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Traits of Leaders in Movement Initiation: Classification and Identification

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1 Introduction

Leadership is one of the processes that both human and other social animal species use to solve complicated tasks to collectively achieve a goal [6, 8]. Understanding how leadership emerges and what behavioral mechanisms translate into leadership provides an insight into the complex organization of problem-solving and decision-making strategies in nature. In a movement context, leaders are individuals who successfully initiate movement, which the group then follows [6, 18]. In the leadership inference literature, there are many approaches for inferring leadership based on social network action-log data [9, 10], position-tracking information [2, 3, 13, 16], and others. There are also many approaches that define the traits of a "leader" a priori and extract data from individuals that fit the model's definition, such as influence maximization model [10, 11], implicit leadership model [6, 19], and flock model [3]. However, are these traits evident in real instances of leadership? Conversely, are the individuals defined by these traits indeed leaders? In this work, we propose an explicit framework for testing hypotheses about the behavioral traits of leaders by combining leader identification approaches with leadership characterization. We first identify leaders and then evaluate behavioral traits that purportedly characterize a leader. In the context of movement initiation, we focus on three behavioral traits commonly assumed to be associated with leadership of group movements: (1) being at the front of the group [3], (2) being the first to start

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moving [6], and (3) being the first to move in the new direction [4]. The framework is general enough to incorporate any set of traits as the set of hypotheses of leader characterization.

> LEADER TRAIT CHARACTERIZATION PROBLEM: **Given a time series of individual activities and target traits, the goal is to find a set of leaders during decision-making periods and a set of traits that best characterize these leaders.**

We propose a two-step approach for the LEADER TRAIT CHARACTERIZATION PROBLEM:

1. We find instances of leadership and identify leaders, using an agnostic and assumption-free leadership inference framework FLICA [1, 2];
2. We evaluate traits of interest for all identified leaders and perform hypothesis testing to infer which traits are significant.

In the following section, we describe the FLICA approach for leadership inference as well as the justification and the approach for testing particular behavioral traits—position, velocity, and directionality—to characterize leaders.

We demonstrate our approach using simulation datasets for sensitivity analysis and a publicly available position tracking dataset from a troop of wild olive baboons (*Papio anubis*) from Mpala Research Centre, Kenya [7, 17], as well as a fish school of golden shiners (*Notemigonus crysoleucas*), which is another publically available dataset [16]. Our results show that the framework is robust to noise in the classification task of trait models. In baboons, movement initiators are not the first to move but, instead, are the first to explore new areas and that the group quickly aligns itself with the direction of the leader's movement. On the contrary, in a school of fish, movement initiators are the first to move and the first to explore new areas before others, and the group then too aligns its direction with initiators quickly.

2 The Proposed Approach

As stated earlier, there are many aspects of leader identity that may be used as the defining traits of the leader: the individual may be the oldest, biggest, wisest, or loudest. However, here we focus on the *behavioral* aspects of successful leadership, particularly in movement initiation. Are the leaders the ones who move first, move in the new direction, stay at the front, etc.? These are aspects of leadership behavior that also are inferable from the spatiotemporal time series data directly. We use the notion of a convex hull of the variable of interest for the group versus an individual, particularly the leader individual.

2.1 Bidirectional Agreement in Multi-Agent Systems

The use of convex hull to analyze traits of a leader in this paper is motivated by the work on bidirectional agreement dynamics in multi-agent systems by Chazelle [5]. Chazelle showed that in a multi-agent system, the states of all individuals converge to a group consensus if each individual changes its state for each time step under what he calls the "bidirectional agreement condition." The bidirectional agreement condition constrains an individual's choice of the state at each time step within the convex hull of the states of its neighbors (in the arbitrary agent network) in the previous time step. Thus, to break the group consensus state, some individual must break the convex hull condition at some time point. In the collective movement context, a state of an individual at time t can be the individual's position, direction of movement, velocity, or acceleration.

Initially, all individuals' state is within the convex hull of the group's state. However, when the group initiates movement, the group changes its state from the initial state to unstable state. Leaders who initiate movement must break the convex hull of the group state to change the state of the group from one state to another state. After movement initiation, under "bidirectional agreement condition," the group converges to a stable state, and everyone stays within the convex hull of the group's state again unless the convex hull is breached. In other words, we hypothesize that leaders are the state changers who start breaking the convex hull before others, and we test that hypothesis. For example, suppose we define a state as a position of each individual. Initially, by definition, all individuals are within the convex hull of an individual's positions. When leaders initiate movement by leading at the front, they must step outside the convex hull of group's positions.

We can define individual states to be any variable directly derivable from the time series data, including individual positions, velocities, or directions. However, the question is which of these variables' convex hull of the group states that leaders actually break when they initiate movement that everyone follows. In this paper, we consider the type(s) of convex hull that leaders break as behavior traits, and we aim to infer these traits from time series data of group movement.

2.2 Bidirectional Agreement Condition

First, we start with the one-dimensional states of bidirectional agreement condition. At any time t, suppose $S_t = \{s_{1,t}, \ldots, s_{n,t}\}$ is a set of individual states at current time where $s_{i,t} \in \mathbb{R}$ is a one-dimensional state of individual i at time t, $m_{i,t}$ is a point in S_t that is closest to $s_{i,t}$ but has a value at most $s_{i,t}$, and $M_{i,t}$ is a point in S_t that is closest to $s_{i,t}$ but has a value at least $s_{i,t}$ and a constant $\rho \in (0, 1/2]$. The work by Chazelle [5] defines a bidirectional agreement condition as follows:

$$(1-\rho)m_{i,t} + \rho M_{i,t} \leq s_{i,t+1} \leq \rho m_{i,t} + (1-\rho)M_{i_t}. \tag{1}$$

The interpretation is that if all individuals change their state within the bound of their neighbor's states, the group will converge to a collective state, which is a stable state. In high-dimensional states, the bidirectional agreement condition still requires individuals to change their state from time to time within the bound of their neighbor's states, which is a convex hull of neighbor's states to make the group converge to a stable state. In other words, if all individual states always stay within their neighbor-state convex hull, then the group converges to a single point of collective state and stays there forever.

2.3 Leaders as State Changers

When leaders initiate a group movement, if leaders are state changers, then leaders are necessary to be the first who break the bidirectional agreement condition or step outside group's state convex hull. Figure 1 shows an example of state-changing situation in two-dimensional state. Suppose U is a state changer and a leader, while U initiates movement, U steps outside the group's state convex hull, which means U breaks the bidirectional agreement condition. In this paper, we infer whether leaders who initiate movement are state changers by observing the association between

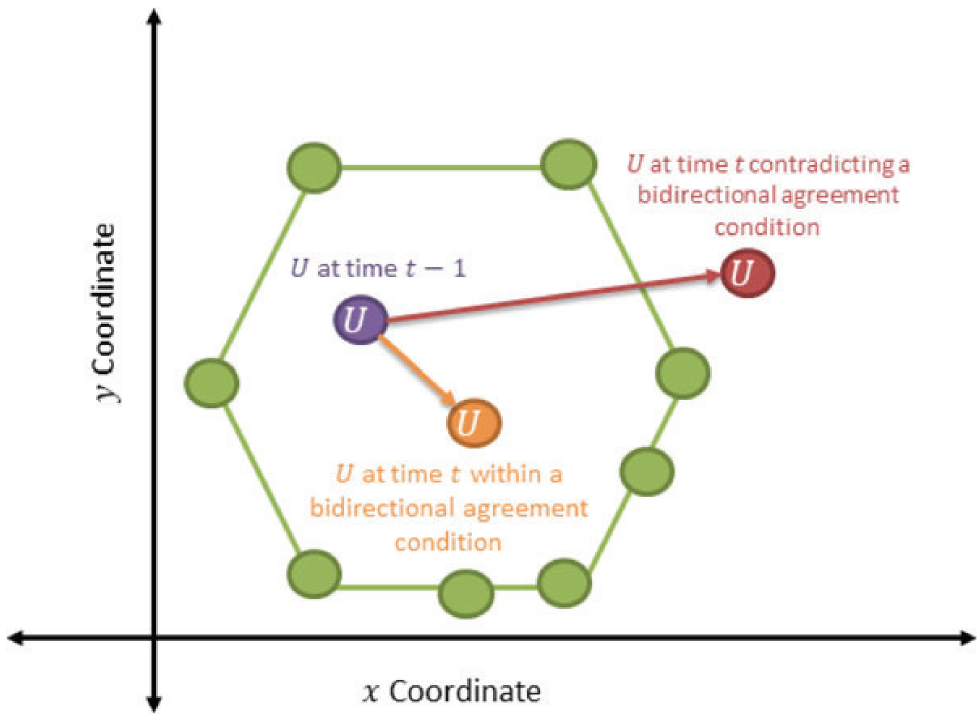

Fig. 1 An example of state-changing situation in the two-dimensional space. The green nodes are states of individuals at time $t - 1$, and the green polygon is a neighbor-state convex hull of individual U. If U changes its state under the bidirectional agreement condition, then the next state of U is always in the convex hull (orange). On the contrary, if U steps outside the convex hull (red) to make a group changes its state, then it breaks the bidirectional agreement condition

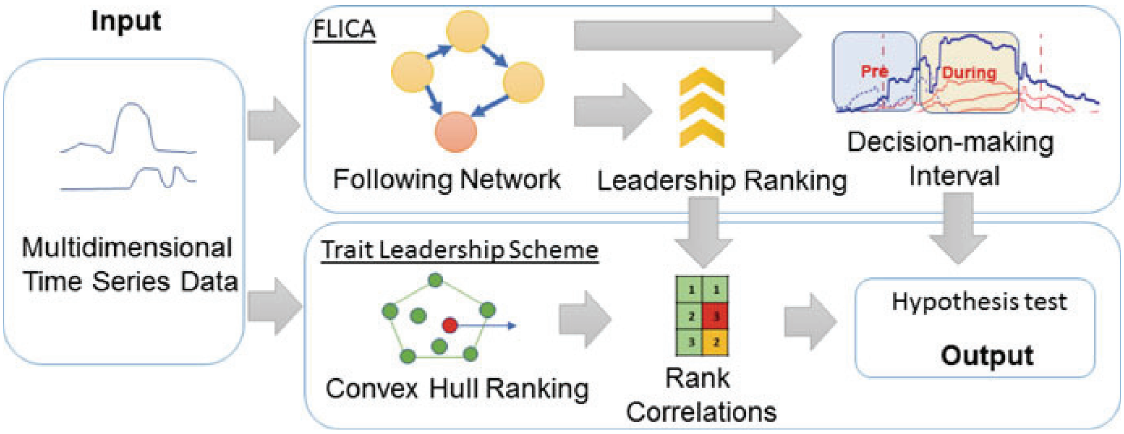

Fig. 2 High-level overview of trait leadership scheme using FLICA ([1, Figure 1] used and modified with permission). An arrow between elements represents a relationship that an element at the rear of the arrow is the input of an element at the head of the arrow. For example, rank correlations are calculated by taking leadership ranking and convex hull ranking as inputs

the time that individuals break the convex hull of states (velocity, position, and direction) compared to individual leadership ranking.

2.4 Approach Overview

The high-level overview of the proposed leadership trait characterization scheme using FLICA [1, 2] is shown in Fig. 2. Given a time series of GPS positions of baboons, we use FLICA to report a dynamic following network, leadership ranking, and decision-making intervals. Then, in this work, we propose measures of velocity, position, and direction convex hull containment as traits of leadership and conduct the experiments to find any significant positive/negative correlations between leadership ranking and those measures.

Let $D = \{Q_1, \ldots, Q_n\}$ be a set of time series of positions where D consists of n time series where each $Q_i \in D$ has length T (the number of time steps) and each Q_{it} is a position coordinate of the individual i at time t.

2.5 FLICA

Construction of the Network of Following Relations To infer the following relations between time series, dynamic time warping (DTW) [15] was deployed to measure the similarity between the shape (not the exact position) and the shift in the trajectories. Figure 3 demonstrates an example of following relation inference between time series Q and U. In this figure, U follows Q which has time delay Δt. By considering an optimal warping path within DTW, we can infer a following

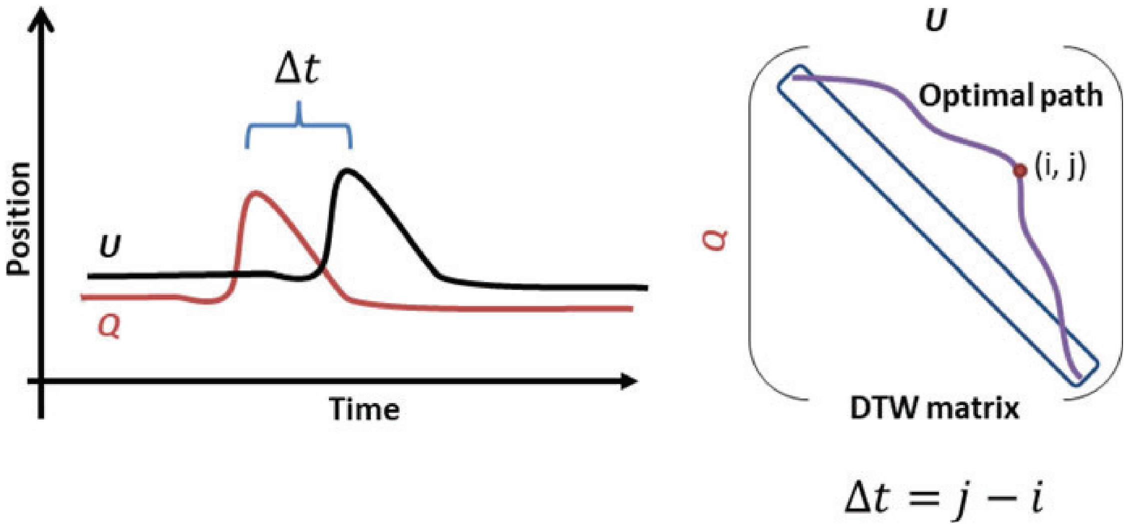

Fig. 3 An example of a following relation between time series U and Q. (Left) Time series U follows Q. (Right) The optimal warping path on the DTW dynamic programming matrix, shifting U backward in time onto Q

relation between time series. For each pair of time series, we calculate an average of differences between indices within an optimal warping path of DTW to extract the average time delay between time series. If a time delay is positive, then U follows Q, negative if Q follows U, and zero if neither U nor Q follows each other.

To construct a dynamic following network, we split time series into subintervals to infer following networks, and then we combine following networks from these subintervals to be a single dynamic following network. Let ω be a time window that defines a subinterval and $\delta = 0.1\omega$ be a sliding window parameter. Let a kth window be an interval given by $w(k) = [k \times \delta, k \times \delta + \omega]$. A following network $G_k = (V, E_k)$ at time interval $w(k)$ is a directed graph where V is a set of time series nodes (that do not change), which has one-to-one mapping to each individual time series, and E_k is a set of following edges. If $e(U, Q) \in E_k$, then a time series U follows Q at time interval $w(k)$. FLICA computes the networks of following relations from $w(0)$ to $w(m)$ to cover the entire time series intervals T, and then it reports the entire dynamic following network.

Leadership Ranking To rank a degree of leadership, given a following network, FLICA uses PageRank [14]. An individual with a high PageRank score has many followers and is followed by individuals who themselves have many followers, which matches the intuitive notion of leadership. We prefer PageRank rather than any other centrality measure, such as degree or betweenness, because PageRank captures the transitive nature of following relations and the end-to-end aspect of the individual to leader path. Leaders are nodes that have a high number of reachable paths to them in a following network. Ultimately, leaders are the nodes to whom *everyone* has a reachable path. Hence PageRank is the most appropriate measure in our leadership setting.

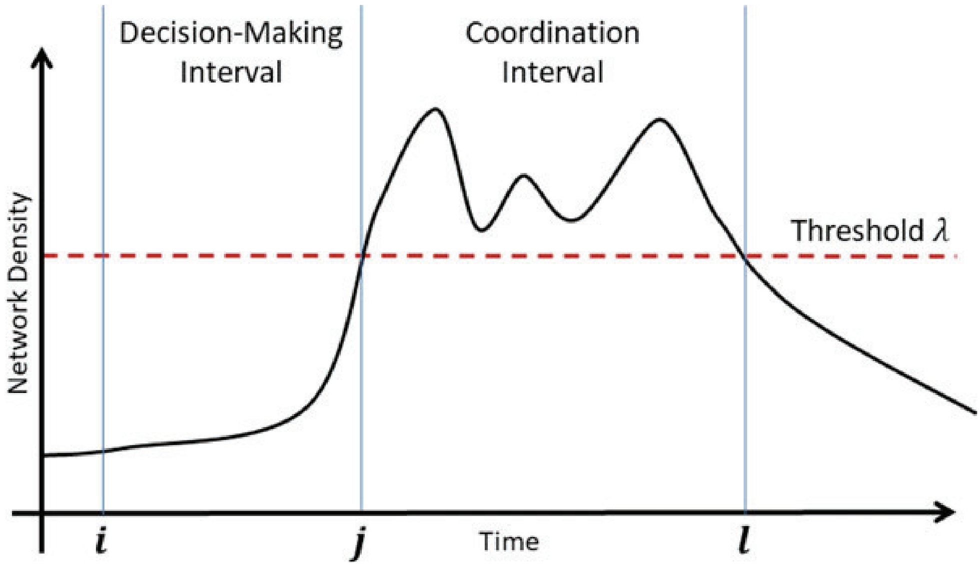

Fig. 4 An example of a coordination event identified based on a network density time series ([1, Figure 4] used and modified with permission). Based on a threshold λ, we identify a decision-making interval, followed by a coordination interval

Decision-Making Interval Detection After constructing a dynamic following network, FLICA uses network density as the measure of the level of coordination within the group to extract intervals of coordinated activity and the decision-making period that preceded it. Let λ be a decision-making threshold parameter. We set a threshold of network density at a percentile λth for the values of dynamic following network densities to separate between two types of intervals: coordination events and non-coordination interval. Figure 4 shows the example of coordination event that has its peak greater than λ. In other words, a coordination event represents an interval which has a high number of following edges. The intervals outside coordination events are non-coordination intervals. The interval directly preceding the coordination event is referred to as a decision-making interval; FLICA reports the initiator of each of these coordinated events. Now, for each instance of coordination and the resulting of significantly following in the group, FLICA reports the ranks of all the individuals during the initiation of that event. We use that ranking to evaluate the corresponding ranking by the proposed leadership traits.

2.6 *Leadership Trait Characterization Scheme*

The Quantification of the Traits of Interests We focus on three common characterizations of a leader: being the first to move, being at the front of a group, and being the first to move in the new direction. We use the notion of the convex hull to measure the similarity of the trait value for an individual versus the group as a whole.

First, to measure the notion of being the *first to move*, we need to consider the velocity of all individuals at the previous time step. If any individual moves before others, its velocity is higher than others' velocity at the previous time step. That is, it is higher than the maximum previous velocity of any individual, or, to put it in other words, it is outside of the convex hull of the velocities in the previous time step in the positive direction (since velocity is a one-dimensional measure).

Second, to measure the notion of being at the *front of the group*, we need to consider both direction of individuals and their positions. If any individual moves toward the front of the group, then its direction of movement is the same as the group's direction, but its position is outside the group's area of the previous time step. That is, the coordinates of the individual at the front of the group are outside of the convex hull of the coordinates of the individuals in the previous time step but aligned with the direction vector of the group.

Third, to measure the notion of being the *first to move in the new direction*, we need to consider direction vectors of all the individuals. If any individual moves in the new direction, which is not the same as the group's direction, then the angle between its current direction vector and the group's direction vector at the previous time step must be high. That is, the current (angle of the) direction vector of the individual is outside the convex hull of the direction vectors of the individuals in the previous time step.

Convex Hull Ranking Measures For each of the three measures, we construct the convex hull in each time step and rank the individuals by the frequency with which their value in the current step is outside the convex hull of the values of all the individuals in the previous step for the same measure.

The velocity convex hull (VCH) ranking score measures the frequency with which the discrete time series derivative (dQ_i/dt) associated with an individual i is outside the bounds of the population's (including i) discrete derivative interval in the previous time step. The highest rank of this measure indicates an individual who is the first to move in the group. Let $n \times T$ matrix VCH be a velocity convex hull score matrix where $\text{VCH}(Q_i, t) = 1$ if a time series Q_i at time t has its velocity greater than a maximum velocity of the entire group at time $t - 1$ and $\text{VCH}(Q_i, t) = -1$ if Q_i has its velocity less than a minimum velocity of the entire group at time $t - 1$, and otherwise $\text{VCH}(Q_i, t) = 0$.

The position convex hull (PCH) ranking score measures the frequency with which a position and direction associated with an individual i are outside the bounds of the population's position convex hull in the previous time step. A high rank of this measure indicates which individual first explores a new area before others. Let $n \times T$ matrix PCH be a position convex hull score matrix where $\text{PCH}(Q_i, t) = 1$ if a time series Q_i at time t has its direction toward the group's direction and i's position at time t is outside the group's position convex hull at time $t - 1$ (see Fig. 5). In contrast, if Q_i is outside the convex hull but moving in the opposite way of group direction, then $\text{PCH}(Q_i, t) = -1$ (see Fig. 5); otherwise $\text{PCH}(Q_i, t) = 0$. We consider that i is moving toward the group direction if the angle between i's

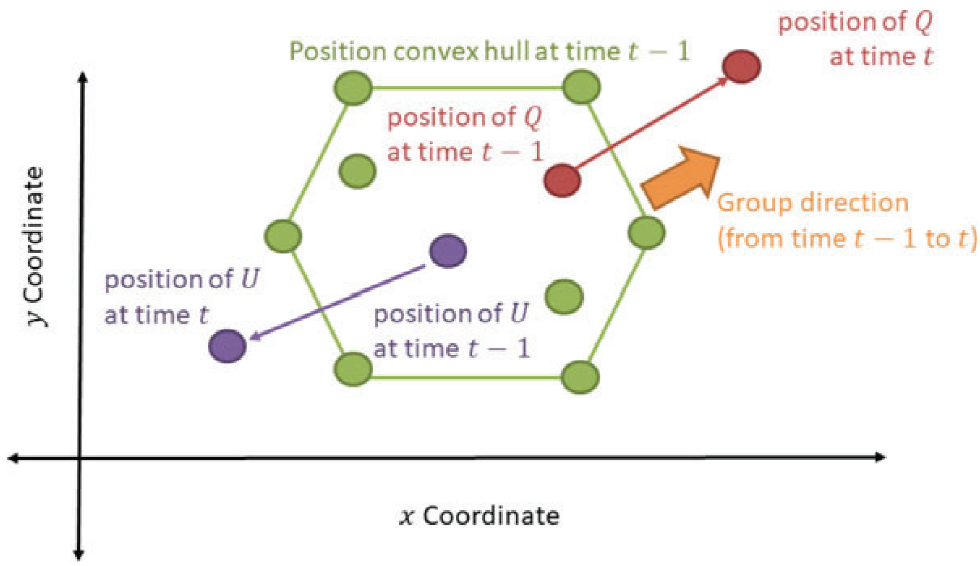

Fig. 5 An example of position convex hull. Each point represents an individual, and the polygon represents a convex hull boundary at time $t-1$. In this example, Q steps outside the convex hull at time t toward the group direction, while U steps outside the convex hull in the opposite direction. In this case, Q gets a score +1, and U gets a score -1 for time step t. If an individual is still in the convex hull, it gets zero score

direction vector and group's direction vector is between $-90°$ and $90°$. Otherwise, we consider that i is moving in the opposite direction from its group movement.

The direction convex hull (DCH) ranking score measures the frequency with which the angle between individual's direction vector and group's direction vector is outside the bound of the set of angles between each individual and the group's direction vector in the previous time step. A high rank of this measure indicates an individual that frequently deviates from the group's direction of travel. Let $n \times T$ matrix DCH be a direction convex hull score matrix where $\mathrm{DCH}(Q_i, t) = 1$ if a time series Q_i at time t has its individual-group direction angle greater than a maximum individual-group direction angle of the entire group at time $t-1$ and $\mathrm{DCH}(Q_i, t) = -1$ if i has its individual-group direction angle lower than a minimum individual-group direction angle of the entire group at time $t-1$, and otherwise $\mathrm{DCH}(Q_i, t) = 0$.

Rank Correlation We deploy the Kendall rank correlation coefficient $\tau(x, y)$ [12] to infer correlation between PageRank leadership ranking (see paragraph "Leadership Ranking") and the convex hull ranking.

Further, for any given threshold λ used to determine coordination events, we focus on only decision-making intervals to measure the rank correlations. We define two levels of analysis: **time-point level** and **interval level**. First, for a **time-point-level** correlation, we compute a rank correlation for each time t within any decision-making interval as follows.

Let $R_{\mathrm{PR},t} = \mathrm{argsort}(\mathrm{PR}(:, t))$ be a PageRank rank ordered list at time t such that $R(Q_i)_{\mathrm{PR},t} = q$ if an individual i is at qth rank at time t and $R(Q_i)_{\mathrm{PR},t} = 1$ if i is a leader at time t. Note that we always use argsort to represent the descending sort order for the entire paper since the higher score implies the better rank.

$R_{\mathrm{VCH},t} = \mathrm{argsort}(\mathrm{VCH}(:, t))$, $R_{\mathrm{PCH},t} = \mathrm{argsort}(\mathrm{PCH}(:, t))$, and $R_{\mathrm{DCH},t} = \mathrm{argsort}(\mathrm{DCH}(:, t))$ are velocity, position, and direction convex hull rank order lists, respectively. A rank correlation between PageRank and VCH is $\tau(R_{\mathrm{PR},t}, R_{\mathrm{VCH},t})$. A rank correlation between PageRank and PCH is $\tau(R_{\mathrm{PR},t}, R_{\mathrm{PCH},t})$. And a rank correlation between PageRank and DCH is $\tau(R_{\mathrm{PR},t}, R_{\mathrm{DCH},t})$. We define a set of time-point PageRank-VCH correlations as follows:

$$\Phi_{\mathrm{PR,VCH}} = \{\tau(R_{\mathrm{PR},t_1}, R_{\mathrm{VCH},t_1}), \tau(R_{\mathrm{PR},t_2}, R_{\mathrm{VCH},t_2}), \ldots\} \tag{2}$$

where t_i is a time point within any decision-making interval. Similarly, we can also define a set of time-point PageRank-PCH correlations and PageRank-DCH correlations in the similar way.

$$\Phi_{\mathrm{PR,PCH}} = \{\tau(R_{\mathrm{PR},t_1}, R_{\mathrm{PCH},t_1}), \tau(R_{\mathrm{PR},t_2}, R_{\mathrm{PCH},t_2}), \ldots\} \tag{3}$$

$$\Phi_{\mathrm{PR,DCH}} = \{\tau(R_{\mathrm{PR},t_1}, R_{\mathrm{DCH},t_1}), \tau(R_{\mathrm{PR},t_2}, R_{\mathrm{DCH},t_2}), \ldots\} \tag{4}$$

Second, for an **interval-level** rank correlation, we compute the representative correlation of the entire decision-making interval for each coordination event. Let $I = (i, j, l)$ be any coordination event, and we define $\tilde{R}_{\mathrm{PR},I} = \mathrm{argsort}(\sum_{t\in[i,j]} R_{\mathrm{PR},t})$ as a PageRank rank ordered list during decision-making interval of coordination event I. $\tilde{R}_{\mathrm{VCH},I} = \mathrm{argsort}(\sum_{t\in[i,j]} R_{\mathrm{VCH},t})$, $\tilde{R}_{\mathrm{PCH},I} = \mathrm{argsort}(\sum_{t\in[i,j]} R_{\mathrm{PCH},t})$, and $\tilde{R}_{\mathrm{DCH},I} = \mathrm{argsort}(\sum_{t\in[i,j]} R_{\mathrm{DCH},t})$ are defined to be VCH, PCH, and DCH rank ordered lists of I, respectively. The PageRank-VCH rank correlation at decision-making interval of I is $\tau(\tilde{R}_{\mathrm{PR},I}, \tilde{R}_{\mathrm{VCH},I})$, the PageRank-PCH rank correlation is $\tau(\tilde{R}_{\mathrm{PR},I}, \tilde{R}_{\mathrm{PCH},I})$, and the PageRank-DCH rank correlation is $\tau(\tilde{R}_{\mathrm{PR},I}, \tilde{R}_{\mathrm{DCH},I})$. We define a set of interval PageRank-VCH correlations as follows:

$$\tilde{\Phi}_{\mathrm{PR,VCH}} = \{\tau(\tilde{R}_{\mathrm{PR},I_1}, \tilde{R}_{\mathrm{VCH},I_1}), \tau(\tilde{R}_{\mathrm{PR},I_2}, \tilde{R}_{\mathrm{VCH},I_2}), \ldots\} \tag{5}$$

where I_i is an ith coordination event. We can also define PageRank-PCH correlations and PageRank-DCH correlations in the similar way.

$$\tilde{\Phi}_{\mathrm{PR,PCH}} = \{\tau(\tilde{R}_{\mathrm{PR},I_1}, \tilde{R}_{\mathrm{PCH},I_1}), \tau(\tilde{R}_{\mathrm{PR},I_2}, \tilde{R}_{\mathrm{PCH},I_2}), \ldots\} \tag{6}$$

$$\tilde{\Phi}_{\mathrm{PR,DCH}} = \{\tau(\tilde{R}_{\mathrm{PR},I_1}, \tilde{R}_{\mathrm{DCH},I_1}), \tau(\tilde{R}_{\mathrm{PR},I_2}, \tilde{R}_{\mathrm{DCH},I_2}), \ldots\} \tag{7}$$

Leadership Model Classification Using Trait-Rank Correlation For model classification, we use three interval-level rank correlations from paragraph "Rank

Correlation” as features to train a classifier. For each dataset, our framework provides a vector of features $\boldsymbol{v} = (\tau(\tilde{R}_{\mathrm{PR}}, \tilde{R}_{\mathrm{VCH}}), \tau(\tilde{R}_{\mathrm{PR}}, \tilde{R}_{\mathrm{PCH}}), \tau(\tilde{R}_{\mathrm{PR}}, \tilde{R}_{\mathrm{DCH}}))$, which represents trait characteristic of leadership model. We use multiclass support vector machine (SVM) as our main classifier.

3 Experimental Setup

3.1 Trait of Leadership Model

In this section, we provide three different models of trait leadership. We use these models to demonstrate that our rank correlations in paragraph “Rank Correlation” can be used as features to classify these models, which have different traits of leadership. All these models are in two-dimensional space. Initially, there are 20 individuals within a unit cycle. Positions of individuals are uniformly distributed within this unit cycle. Then the group moves toward a collective target.

Moving First Model In this model, high-rank individuals move earlier than low-rank individuals. A leader moves toward target trajectory, and everyone follows its hierarchy. We have ID(1) as a leader. ID(k) moves first; then it is followed by ID($k + 1$) with a constant time delay. The acceleration of movement for all individuals is constant. We aim to use this model as a representative model that high-rank individuals always move earlier than low-rank individuals. For this model, we set the initial velocity at 1 unit/time step and acceleration at 0.001 unit/time step^2.

Moving Front Model This model also has an ordered hierarchy of following the same as the previous model. Nevertheless, there is no order of movement initiation. In other words, all individuals have uniformly time delay before they start moving. The group moves along a target trajectory with a constant velocity, and a leader is always in the front of the group followed by high-rank individuals. Lower-rank individuals follow higher-rank individuals. We aim to use this model as a representative model that high-rank individuals always explore new areas before low-rank individuals. For this model, we set the initial velocity at 1 unit/time step and acceleration at 0 unit/time step^2.

Reversible Agreement Model Compared to previous models, this model has no leader and any following hierarchy. All individuals move toward the average of group’s direction with a constant velocity. This model is one of the bidirectional agreement systems that have convergence property [5]. In our case, all individual’s directions converge to an average group direction, which implies the existence of coordinated movement of the group. We aim to use this model as a representative model that the group has coordinated movement without leadership hierarchy. We expect that any leadership model classification should be able to at least distinguish between leadership models and this non-leadership model. For this model, we set the initial velocity at 1 unit/time step and acceleration at 0 unit/time step^2.

3.2 Datasets

Simulation Datasets for Sensitivity Analysis We create simulation datasets with the difference level of noises. We have two types of noise here: direction noise and position noise. For direction noise, instead of moving to a target direction at degree D, an individual moves toward direction $D + a$. The direction noise a is drawn randomly from normal distribution with zero mean and γ standard deviation where $\gamma \in \{0, 1, 10, 30, 60\}$. For position noise, suppose (x, y) is the next position that an individual should move to, with position noises, the actual position that the individual moves is $(x + b_1, y + b_2)$. The position noises b_1, b_2 are drawn randomly from a normal distribution with zero mean and β standard deviation where $\beta \in \{0.0001, 0.001, 0.01, 0.1, 1\}$.

For each noise setting (γ, β), we create 100 for each trait of leadership model. Each dataset contains 20 time series of individuals, which have the length as 300 time steps. In total, since we have three leadership models and 25 possible different (γ, β), we have 7500 datasets.

Simulation Datasets for Degree of Hierarchy Structure Analysis We use simulated datasets that can be found in [2]. There are three leadership models we use in this paper: dictatorship, hierarchical model, and random model. Each model consists of 100 datasets. Each dataset has two-dimensional time series of 20 individuals. Each time series has its length at 12,000 time steps. There are 20 coordination events within each dataset.

Initially, all individuals are at their starting point. In dictatorship model (DM), a leader moves first, and then everyone else follows its leader with some time delay. In hierarchical model (HM), there are four high-rank individuals, ID1, ID2, ID3, and ID4. Other non-high-rank individuals get assigned by their leaders to be one of the high-rank individuals. ID1 is a global leader of all high-rank individuals that always moves first. ID2 and ID3 follow ID1 with some time delay. Then ID4 follows ID1. Lastly, the followers of ID1, ID2, ID3, and ID4 follow their leaders.

For the random model, all individuals move together toward a target direction. However, these individuals never follow any specific individuals. Hence, there are coordination events in this model, but there are no leaders.

Baboon Dataset The baboon dataset is a publically available dataset that contains GPS tracking information from 26 members of an olive baboon (*Papio anubis*) troop recorded from 6 a.m. to 6 p.m. between August 01, 2012 and August 10, 2012. These baboons live in the wild at the Mpala Research Centre, Kenya [7, 17]. For each individual, the GPS collar recorded its latitude and longitude position for every second. Then these latitude and longitude time series were converted to be the $X - Y$ coordinate trajectories.

We preprocessed the GPS data, removing some individuals whose GPS collars were active only in a short period of time. The final dataset consists of 16 individuals, each of whose trajectory has a length of 419,095 time steps. The composition of the baboon group includes individuals who vary in sex (male and female) and age (juvenile, subadult, and adult). This dataset includes a variety of baboon activities, including sleeping, foraging, traveling, and resting.

Fish Dataset The fish school dataset of golden shiners (*Notemigonus crysoleucas*) is another publically available dataset. The two-dimensional fish movement trajectories are recorded by video in order to study information propagation over the visual fields of fish [16]. The number of individuals within each population is 70 individuals, but only 10 individuals are labeled with a trained class. A trained fish is able to lead the school to feeding sites. The dataset contains 24 coordination events. The fish trajectories have their length between 550 and 600 time steps. Our task is to identify the traits of trained fish.

3.3 Sensitivity Analysis in Model Classification

We separate simulation dataset into the groups based on the value of noise setting (γ, β). For each group, it consists of 100 datasets of moving first model, 100 datasets of moving front model, and 100 datasets of reversible agreement model. We report tenfold cross-validation of model classification for each group of datasets having the same noise level (γ, β). We also report the rank correlation between the ground-truth leadership rank and inferred leadership rank from our framework to measure the ability of leadership inference within difference level of noises.

3.4 Hypotheses Tests

In this section, we aim to design a hypothesis testing scheme to address three hypotheses: (1) individuals who act as leaders (identified by FLICA) are individuals who move first, initiating their movements before others in their group in the decision-making period prior to coordinated movement, (2) individuals who act as leaders are individuals who always explore a new area before others prior to a coordinated movement, and (3) individuals who act as leaders are individuals who always align with the group direction. We define leaders as individuals who possess a highly ranked position in a PageRank rank ordered list. The hypothesis testing methods we used can be categorized into two categories: zero mean/median test and normality test. For the zero mean/median hypothesis test, we aim to test whether a positive/negative correlation exists between PageRank and convex hull ranking in both time-point and interval levels. For normality tests, we aimed to determine whether correlation samples come from a normal distribution. If not, the interpretation of tests which assume normally distributed data, e.g., t-test, should be considered carefully. The full list of hypothesis testing methods we used is in Table 1. We set significance level at $\alpha = 0.001$ for all tests.

Table 1 Description of hypothesis tests used in this paper

	Method	Null hypothesis H_0
zero mean/median hypothesis test	t-Test	A sample has a normal distribution with zero mean and unknown variance
	Sign test	A sample has a distribution with zero median
	Wilcoxon signed-rank test	A sample has a symmetric distribution around zero median
Normality test	Kolmogorov-Smirnov test	A sample comes from a normal distribution
	Chi-square goodness-of-fit test	A sample comes from a normal distribution with a mean and variance estimated from a sample itself
	Jarque-Bera test	A sample comes from a normal distribution with an unknown mean and variance
	Anderson-Darling test	A sample comes from a normal distribution

A significance level has been set at $\alpha = 0.001$ for all experiments

3.5 Parameter Setting

For simulation datasets, we set the time window $\omega = 60$ and $\delta = 6$, which is the optimal setting since the simulation dataset has time delay less than 5 time steps by design. For the analysis in baboon dataset, we set the time window $\omega = 240$ and $\delta = 24$. For the fish datasets, we set the time window $\omega = 285$ and $\delta = 28$. Both parameter settings of baboon and fish datasets are set based on the fact that these settings can infer the highest number of following relations per following group on average. The time sliding window parameter δ serves to trade off computation versus the sampling rate of the time series process. The FLICA has time complexity $\mathcal{O}(n^2 \times t \times \omega)$. The network density decision-making threshold λ was set at 25th, 50th, 75th, and 99th percentile of network density values for the baboon dataset to detect decision-making intervals. For the simulation and fish datasets, we already have the decision-making intervals, so we do not need to set λ.

4 Results

4.1 Traits of Leader Classification: Sensitivity Analysis

Figure 6 shows the result of sensitivity analysis in the model classification. Loss values of tenfold cross-validation are shown in the figure at the top. A loss value is a percentage of datasets that the classifier predicted them into wrong classes. Figure 6 (top) shows that when the level of noises increases, classifier produces more errors. According to the cross-validation result, our framework can distinguish between

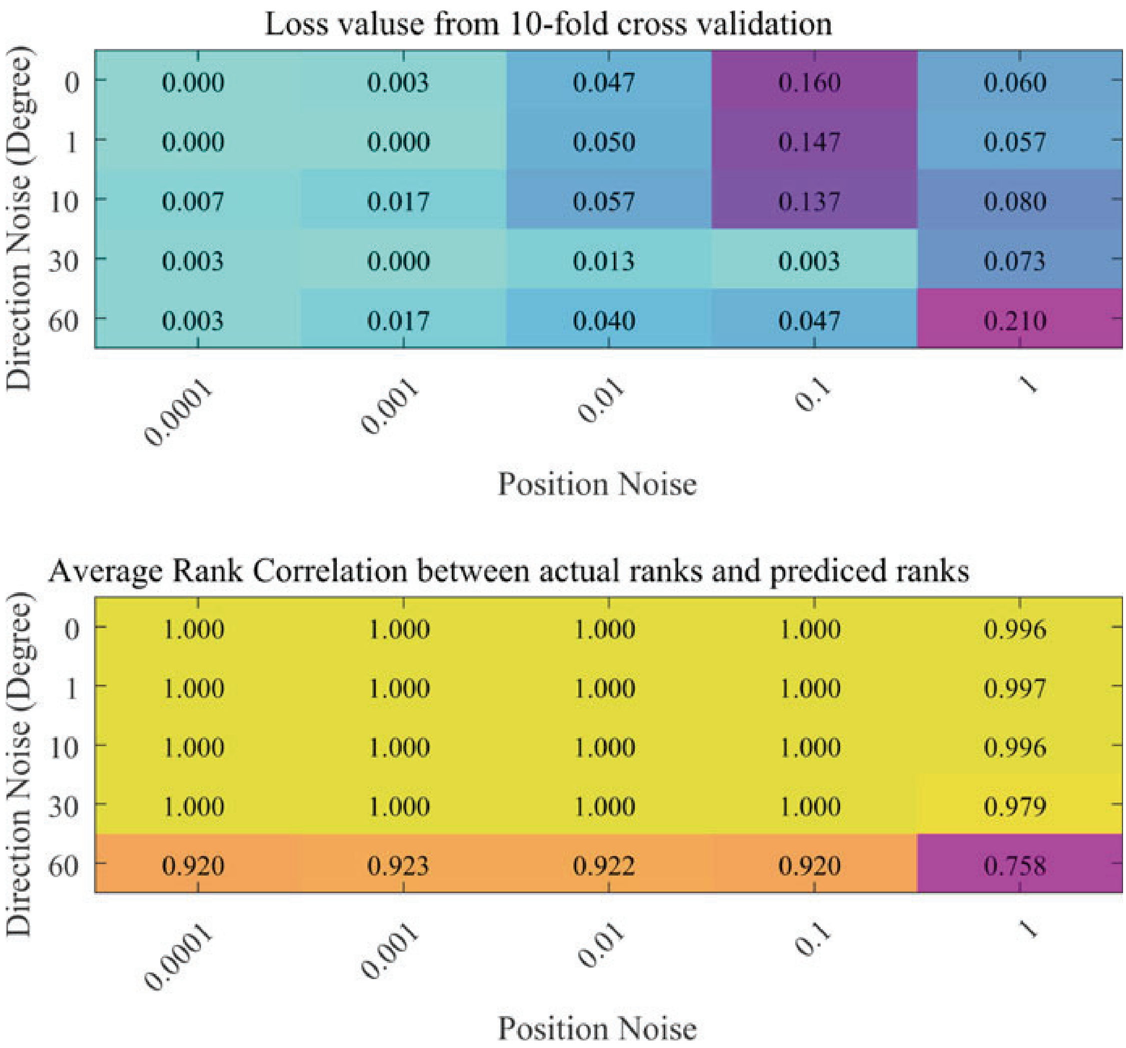

Fig. 6 Sensitivity analysis in model classification task from simulation datasets with different noise levels. (Top) Tenfold cross-validation loss values. Each element in the table represents the loss value of each noise setting (γ, β). (Bottom) Rank correlation between actual leadership ranking and predicted ranking from moving first and moving front models

leadership models (moving first and moving front models) and non-leadership model (reversible agreement model). Additionally, the result suggests that position noise affected the classification result than direction noise. When the position noise level reach at 1, which is the diameter of group movement, the leadership rank is less consistent with the ground-truth rank (Fig. 6 bottom). This indicates that both leadership ranking and traits of leadership inference are hard to perform under high-level of position noise. In general, this result shows that our framework performs accurately even if an input data is noisy until a certain degree of noises.

4.2 Trait Identification of Baboon Movem

The distributions of rank correlations inferred from the baboon dataset are in Fig. 7. At the time-point level, the distribution of PageRank and velocity convex hull (PR-VCH) correlation $\Phi_{\mathrm{PR,VCH}}$ is at the top left of the figure, the distribution of PageRank and position convex hull (PR-PCH) correlation $\Phi_{\mathrm{PR,PCH}}$ is at the top middle, and the distribution of PageRank and direction convex hull (PR-DCH) $\Phi_{\mathrm{PR,DCH}}$ is at the top right of the figure. For the interval-level correlations, the distribution $\tilde{\Phi}_{\mathrm{PR,VCH}}$ is at the bottom left, the distribution $\tilde{\Phi}_{\mathrm{PR,PCH}}$ is at the bottom middle, and the distribution $\tilde{\Phi}_{\mathrm{PR,DCH}}$ is at the bottom right. Table 2 illustrates the means and standard deviations of these correlation distributions.

Figure 7 and Table 2 suggest that there is no correlation between PageRank and velocity convex hull ranking at both the time-point level (Fig. 7's top left) and the interval level (Fig. 7's bottom left). In contrast, positive correlations exist between PageRank and position convex hull in both levels (Fig. 7, top and bottom middle). Moreover, negative correlations exist between PageRank and direction convex hull in interval level (Fig. 7, bottom right). When we set a higher percentile threshold, we get stronger coordination events; a stronger coordination event has a higher number of following relations. Both Fig. 7 and Table 2 illustrate that the rank correlations between PageRank and position convex hull ranking are higher, while the rank correlations between PageRank and direction convex hull ranking are lower

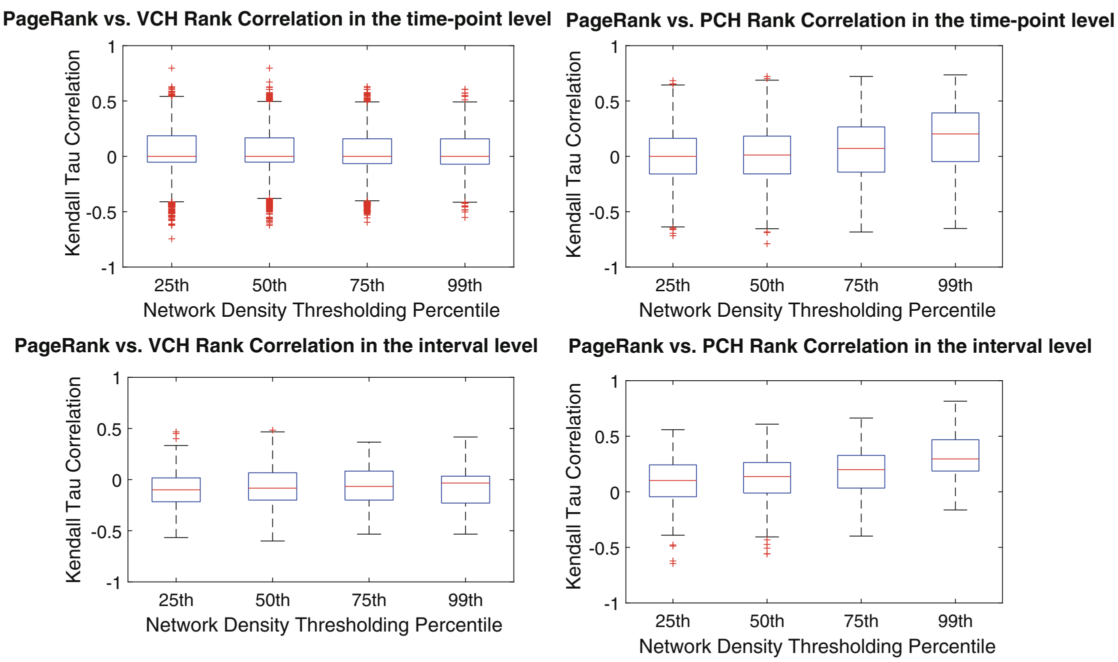

Fig. 7 Comparison of PR-VCH, PR-PCH, and PR-DCH rank correlations under different thresholds. For PR-VCH correlation, the results in both time-point level (top left) and interval level (bottom left) show that there are no strong correlations between leadership and VCH ranking. In contrast, leadership and PCH rankings have positive correlations in both time-point level (top middle) and interval level (bottom middle), as well as PR-DCH correlation which has a negative correlation at the interval level (bottom right)

Table 2 PR-VCH, PR-PCH, and PR-DCH rank correlations from the baboon dataset under different thresholds

		Time-point level		Interval level	
	Percentile	Mean	STD	Mean	STD
PR-VCH corr.	25th	0.03	0.20	−0.09	0.18
	50th	0.03	0.19	−0.07	0.18
	75th	0.03	0.19	−0.06	0.19
	99th	0.03	0.19	−0.07	0.21
PR-PCH corr.	25th	0.00	0.23	0.09	0.20
	50th	0.01	0.25	**0.12**	0.21
	75th	0.06	0.27	**0.18**	0.22
	99th	**0.15**	0.30	**0.32**	0.22
PR-DCH corr.	25th	−0.0082	0.1739	0.03	0.22
	50th	−0.0098	0.1745	−0.01	0.23
	75th	−0.0251	0.1818	−0.09	0.25
	99th	−0.0552	0.1827	**−0.24**	0.24

when we set a stronger threshold. However, there is not a large difference in the correlations of PageRank and velocity convex hull when we varied the threshold value.

Based on this result, due to weak correlations of PR-VCH in both time-point and interval levels and PR-DCH in the time-point level, we decided to conduct the hypothesis tests only in the PR-PCH rank correlation samples in both levels while conducting the hypothesis tests for PR-DCH in the interval level.

The result of these hypotheses tests are shown at Table 3. In the aspect of normality test results, correlations at time-point level of PCH are less normal compared to the PCH's and DCH's correlations at the interval level. This implies that the result of t-test at the PCH's time-point level should be interpreted carefully.

In the aspect of zero mean/median hypothesis test, with the significance level at $\alpha = 0.001$, PageRank and position convex hull ranking have positive correlations far from 0. This implies that individuals who act as leaders tend to explore new areas before other individuals during decision-making intervals.

In contrast, PageRank and direction convex hull ranking have negative correlations far from 0 at the 75th and 99th percentile thresholds. This implies that individuals who act as leaders tend to align with the group's direction (or, more intuitively, the group is aligned with the leader's direction), while non-leaders frequently attempt to change the direction, but nobody follows. In other words, high-rank individuals control the group direction, and this is why they are almost always inside the direction convex hull. When high-rank individuals move in any given direction, the group follows almost immediately, and this makes the group's direction the same as the leading individuals' direction.

Finally, for interval-level correlations of PR-PCH and PR-DCH ranking, we reported the normal confidence intervals at Table 4. We only reported the confidence intervals of interval-level correlations because of the normality test results; time-

Table 3 Hypothesis test results of PR-PCH and PR-DCH correlation

	Tests/percentile THS	PR-PCH corr. at a time-point level				PR-PCH corr. at an interval level				PR-DCH corr. at an interval level			
		25th	50th	75th	99th	25th	50th	75th	99th	25th	50th	75th	99th
Zero mean/ median test	t-Test	0	1	1	1	1	1	1	1	0	0	1	1
	Sign test	0	1	1	1	1	1	1	1	0	0	1	1
	Wilcoxon signed-rank test	0	1	1	1	1	1	1	1	0	0	1	1
Normality test	Kolmogorov-Smirnov test	1	1	1	1	1	1	1	1	1	1	1	1
	Chi-square test	1	1	1	1	0	0	0	0	0	0	0	0
	Jarque-Bera test	1	1	1	1	0	0	0	0	0	0	0	0
	Anderson-Darling test	1	1	1	1	0	0	0	0	0	0	0	0

The zero value implies that a test fails to reject H_0, while one implies a test successfully rejects H_0 with $\alpha = 0.001$

Table 4 Normal confidence intervals of PR-PCH and PR-DCH correlations from the baboon dataset at the interval level with $\alpha = 0.001$

	Percentile threshold	Normal confidence interval			
		Mean μ		STD	
		Lower bound	Upper bound	Lower bound	Upper bound
PR-PCH	25th	0.06	0.13	0.18	0.23
	50th	0.08	0.16	0.18	0.23
	75th	0.13	0.23	0.19	0.25
	99th	0.19	0.44	0.16	0.35
PR-DCH	25th	−0.01	0.07	0.20	0.25
	50th	−0.05	0.03	0.21	0.27
	75th	−0.14	−0.04	0.21	0.29
	99th	−0.38	−0.11	0.17	0.37

point level of PR-PCH correlation distributions seems not to be normal (see Table 3), while the rest of the cases are normal. All normal confidence intervals of PR-PCH correlation distributions have their lower bound greater than 0, while the upper bounds of PR-DCH correlation at the 75th and 99th percentile thresholds are below 0. This supports the hypotheses that there exist a positive correlation between PageRank and PCH ranking and a negative correlation between PageRank and DCH at the interval level.

4.3 Trait Identification of Fish Movement

The distributions of rank correlations inferred from the fish dataset are in Fig. 8. At the time-point level, the distributions of $\Phi_{\mathrm{PR,VCH}}$, $\Phi_{\mathrm{PR,PCH}}$, and $\Phi_{\mathrm{PR,DCH}}$ are at the left of the figure, while the right of the figure contains rank correlations at the interval level. Table 5 illustrates the means and standard deviations of these correlation distributions.

At the time-point level, Fig. 8 and Table 5 suggest that there is no correlation between PageRank vs. VCH ranking and PageRank vs. DCH ranking, while we have positive rank correlations of PageRank vs. PCH on average. At the interval level, $\tilde{\Phi}_{\mathrm{PR,VCH}}$ and $\tilde{\Phi}_{\mathrm{PR,PCH}}$ have positive values on average, while $\tilde{\Phi}_{\mathrm{PR,DCH}}$ has negative values on average.

Based on this result, due to the weak correlations of PR-VCH and PR-DCH in a time-point level, we decided to conduct the hypothesis tests only in the PR-PCH rank correlation samples in both levels while conducting the hypothesis tests for PR-VCH and PR-DCH in the interval level.

The result of hypotheses tests from the fish dataset is shown in Table 6. In the aspect of normality test results, correlations at time-point level of PCH are less

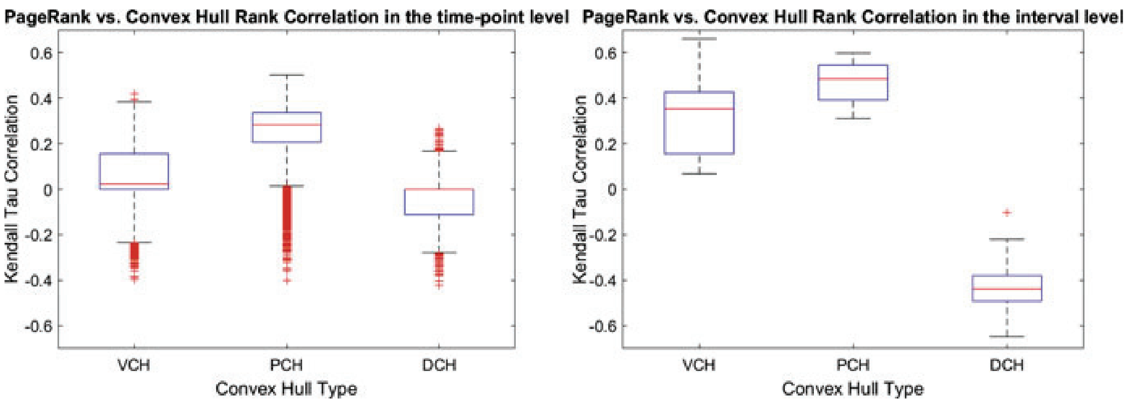

Fig. 8 Comparison of PR-VCH, PR-PCH, and PR-DCH rank correlations in both time-point and interval levels from the fish movement dataset. In the time-point level (left), the result shows that leadership vs. VCH and leadership vs. PCH rankings have positive correlations, while leadership and DCH has negative correlation. In the interval level (right), leadership vs. VCH and leadership vs. PCH rankings have stronger positive correlations than time-point level, while leadership and DCH also have stronger negative correlation

Table 5 PR-VCH, PR-PCH, and PR-DCH rank correlations from the fish dataset

	Time-point level		Interval level	
	Mean	STD	Mean	STD
PR-VCH corr.	0.05	0.12	0.32	0.16
PR-PCH corr.	0.26	0.12	0.47	0.09
PR-DCH corr.	−0.05	0.08	−0.43	0.12

Table 6 Hypothesis test results of PR-VCH, PR-PCH, and PR-DCH correlations in time-point level and interval level from the fish movement dataset

		Time-point level	Interval level		
		PR-PCH corr.	PR-VCH corr.	PR-PCH corr.	PR-DCH corr.
Zero mean/ median test	t-Test	1	1	1	1
	Sign test	1	1	1	1
	Wilcoxon signed-rank test	1	1	1	1
Normality test	Kolmogorov-Smirnov test	1	1	1	1
	Chi-square goodness-of-fit test	1	0	0	0
	Jarque-Bera test	1	0	0	0
	Anderson-Darling test	1	0	0	0

The zero value implies that a test fails to reject H_0, while one implies a test successfully rejects H_0 with $\alpha = 0.001$

Table 7 Normal confidence intervals of PR-VCH, PR-PCH, and PR-DCH correlations from the fish dataset at the interval level with $\alpha = 0.001$

	Normal confidence interval			
	Mean μ		STD	
	Lower bound	Upper bound	Lower bound	Upper bound
PR-VCH	0.20	0.45	0.11	0.30
PR-PCH	0.41	0.54	0.06	0.16
PR-DCH	−0.52	−0.34	0.08	0.21

normal compared to the VCH's, PCH's, and DCH's correlations at the interval level. This implies that the result of t-test at the PCH's time-point level should be interpreted carefully.

In the aspect of zero mean/median hypothesis test, with the significance level at $\alpha = 0.001$, both PageRank vs. velocity convex hull ranking and PageRank vs. position convex hull ranking have positive correlations far from zero on average. The result demonstrates that individuals who act as trained fish tend to move earlier and explore new areas before other individuals during coordination events. On the contrary, PageRank vs. Direction convex hull ranking has negative correlations far from zero on average. The result implies that when trained fish moves in any given direction, the group follows almost immediately, and this makes the group's direction the same as a trained fish's direction.

We also reported the normal confidence intervals at Table 7. We only reported the confidence intervals of interval-level correlations because of the normality test results; time-point level of PR-PCH correlation distributions seems not to be normal (see Table 6), while the rest of the cases are normal.

According to Table 7, the normal confidence intervals of PR-VCH and PR-PCH correlation distributions have their lower bound greater than zero, while the upper bound of PR-DCH correlation is below zero. This supports the hypotheses that there exist a positive correlation between PageRank and VCH ranking as well as PageRank and PCH ranking, while there is a negative correlation between PageRank and DCH at the interval level.

4.4 Traits of Leaders as Measure of Degree of Hierarchy Structure

Another application of trait-rank correlations we proposed here is to use these correlations to measure the degree of hierarchy structure in the datasets. The hierarchy structure here is the order of early movement. If the datasets contain a high degree of order of movement, then some specific individuals (e.g., high-rank individuals) always move before other individuals. In contrast, if datasets contain no order of movement, then there is no specific order of individuals who move before others. Figure 9 illustrates the distributions of PR-VCH rank correlations of datasets (paragraph "Simulation Datasets for Degree of Hierarchy Structure Analysis") from three leadership models. As we expected, since hierarchical model has a higher degree of structure than the dictatorship model, hence, it has the highest value of PR-VCH rank correlations. The dictatorship model has the second highest value of PR-VCH rank correlations since there is a weak order of early movement; a leader

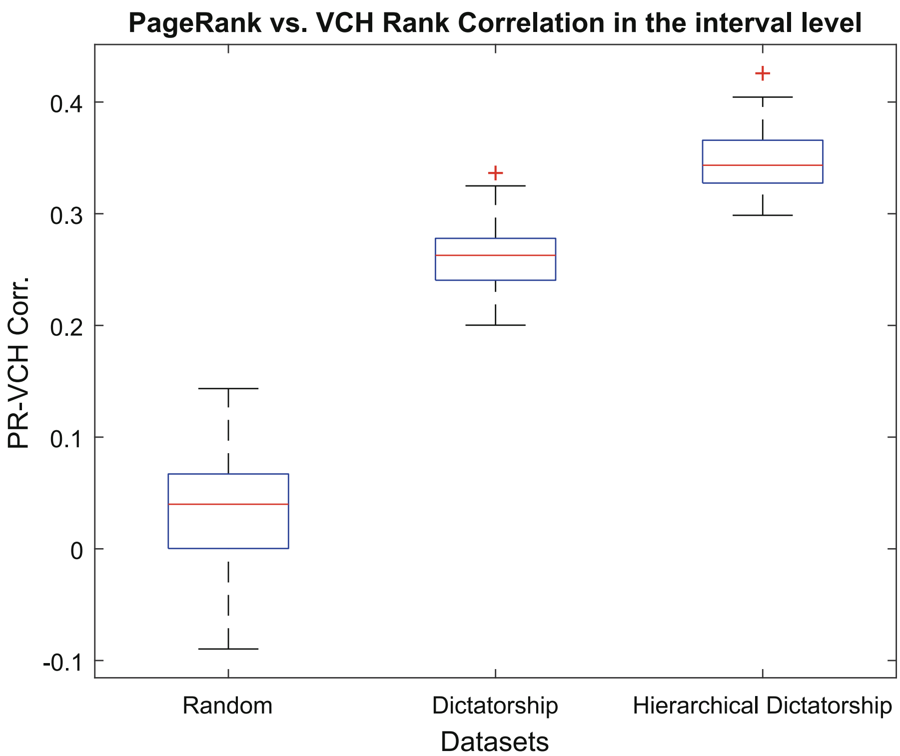

Fig. 9 The distributions of PR-VCH rank correlations of datasets from three leadership models. The higher-rank correlation implies the higher degree of hierarchy structure of early movement order in the model

Table 8 The means and standard deviations of PR-VCH rank correlations from both simulated and biological datasets

Datasets	Mean	STD
Hierarchical model	0.35	0.03
Dictatorship model	0.26	0.03
Random	0.04	0.05
Baboon (25th THS)	−0.09	0.18
Baboon (50th THS)	−0.07	0.18
Baboon (75th THS)	−0.06	0.19
Baboon (99th THS)	−0.07	0.21
Fish	0.32	0.16

always moves first. Lastly, the random model has the PR-VCH rank correlations around zero since it has no order of early movement.

In conclusion, the result implies that the higher-rank correlation implies the higher degree of hierarchy structure of early movement order in the model.

Table 8 shows the mean and standard deviation of PR-VCH rank correlations from both simulated and biological datasets. The result shows that baboon datasets have PR-VCH rank correlations nearly zero in all threshold of coordination events, while fish datasets have PR-VCH rank correlations nearly the Hierarchical model's correlations. This implies that baboons have no hierarchy of early movement, while schools of fish have pretty high degree of movement order.

5 Conclusions

In this paper, we proposed a framework for testing the correspondence between behavioral traits and leader individuals in the context of movement initiation. We focused on three hypotheses. First, individuals who act as leaders tend to move before others in their group in the period preceding coordinated movement. Second, individuals who act as leaders tend to move into new areas before others prior to a coordinated movement. Third, individuals who act as leaders tend to set the group's direction of travel. We constructed a dynamic following network and used the simple notion of convex hull as the measure of degree of difference of the velocity, position, and direction of an individual from its group. We use proposed traits of leaders for model classification. We evaluated the classification task on simulated movement data. We tested our proposed approach in baboon movement and fish movement datasets using the time series leadership inference framework, FLICA.

We found that during baboon decision-making intervals before a period of coordinated troop movement, there was a positive correlation between an individual's leadership ranking and the frequency with which an individual decided to step outside the group to explore a new area. Moreover, there was a negative correlation between leadership ranking and the frequency which individuals misaligned with the group's direction. We drew this conclusion from the hypothesis testing of the distribution of correlations between leadership ranking and convex hull measures, constructed by the proposed framework. However, there was no strong correlation between the frequency of early movement and leadership ranking.

In the fish dataset, we found that there were a positive correlation between the leadership ranking and the order of movement ranking, as well as leadership vs. order of exploring new areas ranking. On the contrary, on average, there was a negative correlation between the leadership ranking and the frequency which individuals misaligned with the group's direction. These results suggest that trained fish seems to move earlier than other fish to the new area and the untrained fish aligns with trained fish quickly.

Our work establishes a general framework to draw conclusions about leadership characteristics of individuals initiating movement and to test long-standing common assumptions about the behavioral traits the leaders possess. Our framework is sufficiently general to be applied to any movement dataset and any set of traits directly computable from the data.

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