Three Essays in Economics

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Chapter 1

Introduction

1.1 The dissertation in a nutshell

This dissertation consists of three self-contained research papers and has been written during my studies in the Doctoral Programme in Quantitative Economics and Finance at the University of Konstanz. The first paper (Chapter 2) is a joint work with Volker Hahn. We examine the impact of prices with special endings, also referred to as price points, on patterns of microeconomic price adjustment. It includes a comprehensive appendix which lays the foundation for a revised version of the paper. The second paper (Chapter 3) is a single-authored work and studies the interaction between positive trend inflation and real rigidities. The third paper (Chapter 4) is a joint work with Carl Maier in which we focus on information acquisition on posted offer markets.

The research question examined in Chapter 2 is to what extent, if at all, prices with special endings such as $0.99 or $5.00, also referred to as price points, can explain the observed price dynamics at the micro level if one abstracts from conventional sources of price stickiness such as menu costs. We present a macroeconomic model with positive trend inflation in which nominal rigidities result from price points and sticky information. We argue that a model variant that allows for a general distribution of price points is consistent with many stylized facts of price setting found in the micro data. More specifically, it makes empirically reasonable predictions concerning the duration of price spells, the shape of the hazard function, the fraction and the size of price decreases and, in particular, the relationship between price changes and inflation. Our model captures several facts which the Calvo sticky-price model cannot explain and generates plausible aggregate effects of monetary policy.

Real rigidities, i.e. mechanisms dampening the magnitude of price changes conditional on price adjustment, are typically seen as key in explaining the long-lasting effects
of monetary policy. In Chapter 3 we show that different sources of real rigidities which are equivalent under zero trend inflation lead to markedly different implications for dynamics of inflation, employment and the effectiveness of monetary policy when inflation follows a positive trend. Our contribution is twofold. First, we use theory and data to infer the degrees of real rigidities by assessing their empirical performance in matching the US inflation dynamics in the context of a New Keynesian model with Calvo pricing. Second, we document a potentially negative impact of trend inflation on the key ability of real rigidities in amplifying the real effects of monetary policy. Although we infer strong degrees of real rigidities from the data, the real effects of monetary disturbances are comparable to a model without real rigidities.

In Chapter 4 we examine the phenomenon of a large number of unsuitable applications on posted offer markets observed in the data. To this end, we propose a microeconomic model in which sellers are capacity constrained and buyers do not know ex ante whether a given offer suits them or not. Buyers endogenously decide to acquire information or to apply despite being uninformed. Our model has clear welfare implications in favor of informed signaling and shows that falling transaction costs can decrease market efficiency. We argue that the generally held view that online markets are more efficient than traditional markets may be misleading.

1.2 Zusammenfassung


Im Kapitel 2 präsentieren wir ein makroökonomisches Modell mit positiver Trendinflationsrate, in welchem nominale Rigiditäten aus den Schwellenpreisen, d.h. Preisen mit
besonderen Endungen, wie z.B. $0.99 or $5.00, und der starren Informationsbeschaf-
fung hervorkommen. Wir zeigen, dass insbesondere die Modellvariante mit einer allge-
meinen Verteilung der Schwellenpreise mit der empirischen Evidenz der Preissetzung
konsistent ist. Unser Modell macht empirisch plausible Vorhersagen über die Dauer
der Preise, die Form der Hazardfunktion, den Anteil und die Größe der Preissenkungen
und insbesondere über die Beziehung zwischen der Inflation und der Preisanpassung.
Mehrere empirische Befunde, welche das Calvo Modell nicht replizieren kann, kön-
nen durch unser Modell gut erklärt werden. Zudem generiert es plausible aggregierte
Effekte der monetären Politik.

Das Kapitel 3 zeigt, dass positive Trendinflation zu unterschiedlichen Implikationen
verschiedener realer Rigiditäten bezüglich der Dynamik der Inflation, der Beschäfti-
Erstens nutzen wir die Theorie und die Daten, um das Ausmaß verschiedener realer
Rigiditäten zu schätzen. Hierzu nutzen wir die aggregierte Inflationsrate in den USA
und das neukyesianische Modell mit Calvo-Preissetzung. Zweitens dokumentieren
wir einen negativen Einfluss der Trendinflationsrate auf die Schlüsselfigenschaft der
realen Rigiditäten, die realen Effekte der Geldpolitik zu verstärken. Obwohl wir rela-
tiv starke reale Rigiditäten in den Daten feststellen, sind die realen Auswirkungen der
Geldpolitik vergleichbar zu einem Modell ohne reale Rigiditäten.

Im Kapitel 4 untersuchen wir das Phänomen der großen Anzahl an unpassenden Be-
werbungen auf Inserat-Märkten, auf welchen ein Interessent dem Anbieter zuvor sein
Interesse signalisieren muss. Zu diesem Zweck entwickeln wir ein mikroökonomisches
Modell, in welchem die Verkäufer kapazitätsbeschränkt sind und die Käufer ex-ante
nicht wissen, ob das inserierte Angebot ihnen zusagt. Die Käufer entscheiden, ob sie
sich über das Angebot genauer informieren oder sich uninformiert bewerben. Unser
Modell liefert eindeutige normative Aussagen zugunsten der Informationsbeschaffung
vor jeder Bewerbung und zeigt, dass fallende Transaktionskosten die Markteffizienz und
soziale Wohlfahrt reduzieren können. Unsere Schlussfolgerung ist, dass die allgemeine
Ansicht, dass Online-Märkte effizienter als traditionelle Märkte sind und die Wohlfahrt
verbessern, irreführend sein kann.
Abstract
This paper proposes a macroeconomic model with positive trend inflation that involves an important role for price points as well as sticky information. We argue that, in particular, a variant of our model that allows for a general distribution of price points is successful in explaining several stylized facts of individual price setting. More specifically, it makes empirically reasonable predictions with regard to the duration of price spells, the sizes of price increases and decreases, the shape of the hazard function, the fraction of price changes that are price increases, and the relationship between price changes and inflation. Moreover, our model implies plausible aggregate effects of monetary policy in contrast to a model with a prominent role for price points but no information rigidities.
2.1 Introduction

Understanding the nature of microeconomic price rigidities is central to monetary economics. The leading paradigm in monetary economics, the new Keynesian model, starts from the premise that extended spells of constant prices point to the existence of price-adjustment costs. These costs are considered to be instrumental for rationalizing why monetary policy has real effects. The present paper explores an alternative mechanism that can explain spells of constant prices: price points.

As discussed in more detail in Section 4.2, there is strong evidence in favor of the relevance of price points, as some prices are chosen much more frequently compared to other prices (see Kashyap (1995), Blinder et al. (1998), Dhyne et al. (2006), Levy et al. (2011) and Chen et al. (2017)). In particular, Knotek (2016) considers a model with traditional menu costs and additional costs that accrue to firms when they choose prices that are not price points. According to his estimation, menu costs are effectively irrelevant as a source of price rigidity. He shows that, as a consequence, monetary shocks have almost no effects on real variables.

The present paper takes a standard macroeconomic model with positive trend inflation as a starting point and adds the following two modifications. First, we abstract from menu costs and impose a price-point restriction (PPR), i.e. a requirement that firms may only select prices from a discrete set of price points. Second, because a model with the PPR but without additional costs of adjusting prices would have the counterfactual implication that monetary policy is completely ineffective, we incorporate sticky information as in Mankiw and Reis (2002) into our model.

As a result, our model can not only generate spells of constant prices, but is also compatible with the empirical regularity that monetary policy can affect output and other real variables in the short run as well. While Knotek (2016) analyzes the potential of price points to explain the empirically observed durations of price spells and distribution of price endings, we also explore whether our model is in line with several other stylized facts of microeconomic price adjustment, which were documented e.g. by Klenow and Kryvtsov (2008) (henceforth: KK) and Nakamura and Steinsson (2008) (henceforth: NS).

In particular, we derive the following findings for our main model with the PPR as

---

2Price-adjustment costs are often modeled in a shorthand manner via time-dependent pricing.
3See Woodford (2003) for a textbook treatment of the new Keynesian model.
4A positive rate of trend inflation allows us to study the differences in magnitudes of price increases and decreases.
5For a comprehensive review of the literature on individual price dynamics see Klenow and Malin (2010) and Nakamura and Steinsson (2013).
well as sticky information (henceforth: PP) and a benchmark model without the PPR but sticky prices à la Calvo (1983) (henceforth: SP). Both models are consistent with the following stylized facts, which are reported in KK: First, prices stay constant on average for 2-3 quarters while, second, duration spells are significantly variable. Third, the magnitude of relative price changes is 11% on average. Fourth, the intensive margin dominates the variance of inflation. Fifth, the frequency of price increases co-varies strongly with inflation.

The PP model outperforms the SP model along several dimensions: First, the magnitude of price decreases is larger than the magnitude of price increases (Burstein and Hellwig (2007), KK). Second, prices move back and forth between a few rigid values (Eichenbaum et al., 2011; Knotek, 2016). Third, the frequency of price changes co-varies with inflation (KK). Fourth, the frequency of price decreases hardly changes with inflation (NS).

The basic PP model has two major shortcomings. In particular, the hazard curve of price adjustments has a maximum at around 6 to 7 quarters, although the data suggest that hazard curves are roughly flat (KK, NS). Moreover, a plot of the magnitude of relative price changes as a function of the age of the price reveals a minimum at around 6 to 7 quarters, which is at odds with the empirical finding that this curve should be approximately flat as well. Both problems can be traced back to the assumption made in our basic PP model that all price points are located on an equally spaced grid. As a consequence, we introduce an extended PP model with a more general distribution of price points in Section 2.7 and demonstrate that this extension alleviates both shortcomings of the basic PP model.

The remainder of our paper is organized as follows. In Section 4.2, we review the empirical evidence on price points. Section 2.3 presents our model. Analytical results for log-linearized versions of our model are derived in Section 3.3. Our simulation strategy is described in Section 2.5 and our main findings about price dynamics are presented in Section 2.6. We discuss the extension to our basic PP model that allows for a richer distribution of price points in Section 2.7. Section 2.8 studies the impact of monetary shocks on aggregate variables. In Section 2.9, we discuss the relationship between our PPR and menu costs. Section 2.10 concludes and provides a brief discussion of a supplementary analysis in the Appendices B and C. In particular, in the version of the paper presented in this chapter we rely on empirical evidence about microeconomic price setting for the U.S. from different data sources. In order to avoid issues concern-
2.2. EVIDENCE ON PRICE POINTS

In the last two decades, a rich literature documenting the dynamics of individual prices has emerged. One of the striking regularities observed in the data is the presence of price points, i.e. prices with special endings, for instance the digits 5 or 9, which are used substantially more often than other prices.

Several cognitive and behavioral mechanisms have been proposed as a rationale for price points.\(^8\) One reason for firms to choose threshold prices like $1.99 may be that consumers perceive the difference between $1.99 and $2.00 to be larger than, say, the difference between $1.98 and $1.99. Hence demand may drop disproportionately when firms raise their price from $1.99 to $2.00, which makes it comparably likely that they choose $1.99. A related concept is that of convenient prices, i.e. prices chosen because they require few pieces of money or little change (see Knotek (2008)). This concept helps to explain why certain goods like newspapers are often sold at prices such as $1.00, $1.50, or $2.00. Alternatively, restricting prices to pre-specified sets of prices may be a means of simplifying decision problems for boundedly rational firms or consumers.\(^9\) There are other areas where similar simplifications are common. For example, most people would probably use “simple” times like 6:30 a.m. rather than times like 6:33 a.m. when setting their alarm clocks. For the purpose of our paper, it is only important that firms prefer a certain class of prices to other prices; the exact mechanism why these prices are preferred is not relevant.

Early evidence on the role of price points for price rigidity stems from Kashyap (1995), who analyzes prices in retail catalogues, and Blinder et al. (1998), who conduct a survey on price stickiness among U.S. firms. More recently, Levy et al. (2011) use both scanner and online prices in the U.S. to document that prices with 9-endings occur more frequently than other prices, that they are less likely to change and that

\(^8\)For surveys see Monroe (1973) and Hamadi and Strudthoff (2016).  
\(^9\)It may be noteworthy that simplifying price-setting by choosing only price points would plausibly affect a supermarket’s profits only to a negligible extent.
the magnitude of price changes is larger for these prices in comparison to prices with non-9-endings.\textsuperscript{10}

While the literature on price points has focused on supermarket scanner data and online markets so far, price points are important for a broad set of consumer prices as well. Figure 2.1, which shows the distribution of last digits of consumer prices in the United Kingdom, demonstrates that the distribution clearly differs from a uniform distribution, which one would plausibly expect if price points played no role.\textsuperscript{11} Interestingly, while “9” occurs comparably often as a last digit, the most frequently chosen last digit is “0.” This could point to the relevance of convenient prices or the existence of large threshold prices like £49.00, where the last digit before the decimal point is “9.”\textsuperscript{12} A closer look at the data reveals that for some categories of products in the ONS database, “9” is the most frequent last digit, whereas “0” occurs most often for other product categories.\textsuperscript{13}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2_1.png}
\caption{Distribution of final digits for all consumer prices in the U.K. from February 1996 to December 2016, all prices weighted with the weights used for the construction of the CPI. Source: Office of National Statistics (ONS), own calculations}
\end{figure}

\textsuperscript{10}Price points have been found to be empirically relevant in other countries too. Dhyne et al. (2006) summarize the evidence from the Eurosystem’s Inflation Persistence Network and document that across various European countries price points matter for the frequency of price changes. Recently, Chen et al. (2017) provide additional evidence for the relevance of price points. They show that after the Israeli parliament restricted prices to have a zero ending in January 2014, 90-ending prices became the new price points.

\textsuperscript{11}We use monthly price quotes collected for the consumer price index micro dataset of the United Kingdom’s Office for National Statistics (ONS). A detailed description of this data set can be found in Appendix B.

\textsuperscript{12}Even prices that do not obviously qualify as price points, like a price of $1.34, may be the result of price adjustments in discrete steps. For example, a price of $1.34 could be the result of a 10% discount on an original price of $1.49.

\textsuperscript{13}See Appendix C.2.
It may also be instructive to examine the most frequently chosen prices in the ONS database. As can be seen from Table 2.1, most of the fifteen most frequently used prices end with “99,” “00,” or “50.” It may also be noteworthy that the three largest prices in this list, £10, £20 and £25, appear to be rather special. Moreover, indirect evidence suggesting the relevance of price points stems from the observation that prices jump discontinuously between a few fixed values (NS, Eichenbaum et al. (2011), Knotek (2016)). This pattern cannot be reconciled with simple menu-cost models easily.

Table 2.1: Most frequent consumer prices in the UK from February 1996 to December 2016, all prices weighted. Source: ONS, own calculations

<table>
<thead>
<tr>
<th>rank</th>
<th>price</th>
<th>relative freq.</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.92%</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>0.84%</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
<td>0.82%</td>
</tr>
<tr>
<td>4</td>
<td>1.99</td>
<td>0.81%</td>
</tr>
<tr>
<td>5</td>
<td>3.99</td>
<td>0.79%</td>
</tr>
<tr>
<td>6</td>
<td>10.00</td>
<td>0.79%</td>
</tr>
<tr>
<td>7</td>
<td>4.99</td>
<td>0.76%</td>
</tr>
<tr>
<td>8</td>
<td>3.00</td>
<td>0.72%</td>
</tr>
<tr>
<td>9</td>
<td>5.00</td>
<td>0.71%</td>
</tr>
<tr>
<td>10</td>
<td>2.99</td>
<td>0.70%</td>
</tr>
<tr>
<td>11</td>
<td>25.00</td>
<td>0.70%</td>
</tr>
<tr>
<td>12</td>
<td>9.99</td>
<td>0.69%</td>
</tr>
<tr>
<td>13</td>
<td>7.99</td>
<td>0.69%</td>
</tr>
<tr>
<td>14</td>
<td>2.50</td>
<td>0.66%</td>
</tr>
<tr>
<td>15</td>
<td>1.50</td>
<td>0.63%</td>
</tr>
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2.3 Model

Having discussed the empirical evidence in favor of price points, we now try to assess their implications for price dynamics. For this purpose, we propose two variants of a textbook macroeconomic model (see Woodford (2003)) with positive trend inflation and idiosyncratic productivity shocks as in Gertler and Leahy (2008).\footnote{Idiosyncratic productivity shocks are necessary to generate the large price changes observed in the data (see Golosov and Lucas (2007)).} In our main model, which we label PP, we consider firms that can choose prices only from a discrete set of price points, i.e. subject to a price-point restriction (PPR). In addition, firms’ pricing behavior is subject to information rigidities à la Mankiw and Reis (2002). As a benchmark, we examine a second version of our model. This version incorporates
neither price stickiness nor a PPR but involves sticky prices as in Calvo (1983). We label the sticky-price model SP in the following.

Time is discrete and denoted by \( t = 0, 1, 2, \ldots \). The model is populated by households and monopolistically competitive firms. We provide details about each of these groups in turn.

2.3.1 Households

There is a continuum of households that own identical shares of all firms and receive firms’ profits as dividends. The households’ instantaneous utility function in period \( t \) is

\[
u(C_t, M_t/P_t, N_t) = \ln \left( C_t \left( \frac{M_t}{P_t} \right)^\nu \right) - \frac{N_t^{1+\varphi}}{1 + \varphi}, \tag{2.1}\]

where \( \nu \) and \( \varphi \) are positive parameters, \( M_t \) represents nominal money holdings, \( P_t \) is the aggregate price level, \( N_t \) stands for the household’s supply of labor and \( C_t \) is a consumption basket. \( C_t \) is given by a Dixit-Stiglitz aggregator function

\[
C_t = \left[ \int_0^1 (C_{j,t})^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}, \tag{2.2}\]

where \( C_{j,t} \) denotes the quantity of good \( j \in [0,1] \) consumed in period \( t \) and \( \varepsilon \) (\( \varepsilon > 1 \)) stands for the elasticity of substitution between the differentiated goods.

Utility in future periods is discounted by the factor \( \beta \in (0,1) \). In each period \( t \), the real flow budget constraint is

\[
\int_0^1 \frac{Q_{j,t} C_{j,t} \, dj}{P_t} + \frac{M_t - M_{t-1}}{P_t} + \frac{1}{R_t} B_t - B_{t-1} = W_t \frac{N_t}{P_t} + T_t, \tag{2.3}\]

where \( Q_{j,t} \) denotes the price of one unit of good \( j \), \( B_t \) bond holdings, \( R_t \) the nominal interest rate, \( W_t \) the nominal wage, and \( T_t \) a real transfer, which includes the profits of firms and the government’s seigniorage revenues. Bonds are in zero net supply. The aggregate price level \( P_t \) is given by

\[
P_t = \left[ \int_0^1 (Q_{j,t})^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}. \tag{2.4}\]

The growth rate of the nominal money stock, which is denoted by \( g_t^m \), follows an exogenously given stationary stochastic process. We allow for a positive unconditional mean of \( g_t^m \), which enables us to model a positive inflation trend.
2.3.2 Firms

The economy is populated by a continuum of monopolistically competitive goods producers, indexed by $j \in [0, 1]$. Each firm $j$ produces the individual good $j$ and sells it directly to consumers. The production function is of the form

$$Y_{j,t} = X_{j,t}^\gamma N_{j,t},$$

with $\gamma \in (0, 1]$ and where $X_{j,t}$ is an idiosyncratic productivity level and $N_{j,t}$ is the labor input of firm $j$ at time $t$. For $\gamma < 1$, there are decreasing returns to scale, which could also be interpreted as the production function being of the Cobb-Douglas type but with fixed capital.

In every period, each firm $j$ is hit by a productivity disturbance with probability $1 - \alpha$. When this happens, the firm survives with probability $\tau$. For surviving firms, productivity changes according to $X_{j,t} = X_{j,t-1} e^{\xi_{j,t}}$, where $\xi_{j,t}$ is an i.i.d. firm-specific shock that is uniformly distributed over the support $[-\frac{1}{2}, +\frac{1}{2}]$. If a firm does not survive, which happens with probability $1 - \tau$, conditional on a shock, it is immediately replaced by a new firm with productivity one, i.e. $X_{j,t} = 1$. This can be interpreted as product substitutions. We note that the main purpose of the assumption that firms may die with probability $1 - \tau$ is to guarantee a stationary distribution of productivities across firms as in Gertler and Leahy (2008). For our calibration, it will be convenient to introduce $\theta := (1 - \alpha)(1 - \tau)$ as the probability that a given firm exits in a given period.

In our main model (PP), firms receive information about the aggregate monetary disturbance and the idiosyncratic productivity shock in periods in which they are hit by an idiosyncratic shock. In all other periods, they must act on outdated information. While firms can adjust the prices of their outputs in every period, they must choose these prices subject to a price-point restriction (PPR), as will be explained in the following.

We assume that each firm $j$ chooses the price for a quantity $U_j$ of the good, where we use $\hat{Q}_{j,t}$ to denote this price. $U_j$ is constant over time and exogenous for each firm $j$. Thus the price of quantity $U_j$, $\hat{Q}_{j,t}$, and the price per unit of the good, $Q_{j,t}$, are related via $\hat{Q}_{j,t} = U_j Q_{j,t}$. We can think of the $U_j$'s as different package sizes of the

---

15This assumption about when firms receive information updates is closely related to the modeling strategy in Gertler and Leahy (2008), who assume that firms face costs of information acquisition that are too large for firms to search for information in the absence of idiosyncratic shocks but small enough such that firms always acquire information when they are hit by a shock. Klenow and Willis (2007) also consider a model where firms update their information about idiosyncratic shocks more frequently than their information about aggregate disturbances.
differentiated products.

The PPR implies that each firm $j$ can only choose a log price $\tilde{q}_{j,t} := \ln(\tilde{Q}_{j,t})$ that lies in the set of price points $\Delta \cdot Z$, where $Z$ is the set of positive and negative integers and $\Delta$ is the exogenously given relative distance between price points, which is identical for all firms. Moreover, we assume that the natural logarithms of the firms’ $U_j$’s, which are denoted by $u_j$’s, are uniformly arranged on the interval $[0, \Delta]$. Let $q_{j,t}$ be the natural logarithm of the per-unit price $Q_{j,t}$. As $q_{j,t} = \tilde{q}_{j,t} - u_j$ and $\tilde{q}_{j,t} \in \Delta \cdot Z$, the log per-unit price $q_{j,t}$ can only be chosen such that $q_{j,t} \in \Delta \cdot Z - u_j$. Firm $j$’s profits in period $t$ are given by the difference between revenues and total labor costs,

$$\Pi_{j,t} = \frac{Q_{j,t} Y_{j,t}}{P_t} - \frac{W_t}{P_t} N_{j,t}.$$ 

(2.6)

Finally, a few comments on our assumptions regarding price points are in order. First, we would like to stress that we take the relevance of price points as given and introduce them into our model as an exogenous constraint on firms’ price setting. Second, we note that our assumption about the relative distance between price points being constant is in line with the observation that, for $0.89$, the next price point would be $0.99$ but for a price point of $8.99$, the next price point would plausibly be $9.99$.\textsuperscript{16} Hence, the assumption of constant relative differences between price points is plausible to be a reasonable first approximation. This assumption will be modified in Section 2.7. Third, the assumptions that prices refer to fixed quantities $U_j$ of goods and that the $U_j$’s are uniformly arranged have the plausible consequence that the fraction of firms choosing a price point below the price they would charge in the absence of a PPR and the fraction of firms choosing a higher price than they would select without a PPR are constant over time.\textsuperscript{17}

In our benchmark case with sticky prices (SP), we abstract from information rigidities. Firms do not face a PPR but can only adjust their prices when they are hit by an idiosyncratic productivity shock, which happens with exogenous probability $1 - \alpha$. Thus firms in the SP face price stickiness as in Calvo (1983).

\textsuperscript{16}This argument is also supported by the evidence presented in Levy et al. (2011). They find that for small prices, prices with 9s in the penny and dime digits are particularly persistent. For more expensive products, they observe more persistence of prices with 9s in the $1, \$10,$ and $\$100$ digits.

\textsuperscript{17}Otherwise, under positive inflation there would be discontinuous jumps in the price level in periods where a large fraction of firms adjusted their price upwards to the next price point.
2.4 Solution

2.4.1 Common equations for the PP and the SP model

In the following, we consider log-linearized versions of the PP and the SP model. In both scenarios, the equations describing the optimal behavior of households, which are stated in Appendix D.1, have well-known log-linear approximations around the steady state

\[ w_t - p_t = \ln \left( \frac{W}{P} \right) + \varphi \hat{N}_t + \hat{Y}_t, \]  
\[ \hat{Y}_t = - \left( \hat{R}_t^n - \mathbb{E}_t \left[ \hat{\pi}_{t+1} \right] \right) + \mathbb{E}_t \left[ \hat{\pi}_{t+1} \right], \]  
\[ m_t - p_t = \ln \left( \frac{M}{P} \right) + \hat{Y}_t - \frac{1}{\hat{R}_t^n} \hat{R}_t^n, \]  
\[ \hat{Y}_{j,t} = - \varepsilon \left[ q_{j,t} - p_t + \ln \left( \frac{Q}{P} \right) \right] + \hat{Y}_t, \]

where here and henceforth small letters denote log levels, variables with a bar denote steady-state levels, and variables with a “hat” stand for relative deviations from the steady state.\(^{18}\)

2.4.2 Equations specific to the PP model

In the model with price points and information frictions, a firm hit by an idiosyncratic shock \( i \) periods ago sets the following price for one unit of its good:

\[ q_{j,t}^{PP} = \mathcal{T}_j \left\{ \mathbb{E}_{t-i} \left[ q_{j,t}^{PP} \right] \right\} \]
\[ = \mathcal{T}_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left[ -\frac{1}{\gamma} x_{j,t} + \mathbb{E}_{t-i} \left[ \hat{u}c_t \right] \right] + \mathbb{E}_{t-i} \left[ p_t \right] \right\}, \]

where \( \mathcal{T}_j : \mathbb{R} \to \mathbb{R} \) is an operator which maps the hypothetical, optimal price of producer \( j \) in the absence of the PPR, \( \mathbb{E}_{t-i} \left[ q_{j,t}^{PP} \right] \), to the closest corresponding price point \( q_{j,t}^{PP} \in \Delta \cdot Z - u_j \). \( \hat{u}c_t \) denotes the relative deviation of aggregate unit labor costs from their steady-state value. For details of the derivation see Appendix A.2. It may be worth stressing that \( q_{j,t}^{PP} \) is the price for one unit of the consumption good. The price actually chosen by the firm for a package of log size \( u_j \) is \( \tilde{q}_{j,t}^{PP} = q_{j,t}^{PP} + u_j \).

For the scenario with information frictions, our model results in a sticky-information

\(^{18}\)We have used \( \hat{Y}_t = \hat{C}_t \) for the derivation of (3.9)-(3.12).
Phillips curve à la Mankiw and Reis (2002):

\[
\hat{\pi}_t = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \frac{(1 - \alpha^{PP})}{\alpha^{PP}} \hat{uc}_t + (1 - \alpha^{PP}) \sum_{i=0}^{\infty} \left( \alpha^{PP} \right)^i E_{t-1-i} \left( \hat{\pi}_t + \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left( \hat{uc}_t - \hat{uc}_{t-1} \right) \right)
\]

(2.12)

where we have added the superscript \( PP \) to the parameter \( \alpha \) in the PP model. The derivation of (2.12) can be found in Appendix A.2.

### 2.4.3 Equations specific to the SP model

In the SP model, the price setting equation is given by

\[
\psi_{j,t}^{SP} = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left( \hat{\psi}_t - \hat{\phi}_t \right) - \frac{1}{\gamma + \varepsilon(1 - \gamma)} x_{j,t} + p_t + \ln \left( \frac{Q}{P} \right),
\]

(2.13)

where we have introduced the superscript \( SP \) for the SP model. The auxiliary variables \( \hat{\psi}_t \) and \( \hat{\phi}_t \) are given by

\[
\hat{\psi}_t = \left( 1 - \alpha^{SP} \beta \pi^e \right) \left[ \hat{uc}_t - \hat{s}_t \right] + \alpha^{SP} \beta \pi^e \left[ E_t \hat{\psi}_{t+1} + \frac{\varepsilon}{\gamma} E_t \hat{\pi}_{t+1} \right],
\]

(2.14)

\[
\hat{\phi}_t = \alpha^{SP} \beta \pi^e \left[ E_t \hat{\phi}_{t+1} + (\varepsilon - 1) E_t \hat{\pi}_{t+1} \right].
\]

(2.15)

The deviation \( \hat{s}_t \) of price dispersion from its steady-state value is given by

\[
\hat{s}_t = \frac{\varepsilon (\pi^e - \pi^e-1)}{\gamma (1 - \alpha^{SP} \pi^e-1)} \alpha^{SP} \hat{\pi}_t + \alpha^{SP} \pi^e \hat{s}_{t-1}.
\]

(2.16)

The associated Phillips curve with trend inflation is then

\[
\hat{\pi}_t = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \frac{(1 - \alpha^{SP} \pi^e-1)(1 - \alpha^{SP} \beta \pi^e)}{\alpha^{SP} \pi^e-1} \left[ \hat{uc}_t - \hat{s}_t \right]
\]

(2.17)

\[
+ \beta \left[ 1 + \frac{1 - \alpha^{SP} \pi^e-1}{\gamma + \varepsilon(1 - \gamma)} \left( \pi^e \pi^e (1 - \gamma) - 1 \right) \right] E_t \hat{\pi}_{t+1}
\]

\[
+ \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \alpha^{SP} \beta \left( \pi^e - \pi^e-1 \right) \frac{1 - \alpha^{SP} \pi^e-1}{\alpha^{SP} \pi^e-1} E_t \hat{\psi}_{t+1},
\]

where \( \hat{\psi}_t \) is given by equation (3.66).

\[\text{[19]}\text{The derivation is standard. It is contained in a separate online appendix.}\]
2.5 Simulation Strategy

The main objective of this paper is to simulate the individual price dynamics implied by the PP model and the SP model in order to assess how well these models can explain the empirical findings about price-setting documented by KK, NS, and others. In this section, we explain our simulation strategy and the calibration of our model.

We compute the individual price dynamics for the PP and the SP model variants using the price-setting equations (2.11) and (3.78), respectively. More specifically, we simulate the prices set by 100,000 firms for the time period 1988Q1 - 2004Q4, which is the period considered by KK and NS. While the idiosyncratic shocks are generated by a random number generator in this simulation exercise, we use realized values for the current and past CPI and unit labor costs. As firms’ optimal prices also depend on current and lagged expectations of unit labor costs and the price level, we follow Sbordone (2002) and Dupor et al. (2010) and estimate a vector autoregressive model to generate the corresponding forecasts. We use data from 1983Q1-2004Q4 for our VAR model, which enables us to calculate lagged expectations of current economic variables for the entire period 1988Q1-2004Q4.\textsuperscript{20} The forecasting model includes CPI quarter-on-quarter inflation rates and unit labor costs.\textsuperscript{21,22} The time unit is a quarter, as data on unit labor costs is not available at shorter time intervals.

We would like to comment on a difference between the simulations for the PP model and the ones for the SP model. As in our simulations firms utilize information about aggregate unit labor costs but do not observe a direct measure of costs on the individual firm level, they rely on a relationship between individual and aggregate costs that involves the measure of price dispersion $\hat{s}_t$.\textsuperscript{23} In the PP model, $\hat{s}_t = 0$ is satisfied in every period $t$ for a linear approximation around the steady state, which entails that firms do not have to calculate $\hat{s}_t$ when computing their own costs from aggregate unit labor costs. By contrast, $\hat{s}_t = 0$ does not hold in the SP model under the assumption of a positive inflation rate in the steady state. Therefore, firms use (3.71) to compute $\hat{s}_t$ in our simulations. In this sense, our simulations involve an additional condition for the SP model compared to the PP model. We have confirmed that all our results about individual price dynamics in the SP model are virtually unaffected if we make the assumption that firms (erroneously) use $\hat{s}_t = 0$ in the SP model as well.

For our calibration, we proceed as follows. First, we rely on external information to

\textsuperscript{20}This is necessary for the PP model.
\textsuperscript{21}We follow Galí and Gertler (1999) and measure log real unit labor cost as the logarithm of the ratio of nominal compensation per hour to nominal output per hour in the non-farm business sector.
\textsuperscript{22}We use a VAR model of order two, which is suggested by the standard information criteria.
\textsuperscript{23}The details of the derivations are laid out in a separate online appendix.
2.5. SIMULATION STRATEGY

calibrate $\beta$, $\varphi$, $\varepsilon$, $\gamma$, $\theta$, and $\Delta$.\textsuperscript{24} Since we use quarterly data, we choose a discount factor of $\beta = 0.99$. In line with Gertler and Leahy (2008), we utilize $\varphi = 1$ for the inverse of the Frisch elasticity of the labor supply, and $\varepsilon = 11$, which implies a steady-state markup of 10% over marginal costs. We set $\gamma$ to the labor income share of approximately 62% (Elsby et al., 2013). For the exit probability $\theta$, we select $\theta = 0.087$, which is in line with the evidence from KK and NS that the monthly rate of forced item substitutions is around 3%. To calibrate $\Delta$, which is only relevant for the PP model, we refer to the following observations. First, Levy et al. (2011) find that the price points in cents contained in the data set from Dominick’s supermarkets end with the digit 9. Moreover, we note that the modal price in this data set is $1.99, which implies that the two closest price points are $1.89 and $2.09. As the relative differences of these prices from $1.99 are approximately 5%, we set $\Delta = 0.05$.\textsuperscript{25}

<table>
<thead>
<tr>
<th>data</th>
<th>targeted period</th>
<th>1988Q1 - 2004Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean price change (KK)</td>
<td></td>
<td>11.3%</td>
</tr>
<tr>
<td>VAR period</td>
<td>1983Q1 - 2004Q4</td>
<td></td>
</tr>
<tr>
<td>mean q-o-q inflation</td>
<td></td>
<td>0.76%</td>
</tr>
<tr>
<td>annualized mean inflation</td>
<td></td>
<td>3.08%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>calibrated externally</th>
<th>$\beta$</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.087</td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>calibrated internally</th>
<th>$\alpha^{PP}$</th>
<th>0.73</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{SP}$</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>$\chi^{PP}$</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td>$\chi^{SP}$</td>
<td>1.84</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Calibration summary

Second, parameters $\alpha$ and $\chi$ are calibrated to values that differ across the two sce-

\textsuperscript{24}The utility parameter $\nu$ does not affect our simulations.

\textsuperscript{25}Our results are qualitatively robust to reasonable changes in this parameter value. The results are available upon request. A scenario with multiple values of $\Delta$ is considered in Section 2.7.
To determine the respective values of $\alpha$, we implement the procedure that was introduced by Sbordone (2002) and was also used by Dupor et al. (2010), more recently. For this purpose, we use equations (2.12) and (2.17) to compute the model-implied inflation in the PP and the SP model as functions of $\alpha$, where current and lagged real unit labor costs are set to the values in the data and the VAR model is used to generate forecasts of inflation and real unit labor costs.\footnote{As is common in the literature, we restrict the number of lagged expectations included in the sticky-information Phillips curve. In particular, we consider only expectations that are formed up to 19 periods ago. For $\alpha^{PP} = 0.73$, the value implied by our calibration exercise, only 0.2% of all firms have not updated their information after 19 periods. We proceed analogously when computing $\hat{s}_t$ and $\hat{\psi}_t$ for the SP model.} We then select $\alpha$ by minimizing the sum of quadratic deviations of realized inflation rates from inflation rates predicted by the respective Phillips curve, i.e. (2.12) for the PP model and (2.17) for the SP model, for the time horizon 1988Q1-2004Q4. This procedure results in $\alpha^{PP} = 0.73$ and $\alpha^{SP} = 0.67$ for the two scenarios.

It remains to calibrate $\chi$, the width of the support for the idiosyncratic shocks, where we also allow for different values for the two scenarios. We apply the simulated method of moments to equations (2.11) and (3.78) and determine these parameters by targeting the mean magnitude of price changes, which is 11.3% in US CPI data (see Table III in KK). This procedure results in $\chi^{PP} = 2.20$ and $\chi^{SP} = 1.84$. Table 2.2 summarizes our calibration.

### 2.6 Individual Prices

We now turn to our simulation results regarding individual price dynamics. These simulations show that our PP model can explain several pieces of the evidence on individual price dynamics at least as well as the SP model. In Section 2.7, we will demonstrate that the PP model is even more successful in explaining the stylized facts of individual price dynamics when we allow for a more general distribution of price points.

#### 2.6.1 Duration of price spells and the magnitude of price changes

**Duration of price spells** How often do prices change? Empirical studies find that the mean duration of regular prices is roughly three quarters, depending on the sample period, the weighting of prices and the treatment of product substitutions. For
example, KK estimate the frequencies of price adjustments for different categories of products and obtain that the mean of the implied durations is 2.9 quarters.\textsuperscript{28,29} As shown in Table 2.3, even though this statistic has not been targeted, both models can match this evidence very well. The PP model implies a mean duration of 2.9 quarters exactly as in KK, and the SP model involves a slightly higher duration of 3.2 quarters. Despite the higher value of $\alpha$ in the PP model, the duration of price spells is shorter in the PP scenario since firms can change their prices not only when they are hit by an idiosyncratic shock but can adjust their prices freely even in periods where they receive no new information.

\begin{table}[h]
\begin{center}
\begin{tabular}{l|c|c|c}
 & KK & PP & SP \\
\hline
mean price duration in quarters & 2.9 & 2.9 & 3.2 \\
std. dev. of price dur.'s in q.'s & 1.7 & 1.9 & 2.7 \\
mean magn. of changes (targeted) & 11.3\% & 11.3\% & 11.3\% \\
median magn. of changes & 9.7\% & 10.0\% & 10.2\% \\
mean price increases & 10.6\% & 9.9\% & 12.3\% \\
mean price decreases & 13.3\% & 14.3\% & 10.1\% \\
share of price decreases & 43.4\% & 31.4\% & 43.7\% \\
\hline
\end{tabular}
\end{center}
\caption{Simulation results}
\end{table}

**Variance of price durations**  KK document that the standard deviation of durations between price adjustments for a given item in the BLS data is around 1.7 quarters (see their Table V). Our simulations yield 1.9 for the PP case and 2.7 for the SP case. Due to the idiosyncratic productivity shocks that arrive with a fixed probability in every period, both models can generate variances of price spells that are broadly consistent with the empirical evidence, although the SP model generates price spells that are somewhat more volatile than in the data.

**Magnitude of price changes**  The empirical evidence that prices change by much more than necessary to catch up with inflation has been emphasized since Bils and Klenow (2004). As argued by Golosov and Lucas (2007), a model has to involve idiosyncratic shocks in order to be able to explain this pattern. As both model variants include idiosyncratic shocks, we are able to hit the calibration target of an average

\textsuperscript{28}See the implied durations for regular prices in their Table I. This value includes ends of price spells due to product substitutions.

\textsuperscript{29}Gorodnichenko et al. (2018) find that, even in online markets, where physical price adjustment costs are negligible, prices remain fixed for comparably long periods.
magnitude of price changes of 11.3% in both cases.\textsuperscript{30} In both models and in the data, the median magnitude of relative price changes is smaller than the mean.

**Magnitude of price changes for increases vs. decreases** It is a puzzling asymmetry in empirical data that the magnitude of price decreases tends to be larger than the size of price increases. Burstein and Hellwig (2007) document this fact for the Dominick’s database and KK provide evidence that for regular prices in the BLS data set, increases average 10.6% whereas decreases average 13.3%.

Table 2.3 shows that the PP model outperforms the SP model in this regard since, in contrast with the SP model, the PP model generates larger price decreases than increases. In particular, the PP model generates average increases of 10% and average decreases of 14% similarly to the data whereas the SP model implies average increases of 12% and average decreases of only 10%.

How can this relative success of the PP model be explained? Roughly speaking, most of the price decreases in the PP model are driven by idiosyncratic productivity shocks, which have a comparably large variance. By contrast, increases in prices also occur because of the positive trend in inflation. These increases are small, as firms adjust their price upwards by $\Delta$ in these cases, which is the smallest possible price change in the PP model. As a consequence, increases are smaller on average than decreases.

In the SP model, price increases are larger than decreases on average because every time a firm is allowed to adjust its price, the new price is determined by two main factors: the idiosyncratic productivity shock, which has zero expected mean, and the change in the price level since the price was adjusted last, which is positive under a positive trend inflation rate. It is clear that the resulting price change involves larger average increases than decreases.

**Fraction of price changes that are price decreases** KK find that 43.4\% of all price changes are price decreases (see their Table VI). Both models considered in this paper are broadly in line with this finding. The SP model predicts a fraction of 44\% and the PP model implies that 32\% of all price changes are price decreases. While the value predicted by the PP model is lower than the one found in KK, it is still in line with the empirical findings in other papers. For example, for essentially the same data that is used in KK, NS report a value of roughly one third.\textsuperscript{31}

\textsuperscript{30}If we exclude the relatively large price changes that occur when products are replaced, this value drops to 9.7\% in the PP and to 9.9\% the SP model. This is in line with the finding reported in Footnote 9 in KK that price changes are 1 to 2 percentage points smaller if price changes at substitutions are excluded.

\textsuperscript{31}These differences arise because KK and NS proceed slightly differently, e.g. when removing price changes due to sales. For more detailed information on the differences in findings between KK and
2.6. INDIVIDUAL PRICES

**Price points** Data on individual prices indicate that prices move back and forth between a few rigid values (NS, Eichenbaum et al. (2011), Knotek (2016)). Obviously this fact is matched by the PP model by construction. Nevertheless it is still one of the most puzzling empirical observations and it is not trivial for macro models to be consistent with this pattern (see Kehoe and Midrigan (2015)).

2.6.2 Frequency and magnitude of price changes as functions of the age of the price

**Hazard rates** Simple state-dependent pricing models like menu-cost models typically predict an increasing hazard curve, i.e. an increasing probability of a price change as a function of the duration of a price spell.\(^{32}\) As shown by KK, NS and Klenow and Malin (2010), this implication is not supported by the data.\(^{33}\)

![Figure 2.2: Hazard rates](image)

Time-dependent pricing models based on Calvo pricing produce flat hazard curves by construction, which can be seen from Figure 2.2 in the case of our SP model. By contrast, while the hazard curve in the PP model is rather flat up to the fourth and fifth quarter, it features a significant peak around the seventh quarter. The reason for this outcome is straightforward. In the PP model there are two main reasons for price changes: First, idiosyncratic shocks may occur. These shocks alone would produce a flat hazard curve. Second, in the absence of idiosyncratic shocks, firms form expectations about changes in the aggregate price level, which require an adjustment of

\(^{32}\)NS, see NS.

\(^{33}\)NS highlight that the hazard function can take many different forms in models with idiosyncratic shocks.

Estimating the shape of hazard functions is empirically challenging because of substantial heterogeneity in the frequency of price adjustments across goods.
their prices from time to time due to the positive level of trend inflation. As the average quarter-on-quarter inflation rate is 0.76% for our sample and the relative difference between price points is \( \Delta = 5\% \), one would expect that firms are comparably likely to adjust their prices after \( \frac{5}{0.76} \approx 6.6 \) quarters, which is exactly what Figure 2.2 shows.

The peak of the hazard curve can be seen as an artifact of our assumption that the relative distance between price points is identical in all cases, which is arguably not particularly realistic. Consider for example, the price point $0.99. The next price point would be $1.09, which would imply a relative difference of roughly 10%, compared to the difference of 5% between $1.99 and $2.09. We take up this idea in the subsequent Section 2.7 and show that the hazard curve becomes significantly flatter once we allow for a richer distribution of \( \Delta \)'s.

**Size of price changes as a function of age** Another important empirical fact is that the mean magnitude of relative price changes is approximately independent of the time since the last adjustment (KK, Klenow and Malin (2010)). This empirical finding is at odds with the typical prediction of time-dependent pricing models that prices are adjusted more strongly if they have not been adjusted for a longer time period.\(^{34}\)

Interestingly, although our SP model falls into the class of time-dependent pricing models, it is quite successful in replicating the empirical finding under consideration, as can be seen from Figure 2.3. The success of the SP model is due to the fact that our calibration selects a comparably large variance of idiosyncratic shocks. Hence the size of price changes is mostly driven by the realization of the idiosyncratic shock and hardly influenced by the comparably modest changes in the price level that occurred since the price was last adjusted.

By contrast, the PP model implies a sizable trough at around 7 quarters. The intuition is straightforward. As we have explained in our previous discussions, positive trend inflation causes firms in the PP model to adjust prices upwards by \( \Delta \) from time to time even in the absence of idiosyncratic shocks. Due to our assumption of a single value of \( \Delta \), this occurs always after approximately 7 quarters.\(^{35}\) It appears plausible that a more general specification of the set of price points would thus improve the performance of the PP model substantially. In fact, we will show in Section 2.7 that this conjecture is correct.

---

\(^{34}\)This prediction can be understood by noting that the optimal price drifts away from the current price as time passes and therefore larger adjustments are necessary for prices that have not been adjusted for a long time.

\(^{35}\)It is noteworthy that Figure 2.3 shows that, at the trough of the graph for the PP model, prices change by only slightly more than \( \Delta = 5\% \) on average.
2.6. INDIVIDUAL PRICES

Figure 2.3: The absolute value of relative price changes conditional on the price’s age

2.6.3 Price changes and inflation

<table>
<thead>
<tr>
<th>variable</th>
<th>source</th>
<th>mean</th>
<th>std dev</th>
<th>correlation</th>
<th>( \beta_\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( fr_t )</td>
<td>KK</td>
<td>26.60</td>
<td>3.2</td>
<td>0.25</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>PP</td>
<td>33.44</td>
<td>1.15</td>
<td>0.49</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>30.17</td>
<td>0.12</td>
<td>-0.05(^\dagger)</td>
<td>-0.02(^\dagger)</td>
</tr>
<tr>
<td>( dp_t )</td>
<td>KK</td>
<td>0.98</td>
<td>1.19</td>
<td>0.99</td>
<td>3.55</td>
</tr>
<tr>
<td></td>
<td>PP</td>
<td>2.20</td>
<td>0.67</td>
<td>0.76</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>2.40</td>
<td>0.92</td>
<td>0.88</td>
<td>2.31</td>
</tr>
</tbody>
</table>

Table 2.4: Time-series moments

Notes: \( fr_t \) = the fraction of items with changing prices, \( dp_t \) = the average relative price change; values marked with a \(^\dagger\) are highly insignificant as both p-values are approximately 0.7.

Relationship of frequency and size of price adjustment with inflation  KK find that the overall frequency of price changes co-moves with inflation. In particular, they find that, for monthly data, the correlation between the frequency of price adjustment and inflation equals 0.25. The positive correlation between the frequency of price adjustment and inflation is strong evidence in favor of state-dependent pricing and indeed the PP model can match this evidence qualitatively, as can be seen from Table 2.4. Similarly, estimating

\[
fr_t = \beta_\pi \pi_t + b + \epsilon_t \tag{2.18}
\]
with error terms $\epsilon_t$ and intercept $b$ via OLS produces a significant positive coefficient $\beta_\pi = 1.61$, which is qualitatively in line with the value of 2.38 found in KK.$^{36,37}$ By contrast, the SP model assumes a fixed probability of price adjustment and thus implies no correlation of the frequency of price adjustment with inflation.

KK find a large positive correlation between inflation and the mean price change. Both models are compatible with this finding. Using the average of relative price changes, $d_p_t$, as a dependent variable in (2.18) produces significant positive coefficients $\beta_\pi$ in all three cases KK, PP, SP. To sum up, while only the PP model can correctly predict that the frequency of price adjustments co-moves with inflation, both models are in line with the data when it comes to the correlation between the average size of price changes and inflation.

**Relationship of frequencies of price increases and price decreases with inflation**  NS and, more recently, Nakamura et al. (2018) have documented that the frequency of price increases changes substantially over time and co-varies with inflation. By comparison, the frequency of price decreases is more stable.

![Figure 2.4: Frequency of price increases: blue/black solid line; frequency of price decreases: blue/black dashed line; inflation: red line.](image)

Figure 2.4 shows that this pattern can be reproduced qualitatively by the PP model but not by the SP model. In the SP model, when an idiosyncratic shocks hits a firm, the price is adjusted in response to the size of the idiosyncratic shock and changes in the aggregate price level. As a consequence, a comparably large fraction of price changes are increases when inflation is high. As the frequency of price changes is fixed

---

$^{36}$The coefficients are not directly comparably as KK use monthly data for which short-term fluctuations due to sales have been filtered out, whereas we consider quarterly data.

$^{37}$Klenow and Malin (2010) also find that the frequency of price changes co-varies with inflation.
by assumption, a higher frequency of price increases automatically translates into a lower frequency of price decreases.

In the PP model, the current inflation rate mainly affects the frequency of the small positive adjustments that are caused by expected increases in the price level. In periods of high inflation, there are more of these increases and thus the frequency of price increases is higher. By comparison, the frequency of price decreases is affected only to a smaller extent by higher inflation rates, as most price decreases are triggered by negative idiosyncratic shocks.

### 2.6.4 Intensive margin dominates the variance of inflation

A major purpose of studying price dynamics is to understand what drives fluctuations in inflation: Is it that the number of firms that change their prices varies or that firms change prices by different amounts? Put differently, are inflation dynamics driven by the extensive margin (EM) or the intensive margin (IM)? Figure 2.5 shows the time series predictions of the PP and the SP model for the average price adjustment and the frequency of price adjustment against the realized path of inflation. The extensive margin is relatively stable in both scenarios but its correlation with inflation is different across models: 0.49 in the PP and not significantly different from zero in the SP model. In both models and in the data, the intensive margin is more volatile and co-moves closely with inflation, which is in line with KK’s empirical findings.

To further explore this finding, KK use a decomposition of the inflation variance into terms capturing the intensive margin and terms capturing the extensive margin. More specifically, the decomposition is given by

\[
\text{var}(\pi_t) = \underbrace{\text{var}(dp_t) \bar{fr}^2}_{\text{IM term}} + \underbrace{\text{var}(fr_t) dp^2}_{\text{EM terms}} + 2 \bar{fr} dp \text{cov}(fr_t, dp_t) + O_t, \tag{2.19}
\]

where \(dp_t\) is the average relative price change, \(fr_t\) is the fraction of prices in a given period that are adjusted and the values with a bar correspond to time averages. \(O_t\) are higher-order terms that are functions of \(fr_t\).

The results from the variance decomposition are displayed in Table 2.5. Obviously,
2.6. INDIVIDUAL PRICES

Figure 2.5: Extensive and intensive margins vs. inflation

Notes: The blue solid line is the realized path of annualized inflation. The red dotted line represents the extensive margin, i.e. the frequency of price adjustments ($f_r$), divided by 10. The black dashed line stands for the intensive margin, i.e. the mean relative price change ($d_p$).

The SP model assigns all fluctuations in the inflation rate to the intensive margin because the frequency of price adjustments is fixed by assumption. The PP model also attributes positive weight to the extensive margin.

<table>
<thead>
<tr>
<th></th>
<th>IM (in percent)</th>
<th>EM (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KK</td>
<td>91</td>
<td>9</td>
</tr>
<tr>
<td>PP</td>
<td>83</td>
<td>17</td>
</tr>
<tr>
<td>SP</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.5: Variance decomposition: extensive margin vs. intensive margin

For completeness, we also consider another decomposition proposed by KK, which addresses the question whether fluctuations in inflation are the consequences of changes in price increases or price decreases. KK note that inflation can be written as

\[ \pi_t = f r_t^+ d p_t^+ - f r_t^- d p_t^- , \]  

(2.20)

where $f r_t^+$ and $f r_t^-$ denote the fractions of price changes that are increases or decreases at time $t$, respectively, and $d p_t^+$ and $d p_t^-$ denote the average magnitudes of increases and decreases. With the help of (2.20), the variance of inflation can be expressed in
the following way

\[
\text{var}(\pi_t) = \underbrace{\text{var}(fr^+_t dp^-_t) - \text{cov}(fr^+_t dp^-_t, fr^-_t dp^-_t)}_{\text{POS term}} + \underbrace{\text{var}(fr^-_t dp^-_t) - \text{cov}(fr^+_t dp^-_t, fr^-_t dp^-_t)}_{\text{NEG term}}.
\] (2.21)

As shown in Table 2.6, both models imply reasonable values for the POS and NEG terms defined in (2.21). Around 60% of the variance of inflation can be traced back to changes in price increases, the remaining 40% are due to changes in price decreases.

<table>
<thead>
<tr>
<th></th>
<th>POS (in percent)</th>
<th>NEG (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KK</td>
<td>59</td>
<td>41</td>
</tr>
<tr>
<td>PP</td>
<td>62</td>
<td>38</td>
</tr>
<tr>
<td>SP</td>
<td>59</td>
<td>41</td>
</tr>
</tbody>
</table>

Table 2.6: Variance decomposition: price increases vs. price decreases

2.6.5 Summary

In Table 2.7, we provide an arguably subjective summary of our findings from the comparison of the PP and the SP model. Importantly, while the PP model is quite successful in explaining several stylized facts of price adjustment, it does not imply a flat hazard curve, i.e. Fact 3, and fails to reproduce Fact 5, which involves that the mean magnitude of price changes does not change with the age of the price. We have already mentioned that both of these problems may arise because we have considered a particularly simple distribution of price points until now. In the next section, we will therefore study a version of our PP model that allows for a more general distribution of price points.41

2.7 General Distribution of Price Points

In this section we relax our assumption that the log price points for all firms are distributed on evenly spaced grids with distances that are identical across firms. By

41Alvarez et al. (2016) and Nakamura et al. (2018) document that price dispersion is unresponsive to inflation at low rates of inflation. Thus one might also be interested in the implications of our two models in this regard. For the measure of price dispersion \( s_t \), we have already highlighted that \( \hat{s}_t = 0 \) holds in the PP model for a log-linear approximation. By contrast, the SP model implies a value of \( \hat{s}_t \) that is typically different from zero (see (3.71)).
2.7. GENERAL DISTRIBUTION OF PRICE POINTS

<table>
<thead>
<tr>
<th>Facts</th>
<th>PP</th>
<th>SP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. regular prices stay constant on average for 2-3 quarters</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>2. variable price durations</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>3. flat hazard curves</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>4. large average magnitude of price changes (targeted in both cases)</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>5. size of price changes does not change with price duration</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>6. magnitude of price decreases exceeds the size of increases</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>7. approximately 40% of regular price changes are decreases</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>8. prices move back and forth between a few rigid values</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>9. frequency of price changes co-moves with inflation</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>10. the freq. of price increases co-varies strongly with inflation</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>11. the frequency of price decreases hardly changes with inflation</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>12. intensive margin dominates the variance of inflation</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 2.7: Sticky price economies vs. stylized facts of price adjustment

Notes: Sources of stylized facts: Facts 1-7, 9, 12 can be found in KK, fact 8 stems from Eichenbaum et al. (2011), facts 10 and 11 taken from Nakamura and Steinsson (2008) and Nakamura et al. (2018)

contrast, we assume that there are n different values $\Delta_1, \Delta_2, \ldots, \Delta_n$ of relative differences between price points with n positive weights $\rho_1, \rho_2, \ldots, \rho_n$ satisfying $\sum_{k=1}^{n} \rho_k = 1$. The set of firms $[0, 1]$ can be split into n subsets $[0, \rho_1), [\rho_1, \rho_1 + \rho_2), \ldots, [1 - \rho_n, 1]$. Firms in the kth interval can choose log prices $\hat{q}_t^j$ only from the set $\Delta_kZ$. The package sizes $u_j$ are uniformly arranged on $[0, \Delta_k]$ for these firms. It is straightforward to see that in this variant of our model, a firm’s optimal price continues to be given by (2.11), where $u_j$ and $\Delta$ have to be replaced by the appropriate values. Moreover, inflation still follows the sticky-information Philips curve (2.12).

It remains to determine the values for the $\Delta_k$’s and the $\rho_k$’s. For this purpose, we draw on the frequently used Dominick’s Finer Foods database. First, we define price points as all posted prices that make up at least 10% of all observations in a window of $\pm 10\%$
around the price.\footnote{The price points selected by this procedure represent 62% of all prices. This number is roughly in line with Levy et al. (2011) and Knotek (2016) who find that 9-ending prices account for about two thirds of price observations.} Second, we compute the relative difference when moving upward from one price point to the next one and weight the resulting differences with the relative frequencies of the observed price points. Figure 2.6 shows the different values of $\Delta_k$’s in the Dominick’s database and the corresponding weights $\rho_k$. The weighted average $\sum_{k=1}^{n} \rho_k \Delta_k$ equals approximately 7%. We observe that it is unnecessary to recalibrate $\alpha$ for the general distribution of $\Delta$’s, as aggregate inflation dynamics are unaffected by the distribution of $\Delta$’s. By contrast, we need to adjust the value of $\chi$. Following the same procedure as in Section 2.5, we obtain a value of $\chi = 1.96$ for the generalized PP model.

![Figure 2.6: Distribution of $\Delta$’s constructed from the Dominick’s Finer Foods Database](image)

We are now in a position to simulate price dynamics for our generalized PP model. As can be seen from the left panel of Figure 2.7, the pronounced peak in the hazard curve observed in Figure 2.2 vanishes almost completely. This is due to the fact that in the basic PP model, most firms who have not encountered an idiosyncratic shock for some time adjust their prices after approximately $\Delta/(0.76\%)$ quarters, where 0.76\% is the average quarterly inflation rate in our sample. With different values of $\Delta_k$, these adjustments occur after different numbers of periods $\Delta_k/(0.76\%)$, which smooths out the peak considerably. In a similar vein, the trough for the graph displaying the magnitude of relative price changes as a function of age, which we observed in Figure 2.3, largely disappears. The right panel of Figure 2.7 shows that the generalized PP model becomes now more consistent with the approximately flat profile as documented by KK and others.
2.7. GENERAL DISTRIBUTION OF PRICE POINTS

Figure 2.7: Comparison of the hazard rates and the average magnitude of price changes conditional on the price duration between the SP model and the PP model variant with multiple $\Delta$’s

Table 2.8: Simulation results including a generalized version of the PP model

We would like to mention that the more general distribution of $\Delta$’s does not qualitatively affect our results concerning the other stylized facts. In particular, Table 2.8 compares the main moments from the data with the simulation results for three different models: the PP model, the generalized PP model, and the SP model. It is obvious from the table that the generalized PP model’s predictions are in several cases even slightly closer to the data (KK) than those of the PP model.
2.8 Impulse Responses

We now turn to the aggregate dynamics of the PP and the SP model that are triggered by monetary-policy shocks. Figure 2.8 plots the impulse responses to an unanticipated permanent negative shock to the money supply. In the PP case, i.e. the model variant with price points and information rigidities, the impulse responses are largely identical to those in the standard sticky-information model presented in Mankiw and Reis (2002) for the particular value of $\alpha^{PP}$ resulting from our calibration. The impulse responses for the PP and the SP model are qualitatively similar except for the impulse response of inflation. This response is hump-shaped for the PP model, which Mankiw and Reis (2002) consider to be a major advantage of the sticky-information Phillips curve. We would like to stress again that a variant of our PP model without information stickiness would imply that monetary shocks have no effect on output. To sum up, the PP model does not only involve individual price dynamics that are broadly in line with the microeconomic evidence but entails responses of economic aggregates to monetary shocks that appear plausible as well.

![Figure 2.8: Impulse responses to a permanent negative money supply shock](image)

**Notes:** PP: dashed red lines. SP: black solid lines.

In the literature, there is a controversial debate about whether the sticky-information model or the sticky-price model are more in line with aggregate data. Some authors find that the sticky-price model is superior to the sticky-information model, provided that backward-looking agents are included (see e.g. Kiley (2007)). However, the introduction of backward-looking agents does not have strong microfoundations. Klenow

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43 A minor difference is that Mankiw and Reis (2002) consider $m_t - p_t = \ln \left( \frac{M}{P_t} \right) + \hat{Y}_t$ instead of the money demand (3.11).
and Willis (2007) find that firms’ price decisions depend on outdated information about aggregate shocks, which supports the idea that sticky information is important for understanding inflation dynamics.\textsuperscript{44} Another point of debate is the plausibility of the models’ policy implications in a liquidity trap. Kiley (2016) criticizes the sticky-price model in that regard and argues in favor of the sticky-information model, while Eggertsson and Garga (2017) claim that the sticky-information model may have similar policy implications as its sticky-price counterpart.

Interestingly, Coibion (2010) highlights that sticky-information models typically imply a delayed response of inflation to aggregate shocks, whereas sticky-price models predict a quick response. As empirically the response of inflation to monetary shocks occurs with a lag but the response to productivity shocks occurs fast, the sticky information model can explain the response to monetary shocks but predicts a too sluggish response to aggregate technology shocks.

In our model, there are aggregate monetary shocks but no aggregate technology shocks. If one wanted to include aggregate technology shocks into our model, an interesting avenue would be to take up an idea due to Coibion (2010, p. 100) and to assume that firms become more readily informed about aggregate productivity shocks compared to monetary shocks. Making such an assumption might entail a model with a fast response of inflation to productivity shocks but a delayed response to monetary disturbances.

\section*{2.9 Relationship to Menu-Cost Models}

In this paper, we use a model with Calvo pricing as a benchmark for our comparisons but one might also ask how the PP model would fare against a variant with menu costs. While a rigorous analysis of such a model variant is beyond the scope of this paper, we offer a few thoughts about the relationship between our PPR and menu costs.

In some respects, menu costs and a PPR lead to similar predictions. For example, in the absence of idiosyncratic and aggregate shocks, both modeling approaches entail that all prices move upwards in a step-wise manner under positive trend inflation. Due to the selection effect, which involves that only the prices farthest away from their optimal values adjust, basic menu-cost models predict that changes in money growth rates have no effect on real variables (see Caplin and Spulber (1987)). This is closely related to the observation that our model with a PPR but without information frictions would imply that monetary policy has purely nominal effects. Moreover, some

\textsuperscript{44}Dupor et al. (2010) show that a dual-stickiness model outperforms a hybrid sticky price model. Kaufmann and Lein (2013) find support in favor of a multi-sector sticky price model compared to a model with rational inattention.
empirical findings like the larger sizes of price decreases compared to increases could be explained by menu cost models as well (see NS).

However, menu costs and our PPR do not always lead to identical predictions. Menu costs involve that relatively small price changes within a certain interval do not occur but that all price changes from a continuum outside this interval may occur.\textsuperscript{45} By contrast, a PPR requires that all price changes come in discrete steps. It is because of this difference that Knotek (2016) finds that price points are more relevant for understanding price dynamics than menu costs.

Finally, it is well known that jumps in the price level would induce many firms to adjust their prices simultaneously in a menu-cost model (see Caplin and Spulber (1987, p. 720)). At least temporarily, these coordinated price changes would substantially reduce any price dispersion that is not driven by differences in productivities or similar fundamental factors.\textsuperscript{46} This arguably implausible effect does not occur under a PPR.

\section{Conclusion}

Kashyap (1995), Blinder et al. (1998) and Levy et al. (2011) have identified the empirical regularity that price points are relevant for firms’ price setting decisions. Based on this observation, Knotek (2016) has shown that price points rather than menu costs may be responsible for extended price spells. However, a model where price points are the only source of price stickiness has the implication that monetary policy has no real effects, which contradicts the widespread consensus in monetary economics that central banks can influence real output in the short run.

As a consequence, this paper has proposed a model featuring a prominent role for price points as well information stickiness. Due to the presence of sticky information, monetary policy has real effects in our model. At the same time, our model can reproduce many stylized facts of price-setting, which cannot be easily reconciled with time-dependent pricing models such as those based on Calvo pricing. For example, our model is in line with the findings that the frequency of price adjustment is positively related to inflation and that the magnitude of price decreases exceeds the size of increases. By construction, it is also compatible with the observation that prices jump back and forth between a few rigid values.

One can also look at our approach from a slightly different angle and interpret it as

\textsuperscript{45}For stochastic menu costs, the length of this interval may not be constant.

\textsuperscript{46}In a similar vein, menu cost models would predict that the introduction of a new currency like the Euro in many European countries would lead to a temporary reduction in price dispersion.
a way of making sticky-information models more consistent with empirical findings about price dynamics. For example, Maćkowiak and Wiederholt (2009, p. 798) note that their model of rational inattention is not compatible with spells of constant prices, unless additional frictions such as menu costs are included. The PPR proposed in this paper can be viewed as a simple alternative mechanism that generates sticky individual prices in models of rational inattention.

Our framework could also be used to examine the impact of a change in trend inflation on price dynamics. The PP model would entail that price changes are more frequent and have a smaller mean magnitude if the trend rate of inflation is raised in a comparative statics exercise. This follows from the observation that the comparably small price adjustments that are necessary from time to time to catch up with increases in the price level occur more frequently when inflation is higher. By contrast, the SP model would predict that the frequency of price changes is not affected by changes in trend inflation and that the magnitude of price changes increases with inflation. The evidence presented by Wulfsberg (2016) for Norway, namely that prices change more frequently and in smaller steps in periods of high inflation compared to periods of low inflation, appears to be more in line with the predictions of the PP model. Using our model to carefully examine how changes in trend inflation affect price dynamics would be an interesting avenue for future research.

**Outlook** In the version of the paper presented in this chapter we rely on empirical evidence about microeconomic price setting for the U.S. from different data sources. More precisely, the Dominick’s database consists mostly of grocery items while KK or NS focus on all goods in the CPI. In order to avoid issues concerning the compatibility of moments across different data sets, in the appendix we provide an extension of the paper which is primarily based on aggregating the data sources and focusing exclusively on micro price data in the U.K. In particular, we generate all pricing moments from micro data using the ONS database on our own and do not refer to KK or other sources in the first place.

The extended paper is also calibrated in a different way. More specifically, we do not estimate the probability of being hit by an idiosyncratic productivity shock from the inflation data using the approach in Sbordone (2002). Instead, we calibrate the probability of a firm-specific shock and the support of the shock’s distribution using the method of simulated method of moments to match the quarterly frequency and average absolute magnitude of price adjustment, respectively. We also estimate the distribution of price points for the U.K.

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47 To be more precise, this is true for small and moderate inflation. For very large inflation rates, almost all prices are raised every period. In such a situation, the size of price changes would increase with inflation.
We find that our main result that the combination of price points and sticky information can explain many stylized facts about price setting found in the micro data is robust to changing the focus to the UK. The SP model still predicts that price decreases move too much with inflation, it cannot capture that the frequency of price adjustment moves with inflation, that the magnitude of price decreases is larger than the magnitude of increases and it cannot explain the back-and-forth movement of prices.

Some additional results are noteworthy. In the extended version of the paper, we quantify the back-and-forth movement of prices to examine to which extent the discussed model variants can capture this pattern not only qualitatively but also quantitatively. Following Ilut, Valchev, and Vincent (2016) we compute a measure of price memory, i.e. the probability that when a firm resets the price of its product, the new price is one that was visited in the past. We find that the PP model can explain the back-and-forth pattern very well. In particular, we estimate from the ONS micro data that the probability of revisiting a price which has been posted in the last two years is between 12 and 18%. Our PP model predicts a probability of 17% whereas the SP model predicts zero probability.

For quarterly micro data in the U.K., we find that the hazard rate, i.e. the probability of adjusting a price conditionally on its duration, is increasing. KK document for the monthly prices in the U.S. a flat hazard rate. Our PP model is consistent with this finding whereas the SP model, featuring Calvo pricing with constant probability of price adjustment, cannot replicate this finding by construction.

Overall, the extension of this chapter based on micro price data in the U.K. supports our result that price points can explain patterns of microeconomic price adjustment reasonably well and that results discussed in this chapter continue to hold for alternative calibrations and different time periods.
A. Appendix of Chapter 2

A.1 Households’ Optimality Conditions

In this section, we state the first-order conditions that describe the optimal behavior of households. Minimizing costs for a given size of the consumption basket $C_t$ yields the demand function

$$C_{j,t} = \left( \frac{Q_{j,t}}{P_t} \right)^{-\varepsilon} C_t,$$

where the aggregate price level $P_t$ is given by

$$P_t = \left[ \int_0^1 (Q_{j,t})^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$  \hspace{1cm} (2.23)

The household’s utility maximization problem results in the following standard conditions:

$$\frac{W_t}{P_t} = N_i^\varepsilon C_t,$$  \hspace{1cm} (2.24)

$$\mathbb{E}_t \left[ \beta \frac{C_t}{C_{t+1}} \frac{R^n_t P_t}{P_{t+1}} \right] = 1,$$  \hspace{1cm} (2.25)

$$\frac{M_t}{P_t} = \nu C_t \frac{R^n_t}{R^n_t - 1}. $$  \hspace{1cm} (2.26)

A.2 Firms’ Optimal Pricing Decisions

Price setting in the PP model

In this appendix, we determine firms’ optimal price-setting behaviors. Equation (3.77) and the demand function (3.34) can be used to write the profit function (2.6) as

$$\Pi_{j,t} = \frac{1}{1 + \tau_t} \left( \frac{Q_{j,t}}{P_t} \right)^{1-\varepsilon} Y_t - \frac{W_t A_t}{P_t} \frac{1}{\gamma} Y_t^{-\frac{1}{\gamma}} \left( \frac{Q_{j,t}}{P_t} \right)^{-\frac{1}{\gamma}} Y_t^{-\frac{1}{\gamma}}.$$  \hspace{1cm} (2.27)
A. APPENDIX OF CHAPTER 2

Producer $j$’s first order condition is

$$
1 - \varepsilon \left( \frac{Q_{j,t}^*}{P_t} \right)^{-\gamma} Y_t + \varepsilon \frac{W_t}{P_t} A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} \left( \frac{Q_{j,t}^*}{P_t} \right)^{-\frac{\gamma}{\gamma - 1}} Y_t^{\frac{1}{\gamma}} = 0, \quad (2.28)
$$

from which we obtain the optimal price $Q_{j,t}^*$ that would be chosen if there were no information frictions and if prices were not restricted to the set of price points:

$$
\left( \frac{Q_{j,t}^*}{P_t} \right)^{\frac{\gamma}{\gamma - 1}} = \frac{\varepsilon}{(\varepsilon - 1)\gamma} \left(1 + \tau_t\right) \frac{W_t}{P_t} A_t^{-\frac{1}{\gamma}} X_{j,t}^{-\frac{1}{\gamma}} Y_t^{\frac{1}{\gamma}} \quad (2.29)
$$

A log-linear approximation of this condition yields the following hypothetical optimal log price in the absence of a PPR and information rigidities:

$$
q_{j,t}^{PP} = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left[-\frac{1}{\gamma} x_{j,t} + \hat{\tau}_t + w_t - p_t - \ln \left(\frac{W_t}{P_t}\right) + \frac{1 - \gamma}{\gamma} \hat{Y}_t - \frac{1}{\gamma} \hat{A}_t\right] + p_t \quad (2.30)
$$

We observe that the expression $w_t - p_t - \ln \left(\frac{W_t}{P_t}\right) + \frac{1 - \gamma}{\gamma} \hat{Y}_t - \frac{1}{\gamma} \hat{A}_t$ represents the deviation of aggregate unit labor costs from its steady-state value in this economy, $\hat{w} = w_t - p_t - \ln \left(\frac{W_t}{P_t}\right) + \hat{N}_t - \hat{Y}_t$, since the log-linearized aggregate production function is given by $\hat{N}_t = \frac{1}{\gamma} \hat{Y}_t - \frac{1}{\gamma} \hat{A}_t$.\(^{48}\) Therefore the hypothetical optimal log price in the absence of a PPR and information frictions is given by

$$
q_{j,t}^{PP} = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left[-\frac{1}{\gamma} x_{j,t} + \hat{\tau}_t + \hat{w} \right] + p_t. \quad (2.31)
$$

In the following we analyze optimal price setting behavior under the assumption that firm $j$ can only select price points. Consider a quadratic approximation of the profit function around its maximum. Then, given that firm $j$ has last updated its information in period $t - i$, its profit-maximizing admissible log price $q_{j,t}$ is the element in the set $\Delta \cdot Z - u_j$ that is closest to $E_{t-i}[q_{j,t}^{PP}]$.

Hence, a firm $j$ that has received new information $i$ periods ago selects

$$
q_{j,t}^{PP} = T_j \left\{ \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left[-\frac{1}{\gamma} x_{j,t} + \hat{\tau}_t + \hat{w} \right] + E_{t-i} \left[ p_t \right] \right\}. \quad (2.32)
$$

Note that we have used the fact that $x_{j,t}$ does not change in periods where the firm is

---

48Note that price dispersion $s_t = \int_{t-1}^t \left( \frac{Q_{j,t}}{P_t} \right)^{-\gamma/\gamma} (X_{j,t})^{-1/\gamma} dj$ affects the relationship between employment and output, as $Y_t = (A_t N_t^\gamma)/s_t$ (see Ascari and Sbordone (2014)). However, $s_t$ reaches a minimum in the steady state of the PP model, which means that small perturbations have no first-order effect on $s_t$ and thus $\hat{Y}_t = \hat{A}_t + \gamma \hat{N}_t$ holds approximately. The same result does not hold under Calvo pricing when the steady-state inflation rate is positive.
not subject to idiosyncratic shocks, i.e. \( E_{t-i} [x_{j,t}] = x_{j,t} \).

### Sticky-information Phillips curve

In this appendix, we derive the Phillips curve for the PP model. Recall that firms update their information if and only if they are affected by an idiosyncratic shock, which happens with probability \( 1 - \alpha_{PP} \) in each period.

Let \( q_t \) be the average log price of firms that are hit by an idiosyncratic shock in period \( t \), which implies an update of firms’ information sets. Because the individual differences of \( q_{j,t} \) from \( q^*_{j,t} \) wash out in the aggregation\(^{49}\) and because the average value of \( x_{j,t} \) is always zero, this price can be written as

\[
q_t = \int_0^1 q_{j,t} dj = \int_0^1 q^*_{j,t} dj = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left( \hat{\pi}_t + \hat{u}_t c_t \right) + p_t, \tag{2.33}
\]

where we have utilized equation (2.31).

Moreover, we note that firms choose prices \( E_{t-i}[q_t] \) on average if they were hit by a shock \( i \) periods ago. Hence the log price level can be written as

\[
p_t = (1 - \alpha_{PP}) \sum_{i=0}^{\infty} (\alpha_{PP})^i E_{t-i}[q_t]
= (1 - \alpha_{PP}) \sum_{i=0}^{\infty} (\alpha_{PP})^i E_{t-i} \left[ \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left( \hat{\pi}_t + \hat{u}_t c_t \right) + p_t \right], \tag{2.34}
\]

where we have used (2.33) to replace \( q_t \). Equation (2.34) is equivalent to the expression obtained in Mankiw and Reis (2002, p. 1300). Therefore it can be used to formulate a sticky-information Phillips curve analogous to the one obtained by them:

\[
\hat{\pi}_t = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \frac{1 - \alpha_{PP}}{\alpha_{PP}} \left( \hat{\pi}_t + \hat{u}_t c_t \right)
+ (1 - \alpha_{PP}) \sum_{i=0}^{\infty} (\alpha_{PP})^i E_{t-i} \left[ \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \left( \hat{\pi}_t - \hat{\pi}_{t-1} + \hat{u}_t c_t - \hat{u}_{t-1} c_{t-1} \right) \right] \tag{2.35}
\]

\(^{49}\)Note that \( q_{j,t} = q^*_{j,t} + d_{j,t} \) where \( d_{j,t} \) denotes the distance between the optimal price and the closest price point. We observe that \( d_{j,t} \sim U[-\frac{\Delta}{2}, \frac{\Delta}{2}] \) given that \( u_j \sim U[0, \Delta] \).
B. ONS MICRO PRICE DATABASE

B. ONS micro price database

B.1 Description of the data set

The CPI micro dataset of the United Kingdom’s Office for National Statistics (ONS) is used to compute the official CPI for the UK. It consists of monthly prices of on average (median) 556 (560) goods and services.\(^{50}\) The data set starts in January 1996 and is continuously updated till today. Prices are collected from over 14,000 retail stores and 13 regions across the UK. The goods and services in the CPI are classified according to the Classification of Individual Consumption by Purpose (COICOP) standards in the following way:

- Coicop 2 (Divisions): food and non-alcoholic beverages, health, ...
- Coicop 3 (Classes): bread, meat, fish, ...
- Coicop 4 (Items): large loaf-white-sliced-800g, french baguette, rice-long-grain-white 500g, houseplant, child’s trousers (18 months - 4 years)

Time span One of the main advantages of this dataset is its availability. Although the ONS micro CPI database starts already in January 1996, we follow Kryvtsov and Vincent (2017, henceforth KV) and begin the sample in February 1996. Another reason why to begin in February 1996 is that the stratum weights which are necessary for the calculation of the item indices are provided by ONS only from February 1996 onward. The database is continuously updated till today. Our sample period is February 1996 till December 2016, i.e. 251 months.

House prices are not part of the sample. The housing portion of the consumer prices (mortgage interest payments, house depreciations, insurance and other house purchase fees) is not part of the sample, as these prices are not collected by the third-party contractor. The only related price category is house contents insurance which is part of the COICOP class insurance.

\(^{50}\)ONS uses prices for ca. 700 goods a month, however, it can only practically publish price quote data for prices collected by their third party contractor and not those from its central collection modes.
B.2 Steps of the data preparation process

In the following we list the particular steps of the data preparation process. We elaborate on each of them in subsequent sections.

1. download raw data from the ONS webpage which is not in the time series form
2. establish time series (see subsection B.3)
3. discard all prices which are not used by the ONS for calculating the CPI (see subsection B.4)
4. fill in gaps of up to two months and apply the sales filters (see subsection B.6)
5. establish quarterly time series by selecting the price at the end of a quarter
6. compute time series moments
7. compute the hazard rates
8. compute the distribution of price points

B.3 Time series of individual product prices

As mentioned earlier, the ONS CPI micro data is not provided in the time series form. Therefore, we need to identify unique products and track them over time to establish time series. 52 We define a product as a quintuple of the item ID, the shop ID, the shop type, the region ID and the stratum type. 53

Note that due to confidentiality, ONS is not publishing a variable location. As a consequence, sometimes two or more prices are observed for the same product in the same period. Because in such cases we cannot identify a unique price to continue the product’s time series, we discard the product from our analysis completely.

By doing so we discard 4.3 Mio observations, or 15.5% of price observations. However, even without these observations we are able to replicate the published item indices almost perfectly. This elimination is necessary since otherwise statistics on frequency and magnitude of price adjustment could be spurious.

Table 2.9 provides an overview about the main descriptive statistics for the preprocessed data set after establishing time series.

51 This has no qualitative effect on our results. Quantitatively, the results are very similar and are available upon request.
52 KK call them quote-lines.
53 Our way of identifying and tracking goods over time has been confirmed by ONS.
B. ONS MICRO PRICE DATABASE

Number of all price observations 24,525,632
Number of items 1,233
Number of regions 13
Number of shops 2,770
Number of time series 687,212

Table 2.9: Descriptive statistics - Raw data

B.4 Validation

We use only price quotes which are also used by ONS to compute the official U.K. CPI. ONS applies the following criteria:

1. the posted price as well as the corresponding base price must be valid meaning that the so-called validity status must be 3, 4, 5 or 54 (3 or 4 for base prices),\textsuperscript{54}

2. after a non-comparable substitution the product is excluded for two periods.

In addition, we eliminate price quotes which equal to 0.01 pound. The reason is that these prices do not actually occur in reality but e.g. if some fee is abolished, the statistical office introduces the price of 0.01 to account for the decrease in prices.

Implementation of these criteria reduces the number of observations (Table 2.10), yet does not affect the number of shops, items and regions.

Number of all price observations 22,824,995
Number of items 1,233
Number of regions 13
Number of shops 2,770
Number of time series 682,821

Table 2.10: Descriptive statistics - Validated data

Notes: Sample 1996:02 till 2016:12. ONS validation rules applied.

After validation, the mean (median) number of items per month is 555 (558).

B.5 Substitutions

A product substitution occurs if a product becomes permanently unavailable. Then, if there exists a product with the same basic attributes as the old one, it is marked ‘C’\footnote{Data on the validity status of base prices is missing up to year 2003. Therefore, we assume that all missing base price validity statuses are as if valid. We verify this assumption by replicating the officially published ONS item indices using the micro price data reasonably well.}
comparable and the price chain continues. In the case of a non-comparable product being selected, it is marked ‘N’, the price chain is broken and a new base price imputed.

In the ONS data set, there is information on four types of substitutions: C (comparable), X (comparable on sale), N (non-comparable), Z (non-comparable on sale). Table 2.11 provides an overview about their frequencies in the data. The unweighted share of C substitutions among all valid observations is 6.3%.

<table>
<thead>
<tr>
<th>Frequency of Substitutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of N substitutions</td>
</tr>
<tr>
<td>Number of C substitutions</td>
</tr>
<tr>
<td>Number of X substitutions</td>
</tr>
<tr>
<td>Number of Z substitutions</td>
</tr>
</tbody>
</table>

Table 2.11: Frequency of substitutions

B.6 Sales filters

As highlighted in Bils and Klenow (2004), KK or NS, the extent of price rigidity is highly sensitive to the treatment of temporary price changes. Following the literature we therefore compute the frequency and magnitude of price adjustments not only for posted prices but also for regular prices, i.e. price series net of temporary price changes.

To compute regular prices we apply the following sales filters:

1. **Sales flag filter**
   - if a sales flag "S" is observed, the unobserved regular price during a sale is given by the last observed non-sales price, conditionally on the price change being a price decrease.

2. **V-shaped filter** à la Nakamura and Steinsson (2008, QJE, Five facts about prices)
   - see More Facts About Prices, Supplement to: “Five Facts About Prices: A Reevaluation of Menu Cost Models”, Appendix A
   - a sale episode begins with a price drop and ends as soon as a price increase is registered, as long as this price increase occurs within three months.

   - see More Facts About Prices, Supplement to: “Five Facts About Prices: A Reevaluation of Menu Cost Models”, Appendix B

One general note, none of the filters fills in gaps between prices observations.
C    Evidence for the UK

C.1    Price points in the UK

In this section we estimate the distribution of price points in the ONS database. We use the fact that the data contains information already on the shop and stratum level. The general idea is to estimate the price points distribution (and thus the distribution of $\Delta$’s) for each shop and then to aggregate them up. We apply the same rule as in the main text, i.e. in a neighborhood of $\pm 10\%$ at least 10\% of price observations must be explained by the particular price to be identified as a price point. Figure 2.9 shows the $\Delta$-distribution in the ONS database for regular prices obtained by applying the sales-flag filter (see Section B.6).

Figure 2.9: Aggregate distribution of $\Delta$’s in the ONS database
C.2 Additional evidence on price endings in the ONS database

In the main text we provide evidence on the distribution of price endings and the most frequent prices in the ONS database across categories. However, as generally documented, the heterogeneity across the categories is substantial. To shed light on these issues we provide some additional evidence on price endings by reporting the distribution of price endings (Figure 2.10) and the most frequent prices (Table 2.12) in all twelve COICOP2 categories in the ONS database.

Figure 2.10: Distribution of price endings in COICOP2 categories
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Unweighted fraction among observations</td>
<td>24.9</td>
<td>4.4</td>
<td>17.9</td>
<td>3.9</td>
<td>13.5</td>
<td>1.9</td>
<td>4.2</td>
<td>0.3</td>
<td>11.0</td>
<td>0.1</td>
<td>11.3</td>
<td>8.5</td>
</tr>
<tr>
<td>Share of prices &lt; £100</td>
<td>100</td>
<td>100</td>
<td>95.8</td>
<td>77.1</td>
<td>69.6</td>
<td>90.2</td>
<td>85.2</td>
<td>99.4</td>
<td>86.3</td>
<td>100</td>
<td>99.9</td>
<td>83.0</td>
</tr>
<tr>
<td>Rank 1</td>
<td>0.99</td>
<td>3.99</td>
<td>10.00</td>
<td>30</td>
<td>19.99</td>
<td>45</td>
<td>45</td>
<td>29.99</td>
<td>9.99</td>
<td>2.50</td>
<td>2.00</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>(2.42)</td>
<td>(2.86)</td>
<td>(4.06)</td>
<td>(1.99)</td>
<td>(2.34)</td>
<td>(3.80)</td>
<td>(2.23)</td>
<td>(5.39)</td>
<td>(2.71)</td>
<td>(5.66)</td>
<td>(2.14)</td>
<td>(1.57)</td>
</tr>
<tr>
<td></td>
<td>(2.29)</td>
<td>(1.82)</td>
<td>(3.62)</td>
<td>(1.84)</td>
<td>(1.70)</td>
<td>(3.29)</td>
<td>(2.11)</td>
<td>(4.38)</td>
<td>(2.41)</td>
<td>(5.64)</td>
<td>(1.93)</td>
<td>(1.55)</td>
</tr>
<tr>
<td>Rank 3</td>
<td>0.89</td>
<td>2.99</td>
<td>12.00</td>
<td>35</td>
<td>24.99</td>
<td>50</td>
<td>3.99</td>
<td>34.99</td>
<td>4.99</td>
<td>5.00</td>
<td>1.50</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(1.96)</td>
<td>(1.65)</td>
<td>(3.38)</td>
<td>(1.80)</td>
<td>(1.61)</td>
<td>(3.26)</td>
<td>(2.05)</td>
<td>(4.10)</td>
<td>(2.38)</td>
<td>(5.33)</td>
<td>(1.86)</td>
<td>(1.35)</td>
</tr>
<tr>
<td>Rank 4</td>
<td>1.99</td>
<td>4.49</td>
<td>20.00</td>
<td>40</td>
<td>14.99</td>
<td>40</td>
<td>5.99</td>
<td>2.99</td>
<td>7.99</td>
<td>3.00</td>
<td>1.80</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>(1.53)</td>
<td>(1.03)</td>
<td>(3.32)</td>
<td>(1.62)</td>
<td>(1.47)</td>
<td>(3.19)</td>
<td>(2.05)</td>
<td>(3.84)</td>
<td>(2.36)</td>
<td>(5.12)</td>
<td>(1.61)</td>
<td>(1.29)</td>
</tr>
<tr>
<td>Rank 5</td>
<td>0.69</td>
<td>5.99</td>
<td>35.00</td>
<td>45</td>
<td>9.99</td>
<td>35</td>
<td>40</td>
<td>24.99</td>
<td>5.99</td>
<td>2.00</td>
<td>2.20</td>
<td>4.99</td>
</tr>
<tr>
<td></td>
<td>(1.49)</td>
<td>(0.93)</td>
<td>(2.59)</td>
<td>(1.51)</td>
<td>(1.47)</td>
<td>(2.85)</td>
<td>(2.03)</td>
<td>(3.65)</td>
<td>(1.99)</td>
<td>(5.06)</td>
<td>(1.51)</td>
<td>(1.27)</td>
</tr>
</tbody>
</table>

Table 2.12: Most frequent observations at the category level

Numbers in brackets represent the weighted fractions of a price among all price observations in the given category.
C.3 Patterns of price adjustment in the UK

In this section we report the statistics describing the patterns of price setting in the UK using the ONS database. We focus on quarterly data which were constructed by taking the last monthly price observation in a given quarter. In case of substitutions, the whole quarter is characterized by the substitution flag. We drop three quarters from the sample: Q4 2008, Q1 2010 and Q1 2011. There were changes in the VAT rates in all these quarters.

We report the moments from the data for posted prices as well as filtered prices using the filters described in subsection B.6. In particular, we apply the sales flag filter (henceforth: SF) which only uses information available from ONS and the Kehoe-Midrigan filter (henceforth: KM) for illustrative purposes. We target the frequency and magnitude of SF prices. Noteworthy, price adjustments due to forced substitutions are excluded from all statistics, if not stated otherwise.

We directly compare the moments in the data with the respective moments generated by the following model variants:

1. **PPSI** - this is our main model featuring price points and sticky information. This is the PP model in the main text,
2. **PP** - this model includes only price points but no sticky information,
3. **SP** - this model is the SP model from the main text featuring Calvo pricing,
4. **PPSP** - this model combines Calvo pricing and price points.

For each model we calibrate the probability of being hit by an idiosyncratic productivity shock $\alpha$ and the support of idiosyncratic shocks $\chi$ to match the frequency and the average absolute magnitude of price adjustment in the data, respectively.

<table>
<thead>
<tr>
<th>model</th>
<th>PPSI</th>
<th>PP</th>
<th>SP</th>
<th>PPSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>0.74</td>
<td>0.69</td>
<td>0.57</td>
</tr>
<tr>
<td>$\chi$</td>
<td>2.58</td>
<td>2.40</td>
<td>1.95</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Table 2.13: Calibration of $\alpha$ and $\chi$ for each model variant
Frequency and magnitude of price adjustment

In this section we provide an overview about the unconditional moments of the price setting behavior in the UK. Table 2.14 shows statistics concerning the frequency of price adjustment. Tables 2.15 and 2.16 illustrate the magnitude of price adjustment. Figure 2.11 shows the distribution of price changes in the data and according to the model simulations. Table 2.17 shows the corresponding moments of the distribution of price changes. All tables and figures are self-explanatory and complementary to the discussion of stylized facts of price setting in the main text in Section 2.6.1.

<table>
<thead>
<tr>
<th>Source</th>
<th>mean (fr)</th>
<th>mean (fr^+)</th>
<th>mean (fr^-)</th>
<th>ratio (fr^+/fr^-)</th>
<th>share decr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posted prices</td>
<td>0.295</td>
<td>0.192</td>
<td>0.103</td>
<td>1.72</td>
<td>0.350 (0.357)</td>
</tr>
<tr>
<td>SF</td>
<td><strong>0.259</strong></td>
<td><strong>0.179</strong></td>
<td><strong>0.080</strong></td>
<td><strong>2.2</strong></td>
<td><strong>0.309</strong> (0.317)</td>
</tr>
<tr>
<td>KM-filter</td>
<td>0.232</td>
<td>0.160</td>
<td>0.072</td>
<td>2.2</td>
<td>0.311 (0.318)</td>
</tr>
<tr>
<td>models</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP-SI(^1)</td>
<td>0.259(^2)</td>
<td>0.190</td>
<td>0.068</td>
<td>2.8</td>
<td>0.264 (0.264)</td>
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<tr>
<td>SP</td>
<td>0.259(^2)</td>
<td>0.139</td>
<td>0.120</td>
<td>1.2</td>
<td>0.463 (0.463)</td>
</tr>
<tr>
<td>PP-SP(^1)</td>
<td>0.259(^2)</td>
<td>0.143</td>
<td>0.116</td>
<td>1.2</td>
<td>0.447 (0.463)</td>
</tr>
<tr>
<td>PP(^1)</td>
<td>0.259(^2)</td>
<td>0.180</td>
<td>0.078</td>
<td>2.3</td>
<td>0.303 (0.303)</td>
</tr>
</tbody>
</table>

Table 2.14: Frequency of price adjustments

Notes: Quarterly data. All numbers are fractions. \(fr\) = fraction of items with changing prices, \(fr^+\) = fraction of price increases, \(fr^-\) = fraction of price decreases. Changes due to substitutions are excluded both in frequency and magnitude statistics. ONS sample 1996:Q1 till 2016:Q4. Frequency is computed as an average of quarterly frequencies. We weight the changes with the corresponding CPI item weights augmented for product specific stratum and shop weights. The weights sum up to one in every quarter. Unless stated otherwise, prices in quarters in which there was a VAT change have been discarded.

1 Price points have been identified using quarterly SF prices.
2 Targeted moment.
3 The numbers in brackets are computed as the fraction of price changes among all ever observed price changes, whereas the numbers outside of brackets are averages of quarterly fractions.
## C. EVIDENCE FOR THE UK

### Table 2.15: Magnitude of price adjustment

<table>
<thead>
<tr>
<th>Source</th>
<th>mean</th>
<th>median</th>
<th>std</th>
<th>mean increases</th>
<th>mean decreases</th>
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<tr>
<td><strong>data</strong></td>
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</tr>
<tr>
<td>Posted prices</td>
<td>0.131</td>
<td>0.071</td>
<td>0.167</td>
<td>0.121</td>
<td>0.150</td>
</tr>
<tr>
<td><strong>SF</strong></td>
<td>0.103</td>
<td>0.059</td>
<td>0.135</td>
<td>0.096</td>
<td>0.118</td>
</tr>
<tr>
<td>KM-filter</td>
<td>0.114</td>
<td>0.064</td>
<td>0.147</td>
<td>0.102</td>
<td>0.139</td>
</tr>
<tr>
<td><strong>models</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP-SI</td>
<td>0.103^1</td>
<td>0.085</td>
<td>0.091</td>
<td>0.082</td>
<td>0.161</td>
</tr>
<tr>
<td>SP</td>
<td>0.103^1</td>
<td>0.102</td>
<td>0.060</td>
<td>0.110</td>
<td>0.095</td>
</tr>
<tr>
<td>PP-SP</td>
<td>0.103^1</td>
<td>0.100</td>
<td>0.053</td>
<td>0.108</td>
<td>0.097</td>
</tr>
<tr>
<td>PP</td>
<td>0.103^1</td>
<td>0.085</td>
<td>0.085</td>
<td>0.087</td>
<td>0.141</td>
</tr>
</tbody>
</table>

Notes: Quarterly data. ONS sample 1996:Q1 till 2016:Q2. Absolute size of price changes is computed as a weighted average of all price changes without taking quarterly averages first. Unless stated otherwise, price changes in quarters in which there was a VAT change have been discarded. ^1 Targeted.

### Table 2.16: Small price changes

| Source          | |dp| < 5.0% | |dp| < 2.5% | |dp| < 1% |
|-----------------|----------------|--------|--------|--------|
| **data**        |                |        |        |        |
| Posted prices   | 0.380          | 0.194  | 0.051  |        |
| **SF**          | **0.437**      | **0.224**| **0.060**|        |
| KM-filter       | 0.409          | 0.208  | 0.052  |        |
| **models**      |                |        |        |        |
| PP-SI           | 0.402          | 0.281  | 0.178  |        |
| SP              | 0.245          | 0.121  | 0.049  |        |
| PP-SP           | 0.143          | 0.033  | 0.005  |        |
| PP              | 0.373          | 0.246  | 0.113  |        |

Notes: Quarterly data. All numbers are fractions. Weighted.
Figure 2.11: Distribution of price changes

<table>
<thead>
<tr>
<th>Source</th>
<th>mean</th>
<th>std</th>
<th>skewness</th>
<th>curtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF</td>
<td>0.029</td>
<td>0.167</td>
<td>1.693</td>
<td>14.658</td>
</tr>
<tr>
<td>PP-SI</td>
<td>0.018</td>
<td>0.136</td>
<td>-0.227</td>
<td>3.134</td>
</tr>
<tr>
<td>SP</td>
<td>0.015</td>
<td>0.118</td>
<td>-0.003</td>
<td>1.883</td>
</tr>
</tbody>
</table>

Table 2.17: Moments of the distribution of price changes
Frequency and magnitude of price changes conditional on the age of the price

This section shows the hazard rate, i.e. the probability of price adjustment as a function of the age of the price (Figure 2.12) and the size of price adjustment as a function of the age of the price (Figure 2.13) for the UK. Results presented here are complementary to the discussion in Section 2.6.2.

**Figure 2.12: Hazard rates of price adjustment**

Notes: Including decile fixed effects. In the regression for the slope coefficient we include a dummy for year fixed effects, i.e. for every fourth quarter.
Figure 2.13: Magnitude of price adjustment conditional on age

Notes: Including decile fixed effects. For the SP model, the percentile fixed effects are not taken into account since there is no heterogeneity in the SP model.
Price adjustment and inflation

The results presented in this section are complementary to the Section 2.6.3. Table 2.18 and Figure 2.14 illustrate the comovement of the frequency and the size of price adjustment with inflation. 2.15 shows how the frequency of increases and the frequency of decreases comove with inflation.

<table>
<thead>
<tr>
<th>variable</th>
<th>filter</th>
<th>mean (%)</th>
<th>std dev (%)</th>
<th>correlation</th>
<th>$\beta_x$ (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$fr_t$</td>
<td>data</td>
<td>Posted</td>
<td>29.50</td>
<td>4.39</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SF</td>
<td>25.88</td>
<td>4.71</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KM</td>
<td>23.20</td>
<td>4.09</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>models</td>
<td>PPSI</td>
<td>25.87</td>
<td>2.04</td>
<td>0.41</td>
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<td></td>
<td></td>
<td>SP</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>PP</td>
<td>25.88</td>
<td>7.3</td>
<td>0.68</td>
</tr>
<tr>
<td>$dp_t$</td>
<td></td>
<td>Posted</td>
<td>2.48</td>
<td>1.22</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SF</td>
<td>2.85</td>
<td>1.18</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KM</td>
<td>2.54</td>
<td>1.23</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PPSI</td>
<td>1.79</td>
<td>0.84</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>1.47</td>
<td>0.85</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PP</td>
<td>1.50</td>
<td>1.48</td>
<td>0.65</td>
</tr>
<tr>
<td>$fr_t^+$</td>
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<td>PPSI</td>
<td>19.04</td>
<td>2.01</td>
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<tr>
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<td>SP</td>
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<td></td>
<td>PP</td>
<td>18.04</td>
<td>8.01</td>
<td>0.75</td>
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<tr>
<td>$dp_t^+$</td>
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<td>PPSI</td>
<td>8.26</td>
<td>0.69</td>
<td>0.03</td>
</tr>
<tr>
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<td></td>
<td>SP</td>
<td>10.95</td>
<td>0.51</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PP</td>
<td>9.85</td>
<td>3.01</td>
<td>-0.64</td>
</tr>
<tr>
<td>$fr_t^-$</td>
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<td>PPSI</td>
<td>6.83</td>
<td>0.67</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>12.00</td>
<td>0.84</td>
<td>-0.24</td>
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<tr>
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<td>PP</td>
<td>7.84</td>
<td>2.39</td>
<td>-0.43</td>
</tr>
<tr>
<td>$dp_t^-$</td>
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<td>PPSI</td>
<td>16.09</td>
<td>0.73</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
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<td>-0.35</td>
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<tr>
<td></td>
<td></td>
<td>PP</td>
<td>14.51</td>
<td>1.73</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 2.18: Time-series moments

Notes: $fr$ = fraction of items with changing prices, $dp_t$ = average relative price change (not the absolute value), $fr_t^+$ = fraction of items with rising prices, $dp_t^+$ = size of price increases, $fr_t^-$ = fraction of items with falling prices, $dp_t^-$ = absolute size of price decreases, $pos = fr_t^+ \cdot dp_t^+$, neg = $fr_t^- \cdot dp_t^-$. 
Figure 2.14: Intensive margin (IM) and extensive margin (EM) vs. inflation

Notes: Yearly moving averages. The blue solid line is the realized path of annualized q-o-q inflation for items included in our sample. The red dashed line represents the intensive margin, i.e. the average non-absolute relative price change in every quarter\( (\frac{dp_t}{10}) \). The black dotted line represents the extensive margin, i.e. the frequency of price adjustments in every quarter\( (fr_t) \), divided by 10.
Figure 2.15: Frequency of increases and decreases vs. inflation

Notes: Yearly moving averages. The blue solid line is the realized path of annualized q-o-q inflation for items which are included in our sample(left axis). The red dashed line represents the frequency of price increases ($fr_t^+$). The black dotted line shows the frequency of price decreases ($fr_t^-$).
Decomposition of inflation variance

As discussed in the main text, a major purpose of studying price dynamics is to understand what drives fluctuations in inflation. This section is thus complementary to Section 2.6.4 in the main text.

<table>
<thead>
<tr>
<th></th>
<th>IM (in percent)</th>
<th>EM (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posted</td>
<td>81</td>
<td>19</td>
</tr>
<tr>
<td>SF</td>
<td>64</td>
<td>36</td>
</tr>
<tr>
<td>KM</td>
<td>71</td>
<td>29</td>
</tr>
<tr>
<td><strong>models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPSI</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>SP</td>
<td>96</td>
<td>4</td>
</tr>
<tr>
<td>PP-SP</td>
<td>98</td>
<td>2</td>
</tr>
<tr>
<td>PP</td>
<td>67</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 2.19: Variance decomposition: extensive margin vs. intensive margin

<table>
<thead>
<tr>
<th></th>
<th>POS (in percent)</th>
<th>NEG (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
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<td></td>
</tr>
<tr>
<td>Posted</td>
<td>63</td>
<td>37</td>
</tr>
<tr>
<td>SF</td>
<td>82</td>
<td>18</td>
</tr>
<tr>
<td>KM</td>
<td>73</td>
<td>27</td>
</tr>
<tr>
<td><strong>models</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPSI</td>
<td>61</td>
<td>39</td>
</tr>
<tr>
<td>SP</td>
<td>63</td>
<td>37</td>
</tr>
<tr>
<td>PP-SP</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>PP</td>
<td>81</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 2.20: Variance decomposition: price increases vs. price decreases

Notes: We decompose the model implied inflation \( \pi_t = f_{rt}d_{pt} \). Note that \( f_{rt} \) and \( d_{pt} \) do not include substitutions but the official inflation rate does and thus we focus on price adjustments exclusively from changing prices of the same products.
Back-and-forth movement of prices

As discussed in the outlook in the conclusion, for the UK data we quantify the back-and-forth movement of prices to examine to which extent the discussed model variants can capture this pattern not only qualitatively but also quantitatively. Following Ilut, Valchev, and Vincent (2016) we compute a measure of price memory, i.e. the probability that when a firm resets the price of its product, the new price is one that was visited in the past. We find that the PP model can explain the back-and-forth pattern very well. In particular, we estimate from the ONS micro data that the probability of revisiting a price which has been posted in the last two years is between 12 and 18%. Our PP model predicts a probability of 17% whereas the SP model predicts zero probability.

<table>
<thead>
<tr>
<th>Source</th>
<th>Average number of unique prices</th>
<th>Probability of revisiting a price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>data</td>
<td></td>
</tr>
<tr>
<td>Posted</td>
<td>3.08</td>
<td>0.18</td>
</tr>
<tr>
<td>SF</td>
<td>2.94</td>
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<tr>
<td>KM</td>
<td>2.74</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>models</td>
<td></td>
</tr>
<tr>
<td>PP-SI</td>
<td>2.82</td>
<td>0.17</td>
</tr>
<tr>
<td>SP</td>
<td>2.82</td>
<td>0</td>
</tr>
<tr>
<td>PP-SP</td>
<td>2.72</td>
<td>0.25</td>
</tr>
<tr>
<td>PP</td>
<td>2.74</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2.21: Back and forth movements

Notes: ONS sample 1996:Feb till 2016:Dec. For quarterly statistics, we assume a window of 4 quarters.
Chapter 3

Trend Inflation and Real Rigidities

Abstract

Real rigidities are generally viewed as key in propagating monetary shocks in New Keynesian models. This paper shows that different ways of modeling real rigidities which are equivalent under zero trend inflation yield markedly different implications for the dynamics of inflation, employment and the effectiveness of monetary policy when inflation follows a positive trend. We assess the empirical performance of three widely used types of real rigidities — firm-specific capital allocation, firm-specific wages and kinked-demand curves — in matching the U.S. inflation dynamics. We provide evidence for the presence of strong real rigidities due to firm-specific capital and firm-specific wages but find almost no support for the presence of the kinked-demand curve. Firm-specific wages outperform the kinked-demand curve and firm-specific capital in terms of empirical fit. We document that positive trend inflation reduces the ability of firm-specific factors in prolonging the real affects of monetary disturbances and monetary policy is not much more effective than in a model without real rigidities.
3.1 Introduction

A large empirical literature has found that monetary policy shocks affect real economic activity and inflation for longer periods than the typical duration of nominal prices. As documented by Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) for the US, or as in the first chapter of this thesis for the UK, micro prices change on average every two to three quarters. Christiano, Eichenbaum, and Evans (2005), inter alia, provide evidence for large and sustained real effects of monetary policy for up to three years.

New Keynesian models typically explain the long-lasting effects of monetary policy using a flat short-run Phillips curve with two key ingredients: sticky prices and strategic complementarities in price setting, also referred to as real rigidities.\(^1\) On the one hand price stickiness slows the response of the overall price level to aggregate shocks by assuming that potentially not all prices will adjust. On the other hand real rigidities reduce the response of the newly set prices what prolongs the real effects of shocks beyond the duration of nominal prices.\(^2\)

Examples of real rigidities include sticky intermediate prices (Basu, 1995), a kinked-demand curve (Kimball, 1995), sticky real wages (Blanchard and Galí, 2007) or firm-specific factors (Rotemberg (1996), Woodford (2003), Gertler and Leahy (2008)). Notably, these different real rigidities have observationally equivalent implications for economic dynamics if the model is log-linearized around a zero steady-state inflation (see e.g. Levin, López-Salido, and Yun (2007)).

This paper shows that with positive steady-state inflation the underlying source of real rigidity influences the dynamics of inflation, employment and the effectiveness of monetary policy. We use this observation to assess the empirical performance of three widely used types of real rigidity — a firm-specific capital allocation, firm-specific wages and kinked-demand curves — in matching the US inflation dynamics. We analyze three variants of a textbook macroeconomic model (see Woodford (2003)) which differ solely in the source of real rigidity and compute the inflation rates implied by the Phillips curve for each model variant. The parameter values governing the degree of real rigidity are estimated by minimizing the variance of the deviations of the model-implied inflation rate from the actual inflation rate. We find that firm-specific wages caused by segmented, i.e. firm-specific, labor markets achieve the best empirical fit and

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\(^1\)Even though these two concepts are used in the literature interchangeably, there are some minor differences, see Romer (2011). For the rest of the paper we will stick to the term real rigidity.

\(^2\)The term real rigidity is generally used to capture all mechanisms that mitigate the magnitude of price adjustment conditionally on changing the price. The amplification mechanism of real rigidities is well known at least since the seminal works by Ball and Romer (1990) and Kimball (1995).
significantly outperform the other two types of real rigidity. In other words, a version of the New-Keynesian model with Calvo pricing and firm-specific wages explains the U.S. inflation dynamics significantly better than versions of the model with firm-specific capital and the kinked-demand curve. This results point to the importance of the elasticity of labor supply in understanding the inflation dynamics.

Our finding that economic dynamics depend on the type of real rigidities is complementary to Levin et al. (2007) and Klenow and Willis (2016). Levin et al. (2007) consider a nonlinear New Keynesian model and show that different real rigidities yield markedly different implications for the welfare costs of steady-state inflation and inflation volatility. Klenow and Willis (2016) document that different sources of real rigidity might have different implications for microeconomic dynamics of price adjustment.

We further find that the presence of trend inflation considerably affects the ability of real rigidities in prolonging the real effects of monetary policy. More specifically, we document that positive trend inflation mitigates the persistence effect for the model variants with firm-specific wages and firm-specific capital, whereas the impact of the kinked-demand curve is hardly affected. The basic intuition for this result is that positive trend inflation increases the effective discount factor and price setters care more about the future. This increases the present value of future costs which dampens and offsets the impact of firm-specific inputs to react less strongly to changes in costs.

In contrast, the kinked-demand curve does not affect firms’ cost structure but firms’ revenues if adjusting prices. Therefore, the trend inflation does not mitigate its effect on optimal price setting and thus its ability to prolong the real effects of monetary shocks. Our results have thus important implications for explaining and modeling real effects of monetary policy. With trend inflation, even strong levels of firm-level real rigidities may not be strong enough to increase the effectiveness of monetary policy.

Several papers have already focused on the interaction between trend inflation and real rigidities. Kurozumi and Zandweghe (2016) show that the kinked-demand curve can mitigate the effect of positive trend inflation in inducing equilibria indeterminacy in the New Keynesian model with a Taylor rule. Bakhshi, Khan, Burriel-Llombart, and Rudolf (2007) show that real rigidity due to firm-specific factors has the opposite effect

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3The nonlinearity of real rigidities has been recently emphasized e.g. by Lindé and Trabandt (2018).

4Several papers derive a generalized formulation of the New Keynesian Phillips curve with Calvo pricing under positive trend inflation (see Ascari (2004), Yun (2005), Cogley and Sbordone (2008) and Ascari and Sbordone (2014) for a comprehensive survey) and emphasize that the form of this curve is more forward-looking than its zero inflation counterpart. We find that it is exactly this implication of trend inflation which reduces the ability of some forms of real rigidity to prolong the persistence of responses to shocks.

5Burstein and Hellwig (2007) document a similar result considering a state-dependent pricing model.
and leads to larger regions of the parameter space resulting in equilibria indeterminacy. Amplifies the indeterminacy issue. These opposite results underline the importance of distinguishing between real rigidities on empirical grounds which is one of the main goals of this paper.\footnote{Klenow and Malin (2010) note that the overall empirical evidence on the presence of real rigidities is mixed. Gopinath and Itskhoki (2011) consider real rigidities as the main source of the incomplete pass-through of exchange rate changes to import prices. Eichenbaum and Fisher (2007) and Carvalho et al. (2015) find significant degrees of real rigidity in aggregate data. At the firm level Klenow and Willis (2016) and Kryvtsov and Midrigan (2013) find it difficult to reconcile some types of firm-level real rigidities with large idiosyncratic price changes and inventory-sales ratios, respectively. The mixed evidence might be partly driven by insufficiently taking into account that different sources of real rigidity may have different implications for economic dynamics.}

The remainder of this paper is organized as follows. Section 3.2 presents the model. In Section 3.3 we derive the analytical results for the log-linearized versions of the three model variants with the different types of real rigidities. Our empirical analysis is described in Section 3.4 and our main findings are presented in Section 3.5. Section 3.6 studies the impact of monetary shocks on aggregate variables. Section 4.8 concludes.

### 3.2 Model economies

In this section we present our model framework based on a textbook macroeconomic model (see Woodford (2003)) with positive trend inflation. We specify three model variants which differ solely in the source of real rigidity. The first model variant includes firm-specific wages due to segmented labor markets (henceforth: SLM) as modelled by Gertler and Leahy (2008). The second model variant features decreasing returns to labor inputs (henceforth: DRL), also referred to as fixed capital allocation at the firm level (see Sbordone (2002), Woodford (2003), Eichenbaum and Fisher (2007)). The third model variant exhibits real rigidity due to the kinked-demand curve (henceforth: KDC; see Kimball (1995), Levin et al. (2007), Kurozumi and Zandwaghe (2016)).

#### 3.2.1 SLM model

The model economy consists of a continuum of islands $z \in [0, 1]$. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. Each island is populated by infinitely many households and infinitely many monopolistically competitive firms. We provide details about each of these groups in turn.
Households  On each island $z$, there is a continuum of households of mass unity. Households can supply labor only on this island. All households on all islands own identical shares of all firms and receive firms’ profits as dividends. There is perfect consumption insurance across islands. Each household’s instantaneous utility function in period $t$ is

$$u(C_t, M_t/P_t, N_{z,t}) = \log \left[ \frac{C_t}{M_t} \right]^\nu - \frac{N_{z,t}^{1+\varphi}}{1+\varphi},$$

where $\nu$ and $\varphi$ are positive parameters, $M_t$ denotes nominal money holdings, $P_t$ is the aggregate price level, $N_{z,t}$ stands for the household’s supply of labor, and $C_t$ denotes a consumption basket, which consists of differentiated goods $C^j_{z,t}$. In particular, $C^j_{z,t}$ denotes the consumed quantity of an individual good $j$ produced on island $z$ in period $t$. The consumption basket is given by a Dixit-Stiglitz aggregator function

$$C_t = \left[ \int_0^1 \int_0^1 (C^j_{z,t})^{\frac{1}{1-\varepsilon}} djdz \right]^\frac{1}{1-\varepsilon},$$

where $\varepsilon (\varepsilon > 1)$ stands for the elasticity of substitution between the differentiated goods. The aggregate price level $P_t$ is given by

$$P_t = \left[ \int_0^1 \int_0^1 (Q^j_{z,t})^{1-\varepsilon} djdz \right]^\frac{1}{1-\varepsilon},$$

where $Q^j_{z,t}$ denotes the price set by an individual producer $j$ from island $z$ in period $t$.

Utility in future periods is discounted by the factor $\beta \in (0,1)$. In each period $t$, the real flow budget constraint is

$$\int_0^1 \int_0^1 \frac{Q^j_{z,t}}{P_t} C^j_{z,t} djdz + \frac{M_t - M_{t-1}}{P_t} + \frac{1}{\pi_t} B_t - B_{t-1} \frac{P_t}{P_t} = W_{z,t} N_{z,t} + T_{z,t}$$

where $B_t$ stands for bond holdings, $R^n_t$ the gross nominal bond yield, and $T_{z,t}$ is an island specific real transfer, which includes the profits of firms, the government’s seigniorage revenues, and the transfers from consumption insurance. The island-specific nominal wage is denoted by $W_{z,t}$. Bonds, which are in zero net supply, are traded in an economy-wide market. The gross nominal bond yield, $R^n_t$, is therefore identical across islands.

Firms  Each island $z \in [0,1]$ is populated by a continuum of firms of mass unity, which produce differentiated goods. The firms set their prices and sell their goods directly to consumers across all islands. Given the production function $Y^j_{z,t} = N^j_{z,t}$, where $N^j_{z,t}$ denotes the labor input of firm $j$ on island $z$ at time $t$, the profit of a goods
producer $j$ in period $t$ is given by the difference between revenues and total labor costs

$$\Pi_{z,t}^j = \frac{Q_{z,t}^j}{P_t} Y_{z,t}^j - \frac{W_{z,t}^j}{P_t} N_{z,t}^j.$$  

(3.5)

It is noteworthy that the nominal wage in the firm’s instantaneous profit function given by equation (3.5) is only indexed by $z$ and not by $j$, i.e. it is not firm-specific but only island specific. The segmented labor markets imply inelastic labor supply on each island and thus island specific wages.

The simple intuition for why SLM are a powerful source of real rigidity is the following. Consider a shock that raises a firm’s cost. A firm that re-optimizes its price will respond by planning to increase its price. However, this price increase would reduce demand for this good, which would reduce output and thus labor demand on that island. Because wages are island specific, this leads to a fall in the firm’s costs. This compensates for the initial increase in the firm’s cost and the firm would raise its price by less than it would if labor markets were not segmented.

Goods producers are subject to price stickiness à la Calvo (1983) such that they can change their prices only with probability $1 - \alpha$ in every period. This means that on an island either all prices change or no price changes. The optimization problem of a producer $j$ on island $z$ is thus given by

$$\max_{Q_{z,t}^j} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}^i \alpha^i \left[ \left( \frac{Q_{z,t}^j}{P_{t+i}} \right)^{1-\varepsilon} Y_{t+i}^j - \frac{W_{z,t+i}^j}{P_{t+i}} \left( \frac{Q_{z,t}^j}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i}^j \right],$$

(3.6)

where $\Lambda_{t,t+i} = \beta^i \frac{U'(C_{t+i})}{U'(C_t)} = \beta^i \frac{C_{t+i}}{C_{t+i}}$ denotes the stochastic discount factor between periods $t$ and $t + i$ and where we have used the household’s demand for good $j$ given by

$$Y_{z,t+i}^j = \left( \frac{Q_{z,t}^j}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i}^j.$$

Closing the model We close the model (and also the model variants which will follow) by assuming that the growth rate of the nominal money stock, which is denoted by $g_m^t$, follows an exogenously given stationary stochastic process. We allow for a positive unconditional mean of $g_m^t$, which enables us to model a positive inflation trend. The corresponding seignorage revenues are used as lump-sum transfers to households.

3.2.2 DRL model

In this model version we abstract from the island structure of the economy since there is only one economy-wide labor market implying the same wages paid to all workers.
3.2. MODEL ECONOMIES

Hence, the description of the households’ side of the economy remains the same with the only modification that we consider only one island such that the integrals over the z’s disappear.

**Firms** To introduce decreasing returns to labor inputs, we modify the production function of producer $j$ in period $t$ to $Y_j^t = \left( N_j^t \right)^\gamma$, where $\gamma \in (0, 1)$ and $N_j^t$ denotes the labor input of firm $j$ at time $t$. With $\gamma \in (0, 1)$, marginal costs of each firm are not constant and depend on the firm’s output level. There are decreasing returns to scale, which could also be interpreted as the production function being of the Cobb-Douglas type but with fixed capital.\(^7\) DRL can be considered as a way of modeling that a firm’s capital stock is fixed and cannot be reallocated after a shock.\(^8\) The remaining description of the firms’ side of the economy remains the same.

The basic intuition for why DRL are a source of real rigidity can be described as follows. With firm-specific capital, a firm’s stock of capital is predetermined such that the firm’s marginal cost is an increasing function of its output. In response to positive shock in marginal costs, a firm that re-optimizes its price will respond by planning to raise its price. However, such a price increase would lower demand and thus output, which leads to a fall in the firm’s marginal cost. Therefore, the firm would raise its price by less than if capital were not predetermined.

### 3.2.3 KDC model

**Households** As in Kimball (1995), kinked-demand curves are approximated by concave demand functions for each differentiated good. This is achieved by abstracting from the Dixit-Stiglitz constant-elasticity-of-substitution aggregator. Following Levin et al. (2007) and Kurozumi and Zandweghe (2016), we assume that each household’s function for aggregating the consumption basket is given by

\[
\int_0^1 F \left( \frac{C_j^d}{C_t} \right) dj = 1, \tag{3.7}
\]

\(^7\)We assume the same source of real rigidity also in the models presented in Chapter 1.

\(^8\)See, e.g., Sbordone (2002) and Woodford (2003). Of course, the assumption of completely segmented capital markets might sound dramatic. However, Woodford (2003) and in particular Eichenbaum and Fisher (2007) document that assuming only partly flexible capital allocation yields similar results.
where the function $F$ is given by

\[
F\left(\frac{C^j_t}{C_t}\right) = \frac{\rho}{(1 + \epsilon)(\rho - 1)} \left[ (1 + \epsilon)\frac{C^j_t}{C_t} - \epsilon \right]^{\frac{\rho - 1}{\rho}} + 1 - \frac{\rho}{(1 + \epsilon)(\rho - 1)}
\]  
(3.8)

with $\rho = \varepsilon(1 + \epsilon)$. The parameter $\epsilon$ governs the curvature of the demand curve for each differentiated good $j$. In the special case of $\epsilon = 0$, the aggregation technology in equation (3.7) is reduced to the CES technology in equation (3.2).

As in the description of the other two model variants above, it is useful to sketch the basic intuition why KDC acts as a source of real rigidity. Consider again a rise in firm’s cost, which induces a firm to increase its price. A concave downward sloping demand curve implies that for any given rise in its price, the demand curve for the firm’s good will be more flat and thus more elastic. Therefore, the firm will raise its price by less than in the case of a CES aggregator, which implies than demand is linear.

Our model variant with KDC is the same as the model in Kurozumi and Zandweghe (2016) with the only difference that they close the model with a Taylor rule. They provide a comprehensive exposition of the model description and solution which we abstract from in what follows and rather concentrate thoroughly on the implications of KDC for economic dynamics.

### 3.3 Solution

In this section we present the log-linearized equilibrium conditions for all three model variants.

#### 3.3.1 Equilibrium conditions for the SLM model

The equations describing the optimal behavior of households, which are stated in Appendix D.1, have well-known log-linear approximations around the steady state

\[
w_{z,t} - p_t = \ln \left( \frac{W}{P} \right) + \varphi \hat{N}_{z,t} + \hat{Y}_t, \quad (3.9)
\]

\[
\hat{Y}_t = - \left( \hat{P}_t^n - \mathbb{E}_t [\pi_{t+1}] \right) + \mathbb{E}_t \left[ \hat{Y}_{t+1} \right], \quad (3.10)
\]

\[
m_t - p_t = \ln \left( \frac{M}{P} \right) + \hat{Y}_t - \frac{1}{\hat{R}_t^n - 1} \hat{P}_t^n, \quad (3.11)
\]

\[
\hat{Y}^j_{z,t} = -\varepsilon \left( q^j_{z,t} - p_t \right) + \hat{Y}_t, \quad (3.12)
\]
3.3. SOLUTION

where small letters denote log levels, variables with a bar denote steady-state levels, and variables with a “hat” stand for relative deviations from the steady state. \( \pi_t \) is the relative deviation from the steady-state value of gross inflation \( P_t/P_{t-1} \).

The associated (generalized) New Keynesian Phillips curve with trend inflation and SLM (in terms of unit labor costs) is given by

\[
\hat{\pi}_t = \frac{1}{1 + \varphi \varepsilon} \left[ (1 - \alpha \bar{\pi}^{\varepsilon-1}) \left( \frac{1}{1 + \alpha \beta \bar{\pi}^{\varepsilon(1+\varphi)}} \right) \left[ \bar{u}c_t - (1 + \varphi) \hat{s}_t \right] \right] 
+ \mathbb{E}_t \hat{\pi}_{t+1} \beta \left[ 1 + \frac{\varepsilon(1 + \varphi)}{1 + \varphi \varepsilon} \left( 1 - \alpha \bar{\pi}^{\varepsilon-1} \right) \left( \bar{\pi}^{1+\varphi} - 1 \right) \right] 
+ \mathbb{E}_t \hat{\psi}_{t+1} \omega \left[ 1 - \alpha \bar{\pi}^{\varepsilon-1} \right] \left( \bar{\pi}^{1+\varphi} - 1 \right),
\]

(3.13)

where \( \bar{u}c_t \) denotes the deviation of aggregate unit labor costs from their steady-state value, \( \hat{s}_t \) is the relative price distortion due to inefficient dispersion of prices given by equation (3.14), \( \bar{\pi} \) denotes the trend inflation rate and \( \hat{\psi}_t \) is an auxiliary variable given by equation (3.15):

\[
\hat{s}_t = \alpha \varepsilon \bar{\pi}^{\varepsilon} - \bar{\pi}^{\varepsilon-1} \sum_{i=0}^{N} \left( \alpha \bar{\pi}^{\varepsilon} \right)^i \hat{\pi}_{t-i} + \left( \alpha \bar{\pi}^{\varepsilon} \right)^{N+1} \hat{s}_{t-N-1}
\]

(3.14)

\[
\hat{\psi}_t = \left( 1 - \alpha \beta \bar{\pi}^{\varepsilon(1+\varphi)} \right) \sum_{i=0}^{N} \left( \alpha \beta \bar{\pi}^{\varepsilon(1+\varphi)} \right)^i \left[ \bar{u}c_t - (1 + \varphi) \hat{s}_t \right] 
+ \varepsilon(1 + \varphi) \sum_{i=1}^{N+1} \left( \alpha \beta \bar{\pi}^{\varepsilon(1+\varphi)} \right)^i \mathbb{E}_t \hat{\pi}_{t+i} + \left( \alpha \beta \bar{\pi}^{\varepsilon(1+\varphi)} \right)^{N+1} \mathbb{E}_t \hat{\psi}_{t+N+1}.
\]

(3.15)

Note that if steady state inflation equals zero, i.e. \( \bar{\pi} - 1 = 0 \), the Phillips curve (3.13) reduces to the conventional NKPC with segmented labor markets (3.16), as \( \hat{s}_t \) does not affect the first-order dynamics:

\[
\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \frac{1}{1 + \varphi \varepsilon} \bar{u}c_t + \mathbb{E}_t \beta \hat{\pi}_{t+1}.
\]

(3.16)

The system of equilibrium conditions is given by equations (3.10), (3.11), (3.13) - (3.15) and is closed by the equation governing the evolution of the money stock.

\[9\]We have used \( \bar{Y}_t = \bar{C}_t \) for the derivation of (3.9)-(3.12).
3.3.2 Equilibrium conditions for the DRL model

The equations describing the optimal behavior of households are slightly different than in the SLM model since the island structure has been abolished. Therefore the New Keynesian IS curve, equation (3.10), and the money demand, equation (3.11), stay unchanged whereas the labor supply, equation (3.17), and the goods demand, equation (3.18), are not island specific:

\[
\begin{align*}
\tilde{w}_t - p_t &= \ln \left( \frac{\tilde{W}}{\tilde{P}} \right) + \varphi \tilde{N}_{z,t} + \tilde{Y}_t. \\
\tilde{Y}_t^j &= -\varepsilon (q_t^j - p_t) + \tilde{Y}_t.
\end{align*}
\]

(3.17) \hspace{1cm} (3.18)

The associated New Keynesian Phillips curve with trend inflation and DRL is given by

\[
\hat{\pi}_t = \omega \left[ \frac{(1 - \alpha \hat{\pi}^{\varepsilon-1})(1 - \alpha \beta \tilde{\pi}^{\varepsilon})}{\alpha \hat{\pi}^{\varepsilon-1}} \right] \left[ \tilde{u} c_t - \hat{s}_t \right] \\
+ E_t \hat{\pi}_{t+1} \beta \left[ 1 + \frac{\varepsilon}{\gamma + \varepsilon(1 - \gamma)} (1 - \alpha \hat{\pi}^{\varepsilon-1}) (\tilde{\pi}^{\varepsilon} - 1) \right] \\
+ E_t \hat{\psi}_{t+1} \beta \omega (1 - \alpha \hat{\pi}^{\varepsilon-1}) (\tilde{\pi}^{\varepsilon} - 1),
\]

(3.19)

where \( \omega = \frac{\gamma}{\gamma + \varepsilon(1 - \gamma)} \), \( \hat{s}_t \) denotes the relative price distortion due to inefficient dispersion of prices given by equation (3.20), \( \hat{\pi} \) denotes the trend inflation rate, \( \hat{\psi}_t \) is an auxiliary variable given by equation (3.21):

\[
\hat{s}_t = \alpha \frac{\varepsilon \hat{\pi}^{\varepsilon} - \hat{\pi}^{\varepsilon-1}}{\gamma 1 - \alpha \hat{\pi}^{\varepsilon-1}} \sum_{i=0}^{N} \left( \alpha \hat{\pi}^{\varepsilon} \right)^i \hat{\pi}_{t-i} + \left( \alpha \hat{\pi}^{\varepsilon} \right)^{N+1} \hat{s}_{t-N-1}, \\
\hat{\psi}_t = \left( 1 - \alpha \beta \hat{\pi}^{\varepsilon} \right) \sum_{i=0}^{N} \left( \alpha \beta \hat{\pi}^{\varepsilon} \right)^i \left[ \tilde{u} c_t - \hat{s}_t \right] \\
+ \frac{\varepsilon}{\gamma} \sum_{i=1}^{N+1} \left( \alpha \beta \hat{\pi}^{\varepsilon} \right)^i E_t \hat{\pi}_{t+i} + \left( \alpha \beta \hat{\pi}^{\varepsilon} \right)^{N+1} E_t \hat{\psi}_{t+N+1}.
\]

(3.20) \hspace{1cm} (3.21)

With zero steady state inflation the Phillips curve (3.19) returns to the conventional form with decreasing returns to scale in labor inputs (3.22), as \( \hat{s}_t \) again does not affect the first-order dynamics:

\[
\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \omega \tilde{u} c_t + E_t \beta \hat{\pi}_{t+1}.
\]

(3.22)

The system of equilibrium conditions is given by equations (3.10), (3.11), (3.19) - (3.21).
and is closed by the equation governing the evolution of the money stock.

3.3.3 Equilibrium conditions for the KDC model

As mentioned above, the conditions for the model variant with kinked demand curves are not derived separately since they are provided in Kurozumi and Zandweghe (2016).

As shown by Kurozumi and Zandweghe (2016), log-linearized equilibrium conditions including the generalized New Keynesian Phillips curve with trend inflation and KDC in terms of unit labor costs are given by

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \omega \left(1 - \alpha \bar{\pi}^{-1}\right) \left(1 - \alpha \beta \bar{\pi}^{-1}\right) \left(\frac{\bar{s} c_t - \bar{s} \hat{s}_t}{s + \epsilon}\right), \tag{3.23}
\]

\[
- \frac{1}{\alpha \bar{\pi}^{-1}} \left(\hat{d}_t - \alpha \beta \bar{\pi}^{-1} E_t \hat{d}_{t+1}\right) + \hat{d}_{t-1} - \alpha \beta \bar{\pi}^{-1} \hat{d}_t
- \rho \left(1 - \alpha \bar{\pi}^{-1}\right) \left[\alpha \beta \bar{\pi}^{-1} (\bar{\pi} - 1) (\rho - 1) + \bar{\epsilon} (1 - \alpha \beta \bar{\pi}^{-1})\right] \hat{d}_t + \hat{\phi}_t + \hat{\psi}_t,
\]

\[
\hat{s}_t = \alpha \bar{\pi}^{\rho} \hat{s}_{t-1} + \frac{(\bar{\pi} - 1) \alpha \rho \bar{\pi}^{-1}}{1 - \alpha \bar{\pi}^{-1}} \left(\hat{\pi}_t + \hat{d}_t - \hat{d}_{t-1}\right), \tag{3.24}
\]

\[
\hat{d}_t = \frac{\alpha \bar{\pi}^{-1} \left[1 - \alpha \beta \bar{\pi}^{-1} + \bar{\epsilon} \bar{\pi}^{\rho} (1 - \alpha \beta \bar{\pi}^{-1})\right]}{1 - \bar{\epsilon} \alpha \bar{\pi}^{-1} + \bar{\epsilon} (1 - \alpha \beta \bar{\pi}^{-1})} \hat{d}_{t-1}
- \frac{\bar{\epsilon} \alpha \bar{\pi}^{-1} (\bar{\pi}^{\rho} - 1) (1 - \alpha \beta \bar{\pi}^{-1})}{(1 - \alpha \bar{\pi}^{-1}) \left[1 - \alpha \beta \bar{\pi}^{-1} + \bar{\epsilon} (1 - \alpha \beta \bar{\pi}^{-1})\right]} \hat{\pi}_t,
\]

\[
\hat{\phi}_t = \alpha \beta \bar{\pi}^{-1} E_t \hat{\phi}_{t+1}
+ \beta (\bar{\pi} - 1) (1 - \alpha \bar{\pi}^{-1}) \left[\rho E_t \hat{\pi}_{t+1} + (1 - \alpha \beta \bar{\pi}^{-1}) \left(\frac{\bar{s} c_{t+1}}{s + \epsilon} E_t \hat{s}_{t+1} + \rho E_t \hat{d}_{t+1}\right)\right], \tag{3.26}
\]

\[
\hat{\psi}_t = \alpha \beta \bar{\pi}^{-1} E_t \hat{\psi}_{t+1} + \bar{\epsilon} \beta (\bar{\pi}^{\rho} - 1) (1 - \alpha \bar{\pi}^{-1}) \frac{\bar{\pi}^{\rho}}{\rho - 1 - \bar{\epsilon} (\rho + 1)} \hat{\pi}_{t+1}, \tag{3.27}
\]

where \(\rho = \bar{\epsilon} (1 + \bar{\epsilon}), \bar{\epsilon} = \frac{\bar{\epsilon} (1 - \alpha \beta \bar{\pi}^{-1}) \left[1 - \alpha \beta \bar{\pi}^{-1} (1 - \alpha \beta \bar{\pi}^{-1})\right]^{-\frac{\bar{\epsilon} \rho}{\rho - 1}}}{1 - \alpha \beta \bar{\pi}^{-1} (1 - \alpha \beta \bar{\pi}^{-1})^{-\frac{\bar{\epsilon} \rho}{\rho - 1}}}, \omega = \frac{1 - \bar{\epsilon} \rho}{\rho - 1 - \bar{\epsilon} (\rho + 1)}, \) and the dispersion in the steady state is \(\bar{s} = \frac{1 - \bar{\epsilon} \rho}{1 - \alpha \beta \bar{\pi}^{-1} (1 - \alpha \beta \bar{\pi}^{-1})^{-\frac{\bar{\epsilon} \rho}{\rho - 1}}}.

When considering zero steady state inflation, the Phillips curve (3.23) again reduces to the conventional version with the kinked-demand curve (3.28), as price dispersion variables \(\hat{s}_t\) and \(\hat{d}_t\) become zero:

\[
\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \omega \hat{u} c_t + E_t \beta \hat{\pi}_{t+1}. \tag{3.28}
\]

The system of equilibrium conditions is given by equations (3.10), (3.11), (3.23) - (3.27) and is closed by the equation governing the evolution of the money stock.
3.4 Empirical implementation

After deriving the log-linearized equilibrium conditions in the previous section, we now turn to our estimation procedure. Note that for each model variant we only estimate one parameter which governs the corresponding degree of real rigidity. To this end, we rely on the two-step econometric approach used by Sbordone (2002), Woodford (2001) or Dupor et al. (2010). In the first step, since the Phillips curve contains expectations about future unit labor costs and inflation rates, we estimate a small, unrestricted, bivariate VAR model as a way to generate the corresponding forecasts. We employ a quarterly calibration because data on unit labor costs is available only on a quarterly basis. In the second step, we use the Phillips curve for each model variant to obtain a model-implied quarterly inflation series. In particular, as in Woodford (2001) and Dupor et al. (2010), the corresponding Phillips curves are iterated forward.

Finally, the parameters governing the degree of real rigidity in each model variant, i.e. the inverse of the Frisch elasticity of labor supply $\varphi$ in the SLM model, the concavity parameter of the production function $\gamma$ in the DRL model, and the concavity parameter of the demand curve $\epsilon$ in the KDC model are estimated from the data by minimizing the variance of a distance between the models’ inflation rates and the actual inflation.

Our targeted period is 1965Q1 - 2004Q4, i.e. 160 quarters, which includes the Great Inflation period. We also report results for the period of Great Moderation 1988Q1 - 2004Q4, i.e. 68 quarters. To obtain the model-implied inflation rate we must compute the price dispersion measure $s$, which is a weighted sum of past inflation rates. Therefore, we use earlier data from 1960Q1. For the VAR model we also use data from the sample period 1960Q1 - 2004Q4. The annualized average quarter-on-quarter inflation rate in this period is 1.04% or 4.2%.

We would like to emphasize that the estimation approach used in this paper was initially used to estimate the degrees of price and information stickiness (Sbordone, 2002; Dupor et al., 2010). We use this method to estimate the degrees of real rigidities while calibrating the value of the Calvo parameter $\alpha$ externally. Hereby, we rely on the microeconomic evidence about price adjustment and calibrate the probability of price

---

10 Sbordone (2002) transforms the standard NKPC into a closed-form solution for the logarithm of the price-unit labor cost ratio, taking nominal marginal cost growth as given.

11 We use the fact that the elements of the infinite sums exponentially decrease and cut the sums after 20 periods. Our results are robust to the cut off period.

12 Alternatively, one could also use the GMM method as in Galí and Gertler (1999) or Eichenbaum and Fisher (2007). We rely on the approach highlighted in Sbordone (2002) mainly due to its tractability since the parameters governing the degree of real rigidities do not occur only in the slope of the Philips curve as with zero trend inflation. This complicates the functional form of the Philips curve to be estimated.
3.5 FINDINGS

changes externally. In particular, we follow Klenow and Kryvtsov (2008) who find that the average duration of prices is 2.9 quarters. Therefore, we calibrate $\alpha$ such that $2.9 = \frac{1}{1-\alpha}$, which yields $\alpha = 0.6552$. In Appendix D.2 we conduct a detailed robustness analysis of sensitivity of our results to the choice of $\alpha$.

The remaining two parameters are set to standard values in the literature. We set the discount factor $\beta$ to 0.99 since we consider quarterly data. The elasticity of substitution between differentiated goods in the CES aggregator case is $\varepsilon = 11$, following Gertler and Leahy (2008).

To make statistical inference, we compute the confidence intervals for parameter estimates using a bootstrap method. In particular, in order to maintain the time-series data structure, we generate 20 subsamples of 80 quarters which is the half of the overall sample length and perform the estimation in the bootstrapped time samples. For the shorter sample we also generate 20 subsamples with 34 observations (half of the sample length). We report the 5% and the 95% percentile of the final distribution of the estimates.

3.5 Findings

3.5.1 Results for the sample period 1965-2004

In this section we report and discuss our estimation results for our targeted period from 1965 to 2004 which includes the Great Inflation period. In the subsequent section we report results for a shorter sample period of the Great Moderation from 1988 to 2004. For the sake of completeness, we distinguish between the scenarios with and without trend inflation. As a measure of empirical fit we report the root mean squared error.

As can be seen from Table 3.1, with zero trend inflation, all three sources of strategic complementarity in price setting imply the same degree of real rigidity and therefore the same slope of the short-run Phillips curve, i.e. the parameter on $\tilde{u}_{lt}$ in the corresponding Phillips curve, of 0.023. All three estimates are significantly different from

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13Klenow and Kryvtsov (2008) report this frequency for the period 1988-2004. We use this statistic also for our baseline sample 1965-2004 but perform rigorous robustness analysis. Our main results remain qualitatively the same. Also there is a significant heterogeneity in the frequency of price adjustment across industries and sectors, yet we restrict our attention to a single-sector model.

14The method used to compute the confidence bounds generates overlapping estimation periods. For robustness we have also computed the confidence bounds by splitting the overall sample into subsequent, not overlapping subperiods of, however, much shorter length. For the long sample we have split the sample into five subperiods á 32 observations and for the shorter sample into four subperiods á 17 periods. We find that the confidence bounds do not change much and are qualitatively robust to this modification.
values implying no real rigidity. The same functional form of the Phillips curve in
the absence of positive trend inflation implies that the empirical fit of all three model
variants is identical.

<table>
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Table 3.1: Empirical fit: 1965Q1-2004Q4

Notes: As a measure of empirical fit we employ RMSE multiplied by 1000. PC
slope denotes the slope of the short-run Phillips curve, i.e. the parameter on
current unit labor costs $\hat{w}_t$ in the Phillips curve. Numbers in parentheses show
the 90% confidence intervals.

Including positive trend inflation changes the results remarkably. We find that the
slope of the Phillips curve declines with trend inflation, a result emphasized by Ascari
(2004), Bakhshi et al. (2007) and Cogley and Sbordone (2008). In particular, with no
real rigidity, the slope of the NKPC is 0.18, whereas with the level of trend inflation
in this period the slope decreases to 0.10.

The statistics in Table 3.1 show that the best empirical fit is achieved with real rigidity
via segmented labor markets and a value of the inverse of Frisch elasticity of labor
supply of 0.78. Notably, the presence of trend inflation improves the empirical fit by
20% compared to the case without trend inflation. The empirical fit deteriorates for
the other two sources of real rigidity.

Interestingly, this exercise shows that to achieve the best empirical fit in the KDC
model, the kink in the demand curve should be abolished. This result suggests a challenge for the kinked-demand curve approach which has been widely used in recent literature.\textsuperscript{15} Notably, the slope of the Phillips curve in the KDC model with trend inflation increases, compared to the scenario with zero inflation in steady-state, since the degree of real rigidity due to KDC is zero.

Compared to the zero trend inflation case, with positive trend inflation only, the SLM model exhibits a flatter slope of the Phillips curve. Moreover, even though the empirical fit of the DRL model slightly deteriorates, it is still better than in the case with no real rigidities.\textsuperscript{16}

Our estimates for parameters governing the particular types of real rigidity appear to be in line with values used in the literature. For the KDC approach, Kurozumi and Zandweghe (2016) use $\epsilon = -9$, Dotsey and King (2005) and Eichenbaum and Fisher (2007) employ $\epsilon = -10$ and $\epsilon = -33$. These values seem to be supported by our estimate for the case of zero trend inflation. In the DRL case, $\gamma$ should be close to the labor share in aggregate income. Our estimates of $\gamma = 0.61$ for zero trend inflation and $\gamma = 0.82$ with positive trend inflation are consistent with this view. Finally, when considering the plausibility of our estimates for the degree of segmented labor markets, the crucial parameter $\varphi$ is the inverse of the Frisch-elasticity of labor supply. Our estimates $\varphi$ of 0.65 and 0.78 imply the Frisch-elasticities of 1.5 and 1.3, respectively. (Gertler and Leahy, 2008) argue that a Frisch-elasticity of 1 is a reasonable intermediate range value in the literature. Hence, our estimates are in line with values typically used in the literature.

Figure 3.1 visualizes the empirical fit of the considered sources of real rigidity. The empirical fit of the SLM model markedly outperforms the other model variants at the time of the peak of the inflation rate in the late 70’s and the beginning of the 80’s. The KDC model appears to generate more upward deviations from the realized path of inflation than the other two models.

In the next section we focus on the period of the Great Moderation 1988-2004 which features a more stable path of quarterly inflation.

\textsuperscript{15}See e.g. Kurozumi and Zandweghe (2016) and (Lindé and Trabandt, 2018).

\textsuperscript{16}This can be observed by comparing the KDC model results since due to the estimated zero kink in the demand curve the empirical fit of the KDC model corresponds to a model with positive trend inflation but no real rigidity.
Figure 3.1: Realized and model-implied paths of inflation: 1965-2004

Notes: The blue solid line represents the annualized de-trended inflation rate in the U.S. The red dotted line represents the inflation rate as implied by the GNKPC using the parameter governing the degree of real rigidity which minimizes the variance of deviations from the actual rate.
3.5. FINDINGS

3.5.2 Results for the Great Moderation 1988 - 2004

In this section we provide the results for the period 1988Q1 - 2004Q4 (68 quarters). This period is also the main focus of Klenow and Kryvtsov (2008) who document that the frequency of price adjustment in this time span was on average 2.9 quarters. The VAR period begins five years earlier in 1983Q1.

Table 3.2 shows that the result that SLM yield the best empirical fit is robust to the change of sample period. For the case without trend inflation, the degree of real rigidity which fits the data best is 0.04, i.e. three times as large as for the longer sample 1965-2004. Accordingly, the slope of the Phillips curve is further reduced from 0.02 to 0.008.

<table>
<thead>
<tr>
<th>Trend</th>
<th>Segmented labor markets (1)</th>
<th>Decreasing returns to labor (2)</th>
<th>Kink demand curve (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical fit</td>
<td>no</td>
<td>2.09</td>
<td>2.09</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>2.00 [0.49, 10]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1 [0, 0.75]</td>
<td>0.33</td>
<td>1</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0 [0, 0.75]</td>
<td>0</td>
<td>-20.00 [-35, -3.93]</td>
</tr>
<tr>
<td>PC slope</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Empirical fit</td>
<td>yes</td>
<td>1.82</td>
<td>2.08</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>3.31 [2.35, 3.71]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1 [0.32, 0.61]</td>
<td>0.58</td>
<td>1</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0 [0.32, 0.61]</td>
<td>0</td>
<td>-0.20 [-0.99, -0.33]</td>
</tr>
<tr>
<td>PC slope</td>
<td>0.001</td>
<td>0.012</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Table 3.2: Empirical fit: 1988Q1-2004Q4

Notes: As a measure of empirical fit we employ RMSE multiplied by 1000. PC slope denotes the slope of the short-run Phillips curve, i.e. the parameter on current unit labor costs \( \hat{u} \bar{c}_t \) in the Phillips curve. Numbers in parentheses show the 90% confidence intervals.

Including trend inflation improves the empirical fit, not only of the SLM model variant but also of the DRL model. The fit worsens only in the KDC case. For the period of the Great Moderation we find evidence for a significant kink in the demand curve,
3.5. FINDINGS

even though it is almost negligible. Our estimate of $\epsilon$ is -0.2, much smaller than the values assumed in the literature.\footnote{Note that the confidence interval reported in Table 3.2 does not include the estimate. This is because we use bootstrap confidence intervals which do not guarantee to include the actual estimate.} As for the longer sample including the Great Inflation period, the slope of the Phillips curve declines in comparison to the zero trend inflation case only in the SLM model. The DRL and the KDC model variants induce a rise of the slope. It is important to note that the slope in the DRL case is still far away from the slope without real rigidities (see again the KDC result).

Some thought concerning the plausibility of our estimates are in order. As discussed in the previous section, values of $\epsilon$ as the kink of the demand curve range up to -33, so our estimate for the zero trend inflation scenario of -20 is not an outlier. The estimate of $\gamma$ of 0.33 with zero trend inflation appears to be rather low if thinking about it as a labor share of income. Positive trend inflation implies a more plausible estimate closer to two thirds. Finally, our estimates of $\varphi$ are for the shorter sample larger than one. This implies values for the Frisch-elasticity of labor supply below one (0.5 and 0.3, respectively). These values appear to be more in line with microestimates of the Frisch-elasticity rather with values usually employed in macro studies (see Peterman (2014)). Yet by no means does our estimation procedure deliver values outside of the ranges seen in the literature.

Figure 3.2 visualizes the empirical fit of model-implied inflation rates to the realized path of inflation. In general, the empirical fit is lower than for the longer sample, which follows from the performance of the VAR model for building expectations about unit labor costs and inflation.
3.5. FINDINGS

Segmented labor markets

Decreasing returns to labor

Kinked demand curve

Figure 3.2: Realized and model-implied paths of inflation: 1988-2004

Notes: The blue solid line represents the annualized detrended inflation rate in the U.S. The red dotted line represents the inflation rate as implied by the GNKPC using the parameter governing the degree of real rigidity which minimizes the variance of deviations from the actual rate.
3.6 Implications for the real effects of monetary policy

A major purpose of estimating the degree of real rigidities is to understand their importance for amplifying the real effects of monetary policy. Therefore, in this section we assess the aggregate implications of considered real rigidities by studying the impulse responses for each model variant with levels of real rigidity inferred from the data. We show impulse responses for both the long and the short sample.

3.6.1 Impulse responses in the long sample

As can be seen from Figure 3.3 for the estimates in the longer sample including the Great Inflation, segmented labor markets and decreasing returns to labor inputs imply more persistent effects of monetary disturbances than the kinked demand curve. This result can be traced back to the estimated zero kink in the demand curve.

Noticeably, even though the nature of impulse responses for the SLM and DRS variants are almost identical, the response of the relative price distortion from the labor market clearing condition varies considerably. The relative price distortion is the wedge between aggregate labor input and aggregate output and represents the main reason for a lower output in the presence of trend inflation in steady-state.

The intuition for this result comes from equation (3.71) in the Appendix. The parameter $\gamma$ enters the equation for price dispersion and increases the response of distortion to changes in the inflation rate. Because price dispersion is the main source of welfare costs of inflation in New Keynesian models (see Nakamura et al. (2018)), the source of real rigidity can have important implications for optimal monetary policy.

If we observe a similar response of output in both the SLM and the DRS model but the distortion behaves differently, it must be labor supply which explains the difference. This follows from the aggregate labor supply equation. As we can see in the bottom left panel of Figure 3.3, the labor supply reacts much more strongly than in the other two models. This observation confirms the main motivation of this paper that distinguishing between sources of real rigidity matters. In addition, because the labor supply reacts so differently, it could be used as another criterion how to differentiate between real rigidities in the data by considering empirical impulse responses.
3.6.6. IMPLICATIONS FOR THE REAL EFFECTS OF MONETARY POLICY

Figure 3.3: Impulse responses: 1965-2004

Notes: The shock is a permanent decrease of the nominal money stock by 1%. The vertical axis shows percentage points. Black solid line: fixed capital, i.e. decreasing returns to labor; red dashed line: segmented labor markets; blue dotted line: kinked demand curve. The panel at the right bottom shows the relative price distortion, i.e. the wedge between aggregate output and aggregate labor in the labor market clearing condition.

3.6.2 Impulse responses in the short sample

For the shorter sample 1988-2004, the results change considerably. Most importantly, as discussed in subsection 3.5.2, the estimated degrees of real rigidity increase compared to the longer sample period. In particular, while for the longer sample we estimate \( \phi = 0.78 \), we obtain for the short sample \( \phi = 3.31 \). However, as can be seen from subsection 3.6.2, the persistence of the response for the SLM case is as low as in the KDC model, while the estimate of the kink in the demand curve is only minor (\( \epsilon = -0.2 \) which is though significant but very low). The next section shows why the impulse responses to monetary shocks are not persistent despite strong degrees of rigidity inferred from the data.
3.6. IMPLICATIONS FOR THE REAL EFFECTS OF MONETARY POLICY

![Impulse response graphs](image)

Figure 3.4: Impulse responses: 1988-2004

Notes: The shock is a permanent decrease of the nominal money stock by 1%. The vertical axis shows percentage points. The panel at the right bottom shows the relative price distortion, i.e. the wedge between aggregate output and aggregate labor in the labor market clearing condition.

3.6.3 Impact of positive trend inflation on the ability of real rigidities to prolong real effects of monetary policy

Despite strong firm-level real rigidities the impulse responses might not be more persistent than in a model without real rigidities. The intuition behind the result is based on the more pronounced forward-looking behavior of price-setters under positive trend inflation. We illustrate this observation in Figure 3.5 which shows the impulse responses of output (top line) and inflation (bottom line) for four cases: (1) no trend and no real rigidity, (2) no trend but real rigidity, (3) trend but no real rigidity and (4) trend and real rigidity.

Under SLM and DRS, positive trend inflation mitigates the long-lasting effect of the real rigidity, especially on the response of output. The reduction gets stronger the larger the real rigidity. However, the responses in the KDC model are hardly affected. The intuition for this result is as follows. Firm-specific factors reduce the sensitivity of price setters to changes in the present value of future marginal costs. However, positive
3.6. IMPLICATIONS FOR THE REAL EFFECTS OF MONETARY POLICY

<table>
<thead>
<tr>
<th>Segmented labor markets</th>
<th>Decreasing returns to labor</th>
<th>Kinked demand curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi = 2 )</td>
<td>( \gamma = 0.33 )</td>
<td>( \epsilon = -20 )</td>
</tr>
</tbody>
</table>

Figure 3.5: The effect of positive trend inflation on the persistence effect of real rigidities.

trend inflation increases the effective discount factor since price setters are more forward looking. This, in turn, increases the sensitivity of price setters to their cost structure and mutes the effect of real rigidities due to firm-specific input factors in prolonging the persistence of responses to shocks. This finding has important implications for modeling real effects of monetary policy since in the presence of trend inflation even strong levels of firm-level real rigidities can be too weak to generate much larger aggregate real effects from monetary shocks than a model without real rigidities.

The key is equation (3.66). If \( \varphi > 0 \) or \( \gamma < 1 \), i.e. for the case of real rigidity in the SLM and DRS case, respectively, the impact of future inflation expectations on current inflation increases. If in this equation the parameters \( \varphi \) and \( \gamma \) are set to 0 and 1, respectively, the impulse responses of inflation and output are much more persistent. Equation (3.66) represents the log-linearized form of the numerator in the optimal price equation (3.44) which represents the discounted expected future costs. Hence, SLM and DRS are real rigidities which affect the cost structure of price setters. With positive trend inflation, which increases discounted costs, price setters are going to react more vigorously to changes in their cost structure despite strong real rigidities. In contrast, the KDC does not affect firms’ cost structure but firms’ revenues if adjusting prices.
Therefore, trend inflation does not mitigate its effect on optimal price setting. It would be interesting to assess the responses of real variables to other types of shocks, such as technology and preference shocks, under different types of real rigidity. Presumably, in scenarios in which trend inflation influences the present value of future expected revenues trend inflation would interact with the KDC too.

### 3.7 Conclusion

This paper proposes a method to empirically distinguish between different sources of real rigidity. This is achieved by exploiting the observation that positive trend inflation in the log-linearized New-Keynesian model implies different first-order effects of various types of real rigidities. To estimate the parameter values governing the degrees of real rigidity we use the approach from Sbordone (2002) and Dupor et al. (2010). However, unlike these papers, we fix the level of nominal rigidity based on the empirical evidence and estimate the degrees of various real rigidities. We document that in terms of empirical fit segmented labor markets outperform the other two types of real rigidities.

We show that the presence of positive trend inflation might mitigate the effect of real rigidity in enhancing the persistence of responses to monetary shocks in the presence of segmented labor markets and decreasing returns to scale. This is due to the effect of positive trend inflation increasing the present value of future costs which offsets the impact of firm-specific inputs to react less strongly to changes in costs.

An interesting avenue for further research could be to consider non-linear versions of the Phillips curve, as e.g. emphasized recently by Lindé and Trabandt (2018) for the KDC approach. This modification would stress the importance of non-linearities in price setting. Moreover, to further examine robustness and plausibility of our results it would be interesting to include backward-looking components in the GNKPC as highlighted by Eichenbaum and Fisher (2007) or, as pointed out by Coibion (2010), a different way of building expectations such as out-of-sample forecasts and adaptive expectations or different measures of expectations such as expectations of professional forecasters. Furthermore, the analysis in this paper could be broadened to encompass additional types of real rigidities, especially sticky intermediate prices. Another interesting question would be to analyze the impact of real rigidities on the responses to other types of shocks such as preference shocks or TFP shocks.

\(^{18}\) Ball and Romer (1990) and Klenow and Willis (2016) emphasize the distinction between so-called macro and micro rigidities. All three sources of real rigidity belong to the micro category since they make price-setters less willing to move their relative prices. Macro rigidities such sticky intermediate prices or sticky real wages introduce a sticky component to the cost of all firms and do not make firms less willing to change their relative prices.
Exploring the sectoral heterogeneity of real rigidities such as in Carvalho and Nechio (2016) could also provide additional insights into how to calibrate the strategic complementarities in price setting. Finally, it would also be interesting to analyze the business cycle properties of real rigidities. It sounds plausible to assume that the degree of real rigidity due to the elasticity of labor supply or capital adjustment costs as discussed in this paper can change over the business cycle. This would be useful for understanding the effectiveness of monetary policy in times of booms and busts.
D Appendix of Chapter 3

D.1 Households’ Optimality Conditions

In this section, we state the first-order conditions that describe the optimal behavior of households.

SLM model

Minimizing costs for a given size of the consumption basket $C_t$ yields the demand function

$$C^i_{z,t} = \left( \frac{Q^i_{z,t}}{P_t} \right)^{-\epsilon} C_t, \quad (3.29)$$

where the aggregate price level $P_t$ is given by

$$P_t = \left[ \int_0^1 (Q^i_{z,t})^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}. \quad (3.30)$$

The household’s utility maximization problem results in the following standard conditions:

$$\frac{W_{z,t}}{P_t} = N_{z,t}^{\epsilon} C_t, \quad (3.31)$$

$$\mathbb{E}_t \left[ \beta \frac{C_t}{C_{t+1}} \frac{R^n_t}{P_t} \right] = 1, \quad (3.32)$$

$$\frac{M_t}{P_t} = \nu C_t \frac{R^n_t}{R^n_t - 1}. \quad (3.33)$$

DRL model

Minimizing costs for a given size of the consumption basket $C_t$ yields the demand function

$$C^i_t = \left( \frac{Q^i_t}{P_t} \right)^{-\epsilon} C_t, \quad (3.34)$$
where the aggregate price level $P_t$ is given by

$$P_t = \left[ \int_0^1 (Q_t^z)^{1-\varepsilon} \, dz \right]^{\frac{1}{1-\varepsilon}}. \quad (3.35)$$

The household’s utility maximization problem results in the following standard conditions:

$$\frac{W_t}{P_t} = N_t^{\varepsilon} C_t, \quad \text{(3.36)}$$

$$E_t \left[ \beta \frac{C_t}{C_{t+1}} R_t^n P_t \frac{P_t}{P_{t+1}} \right] = 1, \quad \text{(3.37)}$$

$$\frac{M_t}{P_t} = \nu C_t \frac{R_t^n}{R_t^{n-1}}. \quad \text{(3.38)}$$

### Deriving the Phillips curve

In this section we derive the New Keynesian Phillips curve under Calvo pricing with trend inflation and with two sources of real rigidity: decreasing returns to scale in labor inputs and segmented labor markets. Note that in the main article we have for exposition and clarity purposes strictly distinguished between these two sources of real rigidity. However, the way how we model them is not mutually exclusive such that these two sources of real rigidity can be easily nested in one model. Unfortunately this is not the case for the kinked-demand curve with trend inflation which cannot be easily introduced into the same model. The reason is that the model would not only lose its tractability but is not analytically solvable. To stay parsimonious, for notation details refer to the model description in the main article.

### Profit optimization and price setting

The production function of goods producers on island $z$ is of the form

$$Y_{z,t}^j = (N_{z,t}^j)^\gamma \quad \text{(3.39)}$$

with $\gamma \in (0, 1]$ and their real profit function $\forall \, t \geq 0$ is given by

$$\Pi_{z,t}^j = \frac{Q_{z,t}^j}{P_t} Y_{z,t}^j - \frac{W_{z,t}}{P_t} N_{z,t}^j. \quad \text{(3.40)}$$

Goods producers are subject to stickiness à la Calvo (1983), i.e. they can change their prices only with probability $1 - \alpha$ in every period. Hence, the producer’s optimization
problem on island \( z \) is given by

\[
\max_{Q_{z,t}} \mathbb{E}_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \alpha^i \left[ \left( \frac{Q_{z,t}}{P_{t+i}} \right)^{1-\varepsilon} Y_{t+i} - W_{z,t+i} \left( \frac{Q_{z,t}}{P_{t+i}} \right)^{-\frac{\varepsilon}{\gamma}} Y_{t+i}^{\frac{1}{\gamma}} \right],
\]

(3.41)

where \( \Lambda_{t,t+i} = \beta^i \frac{U'(C_{t+i})}{U'(C_t)} = \beta^i \frac{C_{t+i}}{C_{t+i}} \) denotes the stochastic discount factor between periods \( t \) and \( t+i \) and where we have used the household’s demand for good \( j \) given by

\[
Y_{z,t+i} = \left( \frac{Q_{z,t+i}}{P_{t+i}} \right)^{-\frac{\varepsilon}{\gamma}} Y_{t+i}.
\]

Computing the first order condition with respect to the individual price \( Q_{z,t} \), using \( C_{t+i} = Y_{t+i} \quad \forall i \geq 0 \), and simplifying, results in the following equation for the average optimal price on island \( z \) in period \( t \), \( Q_{z,t}^* \),

\[
\left( Q_{z,t}^* \right)^{\frac{\gamma + (1-\gamma)}{\gamma}} = \varepsilon \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i Y_{t+i}^{\frac{1-\gamma}{\gamma}} P_{t+i}^\gamma}{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i P_{t+i}^{\gamma-1}}.
\]

(3.42)

Note that \( Q_{z,t}^* \) depends on the island specific wage \( W_{z,t} \). For aggregation purposes we would like to replace the island specific wage by an aggregate variable to be able to use the aggregate data on unit labor costs. To this end we combine the household’s labor supply condition, the production function equation (3.39) and the households’ demand equation

\[
\frac{W_{z,t}}{P_t} = N_{z,t} Y_t = Y_{z,t}^\phi Y_t = \left( \frac{Q_{z,t}}{P_t} \right)^{-\varepsilon} Y_t = \left( \frac{Q_{z,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} Y_t^{1+\frac{\varepsilon}{\gamma}}.
\]

(3.43)

We plug this expression into equation (3.42) and receive

\[
\left( Q_{z,t}^* \right)^{\frac{1}{\gamma+1}} = \varepsilon \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i Y_{t+i}^{\frac{1-\gamma}{\gamma}} P_{t+i}^\gamma}{\mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i P_{t+i}^{\gamma-1}},
\]

(3.44)

where \( \omega = \frac{\gamma}{\gamma+\psi(1+\phi-\gamma)} \).

We divide equation (3.42) by \( P_t^{\frac{1}{\gamma}} \) and simplify to get

\[
\kappa_{z,t} := \left( \frac{Q_{z,t}^*}{P_t} \right)^{\frac{1}{\gamma}} = \varepsilon \frac{\psi_t}{(\varepsilon-1) \gamma \phi_t}.
\]

(3.45)
where

\[ \psi_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i Y_{t+i}^{\frac{1+\nu}{\gamma}} \pi_{t,t+i}^{\frac{\nu}{\gamma}}, \]  
(3.46)

\[ \phi_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i \pi_{t,t+i}^{\nu-1}. \]  
(3.47)

In the equations above \( \pi_{t,t+i} \) denotes the inflation between periods \( t \) and \( t + i \). Note that equation (3.46) and equation (3.47) can be rewritten recursively as

\[ \psi_t = Y_t^{\frac{1+\nu}{\gamma}} + \alpha \beta \mathbb{E}_t \left[ \frac{\nu}{\gamma} \pi_{t+1}^{\frac{\nu}{\gamma}} \psi_{t+1} \right], \]  
(3.48)

\[ \phi_t = 1 + \alpha \beta \mathbb{E}_t \left[ \pi_{t+1}^{\nu-1} \phi_{t+1} \right]. \]  
(3.49)

**Price level, aggregation and price dispersion**  
Upon Calvo pricing, the price level given by equation (3.35) evolves according to

\[ P_t = \left[ \alpha P_{t-1}^{1-\epsilon} + (1 - \alpha) (Q_{\ast,t}^{\nu})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \]  
(3.50)

where we use the fact that the average price on an island will be the same on all islands which have been hit by a Calvo shock.

Dividing equation (3.50) by \( P_t \) and defining \( \pi_t = P_t / P_{t-1} \) yields

\[ 1 = \left[ \left( \alpha P_{t-1}^{1-\epsilon} + (1 - \alpha) (Q_{\ast,t}^{\nu})^{1-\epsilon} \right) P_t^{-(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}}, \]  
(3.51)

\[ 1 = \alpha \pi_t^{\nu-1} + (1 - \alpha) \left( \frac{Q_{\ast,t}^{\nu}}{P_t} \right)^{1-\epsilon}, \]  
(3.52)

\[ \kappa_{z,t} = \frac{Q_{\ast,t}^{\nu}}{P_t} = \left[ \frac{1 - \alpha \pi_t^{-1}}{(1 - \alpha)} \right]^{\frac{1}{1-\epsilon}}. \]  
(3.53)

**Price dispersion**  
Labor demand of firm on island \( z \) in period \( t \) is derived from its production function and is given by

\[ N_{z,t} = (Y_{z,t})^{\frac{1}{\gamma}}. \]  
(3.54)

The aggregate labor demand is then given by

\[ N_t = \int_0^1 N_{z,t} \, dz = \int_0^1 Y_{z,t}^{\frac{1}{\gamma}} \, dz = Y_t^{\frac{1}{\gamma}} \int_0^1 \left( \frac{Q_{z,t}^{\nu}}{P_t} \right)^{-\frac{\nu}{\gamma}} \, dz, \]  
(3.55)
where $s_t$ denotes the resource costs due to inefficient price dispersion in the economy. Under the assumption of Calvo pricing, we can rewrite $s_t$ as follows:

$$s_t = (1 - \alpha) \left( \frac{Q_{z,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} + \alpha (1 - \alpha) \left( \frac{Q_{z,t-1}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} + \alpha^2 (1 - \alpha) \left( \frac{Q_{z,t-2}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} + \ldots$$  

(3.56)

Now by expanding all terms by $(P_{t-1}/P_{t-1})^\frac{\varepsilon}{\gamma}$ and by collecting the terms, we end up with a recursive formulation of $s_t$:

$$s_t = (1 - \alpha) \left( \frac{Q_{z,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} + \alpha \pi_t^\frac{\varepsilon}{\gamma} \left[ (1 - \alpha) \left( \frac{Q_{z,t-1}}{P_{t-1}} \right)^{-\frac{\varepsilon}{\gamma}} + \ldots \right],$$

(3.57)

$$= (1 - \alpha) \left( \frac{Q_{z,t}}{P_t} \right)^{-\frac{\varepsilon}{\gamma}} + \alpha \pi_t^\frac{\varepsilon}{\gamma} s_{t-1}.$$  

(3.58)

**Unit labor costs**  
Economy-wide unit labor costs are given by

$$ulc_t = \frac{W_t N_t}{P_t Y_t} = \frac{W_t}{P_t Y_t} \frac{1 - \gamma}{1 - \gamma} s_t,$$  

(3.59)

where we have used equation (3.55).

**Log-linearization**

In what follows, lowercase letters represent the log levels of variables and variables with a “hat” stand for relative deviations from their steady state values.

Log-linearizing equation (3.45) yields

$$\frac{1}{\omega} \hat{k}_{z,t} = \hat{\psi}_t - \hat{\phi}_t.$$  

(3.60)

Log-linearizing equation (3.48) yields

$$\hat{\psi}_t = \left( 1 - \alpha \beta \pi \frac{\varepsilon (1 + \varepsilon)}{\gamma} \right) \frac{1 + \varphi}{\gamma} \hat{\gamma}_t + \alpha \beta \pi \frac{\varepsilon (1 + \varepsilon)}{\gamma} \left( \frac{\varepsilon (1 + \varphi)}{\gamma} E_t \hat{\pi}_{t+1} + E_t \hat{\psi}_{t+1} \right).$$  

(3.61)
Log-linearizing equation (3.59) and yields

\[ \hat{u}_t = \frac{W_t}{P_t} + \frac{1 - \gamma}{\gamma} \hat{Y}_t + \hat{s}_t. \]  

(3.62)

Next, taking the labor supply equation (3.43), aggregating and taking difference yields

\[ \left( \frac{W_{z,t}}{P_t} \right) - \left( \frac{W_t}{P_t} \right) = -\frac{\varepsilon \varphi}{\gamma} \hat{k}_{z,t} - \varphi \hat{s}_t. \]  

(3.63)

Note that the labor supply equation can be rewritten to

\[ \left( \frac{W_{z,t}}{P_t} \right) + \frac{\varepsilon \varphi}{\gamma} \hat{k}_{z,t} = \left( 1 + \frac{\varphi}{\gamma} \right) \hat{y}_t. \]  

(3.64)

Using equation (3.63) and equation (3.64) to simplify equation (3.62) yields

\[ \hat{u}_t = \frac{1 + \varphi}{\gamma} \hat{y}_t + (1 + \varphi) \hat{s}_t. \]  

(3.65)

This expression can be used to rewrite equation (3.61) as

\[ \hat{\psi}_t = \left( 1 - \alpha \beta \frac{\pi \varepsilon (1 + \varphi)}{\gamma} \right) \left[ \hat{u}_t - (1 + \varphi) \hat{s}_t \right] \]  

\[ + \alpha \beta \frac{\pi \varepsilon (1 + \varphi)}{\gamma} \left[ \frac{\varepsilon (1 + \varphi)}{\gamma} \hat{\pi}_{t+1} + \hat{\varepsilon}_t \hat{\psi}_{t+1} \right]. \]  

(3.66)

Log-linearizing equation (3.49) yields

\[ \dot{\hat{\psi}}_t = \alpha \beta \frac{\pi \varepsilon - 1}{1 - \alpha \pi \varepsilon - 1} \hat{\pi}_{t+1} + (\varepsilon - 1) \hat{\varepsilon}_t \hat{\pi}_{t+1}. \]  

(3.67)

Log-linearizing equation (3.53) yields

\[ \hat{k}_{z,t} = \frac{\alpha \pi \varepsilon - 1}{1 - \alpha \pi \varepsilon - 1} \hat{\pi}_t. \]  

(3.68)

Substituting equation (3.68) into equation (3.60) and solving for \( \hat{\phi}_t \) yields

\[ \hat{\phi}_t = \hat{\psi}_t \left( \frac{1}{\omega} - \frac{\alpha \pi \varepsilon - 1}{1 - \alpha \pi \varepsilon - 1} \right). \]  

(3.69)
Using this equation to replace $\hat{\varphi}_t$ and $\hat{\varphi}_{t+1}$ in equation (3.67) leads to

$$
\hat{\varphi}_t = \frac{1}{\omega} \frac{\alpha \pi^{\varepsilon-1}}{1 - \alpha \pi^{\varepsilon-1}} \hat{\pi}_t + \alpha \beta \pi^{\varepsilon-1} (\varepsilon - 1) E_t \hat{\pi}_{t+1}
$$

$$
+ \alpha \beta \pi^{\varepsilon-1} \left[ E_t \hat{\varphi}_{t+1} - \frac{1}{\omega} \frac{\alpha \pi^{\varepsilon-1}}{1 - \alpha \pi^{\varepsilon-1}} E_t \hat{\pi}_{t+1} \right].
$$

(3.70)

**Price dispersion** Log-linearizing equation (3.58) yields:

$$
\hat{s}_t = \alpha \frac{\pi^{\varepsilon}}{\gamma} - \pi^{\varepsilon-1} \frac{1}{1 - \alpha \pi^{\varepsilon-1}} \hat{\pi}_t + \alpha \pi^{\varepsilon} \hat{s}_{t-1},
$$

(3.71)

where we have used equation (3.68).

**GNKPC** Equation (3.66) and equation (3.70) can be combined to derive the generalized New Keynesian Phillips curve with trend inflation in terms of unit labor costs

$$
\hat{\pi}_t = \omega \left[ \frac{(1 - \alpha \pi^{\varepsilon-1}) \left(1 - \alpha \beta \pi^{\varepsilon(1+\varphi) - \gamma} \right)}{\alpha \pi^{\varepsilon-1}} \right] \left[ \hat{u} c_t - (1 + \varphi) \hat{s}_t \right]
$$

$$
+ E_t \hat{\pi}_{t+1} \beta \left[ 1 + \frac{\varepsilon(1 + \varphi)}{\gamma + \varepsilon(1 + \varphi - \gamma)} (1 - \alpha \pi^{\varepsilon-1}) \left( \pi^{\frac{\varepsilon}{2}} - 1 \right) \right]
$$

$$
+ E_t \hat{\varphi}_{t+1} \beta \omega \left(1 - \alpha \pi^{\varepsilon-1} \right) \left( \pi^{\frac{\varepsilon}{2}} - 1 \right),
$$

(3.72)

where $\hat{\varphi}_t$ is given by equation (3.66).

If the steady state inflation equals zero, i.e. $\bar{\pi} - 1 = 0$, the Phillips curve (3.72) reduces to the conventional NKPC with segmented labor markets and decreasing returns to scale in labor inputs, as $\hat{s}_t$ does not affect the first order dynamics

$$
\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \omega \hat{u} c_t + E_t \beta \hat{\pi}_{t+1}.
$$

(3.73)

**Forward and backward iterations for estimation purposes** Iterating forward equation (3.66) yields

$$
\hat{\varphi}_t = \left(1 - \alpha \beta \pi^{\varepsilon(1+\varphi) - \gamma} \right) \sum_{i=0}^{N} \left( \alpha \beta \pi^{\varepsilon(1+\varphi) - \gamma} \right)^i \left[ \hat{u} c_t - (1 + \varphi) \hat{s}_t \right]
$$

$$
+ \frac{\varepsilon}{\gamma} \sum_{i=1}^{N+1} \left( \alpha \beta \pi^{\varepsilon(1+\varphi) - \gamma} \right)^i E_t \hat{\pi}_{t+i}
$$

$$
+ \left( \alpha \beta \pi^{\varepsilon(1+\varphi) - \gamma} \right)^{N+1} E_t \hat{\varphi}_{t+N+1}.
$$

(3.74)
D. APPENDIX OF CHAPTER 3

Iterating forward equation (3.67) yields

\[ \hat{\phi}_t = (\varepsilon - 1) \sum_{i=1}^{N} (\alpha \beta \pi^{\varepsilon-1})^i \hat{\pi}_{t+i} + (\alpha \beta \pi^{\varepsilon-1})^N \hat{\pi}_{t+N}. \]  

(3.75)

Iterating forward equation (3.71) yields

\[ \hat{s}_t = \alpha \frac{\varepsilon \pi^{\frac{\varepsilon}{\gamma}} - \pi^{\varepsilon-1}}{1 - \alpha \pi^{\varepsilon-1}} \sum_{i=0}^{N} (\alpha \pi^{\frac{\varepsilon}{\gamma}})^i \hat{\pi}_{t-i} + (\alpha \pi^{\frac{\varepsilon}{\gamma}})^{N+1} \hat{s}_{t-N-1}. \]  

(3.76)

D.2 Sensitivity to the value of the Calvo parameter

One of the main goals of this paper is to estimate the degree of different types of real rigidity, using the aggregate inflation dynamics. We rely on the approach in Sbordone (2002) and Dupor et al. (2010). However, unlike these papers, we do not fix the degree of real rigidity and estimate the level of nominal rigidity. We rather keep the level of nominal rigidity constant and estimate the degree of real rigidity. In this section we discuss the implications of imposing the degree of nominal rigidity for our results.

No trend inflation

First, we estimate the degree of real rigidity for different levels of nominal rigidity when the trend inflation is zero. Parameter estimates and confidence bounds are shown in Figure 3.6. We denote the degree of real rigidity by \( \lambda \). Figure 3.6 shows that the estimated degrees of real rigidity vary markedly with the imposed levels of nominal rigidity in a non-linear way.

The intuition for these results is straightforward. The data imply a small and positive link between inflation and the output gap. This can be seen in Table 3.1 and Table 3.2 which show the empirical slopes of the Phillips curve. For high degrees of nominal rigidity it is not necessary to assume high degrees of real rigidity to generate a flat Phillips curve. However, when the imposed level of nominal rigidity is low, which implies more flexible prices, the empirical evidence on the persistence of real effects of monetary policy requires high degrees of real rigidity, i.e. low values of \( \lambda \).

For our baseline value of \( \alpha \approx 0.66 \), we obtained a degree of real rigidity of 0.12 and a slope of Phillips curve of 0.023. Woodford (2003) argues that plausible values of \( \lambda \) are between 0.10 and 0.15 (see also (Coibion, 2010)). According to our estimates, a value

---

19 Similar figure can be found in Coibion (2010) who is however considering the sticky information parameter.
Figure 3.6: Estimated degrees of real rigidity, conditional on the level of nominal rigidity

Notes: Recall that $\lambda$ is defined as the inverse of the degree of real rigidity meaning that smaller values of $\lambda$ imply stronger degrees of real rigidity.

of $\lambda$ of 0.1 implies $\alpha = 0.63$ and a $\lambda$ of 0.15 implies $\alpha = 0.67$ which is close to our baseline calibration.

**Trend inflation**

For the case of positive trend inflation we report the degrees of real rigidity for the SLM scenario since this model variant implies the best empirical fit. In the SLM model $\lambda = \frac{1}{1 + \phi \epsilon}$. As can be seen from Figure 3.7, the relationship between the degree of nominal and real rigidity changes with positive trend inflation. In particular, the previously found negative monotonic relationship, which implies lower degrees of real rigidity if the level of nominal rigidity increases, now partly breaks down.

For a high level of nominal rigidity, i.e. $\alpha$ close to 1, the required values of $\lambda$ do not approach 1 as in the case of zero trend inflation. It follows that the substitutability between nominal and real rigidities in keeping the slope of the NKPC low breaks down. The empirical fit is much better than with zero trend inflation even if, in the case of high degrees of nominal rigidity, the parameter $\lambda$ remains low.
D.3 Magnitude of idiosyncratic shocks

Klenow and Willis (2016) argue that micro rigidities (KDC) necessitate implausible large idiosyncratic productivity shocks in comparison to macro rigidities (sticky intermediate prices) to match individual price dynamics from the US CPI.

In our analysis we examine three types of micro rigidities but in the following we show, that the argument of Klenow and Willis (2016) holds also within the category of micro rigidities. In particular, we show that the estimated results imply that the magnitude of idiosyncratic productivity shocks must be 5x larger in the variant with decreasing returns to scale than in the model variant with segmented labor markets.

To this end we introduce idiosyncratic productivity shocks as in Gertler and Leahy (2008). The economy is populated by a continuum of monopolistically competitive goods producers, indexed by $j \in [0, 1]$. Each firm $j$ produces the individual good $j$ and sells it directly to consumers. The production function is of the form

$$Y_{j,t} = X_{j,t}N_{j,t}^\gamma,$$  (3.77)

with $\gamma \in (0,1]$ and where $X_{j,t}$ is an idiosyncratic productivity level and $N_{j,t}$ is the labor input of firm $j$ at time $t$. For $\gamma < 1$, there are decreasing returns to scale, which
could also be interpreted as the production function being of the Cobb-Douglas type
but with fixed capital.

In every period, each firm $j$ is hit by a productivity disturbance with probability $1 - \alpha$. When this happens, the firm survives with probability $\tau$. For surviving firms, productivity changes according to $X_{j,t} = X_{j,t-1} \xi_{j,t}$, where $\xi_{j,t}$ is an i.i.d. firm-specific shock that is uniformly distributed over the support $[-\frac{1}{2}, \frac{1}{2}]$. If a firm does not survive, which happens with probability $1 - \tau$, conditional on a shock, it is immediately replaced by a new firm with productivity one, i.e. $X_{j,t} = 1$. This can be interpreted as product substitutions. We note that the main purpose of the assumption that firms may die with probability $1 - \tau$ is to guarantee a stationary distribution of productivities across firms as in Gertler and Leahy (2008) on each island.

It can be shown that the log-linearized price-setting equation is given by

$$q_{j,t}^j = \frac{\gamma}{\gamma + \varepsilon(1 + \varphi - \gamma)} \left( \hat{\psi}_t - \hat{\phi}_t \right) - \frac{1}{\gamma + \varepsilon(1 - \gamma)} x_{j,t}^j + p_t + \ln \left( \frac{Q}{P} \right), \quad (3.78)$$

$$\hat{\psi}_t = \left( 1 - \alpha \beta \pi^\varepsilon \right) \left[ u \alpha c_{t-1} - (1 + \varphi) \hat{s}_t \right] + \alpha \beta \pi^\varepsilon \left[ \frac{\varepsilon(1 + \varphi)}{\gamma} \right] \left[ \hat{\pi}_{t+1} + \hat{\pi}_t \right], \quad (3.79)$$

$$\hat{\phi}_t = \alpha \beta \pi^{\varepsilon-1} \left[ \hat{\pi}_t + (\varepsilon - 1) \hat{\pi}_{t+1} \right]. \quad (3.80)$$

Since the parameter in front of $x_{j,t}^j$ depends positively on $\gamma$ but not at all on $\varphi$, it is straightforward that in the DRS scenario the magnitude of real rigidities must be larger to achieve the same magnitude of price changes due to idiosyncratic productivity shocks. Hence, therefore even different micro real rigidities can have different implications for the size of idiosyncratic shocks and this conclusion is not limited to the comparison of micro and macro rigidities as highlighted by (Klenow and Willis, 2016). Given our estimates for the shorter sample, DRS necessitate 5x as large shocks as SLM.
Chapter 4

Endogenous Search Behavior on Posted Offer Markets with Capacity Constrained Sellers

with Carl G. Maier

Abstract

Empirical evidence shows that a low cost of signaling interest in offers, e.g. applying for jobs or apartments, can result in a large number of unsuitable applications. We explain this phenomenon as an equilibrium outcome of a microeconomic model of posted offer markets with capacity constrained sellers and multiple applications. Buyers do not know ex ante whether a given offer suits them or not and endogenously decide to acquire information or to apply uninformed. Our model has clear welfare implications in favor of informed signaling and shows that falling transaction costs can decrease market efficiency and social welfare. We argue that the generally held view that online markets are more efficient than traditional markets may be misleading.

1We would like to thank participants of the German Economic Association (VfS) meeting 2017 in Vienna, the Augustin Cournot Doctoral Days 2017 in Strasbourg, as well as seminar participants at the University of Konstanz for many valuable comments and suggestions.
4.1 Introduction

The last two decades have seen a striking rise of new internet platforms for all kinds of activities including search, e-commerce, online media and job matching. As argued by Levin (2013), internet platforms take advantage of a reduction of a range of economic costs such as the cost of acquiring and providing information and the market participation cost. A natural consequence of low participation costs is that, e.g., more workers will apply for many more jobs. In fact, Autor (2001) and Kuhn (2014) provide evidence for excess applications to online job postings. In particular, employers receive unmanageable numbers of resumes which are partly unsuitable in the sense of coming from underqualified and not suitable candidates.\(^2\)

The labor or rental markets are examples of so-called posted offer markets which require that buyers signal interest for offers. Yet these signals can be unsuitable in two ways. Either an interested buyer does not meet the seller’s requirements or, after learning more about the offer, she loses interest. This paper presents a microeconomic model of posted offer markets which involves costly signaling and incomplete information to study their interaction and implications for market efficiency and welfare. We show that the significant number of inappropriate signals observed in the data can be rationalized as an equilibrium outcome.

The basic structure of our model is the following. On a market, sellers post offers to which buyers can react. Sellers are capacity constrained such that each seller posts only one offer. Furthermore, each buyer wants to buy only one good but is allowed to apply to more than one seller. All buyers are ex-ante uninformed which offers suit their preferences. When observing a posted offer, a potential buyer can pay some research cost to learn whether the offer is suitable for her. Yet, if she does not engage in research, she can still signal her interest. In this case a deal is not always feasible. If a deal between a buyer and a seller is made, the buyer pays the price posted by the seller.

We show that it is rational for buyers not to acquire information and to signal interest despite being uninformed if the cost of applying is low compared to the cost of information. A switch from an equilibrium in which buyers conduct research before signaling their interest to an equilibrium with uninformed signaling can occur even if both of these costs fall. The prices posted by the sellers are linked to the buyers’ information decision. Our model implies that sellers increase prices proactively to induce buyers

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\(^2\)In addition, applications might carry little information concerning the applicants’ determination to accept the offer (job) when given the chance, as can be easily illustrated e.g. by the effort and time to fill faculty positions. Bram (2016) argues that the rental housing markets show similar degrees of glut and unsuitability of applications.
to acquire information. The equilibrium in which buyers do not acquire information and apply uninformed implies higher market congestion and lower social welfare. For plausible parameterizations it is not constrained efficient even though sellers do charge lower prices.

Our paper has clear welfare implications. Increasing participation costs by making signals more costly or lowering information acquiring costs would improve market efficiency and social welfare. We discuss some current real-world approaches in market design aiming at increasing the information content of signals consistent with the results of our model in Section 4.7. In addition, we derive testable predictions such as that sellers proactively increase prices to make buyers applying informed and buyers send uninformed signals if the ratio of signaling costs and costs of information is sufficiently low.

The paper is structured as follows. Section 4.2 provides an overview of the related literature. Section 4.3 introduces and solves a simplified version of the model with a single seller. Section 4.4 interprets and discusses the implications derived from this model. Section 4.5 introduces and solves an extended model with several sellers. In Section 4.6 we discuss the obtained results and Section 4.7 presents the implications of our analysis for market design. Section 4.8 concludes.

### 4.2 Related literature

In this work we analyze posted offer markets with capacity constrained sellers where buyers can simultaneously apply for multiple offers and endogenously decide to acquire information or not. Our work is thus related to various strands of literature focusing on one or more of these characteristics. Yet to the best of our knowledge there is no work analyzing all these characteristics simultaneously.

First, we contribute to the literature on posted offer markets where information is costly to consumers. Our paper is closely related to Burdett and Judd (1983) and Lester (2011). Burdett and Judd (1983) study the impact of incomplete information on price dispersion in equilibrium but abstract from capacity constrained sellers. Lester (2011) shows that the assumption of capacity constrained sellers plays a crucial role and leads to a so-called Lester’s paradox: A larger share of informed buyers leads to higher market prices.\(^3\) The basic intuition for this result is that with capacity constrained sellers, buyers turn their interest to offers with higher prices as they expect fewer competing buyers for these offers. However, both papers assume fixed fractions of informed and

\(^3\)Helland, Moen, and Preugschat (2017) find experimental evidence for this result.
uninformed buyers. Our paper differs by endogenizing information acquisition and allowing for multiple applications. We provide an alternative explanation for Lester’s paradox by showing that the relation between prices and information works in both directions: sellers can increase the information acquisition of buyers by setting higher prices.

Second, we allow buyers to signal their interest simultaneously in multiple offers. Several papers consider posted offer markets and capacity constrained sellers, while allowing for multiple applications. Albrecht et al. (2004) derive a matching function under these assumptions, and Albrecht et al. (2006) assess the impact of multiple applications on wages and matching efficiency and identify multiple applications as inefficient. Whereas these two works focused on undirected search, meaning uniformed buyers and random applications, Galenianos and Kircher (2009) look at directed search. In their model, all postings are observed by the workers and each worker decides how many applications to send. Firms then follow by making a job offer to one of the applicants they have received, if any. Finally, workers that receive one or more offers choose which offer to accept. If a firm’s chosen applicant rejects the job offer then the firm remains unmatched. In contrast to previous findings, they conclude that multiple applications can impair efficiency.

Our paper is closely related to Kircher (2009) who relaxes the assumption that firm cannot approach other applicants in the case a firm’s initial offer was rejected which implies a different matching process. In this set up under non-zero application costs the constrained efficiency of equilibrium is restored in comparison to Galenianos and Kircher (2009) and the outcomes converge to a social optimum if application costs vanish. The driving element of this finding is that sellers can process all applicants in their queues which implies that offers cannot stay vacant due to coordination frictions.

This paper differs from previous studies and in particular from Kircher (2009) in two important dimensions. First, the decision of a buyer to conduct a directed or undirected search is endogenous. We find that the model outcome does not necessarily converge to a social optimum if application costs vanish since buyers may decide to engage in undirected search. Second, the matching technology in case of multiple sellers and multiple offers is different. Kircher (2009) applies a concept of stability which implies that a firm does not leave the job vacant while one of its applicants starts to work at a lower wage or remains unemployed. We abstract from this concept and assume that if a match has been formed, it cannot be dissolved and matched workers cannot be approached by other sellers again. We find that equilibrium with multiple applications but where buyers do not acquire information is not constrained efficient and imply

\[4\text{For a recent comprehensive survey of directed search models see Wright, Kircher, Julien, and Guerrieri (2017).}\]
welfare losses.

Third, Cheremukhin, Restrepo-Echavarria, and Tutino (2013, 2016) consider targeted search in which buyers decide about the intensity and location of their search efforts. Our work can be considered as incorporating the polar cases of a continuum of targeted search: Either buyers do not search at all or they examine every offer with an intensity such that every piece of information is revealed. In contrast to the two mentioned works we allow for multiple applications.

4.3 The single seller model (SSM)

In this section we present a simplified version of our model with a single seller and multiple buyers. This model variant allows us to solve the model analytically and is rich enough to illustrate the basic model mechanism and our results.

4.3.1 Model description

In the SSM model we consider a game played by $B$ buyers and one seller who posts and commits to an offer with the price $P$. All buyers observe the price but differ in their individual willingness to pay, $\chi$, which is a random variable with cumulative density function $F$ and $\chi \in [0, \overline{\chi}]$. For simplicity we set $\overline{\chi} = 1$.

To learn $\chi$ is costly. After observing $P$, every potential buyer can decide to learn her own willingness to pay $\chi$ by paying the research cost $c_R$. If no research is done, potential buyers stay unaware of their realization of $\chi$. Every potential buyer can signal her interest in the offer by paying the signaling cost $c_I$ (independently of conducting research before). Buyers who do not apply drop out of the game.

The seller receives applications but cannot infer which buyers are willing to accept the offer. Therefore the seller will randomly consider one of the applications and pay the $c_A$ to learn this buyer’s realization of $\chi$.\footnote{$c_A$ can be interpreted as costs of interviewing or showing somebody an apartment for rent.} If the considered buyer applied uninformed, she will incur the cost $c_R$ now and learn her realization of $\chi$. If $\chi \geq P$, a deal is achieved and the game ends. If $\chi < P$, the seller continues and processes another application at the cost $c_A$. This process continues until a deal is made or $\chi < P$ is revealed for all interested buyers. If a deal is achieved, the involved buyer realizes the surplus $\chi - P$ and the seller the surplus $P$.\footnote{The term surplus captures the benefits from achieving a deal for the seller and the respective buyer, disregarding the costs they have incurred so far. In contrast, the term payoff is used to refer
To sum up, the game (i.e. the matching process) in the SSM has the following stages:

1. The seller posts an offer with the price $P$.
2. Buyers decide to conduct research.
3. Buyers decide to signal an interest in the offer.
4. The seller randomly selects one application to process.
5. The selected buyer learns her willingness to pay if not done so before signaling her interest and accepts or rejects the offer.
6. If the offer is accepted, a match is established. If the offer is rejected, the seller proceeds with further applications until a deal is made or there are no remaining applications.

### 4.3.2 Equilibrium concept and strategies in the SSM

The equilibria determined in the following are perfect Bayesian equilibria in pure strategies. Following Lester (2011) and as it is common in the directed search literature due to coordination frictions, we focus on symmetric equilibria where all buyers play the same strategy.

Players maximize their expected payoffs. The seller’s strategy is the price $P$ that can be set to any weakly positive number. Buyers decide about doing research and signaling interest. Two strategies of the buyers can be directly ruled out since obvious deviations exist that yield a greater expected payoff. First, doing research and never signaling interest definitely leads to the cost $c_R$ with no chance to realize a surplus, as the buyer cannot be part of a deal. Second, doing research and signaling interest independently of the learned realization of $\chi$ as this would imply that a buyer pays $c_I$ even in the cases where $\chi < P$ is known. The strategy of never doing research and never signaling interest is optimal for all $P > \bar{\chi}$, i.e. if the seller demands a prohibitive price. Yet, it cannot constitute an equilibrium in the SSM, as will become clear. Buyers have therefore two viable strategies: playing $U$, i.e. no research but applying, and playing $I$, i.e. to conduct research and to apply conditionally on the realization of $\chi$.

to the surplus less the costs.
4.3.3 Solution of the SSM

The model is solved by backward induction. First, the buyers’ stage is solved given an arbitrary price \( P \). Second, the price is determined by maximizing the seller’s expected payoff given the buyers’ strategies.

The buyers’ stage in the SSM

Let \( P \) be the price set by the seller. Consider strategy \( I \) first. Buyers learn their willingness to pay \( \chi \) by incurring the cost \( c_I \). We assume that buyers will signal interest if \( \chi > P \). By doing so we neglect the impact of paying \( c_I \) on the initial decision to signal interest. More precisely, if an individual buyer knows \( \chi \), there exists a critical willingness to pay \( \tilde{\chi} \), at which the buyer is indifferent between signaling interest or not. We impose the rule \( \chi > P \) for applying since it makes the main insights of the paper more tractable and sharpens the intuition for our results. We argue that our findings are not driven by this assumption and would be even stronger without it as we discuss in more detail in Appendix E.1.

Assume that all buyers play \( I \). The expected payoff of one individual buyer is given by

\[
E\Pi_B^{I,I} = -c_R + (1 - F(P)) \left( -c_I + PA^{I,I} (E(\chi|\chi \geq P) - P) \right).
\] (4.1)

The interpretation of equation (4.1) is straightforward. When playing \( I \), the buyer pays \( c_R \) directly. \( \chi \geq P \) is true with probability \( 1 - F(P) \), and in this case she will signal interest and pay the signaling cost \( c_I \). With probability \( PA^{I,I} \), she will be approached and asked by the seller to accept the offer and realize the expected surplus \( E(\chi|\chi \geq P) - P \).

The probability of being asked \( PA^{I,I} \) is given by

\[
PA^{I,I} = \frac{1}{(B - 1)(1 - F(P)) + 1}.
\] (4.2)

Because all buyers play \( I \) such that \( \chi \geq P \) holds for all buyers who signal interest, the seller is able to achieve a deal with the first interested buyer he approaches, which implies that an individual buyer’s probability of being asked by the seller equals the probability of being asked first. From the perspective of a buyer who has learned \( \chi \geq P \) and signaled interest, the expected number of interested buyers is given by \( (B - 1)(1 - F(P)) + 1 \) since an individual buyer knows that each of the \( B - 1 \) remaining buyers learns \( \chi \geq P \) with probability \( 1 - F(P) \) and signals interest in this case.
To show that there exists an equilibrium in which every buyer plays $I$ one needs to show that no individual buyer has an incentive to deviate and to play $U$. The deviating buyer’s expected payoff is

$$E\Pi^U_B = -c_I + PA^U_I (-c_R + (1 - F(P)) (E(\chi|\chi \geq P) - P)),$$

(4.3)
The deviating buyer will always pay $c_I$. If she is approached, she has to pay $c_R$. Given $\chi \geq P$, she accepts the offer. Note that all other buyers still play $I$. Therefore, the deviating buyer can also only buy the good if she is approached first, since the seller will achieve a deal with any other interested buyer at first try. Thus $PA^U_I = PA^I_I \equiv PA^I$. Simplifying $E\Pi^I_B \geq E\Pi^U_B$ leads to the inequality

$$c \equiv \frac{c_I}{c_R} \geq \frac{1 - PA^I}{F(P)} \equiv \Omega^I.$$

(4.4)

Note that we set $\frac{c_I}{c_R} \equiv c$ for simplicity. Condition in equation (4.4) characterizes when it is optimal for an individual buyer to play $I$, given all other buyers play $I$. Thus, it constitutes the condition for a symmetric equilibrium on the buyers’ stage where all buyers play $I$.

Next, assume that all buyers play $U$. The expected payoff of one individual buyer in this case and the deviation payoff when only that buyer switches to strategy $U$ are given by

$$E\Pi^U_B = -c_I + PA^U_U (-c_R + (1 - F(P)) (E(\chi|\chi \geq P) - P)),
$$

(4.5)

$$E\Pi^I_B = -c_R + (1 - F(P)) (-c_I + PA^I_U (E(\chi|\chi \geq P) - P)).$$

(4.6)

In the case of $E\Pi^U_B$, cost $c_I$ has to be paid always. $c_R$ occurs when being approached, and only with probability $pr(\chi \geq P)$ is the expected surplus realized. Deviating to $I$ implies that $c_R$ is always incurred. Interest is signaled if $\chi \geq P$, and the expected surplus may be realized if the buyer is approached by the seller (since $\chi \geq P$ is ensured at this point). As before, $PA^I_U = PA^U_U$. Here however, the possibility exists that no deal is reached with a random interested buyer, since $\chi < P$ is possible. So buyers can actually be approached as the second, third, and so on. Being approached second requires three things. The particular buyer must not have been asked first, the willingness to pay of the buyer who was asked first must have been lower than the price, and the considered buyer must have been actually approached second. Similar conditions are required to be approached third and so on. Since the remaining buyers play $U$ by assumption, there will be $B$ interested buyers if the individual buyer signals interest. Thus, a buyer might even be approached as the $B$th by the seller. Let
4.3. THE SINGLE SELLER MODEL (SSM)

\[ PA^{I,U} = PA^{U,U} \equiv PA^U \]
and
\[
PA^U = \frac{1}{B} + \frac{B - 1}{B} F(P) \left( \frac{1}{B - 1} + \frac{B - 1}{B} \frac{2}{B - 1} (F(P))^2 \frac{1}{B - 2} + \ldots \right) \\
= \frac{1}{B} \sum_{n=0}^{B-1} (F(P))^n = \frac{1 - F(P)^B}{B (1 - F(P))}.
\]

(4.7)

Simplifying \( E\Pi_B^{U,U} \geq E\Pi_B^{I,U} \) yields the inequality in equation (4.8), which characterizes a symmetric equilibrium on the buyers’ stage in which every buyer plays \( U \).

\[
c \leq \frac{1 - PA^U}{F(P)} \equiv \Omega^U.
\]

(4.8)

The intuition is straightforward. In equations (4.4) and (4.8), the identity of the individual buyer is arbitrary. Thus, the strategy profile where playing \( I \) constitutes a symmetric Nash equilibrium if \( c \geq \Omega^I \). Analogously, the strategy profile where every buyer plays \( U \) constitutes a symmetric Nash equilibrium with \( c \leq \Omega^U \). Note that \( \Omega^I, \Omega^U \geq 0 \) since \( PA^I, PA^U \leq 1 \). Furthermore, \( \Omega^I \geq \Omega^U \) is always true.\(^7\) Hence, there is no issue of equilibrium selection.

For \( c \in (\Omega^U, \Omega^I) \), no symmetric equilibrium in pure strategies exists. We show that a symmetric mixed strategy equilibrium exists for each \( c \in (\Omega^U, \Omega^I) \).\(^8\) Consider the mixed strategy \( \sigma_r \) where buyers play action \( I \) with probability \( r \) and action \( U \) with probability \( 1 - r \). Obviously \( \sigma_1 = I \) and \( \sigma_0 = U \) are implied. There exists \( r^* \) that characterizes a symmetric Nash equilibrium in the buyers’ stage for every \( c \in [\Omega^U, \Omega^I] \).

Intuitively, \( r^* \) falls with decreasing \( c \) meaning the probability of playing \( I \) decreases as the cost of signaling relative to the cost of research decreases. We show that there is a smooth transition between the two pure strategy equilibria.

\[ r^* \]

Figure 4.1: Equilibrium values \( r^* \) of the mixed strategy \( \sigma_r = \{I, r; U, 1 - r\} \)

\(^7\)Refer to Appendix E.2.

\(^8\)See Appendix E.3 for the formal parts of the discussion on mixed strategy equilibria and asymmetric equilibria.
Figure 4.1 illustrates the equilibria in the buyers’ stage. The mixed strategy equilibria lie between the indicated thresholds $Ω^U$ and $Ω^I$. For $c ≤ Ω^U$ buyers play $U$ and $c ≥ Ω^I$ $I$. Both thresholds $Ω^I$ and $Ω^U$ converge to $\frac{1}{F(P)} = c ≡ Ω$ if the number of buyers converges to infinity (see Appendix E.2).

The seller’s stage in the SSM

In this section we focus on the seller’s behavior and determine the optimal price $P$ given buyers’ optimal strategies. We consider the limit case with the single boundary $Ω$. We show that there exists a unique threshold price $\tilde{P}$ given the cost ratio $c$ such that buyers play $U$ if $P ≤ \tilde{P}$ and they play $I$ if the posted price is above this threshold. This implies that the seller can by posting the price determine buyers’ equilibrium strategy.

Given all buyers playing $I$, the seller will be able to achieve a deal if there is at least one interested buyer, since strategy $I$ ensures $χ ≥ P$. If this is true, the seller picks a random interested buyer, pays $c_A$ once and receives the surplus $P$. The seller’s expected payoff if all buyers play $I$ is given by

$$EΠ^I_S = PD^I (-c_A + P),$$  

where $PD^I$, the probability of achieving a deal given all buyers play $I$, is given by $1 - F(P)^B$. At least one buyer must learn $χ ≥ P$ to signal interest. The seller’s expected payoff if all buyers play $U$ can be constructed accordingly. If all buyers play $U$, all $B$ buyers signal interest. This implies that the seller pays $c_A$ at least once, since he will approach at least one interested buyer. With probability $1 - F(P)$, $χ ≥ P$ holds and a deal is achieved with the buyer approached first and the surplus $P$ is realized. With the counter probability $F(P)$, no deal is possible. Thus the seller picks another interested buyer, incurs cost $c_A$ again and has the same chance of achieving a deal with this new interested buyer. In the worst case the seller will not be able to achieve a deal with any of the $B$ interested buyers. In this case he pays $c_A$ $B$ times. Following from this logic, the seller’s expected payoff given all buyers play $U$ is

$$EΠ^U_S = -c_A + (1 - F(P))P + F(P)\left(-c_A + (1 - F(P))P + F(P)\left(-c_A + (1 - F(P))P + F(P)\left(...\right)\right)\right).$$
and can be simplified to

\[ E\Pi^U_S = \frac{1 - (F(P))^B}{1 - F(P)}(P(1 - F(P)) - c_A). \] (4.10)

Comparing \( E\Pi^I_S \) and \( E\Pi^U_S \) given by equation (4.9) and equation (4.10) shows that \( E\Pi^I_S \geq E\Pi^U_S \) is always true.\(^9\) This is quite intuitive. Whenever buyers play \( U \), the seller needs to identify appropriate buyers, which is costly. If buyers play \( I \), only buyers truly interested in the offer apply. If \( P = 0 \), the following holds:

\[ E\Pi^I_S \bigg|_{P=0} = -c_A, \quad E\Pi^I_S \bigg|_{P=1} = 0, \quad E\Pi^U_S \bigg|_{P=0} = -c_A \quad \text{and} \quad E\Pi^U_S \bigg|_{P=1} = -Bc_A \] (4.11)

This means that the seller is always able to give the good away for free to the first approached buyer. \( P = 0 \) also ensures that all potential buyers signal interest if all play \( I \). Demanding a prohibitive price, he will not receive any applications if the buyers play \( I \). However, all \( B \) buyers declare interest if they play \( U \). Then the seller will approach each of them and pay \( c_A \) every time, but he will not be able to achieve a deal. Evaluating the slopes of the two expected payoffs of the seller at \( P = 0 \) and \( P = 1 \) reveals that

\[
\begin{align*}
\frac{\partial E\Pi^I_S}{\partial P} \bigg|_{P=0} &= 1 > 0, \\
\frac{\partial E\Pi^I_S}{\partial P} \bigg|_{P=1} &= -Bf(1)(1 - c_A) < 0, \\
\frac{\partial E\Pi^U_S}{\partial P} \bigg|_{P=0} &= 1 - f(0)c_A > 0 \quad \text{and} \quad \frac{\partial E\Pi^U_S}{\partial P} \bigg|_{P=1} &= -Bf(1)\left(1 - c_A \frac{B - 1}{2}\right) < 0
\end{align*}
\] (4.13)

where \( f \) is the pdf of \( \chi \). A marginal rise in price from \( P = 0 \) will translate to an equal rise in expected payoff if buyers play \( I \). If buyers play \( U \), the seller now faces the risk of approaching a buyer with \( \chi < P \). This effect is strong if probability mass lies on small realizations of \( \chi \). The positive slopes at \( P = 0 \) and the negative ones at \( P = 1 \) imply that both expected payoffs exhibit at least one interior maximum.\(^{10}\) The prices that maximize \( E\Pi^I_S \) and \( E\Pi^U_S \) are denoted by \( P^*I \) and \( P^*U \). Lastly, \( \frac{\partial E\Pi^I_S}{\partial P} \geq \frac{\partial E\Pi^U_S}{\partial P} \) is true for all \( P \). For any \( P \) that implies \( f(P) \geq 0 \), the inequality is strict. This aspect is crucial and implies the two findings \( P^*I > P^*U \) and \( E\Pi^I_S \geq E\Pi^U_S \) for all \( P \).

---

\(^9\)See Appendix E.4 for the formal parts of the discussion here.

\(^{10}\)The normalization \( \bar{\chi} = 1 \) obviously implies \( P \in [0, 1] \). Thus \( c_A \leq 1 \) such that \( 1 - f(0)c_A > 0 \) follow.
Given the single threshold $\Omega$, the conditions $c < \Omega$ and $c \geq \Omega$ can be interpreted as incentive compatibility constraints and the maximization problem of the seller reads

$$
\max_P E\Pi_S = \begin{cases} 
E\Pi_S^U = \frac{1-\left(F(P)\right)^B}{1-F(P)}(P(1-F(P))-c_A), & \text{for } c < \Omega \\
E\Pi_S^I = (1-F(P)^B)(P-c_A), & \text{for } \Omega \leq c 
\end{cases}.
$$

(4.15)

Since $\Omega = \frac{1}{F(P)}$, the seller can provide incentives for buyers to behave in a certain way. Note that $\Omega$ (and both $\Omega^U$ and $\Omega^I$) fall in $P$; see Appendix E.5. Therefore, the seller will induce buyers to play $I$ if he sets a sufficiently large $P$; conversely, he can induce buyers to play $U$ for low prices. The seller’s influence on the buyers’ behavior is shaped by the relative costs $c$. One critical price $\tilde{P}$ exists for each $c$ that implies $\frac{1}{F(P)} = c$. For any $P < \tilde{P}$ the buyers play $U$ and for $P \geq \tilde{P}$ buyers play $I$.

Figure 4.2 displays the expected payoffs obtained before (left) and the resulting optimal prices $P^*$ (right). Due to $E\Pi_S^I \geq E\Pi_S^U$, setting the price $P^*$ is the best choice if $P^* \geq \tilde{P}$ (e.g. the case of $\tilde{P}^1$ in Figure 4.2). Here the incentive compatibility constraint is not constraining the seller effectively. The seller faces a binding constraint whenever $\tilde{P} > P^*$. He can no longer induce the buyers to play $U$ by setting $P^*$. Furthermore, since $E\Pi_S^I$ falls for all $P > P^*$, the price that maximizes the seller’s payoff while inducing buyers to play $I$ is the lowest possible price, $\tilde{P}$. Two scenarios must be distinguished. First, if $\tilde{P}$ is sufficiently small (e.g. $\tilde{P}^2$ in Figure 4.2), the seller obtains a higher expected payoff by setting $P = \tilde{P}^2$ than by setting the price $P^*$. Note that $P = P^*$ is always feasible and incentive compatible if $\tilde{P} > P^*$ since $P^* < P^*$. Second, if $\tilde{P}$ is sufficiently large (e.g. $\tilde{P}^3$), the seller will no longer try to induce buyers to play $I$ and will set $P = P^*$. The critical $\tilde{P}$ where this change occurs is the one that sets the seller indifferent and satisfies $E\Pi_S^I(\tilde{P}) = E\Pi_S^U(P^*)$. For the parametrization underlying Figure 4.2, this price is $P = 0.89$. For simplicity we denote $c(\tilde{P})$ as the cost ratio that implies the critical price $\tilde{P}$ and $\tilde{P}(c)$ vice versa.

Concluding the solution of the SSM, the optimal prices $P^*$ that are set by the single seller for different values of the cost ratio $c$ are given by

$$
P^*(c) = \begin{cases} 
P^* \text{ for } c \text{ that imply } P^{\text{indif}} < \tilde{P}(c) \text{ (low values of } c), \\
\tilde{P} \text{ for } c \text{ that imply } P^* < \tilde{P}(c) \leq P^{\text{indif}} \text{ (medium values of } c), \\
P^* \text{ for } c \text{ that imply } \tilde{P}(c) \leq P^* \text{ (high values of } c) 
\end{cases}.
$$

(4.16)

where $P^{\text{indif}}$ is the highest price that is set by the seller. It is given by $E\Pi_S^U(P^*) = E\Pi_S^I(P^{\text{indif}})$, since it renders the seller indifferent. Note that buyers play $U$ for $P^* = P^*$ and $I$ in all other cases.
4.4 Findings of the SSM

4.4.1 Pricing in the SSM and Lester’s paradox

Figure 4.3 displays the function $P^*(c)$ given by equation (4.16) for different values of $B$. The general structure of $P^*$ does not depend on the number of buyers; hence, two conclusions follow. First, the single seller tries to provide incentives for buyers to acquire information. As argued before, it falls upon the seller to identify appropriate buyers whenever buyers signal interest uninformed, and this is costly for him. By raising the price, the seller can induce buyers to play $I$. Given any $c$, the seller can always induce buyers to play $I$ by demanding a price that is sufficiently close to $\chi = 1$.

Raising the price lowers the chance that a buyer learns $\chi \geq P$ and declares interest. Thus, the seller faces an increasing probability of being unable to sell his good. When $c$ is low such that the lowest price $P$ that would induce buyers to play $I$ is very large (formally $P = P^{\text{indiff}}$; see Equation equation (4.16)), this negative effect dominates the positive one. In these cases the seller forfeits his objective to induce informed buyers and adapts his pricing to the uninformed buyers by lowering his price to $P^* = P^{*U} < P^{*I}$. This sacrifice of potential surplus is profitable, since a lower price limits the expected number of instances he has to approach a buyer to achieve a deal. This process leads to the second observation. Lester’s paradox states that a higher share

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11 To account for the small numbers of buyers, the graphs were constructed using the thresholds $\Omega^I$ and $\Omega^U$ instead of the limit case. Thus, the implied critical prices $\tilde{P}$ as shown in Figure 4.2 are now split into a lower and upper boundary. We still restrict buyers to play pure strategies such that no pure strategy equilibrium exists on the buyers’ stage for prices between these boundaries. For these we set $\bar{\Pi}_S = 0$ such that the seller never chooses such a price.
4.4. FINDINGS OF THE SSM

Figure 4.3: Equilibrium prices

Notes: Prices $P^*$ in the unique symmetric equilibria given the cost ratio $c$ for different numbers of buyers. Underlying parameters: $c_A = 0.1$ and $\chi \sim U[0, 1] = \beta(1, 1)$.

of informed buyers who can engage in directed compared to undirected search may lead to higher prices on posted offer markets with capacity constrained sellers. The function $P^*(c)$ (Equation equation (4.16)) exhibits exactly these features, although the causation here is reversed. The seller deliberately sets higher prices to ensure that buyers become informed and is not reacting to an exogenous increase in the share of informed buyers.

Comparing the specific shapes of the functions $P^*(c)$ for different numbers of buyers (Figure 4.3) reveals standard features. For given thresholds, the seller is able to charge a higher price if he is confronted with a larger number of buyers. Yet, note that the thresholds where the behavior of the buyers changes depend on the number of buyers. An increase in $B$ raises the thresholds $\Omega^I$ and $\Omega^U$, since the probabilities of being asked fall. Thus it becomes harder to induce the buyers to play $I$. However, this effect can be neglected for $B \geq 10$, as becomes apparent in Figure 4.3.
4.4.2 Efficiency and welfare in the SSM

Figure 4.4 shows the expected payoffs of the seller, an individual buyer and the sum of all payoffs that are implied by the prices $P^*$ displayed in Figure 4.3. Clearly, the shapes of $E\Pi_S$ and $E\Pi_B$ exhibit the features identified above. The seller is worse off if buyers play $U$. He induces buyers to become informed for some $c$ such that his expected payoff declines smoothly. An individual buyer obtains an expected payoff around zero. This result is implied by the model, since buyers are price takers. However, she is better off if she can play $U$ instead of $I$. In particular, the buyers are hurt by the seller’s attempt to keep them from playing $U$. This finding is best visible for $B = 2$ but generally true. Lastly and disregarding the transition from $I$ to $U$, $E\Pi_B$ and therefore also the sum of the expected payoffs rises if $c$ falls. Caused by the falling costs, this increase is best visible in the graph for $B = 50$. Different slopes of the sum of the payoffs are driven by the fact that whenever buyers play $U$, all buyers signal interest unconditionally. Thus, for small $c$ in these instances, all buyers will profit from the falling cost ratio compared to only those who learn $\chi \geq P$ whenever buyers play $I$.

In the following we focus on efficiency and welfare. As a measure of welfare we consider the sum of all expected payoffs as well as the Pareto criterion. An appropriate measure of the efficiency is introduced later. We first address the question how a falling cost ratio $c$ affects welfare (a move along the x-axis). For this exercise only the solid lines in Figure 4.4 are of interest. Whenever the seller is able to set $P^* = P^*I$, a marginal decrease in $c$ implies higher welfare. This is straightforward: The behavior of the agents is unaffected, the same price is posted and at least one cost parameter falls such that overall $c$ falls.

Whenever $P^* = \tilde{P}$ (see equation (4.16)) such that the seller induces buyers to play $I$, a falling cost ratio results in lower welfare. This outcome is due to the increase in the posted price as $P^* = \tilde{P}$ rises if $c$ falls. This represents also a worsening according to the Pareto criterion for both the seller and the buyer.

In the $U$ equilibrium welfare increases if $c$ falls. In this scenario buyers’ expected payoff is larger than in all other scenarios, and buyers profit from a falling cost ratio. The gain of an individual buyer from the transition from $I$ to $U$ is small compared to the loss of the seller. Therefore, an equilibrium where buyers play $U$ can only be welfare maximizing at $c = 0$ and if the number of buyers is sufficiently large. Here, only the graph for $B = 50$ exhibits this feature.

We now turn our focus to the study of welfare given a certain cost ratio. Imagine the term falling cost ratio implies that neither $c_R$ nor $c_I$ rise. Note that Figure 4.4 sets $c_R = 0.01$ such that $c_I$ is varied to imply different cost ratios.
4.4. FINDINGS OF THE SSM

\[ B = 2 \]
\[ c_0 = 0.19 \]
\[ \sum \Pi^{S} \ \Pi^{B} \]
\[ B = 5 \]
\[ c_0 = 0.31 \]
\[ \sum \Pi^{S} \ \Pi^{B} \]
\[ B = 10 \]
\[ c_0 = 0.34 \]
\[ \sum \Pi^{S} \ \Pi^{B} \]
\[ B = 50 \]
\[ c_0 = 0.34 \]
\[ \sum \Pi^{S} \ \Pi^{B} \]

Figure 4.4: Individual expected payoffs and welfare

Notes: Expected payoffs of the seller (red), an individual buyer (blue) and the sum of all expected payoffs \( \Pi_S + B \cdot \Pi_B \) (black) given the cost ratio \( c \) for various numbers of buyers. Dotted graphs assume that buyers always play \( I \). Underlying parameters are the same as in Figure 4.3, furthermore \( c_R = 0.01 \).

the problem of a social planner in this context. We follow the related literature use the concept of constrained efficiency, meaning that the planner’s information mirrors that of the agents in the model. The planner who is constrained in his information has no way to tackle inefficiencies caused by information problems, therefore we let the planner to choose buyers’ strategies such that any occurring coordination problems can be resolved.\(^{13}\) The dotted lines Figure 4.4 correspond to the scenario in which the buyers’ strategy is exogenously fixed such that they always \( I \). Whenever the black dotted lines lie above the solid black lines, the equilibrium behavior is not constrained efficient. This is the case for scenarios with realltively few buyers. If the number of buyers increase, However, for larger \( B \) such that as shown in the bottom left panel of 4.4 for \( B = 50 \), the decentralized allocation is constrained efficient.

\(^{13}\)Importantly, the planner is – like the buyers themselves – unaware of the realizations of \( \chi \). Hence, the planner is unable to improve the quality of the achieved matches.
4.5 The multiple sellers model (MSM)

The assumption of a single seller helps to clarify the price setting mechanism and the influence of the price on the buyers’ behavior. Furthermore, many findings can be drawn from the SSM without considering the extended model. However, the assumption of only one seller constitutes a serious restriction. The MSM model presented in this section can therefore be considered as a robustness check.

The crucial characteristic of the MSM is that we allow buyers to react to more than one offer. Real-world posted offer markets, e.g., the labor or housing markets, are today more and more confronted with multiple applications of buyers as discussed in our motivation. Sellers now face the risk that buyers who have a sufficiently large willingness to pay might turn down their offer because they have already accepted or prefer the offer of another seller. However, this extension makes an analytical solution of our model unfeasible. This fact seems surprising at first glance, yet even the simplest scenario that includes two sellers and two buyers is too complicated to solve for a closed form solution. Appendix E.6 illustrates the complexity that emerges in this simple setting. The combination of multiple applications and the sellers’ ability to move along their queues and ask interested buyers sequentially leads to expected payoffs of buyers that are difficult to handle. As a result, the buyers’ information decision cannot be identified properly, which in turn implies that the sellers’ stage cannot be modeled accordingly. For this reason we solve for buyers’ optimal strategies analytically but for sellers’ optimal price numerically.

4.5.1 Description of the MSM

There are $S$ sellers, each offering one good. Seller $j$ posts price $P_j$. Each of the $B$ buyers again seeks to buy one of the goods. We assume that all buyers have to react to all posted offers in the same way – either by playing $I$ or $U$. This assumption is not uncommon (see, e.g., Bulow and Levin (2006)) and is made for two reasons. First, it keeps the solution of the model tractable. Second, it excludes ex ante competition between sellers in prices that arises if buyers have to make a portfolio choice when deciding where to apply. Notably, this assumption does not eliminate all competition for buyers among sellers, as it still allows ex post competition if a buyer is approached by several sellers. In a wider sense, the assumption allows us to determine the buyers’ behavior analytically, even with an endogenous decision of information acquisition. Solving buyers’ portfolio choice where to apply would make this unfeasible.
reflects our focus on symmetric equilibria, since “vacancies [offers] are equally attractive ex ante” (Albrecht et al., 2006, p. 885).

Again, sellers receive applications from buyers which again carry no information whether the buyer is informed or not. All sellers try to achieve a deal with interested buyers at the same time. The matching mechanism is now as follows:

1. All active sellers approach an interested buyer at random. A seller is active if he has not yet sold his good and if he is confronted with a positive number of interested buyers.

2. The buyers approached decide about accepting the offers of the respective sellers. A buyer approached by one or more sellers will accept the offer that yields the largest weakly positive surplus. A buyer will decline all offers that lead to negative surpluses. Approached buyers who have signaled interest uninformed pay $c_R$ and learn their willingness to pay.

3. Sellers eliminates the buyer from this queue if she has declined the offer.

These three steps are repeated as long as there are active sellers. No active sellers are left if either all sellers have sold their good or the sellers who have not yet sold it have no interested buyers left to approach. This mechanism implies three assumptions.

First, buyers will opt for the offer that yields the largest surplus. Second, buyers do not wait strategically and turn down offers to wait for further approaches of other sellers that may lead to a larger surplus. Third, for simplification, buyers who have already accepted an offer and are approached by a seller have to pay $c_R$ again if they have signaled interest unconditionally to this seller. There are two justifications for this assumption. It takes the MSM closer to the SSM where approached uninformed buyers surely pay $c_R$, and it reduces the complexity of the calculus greatly. Yet the assumption can be debated, since it seems in conflicting with the implicit assumption that established matches cannot be dissolved. However, the cost $c_R$ can be viewed as a loss that comes with turning down an offer. Exemplary, a buyer who is offered a job or a place in a degree program is required to take some action to turn the offer down.

4.5.2 Strategies and equilibrium concept in the MSM

Strategies of buyers are mappings of prices to actions, whereas the former strategies $I$ and $U$ are now actions. The strategies of the sellers are the prices they demand for their respective goods.

The applied equilibrium concept is again the Perfect Bayesian Equilibrium in pure
strategies. The buyers’ strategies must imply Nash equilibria in the buyers’ stage for any set of prices \( \{P_j\}_{j=1}^S \). Furthermore, the set of prices implied by the sellers’ strategies must be Nash as well, such that every seller plays a best response to the prices set by the other sellers and anticipates the buyers’ equilibrium behavior. In the equilibrium, all buyers play the same strategy. In addition, we also assume symmetry in the sellers’ stage and disregard all asymmetric equilibria. This means that in equilibrium all sellers charge the same price.

4.5.3 Solution of the MSM

The solution is again done by backward induction. To distinguish the calculations that follow from the solution of the preceding model, the capital Greek letters are replaced by their lower case counterparts.

The buyers’ stage in the MSM

Let the prices posted by the sellers be \( \{P_j\}_{j=1}^S \). If an individual buyer plays action \( I \) for all offers – she signals interest if her willingness to pay exceeds the price of an offer – and the remaining buyers play the same strategy, the individual buyer’s expected payoff is given by

\[
E\pi_B^{I,I} = -c_R S - c_I \sum_{j=1}^S (1 - F(P_j)) + P_{r_{\text{deal}}}^{I,I} E_j (E(\chi|\chi \geq P_j) - P_j).
\] (4.17)

The interpretation of the expected payoff is straightforward. Always playing action \( I \) implies that \( c_R \) is paid \( S \) times. \( c_I \) is paid if the buyer signals interest. In expectation, this happens \( \sum_{j=1}^S 1 - F(P_j) \) times. With the probability of a deal from the perspective of the buyer \( P_{r_{\text{deal}}}^{I,I} \), the expected surplus is realized. If the buyer changes her strategy and plays \( U \) for all offers, she realizes the expected payoff

\[
E\pi_B^{U,I} = -c_I S - c_R E(\#\text{asked}, I) + P_{r_{\text{deal}}}^{U,I} E_j (E(\chi|\chi \geq P_j) - P_j).
\] (4.18)

In this case \( c_I \) is paid \( S \) times and \( c_R \) is paid each time the buyer is asked by a seller. The expected number of times the individual buyer is asked if all other buyers play action \( I \) for all offers is characterized by \( E(\#\text{asked}, I) \). With the probability of a deal the expected surplus is realized. As in the simpler model, \( P_{r_{\text{deal}}}^{I,I} = P_{r_{\text{deal}}}^{U,I} \) holds, which is quite intuitive. Achieving a deal requires to be asked by a seller and the willingness to pay to be larger than the offered price. If all other buyers always play \( I \), then the individual buyer is not able to affect the probabilities that these two prerequisites
are met by her choice of strategy. Therefore, the last terms can be dropped and the comparison of the two expected payoffs, $E\pi_{B}^{I,I} \geq E\pi_{B}^{U,I}$, boils down to the inequality

$$\frac{c_I}{c_R} \geq \frac{S - E(\# \text{asked}, I)}{\sum_{j=1}^{S} F(P_j)} \equiv \omega^{I}. \quad (4.19)$$

Next, let all other buyers play action $U$ for all offers. The expected payoffs of an individual buyer who also always plays $U$ or deviates to $I$ are given by

$$E\pi_{B}^{U,U} = -c_I S - c_R E(\# \text{asked}, U) + P_{\text{deal}}^{U,U} (E(\chi|\chi \geq P_j) - P_j), \quad (4.20)$$

$$E\pi_{B}^{I,U} = -c_R S - c_I \sum_{j=1}^{S} (1 - F(P_j)) + P_{\text{deal}}^{I,U} (E(\chi|\chi \geq P_j) - P_j), \quad (4.21)$$

where $E(\# \text{asked}, U)$ characterizes the expected number of instances the buyer is asked by sellers if the remaining buyers play $U$. By the logic presented above, $P_{\text{deal}}^{U,U} = P_{\text{deal}}^{I,U}$. The comparison $E\pi_{B}^{U,U} \geq E\pi_{B}^{I,U}$ yields

$$\frac{c_I}{c_R} \leq \frac{S - E(\# \text{asked}, U)}{\sum_{j=1}^{S} F(P_j)} \equiv \omega^{U}. \quad (4.22)$$

Note that both thresholds $\omega^{I}$ and $\omega^{U}$ are the multi-seller counterparts of $\Omega^{I}$ and $\Omega^{U}$ derived in the SSM. Accordingly, the thresholds characterize symmetric pure strategy equilibria in the buyers’ stage. In the limit case with infinitely many buyers,

$$\lim_{B \to \infty} \omega^{I} = \lim_{B \to \infty} \omega^{U} = \frac{S}{\sum_{j=1}^{S} F(P_j)} \equiv \omega \quad (4.23)$$

is also valid here, since in both inequalities the expected number of instances a buyer is asked converges to zero for large values of $B$. In the following, we focus on this limit case.\footnote{Apart from simplicity, one reason is that in the limit case the buyers’ behavior does not depend on the expected number of times a buyer is asked. This dynamic is crucial, since $E(\# \text{asked}, I)$ and $E(\# \text{asked}, U)$ in equation (4.19) and equation (4.22) can only determined ex post using the decision rule based on the limit case. Appendix E.6 illustrates that a general analytical solution for the two expressions cannot be included in the model, even in the simplest case of $S = B = 2$.} Assuming $\chi \sim U[0, 1]$ implies $F(P_j) = P_j$ and therefore $\sum_{j=1}^{S} F(P_j) = SF(\overline{P})$ where $\overline{P}$ is the average posted price $\overline{P} = \frac{1}{S} \sum_{j=1}^{S} P_j$. Hence, the assumption of the uniform distribution implies $\omega = \frac{1}{F(\overline{P})}$. This condition is exactly the same as in model one, with $\overline{P}$ replacing $P$. It allows the interpretation that buyers base their action on an observed market signal given by the average price on the market. As before, a unique critical (average) price $\overline{P}$ exists that implies $\frac{c_I}{c_R} = \frac{1}{F(\overline{P})}$. If we let $\omega = \frac{1}{F(\overline{P})}$, then all buyers will play $U$ as a response to all offers if $\overline{P} < \overline{P}$ (since this implies $\frac{c_I}{c_R} < \frac{1}{F(\overline{P})}$) and $I$ for
\( \tilde{P} < \bar{P} \).

Formally, the strategy followed by the buyers \((\sigma_B)\) is given by the assignment

\[
\sigma_B: a(\bar{P}) = \begin{cases} 
U, & \text{for } \bar{P} < \tilde{P} \\
I, & \text{for } \tilde{P} \leq \bar{P}
\end{cases}
\]

with \( \tilde{P} = F^{-1}\left( \frac{c_R}{c_I} \right) \) (4.24)

where \(a\) characterizes the action that as a response to each offer and \(\bar{P}\) is the market signal, as introduced above. We again make the assumption that buyers play \(I\) whenever \(\bar{P} = \tilde{P}\).

The sellers’ stage in the MSM

First, note that an individual seller’s influence on the behavior of the buyers is limited, since buyers evaluate the market signal \(\bar{P}\). Given \(P_{-m} = \{P_j\}_{j \neq m}\), the prices of all other sellers, buyers might play the same action for all \(P_m \in [0, 1]\) – every possible price that can be set by seller \(m\). Second, \(P_m\), the strategy played by seller \(m\), can only be an element of an equilibrium if it is a best response to \(P_{-m} = \{P_j\}_{j \neq m}\), the prices of all other sellers, and considering the optimal behavior of the buyers characterized by strategy \(\sigma_B^*\).

As stated, we focus on symmetric equilibria where all sellers play the same strategy and thus choose the identical price \(P^*\). We determine this price \textit{numerically}. Let all sellers except seller \(m\) play the same strategy, such that \(P_{-m}\) also characterizes the price set by all sellers \(j \neq m\). Now, \(P_m^*(P_{-m})\) is seller \(m\)’s best response function to the uniform behavior of the remaining sellers. As the identity of seller \(m\) is arbitrary, the price \(P^*\) of a symmetric equilibrium is obviously a fixed point of this best response function \(P_m^*(P_{-m})\). We obtain the solutions of the MSM using a computational approach. The ensuing steps and the underlying logic are described in the following.\(^{16}\)

Although the model described is simple in its structure, it gives rise to highly complex and potentially discontinuous utility functions. The two main drivers of this complexity are the endogenous decisions of information acquisition by buyers and the mechanism by which sellers try to achieve a deal with interested buyers. An analytical solution is complicated but possible in the case of multiple applications and directed search (informed buyers), see Galenianos and Kircher (2009)), or undirected search (uninformed buyers), see Albrecht et al. (2004, 2006). These authors look at one-shot matching attempts only. Sellers who were rejected by the buyer cannot approach another buyer in their queue and are left unmatched. Kircher (2009) includes this feature in his model.

\(^{16}\)For a complete description of the algorithm employed and further remarks see Appendix E.7.
but has to assume continuum of agents.\footnote{This implies that entry of additional sellers does not alter an individual buyer’s utility. This assumption is only true in the limit; therefore, we relax it and focus on “small markets” (Lester, 2011).} Lastly, we are unaware of any analytical solutions of undirected search with multiple applications and sequential matching of sellers and interested buyers. Thus, since we want to find out when buyers decide to switch from acquiring information (directed search) to remaining uninformed (undirected search), the computational approach is chosen.

Concerning the shape of the best response function, two features are certain. First, \(P^*_m > 0\) for all \(P_m \in [0, 1]\). Giving the good away for free is never optimal, since marginally raising the price does not alter the probability of selling it and leads to a strictly larger payoff. Second, \(P^*_m < 1\) for all \(P_m\). Demanding a prohibitively high price eliminates seller \(m\)’s chances to sell the good. Thus, whenever \(P^*_m(P_m)\) is continuous, the existence of at least one fixed point is guaranteed. Recall that the best response function may be discontinuous if seller \(m\) is able to change buyers’ behavior.

Using the symmetric price \(P_m\) and solving \(\bar{P} = \tilde{P}\) for \(P_m\) yields

\[
\tilde{P}_m = F^{-1}\left(\frac{S^{CR}}{c_I} - (S - 1)F(P_m)\right). \tag{4.25}
\]

Here \(\tilde{P}_m\) is seller \(m\)’s critical price that allows him to change the buyers’ behavior given the price that is set by all other sellers. If \(\tilde{P}_m \not\in [0, 1]\), then seller \(m\) cannot influence the buyers’ behavior. This is obviously true for \(\frac{c_I}{c_R} \leq 1\). The numerical determination of the best response function is based on a Monte Carlo approach to determine the expected payoffs of seller \(m\) given \(P_m\) and \(P_m\).

Figure 4.5 features different graphs of expected payoffs of seller \(m\) for varying values of \(P_m\). The figure shows that different values of \(P_m\) lead to different critical prices \(\tilde{P}_m\). The maxima are highlighted; furthermore, at each maximum the behavior of the buyers implied by the respective prices is indicated. Obviously the location of the maxima depends to some extent on the grid of prices used.\footnote{For each \(P_m\) that implies \(\tilde{P}_m \in (0, 1)\), the price grid of seller \(m\) includes \(\tilde{P}_m\) (where all buyers play \(I\)) and a price slightly lower than this critical price.} This limitation can be overcome with a polynomial approximation of the obtained graphs of expected payoffs \(E\Pi_m(P_m|P_m)\) and by considering the maxima of these approximations. Note that for each \(P_m\) that implies \(\tilde{P}_m \in (0, 1)\) the expected payoff is discontinuous and the polynomial fitting has to be split in two parts to account for this. The expected payoffs in Figure 4.5 have not been smoothed by a polynomial approximation. The shape of the graphs exhibited there highlights that the procedure of polynomial approximation does not impact the findings, apart from the desired smoothing to overcome the limitations of a price grid.
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Figure 4.5: Expected payoff of seller $m$ given different $P_{-m}$

Notes: Graphs $E\pi_m(P_m|P_{-m})$ for different values of $P_{-m}$. From top to bottom: $P_{-m} = 1, 0.8, 0.6, 0.45, 0.3$ and $0$. Maxima and the implied buyers’ behavior are highlighted. Underlying parameters: $B = 6$, $S = 3$, $c_A = 0.1$ as well as $\chi \sim U[0, 1]$ and $c_I = 0.02$, $c_R = 0.01$, $c = 2$. MC-configuration: The $P_M$ grid includes 20 values, $P_{-m} \in \{0, 0.05, \ldots, 1\}$, some graphs omitted for clarity. 5000 simulations for each point in the joint strategy space.

The best response function that emerges from the expected payoffs displayed in Figure 4.5 is shown in Figure 4.6. Overall, the best response function obtained resembles the structure of the equilibrium prices of the SSM shown in Figures 4.2 and 4.3. Given the price of other sellers, seller $m$ will, to some extent, provide the incentives for buyers to play $I$ instead of $U$. If $P_{-m}$ becomes too small, providing these incentives becomes impossible or too expensive. The gap in the best response correspondence indicates where this change occurs. Keep in mind that our interest lies in the intersection of the best response correspondence with the 45 degree line, the fixed point. For the costs $c_I = 0.02$ and $c_R = 0.01$, this fixed point lies at $0.7$. The location of the fixed point can be found with greater precision by repeating the first step of determining the expected payoffs $E\Pi_m(P_m|P_{-m})$ for a smaller grid of $P_{-m}$ values around the obtained fixed point candidate $0.7$.

Assuming different cost parameters or changing the numbers of sellers and buyers obviously implies different best response functions. Theoretically three outcomes are possible. First, for some parameters the best response function is continuous, since the considered seller cannot alter the buyers’ behavior. In these cases the existence of a fixed point is guaranteed. If this is not the case and the function exhibits a discontinuity, the overall shape of the best response function does not depend on the
4.5. THE MULTIPLE SELLERS MODEL (MSM)

Figure 4.6: Best response function of seller $m$

Notes: Function $P^*$($P_{-m}$) implied by the expected payoffs in Figure 4.5.

chosen parameters.

Second, if the location of the discontinuity overlaps with the 45 degree line, there may be no fixed point.\(^{19}\) Third, best response functions may exhibit two fixed points as shown in Figure 4.7.\(^{20}\)

Figure 4.7: Best response function of seller $m$ (two fixed points)

Notes: Function $P^*$($P_{-m}$). Underlying parameters: $B = 6$, $S = 3$, $c_A = 0.1$ as well as $\chi \sim U[0, 1]$ and $c_I = 0.013$, $c_R = 0.01$, $c = 1.3$. MC-configuration: The $P_M$ grid includes 20 values, $P_{-m} \in \{0, 0.05, \ldots, 1\}$, some graphs omitted for clarity. 5000 simulations for each point in the strategy space.

The multiplicity in fixed points implies that two Nash equilibria of the game exist for the given model parameters. Note that the buyers always play the same strategy $\sigma_B$, as given by equation (4.24). Due to this fact and because buyers are the second movers, the assumption of subgame perfection rules out incredible threats and a possible second

\(^{19}\)This outcome is highly unlikely. However, see Appendix E.7 for a discussion.

\(^{20}\)Here the underlying expected payoffs $E\pi_m(P_m|P_{-m})$ have been smoothed by a polynomial approximation (compared to the example given by Figures 4.5 and 4.6).
4.6. FINDINGS FROM THE MSM

mover advantage. The first stage of the game can be interpreted as a coordination game between sellers. The realized expected payoffs of seller \( m \) are 
\[
E\pi_m(P_m = P_{-m} = 0.57) = 0.2705 \quad \text{and} \quad E\pi_m(P_m = P_{-m} = 0.77) = 0.4848.
\]
Hence, the equilibrium where \( P^* = 0.77 \) is set by all sellers is payoff dominant over \( P^* = 0.57 \). This pattern is persistent for different model parameters. Given there are two fixed points, there is always one that induces buyers to play \( U \) and one where they play \( I \). As argued, the latter one is always preferred by all sellers, since sellers are better off when confronted with informed buyers. Furthermore, let sellers randomize and play \( P = 0.57 \) and \( P = 0.77 \) with equal probability: \( \sigma_S = \{0.57, 0.5; 0.77, 0.5\} \). The expected payoffs of seller \( m \) who plays a pure strategy and faces only randomizing counterparts are given by 
\[
E\pi_m(P_m = 0.57, P_{-m} = \sigma_S) = 0.4326 \quad \text{and} \quad E\pi_m(P_m = 0.77, P_{-m} = \sigma_S) = 0.4603.
\]
Therefore, the equilibrium with \( P^* = 0.77 \) even risk dominates \( P^* = 0.57 \).

Whenever equilibrium selection is required (these cases are rare), we follow Harsanyi (1995) and choose the risk dominant equilibrium. Hence, the solution of the model is given by \( P^* = 0.77 \) and the buyers’ strategy \( \sigma_B \). It is worth mentioning that concepts of payoff and risk dominance coincide in the majority of cases. Furthermore, given multiplicity in fixed points, risk dominance identifies the price that induces buyers to play \( I \) as the relevant equilibrium in the nearly all cases. Importantly, since welfare is presumably larger there (recall the results derived in the SSM), our findings concerning welfare are not driven by the equilibrium selection.

4.6 Findings from the MSM

4.6.1 Pricing in the MSM

Figure 4.8 exhibits the equilibrium prices that are reached for different numbers of buyers and sellers (its counterpart in the SSM is the right panel of Figure 4.2 and Figure 4.3). The thresholds displayed (vertical dashed lines) at the discontinuities indicate the cost ratios at which the buyers’ behavior changes. Buyers play \( I \) for those larger than the thresholds and \( U \) for the ones lower than the thresholds. The prices obtained exhibit standard properties. Decreasing the number of buyers (rows, from right to left) or increasing the number of sellers (columns, from top to bottom) ceteris paribus leads to falling prices. Furthermore, whenever there is excess demand (more buyers than sellers), the critical cost ratios at which buyers change their behavior from \( I \) to \( U \) are lower compared to cases with excess supply. The intuition for this finding is the higher market power of sellers in these scenarios.
The pattern of prices for different cost ratios is qualitatively the same as in the SSM. Prices are higher if buyers are informed, and sellers try to induce buyers to inform themselves by increasing the price if the cost ratio falls. Compared to pricing of the single seller (Figure 4.3), the most prominent difference is that the critical thresholds of $c$ at which a switch from an $I$ equilibrium to an $U$ equilibrium occurs are much larger here, due to the competition between sellers that is introduced in the MSM. Effectively, this competition creates a coordination problem and impairs sellers’ ability to provide incentives for buyers to stay informed. Collectively raising the posted prices increases the incentives for individual sellers to deviate and set a lower price.

Figure 4.8: Equilibrium prices

Notes: Symmetric equilibrium prices $P^*$ given the cost ratio $c$ for various numbers of buyers and sellers. Underlying parameters: $c_A = 0.1$ and $\chi \sim U[0, 1] = \beta(1, 1)$.
MC-configuration: For each combination of $S$ and $B$ both grids of $P_M$ and $P_m$ include 16 values. 1000 simulations for each point in the strategy space.

\footnote{We have assumed that buyers have to react to all offers. Thus, sellers do not compete for signals of interest and competition means “ex post competition” as characterized in Galenianos and Kircher (2009, p. 455). We are able to obtain equilibria without wage dispersion, since buyers face no portfolio choice and posted prices are binding.}
4.6. FINDINGS FROM THE MSM

The competition between sellers turns out to be more fierce if buyers are uninformed. Facing only uninformed buyers might result in numerous attempts by an individual seller to sell his good. The number of these costly approaches can be lowered by demanding a lower price, such that the chance $\Pr(\chi \geq P) = F(P)$ is increased. If all sellers follow this logic, nobody is better off (except for the buyers), and prices decline. Cases of excessive supply (e.g. for $S = 12, B = 3$ and $S = 12, B = 6$), result in Bertrand competition, and the equilibrium prices given uninformed applications are zero. Bertrand competition does not unfold if buyers are informed. Here it is sufficiently unlikely to lose an interested buyer to another seller. Due to the higher price, the probability $\Pr(\chi \geq P)$ is smaller, such that an interested buyer standing in the queue of a particular seller is unlikely to have expressed interest to another seller.

When comparing the scenario with $S = 6, B = 3$ with the case of $S = 12, B = 6$, we observe that despite the same ratio of sellers over buyers, the equilibrium price with uninformed sellers does not fall to zero. Ergo, it seems that both $S$ and $B$ matter in terms of relative and absolute size. The same inference can be drawn by comparing the scenarios on the diagonal, $S = B = 3, S = B = 6$ and $S = B = 12$ – prices change in spite of the proportional increase in $S$ and $B$.

4.6.2 Efficiency and welfare in the MSM

Figure 4.9 shows the expected payoffs of an individual buyer and an individual seller that result from the equilibrium prices in Figure 4.8. As in the SSM, we plot also the sum of all expected payoffs as a measure of the utilitarian welfare. When analyzing welfare we take into account the varying numbers of buyers and sellers since the attainable social welfare differs with $B$ and $S$.

First, we focus on individual payoffs and welfare if the cost ratio $c$ changes. The discontinuities at the thresholds are of special interest here. The expected payoffs of an individual buyer (the solid blue lines), are qualitatively the same as in the SSM. An individual buyer experiences decreasing expected payoff if the sellers counteract falling cost ratios and raise prices to induce buyers to stick to $I$. Furthermore, every buyer is always better off for cost ratios where the equilibrium action of the buyers is $U$ due to lower equilibrium prices.

The expected profit of one individual seller (the solid red lines) increases due to the price increase associated with decreasing cost ratios. This finding is different than in the SSM model and can be explained by lower competition for buyers as the price decreases.

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22Obviously, another factor that favors this sharp decline in prices is the assumption of a strict participation constraint such that sellers cannot drop out of the game and have to sell their good.
4.6. FINDINGS FROM THE MSM

Figure 4.9: Individual expected payoffs and welfare

Notes: Expected payoffs of an individual seller (red), buyer (blue) and the sum of all expected payoffs $E\pi_S + B \cdot E\pi_B$ (black, right y-axis) for different $B$ and $S$. Dotted graphs assume that buyers always play $I$. Underlying eq. prices and parameters stem from Figure 4.8, furthermore $c_R = 0.01$.

decreases which also increases the probability of achieving a deal. Every seller obtains a strictly smaller payoff whenever buyers are uninformed, compared to cost ratios that lead to informed buyers. Note that these results were derived by employing the limit case for threshold $\omega$. This choice constitutes a slight variation compared to the SSM, where the results were based on the true boundaries $\Omega_I$ and $\Omega^U$. This mismatch is negligible, however, as the boundaries $\omega^I$ and $\omega^U$ are sufficiently close, even for low numbers of buyers and sellers (consider Figure 4.1 in the discussion of the SSM). \[23\]

\[23\] Note that an individual seller’s expected payoff exhibits a discontinuity here that is not true in the SSM. The discontinuity is not caused by the focus on the limit threshold $\omega$. It is created by the
4.6. FINDINGS FROM THE MSM

With more buyers than sellers, the expected social welfare, i.e. the sum of the expected payoffs (the solid black lines), falls due to the price increase. Adversely, if there are more sellers than buyers and prices rise, the sellers’ gains offset the loss of the buyers and welfare increases. In all cases the social welfare is lower for cost ratios when buyers do not engage in research and signal interest uninformed.

Now we turn our focus to efficiency. The solution of a constrained planner is to induce buyers to acquire information all the time. Sellers no longer can and need to affect the behavior of buyers. Sellers compete but are not constrained by a low cost ratio that disciplines them. The expected payoffs and welfare in the planner’s solution (the dotted lines) are compared to their counterparts that arise under decentralized allocation. Notably, this exercise reiterates the findings of the SSM: Welfare is larger in the planner’s solution, and the market solution is not constrained efficient. Again, buyers face an incentive to commit to action $I$ to prevent the rise in prices, which occurs for cost ratios slightly larger than the thresholds. In contrast to the results of the SSM, this does not constitute a Pareto improvement, since sellers profit from the increase in prices.

Before concluding our discussion of the results, we would like to discuss the issue of constrained efficiency for the case of “may” buyers. In the SSM we documented that for the scenario of high excess demand, i.e. many buyers but just one seller, the allocation in the $U$ equilibrium might be constrained efficient implying a higher social welfare than playing $I$ all the time. In Figure 4.10 we display the welfare and payoffs for a larger market with $S = B = 50$.

Figure 4.10: Prices and efficiency in a larger market

Notes: Symmetric equilibrium prices $P^*$ (left) and expected payoffs and welfare (right, colors as above) for $S = B = 50$. Dotted graphs assume that buyers always play $I$.

For sufficiently small cost ratios, the market outcomes in terms of welfare are better than the achieved welfare if buyers are forced to play $I$. Thus there are settings where competition among sellers, which prevents them from coordinating on high prices.
uninformed signaling can be constrained efficient. Forcing buyers to inform themselves may not be efficient if the cost of information is high and the cost of applying nil. This paradigm constitutes a contrast to the notion that “Typically, random search leads to inefficient outcomes […] , while directed search leads to efficient outcomes” (Marinescu and Wolthoff, 2016, p. 23). However, we do not want to put too much emphasis on this finding. To keep the model tractable, we assume that buyers have to assess all offers and sellers have to keep approaching buyers until there are none left or they have sold the good. Thus, we force the maximal amount of (costly) communication, and this assumption becomes more unrealistic if the number of market participants rises. Therefore it would be interesting to extend to model for solving a portfolio problem of a buyer, i.e. to also endogenize the number and location of applications.

4.7 Implications for the market design and welfare

Having identified uninformed signaling as welfare reducing, our model speaks a clear language concerning policy implications. Due to uninformed signaling, applications carry no information about the buyer. This issue can be resolved by making the signals (applications) more costly. This suggestion resembles a notion in the seminal contribution by Spence (1973) who argues that with costly university degrees applicants can send informative signals. The problem in many posted offer markets is that applications carry little information concerning the applicants’ determination to accept the job when given the chance.

Our model predicts that efficiency gains can be realized by increasing the cost ratio $\frac{c_I}{c_R}$. Whether this is achieved by raising $c_I$, lowering $c_R$ or any other way is not crucial. One can think about raising the signaling cost $c_I$ as increasing the opportunity cost of applications. As discussed in Bram (2016), newly emerging dating platforms allow users to send a limited number of “special” signals or charge prices for sending signals in some digital-platform-currency. Alternatively, one could think about a different matching technology in which only sellers may initiate a contact. Another way to increase the informative content of applications is to demand preference orderings from applicants. Visibly placing a particular offer on the first place of a ranking may credibly signal that this is the preferred choice as has been recently introduced on the academic job

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For parametrizations with high excess demand we find constrained efficiency of the $U$ allocation but as discussed in the previous section, for plausible parametrizations we always find that the $U$ is welfare reducing and not constrained efficient.

As can be easily illustrated e.g. by the effort and time to fill faculty positions.
4.8. CONCLUSION

4.8 Conclusion

This paper studies the phenomenon of excess and unsuitable applications on posted offer markets. We propose a microeconomic model in which sellers are capacity constrained, buyers endogenously decide to acquire information and are allowed to submit several applications. A crucial characteristic of our model is the possibility to apply for offers even if a buyer does not know her true willingness to pay. This means that if a buyer does not pay the research cost, she will contact all sellers including those with whom no deal is feasible.

We first derive our findings using a simplified model with one seller that allows us to derive a closed form solution for all equilibrium strategies. In turn we generalize the results by considering a model that includes more sellers. The extended model is solved numerically. The main findings of this paper can be summarized as follows.

First, the buyers’ behavior is characterized by the ratio of signaling costs over research costs. Buyers acquire information and send informed signals if this ratio is sufficiently lower. Lowering $c_R$ has similar implications since it also raises the cost ratio $c$. Effectively, this can be done by making information more accessible and easier to process. This task can be accomplished by sellers when posting an offer in the first place. In our model, sellers are choosing posted prices to influence the behavior of buyers and cannot choose $c_R$ per assumption. However, the buyers’ decision rules given by equations (4.4), (4.8), (4.19) and (4.22) imply that the same results can be obtained if sellers can affect $c_R$.

Hence, our paper implies clear policy implications towards informed signaling and provides a theoretical explanation for many current real-time approaches in market design aiming at increasing the information content of signals.

The computational solution of the MSM allows for testing different policy improvements discussed earlier. For example, limiting the total number of signals of interest is easy to implement. Similarly, it is straightforward that many of our findings can be tested in an experimental setting. Works by authors such as Helland, Moen, and Preugschat (2017) take posted offer markets to the lab. We believe that a test is promising as to what extent the selection of buyers’ search strategies is shaped by the cost structure, how the buyers’ search behavior can be affected by sellers and to what extent sellers would increase prices to induce buyers to inform themselves before applying.
high. If it is sufficiently low, they do not acquire information and send uninformed signals.

Second, sellers always prefer to deal with informed buyers compared to uninformed buyers. In particular, sellers can provide incentives for buyers to stay informed if the crucial cost ratio is falling by demanding higher prices. This allows a new interpretation of Lester’s paradox which states that more informed buyers may imply higher prices in equilibrium. We show that sellers proactively increase prices to induce buyers to acquire information.

Third, market outcomes with uninformed buyers are not constrained efficient for plausible parameterizations. The equilibrium with buyers signaling interest uninformed implies a market congestion and lower social welfare. Uninformed signaling can be interpreted as over-exploitation of the attention of sellers, a public resource.

Finally, as discussed in Section 4.7, our paper has clear market design and welfare implications in favor of informed signaling. It provides a theoretical explanation for many current real-world approaches in market design aiming at increasing the information content of signals and to reduce the market congestion due to excess and inappropriate applications (Autor, 2001; Kuhn, 2014; Bram, 2016). An interesting avenue for future studies would be to test the predictions of our model, in particular, that sellers do proactively increase prices to induce buyers to send informed signals.
Appendix of Chapter 4

E.1 Optimal threshold $\tilde{\chi}$ for strategy $I$

If an individual buyer knows $\chi$, there exists a critical willingness to pay $\tilde{\chi}$, at which the buyer is indifferent between signaling interest or not. In the SSM, this threshold can be determined as shown below.

$$0 = -c_I + PA^{I,I}(\tilde{\chi} - P) \quad \rightarrow \quad \tilde{\chi} = \frac{c_I}{PA^{I,I}} + P$$

Obviously, $\tilde{\chi} > P$ holds for $c_I > 0$ as $PA^{I,I} \in [0, 1]$. The willingness to pay must cover the price and the weighted cost of signaling interest. As explained in Section 4.3, $PA^{I,I}$ obviously depends on $\tilde{\chi}$ as well since all other buyers are assumed to play the same strategy ($I$). Thus $\tilde{\chi}$ is defined by

$$\tilde{\chi} = \frac{c_I}{PA^{I,I}(\tilde{\chi})} - P = c_I ((B - 1)(1 - F(\tilde{\chi})) + 1) - P.$$ 

Different distributional assumptions of $\chi$ affect the critical willingness to pay differently. For $\chi \sim U[0, 1]$, that will be in large parts of this paper, an analytical solution is possible since $F(\chi) = \chi$ and

$$\tilde{\chi} = \frac{c_I B}{c_I (B - 1) + 1} + \frac{P}{c_I (B - 1) + 1}.$$ 

The impact of imposing $\tilde{\chi} = P$ can be assessed by looking at the difference

$$\tilde{\chi} - P = \frac{c_I B}{c_I (B - 1) + 1} + \frac{P}{c_I (B - 1) + 1} - P = \frac{c_I}{c_I (B - 1) + 1} (B - (B - 1)P).$$ 

It reveals that the difference is minuscule for sufficiently small values of $c_I$, relative to $\tilde{\chi}$. In the parameterizations of the model that are introduced after Section 4.3, $c_I \approx 0.01$. Additionally, the difference is smaller if $P$ is large (meaning in the domain where a buyer plays $I$). For other distributions, $\tilde{\chi}$ can be determined computationally.

Last and most important, the assumption that buyers signal interest if $\chi > P$ and not $\tilde{\chi} > P$ works against the findings of our paper. It implies that buyers decide to signal interest, while knowing their willingness to pay, for too long. This means that they signal interest informed for too low willingnesses to pay and thus also for lower prices. Yet, this is exactly what the seller wants to achieve as.
E.2 Boundaries $\Omega^I$ and $\Omega^U$

**Proposition:** $\Omega^I \geq \Omega^U$ is always true for $\Omega^I$ and $\Omega^U$ as defined in equations (4.4) and (4.8).

**Proof:**

\[ \Omega^I = \frac{1 - PA^I}{F(P)} \geq \frac{1 - PA^U}{F(P)} = \Omega^U \quad \rightarrow \quad PA^I \leq PA^U. \]

Using a computational approach reveals that the maximum of $\psi = PA^I - PA^U$ is zero and reached for $B = 1$ or $P = \bar{X}$. Recall

\[ PA^I = \frac{1}{(B-1)(1-F(P)) + 1} \quad \text{and} \quad PA^U = \frac{1 - F(P)B}{B(1-F(P))} \]

given by equation (4.2) and equation (4.7). Hence $PA^I = PA^U$ for either $B = 1$ or $P = \bar{X}$ (implying $F(\bar{X}) = 0$). \(\square\)

**Proposition:** $\lim_{B\to\infty} \Omega^U = \lim_{B\to\infty} \Omega^I = \frac{1}{F(P)} \equiv \Omega$.

**Proof:** Recall

\[ \Omega^I = \frac{1 - PA^I}{F(P)} \quad \text{and} \quad \Omega^U = \frac{1 - PA^U}{F(P)}. \]

$\lim_{B\to\infty} \Omega^U = \lim_{B\to\infty} \Omega^I = \frac{1}{F(P)}$ is implied by $\lim_{B\to\infty} PA^I = \lim_{B\to\infty} PA^U = 0$. This is quite intuitive since due to increased demand, the probability of being approached in both pure strategy equilibriums is decreasing in the number of buyers. \(\square\)

E.3 Equilibria for $c \in (\Omega^U, \Omega^I)$

**Proposition:** On the buyers’ stage, there exists a unique mixed strategy equilibrium for all $c \in [\Omega^U, \Omega^I]$.

**Alternative proposition:** For each $c \in [\Omega^U, \Omega^I]$ and given the mixed strategy $\sigma_r$ (as introduced in Section 4.3.3), there exists a mixture $r^*$ that sets the last buyer indifferent between playing $I$ or $U$ if all other buyers play $\sigma_{r^*}$. Formally: $\exists r^* \in [0, 1]$ s.t. $E\Pi_B^{I, \sigma_{r^*}} = E\Pi_B^{U, \sigma_{r^*}} \forall c \in [\Omega^U, \Omega^I]$. 

Proof: As argued in Section 4.3.3, $PA^{I,\sigma_r} = PA^{U,\sigma_r} \equiv PA^{\sigma_r}$. Thus

$$E\Pi_B^{I,\sigma_r} = E\Pi_B^{U,\sigma_r}$$

$$-c_r + (1 - F(P))(-c_I + PA^{I,\sigma_r}(\ldots)) = -c_I + PA^{U,\sigma_r}(-c_I + (1 - F(P))(\ldots))$$

$$c = \frac{1 - PA^{\sigma_r}}{F(P)} \equiv \Omega^r.$$  

$PA^{\sigma_r}$ is derived in the following. From the perspective of a buyer who has signaled interest, the expected total number of buyers that signal interest is given by $1 + (B - 1)(r(1 - F(P)) + 1 - r) \equiv \eta$. In expectation, each of the remaining $B - 1$ buyers plays $I$ with probability $r$ and will signal interest with probability $1 - F(P)$. If a buyer plays action $U$ (with probability $1 - r$) she will always signal interest. Trivially, $\eta$ converges to the (expected) numbers of buyers that signal interest in the pure strategy cases for $r \to 1$ and $r \to 0$. The same logic as presented in the derivation of $PA^U$ (equation (4.7)) applies. As above, being asked second (third, ... ) requires $\chi < P$ for the buyer(s) approached before. Here, this is only possible if the buyer(s) approached before has played $U$ (with probability $1 - r$). Therefore

$$PA^{\sigma_r} = \frac{1}{\eta} + \frac{\eta - 1}{\eta} (1 - r)F(P) + \frac{\eta - 1}{\eta} \frac{1}{\eta - 1} \frac{\eta - 2}{\eta - 1} (1 - r)^2 (F(P))^2 \frac{1}{\eta - 2} + \ldots ,$$

what can be simplified to

$$PA^{\sigma_r} = \frac{1 - ((1 - r)F(P))^\eta}{\eta (1 - (1 - r)F(P))}.$$  

As hinted above, $\lim_{r \to 1} PA^{\sigma_r} = PA^I$ and $\lim_{r \to 0} PA^{\sigma_r} = PA^U$. Thus $c = \Omega^I$ requires $r^* = 1$ and $c = \Omega^U$ requires $r^* = 0$ for $E\Pi_B^{I,\sigma_{r^*}} = E\Pi_B^{U,\sigma_{r^*}}$ to hold. Easily to see, $PA^{\sigma_r}$ and thus also $\Omega^r$ is smooth for all $r \in [0,1]$. Hence for every $c \in [\Omega^U, \Omega^I]$ there exists at least one $r^*$ that implies $c = \frac{1 - PA^{\sigma_{r^*}}}{F(P)}$. □

E.4 Curvature of $E\Pi^I_S$ and $E\Pi^U_S$

Lemma: $E\Pi^I_S \geq E\Pi^U_S$ (defined in Equations equation (4.9) and equation (4.10)) holds for all $P \in [0,1]$ and any $F$, $B$ and $c_A > 0$.  


Proof:

\[ E\Pi_S^I = (1 - F(P)^B)(-c_A + P) \geq \frac{1 - (F(P))^B}{1 - F(P)}(P(1 - F(P)) - c_A) = E\Pi_S^U \]

\[ (1 - F(P))(P - c_A) \geq (1 - F(P))P - c_A \]

\[ c_A F(P) \geq 0 \]

Lemma: \( E\Pi_S^I = -Bc_A \) for \( P = 1 \).

Proof: From L’Hôspital’s rule follows

\[ \lim_{P \to 1} E\Pi_S^U = \lim_{P \to 1} \frac{1 - (F(P))^B}{1 - F(P)}(P(1 - F(P)) - c_A) = \lim_{P \to 1} BF(P)^{B-1}(-c_A) = -Bc_A \]

such that \( E\Pi_S^I > E\Pi_S^U \) is true for all \( P > 0 \) (see above).

Lemma: \( \frac{\partial E\Pi_S^I}{\partial P} \geq \frac{\partial E\Pi_S^U}{\partial P} \) for all \( P \) and \( B, f, F \).

Proof: First obtain

\[ \frac{\partial E\Pi_S^I}{\partial P} = -BF(P)^{B-1}f(P)(P - c_A) + (1 - F(P)^B) \quad \text{and} \]

\[ \frac{\partial E\Pi_S^U}{\partial P} = 1 - F(P)^B + P \left(-BF(P)^{B-1}f(P)\right) \ldots \]

\[ -c_A \frac{-BF(P)^{B-1}f(P)(1 - F(P)) - (1 - F(P)^B)(-f(P))}{(1 - F(P))^2} \]

Second, evaluating the derivatives at \( P = 0 \) and \( P = 1 \) is simple. Only determining

\[ \left. \frac{\partial E\Pi_S^U}{\partial P} \right|_{P=1} = \lim_{P \to 1} 1 - F(P)^B + P \left(-BF(P)^{B-1}f(P)\right) \ldots \]

\[ -c_A \left( \frac{-BF(P)^{B-1}f(P)}{1 - F(P)} - c_A f(P) \frac{1 - F(P)^B}{(1 - F(P))^2} \right) \]

\[ = -Bf(1) - c_A f(1) \left( \lim_{P \to 1} \frac{B(B - 1)F(P)^{B-2}}{-2} \right) \]

\[ = -Bf(1) + c_A f(1) \frac{B(B - 1)}{2} \]

requires some rearrangement and the use of L’Hôspital’s rule twice for the last summand. Thus the proposition holds strictly at \( P = 1 \). Only \( f(0) = 0 \) implies the weak inequality at \( \frac{\partial E\Pi_S^I}{\partial P} = \frac{\partial E\Pi_S^U}{\partial P} \) at \( P = 0 \).
For any $P \in (0,1)$, consider and rearrange the inequality $\frac{\partial \Psi^U}{\partial P} \geq \frac{\partial \Psi^U}{\partial P}$:

$$-BF(P)^{B-1}f(P)(P - c_A) + 1 - F(P)^B \geq 1 - F(P)^B \ldots$$

$$-PBE(P)^{B-1}\frac{f(P)(1 - F(P)) - (1 - F(P)^B) f(P)}{(1 - F(P))^2}$$

$$BF(P)^{B-1}f(P)(1 - F(P))^2 \geq f(P)BF(P)^{B-1}(1 - F(P)) - (1 - F(P)^B) f(P)$$

$$-BF(P)^B f(P) (1 - F(P)) - F(P)^B f(P) \geq -f(P)$$

$$F(P)^B f(P) (B(1 - F(P)) + 1) \leq f(P)$$

$$f(P)(F(P)^B (B(1 - F(P)) + 1) - 1) \leq 0 \equiv \psi$$

For all $P$ that imply $f(P) = 0$, the inequality is strict. In these cases a marginal raise of the price cannot alter the expected payoff since no buyers are affected by the change. Lastly, a numerical approach reveals that $\psi$ is maximal for $P = 1$ (for all values of $B$) and takes on the value one. □

### E.5 Curvature of $\Omega^I$ and $\Omega^U$

**Proposition:** $\frac{\partial \Omega}{\partial P} < 0$, $\frac{\partial \Omega^U}{\partial P} < 0$ and $\frac{\partial \Omega^U}{\partial P} < 0$ for $P \in [0,1]$.

**Proof:** For $\Omega$ the exercise is simple. Rearrange $\Omega^I$ and receive

$$\Omega^I = 1 - PA^I \frac{1}{F(P)} = 1 - \frac{1}{(B-1)(1-F(P)) + 1}.$$ 

Since $\frac{\partial \Omega^I}{\partial P} = \frac{\partial \Omega^I}{\partial F(P)} \frac{\partial F(P)}{\partial P}$ and $\frac{\partial F(P)}{\partial P} \geq 1$, only $\frac{\partial \Omega^I}{\partial F(P)}$ matters. For brevity, denote $F(P)$ by $F$.

$$\frac{\partial \Omega^I}{\partial F} = -\left( -\frac{1}{((B-1)(1-F) + 1)^2}(-B - 1) \right) = -\frac{B - 1}{(...)^2}$$

For $\Omega^U$, $\Omega^U \xrightarrow{F \to 0} \infty$ is true due to $PA^U \xrightarrow{F \to 0} \frac{1}{B}$ and $\Omega^U \xrightarrow{F \to 1} 0$ is true since $PA^U \xrightarrow{F \to 1} 1$ (using L’Hôpital’s rule). With a computational approach, one can show that $\Omega^U$ is strictly monotonically decreasing in $F$ for $F \in [0,1]$. □
E.6  The MSM with $S = 2$ and $B = 2$

Consider $S = 2$ and $B = 2$. Note that this scenario is even simpler than $S = 2$ and $B = 3$ as analyzed in Lester (2011). The two sellers post prices $P_1$ and $P_2$. For brevity, let $F(P_s) \equiv F_s$ and simplify the expected surpluses $E(\chi | \chi \geq P_s) - P_s \equiv ESP_s$, both for $s = 1, 2$. First, let both buyers play $I$. In this case a single buyer's expected payoff is given by equation (4.26) below.

$$E\tilde{\pi}^I_B = -2c_R + (1 - F_1) \cdot F_2 \left[ -c_I + F_1 \cdot ESP_1 + (1 - F_1) \frac{1}{2} \cdot ESP_1 \ldots \right. \left. \right. \right.$$

$$+ (1 - F_1) \frac{1}{2} (1 - F_2) I(1 = \text{argmin} ESP_s) ESP_1 \ldots$$

$$+ F_1 \cdot (1 - F_2) \left[ -c_I + F_2 \cdot ESP_2 + (1 - F_2) \frac{1}{2} \cdot ESP_2 \ldots \right.$$

$$+ (1 - F_2) \frac{1}{2} (1 - F_1) I(2 = \text{argmin} ESP_s) ESP_2 \ldots$$

$$+ (1 - F_1) \cdot (1 - F_2) \left[ -2c_I + F_1 \cdot F_2 \max_s ESP_s \ldots \right.$$

$$+ (1 - F_1) \cdot F_2 \left( \frac{1}{2} \max_s ESP_s + \frac{1}{2} ESP_2 \right) \ldots$$

$$+ F_1 \cdot (1 - F_2) \left( \frac{1}{2} \max_s ESP_s + \frac{1}{2} ESP_1 \right) \ldots$$

$$+ (1 - F_1) \cdot (1 - F_2) \left( \frac{1}{4} \max_s ESP_s + \frac{1}{4} ESP_1 + \frac{1}{4} ESP_2 + \frac{1}{4} \min_s ESP_s \right) \right]$$

(4.26)

Although the expression is enormous (even for $S = B = 2$), the logic behind it is simple: Playing $I$ for both offers leads to costs $2c_R$ in all cases. The buyer signals interest only in the offer of seller one when learning $\chi \geq P_1$ and $\chi < P_2$ (with probability $(1 - F_1) \cdot F_2$). In this case she pays $c_I$ once and obtains $ESP_1$ surely if the other buyer has not signaled interest in this offer (with probability $F_1$). If the other buyer has learned $\chi \geq P_1$ (with probability $1 - F_1$), seller one will randomly pick one of the two buyers. If the regarded buyer is not picked, she still has a chance to buy the good of seller one and realize $ESP_1$. For this the other buyer must signal interest in the offer of seller two as well (with probability $1 - F_2$), then the other buyer is asked by both sellers first and will accept the offer of seller two if $ESP_2 > ESP_1$. This last condition is captured by the indicator function $I(1 = \text{argmin} ESP_s)$ that is equal to one if the argument is true. Next (the third row), the buyer might learn $\chi < P_1$ and $\chi \geq P_2$ (with probability $F_1(1 - F_2)$) and only signal interest in the offer of seller two. She incurs cost $c_I$ once and the same logic as described applies. Last, the buyer pays $c_I$ twice and signals interest in both offers after learning $\chi \geq P_s$ for $s = 1, 2$ (with probability
If the other buyer has not signaled interest in any offer (probability $F_1 \cdot F_2$), the regarded buyer is approached by both sellers and picks the better offer. If the other buyer has signaled interest in one of the two offers (probabilities $(1 - F_1) \cdot F_2$ and $F_1 \cdot (1 - F_2)$), the regarded buyer can nevertheless be approached by both sellers (probability $\frac{1}{2}$) and pick the better offer. With counter probability she is approached only by the seller where she alone has signaled interest and accepts this offer. In the case where the other buyer signals interest in both offers (probability $(1 - F_1) \cdot (1 - F_2)$), the regarded buyer can (with equal probabilities of $\frac{1}{4}$) be approached by both sellers (and pick the best offer), she can be approached by seller one (two) and accept his offer, or both sellers approach the other buyer first such that the regarded buyer can accept the poorer offer in the second round.

\[
E_{s_B}^{U,I} = -2c_I + F_1 \cdot F_2 \left[ -2c_R + (1 - F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1 - F_2) \cdot ESP_2 \ldots 
+ (1 - F_1) \cdot (1 - F_2) \cdot \max_s ESP_s \right] \ldots 
+ (1 - F_1) \cdot F_2 \left[ -c_R + \frac{1}{2} \left( -c_R + (1 - F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1 - F_2) \cdot ESP_2 \ldots 
+ (1 - F_1) \cdot (1 - F_2) \cdot \max_s ESP_s \right) + \frac{1}{2} (1 - F_2) \cdot ESP_2 \right] \ldots 
+ F_1 \cdot (1 - F_2) \left[ -c_R + \frac{1}{2} \left( -c_R + (1 - F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1 - F_2) \cdot ESP_2 \ldots 
+ (1 - F_1) \cdot (1 - F_2) \cdot \max_s ESP_s \right) + \frac{1}{2} (1 - F_1) \cdot ESP_1 \right] \ldots 
+ (1 - F_1) \cdot (1 - F_2) \left[ \frac{1}{4} \left[ -c_R + (1 - F_1) \cdot ESP_1 \right] \ldots 
+ \frac{1}{4} \left[ -c_R + (1 - F_2) \cdot ESP_2 \right] \ldots 
+ \frac{1}{4} \left[ -2c_R + (1 - F_1) \cdot F_2 \cdot ESP_1 + F_1 \cdot (1 - F_2) \cdot ESP_2 \ldots 
+ (1 - F_1) \cdot (1 - F_2) \cdot \max_s ESP_s \right] \ldots 
+ \frac{1}{4} \left[ I(1 = \argmin ESP_s) \cdot (1 - F_1) \cdot ESP_1 \ldots 
+ I(2 = \argmin ESP_s) \cdot (1 - F_2) \cdot ESP_2 \right] \right] \right]
\]

(4.27)

A similar expression can be derived if one considers a deviation of the regarded buyer to strategy $U$ while the other buyer keeps playing $I$. Refer to equation (4.27) above. Again, the expression is tremendous but the interpretation straightforward: The cost $c_I$ occur with certainty twice when playing $U$. The other buyer does not signal interest to any seller if she learns $\chi < P_s$ for both $s = 1, 2$ (with probability $F_1 \cdot F_2$). Thus
the regarded seller is approached by both sellers and she pays $c_R$ twice. She realizes $ESP_1$ if she learns $\chi \geq P_1$ and $\chi < P_2$ (with probability $(1 - F_1) \cdot F_2$). Vice versa, she realizes $ESP_2$ with probability $F_1 \cdot (1 - F_2)$. If she learns $\chi \geq P_s$ for both $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$) she picks the better offer. If the other buyer learns $\chi \geq P_s$ for only one $s = 1, 2$ and the opposite for the other offer (with probabilities $(1 - F_1) \cdot F_2$ and $F_1 \cdot (1 - F_2)$, third and fifth row), the regarded buyer will surely be approached by the seller where she is the sole interested buyer. Thus cost $c_R$ are incurred at least once. With probability $\frac{1}{2}$, the regarded buyer is also approached by the seller where the other buyer has signaled interest such that $c_R$ is paid a second time. In this case, if the buyer learns $\chi \geq P_s$ for only one $s = 1, 2$ (with probability $(1 - F_s) F_{-s}$), she accepts the offer of seller $s$ and realizes $ESP_s$. The buyer is able to pick the best offer if she learns $\chi \geq P_s$ for both $s = 1, 2$ (probability $(1 - F_1) \cdot (1 - F_2)$). With counter probability of $\frac{1}{2}$, the buyer is asked only by the seller where she is the sole interested buyer and she realizes the respective surplus if $\chi \geq P_s$ is true. Last, the other buyer might learn $\chi \geq P_s$ for both $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$, row seven). In this case, four outcomes (all with equal probability) are possible. First, the regarded buyer is only approached by seller one, she has to pay $c_R$ once and realizes $ESP_1$ if $\chi \geq P_1$ is true (probability $1 - F_1$). Second, the same pattern can emerge if she is only approached by seller two. Third, she is approached by both sellers. In this case she realizes $ESP_1 (EPS_2)$ only if $\chi \geq P_1 (\chi \geq P_2)$ is true (probabilities $(1 - F_1) \cdot F_2$ and $F_1 \cdot (1 - F_2)$). Otherwise she can pick the best offer if $\chi \geq P_s$ is true for both $s = 1, 2$ (with probability $(1 - F_1) \cdot (1 - F_2)$). Fourth, both sellers approach the other buyer first. Here she can accept the offer that was declined by the other buyer is she learns that her willingness to pay exceeds the offer’s price.

It is apparent that the comparison $E\pi_B^{L,I} \geq E\pi_B^{U,I}$ cannot be simplified to a clear-cut threshold that defines where equilibria in which both buyers play $I$ exist. Note that the two similarly gargantuan expressions $E\pi_B^{U,U}$ and $E\pi_B^{L,U}$ need to be derived and compared to identify equilibria where both buyers play $U$. Furthermore, the stage of the sellers requires the identification of the symmetric best response function $P_s^*(P_{-s})$ (that takes the buyers’ optimal behavior into account) to be able to determine the equilibrium price. At the latest at this point it becomes clear that even the simplest scenario $S = B = 2$ cannot be solved analytically. The expressions given by equation (4.26) and equation (4.27) can be simplified to a very large extent by assuming that sellers post identical prices what implies $F_1 = F_2$ and $ESP_1 = ESP_2$. Yet, this simplification implies a different model. It eliminates the competition between the two sellers since the mentioned best response functions do not exist under this assumption. The optimal price that one may obtain given the simplification corresponds to the cartel price.
E.7 Fixed point search algorithm

To obtain the solution of the MSM we use Matlab. The used algorithm is structured as follows:

A Define values the model parameters $S$, $B$, $F$ and the cost parameters $c_R$, $c_A$.

B Define a grid of costs $c_I$ denoted as $C_I$.

C For loop $c_I = 1, 2, \ldots, |C_I|$

   C.1 Set $c_I = C_I(\text{loop } c_I)$ implying the cost ratio $c = \frac{c_I}{c_R}$.

   C.2 Define a grid of prices $P_m$ denoted as $P_m$.

   In the first iteration $P_m$ must include 0 and 1. If $\tilde{P}_m$ lay within the boundaries of $P_m$, the set includes the values $(1 - \epsilon) * \tilde{P}_m$ and $\tilde{P}_m$ itself.

   C.3 For loop $P_m = 1, 2, \ldots, |P_m|$

   C.3.1 Set $P_m = P_m(\text{loop } P_m)$.

   C.3.2 Endogenously define a grid of prices $P_m$ denoted as $P_m$.

   C.3.3 For loop $P_m = 1, 2, \ldots, |P_m|$

   C.3.3.1 Set $P_m = P_m(\text{loop } P_m)$.

   C.3.3.2 Simulate the model $MC$-times (given the current $c_I$, $P_m$, $P_m$).

   C.3.3.3 Average over the $MC$ realized payoffs to get $E\pi_m(P_m | P_m)$.

   [Polynomial approximation accounting for discontinuities]

   C.3.4 Find and save $\arg\max_{P_m} E\pi_m(P_m | P_m) = P_m^*(\text{loop } P_m)$.

   C.4 Combine the obtained $P_m^*$ to get the correspondence $P_m^*(P_m)$.

   [Polynomial approximation accounting for discontinuities]

   C.5 If there exists at least one fixed point.

   C.5.t true: If there are multiple fixed points.

   C.5.t.t true: Equilibrium selection by risk dominance, save the fixed point $P^*(\text{loop } c_I)$.

   C.5.t.f false: Save the fixed point $P^*(\text{loop } c_I)$.

   C.5.f false: No sym. eq. on the buyers’ stage for $c_I$. Save $P^*(\text{loop } c_I) = \text{nan}$.

D Combine the obtained $P^*$ to get the correspondence $P^*(c_I)$.

[Polynomial approximation accounting for discontinuities]

Some more details are in order. First, the criterion to evaluate C.5 is a graphical check whether there exists at least one intersection of $P_m^*(P_m)$ with the 45° line. Second, the polynomial approximations at steps C.3.3.3, C.4 and D help to smooth
the obtained functions and reduce noise from the simulations. In addition one has to select an appropriate degree concerning the polynomial. Higher degrees preserve more variation (or noise) but may not be feasible due to the limited number of data points, lower degrees might actually destroy much of the shape of the underlying function. We choose a degree of 3 (2) if the number of data points is greater or equal than 6 (4) and a linear approximation in the remaining cases.

Third, the relationship between the size of $P_{-m}$ and $P_m$, the number $MC$ and the precision that is reached by the algorithm is of interest, especially as greater accuracy is favorable but requires (much) more time. Here an iterative approach can be employed. Rather coarse grids $P_{-m}$ and $P_m$ as well as fewer simulations can be set to determine the location of a fixed point candidate roughly in a first iteration. In further iterations the grids of prices can be finer and centered around this candidate and more simulations can be run to pinpoint the location of the fixed point. However, this approach comes at the risk of missing or mis-specifying a fixed point candidate. Fourth, a few basic rules can help to identify ill-defined best response correspondences. E.g. for $\hat{P}_m \not\in [0,1]$ the best response function should be smooth since no change in the buyers’ behavior occurs. Also, if there exists a discontinuity for a $c_I$ and this discontinuity vanishes for the next $c_I$ that is marginally larger than the previous one, there is probably something wrong. Lastly, whenever one is unable to determine a fixed point, this is probably caused by too much noise and can be resolved by increasing $MC$ or the grid $P_{-m}$ is too coarse. The reason for this is the following. If $P^*_m(P_{-m})$ exhibits a discontinuity, this can hardly result in the absence of an intersection with the $45^\circ$ line since the prices on the right side of the discontinuity (where buyers play $I$) are in all tackled cases higher than the ones on the left side, see Figure 4.6.

The equilibrium expected payoffs of buyers and sellers for cost ratio $c_I$ are obtained by setting $P_m = P_{-m} = P^*(c_I)$ and executing step C.3.3.2 with a large number of simulations and averaging over the realized payoffs. 10000 simulations guarantee sufficient precision.
Abgrenzung

Das zweite Kapitel “Price Points and Price Dynamics” stellt eine gemeinsame Forschungsarbeit mit Prof. Dr. Volker Hahn (Universität Konstanz) dar. Alle Bestandteile dieser Studie wurden gemeinschaftlich erarbeitet und verfasst.

Ich versichere hiermit, dass ich die Studie im dritten Kapitel “Trend Inflation and Real Rigidities” eigenständig, ohne Hilfe Dritter, verfasst habe.

Bibliography


