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The Reference of Indefinites

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Der unbestimmte Artikel hat die Aufgabe, aus verschiedenen Vertretern einer Gattung einen einzelnen herauszuheben.[1]

1. Introduction

In this paper I argue that indefinite NPs have a more complex referential nature than is usually supposed, and that this structure must be reflected in their semantic representation. According to the classical view due to Frege and Russell, indefinite NPs are represented by existential quantifiers, leaving the syntactic constituent that corresponds to the indefinite NP to be an unstructured variable. I show that this simplified view heavily depends on Frege's distinction between 'Gegenstand' and 'Begriff' ('object' vs. 'concept'). According to Frege natural language expresses the former by using the definite article, whereas the indefinite article indicates the latter. Thus, sentence (1) is represented by the formula (1a) in which the indefinite NP *a man* corresponds to the variable *x* in the argument position of the predicate *walk* and of the predicate *man*. The formula specifies that the intersection of the two sets denoted by the predicates is non-empty. The model-theoretic interpretation links the variable *x* to an object *d* that fulfills both predicates, treating the attributive material *man* on par with the assertive material *walk*. Hence at the representational level, the indefinite NP is not represented as an independent expression. This (mis)conception has been widely accepted in semantics and can still be found in current semantic theories.

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* This paper is the final part of a larger project analyzing the semantics of NPs and pronouns using epsilon terms (Egli 1991). It was financed by the Deutsche Forschungsgemeinschaft. In preceding papers it was first argued that Evans' concept of E-type pronouns can be reconstructed by epsilon terms (cf Egli & von Heusinger 1995). Furthermore, it was shown that definite NPs in general must be represented as terms, in particular as context dependent epsilon terms (cf von Heusinger 1996). Their interpretation depends essentially on the salience structure of a given text or discourse. The final step is presented in this paper, showing that unstructured variables do not suffice for representing indefinites. We rather need a more structured representation which is given by indexed epsilon terms.

I would like to thank B.H. Slater, J. Peregrin and the audience of the Konstanz Workshop "Reference and Anaphorical Relations" for helpful and inspiring comments. B. Mayo helped me with the English prosa.

[1] Behagel (1923, I, 45): "The indefinite article has the function of picking out a single representative from among various representatives of a kind."

(1) A man walks

(1a) $\exists x$ [man(x) & walk(x)]

(1b) The formula *$\exists x$ [man(x) & walk(x)]* is true iff there is an object *d* in the domain of individuals such that *d* is in the extension of the predicate *man* and in the extension of the predicate *walk*.

I want to maintain that indefinite NPs must have a different representation that reflects their referential nature in a more adequate way. I shall argue that indexed epsilon terms give a far better picture of indefinites than unstructured variables can do. Sentence (1) is represented by the formula (1c) in which the epsilon term *$\varepsilon_i x$ man(x)* stands for the corresponding grammatical constituent *a man*. The epsilon operator is interpreted as the operation of selecting one element out of a set. The model theoretic interpretation (1d) links an element to the epsilon term that is selected out of the set defined by the descriptive material of the term. The sentence is true if this chosen element has the property of walking.

(1c) walk(ε_ix man(x))

(1d) The formula *walk($\varepsilon_i x$ man(x))* is true iff the object selected out of the set of men lies in the extension of the predicate *walk*.

The epsilon term stands for an independent expression reflecting the fact that indefinite NPs are referring expressions rather than quantifier phrases. This view is vindicated by independent considerations concerning linguistic, philosophical and semantic arguments. In particular, I shall cite the following points as evidence concerning the interpretation and representation of indefinite NPs:

(i) the historical development and spread of the indefinite article in indoeuropean languages,

(ii) Frege's epistemological distinction of 'object' *vs*. 'concept' compared with the grammatical categorization into definite NPs *vs.* indefinite NPs,

(iii) the relation between the grammatical structure of the sentence and its logical form,

(iv) the fine-grained dependency structure of indefinite NPs and

(v) the information-changing potential of indefinites.

The paper is organized as follows: In the second section I discuss the various grammatical functions that are licensed for nominal expressions. This sketch helps to identify the common mistake in categorizing indefinite NPs as being predicative in nature. Section 3 compares different analyses of the indefinite NP, starting from the traditional grammarian view, passing

through the Fregean logic and ending with the dynamic account. Section 4 gives a short introduction to the classical epsilon calculus and its application to linguistic theory. The representation of indefinite NPs as epsilon terms gives a uniform representation for indefinite NPs and E-type pronouns. It further reconstructs dependency structures, not via quantifiers and scope, but by using dependent terms. However, the classical epsilon calculus must be extended in two ways in order to reflect the referential nature and the context change potential of indefinites. The referential nature of indefinites is captured by the contextual dependency of the indexed epsilon operator, which is introduced in section 5. With these indexed epsilon terms, a clear analysis of the so-called 'asymmetric readings' of donkey sentences becomes possible. The context change potential of indefinite NPs is briefly discussed in section 6. An indefinite NP raises the object it denotes to salience in such a way that a subsequent definite expression can pick up this salient object. This mechanism reconstructs the anaphorical relation between two expression without the pretheoretic strategy of binding.

2. The grammatical modes of nominals

Nominals like nouns, adjectives, verbal nouns, participles etc. can take different grammatical modes or functions in a sentence: They can be used as substantives, attributes or as predicates. I will also speak of 'substantive use', 'attributive use' and 'predicative use'. I assume that these modes each show a characteristic grammatical behavior and can be clearly separated from one another. I further suppose that none of them can be reduced to another one. These different grammatical functions are closely related to the word classes of substantives and adjectives.

In the substantive use illustrated by the examples in (2a)-(2c), nominals are subjects, objects or otherwise arguments of other expressions. In this position, they refer to objects and, therefore, motivate the name 'substantive' for a whole class of nominal expressions. In languages with case they carry the case morphemes, and in languages with articles they can take articles. Nominals in the attributive use are prototypical adjectives or participles as in the examples (3a)-(3c). Attributive expressions denote properties rather than objects and they modify nominals in the substantive use. In the attributive use, they cannot combine with the definite or indefinite article. Nominals in the predicative mode as in (4a)-(4c) behave similarly to verbal predicates in that they assert a property of a given object. Generally, they denote properties and they cannot be introduced by articles, as in the attributive position. However, in certain cases nominals with the article can appear in the predicative mode. The use of the article in such position is highly idiosyncratic and restricted.

(2a) An old *man* sat by the *side* of the *road*.

(2b) *Indefinites* are problematic expressions.

(2c) *manus manum* lavat. ‘hand washes hand’

(3a) An *old* man sat by the side of the road.

(3b) Indefinites are *problematic* expressions.

(3c) amantes *amentes* ‘loving - being crazy’

(4a) He is *rich.*

(4b) Indefinites are problematic expressions.

(4c) homo homini *lupus* (est). ‘man is a wolf to man’

The indefinite article - indicating an indefinite NP - can precede nominals in the substantive and the predicative mode, but never a nominal in the attributive mode.[2] I assume that the use of the indefinite article (like the use of the definite article) is more typical in the substantive use than in the predicative use. This position is motivated on the following two grounds: First the development of the indefinite article in Indo-European languages always started in the (referential) substantive function and only then spread to other functions as discussed in the next section. Second, the rules of application of the indefinite article in the predicative use are very idiosyncratic and not very stable, which was noted early on by Jespersen (1925, 152):

> This will make us understand why it is that predicatives are often used either without any article or with the indefinite article, though the rules are somewhat different in different languages. In English one says:
>
> > John was a sailor, and
> > John was a liar,
>
> where German and Danish would have the indefinite article in the latter sentence, but not in the former, where the predicative denotes a profession: Hans war schneider, Hans war ein lügner; Jens var skrædder, Jens var an lögnhals.

The distribution of the indefinite article shows that it primarily indicates an expression that refers to an object. Only in derived uses of the indefinite article does it preceed other types of expressions, as well.

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[2] The distribution of the definite and indefinite article is similar. Like the indefinite article, the definite article can precede nominals in the substantive and predicative mode, but never nominals in the attributive mode. Both articles do not drastically differ in their distribution in predicate position as shown in (i) and (ii). This indicates that there is no syntactic reason to assume that definites are terms, whereas indefinites are predicates.

(i) He was *a man I was looking for*.

(ii) He was *the man I was looking for*.

3. Indefinite NPs and their representation

In this section I give a short overview of different approaches to and representations of indefinite NPs. Since the treatment of indefinite NPs cannot be separated from anaphorical expressions that are linked to them, this overview must encompass anaphora, too. Christophersen (1939, 32f) criticizes the traditional description of the functions of the indefinite article, as shown in (5a), as too narrow, and proposes the modified version (5b). He calls (i) individualizing and (ii) and (iii) classifying, subsuming the predicative use under (ii):

(5a) *Function of the indefinite article (traditional concept)*

- (i) definite individual use (= *a certain one*)
- (ii) indefinite individual use (= *some one or other*)
- (iii) general use (= *whichever one*)

(5b) *Function of the indefinite article (modified by Christophersen)*

- (i) Introductory use: the center of attention is *one particular* individual and its specific attributes. It may be a really existing or imaginary case.
- (ii) The interest centers round the generic characters of a *single* individual, imaginary or real.
- (iii) Generalization to *several or all* the members of a class.

As in most other languages, the indefinite article in Germanic languages is derived from the same root as the numeral for ‘one’. According to Christophersen (1939, 100ff) in Old English, the indefinite article was first used as an “individualizing pronoun” and it “was never employed in generic statements, hardly ever in the predicative, and only rarely with objects of result (i.e. [...] *to dig a grave,* etc.)”.[3] Du Bois (1980, 208) states that both articles were originally used with referential and specific nouns indicating identifiability. Only later did their application spread to other uses (i.e. non-specific and generic nouns) where they received a secondary meaning. Chesterman (1991, 185) gives a similar statement regarding the development of the indefinite article out of the numeral:

> The numeral first appears with discourse-salient referential indefinites, then with other referentials, then spreads through various non-referential uses until it is finally used with all indefinite singular nouns, generics included, as in English. As Givón comments (1981:35), it is striking that the earliest stage is also found in all Creoles, which is suggestive evidence of the universality of the process.

[3] See Behagel (1925) for the development of the German indefinite article, which is quite similar. Differences appear only in secondary uses as in predicative positions or with abstract nouns.

3.1 The traditional grammarian view

Reflecting on this historical development of the indefinite article, traditional grammarians regard indefinite NPs, like *an old man (...)* in (6), as referential expressions, similar to definite NPs, proper names, and demonstratives: The indefinite NP refers to a physical (or fictional) object. Subsequent anaphorical expressions can denote the same object establishing anaphorical reference.

(6) *An old man with steel rimmed spectacles and very dusty clothes* sat by the side of the road. (...) *He* was too tired to go any further.[4]

In this referential view, indefinite NPs behave like expressions without scope: they are interpreted independently from other expressions, and they do not influence other scope sensitive expressions. Other uses of indefinites as listed under (5a(ii)-(iii)) are derived from the referential one. Anaphorical pronouns can be understood as expressions that 'stand for' or 'go proxy' for the antecedent and, therefore, refer to the same object.

3.2 The Fregean or classical view

Frege was too concerned with ontological and epistemological considerations to realize the grammatical nature of indefinites illustrated above. In 'Über Begriff und Gegenstand' (1892), he accounts for the difference between a 'concept' ('Begriff') and an instantiation of such a concept, i.e. an 'object' ('Gegenstand'). He then correlates both with the grammatical terms 'predicate' and 'argument':[5]

> Der Begriff - wie ich das Wort verstehe - ist prädikativ. Ein Gegenstandsname hingegen, ein Eigenname ist durchaus unfähig als grammatisches Prädikat gebraucht zu werden.

He concludes that the indefinite article marks a name for a predicate or for a concept, whereas the definite article indicates a name for an object:[6]

> Dies stimmt vollkomen mit dem von mir gegebenen Kennzeichen überein, wonach beim Singular der bestimmte Artikel immer auf einen Gegenstand hinweist, während der unbestimmte ein Begriffswort begleitet.

Frege's epistemological distinction between concept and object was codified in his representation of indefinites as existential quantifiers and definites as singular terms splitting the grammatical category of NP into two semantic categories. Russell later accommodated the

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4 Old Man at the Bridge, 1.

5 Frege ([1892] 1980, 67): "The concept - as I understand the word - is predicative. However, a name for an object is certainly unable to be used as a grammatical predicate."

6 Frege ([1892] 1980, 69): "This conforms with my description according to which the definite article in the singular always refers to an object, whereas the indefinite (one) accompanies a concept-word."

representation of definite NPs to that of indefinites, i.e. as quantifiers. Owing to Montague and others, this representation became the standard interpretation of definite and indefinite NPs in formal semantics.

Frege and Russell further noted that certain occurrences of indefinite NPs exhibit a dependency structure which is quite similar to the scope sensitive behavior of the existential quantifier in predicate logic. Their observations concern existential sentences like (7), negation like (8) and ambiguous sentences like (9). The indefinite NP in the existential sentence (7) can be quite appropriately represented by the quantifier in (7a). The representation is true if (7b) holds, i.e. if there is an object that is a pontoon. This holds also for the representation (8a) of the negated sentence (8). Here, the negation gets wider scope than the existential quantifier. The interpretation shows that there is no object corresponding to the indefinite NP in (8). And finally, the two intuitively available readings of (9) differ in the dependency of the indefinite NP on the universal expression. In predicate logic, the two readings are represented by a different order of the operators involved, which determines their interpretation.

(7) There was a pontoon bridge across the river.[7]

(7a) $\exists x$ [pontoon(x) & ...]

(7b) There is a *d* in the domain of individuals such that *d* a pontoon and ...

(8) There was no cast net.[8]

(8a) $\neg\exists x$ [cast_net(x)]

(8b) There is no *d* in the domain of individual such that *d* is a cast net.

(9) Every fisherman is subsidized by a state organization.

(9a) $\forall x$ [fisherman(x) $\rightarrow \exists y$ [state organization(y) & subsidize(y, x)]]

(9b) For all *d* in the domain: if *d* is a fisherman than there is an *e* in the domain such that *e* is a state organization and *e* subsidizes *d*.

(9c) $\exists y$ [state organization(y) & $\forall x$ [fisherman(x) $\rightarrow$ subsidize(y, x)]]

(9d) There is an *e* in the domain such that *e* is a state organization and for all *d* in the domain: if *d* is a fisherman than *e* subsidizes *d*.

Summarizing the classical view, indefinite NPs are represented as existential quantifiers that bind unstructured variables. They are interpreted according to their linear order, which expresses certain dependencies in the choice of the appropriate objects. In simple cases like (9) the configurational relations between the expressions coincide with their dependency structure. However, this linear representation limits the possibilities of representing natural language

[7] Hemingway: Old Man at the Bridge, 1.
[8] Hemingway: The Old Man and the Sea, 5.

expressions in at least three ways. First, indefinite NPs are interpreted according to the position of the corresponding existential quantifier. However, there are indefinite NPs that can get an interpretation independent of any other preceding operator, although they cannot be moved or raised for grammatical reasons (for a more detailed analysis of this point see von Stechow (this volume) and Winter (this volume)). Second, the scope of the classical existential quantifier is always limited to the sentence. But cross-sentence anaphora shows that this cannot be correct. This observation leads to the dynamic approaches discussed in the next section. Third, according to the linear order of quantifiers there is always one dependent quantifier. However, in section 4.4 and 5.4 we discuss cases where a different dependency structure seems to be better justified.

3.3 Discourse referents and the dynamic view

The classical view represents indefinite NPs as existential quantifiers that are scope sensitive in order to explain certain readings and ambiguities. Anaphorical pronouns are reconstructed by bound variables, which seems to be an adequate analysis up to the sentence level (cf Geach 1962). However, Evans (1977) among others has shown that this does not work properly across sentences. In (10a), the last occurrence of the variable *x* cannot be bound by the existential quantifier since it is outside of the quantifier's scope. The same holds for the conditional (11), where the last occurrence of the variable *y* in (11a) cannot be bound by the existential quantifier that is subordinated to the conditional. Another problem is that in the intuitive reading (11b) of sentence (11) the indefinite NP gets universal force.

(10) A fisherman walks. He whistles.

(10a) $\exists x$ [fisherman(x) & walk(x)] & whistle(x)

(11) If a fisherman catches a fish he sells it.

(11a) $\forall x$ [(fisherman (x) & $\exists y$ [fish (y) & catch(x, y]) $\rightarrow$ sell(x, y)]

(11b) $\forall x \forall y$ [(fisherman (x) & fish (y) & catch(x, y)) $\rightarrow$ sell(x, y)]

There are basically two ways out: Either one represents anaphorical pronouns as E-type pronouns, i.e. as complex terms like definite descriptions. Alternatively one can introduce a more flexible concept of 'semantic scope' that allows 'dynamic' binding beyond the syntactic scope of classical predicate logic. This goes back to Karttunen (1976), who introduced the concept of *discourse referent* which in the following years was developed by Kamp (1981) and Heim (1982) into fully fleshed out semantic frameworks, the so-called discourse representation theories (a cover term for both theories). Discourse referents are represented as variables that are associated with conditions or predicates representing the descriptive material of the indefinite NP and the assertion of the matrix sentence. A discourse representation structure is extended by

analyzing subsequent sentences. Variables can be bound by various operators, such as adverbs of quantification (see below), existential closure operations as in (10b) or conditionals as in (11c) The existential text-closure binds all free variables that are not yet bound by other operators. The conditional is represented as an unselectively binding universal operator as in (11c), yielding the classical and so called strong reading of a donkey sentence. The sentence is intuitively true if it holds for every fisherman that he sells each fish he has caught:

(10b) $\exists$ {x | fisherman(x) & walk(x) & whistle(x)}

(11c) $\forall$ {<x, y> | fisherman(x) & fish(y) & catch(x, y)} {<x, y> | sell(x,y)}

4. The classical epsilon calculus

In this section, I review the classical epsilon calculus, which goes back to Hilbert & Bernays (1939). In the next two sections, it is expanded by two additional features: First, we assume a family of context dependent choice functions instead of one undetermined choice function (cf Egli 1991). Secondly, we interpret epsilon terms in a dynamic semantics in order to reconstruct anaphorical links by the context change potential of indefinites (cf Peregrin & von Heusinger 1996).

Hilbert & Bernays replaced the existential and universal quantifiers by epsilon terms for metamathematical reasons. They state the following syntactic equivalencies, the so called Hilbert rules:

(12) $\exists x\, Fx \equiv F\, \varepsilon x\, Fx$ (13) $\forall x\, Fx \equiv F\, \varepsilon x\, \neg Fx$

According to these syntactical definitions for the epsilon operator, the following interpretation becomes most natural: An epsilon term $\varepsilon x\, Fx$ is interpreted in a model *M*, consisting of an individual domain *D*, an interpretation function *I*, and a choice function Φ, as an individual that is assigned to a set *F* by the choice function Φ. A choice function is defined as a function that assigns to each non-empty set one of its elements, and an arbitrarily chosen element to the empty set.

(14) $[\![\varepsilon x\, Fx]\!]^{M,g} = \Phi([\![F]\!]^{M,g})$, with Φ as a function given by M = <D, I, Φ>

(15) $\Phi(s) \in s$ if $s \neq \emptyset$ $\Phi(s) \in D$ if $s = \emptyset$

In this way, the universal and existential quantifiers can be replaced. In the following, we will concentrate on the representation of indefinite NPs. We argue that an epsilon term is more appropriate to mirror the nature of indefinite NPs than an existential quantifier. This was already noted by Hintikka (1976, 209f):

> There exists one particularly natural way of looking at quantifiers which has never been put to use entirely satisfactorily before. It is to consider quantifiers as *singular terms*. It is plain even to a linguistically naked eye that quantifier phrases like 'some man', 'every woman', 'a girl', and even phrases like 'some boy who loves every girl' behave in many respects in the same way as terms denoting or referring to particular individuals. In view of such obvious facts, it seems eminently desirable to try to treat quantifier phrases both syntactically and semantically in the same way as singular terms.

4.1 Grammatical and logical form

The use of epsilon terms for representing indefinite NPs goes back to the first Hilbert rule according to which each existential quantifier can be substituted by an epsilon term. The grammatical constituent *a lion that lived in Africa* in (16) gets its logical counterpart in the term *εx [lion(x) & live_in_Africa(x)]*. The same holds in (17), where the epsilon term *εy [negroni(y) & take(x, y)]* is the formal counterpart of *a Negroni*. It is built from the attributive and predicative material of the sentence. The epsilon terms correspond to paraphrases of the indefinite NPs like: *the Negroni that was eaten by him* etc. The paraphrase can be given without being related to the syntactic structure of the matrix sentence. In contrast, (17a) must be paraphrased as *there is a y such that it is a Negroni and it is eaten by a*.

(16) (Once upon a time) there was a lion that lived in Africa (with all the other lions. The other lions were all bad lions and every day they ate zebras and wildebeests and every kind of antelope. The good lion took only a Negroni)[9]

(16a) $\exists x$ [lion(x) & live_in_Africa(x)]

(16b) lion(a) & live_in_Africa(a) $\qquad a = \varepsilon x$ [lion(x) & live_in_Africa(x)]

(17) He took a Negroni.

(17a) $\exists y$ [negroni(y) & take(a, y)]

(17b) [negroni(b) & take(a, b)] $\qquad b = \varepsilon y$ [negroni(y) & take(a, y)]

Anaphorical pronouns can be analogously represented as epsilon terms that are constructed out of the material from the antecedent sentence (cf. Slater 1988). Indefinite NPs and their corresponding E-type pronouns are represented similarly, and the anaphorical relation is not reduced by binding but by a principle of 'coreference by equivalence of the referring terms'.

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[9] Hemingway: The Good Lion, 1.

(18) There was a lion that lived in Africa. He took a Negroni.

(18a) lion(a) & live_in_Africa(a) & [negroni(b) & take(a, b)

a = εx [lion(x) & live_in_Africa(x)]

b = εy [negroni(y) & take(a, y)]

Finally, dependencies, which are traditionally represented by scope interactions, can be reconstructed by embedding of parameters into epsilon terms. The traditional representation (19a) of the wide scope reading of (19) can be translated into the formula (19b). The complex epsilon term expresses the dependence of the zebra from the choice of the lion by the variable *x*, which is bound by the universal quantifier. In contrast to (19), in (20) there is only one zebra such that it was eaten by all lions. This is traditionally formalized by moving the existential phrase over the universal phrase. However, such movements introduce several problems, described by Winter (this volume). Therefore, the epsilon representation (20c) seems to be superior. The formal equivalent of the indefinite phrase is still *in situ*. The dependency structure is captured by the fact that there is no variable in the epsilon term that is bound from outside. The variable *z* in the term is locally bound by the universal quantifier which also stands inside the term. The paraphrase (20c) motivates this representation. (19d) and (20d) show a simplification of the epsilon formulas indicating the basic structure of formulas without existential quantifiers. The epsilon terms are replaced by a Skolem function or by an individual constant, respectively.

(19) Every lion ate a zebra.

(19a) ∀x [lion(x) → ∃y [zebra(y) & eat(x, y)]]

(19b) ∀x [lion(x) → eat(x, εy [zebra(y) & eat(x, y)]) & zebra(εy [zebra(y) & eat(x, y)])]

(19c) For every lion: he ate a zebra that he ate (and the zebra he ate is a zebra).

(19d) ∀x [zebra(x) → eat(x, f(x)) & zebra(f(x)]

(20) Every lion ate a zebra that was very fat and delicious.

(20a) ∃y [zebra(y) & ∀x [lion(x) → eat(x, y)]]

(20b) ∀x [lion(x) → eat(x, b) & zebra(b)]

b = εy [zebra(y) & ∀z [lion(z) → eat(z, y)]]

(20c) Every lion ate the zebra that was eaten by every lion (and the zebra that was eaten by every lion is a zebra).

(20d) ∀x [lion(x) → eat(x, c) & zebra(c)]

4.2 Generalized dependency structure

The representation of the dependency between linguistic expressions as dependencies among terms in the logical representation allows also an analysis for more complex cases that cannot be represented in a linear sequence of scope relations (cf. Hintikka 1976). Sentence (21) can be represented by the two equivalent PL formulas (22) and (23), which differ only in the order of the existential quantifiers. The two variables representing the two indefinite NPs do not exhibit any structure nor dependency. However, translated into epsilon terms they show the dependency of one term on the other. The order of substitution does not make any difference in building the epsilon terms from existential formulas. We start from inside to outside: (cf. Hilbert & Bernays 1939, 24f and Meyer Viol 1995, 172f):

(21) A lion ate a zebra.

(22) $\exists x\,[Lx \wedge \exists y\,[Zy \wedge Exy]]$

(22a) $a = \varepsilon x\,[Lx \wedge \exists y\,[Zy \wedge Exy]]$

(22b) $La \wedge \exists y\,[Zy \wedge Eay]$

(22c) $b = \varepsilon y\,[Zy \wedge Eay]$

(22d) $La \wedge Zb \wedge Eab$

(22e) $La \wedge Z\varepsilon y\,[Zy \wedge Eay] \wedge E(a, \varepsilon y\,[Zy \wedge Eay])$

(23) $\exists y\,[Zy \wedge \exists x\,[Zx \wedge Exy]]$

(23a) $c = \varepsilon y\,[Zy \wedge \exists x\,[Lx \wedge Exy]]$

(23b) $Zc \wedge \exists x\,[Lx \wedge Exc]$

(23c) $d = \varepsilon x\,[Lx \wedge Exc]$

(23d) $Zc \wedge Ld \wedge Edc$

(23e) $Zc \wedge L\varepsilon x\,[Lx \wedge Exc] \wedge E(\varepsilon x\,[Lx \wedge Exc], c)$

In the epsilon representation, there is a difference in dependency structure of the epsilon terms. In (22d) the epsilon term $\varepsilon y\,[Zy \wedge Eay]$ depends on the epsilon term $a = \varepsilon x\,[Lx \wedge \exists y\,[Zy \wedge Exy]]$, whereas in (23d) the dependency is reversed. Besides these two formulas with epsilon terms, additional formulas having the same logical consequences are possible, which do not have any translation into first order PL formulas. In (24a) both epsilon terms are independent expressions. The nearest approach to this dependency structure is Hintikka's representation with branching quantifiers as in (24b)

(24a) $L(\varepsilon x\,Lx) \wedge Z(\varepsilon y\,Zy) \wedge E(\varepsilon x\,Lx, \varepsilon y\,Zy)$

(24b) $\begin{matrix}\exists x \\ \exists y\end{matrix} > [Lx \wedge Zy \wedge E(x, y)]$

This shows that the epsilon calculus is a conservative extension of PL (cf Meyer Viol 1995), in which Hintikka's notorious examples (25) and his representation with branching quantifiers (25a) or Skolem function (25b) can get a first order representation (25c):

(25) Some relative of each villager and some relative of each townsman hate each other.

(25a) $\begin{matrix}\forall x \exists y \\ \forall z \exists u\end{matrix} > [(V(x) \wedge R(x, y) \wedge T(z) \wedge R(z, u)) \rightarrow H(y, u)]$

(25b) $\exists f\, \exists g\, \forall x\, \forall z\, [(Vx \wedge R(x, f(x)) \wedge Tz \wedge R(z, g(z))) \rightarrow H(f(x), g(z))]$

(25c) $\forall x\, \forall z\, [(Vx \wedge Tz) \rightarrow H(\varepsilon y\, [R(x, y)], \varepsilon u\, [R(z, u)])]$

5 Context and choice

Since Hilbert applied his epsilon terms only to the domain of numbers, a naturally ordered set, no determined choice function was necessary. However, in natural language the objects we refer to are not naturally ordered; rather, the order depends on a particular context. Thus, most attempts to introduce the epsilon into linguistic analysis have failed since they did not consider this context dependency. Only Egli (1991) solved this problem by assuming a family of choice functions for representing definite NPs. Each context *i* has its own choice function Φ_i, and the definite NP *the F* is represented as the indexed epsilon term $\varepsilon_i x\, Fx$, which can be paraphrased with *the selected x in the context i such that x is F* or *the most salient x in i such that x is F.* It is interpreted as the element that results from applying the choice function Φ_i to the set of all Fs. The contribution of the context to the interpretation of the definite NP consists in a salience hierarchy. It is assumed that each context can be associated with an ordering among the elements of subsets of the domain of discourse. This order reconstructs the intuitive idea of a salience hierarchy (cf Lewis 1979; Sgall et al. 1986; Grosz et al. 1995). The definite NP *the F* denotes the most salient F according to the situation *i*. This representation expands the ideas of discourse representation theories to give a more comprehensive picture: a definite NP is not only linked to an already introduced discourse referent; rather, it is the most salient discourse referent so far. Anaphorical NPs can be captured by this analysis without making use of coindexing (cf Peregrin & von Heusinger 1996).

Indefinite NPs are interpreted as epsilon terms, too. However, the choice function does not depend on the actual situation as is the case with definites. Indefinites rather introduce a choice function variable.[10]

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[10] This particular formulation of the behavior of indefinites in the epsilon approach is due to a suggestion of J. Peregrin.

(26) the F: $[\![\varepsilon_i x\ Fx]\!] = \Phi_i([\![F]\!])$ with *i* contextually determined

(27) an F: $[\![\varepsilon_i x\ Fx]\!] = \Phi_i([\![F]\!])$ with *i* free

This treatment is similar to that of discourse representation theories (Heim 1982; Kamp 1981), where indefinites introduce new individual variables or discourse referents, and definites introduce discourse referents that must be linked to already existing discourse referents. The advantages of using choice functions are the following: First, in discourse representation theories the discourse referents of definite NPs and anaphorical pronouns must be identified with an already existing discourse referent. This identification is pretheoretic to the analysis. In the epsilon approach, the denotation of a definite NP is determined by the actual choice function, which is the semantic reconstruction of the salience structure of a discourse. Second, indefinites need not be moved or raised for expressing different dependency behaviors. They rather remain *in situ*, whereas the choice function variable can be bound by other operators. This causes different readings of the indefinite, as it will be shown in the next subsection. Third, the assumption of free choice function variables squares with the theory of free indices of Farkas (1995). Fourth, this view clears the way for a dynamic semantics, in which the contextual change potential is expressed in updating choice functions (see Peregrin & von Heusinger 1996).

5.1 Logical form with indexed epsilon terms

The epsilon term $\varepsilon_i x\ lion(x)$ standing for *a lion* is interpreted as the operation of picking one element out of the set of lions. In absence of any operator we assume an existential closure over context indices or choice functions. Thus, the indefinite NP *a lion* refers to an arbitrarily chosen lion. So far, the formalism is similar to discourse representation theories except that the descriptive material is syntactically included in the term. The classical theory represents indefinite NPs as existential quantifiers in (28a). Discourse representation theories free the quantificational force from the representation of the indefinite. They are represented as free variables in (28b) that are associated with predicates (conditions) by the interpretation rules. The epsilon approach represents indefinite NPs in (28c) as epsilon terms that materialize the descriptive content already in the syntax. Furthermore, the syntax reflects the argument structure of a sentence in a quite natural way. Formula (28c) is not equivalent to (28a) or (28b) since it does not express the existence of lion or a zebra due to the definition of the epsilon in (12). However, it is possible to express the existential claim in (28d):

(28) A lion ate a zebra.

(28a) $\exists x\ \exists y[lion(x) \ \&\ zebra(y) \ \&\ eat(x, y)]$

(28b) $\exists\{<x, y> \mid lion(x) \ \&\ zebra(y) \ \&\ eat(x, y)\}$

(28c) $\exists$ eat(ε_ix lion(x), ε_ky zebra(y))

(28d) $\exists$ eat(ε_ix lion(x), ε_ky zebra(y)) & lion(ε_ix lion(x)) & zebra(ε_ky zebra(y))

E-type pronouns can be also analyzed as epsilon terms that are coindexed with their antecedents. Thus, the E-type pronoun reduces to a complex pronoun of laziness, i.e. a copy of its antecedent. The term does not need any more syntactic material than the head of the antecedent, since the determination of the choice function warrants that the same referent is chosen.

(29) Once upon a time, a lion lived in Africa. He took a Negroni.

(29a) live_in_Africa(ε_ix lion(x)) & take(ε_ix lion, ε_jy negroni(y))

Dependencies on other quantifiers are indicated in the scope relations between the indices (cf Farkas 1995 for an indexical theory of scope) or by the dependency of the epsilon term on another expression, as already illustrated in (19b) and (20b). The two possible readings of (30) are not represented in the classical way by the order of the quantifiers involved, as in (13a) and (32a). They rather get similar logical forms that differ only in the dependency structure encoded in the index *i* of the epsilon term ε_i*y zebra(y)* standing for the domain in which the choice is made. In (31b) the choice is made inside the domain of the universal quantification, i.e. the choice of the zebra depends on the instantiation of the lion already found. Therefore, each lion may eat a different zebra. Alternatively, the epsilon term is extended by the matrix predicate which includes the variable *x* that is bound by the universal quantifier. Therefore, the denotation of the epsilon term depends on the choice of the lion. In (32b) the choice of zebra is not influenced by the universal quantification resulting in one zebra for all lions.[11]

(30) Every lion ate a zebra.

(31a) $\forall$x [lion(x) $\rightarrow$ $\exists$y [zebra(y) & eat(x, y)]]

(31b) $\forall$x [lion(x) $\rightarrow$ $\exists$i eat(x, ε_iy zebra(y))]

(31c) $\exists$i $\forall$x [lion(x) $\rightarrow$ eat(x, ε_iy [zebra(y) & eat(x, y)])]

(32a) $\exists$y [zebra(y) & $\forall$x [lion(x) $\rightarrow$ eat(x, y)]]

(32b) $\exists$i $\forall$x [lion(x) $\rightarrow$ eat(x, ε_iy zebra(y))]

The advantages of the epsilon analysis are the following: First there is a uniform representation of definite and indefinite NPs as terms. The difference lies only in the anchoring of the index. Second, the representations (31b), (31c) and (32b) have the same structure. They differ only in

[11] Note that the formulas (31a) and (32a) are not equivalent with (31b), (31c) and (32b), respectively.

the dependency of the epsilon term on the choice of the lion. Since the epsilon term $\varepsilon_i y$ *zebra(y)* in (32b) is independent of any other parameter, it has a wide scope behavior. Third, island escaping readings of specific indefinites do not pose any problem. Kratzer and Reinhart (cf the discussion of Winter (this volume)) introduce choice functions exclusively for such readings. In the example (33) from Fodor & Sag (1982, 369) the indefinite NP *a student* can take scope over the highest clause despite its position in a scope island. Compare (33) with (34) where the phrase introduced by *each* cannot take scope over the clause despite the wide scope preferences for *each*. In the given epsilon approach, this description with choice functions for specific indefinites falls out without additional assumptions.

(33) John overheard the rumor that *a student* of mine had been called before the dean.

(34) John overheard the rumor that *each of my students* had been called before the dean.

5.2 Dependency and symmetry

The formalism allows us to encode dependencies between indefinite NPs that cannot be described in classical predicate logic, as illustrated by the logical forms (35a-c) of the sentence (35). (35a) is the representation corresponding to (28c) in the classical epsilon calculus, since it does not express any dependence of one term on the other. However, the dependency structure can be encoded in the context index. In (35b) the choice function that selects the zebra depends on the choice function that has already chosen the lion such that the zebra is the zebra that is eaten by the selected lion. One can say that the indefinite NP *a zebra* is specific in respect to the lion. (35c) reflects the opposite dependency structure:

(35) A lion ate a zebra.

(35a) $E(\varepsilon_i x\ Lx, \varepsilon_i y\ Zy)$

(35b) $E(\varepsilon_i x\ Lx, \varepsilon_{f(i)} y\ Zy)$

(35c) $E(\varepsilon_{f(i)} x\ Lx, \varepsilon_i y\ Zy)$

Although these representation do not show any truth conditional effects, combined with other operators, like conditionals and adverbs of quantification, they yield different truth conditions. I assume the analysis of conditionals as quantification over cases following Lewis (1975). Sentence (36) receives the representation (36a) where the unselectively binding operator $\forall$ binds all discourse referents. This is equivalent to the classical formula (36b). The epsilon representation (36c) with the universal quantifier over indices, i.e. choice function variable, is equivalent to (36a) and (36b) if there are hunters and zebras. All three formulas are instances of the so called ‘strong’ reading of a donkey sentence since they universally quantify over both variables.

(36) If a hunter sees a zebra, he chases it.

(36a) $\forall\{x, y \mid \text{hunter}(x) \,\&\, \text{zebra}(y) \,\&\, \text{see}(x, y)\}\ \{x, y \mid \text{chase}(x, y)\}$

(36b) $\forall x\ \forall y\ [(\text{hunter}(x) \,\&\, \text{zebra}(y) \,\&\, \text{see}(x, y) \rightarrow \text{chase}(x, y)]$

(36c) $\forall\ [S(\varepsilon_i x\ Hx, \varepsilon_k y\ Zy) \rightarrow C(\varepsilon_i x\ Hx, \varepsilon_k y\ Zy)]$

However, the analysis (36a) is too coarse-grained, as illustrated by (37). The unselective binder *MOST* binds all cases, i.e. both variables, yielding the logical form (37a). However, this representation does not reflect the intuitive truth conditions of (37), but gives rise to the well known 'proportion paradox' (Egli & Bäuerle 1985). The representation (37a) counter-intuitively becomes true in a situation where 99 farmers have one donkey each and they do not beat their unique donkey and where one farmer beats all of his 100 donkeys.

(37) If a farmer owns a donkey he usually beats it.

(37a) $\text{MOST}\ (\{<x, y> \mid \text{farmer}(x) \,\&\, \text{donkey}(y) \,\&\, \text{own}(x, y)\},\ \{ <x, y> \mid \text{beat}(x, y)\})$

It is intuitively obvious that in this case donkey-owning farmers must be counted rather than farmer-donkey pairs. To repair this, discourse representation theories assume a selective binder instead of an unselective binder as in (37b) or propose an additional existential closure rule hat binds the variable in (37c) before the unselectively binding operator can do so. However, both approaches have their problems: the selectively binding approach must stipulate that the operator chooses which variable can be bound. Furthermore, the other free variable must be bound by another operator.

(37b) $\text{MOST}_x\ (\{<x, y> \mid \text{farmer}(x) \,\&\, [\text{donkey}(y) \,\&\, \text{own}(x, y)]\},\ \{ <x, y> \mid \text{beat}(x, y)\})$

(37c) $\text{MOST}\{x \mid \text{farmer}(x) \,\&\, \exists y\ \text{donkey}(y) \,\&\, \text{own}(x, y)\}\ \{x \mid \exists y\ \text{donkey}(y) \,\&\, \text{own}(x, y) \,\&\, \text{beat}(x, y)\}$

(37d) Most donkey-owning farmers beat a donkey they own.

In the epsilon analysis, we assume that we first make a choice of a farmer and then we make the choice of the donkey. However, the second choice is no longer independent from the first one. The first choice constrains the set of possible candidates for the second choice. We can encode this in the logical form either by an extension of the indefinite NP, as in (38), or as a constraint on the possible choice function. The latter can be accounted for by a Skolem function *f* over indices, as in (38a). This Skolem function ties the choice of a donkey to the choice of the farmer and it expresses a quasi uniqueness condition on the donkey. The result of this is that we must only consider choice functions that vary in the assignments for farmers. If we apply an unselectively binding operator as *MOST* in (39), it can only bind the one free index yielding the intuitively correct asymmetric reading in (39a):

(38) $\exists$i own(ε_ix farmer(x), ε_ky [donkey(y) & owns(ε_ix farmer(x), y)])

(38a) $\exists$i own(ε_ix farmer(x), $\varepsilon_{f(i)}$y donkey(y))

with *f*: a function from a choice function *i* into choice a function *f(i) = i'* such that the choice function *i'* selects a donkey that is owned by the farmer who is selected by the choice *i*.

(39) If a farmer owns a donkey he usually beats it.

(39a) MOST (own(ε_ix farmer(x), $\varepsilon_{f(i)}$y donkey(y)) (beat(ε_ix farmer(x), $\varepsilon_{f(i)}$y donkey(y))

6. The salience change potential of indefinite NPs

Definite NPs refer to objects that are picked out by the actual choice function reconstructing the contextual property of a salience hierarchy. Indefinite NPs do not depend on this salience hierarchy, but rather introduce a new variable for choice functions. This variable can be bound by operators like the conditional, adverbs of quantification or by the existential closure. As soon as the choice function is bound, it can select an object. This object then becomes the most salient element of the given set such that a subsequent definite expression can refer to it. Therefore, an indefinite NP can be seen as being an update function for the current choice function. The indefinite NP *an F* updates the current choice functions for the set of F. Given that *a* stands for the actual choice function, we then can analyze sentence (40) as (40a): The definite NP *the farmer* denotes the object that is chosen by the actual choice function Φ_a, whereas the indefinite NP *a donkey* refers to a donkey that was chosen be a new choice function Φ_i . The donkey that was assigned by this choice function to the set of donkeys becomes the most salient donkey. This is informally encoded in the updated index *a** which stands for the particular updated choice function Φ_{a*} that differs from Φ_a at most in the assignment of a certain donkey to the set of donkey. Therefore, the subsequent definite NP *the donkey* can refer to the now most salient donkey by applying Φ_{a*} to the set of donkeys.

(40) The farmer owns a donkey. The farmer feeds the donkey.

(40a) own(ε_ax farmer(x), ε_iy donkey(y)) & feed(ε_{a*}x farmer(x), ε_{a*}y donkey(y))

Anaphorical pronouns can be analyzed by extending the salience change potential of indefinite NPs. An indefinite NP not only puts the referred object at the top of the named set but also at the top of certain supersets. In (22) the indefinite NP *a farmer* refers to an arbitrarily chosen farmer who becomes the first chosen farmer and the first chosen male object etc. In this way, anaphorical pronouns can be regarded as a special case of impoverished anaphorical definite NPs.

(41) The farmer owns a donkey. He feeds it.

(41a) own(ε_ax farmer(x), ε_iy donkey(y)) & feed(ε_{a*}x farmer(x), ε_{a*}y donkey(y))

7. Summary

Indefinite NPs exhibit a more complex nature than usually assumed. This complex structure should be reflected in their semantic representation in the logical form. It was argued that

(i) the prototypical use of indefinites with the indefinite article are referential indefinites.

(ii) the representation of indefinites with quantifiers is based on a misconception of the function of the article by Frege.

(iii) the representation of indefinite NPs with epsilon terms reflects their complex nature much better than does the representation with variable or discourse referents

(iv) epsilon terms can express more and finer dependency structures than PL formulas can do.

(v) indefinites can depend on other indefinites, causing the so called asymmetric readings in donkey sentences.

References

Behagel, Otto 1923-1932. Deutsche Syntax. 4 vol. Heidelberg: Carl Winter.

Chesterman, Andrew 1991. On Definiteness. A Study with Special Reference to English and Finnish. Cambridge: Cambridge Univ.

Christophersen, Paul 1939. The Articles. A Study of Their Theory and Use in English. Copenhagen: Munksgaard.

Du Bois, John 1980. Beyond Definiteness: The Thrace of Identity in Discourse. In: W. Chafe (ed.). The Pear Stories. Cognitive, Cultural and Linguistic Aspects of Narrative Production. Norwood, N.J.: ABLEX, 203-274. (Advances in discourse presses 3)

Egli, Urs 1991. (In)definite Nominalphrase und Typentheorie. In: U. Egli & K. von Heusinger (eds.). Zwei Aufsätze zur definiten Kennzeichnung. Arbeitspapier 27. Fachgruppe Sprachwissenschaft Universität Konstanz.

Egli, Urs & von Heusinger, Klaus 1995. The Epsilon Operator and E-Type Pronouns. In: U. Egli et al. (eds.). Lexical Knowledge in the Organization of Language. Amsterdam: Benjamins, 121-141.

Evans, Gareth 1977. Pronouns, Quantifiers and Relative Clauses (I). Canadian Journal of Philosophy 7, 467-536.

Farkas, Donka 1995. Evaluation Indices and Scope. Linguistic Research Center. UC Santa Cruz. LRC-95-05

Fodor, Janet & Sag, Ivan 1982. Referential and Quantificational Indefinites. Linguistics and Philosophy 5, 355-398.

Frege, Gottlob [1892] 1980. Über Begriff und Gegenstand. In: Funktion, Begriff, Bedeutung. Fünf logische Studien. Hrsg. u. eingeleitet v. Günther Patzig. 5. Aufl. Göttingen: Vandenhoek & Ruprecht, 66-80. [Zuerst erschienen: Vjschr. f. wissensch. Philosphie 16, 1892, 192-205]

Geach, Peter 1962. Reference and Generality. An Examination of Some Medieval and Modern Theories. Ithaca/N.Y.: Cornell Univ. Pr.

Givón, Talmy 1981. On the Development of the Numeral 'one' as an Indefinite Marker. Fokia Linguistica Historia 2, 35-53.

Grosz, Barbara & Joshi, Aravind & Weinstein, Scott 1995. Centering: A Framework for Modeling the Local Coherence of Discourse. Computational Linguistics 21, 203-225.

Heim, Irene 1982. The Semantics of Definite and Indefinite Noun Phrases. Ph.D. Dissertation. University of Massachusetts, Amherst. Ann Arbor: University Microfilms.

von Heusinger, Klaus 1996. Definite Descriptions and Choice Functions. In: S. Akama (ed.). Logic, Language and Computation. Oxford: Oxford University Press, 61-91.

Hilbert, David & Bernays, Paul [1939] 1970. Grundlagen der Mathematik. Vol. II. 2. ed. Berlin; Heidelberg; New York: Springer.

Hintikka, Jaakko 1976. Quantifiers in Logic and Quantifiers in Natural Languages. In: S. Körner (ed.). Philosophy of Logic. Oxford: Blackwell, 208-32.

Jespersen, Otto [1925] 1963. The Philosophy of Grammar. London: Allen & Unwin.

Kamp, Hans [1981] 1984. A Theory of Truth and Semantic Interpretation. In: J. Groenendijk & T. M. V. Janssen & M. Stokhof (eds.). Truth, Interpretation and Information. Dordrecht: Foris, 1-41.

Karttunen, Lauri 1976. Discourse Referents. In: J. McCawley (ed.). Syntax and Semantics 7: Notes from the Linguistic Underground. New York: Academic Press, 363-385.

Lewis, David 1975. Adverbs of Quantification. In: E. L. Keenan (ed.). Formal Semantics of Natural Language. Cambridge: CUP, 3-15.

Lewis, David 1979. Scorekeeping in a Language Game. In: R. Bäuerle & U. Egli & A. von Stechow (eds.). Semantics from Different Points of View. Berlin; Heidelberg; New York: Springer, 172-187.

Meyer Viol, Wilfried 1995. Instantial Logic. An Investigation into Reasoning with Instances. Amsterdam: ILLC Dissertation Series 1995-11.

Peregrin, Jaroslav & von Heusinger, Klaus 1996. Dynamic Semantics with Choice Functions. In: H. Kamp & B. Partee (eds.). Proceedings of the Workshop 'Context Dependence in the Analysis of Linguistic Meaning'. Prague and Stuttgart 1995.

Sgall, Petr & Hajicová, Eva & Panevová, Jarmila 1986. Meaning of the Sentence in its Semantic and Pragmatic Aspects. Hrsg. v. J. Mey. Dordrecht: Reidel.

Slater, B. H. 1988. Prolegomena to Formal Logic. Aldershot/England: Avebury.