A Datta-Das transistor with enhanced spin control

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We consider a two-channel spin transistor with weak spin-orbit induced interband coupling. We show that the coherent transfer of carriers between the coupled channels gives rise to an additional spin rotation. We calculate the corresponding spin-resolved current in a Datta-Das geometry and show that a weak interband mixing leads to enhanced spin control.

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The pioneering spin-transistor proposal of Datta and Das [1] best exemplifies the relevance of electrical control of magnetic degrees of freedom as a means of spin modulating charge flow. In this device [2], a spin-polarized current [3, 4] injected from the source is spin modulated on its way to the drain via the Rashba spin-orbit [5] (s-o) interaction, Fig. 1(a). The spin transistor operation relies on gate controlling [6] the strength $\alpha$ of the Rashba interaction which has the form $H_R = i\alpha\sigma_y \partial/\partial x$ in a strictly 1D channel [5]. Upon crossing the Rashba-active region of length $L$, a spin-up incoming electron emerges in the spin-rotated state

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} \cos(\theta_R/2) e^{-ik_R L} + e^{ik_R L} \\ -i \cos(\theta_R/2) e^{-ik_R L} + i e^{ik_R L} \\ -i \sin(\theta_R/2) e^{-ik_R L} \\ \sin(\theta_R/2) e^{-ik_R L} \end{pmatrix},$$

where $\theta_R = 2m^*\alpha L/h^2 \equiv 2k_R L$ is the rotation angle and $m^*$ is the electron effective mass [1]. The corresponding spin-resolved conductance is found to be $G_{\uparrow,\downarrow} = e^2 (1 \pm \cos \theta_R)/h$.

Here we extend the above picture by considering a geometry with two weakly-coupled Rashba bands in the quasi-one-dimensional channel, Fig. 1(b). We treat the degenerate $k$ states near the band crossings perturbatively in analogy to the nearly-free electron model [7]. This approach allows for a simple analytical description of the problem. We calculate the spin-resolved current by extending the usual procedure of Datta and Das [1] to account for weakly coupled bands. Our main finding is an additional spin rotation for injected electrons with energies near the band crossing [see shaded region around $\varepsilon_F$ in Fig. 2]. As we derive later on, an incoming spin up electron in channel $a$ emerges from the Rashba region in the rotated state

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} \cos(\theta_d/2) e^{-ik_R L} + e^{ik_R L} \\ -i \cos(\theta_d/2) e^{-ik_R L} + i e^{ik_R L} \\ -i \sin(\theta_d/2) e^{-ik_R L} \\ \sin(\theta_d/2) e^{-ik_R L} \end{pmatrix},$$

where $\theta_d = \theta_R d/k_c$ is the additional spin rotation angle, $d$ the interband matrix element and $k_c$ the wave vector at the band crossing, Fig. 2. From (2) we can find the new spin-resolved conductance

$$G_{\uparrow,\downarrow} = \frac{e^2}{h} \begin{pmatrix} 1 + \cos(\theta_d/2) \cos \theta_R \\ 1 - \cos(\theta_d/2) \cos \theta_R \end{pmatrix}.$$  

We now proceed to derive Eqs (2) and (3).

**Model.** We consider a quasi-one-dimensional wire of length $L$ with two bands $a$ and $b$ described by $\varepsilon_{n,\sigma}(k) = \hbar^2 k^2/2m^* + \epsilon_n, \ n = a, b$ and eigenfunctions $\varphi_{k,n,\sigma}(x,y) = e^{ikx} \phi_n(y) \sigma/\sqrt{\mathcal{T}}, \sigma = \uparrow, \downarrow$.
where the $\phi_n(y)$'s denote the transverse confinement wave functions. In the presence of the Rashba s-o interaction, we can derive a Hamiltonian for the system in the basis of the uncoupled wave functions \{\varphi_{k,n,\sigma}(x,y)\}. This reads,

$$H_R = \begin{pmatrix} \varepsilon_n^a(k) & 0 & 0 & -\alpha d \\ 0 & \varepsilon_n^a(k) & \alpha d & 0 \\ 0 & \alpha d & \varepsilon_n^b(k) & 0 \\ -\alpha d & 0 & 0 & \varepsilon_n^b(k) \end{pmatrix},$$

(4)

where $d \equiv \langle \phi_a(y) \partial / \partial y | \phi_b(y) \rangle$, $\varepsilon_n^a(k) = \hbar^2 (k - sk_R)^2 / 2m^* + \epsilon_n - \epsilon_R$, $\epsilon_R \equiv \hbar^2 k^2_R / 2m^*$, ($s = \pm$, $n = a, b$) and we have considered \(|\sigma\rangle\) to be the eigenbasis of $\sigma_y$. For $d = 0$ the Hamiltonian in (4) is diagonal and yields uncoupled Rashba dispersions $\varepsilon_n^a(k)$ (thin lines in Fig. 2); the corresponding wave functions are $\varphi_{k,n,s}(x,y)$ (here $|\sigma\rangle \rightarrow |s = \pm\rangle = [|\uparrow\rangle \pm i |\downarrow\rangle] / \sqrt{2}$). Note that for $d = 0$ the bands cross for some values of $k$. For instance, for $k > 0$ a crossing occurs at $k_c = (\epsilon_b - \epsilon_a) / 2\alpha$. For non-zero interband coupling $d \neq 0$ \cite{8}, we can diagonalize $H_R$ exactly (see Mireles and Kirczenow in Ref. \cite{8}) to find the new dispersions (thick lines in Fig. 2).

**Bands near $k_c$.** Since we are interested in transport with injection energies near the crossing, we follow below a simpler perturbative approach \cite{7} to determine the energy dispersions and wave functions near $k_c$. Near the crossing we can solve the reduced Hamiltonian

$$\tilde{H}_R = \begin{pmatrix} \varepsilon_n^a(k) & \alpha d \\ \alpha d & \varepsilon_n^b(k) \end{pmatrix},$$

(5)

which to lowest order yields

$$\varepsilon_\pm^{\text{approx}}(k) = \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2} \epsilon_b + \frac{1}{2} \epsilon_a \pm \alpha d.$$

(6)

As shown in the inset of Fig. 2, Eq. (6) describes very well the anti-crossing of the bands near $k_c$.

The corresponding zero-order eigenstates are

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}} [|-\rangle_a \pm |+\rangle_b] = \frac{1}{\sqrt{2}} \left[ \left( \frac{1}{\sqrt{2}} \right)_a \pm \left( \frac{1}{\sqrt{2}} \right)_b \right],$$

(7)

where the sub-indices indicate the respective channel. The analytical form in (6) allows us to determine the wave vectors $k_{c1}$ and $k_{c2}$ in Fig. 2 straightforwardly: we assume $k_{c1} = k_c - \Delta / 2$ and $k_{c2} = k_c + \Delta / 2$ and solve $\varepsilon_+^{\text{approx}}(k_{c1}) = \varepsilon_-^{\text{approx}}(k_{c2})$ (assumed $\sim \varepsilon_F$) to find

$$\Delta = \frac{2m^* \alpha d}{\hbar^2 k_c} = \frac{s k_R}{k_c} d.$$

(8)
Note that to the lowest order used here the horizontal splitting \( \Delta \) is constant and symmetric about \( k_c \).

**Boundary conditions.** We now consider a spin-up electron entering the Rashba-active region of length \( L \) in the wire. Following the usual approach, we expand this incoming state in terms of the coupled Rashba states in the wire. We consider only the states \( k_{c1}, k_{c2}, \) and \( k_2 \) in the expansion

\[
|\Psi\rangle = \frac{1}{2} |\psi_+\rangle e^{ik_{c1}x} + \frac{1}{2} |\psi_-\rangle e^{ik_{c2}x} + \frac{1}{\sqrt{2}} |a\rangle e^{ik_2x}.
\]

(9)

The above ansatz satisfies the boundary conditions for both the wave function and (to leading order) its derivative \( x = 0 \). More explicitly, the velocity operator condition \([10]\) at \( x = 0 \) for an electron with \( k = k_F \) yields

\[
\begin{pmatrix}
  k_F \\
  0 \\
  0
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
  k_c + k_2 & -i(k_c - k_2 - 2k_R) \\
  -i\Delta/2 & 0
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
  k_c + k_2 & 0 \\
  -\Delta/2 & -i\Delta/2
\end{pmatrix}
\]

(10)

where we used \( k_2 - k_c = 2k_R \) (still valid to leading order). The ‘four-vector’ notation in (10) concisely specifies the spin states in channels \( a \) (upper half) and \( b \) (lower half). Note that Eq. (10) is satisfied provided that \( \Delta \ll 4k_F \). This inequality is satisfied in our system for realistic parameters.

Underlying the ansatz in (9) is the assumption of unity transmission through the Rashba region. Here we have in mind the particular spin-transistor geometry sketched in Fig. 1(a): a gate-controlled Rashba-active region of extension \( L \) smaller than the total length \( L_0 \) of the wire. In this configuration, there are only small band offsets (which we neglect) of the order of \( \epsilon_R \ll \epsilon_F \) at the entrance \((x = 0)\) and exit \((x = L)\) of the Rashba region. Hence transmission is indeed very close to unity, see Ref. [9]. The boundary conditions at \( x = L \) are also satisfied.

**Generalized spin-rotated state.** From Eq. (9) we find that a spin-up electron entering the Rashba
region at $x = 0$ emerges from it at $x = L$ in the spin-rotated state

$$
\Psi_{\uparrow,L} = \frac{1}{4} \begin{pmatrix}
    e^{-iL\Delta/2} & e^{iL\Delta/2} \\
    -ie^{-iL\Delta/2} & -ie^{iL\Delta/2} \\
    e^{-iL\Delta/2} & e^{iL\Delta/2} \\
    ie^{-iL\Delta/2} & -ie^{iL\Delta/2}
\end{pmatrix} e^{ik_cL} + \frac{1}{2} \begin{pmatrix}
    1 \\
    i \\
    0 \\
    0
\end{pmatrix} e^{ik_2L},
$$

which is essentially Eq. (2). Observe that in absence of interband coupling (i.e., $\theta_d = 0$) Eq. (11) reduces to the Datta-Das state in (1). An expression similar to (11) holds for the case of an incoming spin-down electron.

**Spin-resolved current.** For $x \geq L$ we have

$$
\Psi_{\uparrow}(x \geq L, y) = \frac{1}{2} \begin{pmatrix}
    e^{-i\theta_R/2} \cos(\theta_d/2) + e^{i\theta_R/2} \\
    -ie^{-i\theta_R/2} \cos(\theta_d/2) + ie^{i\theta_R/2}
\end{pmatrix} e^{i(k_c+k_R)x} \phi_a(y) +
\frac{1}{2} \begin{pmatrix}
    -ie^{i\theta_R/2} \sin(\theta_d/2) \\
    e^{i\theta_R/2} \sin(\theta_d/2)
\end{pmatrix} e^{i(k_c-k_R)x} \phi_b(y),
$$

which describes plane waves in the uncoupled channels $a$ and $b$ arising for an incoming spin-up electron in channel $a$. The total current follows straightforwardly (Landauer-Büttiker) from Eq. (12)

$$
I_{\uparrow,1} = \frac{e}{h} eV [1 \pm \cos(\theta_d/2) \cos \theta_R],
$$

where $eV \ll \varepsilon_F$ is the applied bias between the source and drain. The spin-dependent conductance in (3) follows immediately from (13). Equation (13) clearly shows the additional modulation $\theta_d$ of the spin-resolved current due to s-o induced interband coupling. Figure 3 illustrates the angular dependence of $G_\uparrow$ as a function of $\theta_R$ and $\theta_d$. The s-o mixing angle $\theta_d$ enhances the possibilities for spin control in the Datta-Das transistor.

**Realistic parameters.** For concreteness, let us consider infinite transverse confinement (width $w$). In this case, $\epsilon_b - \epsilon_a = 3\hbar^2\pi^2/2mw^2$ and the interband coupling constant $d = 8/3w$. We choose $\epsilon_b - \epsilon_a = 16\epsilon_R$, which implies (i) $\alpha = (\sqrt{3}\pi/4)\hbar^2/mw = 3.45 \times 10^{-11}$ eVm (and $\epsilon_R \sim 0.39$ meV) for $m = 0.05m_0$ and $w = 60$ nm, (ii) $\varepsilon(k_c) = 24\epsilon_R [\varepsilon_F$ should be tuned to $\sim \varepsilon(k_c)]$, and
(iii) $k_c = 8\epsilon_R / \alpha$. Assuming $L = 69$ nm [Rashba region length, Fig. 1(a)], we find $\theta_R = \pi$ and $\theta_d = \theta_R d / k_c = \pi / 2$, since $d / k_c \sim 0.5$. This is a conservative estimate. In principle, $\theta_d$ can be varied independently of $\theta_R$ via lateral gates which alter $w$. Note also that $\Delta / 4k_F \sim 0.05$ [validity of Eq. (10)] for the above parameters. Finally, we note that the most relevant spin-flip mechanism (Dyakonov-Perel) should be suppressed in quasi-one-dimensional systems such as ours [11]. In addition, thermal effects are irrelevant in the experimentally feasible linear regime [12] we consider here.

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List of references


Figures

**FIG. 1**: Spin transistor geometry with a two-band channel. (a) The length $L$ of the Rashba region is smaller than the total length $L_0$ of the wire. (b) Sketch of energy dispersions in the s-o active region with and without interband coupling (Rashba bands) and away from it (parabolic bands). Note the small band offsets between adjacent regions in the wire.

**FIG. 2**: Band structure in the presence of spin-orbit coupling. In absence of interband mixing the Rashba dispersions are uncoupled (thin solid lines) and cross at, e.g., $k_c$. For non-zero interband coupling the bands anti cross (thick solid lines). The inset shows a blowup of the dispersion region near the crossing: the approximate solution [dotted lines, perturbative approach, Eq. (6)] describes well the energy dispersions near $k_c$. 
FIG. 3: Angular dependence of the spin-down conductance. The additional modulation $\theta_d$ due to s-o interband mixing and $\theta_R$ can be varied independently.
\[ G_\downarrow(e^2/h) \]