate output and demand for capital which in turn drives the interest rate up and formal wage down. There are fewer formal workers but they are on average more productive. The pension expenditure to GDP ratio drops due to the direct decrease in pension expenditures associated with fewer retirees and the increase in output. Since individuals receive labor income at the age of 60-65 they can consume more compared to the no-reform scenario. This reform is the most favorable reform in terms of welfare and the pension expenditure to GDP ratio among the presented alternatives, although it increases inequality.

Finally, I evaluate the reform proposed by the Brazilian government. It combines the measures of Reform 3 and Reform 4 presented above. Specifically, the Brazilian government proposed to increase the retirement age to 65 years and the minimum years of contribution to 25 years. This reform is not directly comparable to the reforms discussed above since it generates a lower deficit. Table 2.6 Column 7 presents the results. The level of informality increases but not substantially. The result is driven by the combined effects of Reform 3 and 4 discussed above. Importantly, output reaches the highest level compared to all above mentioned reforms. The pension expenditure to GDP ratio is the lowest due the substantial decrease in expenditures and significant increase in output. Also in terms of welfare this option outperforms the alternative measures. The welfare analysis, however, abstracts from the transitional dynamics which may have important implications on the ranking of the reforms.

2.5.5 Role of Informality

The results presented above show that individuals react to the pension reforms by reallocating their labor from one sector to the other either as a direct response to the contributions and benefits change or as a response to the price change. In this subsection, I evaluate the role of informality by comparing the status-quo economy with the informal sector to a counter-factual economy where the informal sector is absent. In order to do so, I analyze the percentage changes of the key macroeconomic indicators in both scenarios. For the counter-factual economy with no informal sector I set the cost of informality so high \(c_1 = 100\) that it is never optimal to choose this sector. I keep all other parameters at the benchmark values.

In the absence of informality much lower adjustments are required to cover the addi-
Table 2.7: Role of Informality

<table>
<thead>
<tr>
<th></th>
<th>No Reform</th>
<th>Reform 1</th>
<th>Reform 2</th>
<th>Reform 3</th>
<th>Reform 4</th>
<th>Proposal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demographic change</td>
<td>$\tau_p = 18.9%$</td>
<td>$\downarrow$ pen16%</td>
<td>$H = 22$</td>
<td>$JR = 65$</td>
<td>$JR = 65 &amp; \bar{H} = 25$</td>
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<tr>
<td>$Y$, % change</td>
<td>-</td>
<td>0.68</td>
<td>1.74</td>
<td>2.73</td>
<td>18.00</td>
<td>21.44</td>
</tr>
<tr>
<td>$K$, % change</td>
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<td>-7.53</td>
<td>5.14</td>
<td>7.97</td>
<td>11.95</td>
<td>21.67</td>
</tr>
<tr>
<td>$C$, % change</td>
<td>-</td>
<td>-1.42</td>
<td>-2.47</td>
<td>-2.04</td>
<td>13.52</td>
<td>12.09</td>
</tr>
<tr>
<td>Pen. exp. to GDP, pp change</td>
<td>-</td>
<td>-1.30</td>
<td>-3.81</td>
<td>-3.79</td>
<td>-4.94</td>
<td>-7.53</td>
</tr>
<tr>
<td>Gini, pp change</td>
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<td>-0.13</td>
<td>-0.32</td>
<td>0.92</td>
<td>0.55</td>
</tr>
<tr>
<td>Welfare, cev % change</td>
<td>-</td>
<td>-1.39</td>
<td>-0.16</td>
<td>0.02</td>
<td>3.06</td>
<td>3.16</td>
</tr>
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<table>
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<tr>
<th></th>
<th>No Reform</th>
<th>Reform 1</th>
<th>Reform 2</th>
<th>Reform 3</th>
<th>Reform 4</th>
<th>Proposal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demographic change</td>
<td>$\tau_p = 13.5%$</td>
<td>$\downarrow$ pen12%</td>
<td>$H = 20$</td>
<td>$JR = 62.5$</td>
<td>$JR = 65 &amp; \bar{H} = 25$</td>
</tr>
<tr>
<td>$Y$, % change</td>
<td>-</td>
<td>-2.45</td>
<td>2.05</td>
<td>3.24</td>
<td>8.62</td>
<td>22.21</td>
</tr>
<tr>
<td>$K$, % change</td>
<td>-</td>
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<td>3.88</td>
<td>6.29</td>
<td>6.03</td>
<td>22.00</td>
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<td>$C$, % change</td>
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<td>-5.34</td>
<td>-1.09</td>
<td>0.33</td>
<td>5.1</td>
<td>11.13</td>
</tr>
<tr>
<td>Pen. exp. to GDP, pp change</td>
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<td>0.51</td>
<td>-3.16</td>
<td>-2.67</td>
<td>-2.93</td>
<td>-8.7</td>
</tr>
<tr>
<td>Gini, pp change</td>
<td>-</td>
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<td>-0.34</td>
<td>0.2</td>
<td>-0.07</td>
</tr>
<tr>
<td>Welfare, cev % change</td>
<td>-</td>
<td>-2.69</td>
<td>0.34</td>
<td>0.76</td>
<td>5.02</td>
<td>3.29</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>No Reform</th>
<th>Reform 1</th>
<th>Reform 2</th>
<th>Reform 3</th>
<th>Reform 4</th>
<th>Proposal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demographic change</td>
<td>$\tau_p = 18.9%$</td>
<td>$\downarrow$ pen16%</td>
<td>$H = 22$</td>
<td>$JR = 65$</td>
<td>$JR = 65 &amp; \bar{H} = 25$</td>
</tr>
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<td>$Y$, % change</td>
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<tr>
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<td>11.60</td>
<td>22.00</td>
</tr>
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<td>$C$, % change</td>
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<td>0.28</td>
<td>9.80</td>
<td>11.13</td>
</tr>
<tr>
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<td>-3.55</td>
<td>-3.52</td>
<td>-5.64</td>
<td>-8.7</td>
</tr>
<tr>
<td>Gini, pp change</td>
<td>-</td>
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<td>-0.44</td>
<td>-0.48</td>
<td>0.39</td>
<td>-0.07</td>
</tr>
<tr>
<td>Welfare, cev % change</td>
<td>-</td>
<td>-8.81</td>
<td>0.45</td>
<td>0.97</td>
<td>2.11</td>
<td>3.29</td>
</tr>
</tbody>
</table>

Notes: The table shows the results of the pension system reforms with and without the informal sector. The first sub-table repeats the results of the benchmark economy with informality. The second sub-table presents the effects of the pension system reforms which cover the additional fiscal deficit in the economy without the informal sector. The final sub-table shows the consequences of the reforms in the economy without informality which have the same order of magnitude as the benchmark reforms.
tional pension system deficit. The pension system deficit increases by around 3pp without informality which is comparable to the case with informality. It is sufficient to raise the payroll tax from 11% to 13.5% instead of 18.9%, if individuals have no occupational choice. Pension benefits are required to decrease by only 12% instead of 16%, the required years of contribution have to increase by 5 instead of 7 years, and the retirement age has to be prolonged by 2.5 compared to 5 years. Thus informality amplifies the need for policy interventions.

In order to asses the role of informality, I first analyze the aggregate effects in the economy without tax evasion, if the government adjusts the policy instruments to cover the additional fiscal deficit of 3pp. Table 2.7 presents the results.

The increase in the payroll taxes has a distortionary effect in both cases, with and without informality. Although in the latter case, individuals cannot choose to work informally, they still adjust the intensive margin of their labor supply. Interestingly, in the presence of the informal sector output increases whereas in the absence it decreases by around 2.45%. Without the informal sector, individuals respond to the tax hike by reducing their labor supply. In contrast, with the informal sector, individuals change their occupation rather than their labor supply. The absence of the opportunity to work informally as a response to the tax increase leads to a higher welfare loss of 2.69%. Hence, the informal sector mitigates the distortion induced by taxation.

Apart from the tax hike, other pension reforms result in a higher long-term welfare gain without the informal sector. This outcome is partially driven by the fact that the magnitude of policy instrument adjustments is lower without informality. To disentangle the effect of a lower required policy change from the occupation choice, I additionally analyze the outcome of the reforms given the same adjustments as in the economy with tax evasion discussed above.

Interestingly, with the same magnitude of the policy change, reforms which directly affect the labor supply (Reform 1 and Reform 4) have better aggregate outcomes in the presence of informality due to the additional channel of adjustment. Due to the payroll tax hike, individuals suffer a welfare loss of 8.81% as opposed to 1.39%. The retirement age increase leads to a welfare gain of 3.06% and 2.11% with and without informality, respectively. In contrast, policies targeting pension benefits (Reform 2 and Reform 3) are more successful in the absence of the informal sector. Cutting pension benefits leads to a welfare gain of 0.45% as opposed to a welfare loss of 0.16%, and increasing the pension requirements leads to a welfare gain of 0.97% as opposed to 0.02%. In the absence of informality, the government proposal results in a higher output, lower inequality and a higher long-term welfare gain of 3.29%.
2.6 Conclusions

This paper analyzes the long-run aggregate effects of pension system reforms in countries with informal sectors. I develop an overlapping-generation life-cycle model with heterogeneous agents and incomplete markets where agents endogenously choose their sector of occupation: formal or informal. Informality reduces the tax burden during the working period but lowers future pension benefits.

I calibrate my model to Brazil, a country with a large informal sector which is currently undertaking a pension system reform. The model matches well the share of informal workers and the size of the informal sector, labor supply, as well as the characteristics of the pension system, in particular the replacement rate, the pension-expenditure to GDP ratio and the current pension system deficit. Also inequality measures for the whole population, which were not explicitly targeted, are fairly close to their empirical counterparts.

I find that a demographic change, projected by the Brazilian Institute of Statistics, leads to a significant reallocation of labor from the informal to the formal sector which expands tax revenues without direct government interventions. This mitigates a negative effect on the fiscal balance of an increased dependency ratio.

Next, I evaluate four pension system reforms that compensate the additional deficit induced by the demographic change. In particular, I study (1) the increase of the payroll tax from 11% to 18.9%; (2) the reduction of the pension benefit level by 16%; (3) the rise of the required years of contribution from 15 to 22 years; (4) the increase in the retirement age from 60 to 65 years. Among these alternatives, extending the required years of contribution minimizes the share of informality and inequality. The rise in the retirement age leads to the best outcome in terms of welfare and the pension expenditure to GDP ratio.

The reform proposed by the Brazilian government which suggests increasing the required years of contribution to 25 years and raising the retirement age to 65 years leads to a substantial decrease in the pension expenditure to GDP ratio and a significant increase in output. It is also leads to a welfare gain but amplifies inequality in the long run.

Informality allows individuals to adjust the extensive margin of their labor supply as a response to a pension reform. In the absence of an informal sector, much lower government adjustments are required following the demographic change. An immediate policy implication of this finding is that by ignoring the existence of informality the government may fail to cover the induced fiscal deficit.

The limitation of the analysis is that it abstracts from endogenous retirement and transitional dynamics which are important directions for the future work.
Chapter 2. Pension System Reforms in the Presence of Informality

References


IMF (2016). The Urgent Case for Pension Reform in Brazil, *IMF report*.


Appendix 2.A  Data

2.A.1  Data Sample

My main source of data consists of a panel of individuals of working age, sampled by the labor force survey of Brazil, Pesquisa Mensal de Emprego (PME). PME is designed and conducted by the National Statistics Bureau to follow individuals of the six main metropolitan regions of Brazil. The sample period starts in January 2010 and goes until December 2015. I abstract from earlier periods to capture better the characteristics of a current economy.

For the purpose of this paper, I focus on workers who are household heads of age 20-60. I define an individual to be informal if one is an employee or a domestic worker but did not sign a working contract or the one who is self-employed. These categories do not contribute to the social security system and are relevant for the analysis. I do not consider individuals who reported they are unpaid workers, unemployed or those who did not report their occupation. Summary statistics are provided in Table A.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of informal workers</td>
<td>35.66%</td>
</tr>
<tr>
<td>Ratio of mean labor income (F/I)</td>
<td>1.61</td>
</tr>
<tr>
<td>Ratio of median labor income (F/I)</td>
<td>1.62</td>
</tr>
<tr>
<td>Average hours worked (F)</td>
<td>42.51</td>
</tr>
<tr>
<td>Average hours worked (I)</td>
<td>40.99</td>
</tr>
<tr>
<td>Gini gross income</td>
<td>46.6</td>
</tr>
<tr>
<td>N obs.</td>
<td>752445</td>
</tr>
</tbody>
</table>

2.A.2  Estimating Labor Income Process

To estimate the process that governs labor productivity, I first run cross-sectional regressions of the logarithm of hourly wages on age to define the age-efficiency profile $\varpi_j$. I use a second order polynomial in age, since including higher levels decrease the regression fit (R-squared decreases). The obtained estimates are then normalized such that the efficiency parameter of an individual entering the economy is equal to unity. Figure A.1 visualizes the labor efficiency profile over the life cycle.

The estimation of the stochastic component of labor productivity follows closely the procedure described by Heathcote et al. (2010). First, I regress the logarithm of hourly labor earnings on observable individual characteristics such as education, race and age.
irrespective of individual’s occupation.

\[ \lninc_{i,t} = \alpha_0 + \beta_0 \text{educ}_{i,t} + \beta_1 \text{race}_{i,t} + \beta_2 \text{age}_{i,t} + a_3 \text{age}_{i,t}^2 + \eta_{it}, \]  

(2.11)

where \( i \) is individual and \( t \) is time index respectively. Then, I model the residual \( \eta \) as a first-order auto-regressive process:

\[ \log \eta_{i,t+1} = \rho_\eta \log \eta_{i,t} + \zeta_{i,t+1}, \]  

(2.12)

where \( \zeta_{i,t+1} \sim N(0, \sigma_\eta^2) \). I estimate this process and obtain a persistence parameter \( \rho_{\eta,a} = 0.776 \) and the dispersion parameter \( \sigma_{\eta,a} = 0.382 \) on the annual basis. Since the model period is defined to be five years, I translate the obtained parameters into 5-year equivalents:

\[ \rho_\eta = \rho_{\eta,a}^T, \]  

(2.13)

\[ \sigma_\eta^2 = \sigma_{\eta,a}^2 \sum_{t=0}^{T-1} \rho_{\eta,a}^{2t}, \]  

(2.14)

where \( T = 5 \) is the number of years in the period. Consequently, the \( \rho_\eta = 0.282 \) and \( \sigma_\eta = 0.553 \).

**Appendix 2.B Model**

**2.B.1 Earning Index Calculation**

In the model, I approximate the formula which is used to define pension benefits in Brazil. Importantly, an individual who works informally does not pay social security contributions.
and these years are not counted in the earning history. Hence, I need two state variables
to capture the earning index: \( e \)-average earnings and \( h \)-years of contribution. Hence, \( h \)
counts years of contribution if an individual is formally employed:

\[
h_{t+1} = h_t + 1_F.
\]

Average earnings is then calculated:

\[
e_{t+1} = \frac{(h-1)e_t + y_F}{h}.
\]

Pension benefits \( p(e, h) \) then depend on the average earnings \( e \) and years of contribution \( h \):

\[
p = \begin{cases} 
  p & \text{if } h < \bar{H} \\
  p(e, h) & \text{if } h \geq \bar{H} \\
  \bar{p} & \text{if } e > \bar{e}.
\end{cases}
\]

If an individual contributed less than defined number of years \( \bar{H} \) she gets only a minimum pension benefit \( \bar{p} \) which is the calibrated value. The maximum value of pension benefits \( \bar{p} \) is capped by the Brazilian law and is around 6 times the minimum value. The pension benefit \( p(e, h) \) is placed between \( \bar{p} \) and \( \bar{p} \) and linearly increases with average earning index \( e \).

2.B.2 Additional Figures
Figure B.2: Conditional Survival Rates, World Health Organization, Brazil

Notes: The red solid line shows the conditional surviving rates in Brazil for 2010-2015, provided by the World Health Organization. The blue dashed line illustrates the conditional surviving rates for the economy with a demographic change and is shifted by a factor of $\Delta$ to reach the projected life expectancy and dependency ratio for the year 2050.

Figure B.3: Asset Distribution
Notes: The yellow bars show the share of informal workers in the model, whereas the blue bars show the corresponding shares in the data. The model is not able to replicate the empirical counterpart, partially due to the trend component present in the data. There is a decreasing trend of informality rate over the years in Brazil.
Figure B.6: Asset Policy Comparison

Figure B.7: Consumption Policy Comparison
Figure B.8: Labor Policy Comparison

(a) Formal Workers

(b) Informal Workers
Chapter 3

Education Subsidies in the Presence of Informality
3.1 Introduction

Developing countries feature a high level of informality which is defined as unrecorded production with a purpose of tax and social security avoidance (Schneider et al. (2010)). Empirical studies show that low-educated individuals choose informal employment whereas highly-educated individuals opt for the formal sector where the education premium is higher (Araujo et al. (2013), Lopez Garcia (2015), Fairris and Jonasson (2016), Meghir et al. (2015)). Education subsidies make education more affordable and increase the individual skill level which may foster the formal sector participation.\footnote{More empirical evidence on this is provided in Section 3.3.} However, education subsidies have to be financed either with debt accumulation or additional taxation. In the latter case, an important trade-off arises. On the one hand, the education subsidy allows individuals to accumulate more human capital to become formal. On the other hand, raising taxes to finance the education subsidy may encourage some individuals to become informal.

This paper sheds light on the interaction between education policies, taxation and income distribution in the presence of an informal sector. In particular, I aim to answer several questions: How do education subsidies affect the size of informality? What is the revenue and the welfare-maximizing level of the subsidy? What are the distributional implications of education reforms? What are the consequences of the change in the tax schedule in the presence of educational and occupational decisions?

I build a two-sector overlapping generation model with endogenous educational and occupational decisions. In the first stage of their life, individuals make their educational choice given their inherited assets and innate learning ability. The cost of education depends on the innate learning ability and the amount of time invested in education. A share of education costs is subsidized by the government. While making the educational choice, individuals are aware of the future education premium which differs between the formal and informal sectors. In accordance with empirical evidence, I assume that the education premium is higher in the formal sector (Arbex et al. (2010), Araujo et al. (2013), own calculations). Apart from the difference in the education premia, sectors differ in terms of tax compliance and social security coverage. Formal employees pay progressive income taxes. On the contrary, informal employees do not pay income taxes but bare a fixed utility cost associated with the lack of social security. The government collects tax revenues to finance its expenditures which include education subsidies, government consumption and pension payouts.

I calibrate my model to Brazil, a country with a high level of informality and a low level of educational attainment. I use the Brazilian survey data Pesquisa Mansal de Emprego (PME) which contains detailed information about the individual characteristics, educa-
tional attainment, labor income and hours of work for both workers in the formal and informal sectors. My model matches well the characteristics of the formal and informal workers, the distribution of education within the formal and informal sectors as well as the macroeconomic indicators.

In equilibrium, formal employees have a higher average education level compared to the informal. Interestingly, the prevailing share of formal workers have an average level of education which corresponds to the completed high school degree. Although there is an incentive to invest in education to enjoy a high wage premium in the formal sector, progressive taxation prevents individuals from investing too much.

The increase in the education subsidy makes education more affordable, raises the average level of educational attainment, expands formalization and boosts the tax base. Due to this effect the education subsidy is self-financed in the long run and does not require additional tax reforms. Increasing the subsidy level from the current 26% to 80% reduces informality by around 10 pp, increases income tax revenues by 6 pp and diminishes income inequality measured by a Gini coefficient by almost 10 pp. Moreover, welfare grows by almost 10% measured in consumption equivalent variation. A further increase of the subsidy rate does not improve the aggregate outcomes since for the value of 80% all individuals are highly educated and the upper bound of the education level is reached.

Finally, I analyze the effects of a change in the income tax schedule. Interestingly, if the government aims to expand the tax revenues it should decrease rather than increase the tax level. The low level of taxation raises formalization and the education level, increasing the tax base in the long run. Changing progressivity has ambiguous effects on the tax revenues and leads to moderate changes in education and informality levels.

The rest of the paper is organized as follows. In the next section, I review the related literature. In Section 3.3, I present stylized facts about the educational attainment and informality in Brazil. Next, I discuss the model and the calibration in Section 3.4 and 3.5, respectively. Finally, I present the results and draw conclusions.

3.2 Related Literature

There exists an extensive literature studying the impact of education subsidies on labor participation as well as income distribution and inequality. Krueger and Ludwig (2013) analyze the optimal combination of progressive income taxes and education subsidies in a model with endogenous human capital, borrowing constraints and incomplete financial markets. They find that progressive taxes distort labor supply as well as weaken the incentives to acquire education. They also claim that the latter distortion can potentially be mitigated by an education subsidy. Similarly, Bovenberg and Bas (2005) and De Fraja (2002) study optimal education subsidies and find that education policy offsets some of the
tax-induced distortions on learning. Guvenen et al. (2014) abstract from the question of optimality and focus on the distortionary impact of progressive taxation on human capital accumulation which affects the income distribution and inequality.

The studies mentioned above look at tax distortions and the potential mitigating effects of education subsidies in a theoretical framework suitable for developed countries. However, in developing countries, the existence of a large informal sector may have important implications. First, in the presence of informality, taxes induce not only an intensive but also an extensive margin of the labor supply distortion. Individuals may either decrease their hours of work to pay less taxes or decide to work informally and escape government taxation. Second, there is empirical evidence that low-educated individuals are more likely to be informal (Arbex et al. (2010), Lopez Garcia (2015), Meghir et al. (2015)). A hypothesis in the informality literature is that some individuals are not skilled enough to operate in the formal sector and are rationed out to informality (see Maloney (2004) for a discussion). Another point of view, which recently gains more support, is that unskilled low-educated individuals rationally choose to become informal due to a comparative advantage in this sector (Lopez Garcia (2015), Araujo et al. (2013), Fairris and Jonasson (2016)). In particular, Lopez Garcia (2015) shows that human capital accumulation and preferences for job amenities explain up to 72% of transition between the informal and the formal sector.

The literature focusing on education policies and informality is surprisingly scarce. Haanwinckel and Soares (2017) emphasize that improvements in the labor force schooling and the skill level may be more effective in reducing informality in the long term compared to other commonly used instruments. The authors document that informality is strongly responsive to the composition of the labor force. A labor force with higher levels of schooling generates incentives for firms to grow and formalize which decreases informality in the long run.

El Badaoui and Rebiere (2013) develop a search and matching model of a dual labor market to theoretically analyze the impact of increasing access to education on employment flows. In their model, the formal sector is reserved for educated and trained-on-the-job workers, and the informal sector is accessible to all workers. They find that a rise in access to education reduces the size of the informal sector if financed with an external subsidy. However, if an education subsidy is financed by a tax on formal sector firms, it reduces the labor market efficiency. Similarly, Bobba et al. (2018) build a search and matching model where firms and workers are forming formal and informal matches. Workers choose the level of schooling before entering the labor market. The authors study the role of the dual social security system in Mexico and find that changing the social security system can increase output, schooling and long-term productivity at a small fiscal cost.

The study of D’Erasmo et al. (2014) focuses on the demand side for skilled and unskilled
labor by firms which may operate either formally or informally. They analyze the role of institutions in shaping the demand for human capital and the level of informality. The authors show that countries with a low degree of debt enforcement and high costs of formalization are characterized by relatively lower stocks of skilled workers, larger informal sectors and low efficiency.

Importantly, Berniell (2019) in a similar framework with human capital investments, occupational choice and an informal sector theoretically explains the cross-country differences in the level of entrepreneurship, informality and human capital investment.

In contrast to the studies mentioned above, this paper analyzes education financement in the presence of an informal sector in a quantitative general equilibrium framework. I focus on individual choices and labor supply decisions. Due to a rich heterogeneity of individuals and endogenous educational and occupational choices the model allows to analyze the impact of education policies on inequality and welfare. I also study the impact of progressive taxation in the set-up with endogenous human capital accumulation and endogenous informal employment contributing to the literature described above.

### 3.3 Education System and Informality in Brazil

In this section, I provide empirical facts on the education system, informality and inequality in Brazil.

According to the OECD (2018) report, educational attainment in Brazil is low compared to the OECD average. Only 69% of the 15-19 years old and 29% of the 20-24 years old are enrolled in education compared to the OECD average of 85% and 42%, respectively. Strikingly, over half of Brazil’s adult population (25-64 years old) have not completed upper secondary education, which is more than double the OECD average.

Brazil invests a relatively high share of GDP in education (5.5%), however expenditures per student are significantly lower compared to the most OECD and partner countries. The Brazilian government annually spends around 3800USD per student in primary, secondary, and post-secondary non-tertiary public institutions which is less than a half of the OECD average (OECD (2018)). Brazilian Campaign for the Right to Education (2018) reports that in order to reach the goals stated in the National Education Plan, Brazil has to invest three to five times more in education per student compared to the current investment.

Apart from a low level of education, another important feature of the Brazilian labor market is a high level of informality. Schneider et al. (2010) report that the size of the shadow economy in Brazil is around 32.5% of GDP. According to the Brazilian micro data PME (2010-2015) the share of informal labor is around 33.6% when considering self-employed as informal. Concentrating solely on the employees, around 15.4% are employed informally.
Table 3.1 reports the share of low, middle and highly educated individuals within formal and informal sectors. The low-educated are defined as those who do not have a high-school degree, middle-educated are those who have a high-school degree only and highly educated are those who have acquired tertiary education. Almost a half of those who are formally employed have a high-school degree, around 27% are low educated and around 26% are highly educated. In the informal sector, on the contrary, the majority of workers are low-educated and the share of highly educated individuals is lower compared to the formal sector. Additionally, running a Probit regression reveals that the level of education has a negative significant effect on the probability of operating informally.2 This result is consistent with the empirical studies of Araujo et al. (2013) and Fairris and Jonasson (2016) among others.

Table 3.1: Education Distribution within Sectors

<table>
<thead>
<tr>
<th>Education</th>
<th>Formal</th>
<th>Informal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low, %</td>
<td>27.31</td>
<td>40.81</td>
</tr>
<tr>
<td>Middle, %</td>
<td>46.54</td>
<td>39.31</td>
</tr>
<tr>
<td>High, %</td>
<td>26.15</td>
<td>19.89</td>
</tr>
<tr>
<td>Total, %</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: This table reports the distribution of the education level within the formal and informal sectors. Low educated are those who do not have a high-school degree, middle-educated are those who have a high-school degree only and highly educated are those who have acquired tertiary education. Data source is PME from 2010 to 2015.

Figure 3.1 shows log wages in the formal and in the informal sector for low, middle and highly educated individuals along age. First, for a given education level, gross wages in the formal sector are higher compared to informal wages. Second, a higher education level ensures higher wages, both in the formal and in the informal sector (see Arbex et al. (2010) for a detailed empirical investigation of the education premium in the informal sector). However, the education premium is smaller in the informal sector.3 Araujo et al. (2013) evaluated the Brazilian informal labor market using another data source (PNAD) and documented the same facts about the education premium in the formal and the informal sector as in this section.

Another aspect which I address in this paper is the interaction between education policies and income inequality. Georges and Maia (2017) emphasize that Brazil is one of the most unequal countries in the world in terms of income, wealth and education. They document that increasing the education level of the poor is one of the key factors of decreasing inequality. At the same time, the education wage premium, which has increased

2 See Appendix for the details.
3 See Appendix for the regression analysis.
Figure 3.1: Log Wages in the Formal and Informal Sectors

(a) Formal Sector

(b) Informal Sector

Notes: The figures compare log wages in the formal and in the informal sector for low, middle and highly educated individuals along age. Data source is PME from 2010 to 2015.

In recent years in Brazil, raises the income gap for those with and without higher education. Hence, the impact of an increasing education level on income inequality is ambiguous and depends on the targeted groups.

Following these empirical facts, I construct a dynamic stochastic general equilibrium model with two sectors of production and three education levels. Formal and informal workers face education premia with a higher premium for the formal employees. A part of the education costs is financed by the government.

3.4 The Model

I build a dynamic general equilibrium overlapping generations model in the spirit of Auerbach and Kotlikoff (1987) with income and lifespan uncertainty as in Huggett (1996). Individuals go through three stages of their life: (1) the young, education period; (2) the working period and (3) retirement. In the first stage of their life, individuals make the educational choice given their asset position and innate learning ability. Education is costly and depends on the amount of time invested in education. A fixed share of costs is subsidized by the government. In the second stage of life, individuals make an occupation choice. They may work in either a formal or an informal sector and may switch between them during their working lives. The formal sector features a higher education premium and individuals enjoy higher gross wages compared to informal employees. However, formal workers must pay progressive income taxes. Informal workers, on the contrary, do not pay taxes but bare a fixed utility cost associated with informal employment. This util-
ity cost reflects the lack of social security in the informal sector. Individuals retire when they reach the retirement age. Additionally, there are formal and informal firms that hire formal and informal labor, respectively. The government collects tax revenues to finance its expenditures which include education subsidies, government consumption and pension payouts.

3.4.1 Households

Demographics

The economy is populated by overlapping generations of individuals. Individuals enter the economy at the age of \( j_0 \) and face lifespan uncertainty. The conditional probability of survival from age \( j \) to age \( j + 1 \) is denoted as \( \psi_j \). The maximum possible age is \( j = J \), with \( \psi_J = 0 \). The size of a new cohort grows at a constant rate \( n \). Bequests \( b \) are assumed to be collected and distributed as a lump-sum transfer to the young population. Individuals enter the economy with initial assets \( a_0 \) drawn from a log-normal distribution \( \text{Lognormal}(\mu_a, \sigma_a) \).\(^4\) The interpretation of this assumption is that the young individuals do receive heterogeneous transfers from their parents.

Households differ with respect to their age \( j \), an asset position \( a \), an idiosyncratic productivity \( \epsilon \), an innate learning ability \( e \) and a level of education \( h \) which is defined after the first stage of live and does not change afterwards.\(^5\) Asset markets are incomplete, that is, households cannot insure against shocks to working productivity \( \epsilon \). Moreover, individuals are borrowing constrained \( a' \geq 0 \).

Preferences and Endowments

Households maximize the expected sum of discounted utility given by:

\[
E_{j_0} \sum_{j=j_0}^{J} \beta^{j-j_0} \psi_j u(c_j, 1 - l_j),
\]

where \( \beta \in (0, 1) \) is the time discount factor, \( c_j \) and \( 1 - l_j \) is consumption and leisure at the age of \( j \) respectively. The total time endowment is normalized to one.

\(^4\)Parameters \( \mu_a \) and \( \sigma_a \) are set to match the asset distribution in Brazil and further discussion is provided in Section 3.5.

\(^5\)For simplicity, I abstract from the human capital accumulation during the working life.
Educational Choice

In the first period, based on the innate learning ability $e$ and assets inherited from their parents $a_0$, individuals decide how much time to invest in education $q$.\footnote{Alternatively, one could model investment into an education level. I follow Berniell (2019) and opt for a continuous variable for computational reasons.} Education is costly and the cost of education $\xi(e)$ is heterogeneous for individuals with different ability levels $e$. The higher the learning ability, the lower is the cost reflecting the fact that more able individuals find it easier to acquire a certain level of education, hence $\xi(e)' < 0$. The share $s$ of education expenditures is subsidized by the government. For simplicity individuals do not work in the first period. Thus the first period optimization problem reads as follows:

$$V^1(j, a, \epsilon, e, h) = \max_{c,q,a'} \left( u(c, q) + \beta \psi_j E[V(j + 1, a', \epsilon', e, h)|\epsilon] \right)$$

subject to

$$(1 + \tau_c) c + a' \leq (1 + r(1 - \tau_k))(a_0 + b) - (1 - s)\xi(e)q,$$

where $\tau_c$ and $\tau_k$ are the consumption and capital tax, respectively.

Occupational Choice

Depending on the education investment in the first period, individuals attain a certain level of education $h(q)$ which is a linear function of the invested time $q$. In the second period, each individual can allocate one unit of disposable time to leisure or market work. In every period of the working stage, an individual decides whether to work formally or informally. The maximization problem of the working population is given:

$$V(j, a, \epsilon, e, h) = \max \left\{ V^F(j, a, \epsilon, e, h), V^I(j, a, \epsilon, e, h) \right\},$$

where $V^F(.)$ and $V^I(.)$ are the value functions associated with becoming a formal and an informal worker, respectively.

Formal Workers

In the formal sector, individuals receive income $y^F$ which depends on the individual sector-specific age-dependent labor productivity $\eta_j^F$, the formal market wage $w^F$ and hours of work $l$. The labor productivity $\eta_j^F$ consists of three components, $\eta_j^F = \varpi_j \exp(\epsilon) \chi^F h$. $\varpi$ depends on the age $j$ and reflects the life-cycle component of the labor productivity process which
is estimated from the data and fed exogenously into the model. \(^7\) \(\epsilon\) denotes an idiosyncratic labor productivity and is drawn from a finite-state Markov process with transition probability given by \(F(\epsilon'|\epsilon)\). Finally, \(\chi^F h\) is the sector-specific education premium which depends on the education level of an individual.

Formal workers pay consumption tax \(\tau_c\) and capital tax \(\tau_k\), as well as progressive labor income tax \(\tau_l(.)\) which is a function of gross income \(y^F\). \(^8\) Individuals choose consumption, labor supply and savings in order to maximize their expected lifetime utility subject to their budget constraint.

\[
V^F(j, a, \epsilon, e, h) = \max_{c,l,a'} (u(c, l) + \beta \psi_j E[V(j + 1, a', \epsilon', e, h)|\epsilon])
\]
subject to
\[
y^F = \eta^F_j w^F l,
\]
\[
(1 + \tau_c) c + a' = (1 + r(1 - \tau_k)) a + y^F - \tau_l(y^F).
\]

**Informal Workers**

Informal workers do not pay the labor tax, in contrast to the formal employees. Their income \(y^I\) depends on the market informal wage \(w^I\), their individual productivity \(\eta^I_j\) and labor supply. As before the labor productivity \(\eta^I_j\) consists of three components and the first two are the same as for the formal workers. The education premium \(\chi^I\) is different and is assumed to be lower compared to the formal sector which is in accordance with the empirical evidence discussed in Section 3.3. Additionally, informal workers bare a fixed utility cost of informality \(u_I\) (Loayza (1996), Busato and Chiarini (2004), Fernandez-Bastidas (2018)). This cost captures the lack of social security in the informal sector which I do not model explicitly.

\[
V^I(j, a, \epsilon, e, h) = \max_{c,l,a'} (u(c, l) - u_I + \beta \psi_j E[V(j + 1, a', \epsilon', e, h)|\epsilon])
\]
subject to
\[
y^I = \eta^I_j w^I l,
\]
\[
(1 + \tau_c) c + a' = (1 + r(1 - \tau_k)) a + y^I.
\]

**Retirees**

After reaching the retirement age of \(J_R\) individuals stop working and receive the pension benefit \(p\) which for simplicity is the same for everyone. Retirement excludes the possibility

---

\(^7\)The estimation procedure of the life-cycle component is discussed in Section 3.5.

\(^8\)I use a HSV functional form and calibrate the shape of the tax schedule to match empirical moments from the data.
to work. Retirees choose their consumption and saving in order to maximize:

$$V^R(j, a) = \max_{c, a'} \left( u(c, 0) + \beta \psi_j E \left[ V^R(j + 1, a') \right] \right)$$  (3.9)

subject to

$$(1 + \tau_c)c + a' = (1 + r(1 - \tau_k))a + p.$$  (3.10)

### 3.4.2 Firms

#### Formal Firms

Formal firms hire formal labor $L_F$ and rent capital $K$ in order to produce. A representative formal firm operates under a Cobb-Douglas production function with a technological parameter $A^F$. Formal firms pay wage $w^F$ to their employees. Capital depreciates at the rate $\delta$.

Hence, formal firms maximize their profits by solving the problem of the form:

$$\pi^F = \max_{K, L_F} \left( A^F K^\alpha L_F^{1-\alpha} - w^F L_F - (r + \delta)K \right),$$  (3.11)

where $\alpha \in (0, 1)$ is the share of capital in the production function.

#### Informal Firms

Informal firms hire only the informal labor $L_I$ in order to produce. La Porta and Shleifer (2014) document that informal firms are much more labor intensive than the formal ones. I follow Ihrig and Moe (2004), Busato and Chiarini (2004) and assume that informal firms do not use capital and produce with linear production function in labor and a productivity parameter $A^I$.

Informal firms hire informal labor in order to maximize their profits:

$$\pi^I = \max_{L_I} \left( A^I L_I - w^I L_I \right).$$  (3.12)

### 3.4.3 Government

The government collects revenues from the consumption and capital tax paid by all individuals. Additionally, it receives the income tax payments from formal workers. The revenues are spent on government consumption $G$, education subsidies $E$ and pension payouts $P$. 

...
3.4.4 Timing of Events

The sequence of events in this economy unfolds as follows. Individuals enter the economy at the age of $j = j_0$ with assets $a_0$ inherited from their parents plus a lump-sum transfer of incidental bequests $b$. Additionally, at the beginning of their lives, individuals draw a learning ability from a truncated Lognormal distribution $e \sim \text{Lognormal}(\mu_e, \sigma_e)$, where $e \in (0, 1)$. In each period of the working stage, individuals choose their occupation either in the formal or in the informal sector, $o \in \{F, I\}$. At the age of $j = J_R$ individuals retire and receive their pension $p$. At the age of $j = J$ individuals die with certainty.

3.4.5 Stationary Competitive Equilibrium

For a given set of exogenous demographic parameters $\{n, \psi_j\}$ and a given stream of government expenditures $\{G\}$, a competitive equilibrium is characterized by sequences of household policies and value functions $\{c, l, q, d', o, V_F, V_I, V_R, V\}$ for each state $x \in (j, a, \epsilon, e, h)$, production plans $\{Y, K, L_F, L_I\}$, sequences of taxes $\{\tau_c, \tau_k, \tau_l(\cdot)\}$, education subsidy and pension $\{s, p\}$, sequences of prices $\{w^F, w^I, r\}$, sequences of transfers of accidental bequests $\{b\}$ as well as a distribution of individuals $\mu$ that satisfy the following conditions:

1. Taking prices as well as tax and other policy parameters as given, individuals choose their consumption, labor input, savings, investment in education and occupation in order to maximize their expected discounted utility specified by equations (2), (4), (5), (7) and (9).

2. Factor prices are determined competitively:

$$r = \alpha A^F K^{\alpha-1} L_F^{1-\alpha} - \delta,$$

$$w^F = (1 - \alpha) A^F K^{\alpha} L_F^{-\alpha},$$

$$w^I = A^I.$$

3. The total amount of bequests is equal to the amount of assets left by the deceased:\footnote{In order to get bequests $b$ received by the young individuals, we have to divide the total amount $B$ by the mass of the young individuals.}

$$B = \int_x d'(x)(1 - \psi_j)d\mu(x).$$
4. The labor, capital and the goods markets clear:

\[ L_F = \int_x 1^F \eta^F_j l(x) \ d\mu(x), \]

\[ L_I = \int_x 1^I \eta^I_j l(x) \ d\mu(x), \]

\[ K = \int_x a'(x) \ d\mu(x), \]

\[ Y = \int_x c(x) d\mu + G + \tilde{I}, \]

where \( 1^F \) and \( 1^I \) are the indicator functions for the formal and informal employment. \( \tilde{I} \) is the total investment and \( Y \) denotes total production in the economy comprising production in both sectors.

5. The government budget constraint is balanced:

\[ G + \int_x 1^R p d\mu(x) + \int_x 1^{j=1} s \cdot \xi(e) \cdot q(x) d\mu(x) = \]

\[ \int_x (\tau_k r(a(x) + b) + \tau_e c(x)) d\mu(x) + \int_x 1^F (\tau_l y^F(x)) d\mu(x), \]

where \( 1^R \) is the indicator function for retirees and \( 1^{j=1} \) is the indicator function for the young individuals making their schooling decisions.

6. The distribution of individuals across states is stationary:

\[ \mu(x) = R_\mu [\mu(x)]. \]

3.5 Calibration

My main source of data consists of a panel of working age individuals, sampled by the labor force survey of Brazil, Pesquisa Mensal de Emprego (PME). PME is designed and conducted by the National Statistics Bureau and follows individuals of the six main metropolitan regions of Brazil. The sample period starts in January 2010 and goes until December 2015.\(^{10}\)

For the purpose of this paper, I focus on individuals of age 20-65.\(^{11}\) Following the

---

\(^{10}\)I abstract from earlier periods to better capture the characteristics of the current Brazilian economy.

\(^{11}\)I use this sample to derive targets for the working population in the model.
literature, I define an individual to be working informally if she is an employee but did not sign a working contract (Ulyssea 2018). I do not consider individuals who reported to be unpaid workers, unemployed or self-employed. It is evident that a substantial share of self-employed individuals in Brazil operate informally. However, the effect of education on entrepreneurial ability may be different from the education premium for workers (see Berniell (2019)). Hence, I abstract from self-employed in this study.\footnote{See Di Nola et al. (2018) and Franjo et al. (2019) analyzing tax evasion by self-employed in the U.S. and Brazil, respectively.}

The summary statistics of my sample are presented in the Appendix.

The rest of this section is organized as follows: First, I report parameters which are fixed outside the model. Then I discuss internally calibrated parameters which are set to match selected data targets. Finally, I present the model fit along several dimensions of the data and discuss the model performance.

### 3.5.1 Functional Forms and Externally Fixed Parameters

The demographic structure of the economy is set externally. The population growth $n$, is fixed to 1.8\% which is the number used by Jung and Tran (2012) and describes the annual population growth in Brazil. Since for computational purposes one period in the model is equivalent to five years, the number for a model period is $(1 + n)^5 - 1 = 0.093$ or 9.3\%. The conditional surviving probabilities, $\{\psi_j\}_{j=0}^J$, are taken from the life-tables provided by the World Health Organization.\footnote{The right panel of Figure B.2 shows the conditional surviving rates along age.}

Individuals enter the economy at the age of $j_0 = 20$. I set the maximum age in the economy to 14 periods which is equivalent to 90 years. I set exogenously the retirement age to 60 years which is close to the average retirement age in Brazil.\footnote{On average individuals retire when they are 58 (World Bank (2017)).}

The utility function $u$ is defined as $u(c, 1 - l) = \left(\frac{\nu^{(1-l)\gamma}}{1 - \gamma} \right)^{1 - \sigma}$, where $\nu$ denotes the intratemporal elasticity of substitution between consumption and leisure and $\sigma > 0$ is the relative risk aversion coefficient.\footnote{The advantage of this utility function specification is that it is consistent with the balanced growth path.}

The coefficient of relative risk aversion $\sigma$ is fixed to 2 which is a standard value in the macroeconomic literature. In order to estimate the age-efficiency profile $\varpi_j$, and the stochastic component $\epsilon$, I do the following: First, I regress hourly labor earnings on observable individual characteristics such as education, race and age.\footnote{Note, this is done for the whole population and not separately for formal and informal workers. The reason is that the efficiency profile as well as the idiosyncratic shocks are unconditional on the occupation.}

Next, I model the stochastic part as a first order auto-regressive process: $\epsilon_{i,t+1} = (1 - \rho \epsilon) \mu + \rho \epsilon_{i,t} + \zeta_{i,t+1}$, where $\zeta_{i,t+1} \sim N(0, \sigma^2_{\epsilon})$. I estimate this process and...
Table 3.2: Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>population growth</td>
<td>1.8%</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>( { \psi_j }_{j=0} )</td>
<td>surviving probabilities</td>
<td>Figure B.2</td>
<td>WHO</td>
</tr>
<tr>
<td>( J )</td>
<td>maximum age</td>
<td>14</td>
<td>90 years</td>
</tr>
<tr>
<td>( J_R )</td>
<td>retirement age</td>
<td>8</td>
<td>60 years</td>
</tr>
<tr>
<td>Preference and ability process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>risk aversion</td>
<td>2</td>
<td>standard value</td>
</tr>
<tr>
<td>( \rho_\eta )</td>
<td>persistence</td>
<td>0.7761</td>
<td>micro data (PME)</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>standard deviation</td>
<td>0.3542</td>
<td>micro data (PME)</td>
</tr>
<tr>
<td>Technology and production</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>capital share</td>
<td>0.4</td>
<td>Jung and Tran (2012)</td>
</tr>
<tr>
<td>( A^F )</td>
<td>TFP, formal sector</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>( A^I )</td>
<td>TFP, informal sector</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>Government</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s )</td>
<td>education subsidy</td>
<td>0.2623</td>
<td>OECD</td>
</tr>
<tr>
<td>( \tau_{hsv} )</td>
<td>income tax, progressivity</td>
<td>0.08</td>
<td>Gobetti and Orair (2017)</td>
</tr>
<tr>
<td>( \lambda_{hsv} )</td>
<td>income tax, level</td>
<td>0.85</td>
<td>Gobetti and Orair (2017)</td>
</tr>
</tbody>
</table>

obtain a persistence parameter on annual basis \( \rho_\eta = 0.7761 \) which translates to 0.2816 for a model period of 5 years. The dispersion parameter is \( \sigma_\eta = 0.3542 \) which is equivalent to 0.5389 for a five-year span. The details on the estimation and translation from the annual to a five-year period are provided in the Appendix.

For the cost of education \( \xi(e) \) I follow Krueger and Ludwig (2013) and assume a linear decreasing function: \( \xi(e) = 1 - e \) where \( e \in (0, 1) \). The subsidy rate \( s \) which represents the government share of education financing is calculated in the following way. The OECD (2018) reports that the government of Brazil invests roughly 3800USD per student. Additionally, the fee for tertiary education in Brazil is around 9500EUR\(^{17}\) which is 10686USD.\(^{18}\) Hence, total expenditures per student are 14486\( \$ \) per annum and the government covers \( \frac{3800}{14486} = 0.2623 \) or 26.23\% of the total cost.\(^{19}\)

The parameter \( \alpha \) represents the capital share in the formal sector production and is set to 0.4, which is the value used by Jung and Tran (2012). The total factor productivity of the formal and informal sectors, \( A^F \) and \( A^I \), are normalized to unity.

For the calibration of the progressive income taxation, I follow Heathcote et al. (2017) and consider a functional form:

\[
\tau_l(y) = y - \lambda_{hsv}y^{\tau_{hsv}},
\]

where \( \tau_l(y) \) is the tax rate at the income level \( y \), \( \tau_{hsv} > 0 \) is a measure of progressivity.

\(^{17}\)htts://www.master-and-more.eu. ‘How much does education cost around the world’.

\(^{18}\)This holds true at the exchange rate of 1\$ = 0.89EUR.

\(^{19}\)These calculations are similar to the calculations in Vardishvili and Wang (2019).
of the tax schedule and $\lambda_{hsu}$ is a parameter that governs the average tax rate. I use the
data from Gobetti and Orair (2017) who report average tax rates by income brackets in
Brazil to pin down the shape of the tax function. The recovered function as well as the
comparison to the data are presented in the Appendix.

All externally set parameters are reported on annual basis in Table 3.2.

### 3.5.2 Internally Calibrated Parameters

After calibrating the external parameters, there are 12 remaining parameters that are cali-
brated internally: the discount factor $\beta$, the depreciation rate $\delta$, the weight on consumption
in the utility function $\nu$, the utility cost of informality $u_I$, the ratio of education premia in
the formal and the informal sectors $\chi_F/\chi_I$, where $\chi_I$ is normalized to one, two parameters in
the learning ability distribution $\mu_e$ and $\sigma_e$, two parameters in the initial asset distribution
$\mu_a$ and $\sigma_a$, the pension payout $p$, consumption and capital taxes, $\tau_c$ and $\tau_k$. I calibrate
these parameters to match 12 selected moments from the data listed in Table 3.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.91</td>
<td>interest rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.03</td>
<td>capital-output ratio</td>
</tr>
<tr>
<td>$\nu$</td>
<td>weight on leisure</td>
<td>0.5</td>
<td>labor supply</td>
</tr>
<tr>
<td>$u_I$</td>
<td>cost parameter</td>
<td>0.55</td>
<td>share of informal workers</td>
</tr>
<tr>
<td>$\chi_F/\chi_I$</td>
<td>education premium</td>
<td>1.55</td>
<td>ratio mean income, F/I</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>learning ability, mean</td>
<td>1.5</td>
<td>share of low/mid./highly educated</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>learning ability, st. dev.</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_a$</td>
<td>initial asset distribution, mean</td>
<td>0.3</td>
<td>Bottom 90%</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>initial asset distribution, st. dev.</td>
<td>1.85</td>
<td>Gini wealth</td>
</tr>
<tr>
<td>$p$</td>
<td>pension</td>
<td>0.182</td>
<td>pension exp. to GDP</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>consumption tax</td>
<td>0.1</td>
<td>goods and services tax revenue / GDP</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>capital tax</td>
<td>0.22</td>
<td>property tax revenue$^a$ / GDP</td>
</tr>
</tbody>
</table>

$^a$The major components of property income are interest, dividends, and rent.

It is well understood that all the model parameters affect all the targets, but we can
nonetheless outline which data moment is most informative about a specific parameter.
The interest rate and the capital-output ratio identify the discount factor $\beta$ and the de-
preciation rate $\delta$. The weight on leisure $1 - \nu$ is pinned down to match the average labor
supply in Brazil, measured as the percentage of time endowment devoted to market work.
The utility cost of informality $u_I$ affects the share of informal workers in the economy,
whereas the education premium ratio defines the wage ratio between formal and informal
workers.

The parameters for the distribution of the innate learning ability $\mu_e, \sigma_e$ are set to
match the share of low, middle and highly educated individuals in the data. In the data,
I define as low-educated those who do not have a high-school degree, middle-educated are those who have a high-school degree only, highly-educated are those who have acquired tertiary education. In the model, for simplicity, I define low, middle and highly educated individuals as follows: I take the range of invested time in education of the benchmark economy and split it into three equal parts defining each education category as one part. In the benchmark economy, individuals invest between 46% and 99% of their time endowment into education.\(^{20}\) A low-educated individual is the one who invests between 46% and 63.67% of her time endowment. Middle and highly educated are those who invest between 63.67% and 81.34% and between 81.35% and 99%, respectively.

To capture the heterogeneity of inherited assets at the time when the educational decision takes place, I assume that \(a_0\) is drawn from a log-normal distribution \(\text{Lognormal}(\mu_a, \sigma_a)\). \(\mu_a\) and \(\sigma_a\) are set to match the Gini coefficient and the share of assets held by the bottom 90% of population. The recovered values are \(\mu_a = 0.3\) and \(\sigma_a = 1.85\).

I determine the pension in the economy \(p\) so that the pension expenditure to GDP ratio is matched. Finally, I set the parameters for the consumption and capital tax to 10% and 22%, respectively, to roughly match the tax revenues to GDP ratio for consumption and capital tax.\(^{21}\) Table 3.3 summarizes the recovered values for the internal parameters. The presented values are annualized.

### 3.5.3 Model Fit

Before analyzing the mechanism of the model and discussing the results, I present how the model matches the data and discuss the model’s limitations.

The model fit is presented in Table 3.4. The annual interest rate in the model is close to the annual real interest rate in the data. The latter is taken from the data provided by the Federal Reserve Bank of St. Louis.\(^{22}\) The capital-output ratio as well as the labor supply in the model and in the data are also fairly close. The data target for the capital-output ratio is taken from the empirical study of Filho (2002). Note, that labor supply is calculated as the percentage of time endowment devoted to market work. The data target is calculated using PME for a pooled sample of workers both formal and informal.

The share of the informal employees in the economy and the ratio of formal-to-informal wages are matched well. I derive the share of informal workers from PME using the definition of an informal worker described above. Around 15.4% of employees did not sign a working contract and operate informally. The ratio of mean formal to informal gross

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\(^{20}\)This range is sensitive to the weight of the leisure parameter in the utility function.

\(^{21}\)The empirical counterpart for the consumption tax is the tax on goods and services whereas for the capital tax - the property tax.

\(^{22}\)Note, that I take the net interest rate and adjust for the inflation over the period of 2010-2015. This time range is chosen to stay consistent with the period I use in PME.
### Table 3.4: Model Fit, Targeted Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>4.78%</td>
<td>3.38%</td>
<td>FRED</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>3</td>
<td>2.5</td>
<td>Filho (2002)</td>
</tr>
<tr>
<td>Labor supply</td>
<td>37.1%</td>
<td>36.46%</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Share of informal workers</td>
<td>15.4%</td>
<td>16.02%</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Ratio wages, F/I</td>
<td>1.41</td>
<td>1.57</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Low educated</td>
<td>29.22%</td>
<td>31.96%</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Mid. educated</td>
<td>45.52%</td>
<td>45.62%</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Highly educated</td>
<td>25.26%</td>
<td>22.42%</td>
<td>PME, (2010-2015)</td>
</tr>
<tr>
<td>Bottom 90%</td>
<td>50.00%</td>
<td>50.53%</td>
<td>Statista(^a)</td>
</tr>
<tr>
<td>Gini wealth</td>
<td>72.86</td>
<td>73.83</td>
<td>Fortune(^b)</td>
</tr>
<tr>
<td>Pension exp. to GDP</td>
<td>11%</td>
<td>8.9%</td>
<td>OECD</td>
</tr>
<tr>
<td>Consumption tax revenue / GDP</td>
<td>14.2%</td>
<td>12.31%</td>
<td>IMF(^c)</td>
</tr>
<tr>
<td>Capital tax revenue / GDP</td>
<td>1.3%</td>
<td>2.1%</td>
<td>IMF</td>
</tr>
</tbody>
</table>

\(^a\)https://www.statista.com/statistics/754724/wealth-distribution-income-share-brazil/

\(^b\)http://fortune.com/2015/09/30/america-wealth-inequality/


The wage in the data is around 1.41 which is slightly lower than the model value. Importantly, the shares of low, middle and highly educated individuals are matched well. The definition of education categories in the data and in the model are explained above. Figure B.5 shows the recovered distribution of the innate learning ability.

According to the empirical evidence, the poorest 90% of the population hold around half of the total wealth and the remaining 10% of the population hold another half. Moreover, the wealth Gini coefficient is around 72.86. Figure B.4 shows the recovered distribution of initial assets.

The model generates the pension-expenditure to GDP ratio of 8.9% which is slightly less than the value in the data of 11% reported by the OECD. The consumption and capital tax revenues to GDP ratios in the data and in the model are fairly close. The empirical numbers are reported by the IMF in their Government Finance Statistics. I use the period of 2010 to 2015 to be consistent with the data sample used for the other targets discussed above.

Table 3.5 shows the within sector composition of educational attainment in the data and in the model which was not explicitly targeted. The model share of low, middle and highly educated individuals in the formal sector are close to the data counterpart. However, the model exaggerates the share of low educated individuals in the informal sector.

Table 3.6 reports the inequality statistics for the pre-government labor income for the total population as well as separately for formal and informal employees. On the aggregate level, the model generates higher income inequality compared to the data. However, the
Table 3.5: Distribution of Education within Sectors

<table>
<thead>
<tr>
<th>Education</th>
<th>Formal</th>
<th>Informal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Low, %</td>
<td>27.31</td>
<td>25.47</td>
</tr>
<tr>
<td>Middle, %</td>
<td>46.54</td>
<td>49.49</td>
</tr>
<tr>
<td>High, %</td>
<td>26.15</td>
<td>25.04</td>
</tr>
<tr>
<td>Total, %</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: This table shows the within sector composition of educational attainment in the data and in the model. Low education is defined being below a high-school degree. Middle education corresponds to a high-school degree, whereas high education corresponds to a tertiary degree.

Table 3.6: Inequality Statistics, Pre-government Income

<table>
<thead>
<tr>
<th>Moments</th>
<th>Gini</th>
<th>Mean/Median</th>
<th>Bottom 40</th>
<th>Top 20</th>
<th>Top 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.537</td>
<td>1.5901</td>
<td>0.1187</td>
<td>0.5969</td>
<td>0.4827</td>
</tr>
<tr>
<td>Data</td>
<td>0.435</td>
<td>1.5793</td>
<td>0.1675</td>
<td>0.5203</td>
<td>0.3701</td>
</tr>
<tr>
<td>Formal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.293</td>
<td>0.9343</td>
<td>0.1951</td>
<td>0.3353</td>
<td>0.1842</td>
</tr>
<tr>
<td>Data</td>
<td>0.429</td>
<td>1.5793</td>
<td>0.1708</td>
<td>0.5163</td>
<td>0.3654</td>
</tr>
<tr>
<td>Informal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.423</td>
<td>0.9698</td>
<td>0.0941</td>
<td>0.4175</td>
<td>0.2286</td>
</tr>
<tr>
<td>Data</td>
<td>0.442</td>
<td>1.5918</td>
<td>0.1701</td>
<td>0.5245</td>
<td>0.3802</td>
</tr>
</tbody>
</table>

Notes: This table reports inequality statistics for the pre-government gross income for the total population as well as for the formal and informal workers separately. Data source is PME from 2010 to 2015.
model understates the within sector income inequalities due to a sorting mechanism into the sectors. In the data, the distributions of income in the formal and in the informal sectors are similar and quite dispersed, whereas, in the model, the sector income distributions differ more and are more concentrated within. As a result, the total inequality in the model is driven by the difference between the two sectors, whereas the total inequality in the data is the result of a high within-sector income dispersion. This limitation of the model has to be considered when discussing the impact of education policies on inequality.

3.6 Results

In this section, I first discuss the aggregate performance of the model. In particular, I explain the life-cycle patterns of consumption, labor supply and aggregate savings for the whole population. Next, I present the optimal educational and occupational choices in the model and highlight their interrelation. Finally, I asses the outcome of the policy experiments which include an increase of the education subsidy and a change in the tax schedule. For the latter experiment, I consider both a change in the level of taxation as well as the change in tax progressivity.

3.6.1 Aggregate Performance of the Model

Figure 3.2 summarizes the average life cycle profiles of consumption, labor supply in efficiency units and asset accumulation for the whole population. The outcome is standard for an OLG model. Since the time discount factor of the household is smaller than the interest rate in the economy, the household favors an increasing consumption path over the life cycle. At the same time, individuals save for the retirement period to smooth their consumption. During retirement, individuals receive pension benefits which are lower than their labor income and the average consumption decreases. The labor supply follows a hump shape over the life cycle due to two reasons. On the one hand, the labor ability has an efficiency component which increases with age (Figure B.2) and, therefore individuals intensify their labor supply in the initial stage of their economically active life. On the other hand, since individuals demand an increasing consumption path, they also extend their leisure time. Therefore hours worked drop with age. Individuals save during their working period to insure themselves against negative income shocks as well as finance consumption in the retirement period. Figure 3.2b shows that in the first period, individuals dissave their inherited assets and invest in education. The asset spike before the retirement is explained by the composition effect of formal and informal individuals. Assets, consumption and labor supply separately for the formal and the informal employees are reported in the Appendix 3.B.2.
3.6.2 Educational and Occupational Choice

In this subsection, I explain educational and occupational choices of individuals in the benchmark economy. The left panel of Figure 3.3 shows time investment in education by the asset level for different innate learning abilities. The higher the innate learning ability the lower is the cost of education and individuals study more translating into a higher level of education in the future. Similarly, the higher the inherited amount of assets the higher is the education investment since for wealthy individuals education is more affordable.

It is important to understand the relation between educational and occupational choices. Figure 3.3b depicts occupation choices for different levels of assets and innate learning ability. The value of one refers to the formal occupation whereas the value of zero to the informal occupation. Irrespective of the learning ability, individuals with a high asset level choose to be formal. Individuals with a high learning ability invest in education and become formal. Individuals with a low and middle learning ability are informal for low asset values and are formal for high asset levels. All else equal, if education is more affordable, the investment into education and the formal employment are higher.

Table 3.5 presented above shows that within the formal sector most individuals are middle educated in equilibrium. This outcome is the result of two opposing forces. On the one hand, individuals who choose to become formal invest a lot in education to enjoy a higher education premium. On the other hand, progressive taxation prevents them to

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23 All other variables of the state space are fixed to their average values.
Figure 3.3: Educational and Occupational Choice

Notes: The left panel shows investment in education for different levels of the innate learning ability and assets. The right panel depicts an occupational choice for different innate ability and asset levels. The value of one refers to the formal occupation whereas the value of zero refers to the informal occupation.

invest too much since income is taxed. In the informal sector most individuals are low educated. Within informal occupation there is no incentive to invest much in education since the education premium is low.

3.6.3 Policy Experiments

In this subsection, I analyze the effects of different policy reforms. In the first step, I study the impact of an increase in the education subsidy on educational attainment, occupational choice and aggregate outcomes. Next, I analyze the effects of tax policies on educational and occupational choices with a focus on welfare and distributional implications.

Education Subsidy

In this experiment, I vary the subsidy rate $s$ from 0 to 0.8 which means that the government finances from 0 up to 80% of the education costs. At this stage, I do not redistribute additional revenues or deficits, hence, this experiment is not fiscally neutral. The results are presented in Table 3.7.

The rise in the education subsidy makes education more affordable and individuals invest more in education. If the subsidy is sufficiently high ($s=0.8$), all individuals become highly educated and a further extension in the subsidy does not improve the educational attainment. As educational attainment increases, the share of informal employees drops. Increasing the subsidy from the benchmark value of 0.2623 to 0.8 decreases informal em-
ployment from 16.02% to 5.98%. Note, that even with the highest education level, there are individuals who choose to be informal. With the increasing supply of formal labor the relative market wage of formal workers drops. This closes the gap between formal and informal wages and the income inequality decreases.

Welfare measured in terms of consumption equivalent variation (cev) for the working population\footnote{I report this statistic for working population only, since the young population suffers a substantial disutility due to a very low consumption level and high time investment in education. The value function for the young is times lower compared to the working population and a slight change in their utility drives the total result.} increases as the subsidy is raised. This result is not trivial since is driven by three forces: the utility of consumption, the disutility of labor and the disutility of being informal. The utility of consumption is not a dominating force, since even though there are more formal workers with a higher education premium, their relative market wage drops. The disutility of labor shrinks since higher educated individuals choose to work fewer hours. Clearly with fewer informal employees the disutility of being informal decreases. All in all, increasing the subsidy to 80% increases the welfare by almost 10% measured in cev.

The rise of the subsidy rate increases the education expenditure to GDP ratio from 0.53% to 2.15\%\footnote{Note that the benchmark value of the education expenditure to GDP ratio does not match the data counterpart of 5.5\%. The increase in education expenditures by a factor of four would lead to a new level of around 20\% of GDP.}. At the same time, the income tax revenue to GDP ratio rises by around 6pp and the total tax revenue to GDP ratio including consumption and capital taxes elevates by almost 9pp. Tax revenues expand due to an increase in the tax base from formalization and a rise in income due to a higher education level. In the long run, the education subsidy is self-financed and additional tax revenues can be redistributed back to the population to further boost the welfare.

Table 3.7: Education Subsidy

<table>
<thead>
<tr>
<th>Subsidy, s</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.2623</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low educated, %</td>
<td>39.85</td>
<td>32.11</td>
<td>28.12</td>
<td>31.96</td>
<td>33.84</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mid. educated, %</td>
<td>45.44</td>
<td>52.96</td>
<td>48.54</td>
<td>45.62</td>
<td>43.72</td>
<td>68.86</td>
<td>68.26</td>
<td>48.68</td>
<td>24.7</td>
<td>0</td>
</tr>
<tr>
<td>Highly educated, %</td>
<td>14.71</td>
<td>14.93</td>
<td>23.34</td>
<td>22.42</td>
<td>22.44</td>
<td>31.14</td>
<td>31.74</td>
<td>51.32</td>
<td>75.3</td>
<td>100</td>
</tr>
<tr>
<td>Interest rate, %</td>
<td>3.28</td>
<td>3.38</td>
<td>3.42</td>
<td>3.38</td>
<td>3.34</td>
<td>4</td>
<td>4.16</td>
<td>4.62</td>
<td>5.07</td>
<td>5.5</td>
</tr>
<tr>
<td>$w^f/w^I$</td>
<td>1.59</td>
<td>1.57</td>
<td>1.55</td>
<td>1.57</td>
<td>1.58</td>
<td>1.43</td>
<td>1.39</td>
<td>1.31</td>
<td>1.24</td>
<td>1.18</td>
</tr>
<tr>
<td>Gini, all</td>
<td>0.5392</td>
<td>0.5397</td>
<td>0.5394</td>
<td>0.5354</td>
<td>0.533</td>
<td>0.537</td>
<td>0.5173</td>
<td>0.5083</td>
<td>0.4994</td>
<td>0.4598</td>
</tr>
<tr>
<td>Welfare, cev, %</td>
<td>-1.25</td>
<td>-0.32</td>
<td>0.32</td>
<td>0.00</td>
<td>-0.32</td>
<td>2.94</td>
<td>3.28</td>
<td>5.00</td>
<td>7.51</td>
<td>9.76</td>
</tr>
<tr>
<td>Education exp. to GDP, %</td>
<td>0</td>
<td>0.21</td>
<td>0.41</td>
<td>0.53</td>
<td>0.61</td>
<td>0.79</td>
<td>1.04</td>
<td>1.27</td>
<td>1.64</td>
<td>2.15</td>
</tr>
<tr>
<td>Tax rev. income to GDP, %</td>
<td>18.53</td>
<td>19.01</td>
<td>19.4</td>
<td>19.08</td>
<td>19.22</td>
<td>21.32</td>
<td>22.01</td>
<td>22.53</td>
<td>24.28</td>
<td>25.48</td>
</tr>
<tr>
<td>Tax rev. to GDP, %</td>
<td>33.03</td>
<td>33.58</td>
<td>33.86</td>
<td>33.56</td>
<td>33.63</td>
<td>36.56</td>
<td>37.37</td>
<td>38.71</td>
<td>40.49</td>
<td>42.16</td>
</tr>
</tbody>
</table>

Notes: This table reports the effects of the education subsidy change. The benchmark value of the subsidy rate is 26.23%. All other policy parameters are kept unchanged.
Tax Scheme

Alternatively, the government may change the level of taxes or the magnitude of tax progressivity to collect more tax revenues and affect the educational and occupational choices indirectly. Table 3.8 shows an experiment of a decrease in the tax level. Note that the parameter $\lambda_{hsv}$ in the tax function defines the level of taxation. The higher the $\lambda_{hsv}$ the lower is the average tax rate.

Decreasing the level of taxation fosters formalization and increases the education level. As a result, the tax base expands and the tax revenue to GDP ratio rises. The increase in income and the reduction of informality leads to less inequality and higher welfare. Hence, if the government aims to expand tax revenues, it should decrease rather than increase the level of taxation.

Table 3.8: Level of Income Tax

<table>
<thead>
<tr>
<th>Tax level, $\lambda_{hsv}$</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of informal workers, %</td>
<td>27.32</td>
<td>21.73</td>
<td>16.02</td>
<td>9.76</td>
</tr>
<tr>
<td>Low educated, %</td>
<td>37.27</td>
<td>33.35</td>
<td>31.96</td>
<td>11.71</td>
</tr>
<tr>
<td>Mid. educated, %</td>
<td>48.29</td>
<td>52.04</td>
<td>45.72</td>
<td>65.4</td>
</tr>
<tr>
<td>Highly educated, %</td>
<td>14.44</td>
<td>14.61</td>
<td>22.42</td>
<td>22.89</td>
</tr>
<tr>
<td>Interest rate, %</td>
<td>1.68</td>
<td>2.41</td>
<td>3.38</td>
<td>4.65</td>
</tr>
<tr>
<td>$w^F/w^I$</td>
<td>2.19</td>
<td>1.86</td>
<td>1.57</td>
<td>1.31</td>
</tr>
<tr>
<td>Gini, all</td>
<td>0.5532</td>
<td>0.5515</td>
<td>0.5371</td>
<td>0.4843</td>
</tr>
<tr>
<td>Welfare, cev, %</td>
<td>-3.37</td>
<td>-1.25</td>
<td>0.00</td>
<td>3.62</td>
</tr>
<tr>
<td>Education exp. to GDP, %</td>
<td>0.69</td>
<td>0.63</td>
<td>0.53</td>
<td>0.51</td>
</tr>
<tr>
<td>Tax rev. income to GDP%</td>
<td>15.1</td>
<td>17.44</td>
<td>19.08</td>
<td>23.6</td>
</tr>
<tr>
<td>Tax rev. to GDP, %</td>
<td>26.61</td>
<td>29.89</td>
<td>33.56</td>
<td>40.62</td>
</tr>
</tbody>
</table>

Notes: This table reports the effects of the tax level change, $\lambda_{hsv}$. The higher is the parameter $\lambda_{hsv}$, the lower is the income tax level. All other policy parameters are kept unchanged.

Finally, I change the progressivity $\tau_{hsv}$ of the tax function and analyze the consequences in terms of the aggregate outcomes. A higher $\tau_{hsv}$ implies that the poor are taxed less and the rich are taxed more.

Lower taxation of the poor creates incentives for individuals with the low innate learning ability to invest in education and formalize. The share of low educated individuals decreases by around 2pp and the share of the informal workers drops from 16% to 12%. Interestingly, the share of highly educated individuals slightly drops due to the fact that higher tax progressivity discourages human capital accumulation. Expanded formalization and a higher gross income increase welfare by more than 4% measured in cev. However, the gross income inequality measured by the Gini coefficient rises as a response to a higher wage difference between the formal and informal sectors. The tax revenue to GDP ratio first increases with a higher progressivity and then drops showing important trade-offs.
Although formalization expands the tax base at first, the tax distortions on the human capital accumulation and the intensive margin of the labor supply bring the tax revenues down.

Table 3.9: Income Tax Progressivity

<table>
<thead>
<tr>
<th>Tax progressivity, $\tau_{hsv}$</th>
<th>0.01</th>
<th>0.08</th>
<th>0.18</th>
<th>0.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of informal workers, %</td>
<td>20.41</td>
<td>16.02</td>
<td>12.74</td>
<td>12.08</td>
</tr>
<tr>
<td>Low educated, %</td>
<td>31.84</td>
<td>31.96</td>
<td>31.64</td>
<td>30.01</td>
</tr>
<tr>
<td>Mid. educated, %</td>
<td>45.74</td>
<td>45.62</td>
<td>45.94</td>
<td>47.61</td>
</tr>
<tr>
<td>Highly educated, %</td>
<td>22.42</td>
<td>22.42</td>
<td>22.42</td>
<td>22.38</td>
</tr>
<tr>
<td>Interest rate, %</td>
<td>3.37</td>
<td>3.38</td>
<td>3.29</td>
<td>3.09</td>
</tr>
<tr>
<td>$w^F/w^I$</td>
<td>1.57</td>
<td>1.57</td>
<td>1.59</td>
<td>1.64</td>
</tr>
<tr>
<td>Gini, all</td>
<td>0.5303</td>
<td>0.5371</td>
<td>0.5405</td>
<td>0.5426</td>
</tr>
<tr>
<td>Welfare, cev,%</td>
<td>-0.94</td>
<td>0.00</td>
<td>1.94</td>
<td>4.30</td>
</tr>
<tr>
<td>Education exp. to GDP, %</td>
<td>0.52</td>
<td>0.53</td>
<td>0.55</td>
<td>0.58</td>
</tr>
<tr>
<td>Tax rev. income to GDP%</td>
<td>18.74</td>
<td>19.08</td>
<td>18.93</td>
<td>18.33</td>
</tr>
<tr>
<td>Tax rev. to GDP, %</td>
<td>33.02</td>
<td>33.56</td>
<td>33.63</td>
<td>33</td>
</tr>
</tbody>
</table>

Notes: This table reports the effects of the income tax progressivity change, $\tau_{hsv}$. The higher is the parameter $\tau_{hsv}$, the higher is the income tax progressivity. All other policy parameters are kept unchanged.

3.7 Conclusions

In this paper, I examine the effects of education and tax policies in the presence of a substantial informal sector in a general equilibrium life-cycle model. I define the optimal education subsidy rate which maximizes welfare and minimizes inequality.

In the model, individuals endogenously choose how much to invest into education taking into account the cost of education and their future prospects given their education level. The cost of education depends on the innate learning ability, time invested in education and the share which is subsidized by the government. Individuals are also aware of the fact that in the future they can choose to operate either in the formal or in the informal sector. The education premium is higher in the formal sector but individuals have to pay progressive taxes. Informal employees have a low education premium, do not pay taxes and bare a utility cost of informality associated with the lack of social security.

I calibrate my model to Brazil which features a high level of informality and a low level of educational attainment. The latter means that there is room for more human capital accumulation. My model matches well the aggregate macroeconomic indicators, the share of the informal employment, the distribution of educational attainment in the aggregate as well as within formal and informal sectors.
In equilibrium, the majority of formal workers have an average education level\(^{26}\) whereas the majority of informal workers are low educated. Although there is an incentive to invest in education to enjoy a high education premium in the formal sector, progressive taxation prevents individuals from investing too much. Hence, formal occupation features an average education level. On the contrary, the individuals with a low level of education prefer the informal sector due to an expected low education premium in any of the sectors and the privilege of no taxation in the shadow.

An increase in the education subsidy makes education more affordable, raises the average level of educational attainment, boosts formalization and the tax base. Due to this effect, the education subsidy is self-financed in the long run and does not require additional tax hikes. Raising the subsidy level from the current 26% to 80% reduces informality by around 10\(^{pp}\), increases income tax revenues by 6\(^{pp}\) and lowers income inequality measured by a Gini coefficient by almost 10\(^{pp}\). Moreover, welfare increases by almost 10% measured in consumption equivalent variation. Further expansion of the subsidy does not improve the aggregate outcomes since for the subsidy rate of 80% all individuals are highly educated and the upper bound of education is reached.

Although, the education subsidy is self-financed, an interesting question is what are the effects of the change in the income tax schedule. Interestingly, if the government aims to increase the tax revenues it should decrease the tax level rather than increase it. Lower taxes raise formalization and the education level, increasing the tax base in the long run. The change in progressivity has ambiguous effects on tax revenues and leads to moderate changes in the education and informality levels.

To conclude, in a developing country with a high level of informality, education policies have positive aggregate effects in the long run. Due to a positive effect on the tax base there is room for a self-financed education subsidy. This analysis abstracts from the transitional dynamics which may have important implications for the welfare conclusions.

\(^{26}\) The average education level is defined as a finished high-school degree but no acquired tertiary education.
References


Chapter 3. Education Subsidies in the Presence of Informality


Vardishvili, O. and Wang, F. (2019). Education Affordability and Income Inequality, *manuscript, European University Institute*.

Appendix 3.A  Data

3.A.1 Data Sample and Main Empirical Findings

My main source of data consists of a panel of individuals of working age, sampled by the labor force survey of Brazil, Pesquisa Mensal de Emprego (PME). PME is designed and conducted by the National Statistics Bureau to follow individuals of the six main metropolitan regions of Brazil. The sample period starts in January 2010 and goes until December 2015. I abstract from earlier periods to better capture the characteristics of a current economy.

For the purpose of this paper, I focus on 20-65 years old employees who are household heads. I define an individual to be informal if one is an employee but did not sign a working contract. I do not consider individuals who reported they are unpaid workers, self-employed, unemployed or those who did not report their occupation. It is evident that a substantial share of self-employed individuals in Brazil operate informally. However, the effect of education on entrepreneurial ability may be different from the education premium for workers (see Berniell (2019)). Hence, I abstract from self-employed in this study.

Table A.1 shows that there are around 85% of formal and 15% informal employees in the sample. Around 29% of total employees do not have a high-school degree, around 45% have a high-school degree and around 25% have acquired tertiary education.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>84.61</td>
<td>15.39</td>
<td>29.22</td>
<td>45.52</td>
<td>25.26</td>
</tr>
</tbody>
</table>

Table A.2 shows the results for a Probit regression for the pooled sample. The education level has a statistically significant negative effect on the probability of operating informally. Table A.3 presents empirical evidence on the education premium for both formal and informal employees. The coefficients for education are positive and statistically significant for formal as well as informal workers. However, the education premium is larger in the formal sector.


To estimate the process that governs labor productivity, I first run cross-sectional regressions of the logarithm of hourly wages on age to define the age-efficiency profile $\pi_j$. I use a second order polynomial in age, since including higher levels decrease the regression
Table A.2: Probit

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>informal</td>
<td></td>
</tr>
<tr>
<td>educ</td>
<td>-0.116***</td>
<td>(-84.48)</td>
</tr>
<tr>
<td>age</td>
<td>-0.0477***</td>
<td>(-50.25)</td>
</tr>
<tr>
<td>age2</td>
<td>0.000524***</td>
<td>(43.87)</td>
</tr>
<tr>
<td>gender</td>
<td>0.0656***</td>
<td>(21.24)</td>
</tr>
<tr>
<td>race</td>
<td>-0.00149</td>
<td>(-1.26)</td>
</tr>
<tr>
<td>region</td>
<td>-0.00630***</td>
<td>(-18.92)</td>
</tr>
<tr>
<td>cons</td>
<td>0.584***</td>
<td>(25.71)</td>
</tr>
</tbody>
</table>

| N        | 1074916  |

* t statistics in parentheses
** p < 0.05, *** p < 0.001

Table A.3: OLS

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>logw, Formal</td>
<td>logw, Informal</td>
</tr>
<tr>
<td>educ</td>
<td>0.293***</td>
<td>0.249***</td>
</tr>
<tr>
<td></td>
<td>(548.09)</td>
<td>(169.28)</td>
</tr>
<tr>
<td>age</td>
<td>0.0163***</td>
<td>0.0167***</td>
</tr>
<tr>
<td></td>
<td>(42.44)</td>
<td>(16.64)</td>
</tr>
<tr>
<td>age2</td>
<td>0.0000797***</td>
<td>0.0000349*</td>
</tr>
<tr>
<td></td>
<td>(16.49)</td>
<td>(2.77)</td>
</tr>
<tr>
<td>race</td>
<td>-0.0584***</td>
<td>-0.0590***</td>
</tr>
<tr>
<td></td>
<td>(-127.54)</td>
<td>(-45.22)</td>
</tr>
<tr>
<td>region</td>
<td>0.00879***</td>
<td>0.0190***</td>
</tr>
<tr>
<td></td>
<td>(69.00)</td>
<td>(20.00)</td>
</tr>
<tr>
<td>cons</td>
<td>5.068***</td>
<td>4.746***</td>
</tr>
<tr>
<td></td>
<td>(562.52)</td>
<td>(197.15)</td>
</tr>
</tbody>
</table>

| N        | 893448    | 146519   |

* t statistics in parentheses
** p < 0.05, *** p < 0.001
fit (R-squared decreases). The obtained estimates are then normalized such that the efficiency parameter of an individual entering the economy is equal to unity. Figure B.2b visualizes the labor efficiency profile over the life cycle.

The estimation of the stochastic component of labor productivity follows closely the procedure described by Heathcote et al. (2010). First, I regress the logarithm of hourly labor earnings on observable individual characteristics such as education, race and age irrespective of individual’s occupation.

\[
\ln in\text{c}_{i,t} = \alpha_0 + \beta_0 \text{educ}_{i,t} + \beta_1 \text{race}_{i,t} + \beta_2 \text{age}_{i,t} + a_3 \text{age}^2_{i,t} + \eta_{it}, \tag{3.13}
\]

where \(i\) is individual and \(t\) is time index respectively. Then, I model the residual \(\eta\) as a first-order auto-regressive process:

\[
\log \eta_{i,t+1} = \rho \log \eta_{i,t} + \zeta_{i,t+1}, \tag{3.14}
\]

where \(\zeta_{i,t+1} \sim N(0, \sigma^2_{\eta})\). I estimate this process and obtain a persistence parameter \(\rho_{\eta,a} = 0.776\) and the dispersion parameter \(\sigma_{\eta,a} = 0.354\) on the annual basis. Since the model period is defined to be five years, I translate the obtained parameters into 5-year equivalents:

\[
\rho_{\eta} = \rho_{\eta,a}^T, \tag{3.15}
\]

\[
\sigma^2_{\eta} = \sigma^2_{\eta,a} \sum_{t=0}^{T-1} \rho^2_{\eta,a}, \tag{3.16}
\]

where \(T = 5\) is the number of years in the period. Consequently, the \(\rho_{\eta} = 0.282\) and \(\sigma_{\eta} = 0.539\).

3.3.3 Estimating Tax Progressivity

For the calibration of the progressive income taxation, I follow Heathcote et al. (2017) and consider a functional form:

\[
\tau_l(y) = y - \lambda_{hsv} y^{\tau_{hsv}},
\]

where \(\tau_l(y)\) is the tax rate at the income level \(y\), \(\tau_{hsv} > 0\) is a measure of progressivity of the tax schedule and \(\lambda_{hsv}\) is a parameter that governs the average tax rate. I use the data from Gobetti and Orair (2017) who report average tax rates by income brackets in Brazil to pin down the shape of the tax function. The procedure goes as follows: (1) given a gross income bracket and a corresponding tax rate, I calculate net income for each bracket in the data; (2) using a specified functional form, I calculate net income in the model: \(\hat{y}^F = \lambda_{hsv} y_F^{1-\tau_{hsv}}\). (3) I pin down both parameters \(\lambda_{hsv}\) and \(\tau_{hsv}\) to minimize the distance
between net to gross income ratio along income brackets in the model and in the data.

Appendix 3.B  Model

3.B.1 Computational Algorithm

In this subsection, I outline the computational algorithm employed for this project.

1. Given guesses for capital and labor input as well as tax rates, compute factor and consumer prices.

2. Given prices and public transfers, determine household policy functions by backward iteration.

3. Compute the distribution of households over the state space.

4. Aggregate household decisions.

5. Calculate the absolute value of the relative difference between demand and supply for capital and labor. If the difference is small enough the equilibrium is found and we can stop the iteration procedure. If not, start again at point 1.

3.B.2 Additional Figures
Figure B.2: Surviving Probabilities and Labor Efficiency Profile

(a) Conditional Survival Rates

(b) Labor Efficiency Profile over the Life-Cycle

Figure B.3: Informality and Education
Figure B.4: Initial Asset Distribution

Figure B.5: Initial Learning Ability Distribution
Figure B.6: Asset Holdings Comparison

Notes: The left panel compares total asset holdings of formal and informal workers over the life-cycle whereas the right panel compares the respective means, measured in model units.

Figure B.7: Consumption Comparison

Notes: The left panel compares total consumption whereas the right panel compares total labor supply in efficiency units of formal and informal workers over the life-cycle.
Gesamtzusammenfassung


Sektors. Ich konzentriere mich auf Brasilien und bewerte die Auswirkungen von Bildungsubventionen auf die Gesamtwirtschaft. Die Erhöhung des Bildungszuschusses verbessert das Bildungsniveau, erweitert die Formalisierung und erhöht das Wohlbefinden. Die Subventionsrate, die 80% der Bildungskosten abdeckt, maximiert das Wohlbefinden und der allgemeine Wohlfahrtsgewinn liegt bei etwa 10%, gemessen in einer konsumäquivalenten Variation. Aufgrund eines deutlich positiven Effekts auf die Steuerbemessungsgrundlage finanziert sich der Bildungszuschuss langfristig selbst.

Complete References


IMF (2016). The Urgent Case for Pension Reform in Brazil, *IMF report*.


Eigenabgrenzung

Das erste Kapitel habe ich zusammen mit Alessandro Di Nola (Universität Konstanz), Almuth Scholl (Universität Konstanz) und Georgi Kocharkov (Deutsche Bundesbank) verfasst. Meine individuelle Leistung bei der Erstellung dieses Kapitel beträgt 25%.

Ich versichere hiermit, dass ich Kapitel 2 und 3 ohne Hilfe Dritter verfasst habe.