Sovereign and Private Default Risks over the Business Cycle

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Sovereign and Private Default Risks over the Business Cycle*

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Abstract

Sovereign debt crises are often accompanied by deep recessions with sharp declines in external credit to the private sector. In a sample of emerging economies we find that both, sovereign and private risk premia are counter-cyclical. This paper presents a model of a small open economy that accounts for these empirical regularities. It includes private firms which finance a fraction of imports by external debt and are subject to idiosyncratic and aggregate productivity risk, and a government which borrows internationally and taxes firms to finance public expenditures. The model gives rise to endogenous private and sovereign credit spreads and a dynamic feedback mechanism between sovereign and private default risks through the endogenous response of fiscal policy to adverse productivity shocks.

JEL classification: E32, E62, F34

Keywords: Sovereign default; Corporate borrowing; Risk premia

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1 Introduction

Sovereign default is a recurrent phenomena in emerging economies. Default episodes are typically associated with severe recessions, characterized by sharp drops in output, investment, and imports. Recent empirical studies further suggest that sovereign default is followed by worsening conditions for external finance for the private sector. Arteta and Hale (2008) show that foreign credit to non-financial firms contracts sharply in the aftermath of sovereign debt crises. A˘ gca and Celasun (2012) find that higher external sovereign debt in emerging markets translates into higher borrowing costs for the private sector, and much more so in countries that have experienced sovereign default episodes in the past. What is the explanation for these links between sovereign debt and the private sector’s access to foreign credit? And how do these relations affect macroeconomic conditions around default episodes?

To address these questions, in this paper we investigate the dynamic interrelations between sovereign and private credit risks in emerging economies and their effects on macroeconomic outcomes. We first document several business-cycle facts about risk premia on sovereign and private external debt for a sample of emerging and developed economies, and we consider the behavior of these risk premia during Argentina’s sovereign debt crisis in 2002. In line with earlier literature, the default crisis was accompanied by a strong decline in imports and by an increase of sovereign and private risk premia. But while the sovereign premium rose persistently, the increase of the private premium was more transitory. Over the business cycle, private and sovereign premia are countercyclical, which is a robust phenomenon that is independent of the inclusion of default episodes.

We then build a dynamic, stochastic model of a small open economy to account for these observations. Our modeling approach follows the recent literature on sovereign debt in incomplete markets economies with an endogenous default choice of the government (e.g. Aguiar and Gopinath (2006), Arellano (2008)). The model has domestic households, domestic firms producing final goods and intermediate goods, a domestic government and foreign lenders. Final goods firms produce the output with imperfectly substitutable domestic and foreign intermediate goods. A fraction of imported intermediate goods must be financed by external credit. Since firms face idiosyncratic and aggregate productivity shocks, their credit is subject to default risk, so that risk-neutral international investors charge a risk premium on private debt. Households value private consumption, leisure and a public good provided

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1See also Das et al. (2010) who obtain similar results for a broader data set.
by the government. The government acts in the interest of domestic households, it
levies a linear sales tax and borrows internationally so as to smoothen the provision
of the public good and to balance fluctuations in tax revenues over the business cycle.
In any period, the government has the option to default on the outstanding debt,
which gives rise to an endogenous risk premium on sovereign debt. In the event of
default, the government is temporarily excluded from international financial markets
in which case it must finance public expenditures solely from taxes.

In a quantitative application to Argentina we show that our theoretical frame-
work mimics the empirical facts of a typical emerging market economy, in particular
countercyclical sovereign and private spreads, volatile imports and deep recessions
in default. Moreover, fiscal policy is procyclical, in line with the empirical evidence
(e.g. Talvi and Vegh (2005), Ilzetzki and Vegh (2008)), and the endogenous policy
response is critical for the cyclical dynamics of private spreads in our model.

Similar to Cuadra et al. (2010), the procyclicality of fiscal policy is a direct con-
sequence of countercyclical sovereign default risk. If the economy enters a recession,
external public debt becomes more expensive due to the higher default risk, and this
induces the government to raise taxes so as to finance public expenditures. This
procyclicality of fiscal policy is crucial for countercyclical private default risk and for
macroeconomic amplification in our model: Higher taxes in recessions depress firms’
profitability even further and induce more firms to default on their external debt.
In turn, higher private spreads reduce import demand which deepens the recession
and thus amplifies sovereign default risk. In case of a default, by not repaying debt
the government is able to reduce taxes which stimulates profitability and reduces the
private risk premium. Consequently, in line with the data, the increase of the private
premium is of a transitory nature.

Our simulation results suggest that the dynamic feedback mechanism between
sovereign and private credit risks is quantitatively important: Relative to a bench-
mark model without endogenous private default risk, the downturns in output and
imports during a default event are amplified by about 2.5 and 13.5 percentage points,
respectively.

Closely related to our framework, Mendoza and Yue (2012) consider a model in
which firms produce final output from domestically produced and imported inter-
mediate goods. A share of the imported intermediate goods is financed by external
debt.\(^2\) Different from ours, however, they assume that firms are always able to bor-

\(^2\)The broader literature on sovereign debt in quantitative macroeconomic models considers polit-
row at the risk-free rate, which is at odds with the evidence.\footnote{In an earlier working paper version (NBER Working Paper No. 17151), Mendoza and Yue (2012) assume that private firms borrow at the same rate as the government and default simultaneously which is also counterfactual. Pancrazi et al. (2015) consider a dynamic general equilibrium model with sovereign default and private credit granted by domestic banks that borrow internationally. As Mendoza and Yue (NBER Working Paper), they assume that banks borrow at the same rate as the government from international creditors.}

Furthermore, all firms and the government are simultaneously excluded from international financial markets if the government chooses to default. This in turn causes firms to substitute away from imports towards domestically produced goods, generating output costs that are required to account for countercyclical sovereign spreads. Our model also has a contraction in imports and therefore an endogenous output cost of sovereign default, but this factual reaction does not require to shut all firms out of financial markets together with the defaulting government. Moreover, our model has a direct impact of sovereign spreads on private spreads via the endogenous reaction of fiscal policy.

We are abstracting from the role of domestically held debt, often on the balance sheet of banks, that is also discussed as a potential source of amplification. For instance, in theoretical contributions based on finite–horizon economies, Brutti (2011) and Gennaioli et al. (2014) argue that sovereign default harms the balance sheets of domestic banks or private investors, which triggers contractions in credit and investment. Engler and Große-Steffen (2016), Niemann and Pichler (2016), and Sosa-Padilla (2012) build quantitative stochastic general equilibrium models with a similar feature. While this channel is presumably important in countries where a large share of government debt is held domestically, it may be less relevant for most emerging markets. Further, Arteta and Hale (2008) show that the decline in external credit during sovereign debt crises is concentrated in the non-financial sector, which motivates why we abstract from financial intermediaries in our model.

Besides the banking channel, other contributions also consider spill-over effects of sovereign default on external credit of firms. Andreasen (2015) suggests a signaling mechanism, based on the idea that the government’s repayment decision provides new information regarding the institutional quality (such as recovery rates) which
affects the financial conditions of private firms. Sandleris (2014) argues that sovereign default can trigger a collapse of private credit even when no debt is held domestically. In his model, a sovereign default reduces the firms’ collateral value, which limits their borrowing capacity. Arellano and Kocherlakota (2014) propose a reverse mechanism: due to informational frictions in private credit markets, private default crises can emerge as a coordination equilibrium which possibly triggers a sovereign debt crisis.

The remainder of this paper is organized as follows. In the next section, we document empirical evidence about sovereign and private credit risk in emerging market and developed economies. In Section 3, we describe the model framework, define the recursive equilibrium and explain the main determinants of sovereign and private default risks. In Section 4 we calibrate the model to the Argentine economy in order to illustrate the quantitative significance of the interplay between sovereign and private credit risks that our model generates. Section 5 concludes.

2 Empirical Facts

2.1 Cyclical Properties

We document empirical regularities of private and sovereign default risks considering emerging market economies as well as developed economies. Our sample of emerging market economies consists of Argentina, Brazil, Chile, Ecuador, Korea, Malaysia, Mexico, Peru, Philippines, Russia and Venezuela, while our sample of developed small open economies includes Australia, Canada, Netherlands, New Zealand, Sweden and Switzerland. Our sample covers the period from the early 1970s (early 1990s) until the second quarter of 2013 for the developed (emerging markets) economies. Due to data availability the sample periods for the individual countries differ in their starting and end dates. More details on calculations, data sources and further statistics can be found in Appendix B and Appendix C.

We follow Arellano and Kocherlakota (2014) and calculate private risk premia as the spread between the dollar domestic lending rate and the interest rate on a US bond with similar maturity. If foreign currency lending rates are not available, we use the spread between the local currency domestic lending rate and the local currency

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4 The choice of countries is based on the sample analyzed by Neumeyer and Perri (2005), but we add emerging market economies for which we have at least ten years of data. Moreover, we add Switzerland to the sample of developed economies.
<table>
<thead>
<tr>
<th>Country</th>
<th>$E(s)$</th>
<th>$E(s^p)$</th>
<th>$\sigma(s)$</th>
<th>$\sigma(s^p)$</th>
<th>$\rho(s, s^p)$</th>
<th>$\rho(s, y)$</th>
<th>$\rho(s^p, y)$</th>
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<td>4.81</td>
<td>0.87***</td>
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<td>-0.81***</td>
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<td>9.39</td>
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<td>1.96</td>
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<td>0.88</td>
<td>0.63**</td>
<td>-0.40***</td>
<td>-0.34***</td>
</tr>
<tr>
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<td>1.84</td>
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<td>-0.18</td>
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<td>1.32</td>
<td>1.49</td>
<td>0.16</td>
<td>-0.21</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

Table 1: Sovereign and Private Interest Spreads over the Business Cycle

Notes: $s$ denotes the annualized sovereign interest spread while $s^p$ refers to the annualized private interest spread. $y$ denotes real GDP. Risk premia are demeaned and GDP is log-linearly detrended before business-cycle statistics are calculated. Argentina, Ecuador, Russia and Venezuela had default episodes during the time period that we consider. To calculate business-cycle statistics we exclude the default events and consider the following restricted samples: Argentina 1994Q1-2001Q4, Ecuador 2000Q4-2008Q3, Russia 2000Q4-2012Q3, Venezuela 1999Q1 - 2004Q4. Significance is denoted by stars (** 10%, *** 5%, **** 1%).
domestic deposit rate. Sovereign risk premia are obtained from the Emerging Market Bond Index (EMBI). We use the risk premium calculated by JP Morgan instead of the difference between the EMBI yield to maturity and a US bond, because JP Morgan’s risk premium is adjusted for different payment streams.

In Table 1 we summarize the business-cycle properties of sovereign and private interest spreads for our samples of emerging market and developed small open economies, excluding default events in order to provide meaningful comparisons. The business-cycle statistics reveal several empirical regularities. First, the sovereign risk premium tends to be lower and less volatile than its private counterpart, both in emerging markets and developed economies. This result is in line with the hypothesis that the sovereign rating provides a ceiling to private company ratings. Second, private and public risk premia in emerging market economies are higher than in developed economies. Third, the sovereign interest spread is strongly countercyclical in emerging markets while the cyclical behavior is less pronounced in developed economies. Private interest rate spreads tend to be countercyclical, too, but to a lesser extent compared to the sovereign spreads.

2.2 Dynamics Around Default

While our analysis of the cyclical properties of sovereign and private risk premia explicitly abstracts from default events, we now focus on the dynamics of key macroeconomic variables during the sovereign default episode of Argentina (2002Q1). We consider GDP, imports and the sovereign and private interest spread. In Figure 1 we plot the patterns of the variables twelve quarters before and after the default event in quarter $t = 0$. GDP and imports are shown as percentage deviations from a linear trend while the premia are depicted in percent.

First, we observe a deep recession with a substantial drop in imports around the default event. Relatedly, Gopinath and Neiman (2014) show that the recession in

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5For details see Table 6 in Appendix C.
6The difference between EMBI spreads and the difference between the EMBI yield to maturity and a five-year US bond is negligible.
7See also Borensztein et al. (2013) who provide further evidence on the sovereign ceiling.
8Ultimately, for the quantitative evaluation of our model, we are interested in imports of intermediate goods, but since data on them is available only on an annual basis, we use overall imports as a proxy. Following Mendoza and Yue (2012) in the definition of intermediate goods, we calculate the share of intermediate goods in imports. On average these imports account for around 58% of total imports in emerging markets and around 48% in developed economies.
GDP Imports

Sovereign Premium Private Premium

Figure 1: Default Dynamics

Notes: The figure shows the dynamic patterns of GDP, imports, the annualized sovereign premium and the annualized private premium 12 quarters before and after Argentina’s default in 2002. GDP and imports are log-linearly detrended, using data until the default quarter. The sovereign premium is the EMBI Global spread, and the private premium is calculated as the difference between short-term bank credit interest rate in US$ and the 3-month US T-Bill.
the course of Argentina’s default was accompanied by a substitution of imported intermediate goods by domestic intermediate goods. Furthermore, we observe a strong and persistent increase of the sovereign risk premium during the default episode. Private credit costs increase as well, but the rise in the private spread is less pronounced and more transitory. These findings are in line with Ağca and Celasun (2012) who show that private credit costs increase during sovereign debt crises.

3 A Model of Sovereign and Private Default Risk

We describe a stochastic dynamic general equilibrium model of a small open economy. The economy is hit by aggregate productivity shocks and has many firms who are subject to idiosyncratic productivity shocks. These firms borrow internationally and decide to default if productivity is sufficiently low, which generates an endogenous premium on private external debt. The government borrows abroad to smoothen the provision of a public good, and it also has the option to voluntarily default on its debt, which gives rise to a risk premium on public debt.

Our small open economy comprises four types of agents: a representative household, final goods firms, intermediate goods firms and the domestic government. Foreign investors lend to the government and to private firms.

The domestic household owns firms and supplies labor. All firms are perfectly competitive. Final goods firms produce output from two differentiated intermediate goods. One of them is an import good, the other is produced domestically by intermediate goods firms employing labor. A fraction of imported intermediate goods is financed by external private debt. After making import and borrowing decisions, final good firms are hit by idiosyncratic productivity shocks. Instead of redeeming their debt, firms can opt to default if the continuation profit is negative.

The representative household enjoys utility from consumption, leisure and from a public good. The government provides the public good, taxes sales and finances deficits by issuing external debt. Following Arellano (2008), if the government chooses to default, it is excluded from international borrowing for a stochastic number of periods. In addition, during exclusion, the country suffers an output loss which is exogenous to the model.

The timing within each period is as follows. First, aggregate productivity is

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9 For simplicity, a sales tax is the only tax instrument of the government. Other distortionary taxes should have similar implications, as long as they reduce the profitability of firms.
realized, the government decides whether to default on its external debt and it adjusts current policies, i.e. the sales tax, expenditures for the public good and debt issuance. Final goods firms make import decisions and borrow abroad. Intermediate goods firms hire workers and produce. Second, final goods firms are hit by idiosyncratic productivity shocks and decide whether to default or to repay their outstanding debt obligations. Active (non-defaulting) firms buy domestic intermediate goods and produce output.

3.1 Households

The representative household has preferences

$$E_0 \sum_{t \geq 0} \beta^t u(c_t - v(\ell_t), g_t),$$

where $c_t$ and $g_t$ are consumption of the private and the public good, and $\ell_t$ is labor supply. $0 < \beta < 1$ denotes the discount factor, $u$ is strictly increasing and concave and $v$ is strictly increasing and convex. The household does not borrow or lend internationally and thus consumes all labor and profit income. Therefore the household’s budget constraint is $c_t = \Pi_t + w_t \ell_t$, where $\Pi_t$ are aggregate profits and $w_t$ is the real wage. As implied by this notation, the gross price of the consumption good is normalized to unity.

3.2 Firms

Intermediate goods firms produce the domestic intermediate good from labor with linear technology $m_t = \ell_t$. Since firms operate under perfect competition, the price of the domestic intermediate good equals the real wage $w_t$.

Let $s_t \in \{N, D\}$ denote whether the country is in a normal state ($N$) or in a sovereign default episode ($D$). Final goods firms produce output from domestic and foreign intermediate goods, $m_t$ and $m^*_t$ with technology $x_t z_t f(m_t, m^*_t)$ for $s_t = N$. $x_t$ and $z_t$ are idiosyncratic and aggregate productivity, respectively, and $f$ has constant returns to scale and is concave. While $z_t$ follows a Markov process and is known at the beginning of the period, idiosyncratic productivity is realized after firms make import decisions and borrow. Then $x_t$ is drawn i.i.d. from cumulative distribution function $X(\cdot)$. With $\tau_t$ denoting the sales tax, we write the firm’s net revenue as $x_t \tilde{z}_t f(m_t, m^*_t)$ with $\tilde{z}_t \equiv (1 - \tau_t) z_t$. If the country is in a sovereign default episode, $s_t = D$, the country faces exogenous output costs such that output
is \( x_t h(z_t) f(m_t, m^*_t) \leq x_t \hat{z}_t f(m_t, m^*_t) \) with function \( h(.) \leq 1 \), and net revenue is \( x_t \hat{z}_t f(m_t, m^*_t) \) with \( \hat{z}_t \equiv (1 - \tau_t)h(z_t) \).

Imported intermediate goods are bought at the world price \( p^*_t \). We assume that firms must finance the fraction \( \xi \) of imports by external debt and the remaining fraction by domestic funds. International credit markets are incomplete, so that external private debt has gross interest rate \( R_t \) which is unconditional on idiosyncratic productivity realizations and which reflects the firms’ default risk. If a firm imports \( m^*_t \), its external debt is \( R_t \xi p^*_t m^*_t \). After idiosyncratic productivity \( x_t \) is realized, the firm has two options.

Either it stays in business and repays the international debt. Alternatively, the firm opts to default if the profit value \( x_t \hat{z}_t f(m_t, m^*_t) - w_t m_t - R_t \xi p^*_t m^*_t \) is negative. A continuing firm buys domestic intermediate goods proportional to imports, \( m_t = \Phi(\frac{x_t \hat{z}_t}{w_t}) m^*_t \), where \( \Phi \) is an increasing function. Its profit before interest payments is also linear in \( m^*_t \), namely \( \pi(x_t \hat{z}_t, w_t) m^*_t \), where \( \pi \) is increasing (decreasing) in the first (second) argument. Then, the firm defaults if \( x < \overline{x}_t \) with default threshold defined by

\[
\pi(\overline{x}_t \hat{z}_t, w_t) = R_t \xi p^*_t .
\]

Evidently, \( \overline{x}_t \) decreases in \( \hat{z}_t \) and increases in \( (w_t, R_t, p^*_t) \), but it is independent of the amount of imports. In general equilibrium, however, there is an indirect effect of the import volume on default risk via domestic intermediate goods and labor markets.

At the beginning of the period, final goods firms choose imports \( m^*_t \) to maximize the expected profit value

\[
\int_{\overline{x}_t}^{\infty} [\pi(x \hat{z}_t, w_t) - R_t \xi p^*_t] m^*_t \, dX(x) - (1 - \xi) p^*_t m^*_t .
\]

Because this objective is linear in \( m^*_t \), the first-order condition implies zero expected profits,

\[
(1 - \xi) p^*_t = \int_{\overline{x}_t}^{\infty} [\pi(x \hat{z}_t, w_t) - R_t \xi p^*_t] \, dX(x) .
\]

While some firms default, new firms may enter the economy in any period. Due to the constant-returns specification, the number of firms is irrelevant. Without loss of generality, we set the mass of firms to one and interpret \( m^*_t \) as either aggregate or firm-level imports.

\[\text{To keep the model simple, we do not distinguish between domestic equity or debt. Depending on the interpretation, default entails a loss to shareholders (or domestic creditors) equal to } (1 - \xi) p^*_t m^*_t.\]
3.3 International Investors

Risk-neutral international investors have access to an international bond market with constant gross interest rate $\bar{R}$. They lend to domestic firms if the expected gross return of credit equates the safe return. We assume that in the event of a private default, lenders are able to recover a fraction $\eta$ of the value of credit-financed import goods, where parameter $\eta$ reflects the institutional features of the country, such as the quality of legal enforcement.

The investors’ arbitrage condition therefore states that

$$\bar{R} = R_t \left[ 1 - X(\bar{x}_t) \right] + \eta X(\bar{x}_t) ,$$

where $X(\bar{x}_t)$ is the default probability of final goods firms.

3.4 Private Sector Equilibrium

Note that labor supply is $(v')^{-1}(w_t)$ and that labor demand is equal to the demand for domestic intermediate goods. The labor market clears in period $t$ if

$$\int_{x_t}^{\infty} \Phi \left( \frac{z_t x}{w_t} \right) m_t^* dX(x) = (v')^{-1}(w_t) = \ell_t .$$

Households consume all their income, and since aggregate profit income is zero,

$$c_t = w_t \ell_t .$$

Given current aggregate productivity $z_t$, the sales tax $\tau_t$, and the default state $s_t$, the private-sector equilibrium $(w_t, x_t, \ell_t, m_t^*, c_t)$ solves the six equations\textsuperscript{11} (1)–(5). We write $c_t = C(z_t, \tau_t, s_t)$ and $\ell_t = L(z_t, \tau_t, s_t)$ for equilibrium consumption and employment, and we assume that a solution of the private-sector equilibrium exists for the range of admissible values for $(z_t, \tau_t, s_t)$.\textsuperscript{12} We further write aggregate output as $y_t = Y(z_t, \tau_t, s_t)$, which is

$$y_t = \int_{x_t}^{\infty} [z_t + I_{s_t=D}(h(z_t) - z_t)] x f \left( \Phi \left( \frac{z_t x}{w_t} \right), 1 \right) m_t^* dX(x) ,$$

\textsuperscript{11}Equation (1) presupposes default in equilibrium. There can also be a boundary solution where $x$ is at the infimum of the support of $X(.)$ and (1) holds with inequality.

\textsuperscript{12}We also make sure that the equilibrium interest rate is the stable solution of equation (3); namely, deviations to a lower rate may not raise investors’ expected return. This requirement is meaningful because (3) typically has two solutions, the larger of which is unstable.
and we denote the private sector interest rate by \( R_t = \mathcal{R}(z_t, \tau_t, s_t) \).

Because of our assumption that private credit is repaid at the end of the period, the private sector equilibrium does not depend on any endogenous state variables, such as the firms’ net worth, which greatly simplifies the model. Including such state variables would complicate the solution of the model considerably, as it would involve intertemporal decisions of firms that have to forecast future tax policies of the government.

### 3.5 The Government

The government maximizes expected utility of the representative household without commitment over future policy choices. At the beginning of period \( t \), it decides whether to default on its external debt obligation. If it does so, it is excluded from international borrowing in the default period. In any future period, it regains access to international credit with exogenous probability \( \theta \). In a period of market exclusion, the government finances expenditures for the public good by the sales tax revenues, \( g_t = \tau_t y_t \). If the government can borrow internationally, it issues new debt \( b_{t+1} < 0 \) at price \( q(z_t, b_{t+1}) \), facing the flow budget constraint \( g_t = \tau_t y_t + b_t - q(z_t, b_{t+1})b_{t+1} \).

The price of debt reflects the default-risk adjusted rate of return of foreign lenders. The government takes the private sector’s responses as given.

The relevant state variables for the government at the beginning of any period are \((z, b, s)\), with \( s \in \{N, D\} \). The government’s value function in any period with access to international financial markets is

\[
V(z, b, N) = \max \left\{ V^N(z, b), V^D(z) \right\},
\]

where \( V^N \) (\( V^D \)) are continuation utilities after no default (default). If the government stays solvent, the recursive formulation of its problem is

\[
V^N(z, b) = \max_{g, \tau, b_+} u(c - v(\ell), g) + \beta \mathbb{E}_z V(z_+, b_+, N),
\]

subject to

\[
g = \tau y + b - q(z, b_+)b_+,
\]

\[
c = C(z, \tau, N), \quad \ell = L(z, \tau, N), \quad y = Y(z, \tau, N).
\]

The first condition is the budget constraint of the government. The other three equations express the private-sector equilibrium in reduced form.
If the government has defaulted in some period and is excluded from international bond markets, the recursive problem is

\[ V^D(z) = \max_{g, \tau} u(c - v(\ell), g) + \beta E_z \left[ \theta V(z_+, 0, N) + (1 - \theta) V^D(z_+) \right], \]  

subject to \( g = \tau y \) and

\[ c = C(z, \tau, D), \quad \ell = L(z, \tau, D), \quad y = Y(z, \tau, D). \]

The set of default states is

\[ \Sigma^D = \{(z, b) \mid V^D(z) > V^N(z, b)\}. \]

The government’s default probability is

\[ \lambda(z, b_+ \not=) \equiv \text{Prob}\left((z_+, b_+) \in \Sigma^D \mid z\right). \]

International investors lend to the government if

\[ q(z, b_+) = \frac{1 - \lambda(z, b_+)}{R}. \]

The bond price function reflects the endogenous sovereign default risk.\(^{13}\)

### 3.6 Equilibrium Definition

**Definition:** A **recursive equilibrium** is given by

(i) value functions \( V(z, b, s), V^D(z), V^N(z, b) \) and policy functions \( b_+ = B(z, b, s), \tau = T(z, b, s), g = G(z, b, s) \) of the government, solving problems (7)–(9), and a default set \( \Sigma^D \) satisfying (10).

(ii) a pricing function \( q(z, b_+) \) satisfying the arbitrage condition of foreign lenders (11).

\(^{13}\)While the government borrows at the end of period \( t - 1 \) to repay debt in period \( t \), domestic firms borrow within period \( t \). For foreign investors, this difference is irrelevant as long as both loans have the same maturity (i.e., one model period). Even if there was a (small) difference in maturity, this would be reflected in the respective arbitrage conditions, with no further implications for any of our results.
(iii) a private sector equilibrium, defining consumption $c = C(z, \tau, s)$, employment $\ell = \mathcal{L}(z, \tau, s)$, output $y = \mathcal{Y}(z, \tau, s)$, and the private interest rate $R = \mathcal{R}(z, \tau, s)$ for $s = N, D$, satisfying (1)–(6).

A solution to a recursive equilibrium specifies optimal plans for the government and for all private agents in this economy. It includes situations with and without sovereign default. The bond pricing function and the private sector interest rate reflect the risk premia associated with optimal default choices of the government and of the private sector.

4 Quantitative Analysis

In this section, we solve the model numerically to study its quantitative properties. We apply the model to Argentina which is often used as the benchmark for studies on sovereign default given its default history and data availability. We calibrate the model at quarterly frequency and choose parameters to match several empirical targets.

4.1 Calibration

4.1.1 Functional Forms

We choose a CES production function of final goods:

$$f(m, m^*) = \left[ (1 - \omega)(m)^\rho + \omega(m^*)^\rho \right]^{1/\rho},$$

with $\rho < 1$ and $\omega \in (0, 1)$. This implies that the demand for domestic input per unit of foreign input is

$$\Phi(q) = \omega^{1/\rho} \left( q(1 - \omega) \right)^{\frac{\rho}{\rho + 1}} - 1 + \omega^{-1/\rho}, \ q = \frac{x^z w}{w}.$$

Profits (before interest) per unit of imports are

$$\pi(x^z, w) = w \left( \frac{\omega}{1 - \omega} \right)^{1/\rho} \left[ \left( \frac{x^z w}{w} \right)^{\frac{\rho}{\rho + 1}} (1 - \omega)^{\frac{1}{\rho + 1}} - 1 \right] \left( \frac{\rho + 1}{\rho} \right).$$

Both $\Phi$ and $\pi$ are defined for $q = x^z w < (1 - \omega)^{-1/\rho}$ if $\rho > 0$ and for $q = x^z w > (1 - \omega)^{-1/\rho}$ if $\rho < 0$.

Idiosyncratic productivity is uniformly distributed in the interval $[1 - \zeta, 1 + \zeta]$, so that $X(x) = \frac{x - 1 + \zeta}{2\zeta}$. 

14
We employ the GHH preferences (Greenwood et al. (1988)):

\[ u(c, l) = \left( \frac{c - \psi \ell^{1+\psi}}{1 - \gamma} \right)^{1-\gamma} + \alpha \frac{g^{1-\mu}}{1 - \mu}, \]

where \( \gamma > 0 \) and \( \mu > 0 \) denote the parameters of relative risk aversion for private and public consumption and \( \psi > 0 \) is the Frisch elasticity of labor supply. Note that this specification implies that the marginal rate of substitution between private consumption and labor is independent of consumption. \( \alpha \geq 0 \) is a preference weight.

Aggregate productivity follows an AR(1) process:

\[ \log(z_t) = \phi \log(z_{t-1}) + \varepsilon_t, \]

where \( \varepsilon_t \) is i.i.d. \( N(0, \sigma^2_\varepsilon) \).

Following Arellano (2008) we employ asymmetric output costs:

\[ h(z) = \begin{cases} 
\phi E(z) & \text{if } z > \phi E(z) \\
z & \text{else,} 
\end{cases} \]

with \( \phi \in (0, 1) \).

4.1.2 Parameters

All calibrated parameters and the associated targets and sources are listed in Table 2. The parameters of the CES production function \( \omega \) and \( \rho \) are set at the same values as in Mendoza and Yue (2012) who choose these parameters to match regression estimates for the domestic/imported intermediate goods’ demand elasticity.

The technology parameters are set to match the empirical autocorrelation and volatility of the Argentine GDP. The Frisch elasticity is chosen to be 2.22 which is a standard value in international macroeconomics (see also Mendoza and Yue (2012), Neumeyer and Perri (2005) and Cuadra et al. (2010)). The coefficient of risk aversion with respect to private consumption is set to \( \gamma = 2 \). The coefficient of risk aversion with respect to government consumption is calibrated to \( \mu = 7 \) to match the volatility of public consumption. The preference parameter that refers to the weight on public consumption is chosen to generate the empirical mean share of public consumption (12.9%).

As in Mendoza and Yue (2012) the share of credit-financed imports is set to match a 6% share of working capital in GDP. The choice of the risk-free interest
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target/Source</th>
<th>Endogenous Private Interest Rate</th>
<th>Exogenous Private Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frisch elasticity</td>
<td>$\psi$ Mendoza and Yue (2012)</td>
<td>2.22</td>
<td>2.22</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$ standard value</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\mu$ volatility of government consumption</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Weight on public good</td>
<td>$\alpha$ government consumption share</td>
<td>0.00003</td>
<td>0.00002</td>
</tr>
<tr>
<td>Weight on imports</td>
<td>$\omega$ Mendoza and Yue (2012)</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>CES curvature</td>
<td>$\rho$ Mendoza and Yue (2012)</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Share of credit-financed imports</td>
<td>$\xi$ working-capital to GDP</td>
<td>0.495</td>
<td>0.495</td>
</tr>
<tr>
<td>Import price</td>
<td>$p^*$ import share</td>
<td>2.979</td>
<td>1.144</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$\bar{R}$ standard value</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$ debt-service to GDP</td>
<td>0.897</td>
<td>0.932</td>
</tr>
<tr>
<td>Reentry probability</td>
<td>$\theta$ sovereign spread</td>
<td>0.219</td>
<td>0.163</td>
</tr>
<tr>
<td>Output cost</td>
<td>$\phi$ sovereign risk premium pre-default</td>
<td>0.9878</td>
<td>0.9878</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>$\eta$ volatility of private spread</td>
<td>0.52</td>
<td>-</td>
</tr>
<tr>
<td>Dispersion of idios. shocks</td>
<td>$\zeta$ private spread</td>
<td>0.195</td>
<td>-</td>
</tr>
<tr>
<td>Persistence of $z$</td>
<td>$\varphi$ autocorrelation of output</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Variance of $z$ shocks</td>
<td>$\sigma_z$ volatility of output</td>
<td>0.002</td>
<td>0.00285</td>
</tr>
</tbody>
</table>

Table 2: Parameter Choices
rate corresponds to a standard value in international macroeconomics. The import price is set to a value that implies a 12% share of imports as observed in the data. The parameter \( \zeta \) which determines the variance of idiosyncratic productivity shocks is chosen to match an annual mean private spread of about 8 percent. Since the recovery rate \( \eta \) is critical for the response of the private interest rate with respect to changes in fundamentals, we calibrate this parameter to generate the empirical standard deviation of private spreads (cf. Table 1).

We follow Arellano (2008) and set the discount factor \( \beta \) so as to match the average public debt service payments as a share of GDP (3%). In our calibration, the probability of re-entering international financial markets after a default equals 0.219 to generate the mean sovereign premium in the data. This value is in the range of what Aguiar and Gopinath (2006) and Arellano (2008) consider. The exogenous output cost is chosen to match the sovereign risk premium in the period before the default takes place (15%, cf. Figure 1).

### 4.2 Results

We now study the quantitative properties of our simulated model economy. First, we analyze the properties of the policy functions to highlight the main economic mechanism that drives the interaction between sovereign and private default risks. Second, we discuss the business-cycle properties of the simulated model economy. Third, we perform an event study and explore the macroeconomic dynamics before and after a sovereign default. Finally, we assess the quantitative importance of the endogenous feedback mechanism between sovereign and private default risks.

#### 4.2.1 Policy Functions

We first shed light on the optimal decision of the government whether to repay or to default on its outstanding external debt and the implied sovereign credit costs. In Figure 2 we consider realizations of aggregate productivity between \( \pm 1\% \) and show optimal debt and tax policies together with the associated quarterly private interest rate spreads.

The upper left panel of Figure 2 shows the sovereign bond price \( q(z, b_v) \). It is evident that, first, the bond price is decreasing in debt. For low levels of debt the government always repays and the bond price is equal to the inverse of the risk-free rate. With higher debt levels, foreign creditors incorporate the rising sovereign
default probability in their pricing decision and charge a larger risk premium on public debt. Second, the bond price decreases if the economy is hit by adverse aggregate productivity shocks. Since a government is less able to service its external debt in bad times, the sovereign premium reflects the increased risk of a sovereign default.

The upper right panel of Figure 2 shows the government’s debt policy. For high levels of public external debt and in times of recessions the government becomes borrowing constrained due to prohibitive external credit costs. The lower right panel reveals that the optimal sales tax is increasing in debt and decreasing in aggregate productivity. This pattern implies that fiscal policy is procyclical.\footnote{A similar procyclical pattern can be observed in the policy functions for government expenditures.} In times of recessions the government can become borrowing constrained so that it raises taxes to
finance public expenditures. In default, however, public debt is not repaid such that the government is able to reduce the tax rate. The theoretical prediction regarding the cyclical properties of taxes is in line with the broad empirical literature that shows that developing countries and emerging market economies are characterized by procyclical fiscal policies, see, e.g., Talvi and Vegh (2005), Ilzetzki and Vegh (2008). The procyclical pattern of taxes is crucial for countercyclical private interest rates. In recessions the tax rise amplifies private default risk and increases the private interest rate which is shown in the lower left panel of Figure 2.

4.2.2 Cyclical Properties

In Table 3 we show the business-cycle properties implied by our theoretical framework. We report the statistical moments based on simulated time series that exclude default events. All variables are logged before they are linearly detrended, except the tax rate, the sovereign premium, and the private premium.

A comparison of the simulated and the empirical cyclical properties reveals that our model captures the co-movements between the variables and the overall macroeconomic volatility. In particular, the model replicates the countercyclicity of sovereign and private premia and the procyclicality of consumption, imports, and public expenditures.

The model also mimics the empirical fact that imports are more volatile than output. While the model replicates the volatility of the private premium, it overstates the standard deviation of the sovereign premium. The high volatility of sovereign credit costs is due to the occurrence of “near default states” in which adverse realizations of aggregate productivity substantially increase the default risk so that foreign creditors charge high sovereign premia. In spite of high credit costs, the government finds it still optimal to repay because the exclusion from international financial markets imposes a severe punishment.

4.2.3 Dynamics Around Default

To understand the interaction of sovereign and private default risk and their impact on macroeconomic outcomes, we perform an event study and show the dynamics of the economy six quarters before and after a sovereign default. We assume that the government is in a good credit standing in \( t < 0 \) but defaults at date \( t = 0 \). In Figure 3 the solid lines show the percentage deviations from a linear trend for output,
Table 3: Business-Cycle Statistics

Notes: This table reports the business-cycle statistics of output $y$, consumption $c$, labor $l$, public expenditures $g$, imports $m$, the annualized sovereign spread $s$ and the annualized private spread $s^p$. All variables are logged except taxes and premia before they are linearly detrended. Statistics of the theoretical model refer to a simulation of 50,000 quarters where the first 15,000 quarters are discarded. Default episodes (including one quarter before the default event and the subsequent quarters without external borrowing) are excluded. The first column refers to the Argentine data while the second column refers to the simulated time series generated by the model. The third column refers to the model outcome if the private interest rate is exogenously fixed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Argentina</th>
<th>Endogenous Private Interest Rate</th>
<th>Exogenous Private Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>4.94</td>
<td>5.04</td>
<td>4.93</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.05</td>
<td>0.90</td>
<td>1.06</td>
</tr>
<tr>
<td>$\sigma(g)/\sigma(y)$</td>
<td>0.47</td>
<td>0.47</td>
<td>0.44</td>
</tr>
<tr>
<td>$\sigma(m)/\sigma(y)$</td>
<td>3.09</td>
<td>2.58</td>
<td>1.68</td>
</tr>
<tr>
<td>$\sigma(s)$</td>
<td>2.75</td>
<td>7.99</td>
<td>9.82</td>
</tr>
<tr>
<td>$\sigma(s^p)$</td>
<td>4.81</td>
<td>4.93</td>
<td>–</td>
</tr>
<tr>
<td>$\rho(c, y)$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(g, y)$</td>
<td>0.64</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho(m, y)$</td>
<td>0.95</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(s, y)$</td>
<td>-0.85</td>
<td>-0.72</td>
<td>-0.46</td>
</tr>
<tr>
<td>$\rho(s^p, y)$</td>
<td>-0.81</td>
<td>-0.92</td>
<td>–</td>
</tr>
<tr>
<td>$\rho(s, s^p)$</td>
<td>0.87</td>
<td>0.80</td>
<td>–</td>
</tr>
<tr>
<td>$E(s)$</td>
<td>5.97</td>
<td>5.99</td>
<td>5.91</td>
</tr>
<tr>
<td>$E(s^p)$</td>
<td>8.45</td>
<td>8.74</td>
<td>8.45</td>
</tr>
<tr>
<td>$E(b/y)$</td>
<td>-3.03</td>
<td>-3.01</td>
<td>-2.94</td>
</tr>
<tr>
<td>$E(m/y)$</td>
<td>11.76</td>
<td>11.80</td>
<td>11.73</td>
</tr>
<tr>
<td>$E(g/y)$</td>
<td>12.80</td>
<td>12.96</td>
<td>12.94</td>
</tr>
</tbody>
</table>

Table 3: Business-Cycle Statistics

Notes: This table reports the business-cycle statistics of output $y$, consumption $c$, labor $l$, public expenditures $g$, imports $m$, the annualized sovereign spread $s$ and the annualized private spread $s^p$. All variables are logged except taxes and premia before they are linearly detrended. Statistics of the theoretical model refer to a simulation of 50,000 quarters where the first 15,000 quarters are discarded. Default episodes (including one quarter before the default event and the subsequent quarters without external borrowing) are excluded. The first column refers to the Argentine data while the second column refers to the simulated time series generated by the model. The third column refers to the model outcome if the private interest rate is exogenously fixed.
labor, consumption, imports and public expenditures while the tax rate as well as the sovereign and private spreads are in percent.

The dynamic patterns suggest that the economy is in a recession prior to a sovereign default. Low output raises the risk of a sovereign default which is reflected by the sovereign interest spread in the quarters before the default takes place. Due to high credit costs, the government becomes borrowing constrained and raises the tax rate to finance public expenditures. This leads to an endogenous amplification mechanism: Higher taxes in a recession lower the profitability of private firms so that the risk of a private default increases. Foreign creditors incorporate the default risk in their pricing decision and charge a larger risk premium on private external debt. Import demand falls and the recession deepens. In turn, low output reinforces the risk of a sovereign default, and the sovereign premium increases further. After the default, the government is excluded from international financial markets. Since debt service obligations are not fulfilled, the government budget constraint relaxes such that the tax rate can be reduced. The private premium decreases and imports as well as output recover.

Overall, the model generates dynamics around default events that are in line with the empirical evidence observed in Argentina. In particular, given the calibrated sovereign premium of 15% in the period before the default, the model replicates the high private premium of about 20% as seen in the Argentine data. Note that our theoretical model assumes that after a default the government is excluded from international financial markets; this is why the sovereign premium is infinite in the period after the default. The increase in the private premium, however, is of a more transitory nature which is in line with the empirical pattern in Argentina after the default in 2002.

The model also replicates the fact that the import drop is a multiple of the output drop. Note, however, that the model underestimates the reductions in output and imports. During the default in Argentina output (imports) decreased by 18% (74%) while in the simulated economy we observe a decline of 10% (25%). The empirical patterns of output and imports are affected by additional factors that are not included in our model, such as exchange rate dynamics and the sovereign-bank nexus as emphasized in, e.g., Engler and Große-Steffer (2016), Niemann and Pichler (2016), and Sosa-Padilla (2012). Therefore, our model isolates the endogenous amplification generated by the dynamic interaction between sovereign and private default risks through the response of fiscal policy to adverse productivity shocks.
Figure 3: Default Event

Notes: This figure shows the dynamic pattern of output $y$, consumption $c$, labor $l$, imports $m$, public expenditures $g$, taxes $\tau$, the annualized sovereign spread $s$ and the annualized private spread $s^p$ six quarters before and after a default. The government is in a good credit standing in $t < 0$ and defaults at quarter $t = 0$. All variables are shown as percentage deviations from their linear trend, except the tax rate and the premia which are shown in percent, based on a simulation of 50,000 quarters where the first 15,000 quarters are discarded. For all variables the mean values over all default events are shown. The dashed lines refer to the dynamics if the private interest rate is exogenously fixed.
4.2.4 The Quantitative Impact of Endogenous Private Default Risk

To highlight the quantitative importance of the dynamic feedback mechanism between sovereign and private default risks, we exogenously fix the private interest rate at the empirically observed value of 8.45%. To facilitate a comparison with our benchmark model, we re-calibrate the parameters to match the empirical targets as listed in Table 2. In particular, we adjust the preference weight $\alpha$ and the import price $p^*$ to match the government consumption share and the import share. Since a fixed private interest rate shuts down the endogenous amplification mechanism, a higher standard deviation of the exogenous productivity shock is now required to match the volatility of the Argentine GDP. To generate the mean of the Argentine sovereign spread and empirical debt service payments, we adjust the re-entry probability $\theta$ and the rate of time preference $\beta$. Importantly, we keep the exogenous output cost at the same level as before to evaluate the quantitative size of the amplification effect generated by endogenous private interest rates.

The dashed lines in Figure 3 show the dynamics around a default event for a fixed private interest rate. As before, in the quarters before the default, low output raises the sovereign spread such that the government becomes borrowing constrained. To finance public expenditures the government raises the tax rate. Since the private interest rate is exogenously fixed, the amplification mechanism is absent and imports and output fall less strongly. Consequently, the increase in the sovereign spread is less pronounced. In the absence of endogenous private interest rates, output and imports decrease by about 7.5% and 12% and the sovereign premium equals 12.5% in the quarter before the default. Thus, the dynamic feedback between sovereign and private default risk amplifies the downturns in output and import by 2.5 and 13.5 percentage points and raises the sovereign premium by 3 percentage points.

5 Conclusions

In this paper we analyze how sovereign and private default risks interact. We develop a stochastic general equilibrium model of a small open economy featuring endogenous private and sovereign default risks. Private sector firms use imperfectly substitutable domestic and imported intermediate goods to produce a final consumption good, where part of the imports need to be financed by external debt. The economy also features a benevolent government providing a public good, financed by taxing firms and borrowing from abroad. The model can account for several empirical regularities
in emerging market economies, namely countercyclical private and sovereign risk premia, procyclical fiscal policy, and deep recessions with large drops in imports during default events.

Our results suggest that fiscal policy creates a link between sovereign and private default risks and provides an amplification mechanism reinforcing the effects of adverse productivity shocks. Whenever the government faces higher borrowing costs in a recession, it raises taxes so as to reduce external credit costs which decreases firms' profitability and leads to higher private default risk. In turn, firms cut their demand for imported inputs which deepens the recession. While our study highlights this particular mechanism, other channels may also be relevant for future research. For instance, little is known about how exchange rate movements (Asonuma, 2014), debt renegotiations (Yue, 2010), long run growth (Gorneman, 2014) or secondary debt markets interact with private and sovereign credit risks.

References


Appendix A: Numerical Algorithm

The private-sector equilibrium can be calculated on a grid for \((z, \tau)\), without knowing the government’s policy functions. These solutions are used to solve for the government’s problem and the risk-neutral pricing of government bonds via value function iteration.

The numerical algorithm builds on Hatchondo et al. (2010) and employs cubic spline interpolations so that optimal policies are chosen from a continuous set and productivity realizations are allowed that do not lie on the grid. We approximate the equilibrium as the equilibrium of the finite-horizon economy and iterate simultaneous on the value and the bond price functions.

We define evenly distributed grid vectors for bond holdings \(b \in [\bar{b}, \tilde{b}]\) and productivity realizations \(z \in [\underline{z}, \overline{z}]\). Let \(V^{N(0)}(z, b)\) and \(V^{D(0)}(z)\) denote the initial guesses for the value functions. For every grid point \((z, b) \in [\bar{b}, \tilde{b}] \times [\underline{z}, \overline{z}]\) and given the initial guesses \(V^{N(0)}(z, b)\) and \(V^{D(0)}(z)\) we first find candidate values for \(\tau^{(0)}\) and \(b^+(0)\) by employing a global search procedure. These candidate values are then taken as initial guesses for the FORTRAN optimization routine BCPOL from the IMSL library to find \(\tau^{(0)}\) and \(b^+(0)\) via (8), (9) where \(V^{0}_{(0)}(z, b, s)\) satisfies equation (7). Given the initial guess, equations (10) and (11) determine the the default probability \(\lambda^{(0)}(z, b^+(0))\) and the bond price function \(q^{(0)}(z, b^+(0))\), respectively. Expected continuation values are computed using Gauss-Hermite quadrature points and weights. To evaluate the expected continuation values for policies and productivity realizations that do not lie on the grid we employ cubic spline interpolations using the FORTRAN CSDEC routine from the IMSL library. The solutions found at each grid point are used to update the value functions \(V^{N(1)}(z, b)\) and \(V^{D(1)}(z)\). We iterate until the value functions converge.

Appendix B: Further Empirical Findings

Table 4 reports the business-cycle statistics of the countries contained in our sample. Default episodes are excluded. We observe that emerging market economies are more volatile than developed economies. Emerging markets also show excess volatility of private and government consumption, whereas in developed economies only government consumption is more volatile than GDP on average. Furthermore, in both country groups imports and exports are on average two to three times more volatile than GDP. Our observations are in line with results found by Neumeyer and
Argentina  4.94  1.05  0.47  3.09  1.49  
Brazil     3.22  1.57  0.92  5.67  3.11  
Chile      2.65  1.62  1.30  3.74  2.79  
Ecuador    1.93  0.66  0.85  2.98  2.47  
Korea      13.08 0.89  0.45  1.13  0.93  
Malaysia   4.89  1.23  1.74  2.50  2.68  
Mexico     3.05  1.26  1.12  4.48  3.61  
Peru       14.34 0.81  1.53  1.70  1.32  
Philippines 7.57  0.62  1.71  2.22  1.45  
Russia     5.25  1.00  1.13  2.86  1.52  
Venezuela  7.71  0.73  0.59  2.59  1.53  
Australia  2.35  1.07  1.15  4.71  4.16  
Canada     3.22  0.80  1.20  2.85  4.76  
Netherlands 4.07  1.37  0.64  1.76  1.60  
New Zealand 3.65  1.04  1.02  2.05  1.54  
Sweden     3.07  0.60  0.41  2.57  3.01  
Switzerland 2.22  0.60  2.67  2.27  2.49  

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<th>$\sigma(g)/\sigma(y)$</th>
<th>$\sigma(m)/\sigma(y)$</th>
<th>$\sigma(x)/\sigma(y)$</th>
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<th>$\sigma(g)/\sigma(y)$</th>
<th>$\sigma(m)/\sigma(y)$</th>
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</table>

Table 4: Business-Cycle Statistics

Notes: $y$ refers to real GDP, $c$ and $g$ denote real consumption and real public expenditures, respectively. $m$ and $x$ are real imports and exports. All time series are log-linearly detrended.
Appendix C: Data Sources and Calculations

The sample of emerging markets economies is chosen according to JP Morgan’s characterization and data availability. More precisely, we restrict the sample to include countries for which at least 10 years of quarterly data are available until 2013Q2. Korea is the only exception from this rule. We add Korea because it is also in the sample of Neumeyer and Perri (2005).

All data are taken from national sources if available, otherwise we use data from international organizations. Only in cases where no other source was available we use commercial data providers like Oxford Economics. Detailed information on data sources and adjustments are summarized in the tables below.

When data are not seasonally adjusted, we employ the Census X12 method from the U.S. Census Bureau. Most nominal time series are deflated using the GDP deflator. For imports and exports we use the import and export price deflators.

For the calculation of correlations and volatilities we are interested in the cyclical components of the respective series. In order to get the cyclical components of GDP and imports, we subtract a linear trend from the logged series. For the risk premia the raw series are demeaned.

We follow Neumeyer and Perri (2005) and Arellano and Kocherlakota (2014) and use the emerging markets sovereign premia provided by JP Morgan. For developed economies, sovereign premia are calculated by subtracting the medium term US bond yield from the respective countries’ medium term bond interest rate.

In most cases private premia are calculated by subtracting the local currency deposit rate from the lending rate. Whenever the US$ lending rate is available, a US government debt interest rate with similar maturity is used as the risk-free rate.\footnote{See Table 6 for a detailed description of how the risk premia are calculated.}

Tables 5 and 6 give more details on data sources and transformations, where ‘SA’ stands for ‘seasonally adjusted’ and ‘R’ denotes series transformed into real terms.

In order to show that total imports are an acceptable proxy for intermediate good imports we calculate their share in total imports. In this we follow Mendoza and Yue (2012) and define intermediate goods imports as all imports falling into the following product categories of the COMSTAT dataset: (111*) Food and beverages, primary, mainly for industry; (121*) Food and beverages, processed, mainly for industry;
(21*) Industrial supplies not elsewhere specified, primary; (22*) Industrial supplies not elsewhere specified, processed; (31*) Fuels and lubricants, primary; (322*) Fuels and lubricants, processed (other than motor spirit); (42*) Parts and accessories of capital goods (except transport equipment); (53*) Parts and accessories of transport equipment. On average these imports are responsible for around 58% of total imports in emerging economies and around 48% in developed economies.
<table>
<thead>
<tr>
<th>Country</th>
<th>Data Source</th>
<th>Sample</th>
<th>Currency</th>
<th>Adjustment</th>
<th>Basis Year Deflator</th>
<th>Information</th>
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Table 5: Data Sources for GDP and Imports.
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**Table 6: Data Sources for Interest Rates.**