Splitting *at most*[1]

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1. Introduction

Negative quantifiers are known to give rise to split scope readings, where another operator, in particular a modal, takes scope in between the negative and the existential meaning component. As illustrated in the paraphrases of the following examples from German and English, the negation takes wide scope over the modal while the existential is interpreted with narrow scope under the *de dicto* reading.[2]

(1)   Bei der Operation   muss   **kein**   Anästhesist   anwesend sein.   [German]
      at   the surgery   has-to no   anesthetist   present   be
       'It is not required that an anesthetist be present during the surgery.'

(2)   The company need fire **no** employees.   [from Potts 2000]
      'The company is not obligated to fire any employees.'

The most prominent reading of sentence (1), for instance, is the one paraphrased saying that the presence of an anaesthetist is not obligatorily required. It is not about a particular anaesthetist, but rather about the presence of some anaesthetist or other, corresponding to a *de dicto* reading of the indefinite. At the same time, the necessity modal is interpreted in the scope of negation, expressing the absence of an obligation. This split reading cannot be derived under the standard analysis of negative indefinites as negative quantifiers where the negation and the existential quantifier form a lexical unit. Several analyses have been proposed, some maintaining the assumption that negative quantifiers are lexical units (DE SWART 2000; ABELS & MARTÍ 2010) others decomposing them into a negation and an indefinite (among others JACOBS 1980; RULLMANN 1995; PENKA 2011).

As has been observed by DE SWART (2000) and ABELS & MARTÍ (2010), split scope readings do not only arise with negative quantifiers, but also with other downward monotonic quantifiers like *few* and numerals modified by *fewer than* or *at most*. This is illustrated in the following examples.

[2] Since the focus of the paper is on split scope readings, only these readings are paraphrased here and in the following examples although other readings may be available as well.

(3)   The inmates of this prison are allowed to write **few** letters.

'The inmates of this prison are not allowed to write more than a small number of letters.'

(4)   At MIT one needs to publish **fewer than three** books in order to get tenure. [HACKL 2000]

'At MIT one doesn't need to publish more than two books in order to get tenure.'

(5)   A muslim can marry **at most four** women.   [DE SWART 2000]

'A muslim cannot marry more than four women.'

The fact that downward monotonic quantifiers in general give rise to split scope readings has been taken as an argument against approaches that derive split scope readings of negative indefinites by decomposing the quantifiers into a negation and a quantificational part. Presupposing that split scope is a unified phenomenon, DE SWART (2000) argues that the decomposition required is not always morphologically transparent (*fewer than three* for instance would have to be decomposed into negation and *more than two*), which makes the analysis implausible.

This paper focuses on split readings arising with *at most*. Considering *at most* is particularly interesting and instructive with respect to the question whether split scope should receive a unified analysis because *at most* gives rise to a split scope reading in combination with possibility modals, but not with necessity modals. This contrast in the availability of split readings, which does not arise for any other downward entailing quantifier, is illustrated in (6) and (7). Under its most salient reading sentence (6) expresses prohibition of checking out more than ten books. This corresponds to the split reading: to express prohibition, the possibility modal is interpreted in the scope of negation ('It is not allowed' <==> 'It is prohibited'), while *ten books* is interpreted with narrow scope under the modal to express a *de dicto* reading (the sentence is not about particular books, but rather expresses that taking out any set of more then ten books is prohibited).

(6)   You **can** check out **at most ten** books from the library.

'You are not allowed to check out more than ten books from the library.'

(7)   ?You **have to** read **at most ten** books for this class.

<=/=> You are not required to read more than ten books for this class.

In contrast, the split reading is not available for (7) with a necessity modal.[3] If it were, the sentence could be understood as expressing permission of reading ten or fewer books: Interpreting the necessity modal in the scope of negation would express absence of an obligation corresponding to permission of doing the opposite ('You are not required to read more than ten books' <==> 'You are allowed to read ten or fewer books'). But intuitively, this is not a possible reading of (7). What the sentence actually means is not easy to say either, because sentences where *at most* occurs under a necessity modal in general are odd and are harder to interpret than combinations of *at most* with possibility modals (see experimental studies by MCNABB & PENKA 2014a,b).

In order to explain this asymmetry in the availability of split readings arising with *at most,* I build on a recent approach to the semantics of superlative modifiers like *at least* and *at most* (BÜRING 2008; CUMMINS & KATSOS 2010; SCHWARZ 2011 and 2013; KENNEDY 2013). It has been proposed to account for a characteristic of *at least* and *at most* that makes them a particularly interesting object of study in semantics and pragmatics, namely the fact that they give rise to ignorance inferences. (That is by using *at least* and *at most*, a speaker generally conveys that she is unsure about the precise value under discussion; see section 2.) But in its basic version this approach does not account for the interaction of *at most* with modals and cannot derive the spilt reading of *at most* under possibility modals. I propose a modified analysis of *at most* where *at most* is decomposed into its positive antonym *at least* and an antonymizing operator defined in terms of degree negation (BÜRING 2007, HEIM 2008). This analysis provides a principled account of split readings arising with *at most* as well as ignorance inferences and their (non-) obviation under modals.

The paper is organised as follows: Section 2 sets the stage for the analysis by discussing superlative modifiers and the ignorance inferences they give rise to. It also introduces a pragmatic approach to ignorance inferences analysing them as quantity implicatures and shows that it successfully accounts for the interaction of *at least* with modals, but not for *at most.* In section 3, I propose to decompose *at most* into an antonymizing operator and *at least* and show that this successfully accounts for the interaction

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[3] The sentence has a reading – the so called speaker insecurity reading – which might at first glance seem to express negation of the obligation of reading more than ten books. However, there is a crucial difference between this reading and the split scope reading, as will become clearer in the discussion in section 2 and 3: whereas sentence (6) under the split reading can be used to state a rule, (7) cannot.

of *at most* with modals under the pragmatic approach. Section 4 addresses the question whether split scope of different downward entailing quantifiers should receive a unified analysis. Section 5 summarizes and concludes.

2. Superlative modifiers and ignorance inferences

2.1. Speaker insecurity and authoritative readings of *at least* and *at most*

The superlative modifiers *at least* and *at most* have recently received a lot of attention in the semantics and pragmatics literature. What makes them particularly interesting is the fact that in most contexts, they imply speaker ignorance, i.e. they convey that the speaker is not sure about the precise value under discussion (see GEURTS & NOUWEN 2007, NOUWEN 2010). Sentence (8), for instance, conveys that the speaker is not sure how many beers exactly John had last night. The only thing she is sure about is that the number is not less than three. But for all she knows, John might have had four or more beers.

(8)   John had **at least** three beers last night.

In certain environments, however, the implication of speaker uncertainty is absent. In particular, it has been observed that ignorance inferences can be suppressed in certain combinations of *at least* and *at most* with modals (see GEURTS & NOUWEN 2007). Sentence (9), where *at least* occurs under a necessity modal, has a reading which BÜRING (2008) calls authoritative. Under this reading, (9) does not convey speaker ignorance, but rather expresses that 10pp is the required minimum length of the paper. This reading is graphically illustrated in (9a), where '----' signifies the range of permissible paper lengths — which I will also refer to as deontic range — and 10pp is its lower bound.

(9)   The paper **has to** be **at least** 10 pages long.
    a.   '10 pages is the required minimum length of the paper'

            [-----------   authoritative reading
            10pp

    b. 'According to what the speaker knows, the required minimum length might be 10
        pages or it might be more.'

            [//////////-----------   speaker insecurity reading
            10pp

Sentence (9) has another reading conveying speaker ignorance, which can be brought out by prefixing the utterance with "I don't know exactly. But I think …". Under this reading,

4

which BÜRING (2008) calls speaker insecurity reading, the speaker is unsure about the required minimum length of the paper. For all she knows, the lower bound of permissible paper lengths could be 10pp or more. This reading is graphically illustrated in (9b), where '/////' signifies the epistemic range, i.e. the values that for all the speaker knows might or might not be permissible. To bring out the difference between the two readings, consider a situation in which the regulations specify that only papers that are 15pp or longer will be accepted. In this situation, the sentence is judged false under the authoritative reading (9a), whereas under the speaker insecurity reading (9b) the speaker cannot be blamed for making a false statement.

Not all combinations of superlative modifiers and modals allow for both the authoritative and the speaker insecurity reading. GEURTS & NOUWEN (2007) observe that *at least* in combination with necessity modals and *at most* in combination with possibility modals have both the authoritative and the speaker insecurity reading, whereas possibility modals plus *at least* and necessity modals plus *at most* allow for the speaker insecurity reading only. (The question which readings are available for the combination of necessity modal and *at most* is actually more complex and will be discussed in more detail in section 3.4.) The readings that GEURTS & NOUWEN claim to be available for the different combinations are summarized in (9) to (12).

(10)   The paper **can** be **at least** 10 pages long.

   'According to what the speaker knows, the maximally allowed length might be 10 pages or more.'

                   ----------[//////////   speaker insecurity reading only
                        10pp

(11)   The paper **has to** be **at most** 10 pages long.

   'According to what the speaker knows, the required minimum length might be 10 pages or less.'

                   //////////]----------   speaker insecurity reading only

                        10pp

(12)   The paper **can** be **at most** 10 pages long.

a. '10 pages is the maximally allowed length of the paper.'

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10pp   authoritative reading

b. 'According to what the speaker knows, the maximally allowed length might be 10 pages or less.'

$$----------/////////]$$

10pp   speaker insecurity reading

The ignorance implications of *at least* and *at most* and their interaction with modals are currently subject to a lot of work in semantics and pragmatics (amongst others GEURTS & NOUWEN 2007; BÜRING 2008; CUMMINS & KATSOS 2010; NOUWEN 2010 and 2015; SCHWARZ 2011 and 2013; COHEN & KRIFKA 2014; COPPOCK & BROCHHAGEN 2013; KENNEDY 2013). But none of the analyses proposed so far fully accounts for the interaction of *at least* and *at most* with modals. In this paper, I elaborate an approach that seems particularly attractive and derives ignorance as quantity implicatures. As we will see, this approaches in its basic version does not account for the interaction of *at most* with modals and cannot derive the spilt reading of *at most* under possibility modals.

2.1. Ignorance inferences as quantity implicatures

One approach, pioneered by BÜRING (2008) and further developed BY CUMMINS & KATSOS (2010), SCHWARZ (2011, 2013) and KENNEDY (2013), derives ignorance inferences of superlative modifiers in the pragmatic component. Building on a parallel to ignorance inferences arising with disjunction, which are generally taken to be derived via Gricean reasoning, the ignorance inferences triggered by *at least* and *at most* are analyzed as quantity implicatures. SCHWARZ (2011, 2013) spells out an analysis of superlative modifiers as degree operators and shows that ignorance inferences of *at least* and *at most* can be derived in the same way as the ignorance implications of *or* in a neo-Gricean framework (SAUERLAND 2004).[4] The essential ingredients of Schwarz' analysis are the following: In the semantics, *at least* and *at most* are analyzed as degree operators. As shown in (13) *at least* and *at most* take

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[4] MAYR (2013) and SCHWARZ (2013) note that Sauerland's algorithm needs to be revised and based on the notion of Innocent Exclusion (FOX 2007) in order to prevent the generation of unattested scalar implicatures for scalar modifiers. I neglect this issue for the purpose of this paper and circumvent the problem for Sauerland's basic algorithm by considering just those scalar alternatives that asymmetrically entail the assertion and where the number is closest to the modified numeral.

a degree (of type d) and a degree property (of type dt) as arguments and express non-strict comparison between the degree and the maximal degree of which the degree property holds.

(13) a. $[\![$at least$]\!] = \lambda d_d.\ \lambda D_{dt}.\ max(D) \geq d$

   b. $[\![$at most$]\!] = \lambda d_d.\ \lambda D_{dt}.\ max(D) \leq d$

In the pragmatics, utterances with *at least* or *at most* trigger scalar alternatives which are the result of substituting (i) the modified number by other numerals or measure phrases and (ii) *at least* and *at most* by each other or *exactly*.

(14) $[\![$exactly$]\!] = \lambda d_d.\ \lambda D_{dt}.\ max(D) = d$

With these assumptions ignorance inferences are generated for unembedded occurrences of *at least* and *at most* in SAUERLAND'S (2004) system, where scalar implicatures and ignorance inferences are two sides of the same coin. Scalar implicatures arise if primary implicatures of the form "the speaker is not certain that φ", where φ is a stronger scalar alternative, can be strengthened to secondary or scalar implicatures of the form "the speaker is certain that not φ". Ignorance inferences arise if the stronger alternatives cannot simultaneously be false while the assertion is true, or putting it differently, the assertion is equivalent to the disjunction of the stronger alternatives. In this case the alternatives are called 'symmetric'. The derivation of ignorance inferences is illustrated in the following for example (15). Since according to the meaning rule in (13) *at least 10pp* is a quantifier over degrees, it has to undergo QR resulting in the LF (16a). Under the standard semantics of gradable adjectives (HEIM 2000), shown in (16b) for *long* the truth conditions in (16c) are derived, according to which the length of the paper is 10pp or more.

(15) The paper is **at least** 10 pages long.

(16) a. [at least 10pp] [λd [the paper is d long]]

   b. $[\![$long$]\!] = \lambda d_d.\ \lambda x_e.\ \text{LENGTH}(x) \geq d$

   c. $max\{d: \text{LENGTH}(\text{the paper}) \geq d \} \geq 10pp \iff$

      $\text{LENGTH}(\text{the paper}) \geq 10pp$ \qquad henceforth abbreviated as

      $max\{d: long(p,d)\} \geq 10pp$

In the pragmatics, following Grice, this meaning is considered against alternative assertions the speaker could have made instead. The scalar alternatives of (15) correspond to all the sentences where the numeral modifier *at least* is substituted by *exactly* or *at most* and/or the

measure phrase *10pp* is substituted by other paper lengths, cf. (17). Out of these, the alternatives that are more informative, i.e. asymmetrically entail the assertion, are the ones formed by substituting either *at least* by *exactly* or *10pp* by *11pp*, as shown in (18).

(17) Scalar alternatives to (15):

The paper is *NumMod n* pages long.   where *NumMod* ∈ {*at least, exactly, at most*}

$n \in \{ \ldots, 9, 10, 11, \ldots \}$

(18) Stronger scalar alternatives:

a. The paper is **exactly** 10 pages long.   *at least* ➡ *exactly*

$\max\{d: \text{long}(p,d)\} = 10pp$

b. The paper is at least **11** pages long.   *10pp* ➡ *11pp*

$\max\{d: \text{long}(p,d)\} \geq 11pp$ <==>*[5]

$\max\{d: \text{long}(p,d)\} > 10pp$

Because the stronger alternatives in (18) are symmetric, i.e. one of them has to be true for the assertion to be true, none of the primary implicatures can be strengthened to secondary/scalar implicatures, because this would contradict the conjunction of the assertion and all the primary implicatures. Instead, the assertion A and the primary implicatures PI1 and PI2 taken together entail possibility implicatures. This is shown in (19), where using GAZDAR'S (1979) notation, $K\varphi$ corresponds to 'the speaker knows/ believes φ' and $P\varphi$ to 'the speaker considers φ possible'.

(19) a. Assertion: $K \max\{d: \text{long}(p,d)\} \geq 10pp$ A

b. Primary implicatures: $\neg K \max\{d: \text{long}(p,d)\} = 10pp$ PI1

$\neg K \max\{d: \text{long}(p,d)\} > 10pp$ PI2

c. Possibility implicatures: $P \max\{d: \text{long}(p,d)\} = 10pp$ (follows from A + PI2)

$P \max\{d: \text{long}(p,d)\} > 10pp$ (follows from A + PI1)

The primary and possibility implicatures together correspond to ignorance inferences, which are of the form $P\varphi$ & $P\neg\varphi$ (note that $\neg K\varphi$ is equivalent to $P\neg\varphi$). According to them, the speaker does not know whether the paper is exactly 10pp long or whether the paper is more than 10pp long. Together with the assertion, this correctly reflects the meaning of

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[5] The symbol <==>* is used when the equivalence is based on the simplifying assumption that the relevant scale is discrete, i.e. that only full-page lengths are considered. I make this assumption for expository reasons; the analysis works in the same way when dense scales are considered.

sentence (15).

(20) Ignorance implicatures generated:
a. $P \max\{d: \text{long}(p,d)\} = 10\text{pp} \ \& \ P \ \neg\max\{d: \text{long}(p,d)\} = 10\text{pp}$
b. $P \max\{d: \text{long}(p,d)\} > 10\text{pp} \ \& \ P \ \neg\max\{d: \text{long}(p,d)\} > 10\text{pp}$

Ignorance inferences for unembedded occurrences of *at most* are generated in the same way, the only difference being that now the alternative with a lower numeral is symmetric to the alternative where *at most* is substituted by *exactly*.

(21) The paper is **at most** 10 pages long.
$\max\{d: \text{long}(p,d)\} \leq 10\text{pp}$

(22) Stronger scalar alternatives:
a. The paper is **exactly** 10 pages long.   *at most � exactly*
$\max\{d: \text{long}(p,d)\} = 10\text{pp}$

b. The paper is at most **9** pages long.   *10pp � 9pp*
$\max\{d: \text{long}(p,d)\} \leq 9\text{pp}$ $\ <==>*$
$\max\{d: \text{long}(p,d)\} < 10\text{pp}$

(23) Ignorance implicatures generated:
a. $P \max\{d: \text{long}(p,d)\} = 10\text{pp} \ \& \ P \ \neg\max\{d: \text{long}(p,d)\} = 10\text{pp}$
b. $P \max\{d: \text{long}(p,d)\} < 10\text{pp} \ \& \ P \ \neg\max\{d: \text{long}(p,d)\} < 10\text{pp}$
'The speaker doesn't know whether the paper is exactly 10pp long or whether the paper is less than 10pp long.'

The neo-Gricean analysis thus accounts for the fact that unembedded occurrences of superlative modifiers give rise to ignorance inferences. It also makes certain predictions for the interaction of superlative modifiers with modals, which are discussed in the following subsections.

2.2. Interaction of superlative modifiers with necessity modals

Since superlative modifiers are analyzed as degree operators, two different scope orders are possible when they interact with modals. If a superlative modifier is interpreted in the scope of a necessity modal, the stronger scalar alternatives in (26) are not symmetric. That is, they can simultaneously be false while the assertion is true. For example (24) this is the case if the

9

permissible paper length corresponds to a range including 10pp and more. Because the alternatives are not symmetric, primary implicatures get strengthened to scalar implicatures in SAUERLAND'S (2004) system, as shown in (27).

(24) The paper **has to** be **at least** 10 pages long.

(25) a. has to [[at least 10pp] λd [the paper be d long]]   □ > *at least*

b. □ max{d: long(p,d)} ≥ 10pp

'In all the acceptable worlds, the length of the paper is 10pp or more.'

(26) Stronger scalar alternatives:

a. □ max{d: long(p,d)} = 10pp   *at least ➡ exactly*

'In all the acceptable worlds, the length of the paper is exactly 10pp.'

b. □ max{d: long(p,d)} ≥ 11pp <==>*   *10pp ➡ 11pp*

□ max{d: long(p,d)} > 10pp

'In all the acceptable worlds, the paper is longer than 10pp.'

(27) a. Assertion: K □max{d: long(p,d)} ≥ 10

b. Primary implicatures: ¬ K □max{d: long(p,d)} = 10pp

¬ K □max{d: long(p,d)} > 10pp

c. Scalar implicatures: K ¬□max{d: long(p,d)} = 10pp

K ¬□max{d: long(p,d)} > 10pp

According to the scalar implicatures generated, the speaker is sure that the paper does not have to be exactly 10pp long and that the paper does not have to be more than 10pp long. Together with the asserted content this is true iff the permissible paper lengths correspond to a range of values whose lower bound is 10pp. This corresponds to the authoritative reading illustrated in (28).

(28)   [-----------   authoritative reading
                     10pp

If a superlative modifier takes wide scope over a necessity modal, the speaker insecurity reading results. Although the scope order *at least* > □ is truth-conditionally equivalent to □ > *at least* (see HEIM 2000), the pragmatic reasoning is different. Because wide scope of *at least* and *exactly* in the alternatives leads to symmetric alternatives – just

as in the case of unembedded occurrences – ignorance inferences rather than scalar implicatures are generated.

(29) a. [at least 10pp] λd [has to [the paper be d long]]   *at least > □*

  b. max{d: □ long(p,d)} ≥ 10pp

    'The required minimum length of the paper is 10pp or it is more.'

(30) Stronger scalar alternatives:[6]

  a. max{d: □ long(p,d)} = 10pp   *at least ➡ exactly*

    'The required minimum length of the paper is exactly 10pp.'

  b. max{d: □ long(p,d)} ≥ 11pp <==>* max{d: □ long(p,d)} > 10pp   *10pp ➡ 11pp*

    'The required minimum length of the paper is more than 10pp.'

(31) Ignorance implicatures generated:

  a. P max{d: □ long(p,d)} = 10pp & P ¬ max{d: □ long(p,d)} = 10pp

  b. P max{d: □ long(p,d)} > 10pp & P ¬ max{d: □ long(p,d)} > 10pp

These ignorance implicatures express that the speaker is unsure about the required minimum length of the paper; she does not know whether it is exactly 10pp or more than 10pp. Together with the asserted content, this corresponds to the speaker insecurity reading.

(32)   [/////////----------   speaker insecurity reading
                    10pp

    Under the neo-Gricean account the two readings of sentences with a necessity modal and *at least* come down to a difference in scope: The authoritative reading arises if the superlative modifier is interpreted in the scope of the necessity modal, and the speaker insecurity reading results from wide scope of the superlative modifier. As evidence for this scopal ambiguity, BÜRING (2008) observes that only the authoritative reading is available if movement of *at least* over the modal is blocked for independent reasons, in particular if *at least* is contained within a finite clause. Sentence (33) only has the authoritative reading (28), and the speaker insecurity reading (32) is absent. It thus contrasts with the minimally different (24), where *at least* occurs in an infinitival and both readings are available.

(33) It is **required** that the paper be **at least** 10 pages long.

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[6] For reasons of space, from now on I only state the truth-conditions of the alternatives along with the substitutions by which they are generated from the LF under consideration.

The derivations for *at most* are again parallel to those for *at least.* If *at most* takes narrow scope under the necessity modal as in (34) the stronger scalar alternatives are not symmetric and the authoritative reading is derived, as shown in (35) to (38).

(34)   The paper **has to** be **at most** 10 pages long.

(35)   a. has to [[at most 10pp] λd [the paper be d long]]   □ > *at most*

   b. □ max{d: long(p,d)} ≤ 10pp

   'In all the acceptable worlds, the length of the paper is 10pp or less.'

(36)   Stronger scalar alternatives:

   a.   □ max{d: long(p,d)} = 10pp   *at most* ➡ *exactly*

      'In all the acceptable worlds, the length of the paper is exactly 10pp.'

   b. □ max{d: long(p,d)} ≤ 9pp <==>*   *10pp* ➡ *9pp*

      □ max{d: long(p,d)} < 10pp

      'In all the acceptable worlds, the paper is shorter than 10pp.'

(37)   Scalar implicatures generated:

   a. K¬□max{d: long(p,d)} = 10pp

   b. K¬□max{d: long(p,d)} < 10pp

    'The speaker is sure that the paper doesn't have to be exactly 10pp long and that the paper doesn't have to be less than 10pp long.'

(38)   -----------]   authoritative reading
                                      10pp

If *at most* takes wide scope, the stronger scalar alternatives are symmetric and the speaker insecurity reading is derived, as shown in (39) to (42).

(39)   a. [at most 10pp] [λd [has to [the paper be d long]]]   *at most* > □

   b.   max{d: □ long(p,d)} ≤ 10pp

      'The required minimum length of the paper is 10pp or less.'

(40)   Stronger scalar alternatives:

a. $\max\{d: \square \text{ long}(p,d)\} = 10pp$ *at most* ➡ *exactly*

'The required minimum length of the paper is exactly 10pp.'

b. $\max\{d: \square \text{ long}(p,d)\} \leq 9pp$ <==>*   *10pp* ➡ *9pp*

   $\max\{d: \square \text{ long}(p,d)\} < 10pp$

'The required minimum length of the paper is less than 10pp.'

(41)   Ignorance implicatures generated:

a. $\mathsf{P} \max\{d: \square \text{ long}(p,d)\} = 10pp \,\&\, \mathsf{P} \neg \max\{d: \square \text{ long}(p,d)\} = 10pp$

b. $\mathsf{P} \max\{d: \square \text{ long}(p,d)\} < 10pp \,\&\, \mathsf{P} \neg \max\{d: \square \text{ long}(p,d)\} < 10pp$

'The speaker doesn't know whether the required minimum length of the paper is exactly 10pp or whether the required minimum length of the paper is less than 10pp.'

(42)   //////////]-----------   speaker insecurity reading
                               10pp

In general, ignorance inferences are obviated if a superlative modifier is interpreted in the scope of an operator that breaks symmetry. This arguably also accounts for other cases of ignorance obviation, e.g. under universal quantifiers (SCHWARZ, 2011) and generics (NOUWEN 2010). However, we will see in the next section that a possibility modal does not break symmetry.

2.3. Interaction of superlative modifiers with possibility modals

While we just saw that a necessity modal breaks symmetry if a superlative modifier is interpreted in its scope and scalar rather than ignorance implicatures arise, a possibility modal does not break symmetry. Even if the possibility modal takes wide scope over a superlative modifier, the stronger scalar alternatives cannot simultaneously be false while the assertion is true. Therefore ignorance inferences are generated. This is shown in the following for *at most* (the derivations for *at least* are again parallel).

If *at most* is interpreted in the scope of the modal as in (44), a reading results which is weak for several reasons. For one thing the truth conditions merely say that there is an acceptable world where the length of the paper is 10pp or less. This allows for other lengths also being permissible.

(43)   The paper **can** be **at most** 10 pages long.

(44)   a. can [[at most 10pp] [λd [the paper be d long]]]   ◇ > *at most*

b. ◇ max{d: long(p,d)} ≤ 10pp

    'There is an acceptable world where the length of the paper is 10pp or less.'

In the pragmatic component, strong ignorance inferences are generated:

(45)   Stronger scalar alternatives:

    a. ◇max{d: long(p,d)} = 10pp   *at most ➨ exactly*

    'There is an acceptable world where the length of the paper is exactly 10pp.'

    b. ◇max{d: long(p,d)} ≤ 9 pp   <==>*   *10pp ➨ 9pp*

      ◇max{d: long(p,d)} < 10pp

    'There is an acceptable world where the length of the paper is less than 10pp.'

(46)   Ignorance inferences generated:

    a. P ◇ max{d: long(p,d)} = 10pp & P ¬◇max{d: long(p,d)} = 10pp

    b. P ◇ max{d: long(p,d)} < 10pp & P ¬◇ max{d: long(p,d)} < 10pp

According to these ignorance inferences the speaker does not know whether the paper can be exactly 10pp long or whether the paper can be less than 10pp long. Thus for all the speaker knows, the maximally allowed length might be 5pp or the required minimum length might be 10pp. Since this does not exclude a lot of epistemic alternatives, this reading is very weak. This weak reading might not be detectable because there is another reading with stronger truth conditions and sensible ignorance inferences, derived from an LF where the superlative modifier takes wide scope as in (47):

(47)   a. [at most 10pp] λd [allowed [the paper be d long]]   *at most > ◇*

    b. max{d: ◇long(p,d)} ≤ 10pp

      'The maximally allowed length of the paper is 10pp or less.'

(48)   Stronger scalar alternatives:

    a. max{d: ◇long(p,d)} = 10pp   *at most ➨ exactly*

      'The maximally allowed length of the paper is exactly 10pp.'

    b. max{d: ◇long(p,d)} ≤ 9pp   <==>*   *10pp ➨ 9pp*

      max{d: ◇long(p,d)} < 10pp

      'The maximally allowed length of the paper is less than 10pp.'

(49)   Ignorance implicatures generated:

a. P max{d: ◇long(p,d)} = 10pp & P ¬max{d: ◇long(p,d)} = 10pp

b. P max{d: ◇long(p,d)} < 10pp & P ¬max{d: ◇long(p,d)} < 10pp

According to these ignorance inferences the speaker is not sure whether the maximally allowed length of the paper is exactly 10pp or whether the maximally allowed length is less than 10pp. Together with the asserted content, this corresponds to the attested speaker insecurity reading illustrated in (50).

(50)   ----------//////////]   speaker insecurity reading
                              10pp

2.4. Summary of predictions of the neo-Gricean account

In sum, the neo-Gricean account of ignorance inferences makes the following predictions regarding the interaction of superlative modifiers with modals: If *at least* or *at most* are interpreted with wide scope over a necessity or possibility modal, the speaker insecurity reading conveying speaker ignorance is derived. The authoritative reading results if *at least* or *at most* are interpreted in the scope of a necessity modal. Narrow scope under a possibility modal leads to a reading with strong ignorance inferences.

This correctly accounts for the readings observed for *at least*. As discussed in section 2.4 *at least* gives rise to an authoritative reading in combination with a necessity modal (cf. (9)), but not in combination with a possibility modal (cf. (10)). *At most*, in contrast, gives rise to an authoritative split reading in combination with a possibility modal (cf. (12)). While the truth-conditions derived from an LF where *at most* takes wide scope correspond to the split reading excluding worlds where the paper is longer than 10pp, the reading is not authoritative because ignorance inferences are generated in the pragmatic component. Therefore the neo-Gricean analysis, while successful for *at least*, does not account for the interaction of *at most* with modals.

3. A decompositional analysis of *at most*

3.1. Decomposing *at most*

In order to account for the split reading of *at most* and the particular pattern of interaction with modals, I propose that *at most* is morpho-syntactically complex. Building on the idea that marked members of antonym pairs like *short* (vs. *long*) or little (vs. *much*) are generally decomposed in the syntax into an antonymizing operator and the corresponding positive

antonym (BÜRING 2007; HEIM 2008; ALXATIB 2013), I propose that *at most n* is decomposed into an antonymizing operator ANT and *at least n*:

(51)   at most n = [[ANT-n]$_{d(dt)}$-at least]$_{(dt)t}$

As meaning for *at least* I adopt the degree operator semantics proposed by SCHWARZ (2011) and KENNEDY (2013), repeated as (52) from (13a) above.

(52)   [[at least]] = $\lambda d_d.\ \lambda D_{dt}.\ max(D) \geq d$

For the semantics of the antonymizing operator ANT, an obvious candidate would be HEIM'S (2006) degree operator *little,* which is defined in (53) and expresses degree negation (meaning that a degree property D does not hold to degree d) (see also Alxatib 2013).

(53)   [[little]] = $\lambda d_d.\ \lambda D_{dt}.\ \neg D(d)$ [Heim 2006]

Heim uses this meaning of *little* to derive the split reading of sentences like (54) from the LF where *little* takes inverse scope over the modal. (This arguably also works for split scope of *few* as in example (3).)

(54)   We can grow very **little** before we run out of space.   [Heim 2006]
        'It is not possible for us to grow more than very little before we run out of space.'

With this definition of *little,* however, *at most n* cannot be decomposed into *little* plus *at least n*. Instead, *at most n* would have to be decomposed into *little* plus *more than n*, which is not in line with the idea that negative antonyms involve their positive counterparts.

But there are in fact independent reasons why Heim's definition of *little* needs to be revised (see also BECK 2012). The definition in (53) does not derive the intended meaning of sentences like (55), where *that* serves as direct degree argument of *weigh* and anaphorically picks up the measure phrase *40kg*. With the definition of *little* in (53) and the meaning of *weigh* in (56b) in analogy to the standard semantics of gradable adjectives, the truth conditions in (56c) are derived, according to which Sue weighs less than 40kg. But this does not correctly render the meaning of sentence (55). Intuitively, (55) is perfectly compatible with Sue weighing exactly 40kg, and in order for the presupposition of *too* to be fulfilled, Sue needs to have the same weight as Mary, i.e. 40kg.

(55)   [Mary only weighs 40kg.] Sue weighs that little too.

(56)   a. [that little] [λd [Sue d-weighs]]

    b. $[\![weigh]\!] = \lambda d_d. \lambda x_e.$ WEIGHT$(x) \geq d$

    c. $\neg[$WEIGHT$(s) \geq 40kg]$ <==>

        WEIGHT$(s) < 40kg$

To derive the correct meaning for sentence (55), we need a definition of *little* in which only higher degrees are negated, but not the degree contributed by the first argument:

(57) $[\![little]\!] = \lambda d_d. \lambda D_{dt}. \forall d' > d: \neg D(d')$

With this revised definition we get the truth conditions (58), according to which Sue does not weigh more than 40kg. After strengthening by scalar implicature the meaning is that Sue's weight is exactly 40kg, which correctly captures the meaning of sentence (55).[7]

(58) $\forall d' > 40kg: \neg[$WEIGHT$(s) \geq d']$ <==>

        $\neg[$WEIGHT$(s) > 40kg]$

I thus conclude that (57) is the correct definition of *little* (it also works for the split scope cases HEIM (2006) discusses) and for the antonymizing operator ANT more generally.[8] Note that this renders the antonymizing operator ANT equivalent to the non-complex meaning SCHWARZ (2011, 2013) assigns to *at most*, cf. (13b) from above (see also BECK 2012).

(59) $[\![ANT]\!] = \lambda d_d. \lambda D_{dt}. \forall d' > d: \neg D(d')$

(13b) $[\![at\ most]\!] = \lambda d_d. \lambda D_{dt}. \max(D) \leq d$

Since in the analysis I propose, *at most* consists of the antonymizing operator ANT (corresponding to Schwarz' *at most*) and *at least* more complex LFs are generated than under the analysis of Schwarz. This has important consequences for the interaction with modals, as we will see shortly.

3.2. Alternatives and ignorance inferences of *at most*

Before we can turn to the interaction with modals, we first need to address the question what consequences the decompositional analysis has for the pragmatics. We need to know in

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[7] See RETT (2007) on how to derive the fact that negative antonyms are generally evaluative, e.g. that (55) conveys that 40kg falls below the standard weight of women comparable to Sue. Incidentally, for Rett's analysis to work, the antonymizing operator also needs to be defined as in (57) rather than as in (53).

[8] Under the decompositional approach to antonyms, *little* would be ANT plus *much*. Since *much* is semantically vacuous, the meaning of ANT corresponds to the meaning of non-decomposed *little*.

particular what the scalar alternatives are for an utterance with *at most*. In this respect I follow KATZIR (2007) and FOX & KATZIR (2011), who argue that alternatives are structurally defined and generated by substitution of lexical categories and deletion. In particular, I assume that the scalar alternatives for an utterance with *at most* are generated by (i) substituting numerals or measure phrases by each other; (ii) substituting *at least* by *exactly* (see SCHWARZ 2011) and (iii) deleting ANT (see ALXATIB 2013). In addition, I adopt the common assumption that modals are substituted in the alternatives.

With these assumptions about the meaning of *at most* and scalar alternatives, ignorance inferences for unembedded occurrences of *at most* are generated in the same way as above: the stronger scalar alternatives, which are the same as under SCHWARZ' (2011) analysis, are symmetric and thus ignorance inferences are generated. This is illustrated in the following for sentence (60).

(60)   The paper is **at most** 10 pages long.

(61)   a. ANT-10pp [λd_2 [at least-d_2 [λd_1 [the paper is d_1-long]]]]

      b. $\forall d' > 10pp: \neg$ [max{d: long(p,d)} $\geq$ d]   <==>

        $\neg$ max{d: long(p,d)} > 10pp

(62)   Scalar alternatives:

The paper is *Polarity NumMod n* pages long.	where *Polarity* $\in$ { ANT, $\varnothing$ }
	NumMod $\in$ { *at least, exactly* }
	$n \in$ { …, 9, 10, 11, …}

(63)   Stronger scalar alternatives:

      a. The paper is **exactly**10 pages long.   ~~ANT~~, *at least* ➡ *exactly*

        max{d: long(p,d)} = 10pp

      b. The paper is at most **9** pages long.   *10pp ➡ 9pp*

        $\neg$ max{d: long(p,d)} > 9pp   <==>*

        max{d: long(p,d)} < 10pp

(64)   Ignorance inferences generated:

    a. P max$\{d: long(p,d)\} = 10pp$ & P ¬max$\{d: long(p,d)\} = 10pp$

    b. P max$\{d: long(p,d)\} < 10pp$ & P ¬max$\{d: long(p,d)\} < 10pp$

'The speaker isn't sure whether the paper is exactly 10pp long or whether the paper is less than 10pp long.'

3.3. Interaction of *at most* with possibility modals

In the discussion of the interaction of *at most* with modals, let us start with possibility modals, for which the basic neo-Gricean account cannot derive the attested split reading. Recall that the split reading, which is the most prominent reading of sentence (65), is authoritative and expresses prohibition of the paper being longer than 10pp. Under the decompostional analysis of *at most*, the three different scope orders in (66) are possible for this sentence.

(65)   The paper **can** be **at most** 10 pages long.

(66)   a. can [ANT-10pp [λd_2 [at least-d_2 [λd_1 [the paper be d_1-long]]]]] $\Diamond > $ ANT $> $ *at least*

       b. ANT-10pp [λd_2 [at least-d_2 [λd_1 [can [the paper be d_1-long]]]]] ANT $> $ *at least* $> \Diamond$

       c. ANT-10pp [λd_2 [can [at least-d_2 [λd_1 [the paper be d_1-long]]]]] ANT $> \Diamond > $ *at least*

Crucially, the decompositional analysis makes available the LF (66c) where ANT takes wide and *at least* takes narrow scope with respect to the modal. Under this scope order the alternatives are not symmetric and thus scalar implicatures are generated resulting in the authoritative reading, as shown in detail in the following.

(67)   a. ANT-10pp [λd_2 [can [at least-d_2 [λd_1 [the paper be d_1-long]]]]]   ANT $> \Diamond > $ *at least*

       b. $\forall d' > 10pp:$ ¬$\Diamond$ max$\{d: long(p,d)\} \geq d$ <==>

          ¬$\Diamond$ max$\{d: long(p,d)\} > 10pp$

       'There is no acceptable world where the length of the paper is more than 10pp.'

This LF already looks promising in terms of its truth conditions, according to which the paper is not allowed to be longer than 10pp. This is definitely part of the meaning intuitively conveyed by sentence (65) under the split reading. In addition, pragmatic inferences arise by considering the scalar alternatives of the following form:

(68)   Scalar alternatives:

The paper is *Polarity Modal NumMod n* pages long.   where *Polarity* $\in$ {ANT, $\varnothing$}

$Modal \in$ {allowed, required}

$NumMod \in$ {at least, exactly}

$n \in$ {…, 9, 10, 11, …}

We now have to consider eight scalar alternatives. It turns out that out of these, only the two shown in (69) asymmetrically entail the assertion.[9]

(69)   Stronger scalar alternatives:

    a. $\neg \Diamond$ max{d: long(p,d)} > 9pp <==>*   *10pp* ➡ *9pp*

          $\square$ max{d: long(p,d)} < 10pp

    b. $\square$ max{d: long(p,d)} = 10pp   ~~ANT~~, $\Diamond$ ➡ $\square$, *at least* ➡ *exactly*

Crucially, (69a) generated by substituting the numeral with a lower value does not have a symmetric counterpart, i.e. there is no alternative which together with (69a) exhausts the assertion. This is due to the fact that ANT, which has the semantics attributed by SCHWARZ (2011) to *at most*, can be deleted in the alternatives but not substituted by *exactly*. The alternative (69a) thus leads to the scalar implicature (70a), according to which the speaker is sure that the paper does not have to be shorter than 10pp. In addition, the alternative (69b) also leads to a scalar implicature (70b), according to which the speaker is sure that the paper does not have to be exactly 10pp long.

(70)   Scalar implicatures generated:

    a.   K $\neg\square$ max{d: long(p,d)} < 10pp

    b.   K $\neg\square$ max{d: long(p,d)} = 10pp

Taken together, the assertion and the scalar implicatures express that the permissible paper lengths correspond to a range of values whose upper bound is 10pp. In other words, we derive the authoritative reading (71), which corresponds to the split reading of (65).

(71)   ----------]   authoritative reading
                              10pp

    This shows that the decompostional analysis can derive the split (authoritative)

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[9] The truth conditions in (69a) can be derived by two different substitutions: either by substituting *10pp* by *9pp* (as shown) or by substituting *at least* by *exactly* and *10pp* by *9pp*. In case of equivalent alternatives, I only consider the one requiring the fewest substitutions.

reading for the combination of *at most* with a possibility modal, which other analyses fail to account for. In the analysis I propose, the split reading is derived from an LF where ANT takes wide scope. This leads to the prediction that the split (authoritative) reading should not be available if movement out of the scope of the modal is blocked for independent reasons. Evidence that this prediction is borne out comes from sentences like (72), where *at most* is embedded in a finite clause. While the sentence is less than perfect and hard to interpret, it seems clear that it does not have the split (authoritative) reading, according to which 10pp is the maximally allowed length of the paper.[10]

(72)   It is **permitted** that the paper is **at most** 10 pages long.

In addition to the split authoritative reading, the other two readings that the basic neo-Gricean approach discussed in section 2 derives are also generated from the other two available LFs (66a) and (66b). In general, if ANT and *at least* take adjacent scope, the same pragmatic inferences and readings are derived as for SCHWARZ' (2011) non-decomposed *at most*. If both ANT and *at least* are interpreted in the scope of the possibility modal, we get the same symmetric stronger alternatives (74a) and (74b) as in (45). These lead to the ignorance inferences (75). Because we now also consider alternatives which are generated by replacing the possibility modal with a necessity modal, we also derive the scalar implicatures (76) on the basis of the non-symmetric stronger alternatives (74c) and (74d).

(73)   a. can [ANT-10pp [λd_2 [at least-d_2 [λd_1 [the paper be d_1-long]]]]] $\Diamond$ > ANT > *at least*

   b. $\Diamond \forall d' > 10pp: \neg[\max\{d: long(p,d)\} \geq d']$ $\Longleftrightarrow$

   $\Diamond \max\{d: long(p,d)\} \leq 10pp$

   'There is an acceptable world where the length of the paper is 10pp or less.'

(74)   Stronger scalar alternatives:

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[10] The absence of the authoritative reading if wide scope of the modal is enforced also provides an argument against attributing obviation of ignorance inferences under possibility modals to a Free Choice effect (see COPPOCK & BROCHHAGEN (2013) and NOUWEN (2015) for proposals in this direction). Current analyses of Free Choice effects arising with disjunction and indefinites (FOX 2007 among others) derive Free Choice permission readings from an LF where the possibility modal takes wide scope. If the authoritative reading of *at most* was due to a Free Choice effect, then the fact that (72) does not have the authoritative reading is unexpected.

a. $\diamond \max\{d: long(p,d)\} \leq 9pp \iff *$ ~~~~~~~~~~~~~~ *10pp ➡ 9pp*

$\diamond \max\{d: long(p,d)\} < 10pp$

b. $\diamond \max\{d: long(p,d)\} = 10pp$ ~~~~~~~~~~~~~ ~~ANT~~, *at least ➡ exactly*

c. $\square \max\{d: long(p,d)\} \leq 10pp$ ~~~~~~~~~~~~~~~~ $\diamond ➡ \square$

d. $\square \max\{d: long(p,d)\} = 10pp$ ~~~~~~~~~~~~~ ~~ANT~~, $\diamond ➡ \square$, *at least ➡ exactly*

(75) Ignorance Inferences generated:

a. $\mathsf{P} \diamond \max\{d: long(p,d)\} < 10pp \ \& \ \mathsf{P} \neg \diamond \max\{d: long(p,d)\} < 10pp$

b. $\mathsf{P} \diamond \max\{d: long(p,d)\} = 10pp \ \& \ \mathsf{P} \neg \diamond \max\{d: long(p,d)\} = 10pp$

'The speaker does not know whether the paper is allowed to be exactly 10pp long and whether the paper is allowed to be less than 10pp long.'

(76) Scalar implicatures generated:

a. $\mathsf{K} \neg \square \max\{d: long(p,d)\} \leq 10pp$

b. $\mathsf{K} \neg \square \max\{d: long(p,d)\} = 10pp$

'The speaker is sure that the paper does not have to be exactly 10pp long and that the paper does not have to be 10pp long or shorter.'

As discussed for Schwarz' analysis above, the derived ignorance inferences are strong. Although the truth conditions and scalar implicatures taken together now entail that the paper does not have to be one specific length, the resulting reading is still weak. In any case, it does not express an upper or lower bound to the lengths that are permissible according to the knowledge of the speaker. Because this reading is not informative, I assume, following Schwarz, that it is not detectable.

The speaker insecurity reading is derived from the LF (66b), where both ANT and *at least* take scope under the possibility modal. In this case, the derivation is completely parallel to (47)–(49) above: There are two symmetric stronger alternatives, which lead to ignorance implicatures.

(77) a. ANT-10pp [λd_2 [at least-d_2 [λd_1 [can [the paper be d_1-long]]]]] ~~ ANT $> at least > \diamond$

b. $\forall d' > 10pp: \neg[\max\{d: \Diamond long(p,d)\} \geq d']$ $\Longleftrightarrow$

  $\neg[\max\{d: \Diamond long(p,d)\} > 10pp] \Longleftrightarrow$

  $\max\{d: \Diamond long(p,d)\} \leq 10pp$

  'The maximally allowed length of the paper is 10pp or less.'

(78)   Stronger scalar alternatives:

  a. $\max\{d: \Diamond long(p,d)\} \leq 9pp$ $\Longleftrightarrow$*   *10pp ➡ 9pp*

    $\max\{d: \Diamond long(p,d)\} < 10pp$

  b. $\max\{d: \Diamond long(p,d)\} = 10\,pp$ ~~ANT~~*, at least ➡ exactly*

(79)   Ignorance inferences generated:

  a. $P \max\{d: \Diamond long(p,d)\} = 10pp$ & $P \neg\max\{d: \Diamond long(p,d)\} = 10pp$

  b. $P \max\{d: \Diamond long(p,d)\} < 10pp$ & $P \neg\max\{d: \Diamond long(p,d)\} < 10pp$

  'The speaker is not sure whether the maximally allowed length is exactly 10pp or less than 10pp.'

(80)   ----------/////////]   speaker insecurity reading
              10pp

3.4. Interaction of *at most* with necessity modals

To complete the discussion of the readings the decompositional analysis makes available, we also need to re-consider the interaction of *at most* with a necessity modal, as in (81). Crucially, we need to make sure that we do not derive an unattested split-scope reading granting permission for the paper to be 10pp or shorter. The three scope orders in (82) have to be considered.

(81)   The paper **has to** be **at most** 10 pages long.

(82)   a. has to [ANT-10 [λd_2 [at least-d_2 [λd_1 [the paper be d_1-long]]]]]   □ > ANT > *at least*

    b. ANT-10 [λd_2 [at least-d_2 [λd_1 [has to [the paper be d_1-long]]]]]   ANT > *at least* > □

    c. ANT-10 [λd_2 [has to [at least-d_2 [λd_1 [the paper be d_1-long]]]]]   ANT > □ > *at least*

The LF (82c) would seem to be the basis for the split scope reading, according to which the paper does not have to be longer than 10pp and which is in fact not attested. But while the truth-conditions (83b) express permission for the paper to be no longer than 10pp, strong ignorance inferences based on the symmetric alternatives (84a) and (84b) are generated in

the pragmatic component. As before, I assume that the strong ignorance reading is masked by the existence of the speaker insecurity reading with sensible ignorance inferences.

(83) a. ANT-10pp [λd_2 [has to [at least-d_2 [λd_1 [the paper be d_1-long]]]]]ANT $> \square > $ *at least*

  b. $\forall d' > 10pp: \neg\square \max\{d: long(p,d)\} \geq d$ $\Longleftrightarrow$

   $\neg\square \max\{d: long(p,d)\} > 10pp$

   'The paper does not have to be more than 10pp long.'

(84) Stronger scalar alternatives:

  a. $\neg\square \max\{d: long(p,d)\} > 9pp$ *10pp* ➡ *9pp*

  b. $\diamondsuit \max\{d: long(p,d)\} = 10pp$ ~~ANT~~, $\square$ ➡ $\diamondsuit$, *at least* ➡ *exactly*

  c. $\neg\diamondsuit [\max\{d: long(p,d)\} > 10pp]$ $\square$ ➡ $\diamondsuit$

(85) Ignorance inferences generated:

  a. $P \neg\square \max\{d: long(p,d)\} > 9pp \,\&\, P \square \max\{d: long(p,d)\} > 9pp$

  b. $P \diamondsuit \max\{d: long(p,d)\} = 10pp \,\&\, P \neg\diamondsuit \max\{d: long(p,d)\} = 10pp$

   'The speaker is not sure whether the paper is required to be longer than 9pp and she is not sure whether the paper is allowed to be exactly 10pp long.'

(86) Scalar implicature generated:

   $K \diamondsuit \max\{d: long(p,d)\} > 10pp$

   'The speaker is sure that the paper is allowed to be longer than 10pp.'

The speaker insecurity reading is derived from the LF where ANT and *at least* take adjacent scope over the modal (which makes the derivation again equivalent to the one for non-decomposed *at most*):

(87) a. ANT-10pp [λd_2 [at least-d_2 [λd_1 [has to [the paper be d_1-long]]]]] ANT $>$ *at least* $> \square$

  b. $\forall d' > 10pp: \neg[\max\{d: \square long(p,d)\} \geq d']$ $\Longleftrightarrow$

   $\neg[\max\{d: \square long(p,d)\} > 10pp]$ $\Longleftrightarrow$

   $\max\{d: \square long(p,d)\} \leq 10pp$

   'The required minimum length of the paper is 10pp or less.'

(88) Relevant scalar alternatives:

a. max{d: □ long(p,d)} ≤ 9pp   <==>*   *10pp ➥ 9pp*

   max{d: □ long(p,d)} < 10pp

b. max{d: □ long(p,d)} = 10pp   ~~ANT~~*, at least ➥ exactly*

(89)   Ignorance inferences generated:

a. P max{d: □ long(p,d)} < 10pp & P ¬max{d: □ long(p,d)} < 10pp

  b. P max{d: □ long(p,d)} = 10pp & P ¬max{d: □ long(p,d)} = 10pp

 'The speaker is not sure whether the required minimum length is exactly 10pp or less than 10pp.'

(90)   //////////]----------   speaker insecurity reading
                10pp

Finally, if both ANT and *at least* are interpreted in the scope of the necessity modal, the authoritative reading results:

(91)   a. has to [ANT-10pp [λd₂ [at least-d₂ [λd₁ [the paper be d₁-long]]]]] □ > ANT > *at least*

b. □ ∀d' > 10pp: ¬[max{d: long(p,d)} ≥ d']   <==>

  □ ¬[max{d: long(p,d)} > 10pp] <==>

  □ max{d: long(p,d)} ≤ 10pp

  'In every acceptable world, the length of the paper is 10pp or less.'

(92)   Stronger scalar alternatives:

a. □ max{d: long(p,d)} ≤ 9pp   <==>*   *10pp ➥ 9pp*

  □ max{d: long(p,d)} < 10pp

b. □ max{d: long(p,d)} = 10pp   ~~ANT~~*, at least ➥ exactly*

(93)   Scalar implicatures generated:

a. K ¬□ max{d: long(p,d)} < 10pp

b. K ¬□ max{d: long(p,d)} = 10pp

'The speaker is sure that the paper does not have to be less than 10pp long and that the paper does not have to be exactly 10pp long.'

(94)   ----------]   authoritative reading
                10pp

The decompositional analysis thus inherits from the neo-Gricean account the prediction that

for the combination of *at most* with a necessity modal both the speaker insecurity and the authoritative reading is available. But intuitively only the speaker insecurity reading seems to be available for sentence (81) and the authoritative reading (94) according to which 10pp is the maximally allowed length is not possible. Thus the approach seems to overgenerate readings. But at closer inspection, the unavailability of the authoritative reading of (81) might in fact be due to independent factors. It is instructive to observe that the authoritative reading is readily available if *at most* is embedded in a finite clause under a necessity modal. This is illustrated by the sentences in (95) gathered from the internet. Intuitively, they do not express speaker ignorance, but rather report or set the upper bound of the range of permissible values:

(95)   a. [I am looking for suggestions for a dorm room microwave for my son.]

The college **requires** that it be **at most** 1 cu feet in volume and **at most** 800 Watts.[11]

'1 cu feet is the maximally allowed volume and 800 Watts is the maximally allowed power.'

b. This algorithm **requires** that variables be used **at most** once.[12]

'The maximally allowed number of variable uses is one.'

Data like (95) suggest that the authoritative reading is available, and in fact the only reading possible, if there is no choice but to interpret the modal with widest scope. Thus, I take it to be a welcome prediction that wide scope of a necessity modal results in the authoritative reading.

The question remains, however, why the authoritative reading does not seem to be available when *at most* is contained within an infinitival complement of a necessity modal as in (81). A way to explain this could be to relate the scopal interaction of modals with the antonymizing operator to the well-known scope preferences modals show vis-à-vis sentential negation. Since the modal verb *have to* in (81) takes narrow scope with respect to negation (see e.g. IATRIDOU & ZEIJLSTRA 2009), this would predict that *have to* also takes narrow scope with respect to ANT, making the authoritative reading unavailable. A further prediction would be that the authoritative reading is readily available if *at most* is combined with *must*, which scopes over negation. In fact, the authoritative reading seems to be more easily

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available when at *at most* is combined with *must* than when it is combined with *have to*. But still, the combination does not seem to be particularly natural. This might have to do with the fact that the authoritative reading expressed by *at most* plus necessity modal is the same as the one expressed by the combination of *at most* with a possibility modal. Since both possibility modals and *at most* are naturally used to express an upper bound, this combination might seem more natural than combing *at most* with necessity modals, which naturally express lower bounds (see BREHENY 2008). In order to see whether the parallel between sentential negation and the antonymizing operator in their interaction with modals can be upheld, more modals would have to be considered as well as other constructions arguably involving ANT or *little* (in particularly the constructions HEIM (2006) and BÜRING (2007) discuss).

To summarize the discussion of the readings when *at most* is combined with a necessity modal, the decompositional analysis derives the speaker insecurity and the authoritative reading. The fact that the authoritative reading does in many cases not show up is likely to be attributable to independent reasons. As desired, it is not possible to derive a split (authoritative) reading, which would express permission for the paper being 10pp or shorter.

4. A unified analysis of split scope?

This paper started from the observation that a split scope reading arises if *at most* is combined with a possibility modal, but not if it is combined with a necessity modal. This sets *at most* apart from other downward monotonic quantifiers like negative indefinites and numerals modified by *fewer than*, which show no asymmetry in terms of split readings across the two types of modals. An investigation of *at most* can therefore provide important evidence for the adequate analysis of split scope, and in particular contribute to the debate whether split scope of all downward monotonic quantifiers should receive the same analysis.

The asymmetry between possibility and necessity modals is unexpected under unified accounts of split scope readings (DE SWART 2000 and ABELS & MARTÍ 2010) since they apply independently of the type of modal. But the asymmetry can be explained under the analysis I propose, where *at most* is decomposed into an antonymizing operator and *at least.* This provides an argument for deriving split readings arising with different quantifiers in a case-by-case fashion. Decomposing e.g. negative indefinites into a sentential negation and an indefinite (PENKA 2011 among others), and comparative quantifiers into a comparative operator and a quantificational part (HACKL, 2000) allows for accounting for the particular

patterns of split readings these quantifiers exhibit.

In principle, it would be possible to extend the analysis in terms of an antonymizing operator to other quantifiers exhibiting split scope readings. The negative determiner *no*, for instance, could be analyzed as meaning 'no more than zero' and consisting of the antonymizing operator and HACKL'S (2000) gradable determiner MANY, as shown in (96). Sentence (97a) would then receive the truth conditions (97b) saying that there does not exist a set consisting of students who failed with a cardinality greater than zero.

(96)   a. *no* = $[[\text{ANT-}0]_{d(dt)}\text{- MANY}_{d((et)(et))}]$

(97)   a. No student failed.

        b. $\forall d > 0: \neg \exists X [\ |X| = d\ \&\ students(X)\ \&\ failed(X)]$

But although theoretically possible, such an analysis for negative indefinites does not seem adequate. In contrast to numerals modified by *at most* or *fewer than*, negative indefinites do not display overt degree morphology. Hence analysing them in terms of degree semantics does not seem justified if the morpho-syntactic composition of quantifiers is taken seriously. Moreover, negative indefinites in many languages show an interaction with sentential negation and give rise to negative concord. Thus, relating them to sentential negation seems more adequate than relating them to the antonymizing operator.

These considerations point to the conclusion that split scope of different types of downward monotonic quantifiers should not be subsumed under a unified analysis but rather be analysed in a case-by-case fashion.

5. Summary and conclusion

The aim of this paper was to account for the specific pattern of interaction with modals that *at most* exhibits in terms of split scope readings. Split scope readings are available for *at most* in combination with possibility modals, but not necessity modals. Embedding the discussion of split scope readings of *at most* in the recent semantic and pragmatic literature on superlative modifiers, it was noted that the split reading of *at most* under possibility modals is authoritative where the ignorance inferences usually triggered by superlative modifiers are obviated.

Building on the neo-Gricean approach to ignorance inferences, which successfully accounts for the interaction of *at least* with modals, but in its basic version fails for *at most*, I

showed that the neo-Gricean approach successfully accounts for the interaction of *at most* with modals if it is supplemented with the idea that *at most* is decomposed into an antonymizing operator, defined in terms of degree negation, and *at least*. The analysis explains the fact that *at most* gives rise to ignorance inferences when it occurs unembedded or in combination with a necessity modal, but can obviate ignorance inferences in combination with a possibility modal to yield an authoritative split reading.

Several consequences emerge from this analysis bearing on issues that are currently under debate in the semantics and pragmatics literature. First, it shows that split readings arising with *at most* can be derived under an approach where *at most* is decomposed into a negation and another meaning component. This decomposition also seems to be on a principled basis, as the second meaning component is the corresponding positive antonym *at least*. But crucially, in contrast to negative indefinites like *no doctor*, the negation involved is not sentential negation but rather degree negation that figures in the formation of antonyms of gradable adjectives. This suggests that instead of a unified analysis, split scope of downward monotonic quantifiers should be analysed in a case-by-case fashion taking the morphological make-up of the respective quantifier seriously. Second, the particular pattern of interaction with modals which *at most* exhibits lends support to the idea that negative antonyms are generally decomposed in the syntax into an antonymizing operator, defined in terms of degree negation, and the corresponding positive antonym. This pattern of interaction also has consequences for the question what triggers this decomposition, or putting it differently, which element of a pair of antonyms should be analysed as the negative one. For the antonym pair *at least — at most* it seems that semantic rather than morphological properties are decisive. The evidence coming from the interaction with modals suggests that it is *at most* which is more complex and composed of the antonymizing operator, rather than *at least,* although the former is morphologically based on the positive form *much* and the latter on the negative form *little.* It might seem that *at least* is the more likely candidate for decomposition, since it already contains *little*, which is semantically equivalent to the antonymizing operator. But the readings that are available in combination with modals suggest that it is the downward monotonic modifier *at most* that involves the antonymizing operator.

In sum, the investigation of split readings of *at most* in this paper contributes to current debates in the semantics and pragmatics literature in several respects. First, it lends support to the idea that negative antonyms are decomposed in the syntax into an antonymizing operator, defined in terms of degree negation, and the corresponding positive antonym. Second, it

shows that a pragmatic account of the ignorance inferences of superlative modifiers can explain the *prima facie* puzzling interaction of *at least* and *at most* with modals. Finally, it provides evidence against a unified analysis of split scope readings and in favour of a case-by-case analysis for different types of downward monotonic quantifiers.

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