Three Essays on Education Investments

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To my mother,
my father,
and my brother.

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Summary

This dissertation consists of three independent research papers that I wrote as part of my studies in the “Doctoral Programme in Quantitative Economics and Finance” at the University of Konstanz. The three papers address different phenomena in education investments: Chapters 1 and 3 deal with differentials in education investments while Chapter 2 focuses on the effect of technological change on human capital accumulation. In particular, Chapter 1 explains why education investments differ between boys and girls more in some countries than in the others. Chapter 2 studies the effect of a persistent skill-biased technology shock on the human capital accumulation path of a society. The third chapter examines the reasons that underlie differentiated tuition fees among domestic and foreign students and their implication in terms of welfare effects.

The work presented in Chapter 1 is motivated by empirical observations that girls receive less education than boys in some countries. Moreover, this gender difference in education tends to be larger in countries with a higher fertility rate and lower GDP per capita. In order to provide theoretical explanations for these phenomena, I develop a two-sex overlapping-generations model with endogenous education and fertility decisions. The central idea is that women tend to receive relatively lower education because they bear most of the child raising work and hence spent less time in the labor market compared to men. An outcome with differentiated education investments occurs although individuals of both sexes are endowed with similar innate abilities and are not discriminated by their parents. I further explain that the differential in education investments is increasing in the number of children per couple because a higher fertility rate implies that women are constrained to spend more time raising children at home. Furthermore, parents face a quantity-quality trade-off in the model which means that they either choose to have many but less educated children or fewer but well educated children.

The model induces multiple steady states which can explain the empirically observed variations across countries in gender difference in education, fertility and income per capita. Indeed, there exists one steady state with well educated parents who choose to have few and well educated children. The low fertility rate provides that the gender difference

in education is small in this steady state. In a second steady state, parents are less educated and choose to have many and less educated children. In this steady state, the gender difference in education is larger due to the higher fertility rate. I also provide a numerical example showing that my theoretical results fit well to explain empirical observations.

In Chapter 2, I turn to investigate the effect of technological change on the accumulation of human capital and compare this effect across economies which differ in their fertility rates and education costs. I focus on an exogenous and persistent technology shock which is assumed to be skill-biased towards the productivity of skilled workers compared with that of unskilled workers. I set up a one-sex overlapping-generations model with endogenous education decision. Education investments have to be financed by the parent since the capital market is assumed to be imperfect. Individuals who receive education in the first period of their life become skilled in the second period while the others who do not receive education stay unskilled.

The occurrence of a skill-biased technological change induces the skilled wage rate to rise while the unskilled wage rate falls. These changes in the wage structure cause ambiguous effects on education decisions. On the one hand, the increasing skill premium makes investments in education more attractive. On the other hand, the growing education costs relative to income of unskilled parents make education investments become more expensive and hence less attractive for this category of parents. I demonstrate that the net effect of the technology change on human capital accumulation depends then on the development stage of the economy. If the economy is less developed (i.e., the number of unskilled parents is high and hence, average human capital is low), the negative cost effect dominates which implies that human capital accumulation slows down. On the contrary, when the economy is well developed (i.e., the number of skilled parents and average human capital are high), the positive incentive effect dominates and human capital accumulation accelerates.

Given these results, I show that economies with a high fertility rate and/or high education costs face a slow down in human capital accumulation in the short-run and a reduction in the average human capital in the long-run (i.e., the steady state). In contrast, economies with a low fertility rate and/or low education costs may face a slow down in human capital accumulation in the short-run but clearly see a rise in the average human capital in the long-run.

In Chapter 3, I examine determinants of tuition fee differentials among foreign and domestic college students and question whether differentiating tuition fees improves the welfare of a country. Moreover, the chapter aims to predict the future evolution in tuition fee differentials. I base the analysis upon a two-country model with mobile students and mobile workers. The governments of both countries provide publicly funded education and choose tuition fees for domestic and for foreign students and the tax rate in their jurisdiction. In contrast, the quality of education is considered to be exogenous in the model. The objective of a government is to maximize local GDP which is non-discriminating between the groups of domestic and foreign students. This objective function allows me to investigate other determinants of tuition fee differentials besides the pure preference for one student group.

I identify three factors which drive the tuition fee differential, namely, student mobility, worker mobility and education quality. Increasing student mobility affects the tuition fee differentials positively because it becomes easier to attract foreign students while it becomes more difficult to attract domestic students. However, the mobility of workers (i.e., migration flows after graduation) does not affect tuition fee differentials when both natives and foreigners have similar migration patterns. It only does when foreigners are more likely to migrate after graduation (there is evidence for such behavior in reality). The reason is because governments charge higher fees to the student group which is expected to be more mobile after graduation in order to compensate for the lost returns to education investments in case students migrate after their studies. As for the education quality, I find that improving the quality of education in a country widens the tuition fee differential in that region.

Based on the results of the welfare analysis, I find that the welfare of a country is maximized when tuition fees are not differentiated. The reason is because differentiating tuition fees distorts the student migration decision and leads to too much or too little migration (depending on the sign of the tuition fee differential). This result provides an economic argument for the European Union's directive that member countries are not allowed to differentiate tuition fees among students coming from EU member countries.

Zusammenfassung

Die vorliegende Dissertation besteht aus drei eigenständigen Forschungspapieren, die ich während meiner Studienzeit im "Doctoral Programme in Quantitative Economics and Finance" an der Universität Konstanz geschrieben habe. Die drei Papiere behandeln verschiedene Phänomene im Bereich der Bildungsinvestitionen: Kapitel 1 und 3 beschäftigen sich mit Ungleichheiten in den Bildungsinvestitionen und Kapitel 2 untersucht, wie technologischer Wandel die Akkumulation von Humankapital beeinflusst. Das erste Kapitel gibt eine Antwort auf die Frage, warum Bildungsinvestitionen in manchen Ländern stärker zwischen den Geschlechter differieren als in anderen. Im zweiten Kapitel untersuche ich, wie sich ein Technologieschock, der unterschiedlich auf die Produktivität von gut und weniger gut ausgebildeten Arbeitskräften wirkt, auf die Akkumulation von Humankapital und damit die ökonomische Entwicklung einer Gesellschaft auswirkt. Im dritten und letzten Kapitel untersuche ich die Ursachen und die Wohlfahrtseffekte von differenzierten Studiengebühren, wobei die Studiengebühren für In- und Ausländer separat von der Regierung eines Landes gesetzt werden können.

Die Motivation für das erste Kapital liegt in der empirischen Beobachtung, dass Söhne in manchen Ländern eine signifikant bessere Ausbildung als Töchter erhalten. Diese Differenz in der Bildung zwischen den Geschlechtern ist in Ländern mit hoher Fertilität und geringerem BIP pro Kopf stärker ausgeprägt als in anderen. Um dieses Phänomen ökonomisch erklären zu können, führe ich ein Modell mit überlappenden Generationen und endogenen Geburten- und Bildungsentscheidungen ein, in dem explizit zwischen den zwei Geschlechtern unterschieden wird. Die treibende Kraft im Modell ist folgende: Frauen übernehmen in der Regel den größten Teil der Kinderbetreuungsarbeit und arbeiten daher weniger auf dem Arbeitsmarkt als Männer. Die unterschiedliche Lebensarbeitszeit lässt die Erträge von Bildungsinvestitionen in Jungen und Mädchen voneinander abweichen und führt selbst dann zu den bekannten Unterschieden in den Bildungsinvestitionen zwischen den Geschlechtern, wenn beide Geschlechter mit identischen Fähigkeiten geboren wurden und Eltern keines der Geschlechter bevorzugen. Die Differenz in den Bildungsausgaben steigt mit der Anzahl der Kinder je Familie, weil Frauen umso länger zu Hause bleiben und die Kinder betreuen, je mehr Kinder in einer Familie geboren werden.

Im Modell stehen die Eltern einem Zielkonflikt zwischen der Qualität und der Quantität ihrer Kinder gegenüber („quantity-quality trade-off"). Dieser Zielkonflikt führt dazu, dass Eltern sich entweder für viele Kinder mit geringer Bildung oder wenigen Kindern mit guter Bildung entscheiden. In Verbindung mit der Geburtenrate könnten sich verschiedene langfristige Gleichgewichte ergeben, welche die Variation in den geschlechtsspezifischen Bildungsunterschieden erklären würden. Tatsächlich ergeben sich im Modell zwei langfristige Gleichgewichte. In einem Gleichgewicht sind die Eltern gut ausgebildet und entscheiden sich für wenige Kinder, welche eine gute Ausbildung erhalten. Die nächste Elterngeneration ist somit wiederum gut ausgebildet und trifft identische Entscheidungen, wie ihre Eltern in der Vorperiode. Aufgrund der zu erwartenden geringen Geburtenrate sind die Bildungsunterschiede zwischen den Geschlechtern gering. In dem anderen langfristigen Gleichgewicht sind die Eltern weniger gut ausgebildet und haben viele Kinder, die vergleichsweise wenig Bildung erhalten. Die hohe Geburtenrate führt hier zu großen Unterschieden in der Bildung zwischen Söhnen und Töchtern. In einem numerischen Beispiel zeige ich am Ende des ersten Kapitels, dass die theoretisch hergeleiteten Resultate durchaus die empirischen Beobachtungen replizieren können. Eine Ausnahme betrifft den Grad der im Modell generierten Bildungsdifferenzen zwischen den Geschlechtern, die höher sind als in der Wirklichkeit.

Im zweiten Kapitel untersuche ich, welchen Effekt technologischer Wandel auf die Bildungsentscheidungen und damit die Akkumulation von Humankapital hat. Dabei berücksichtige ich Unterschiede zwischen den Ländern in der Geburtenrate und den Bildungskosten. Der Fokus liegt dabei auf einem einmaligen aber dauerhaften Technologieschock, der die Produktivität von qualifizierten, gut ausgebildeten Arbeitskräften stärker erhöht als die Produktivität von gering qualifizierten Arbeitern („skill-biased technological change"). Um diesen Effekt zu untersuchen, entwickle ich ein Unisex-Modell mit überlappenden Generationen und endogener Bildungsentscheidung. Aufgrund der Annahme von imperfekten Kapitalmärkten müssen die Bildungsinvestitionen von den Eltern finanziert werden. Bildung ist in diesem Modell nicht kontinuierlich, sondern führt vielmehr zu einem Sprung in der für den Arbeitsmarkt relevanten Qualifikation.

Tritt ein qualifikationsspezifischer Technologieschock auf, steigt der Lohnsatz für qualifizierte Arbeitskräfte und sinkt der Lohnsatz für gering qualifizierte Arbeitnehmer. Diese Veränderung der Lohnstruktur erzeugt gegenläufige Effekte auf die Bildungsentscheidungen der Haushalte. Einerseits steigt aufgrund der höheren Lohnspreizung der Anreiz, in Bildung zu investieren. Andererseits erhöhen sich auch die Ausbildungskosten relativ zum Einkommen gering qualifizierter Eltern, was die Investitionen in die Ausbildung der Kinder für diese Eltern weniger attraktiv macht. Der Nettoeffekt des Technologieschocks auf die Humankapitalakkumulation hängt in dieser Situation vom Entwicklungsstand der Volkswirtschaft ab. In einer weniger gut entwickelten Volkswirtschaft mit einer hohen Zahl von gering qualifizierten Eltern und dadurch einem niedrigen durchschnittlichen Humankapitalniveau überwiegt der negative Kosteneffekt. Folglich sinken in einer solchen Gesellschaft die aggregierten Investitionen in die Ausbildung der Kinder und wird sich die Akkumulation von Humankapital verlangsamen. In entwickelten Volkswirtschaften mit einer hohen Zahl von qualifizierten Haushalten und einem hohen durchschnittlichen Humankapital dominiert jedoch der positive Anreizeffekt. In diesen Gesellschaften wird somit mehr in die Ausbildung der Kinder investiert und beschleunigt sich die Akkumulation von Humankapital.

Ausgehend von diesen Ergebnissen zeige ich, dass Volkswirtschaften mit hoher Geburtenrate und/oder hohen Ausbildungskosten nach dem Auftreten des Technologieschocks auf kurze Sicht eine Verlangsamung der Humankapitalakkumulation und auf lange Sicht ein Reduktion im durchschnittlichen Humankapital befürchten müssen. Volkswirtschaften mit geringer Geburtenrate und niedrigen Ausbildungskosten jedoch mögen auf kurze Sicht eine Verlangsamung erfahren, auf lange Sicht profitieren sie jedoch bezogen auf das durchschnittliche Humankapital.

In Kapitel 3 beschäftige ich mich mit den Gründen für zwischen In- und Ausländern differenzierten Studiengebühren und der Frage, ob differenzierte Studiengebühren die Wohlfahrt einer Gesellschaft steigern können oder nicht. Zudem versuche ich, die Entwicklung von differenzierten Studiengebühren in der Zukunft vorherzusagen. Als Grundlage für die Analyse verwende ich ein Zwei-Länder-Modell mit mobilen Studenten und mobilen Arbeitskräften. Die Regierungen der beiden Länder stellen öffentlich finanzierte Hochschulbildung bereit und entscheiden unabhängig voneinander über die Studiengebühren für in- und ausländische Studenten und den Steuersatz im eigenen Land. Die Qualität der Hochschulbildung ist hingegen exogen gegeben. Das Ziel jeder Regierung ist die Maximierung des lokalen BIPs. Da das BIP lediglich von der Anzahl und der Qualität, nicht jedoch von der Herkunft der arbeitenden Bevölkerung abhängt, diskriminiert dieses Ziel a priori keine der Studentengruppen. Ich habe dieses Ziel gewählt, um Faktoren für differenzierte Studiengebühren, die über die reine Präferenz für eine Studentengruppe hinausgehen, zu bestimmen und zu untersuchen.

Ich identifiziere in diesem Kapitel drei Faktoren, welche die Differenz in den Studiengebühren beeinflussen: die Mobilität der Studenten, die Mobilität der Arbeiter und die Qualität der Bildung. Eine steigenden Mobilität der Studenten vergrößert den Abstand der Studiengebühren zwischen In- und Ausländern. Grund für diese Entwicklung ist, dass ausländische Studenten einfacher und inländischer Studenten schwieriger von einem Land angezogen bzw. gehalten werden können, wenn Studenten mobiler werden. Die Mobilität der Arbeiter wirkt sich nur dann auf die Differenz in den Studiengebühren aus, wenn sie sich zwischen den Studentengruppen unterscheidet. Da in der Regel ausländische Studenten nach dem Abschluss ihres Studiums mobiler als die einheimischen Studenten sind, führt eine steigende Arbeitermobilität zu einer Ausweitung der Differenz in den Studiengebühren. Die Ursache für dieses Verhalten liegt in der öffentlich bereitgestellten Bildung. Wenn nun ein ehemaliger Student das Studienland verlässt, zahlt sich die Investition in seine Ausbildung für das Land, welches die Bildung finanziert hat, nicht in Form eines höheren BIPs aus. Dieses Land wird daher eine höhere Studiengebühr festsetzen, wenn es die Emigration nach dem Studienabschluss für wahrscheinlich oder wahrscheinlicher hält. Eine Erhöhung der Ausbildungsqualität führt ebenfalls zu einer Ausweitung der Studiengebührendifferenz zwischen in- und ausländischen Studenten.

Ausgehend von den theoretisch hergeleiteten Resultaten zeige ich, bezogen auf die Wohlfahrtsanalyse, dass beide Länder profitieren, wenn sie auf die Differenzierung von Studiengebühren verzichten würden. Der Grund liegt in der von differenzierten Studiengebühren ausgehenden verzerrenden Wirkung auf die Migrationsentscheidungen, die, je nach Vorzeichen der Differenz in den Studiengebühren, zu zu hoher oder zu niedriger Migration führt. Dieses Resultat liefert somit eine ökonomische Begründung für die in der Europäischen Union eingeführten Regel, dass Studiengebühren zwischen EU-Inländer nicht differenzierten werden dürfen.

Chapter 1

Why Does Eric Go to School and Emily Does Not?

1.1 Introduction

In a number of countries, girls receive substantially less education than boys of the same generation (Filmer, 2000). Moreover, this difference seems to vary across countries. There is evidence of a negative relationship between gender differences in education and fertility rate as well as a positive link between such differences and income per capita. In other words, a country with higher per capita income and a lower fertility rate tends to offer more equal education opportunities for its boys and girls. This evidence is illustrated in Figures 1.1 and 1.2 where the gender difference in education is measured throughout this chapter as the ratio of female to male students in primary, secondary and tertiary education, referred to as the gender parity index (GPI).[1] The objective of this work is to investigate the relationship among gender difference in education, fertility rate and income per capita, and to explain why gender difference in education varies across countries.

Aiming to reduce gender differences in education is of particular interest not only because of equity consideration, but also because it promotes economic development. Klasen (2002) suggests that gender differences may have an effect on long-run economic growth in various ways, e.g. through lowering average human capital, raising population growth and reducing investment in physical capital. Indeed, a number of studies find that gender differences in education slow down economic growth (e.g., Klasen, 1999, 2002; Klasen and Lamanna, 2009; Knowles et al., 2002; Schultz, 2002). The underlying message is that societies which do not invest equally in the schooling of their boys and girls pay a price in

[1] Data for Figures 1.1 and 1.2 were collected by the author for 62 countries in the year 2010 from UNData. See Appendix 1.8.1 for more details.

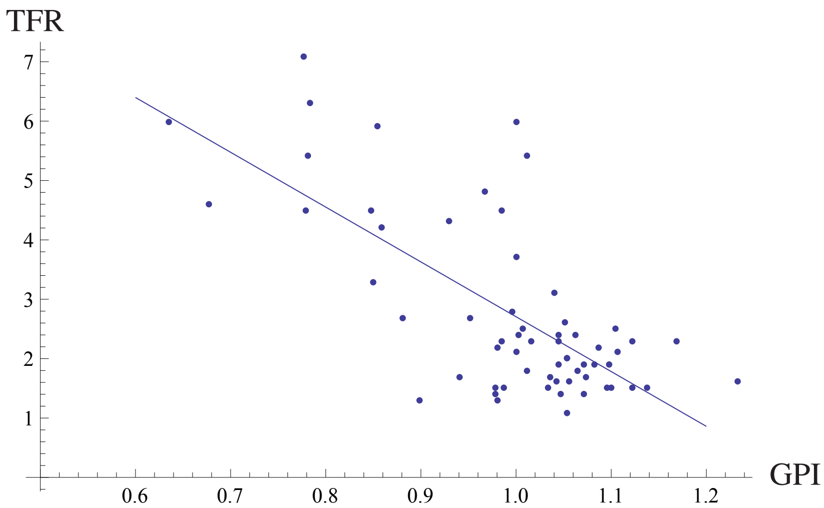


Figure 1.1: The relationship between the total fertility rate (TFR) and the gender parity index (GPI).

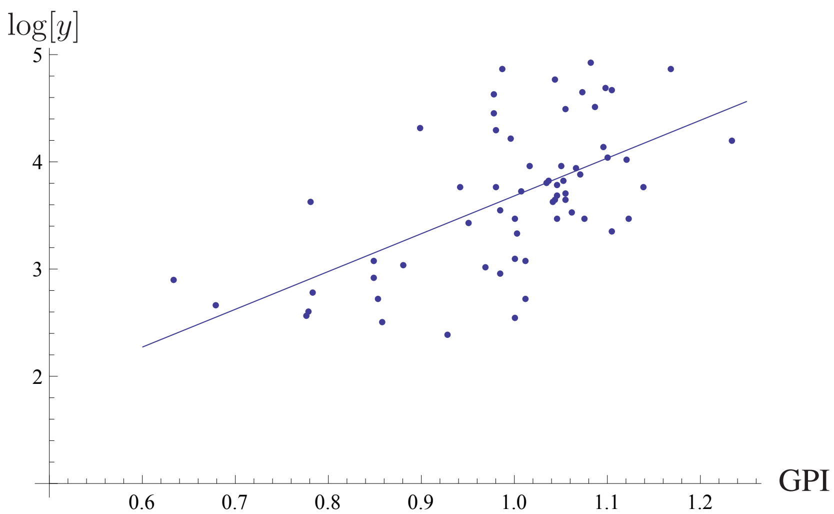


Figure 1.2: The relationship between logged income per capita ($\log[y]$) and the gender parity index (GPI).

terms of slower growth and reduced income (Dollar and Gatti, 1999).

Whether it is rather gender inequality in primary, secondary or tertiary education that affects growth is not much addressed in the literature. The reason might be that it is difficult to disentangle the effect of primary, secondary and/or tertiary education since any difference in primary education also causes inequality in the later education steps. The literature contributions mentioned above measure gender inequality either by the years of schooling or the attainment of primary and secondary education for boys and girls. So from this, one can deduct that the focus is rather on secondary education since this education level may be the most important factor for the evolution of a qualified labor force that contributes to economic growth. In politics, the focus has been on both primary and secondary education (see the Millennium Development Goals). One reason why gender inequality in primary education is more in the focus of politicians may be that gender inequality in secondary education can effectively be targeted only when gender equality in primary education has been achieved.

Various reasons can explain the emergence of gender difference in education; among them are gender-specific return on education (Davies and Zhang, 1995; Alderman and Gertler, 1997), gender-specific education costs (Alderman and King, 1998) and biased parental preferences towards one of the sexes (Davies and Zhang, 1995; Alderman and Gertler, 1997).[2] In particular, Davies and Zhang (1995) and Alderman and Gertler (1997) are interested in the effects of earning differentials (i.e. a gender wage gap) and biased parental preferences; they show that the sex with higher potential earnings and/or the preferred sex tend to receive better education. Alderman and King (1998), who look at the effect of gender-specific education costs, show that the sex with higher costs tends to receive less education.

The theoretical explanations presented above rely on exogenous differences between sexes. It is however debatable whether these differences are empirically observable and relevant in explaining gender difference in education and its variation across countries. In particular, empirical evidence on biased parental preference and gender-specific education costs appear to be weak.[3] Regarding the gender wage gap argument, although a large

[2]Most models assume (implicitly) an imperfect capital market setting indicating that children are unable to finance their education by borrowing. As a consequence, altruistic parents are the one who finance and decide on the education of their children. This is why parental preferences influence the decision on children's schooling.

[3]If at all significant, both factors seem to be valid in a few lower-income countries only which does not allow me to generalize the existence of such differences between sexes for all countries worldwide. Regarding the parental preference argument, Dollar and Gatti (1999) consider religious affiliation among other factors that influences gender difference in education. In a panel of 127 countries for the period of 1975-1990, they find that Protestants and Shintos are among the few religions that positively affect school attainment of girls. Whether parental preferences are biased towards one sex is also debated in sociology; a widely accepted

body of literature confirms that wage rates differ between men and women in both developed (Blau and Kahn, 1992, 1996, 2001) and developing countries (Appleton et al., 1999; Jurajda, 2003; Newell and Reilly, 1996; Pham and Reilly, 2007), its impact on gender difference in schooling is still questionable. Case studies on this impact have been applied to a few lower-income countries (Davies and Zhang, 1995; Alderman and Gertler, 1997) which do not allow generalization of the gender wage gap argument across all countries. Moreover, the argument may not apply for developed countries where gender wage gaps can be observed but an educational gap between boys and girls is not existing (in some developed countries, girls receive even slightly better education). So in order to explain cross-country variations of the gender gap in education, another explanation approach needs to be considered.

Unlike the previous theoretical models, Lagerlöf (2003) develops a setup which does not rely on any assumed difference between sexes. Parents are supposed to take into account the expected human capital of future spouses which makes education decisions interdependent between families. This coordination problem leads to gender difference in education although both sexes are *a priori* fully identical and do not face any type of discrimination. The reason is that parents choose less education for their children when they expect that the children's future spouses will be well educated. However, the disadvantage of Lagerlöf's model is that it generates a continuum of equilibria in the game played between families, where equilibria range between two extreme cases: parents only invest in the schooling of either boys or girls. In particular, the model fails to explain why a society selects to be in a certain equilibrium and can explain the cross-country variations in gender difference only as the result of a pure coincidence.

This chapter extends Lagerlöf's model and contributes to the existing literature in two ways. On the one hand, it develops a tractable framework with two long-run equilibria. In order to explain the link between fertility and gender inequality in education, I include the assumption that women raise children alone. This assumption allows the return of investment in education to differ between sexes, according to which one equilibrium is selected from the continuum of equilibria in the original Lagerlöf model. In combination with the trade-off between quantity and quality of children[4], the chapter links income per capita

consensus has not been reached yet. The study on household surveys of 246 Bolivian families between 2002-2003 by Godoy et al. (2006), for instance, suggests that parental preference can be biased towards boys or girls depending on whether husband or wife is dominant in the family. As far as the costs of education are concerned, Alderman et al. (2001) provide evidence from Pakistan where education costs for girls can be higher because the distance to school makes parents worry more for their daughters than sons.

[4]The quantity-quality trade-off literature, introduced by Becker (1960, 1991) and Becker and Lewis (1973), argues that parents face a trade-off between the number and the education of their children. In particular, parents with less human capital (and thus low income) have a comparative advantage in child quantity whereas educated parents (and thus high income) have a comparative advantage in child quality.

with gender difference in education via fertility. On the other hand, the chapter provides an explanation of cross-country variations in gender inequality through the existence of multiple steady states, without relying on any a priori heterogeneity between countries.

Specifically, I propose a two-sex overlapping-generations model with endogenous fertility and education decisions, where a couple maximizes joint utility taking into account the human capital of futures spouses. Both sexes are endowed with identical innate abilities and do not face any type of discrimination; yet, women are assumed to raise children alone. In this framework, the gender difference in education arises because the marginal return on education differs between sexes. The gender difference depends on the expected future fertility rate, i.e., the higher future fertility, the wider the education gap between boys and girls. The model generates two steady states, an outcome which corresponds to the actually observed variations in gender differences across countries. One steady state comprises relatively equal education for the two sexes, low fertility and high income per capita. The other steady state involves less equal education, higher fertility and lower income per capita.

In the remaining part of the Chapter, I describe the setup of the model in Section 2 and study optimal behavior of parents in Section 3. I solve for potential steady states in Section 4 and discuss the dynamics of the model in Section 5. Section 6 provides a numerical example and Section 7 concludes.

1.2 Model Description

I consider an overlapping-generations model with individuals living for two periods, childhood and adulthood, and belonging to one of the two sexes, male or female. The two sexes are born with identical abilities but may receive different amount of education in their childhood period. Males and females may therefore become heterogeneous with respect to their accumulated human capital. In adulthood, each individual randomly matches with one individual of the opposite sex and forms a couple.[5] The couple provides labor and decides on the number of children and the education of their children.

I assume that half of each couple's children are daughters and half are sons.[6] I further

The reason is because the relative costs of child quantity to child quality are rising in parents' human capital. As a consequence, the quantity-quality trade-off framework links parent's income and fertility.

[5]As will become apparent later in this chapter, individuals of the same sex and of the same generation are identical in their human capital. Since all potential candidates (i.e. individuals of the other sex) are identical, sorting in the marriage market does not play a role in this framework.

[6]In reality, some parents wish to influence the natural distribution of male and female offspring - for instance by abortion. In assuming that the distribution of the children's sex is exogenously given, I rule out such behavior in the model.

assume that adults of the same sex are endowed with identical human capital in the initial period 0. This assumption implies that all randomly matched couples are homogeneous and make the same decisions regarding the number of children and the investments in children's education. In the next period, adults of the same sex are therefore again identical in their accumulated human capital; moreover, randomly matched couples are homogeneous and make the same decisions. It is therefore sufficient to focus on one representative couple for each period t, which I refer to as couple t in the following.

Couple t derives utility from own consumption c_t and the continuous number of children n_t. Furthermore, the couple cares for the well-being of their children due to an altruistic motive. Couple t's joint utility function is given by

$$U(h_t^m, h_t^f) = \log[c_t] + \beta \log[n_t] + \gamma \frac{U(h_{t+1}^m, \tilde{h}_{t+1}^f) + U(\tilde{h}_{t+1}^m, h_{t+1}^f)}{2}.$$

with $\beta > 0$ (the couple likes children) and $\gamma \in]0, 1)$ (the couple discounts future utility).[7] As for the notation, h_t^m and h_t^f denote husband's and wife's human capital, h_{t+1}^m and h_{t+1}^f denote the expected human capital that each son and each daughter are going to accumulate and $\tilde{h}_{t+1}^m$ and $\tilde{h}_{t+1}^f$ denote the expected human capital of future sons and daughters in-law. Functions $U(h_{t+1}^m, \tilde{h}_{t+1}^f)$ and $U(\tilde{h}_{t+1}^m, h_{t+1}^f)$ denote the utility levels of couples in the next period who are going to be formed by their own children and children of the other families. Variables in the future do not contain an expectation operator because there is no uncertainty in the model and because I assume that parents form rational expectations.

Each partner of the representative couple is endowed with one unit of time. Partners use their time endowment for working, for childbearing and for child raising.[8] Bearing and raising one child requires τ units of time. To simplify the model, I assume that women bear and raise children alone. Hence, women devote $1 - \tau n_t$ units of time to the labor market while men work full-time.

The assumption that women bear and raise children alone is crucial for the results of the chapter. It can be justified by the empirical observation that usually women 'pay' the time cost of raising children. Moreover, I could extend the model and let partners decide on who shall take care of the couple's children. Echevarria and Merlo (1999) show in a bargaining model that in such an extended setting women alone take care of the children. The reason

[7]Logarithmic utility functions over consumption, children's quality and quantity are widely used in the literature for the *one-gender* case. The formulation presented here is a tractable functional form which extends the commonly used version to the case of two genders. Parents derive logarithmic utility from consumption and fertility; additionally, they value the utility or well-being of future generations. Doepke and Tertilt (2009) use in principle a similar functional form, though consumption is split between partners.

[8]Childbearing involves the process of conceiving, carrying and delivering a baby. Child raising (also referred to as parenting or child rearing) is then the process of promoting and supporting the development of a child from birth to adulthood.

is that child bearing is assigned to women for biological reasons; only child raising can be shared between partners. Though child bearing may potentially be a very short time compared to the time which is needed to raise a child, it nevertheless leads to a unique Nash equilibrium with lower education for girls than for boys. This gender difference in education induces couples not to share the child raising work and let women raise children alone. Appendix 1.8.2 provides more details of such an extension and proves that the result of Echevarria and Merlo (1999) also applies to the framework presented in this chapter.

Given the assumption on how partners use their time endowment, the couple's labor income derives as

$$\left[h_t^m + (1 - \tau n_t) h_t^f\right] w_t,$$

where the term in brackets states the human capital supplied to the labor market and where w_t denotes the wage each supplied unit of human capital earns. Partners use their labor income to finance consumption and investments in children's education:[9]

$$c_t + \frac{n_t}{2}(e_t^m + e_t^f) = \left[h_t^m + (1 - \tau n_t) h_t^f\right] w_t$$

with e_t^m and e_t^f denoting the investment in each boy's and each girl's education. Investing in a child's education raises his or her human capital in the next period. Human capital production follows a function with decreasing marginal returns and the parental investment in education as the only input factor:

$$h_{t+1}^i = f(e_t^i) = A\sqrt{e_t^i}$$

with the scale parameter A ($A > 0$) and with $i = m$ for a male and $i = f$ for a female.[10]

Production follows a constant returns-to-scale production function with human capital as the only input factor. The wage rate is thus independent of the human capital input and

[9]Bequests from the parent to the children generation do not exist in this model; investments in education are the only transmission channel between generations. This assumption is made for convenience reasons only and since the focus of this chapter is on investments in education as transmission channel. Introducing bequests does not qualitatively change the results presented in this chapter. The reason is that bequests would be simply a further investment opportunity and parents would equalize the returns of both investment opportunities.

[10]Taking also into account that parental human capital may directly influence the education outcome of their children, for instance by introducing parental human capital in the production function for human capital:

$$h_{t+1}^i = A\sqrt{e_t^i (h_t^m)^\alpha (h_t^f)^{1-\alpha}},$$

does not affect the results presented in this chapter. The reason is because, due to logarithmic utility, parental human capital would simply work as a multiplier and would not have any effect on the optimal decision on children's education.

constant over time:

$$w_t = w \quad \forall\, t.$$

A more detailed setup of the production side is not necessary because the results do not depend on the wage rate.

I define the gender gap in education as the ratio between girls' and boys' education:

$$\Delta e_t \equiv \frac{e_t^f}{e_t^m}.$$

The ratio takes values between zero (maximum inequality) and one (maximum equality) where a higher ratio indicates more equal education investments. I further define a steady state as the situation in which fertility rates and human capital of men and women are constant over time: for all t,

$$\begin{aligned} n_t &= n, \\ h_t^m &= h^m, \\ h_t^f &= h^f \end{aligned}$$

with n denoting the fertility rate, h^m men's and h^f women's human capital in the steady state. In the following, I derive the gender gap in education, fertility, human capital and income per capita in the steady state.

1.3 Optimal Decisions

In each period t, the representative couple chooses the consumption level, the number of children and the investment in children's education in order to maximize utility subject to the budget constraint and the natural constraint on fertility given the mother's time constraint:

$$\max_{c_t, n_t, e_t^m, e_t^f} U(h_t^m, h_t^f) = \log[c_t] + \beta \log[n_t] + \gamma \frac{U(h_{t+1}^m, \tilde{h}_{t+1}^f) + U(\tilde{h}_{t+1}^m, h_{t+1}^f)}{2}$$

$$\text{s.t.} \quad c_t + \frac{n_t}{2}(e_t^m + e_t^f) = \Big[h_t^m + (1 - \tau n_t) h_t^f\Big] w \tag{1.1}$$

$$\text{and} \quad n_t < \frac{1}{\tau}. \tag{1.2}$$

I solve this dynamic optimization problem using the dynamic programming approach. Let me define the value function

$$V(h_t^m, h_t^f) \equiv \max\ U(h_t^m, h_t^f) \text{ s.t. (1.1)}$$

which denotes the maximized utility of a couple who is endowed with $\{h_t^m, h_t^f\}$ units of human capital. Assuming that decisions in the next period are optimally chosen, I use the value function to replace $U(h_{t+1}^m, \tilde{h}_{t+1}^f)$ by $V(h_{t+1}^m, \tilde{h}_{t+1}^f)$ and $U(\tilde{h}_{t+1}^m, h_{t+1}^f)$ by $V(\tilde{h}_{t+1}^m, h_{t+1}^f)$ in the objective function. Note that distinguishing between the human capital of own children (i.e. h_{t+1}^m and h_{t+1}^f) and of their future in-laws (i.e. $\tilde{h}_{t+1}^m$ and $\tilde{h}_{t+1}^f$) is necessary because the couple has control over the human capital of their own children only. Substituting also the budget constraint into the objective function leads to the following Bellman equation:

$$V(h_t^m, h_t^f) = \max\Bigg\{ \log\left[\left[h_t^m + (1-\tau n_t)h_t^f\right]w - \frac{n_t}{2}(e_t^m + e_t^f)\right] + \beta\log[n_t] + \gamma\frac{V(h_{t+1}^m, \tilde{h}_{t+1}^f) + V(\tilde{h}_{t+1}^m, h_{t+1}^f)}{2}\Bigg\}.$$

The optimal decisions of the representative couple are characterized by the following first order conditions:

$$-\frac{\tau h_t^f w + \frac{e_t^m + e_t^f}{2}}{c_t} + \frac{\beta}{n_t} = 0, \tag{1.3}$$

$$-\frac{n_t}{2c_t} + \frac{\gamma}{2}\frac{\partial V(h_{t+1}^m, \tilde{h}_{t+1}^f)}{\partial e_t^m} = 0, \tag{1.4}$$

$$-\frac{n_t}{2c_t} + \frac{\gamma}{2}\frac{\partial V(\tilde{h}_{t+1}^m, h_{t+1}^f)}{\partial e_t^f} = 0. \tag{1.5}$$

In all three FOCs, the first term denotes marginal costs while the second term denotes the marginal benefit of having more children or educating them better. From the FOCs, I derive in the following optimal consumption, optimal fertility, optimal education investments and the optimal gender gap in education.

Using equations (1.1) and (1.3), optimal consumption derives as

$$(c_t)^* = \frac{1}{1+\beta}(h_t^m + h_t^f)w \equiv c(h_t^m, h_t^f). \tag{1.6}$$

The representative couple consumes a fixed proportion of their potential income where potential income is defined as the income level when both partners work full-time, i.e. $(h_t^m + h_t^f)w$. To shorten notation in the following, I define the function $c(h_t^m, h_t^f)$ which describes the consumption level of a couple endowed with the human capital vector $\{h_t^m, h_t^f\}$.

In order to derive optimal fertility and optimal education investments, the two derivatives in (1.4) and (1.5) have to be calculated. Using the envelope theorem, the derivatives derive as

$$\frac{\partial V(h_{t+1}^m, \tilde{h}_{t+1}^f)}{\partial e_t^m} = \frac{1}{c(h_{t+1}^m, \tilde{h}_{t+1}^f)} \frac{\partial h_{t+1}^m}{\partial e_t^m} w,$$

$$\frac{\partial V(\tilde{h}_{t+1}^m, h_{t+1}^f)}{\partial e_t^f} = \frac{1}{c(\tilde{h}_{t+1}^m, h_{t+1}^f)} \frac{\partial h_{t+1}^f}{\partial e_t^f} w(1 - \tau n_{t+1}).$$

Note that the optimal consumption equation (1.6) can be used to replace consumption in period $t + 1$, and note further that consumption in the next period is identical between couples because all couples will be endowed with the same vector of human capital:

$$\tilde{h}_{t+1}^m = h_{t+1}^m \qquad \text{and} \qquad \tilde{h}_{t+1}^f = h_{t+1}^f$$

$$\Rightarrow \quad c(h_{t+1}^m, \tilde{h}_{t+1}^f) = c(\tilde{h}_{t+1}^m, h_{t+1}^f) = \frac{1}{1+\beta}(h_{t+1}^m + h_{t+1}^f)w.$$

Applying these equations to the above derivatives leads to:

$$\frac{\partial V(h_{t+1}^m, \tilde{h}_{t+1}^f)}{\partial e_t^m} = \frac{1+\beta}{h_{t+1}^m + h_{t+1}^f} \frac{A}{2\sqrt{e_t^m}}, \tag{1.7}$$

$$\frac{\partial V(\tilde{h}_{t+1}^m, h_{t+1}^f)}{\partial e_t^f} = \frac{1+\beta}{h_{t+1}^m + h_{t+1}^f} \frac{A}{2\sqrt{e_t^f}}(1 - \tau n_{t+1}). \tag{1.8}$$

Plugging the two derivatives (1.7) and (1.8) back into the FOCs leads to a system of three equations which can be solved for the three unknowns. Optimal fertility and optimal education investments follow

$$(n_t)^* = \left(1 + \frac{h_t^m}{h_t^f}\right) \frac{B(n_{t+1})}{2(2 - \tau n_{t+1})}, \tag{1.9}$$

$$(e_t^m)^* = \frac{\gamma h_t^f w}{B(n_{t+1})}, \tag{1.10}$$

$$(e_t^f)^* = \frac{\gamma h_t^f w}{B(n_{t+1})}(1 - \tau n_{t+1})^2 \tag{1.11}$$

with

$$B(n_{t+1}) = \frac{2(\beta(4-\gamma)-\gamma) - 2(\beta(2-\gamma)-\gamma)\tau n_{t+1} - (1+\beta)\gamma(\tau n_{t+1})^2}{2(1+\beta)\tau}.$$

As a consequence, the optimal gender gap in education is given by

$$(\Delta e_t)^* = (1 - \tau n_{t+1})^2. \tag{1.12}$$

The optimal decisions in equations (1.9) to (1.12) depend on three variables: parents' human capital (h_t^m, h_t^f) and the expected fertility rate in the next period (n_{t+1}). The optimal fertility rate in equation (1.9), for instance, is rising with the father's and falling with the mother's human capital which fits well empirical observations.[11] The reason is because rising father's human capital increases the income of the couple and lets parents invest more in child quantity. When the mother's human capital is rising, a similar income effect is present; yet, the income effect is accompanied by a substitution effect because the marginal costs of having children are rising in the mother's human capital. In this case, the couple shifts funds away from child quantity to child quality.[12]

With respect to education investments, equations (1.10) and (1.11) show that education investments are rising in the mother's human capital due to the above discussed substitution effect.[13] Furthermore, optimal education investments are rising in γ because parents pay more attention to the well-being of their children. The positive effect of w is caused by the fact that investments in children's education have a higher pay-off when the wage rate is rising.

The optimal gender gap in education is falling in n_{t+1} because parents equalize the marginal benefits of education investments when deciding on their children's education.

[11] See, for instance, Butz and Ward (1979), Schultz (1985) and Heckman and Walker (1990).

[12] See Appendix 1.8.3 for more details.

[13] There has been a widely held believe in the literature that mother's schooling has a greater impact on children's education than father's schooling. Heckman and Hotz (1986) and Haveman and Wolfe (1995), for instance, find in their empirical studies a significantly stronger effect of mother's than father's schooling. The result of the model with respect to the optimal education investments is in line with this believe. However, some (more recent) studies do not support this believe. Behrman (1997), for instance, reviews a large number of available studies and does not find "a tendency for much greater impact of mother's schooling than of father's schooling on child education outcomes". Behrman and Rosenzweig (2002) use a data set of identical twins and do not find a significantly stronger effect of mother's schooling.

This effect can be seen when setting (1.4) and (1.5) equal to each other which yields

$$\frac{\partial V(h_{t+1}^m, \tilde{h}_{t+1}^f)}{\partial e_t^m} = \frac{\partial V(\tilde{h}_{t+1}^m, h_{t+1}^f)}{\partial e_t^f}.$$

The benefits derive as the marginal effect of education on children's utility. Since women bear and raise children alone and hence work less than men do, the benefit of investing in a girl's education is lower than the investment in a boy's education. The more children are expected to be born in the next period, the larger is the difference in marginal benefits between sexes and thus the more unequal the investments in children's education.

Note that for any vector $\{h_t^m, h_t^f, n_{t+1}\}$, equations (1.9) to (1.12) uniquely determine the optimal decision of couple t. In period t, parent's human capital $\{h_t^m, h_t^f\}$ is given and known.

1.4 Steady States

In the following, I derive the gender gap in education, fertility, human capital and income per capita in the steady state.

The gender-gap in education in the steady state, denoted by Δe, can be derived by applying the steady state definition to equation (1.12) which leads to

$$\Delta e = (1 - \tau n)^2. \tag{1.13}$$

It is uniquely determined by the fertility rate in the steady state: the higher fertility, the less equal investments in education between sexes.

Steady state fertility is obtained by applying the steady state definition to equation (1.9) and by using (1.13) to replace the human capital ratio $\frac{h^m}{h^f}$ which leads to

$$n = \left(1 + \frac{1}{1 - \tau n}\right) \frac{B(n)}{2(2 - \tau n)}. \tag{1.14}$$

This equation has two potential solutions for n:

$$n_{low} = \frac{2 + \beta(4 - \gamma) - \gamma - \sqrt{D}}{(4 - \gamma)(1 + \beta)\tau}, \tag{1.15}$$

$$n_{high} = \frac{2 + \beta(4 - \gamma) - \gamma + \sqrt{D}}{(4 - \gamma)(1 + \beta)\tau} \tag{1.16}$$

with

$$D = 4 - \beta^2(4-\gamma)^2 + 4\gamma - \gamma^2 - 2\beta(8 - 6\gamma + \gamma^2).$$

The two steady states exist when certain conditions hold, as the following proposition shows.

Proposition 1. *Steady states exist when the following parameter restriction is fulfilled:*

$$\frac{\gamma}{4-\gamma} < \beta < \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}. \tag{1.17}$$

Proof. See Appendix 1.8.4 □

The inequalities in proposition 1 state that β (the weight on fertility in the utility function) should neither be too low nor too high compared to γ (the weight of children's well-being in the utility function). If either fertility or children's well-being has too much weight in the utility function, a steady state does not exist in the model.

Parental human capital in the steady state can be derived by inserting optimal education investments as given in equations (1.10) and (1.11) into the production function of human capital and using the steady state definition. Parental human capital follows

$$h^m = \frac{\gamma A^2 w}{B(n)}(1-\tau n), \tag{1.18}$$

$$h^f = \frac{\gamma A^2 w}{B(n)}(1-\tau n)^2. \tag{1.19}$$

Human capital is increasing in the weight γ, in the productivity of education investments A and the wage w. The fertility rate n, in contrast, has a negative effect on human capital levels (see Appendix 1.8.5 for a proof). The intuition for the negative relationship between steady-state human-capital and fertility is the following. We know that a couple spends a fixed proportion of their potential income on consumption and the rest on children. When the couple invests more in child quantity, investments in child quality are consequently decreasing. The lower investments in children's education in turn imply that future generations accumulate less human capital.

Knowing fertility and human capital in the steady state, I can determine income per capita in the steady state. Income per capita derives from family's labor income divided by the number of family members. Its steady state level is denoted by y and derives as

$$y = \frac{\left[h^m + (1-\tau n)h^f\right]w}{2+n}. \tag{1.20}$$

In order to compare income per capita between the two steady states, I discuss the effect of fertility on income per capita in the following. Fertility affects income per capita in three ways: first, the more children a family has, the higher the number of family members. Second, higher fertility lowers the human capital of both husband and wife as stated above. Third, a larger number of children reduces the wife's human capital supplied to the labor market because she spends more time at home raising the couple's children. All three effects imply that income per capita is decreasing in fertility. As a consequence, income per capita is lower in the steady state with a higher fertility rate.

To summarize the findings regarding steady states: assuming that conditions in (1.17) are satisfied, the model exhibits two steady states.[14] One steady state comprises a low fertility rate, high equality in education between sexes and high income per capita. In the second steady state, the fertility rate is higher, education is less equal between sexes and income per capita is lower than in the first one. The following table summarizes the characteristics of the two steady states:

Steady State	Fertility Rate	Gender Equality in Education is	Income per capita is
1	n_{low}	high	high
2	n_{high}	low	low

Note that gender equality and income per capita vary between steady states. While gender equality is negatively related to the fertility rate, it is positively related to income per capita. In other words, the model can reproduce the observed differences between countries via the multiplicity of steady states. The driving force for this result is the interdependence between the fertility rate and the gender gap in education. In societies with a high fertility rate, future earnings differ a lot between men and women. This difference in future earnings leads to a large difference in education investments between sexes. Since women accumulate only little human capital compared to men, the cost of having children is low and hence, couples choose to have many children in the future. In societies with a low fertility rate, in contrast, future earnings are closer together between sexes and thus education investments in boys and girls are more equal. Since women are relatively well educated, implying that the cost of having more children is high, couples in these societies choose a low fertility rate.

[14] Using the Hessian matrix, I prove that utility is concave and thus maximized in both steady states. Appendix 1.8.6 provides details of the proof.

1.5 Dynamics

In this section, I analyze how the economy reaches a steady given the initial conditions. I focus first on the dynamics of the fertility rate because it is the central variable in the model. I discuss the dynamics of the other variables afterwards.

As derived in equation (1.9), the optimal fertility rate depends on parental human capital and the expected fertility rate of the next period. To shorten notation, I define the function η: for all t,

$$n_t = \left(1 + \frac{h_t^m}{h_t^f}\right) \frac{B(n_{t+1})}{2(2 - \tau n_{t+1})} \equiv \eta\left(\frac{h_t^m}{h_t^f}, n_{t+1}\right).$$

In the initial period, the ratio of parental human capital is given. As a consequence, fertility in period 0 depends on n_1 only:

$$n_0 = \eta\left(\frac{h_0^m}{h_0^f}, n_1\right). \tag{1.21}$$

For all future periods, however, the ratio of parental human capital is endogenously determined in the model. Since today's human capital ratio depends on yesterday's investments in education, I can express the human capital ratio by the gender gap in education as follows:

$$\frac{h_t^m}{h_t^f} = \frac{A\sqrt{e_{t-1}^m}}{A\sqrt{e_{t-1}^f}} = \frac{1}{\sqrt{\Delta e_{t-1}}} = \frac{1}{1 - \tau n_t} \qquad \forall t \geq 1. \tag{1.22}$$

The optimal choice of the fertility rate then follows

$$n_t = \eta\left(\frac{1}{1 - \tau n_t}, n_{t+1}\right) \qquad \forall t \geq 1, \tag{1.23}$$

where the fertility rate n_t depends on the expected fertility rate in the next period n_{t+1} only. By the same analogy, fertility in $t+1$ depends on fertility in $t+2$ and so on. I therefore look for a set of fertility rates which solves the two equations (1.21) and (1.23).

Solving equation (1.23) for n_t yields two possible solutions for any expected n_{t+1}:

$$n_t = \left\{\frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} - D}{4\tau(2 - \tau n_{t+1})}, \frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} + D}{4\tau(2 - \tau n_{t+1})}\right\}$$

with

$$D = \sqrt{[4 + \tau B(n_{t+1}) - 2\tau n_{t+1}]^2 - 16(2 - \tau n_{t+1})\tau B(n_{t+1})}.$$

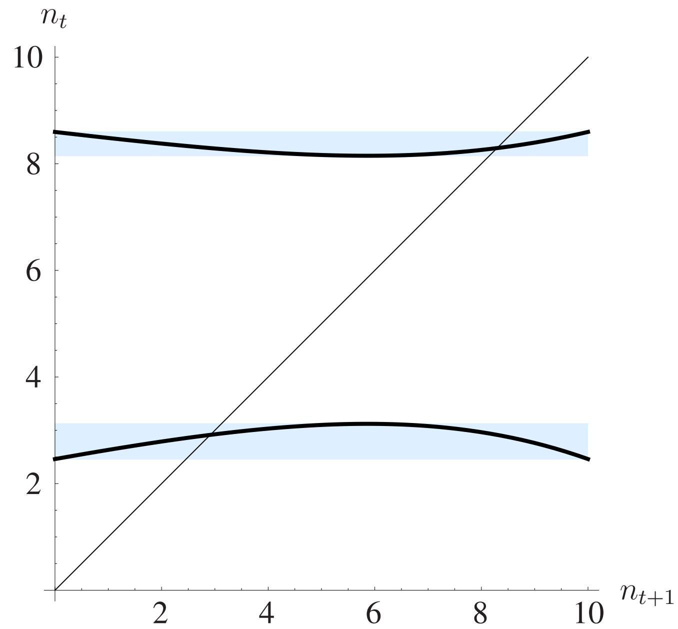


Figure 1.3: Choosing today's fertility given tomorrow's expected fertility.

The two solutions are presented graphically in Figure 1.3.[15] I use this figure in the following to explain how the expected fertility rate for period 1 is determined. Note that the figure describes the relationship between today's and tomorrow's fertility rate for all periods t with $t \geq 1$ and that the fertility rate today (i.e. n_t) is not predetermined.

Consider, for instance, the choice of fertility in the far future, say, in 10 periods. For any fertility rate between zero and $\frac{1}{\overline{\tau}+\tau}$ in period 11, only few choices of n_{10} are optimal as marked by the two shaded areas. By the same logic and going further back in time, the range of optimal fertility rates narrows down further until there are only two options left: the two steady state levels which are given by the intersection of the two curves with the 45-degree line. I therefore conclude that couple 0 can only choose between $n_1 \in \{n_{low}, n_{high}\}$.[16] The dynamic behavior of the fertility rate is then described by

$$n_t = \begin{cases} \eta\left(\frac{h_0^m}{h_0^f}, n\right) & \text{for } t = 0, \\ n & \text{for } t \geq 1, \end{cases}$$

with $n \in \{n_{low}, n_{high}\}$, which in other words means that the economy jumps into its steady state in period 1.

Whether this result is of general nature or applies to certain parameter sets only is

[15]I choose $\beta = 0.3$, $\gamma = 0.5$ and $\tau = 0.1$ which fulfill the conditions of the model.

[16]Note that the fertility rate n_t is not predetermined in Figure 1.3. The only rationally expected fertility rate is then one of the two equilibria levels.

addressed in the following proposition.

Proposition 2. *The dynamics of the fertility rate described by*

$$n_t = \begin{cases} \eta\left(\frac{h_0^m}{h_0^f}, n\right) & \textit{for } t = 0, \\ n & \textit{for } t \geq 1, \end{cases}$$

with $n \in \{n_{low}, n_{high}\}$ *holds for all parameter sets which fulfill* (1.17).

Proof. See Appendix 1.8.7 □

It follows from the above proposition that, as long as steady states exist, the economy jumps directly to one of the steady states.

Knowing the evolution of fertility, I can analyze the dynamic behavior of the other variables. The gender gap in education of period t, for instance, is uniquely determined by the expected fertility rate in $t+1$. Since fertility is in its steady state level from period 1 onward, the gender gap in education is equal to its corresponding steady state level already from period 0 onward:

$$\Delta e_t = \Delta e = (1 - \tau n)^2 \quad \forall t.$$

Parental human capital and income per capita, in contrast, follow different dynamic patterns. I know from the production function of human capital and from optimal education investments that parental human capital follows

$$h_{t+1}^m = A\sqrt{e_t^m} = A\sqrt{\frac{\gamma h_t^f w}{B(n_{t+1})}} \equiv \phi^m(h_t^f, n_{t+1}),$$

$$h_{t+1}^f = A\sqrt{e_t^f} = A\sqrt{\frac{\gamma h_t^f w}{B(n_{t+1})}(1 - \tau n_{t+1})} \equiv \phi^f(h_t^f, n_{t+1}),$$

where the functions $\phi^m(h_t^f, n_{t+1})$ and $\phi^f(h_t^f, n_{t+1})$ denote each man's and each woman's human capital. Note that the function for woman's human capital $\phi^f(h_t^f, n)$ is concave in h_t^f since fertility is constant from period 1 onward. As a consequence, woman's human capital always converges to its steady state value independent of the initial level h_0^f. Man's human capital follows a similar pattern over time. As for the dynamics of income per capita, I deduct that income per capita converges to its steady state level regardless of the initial level because it is mainly determined by the human capital of men and women.

1.6 Numerical Example

A numerical example shall illustrate the differences between the two steady states. I use empirical data to calibrate the model and discuss in the following (i) the choice of parameters and (ii) calculate the variables of interest for the two steady states.

The model has been set up with the following parameter restrictions:

$$\begin{aligned} &0 < A, \\ &0 < w, \\ &0 < \beta, \\ &0 < \gamma < 1, \\ &0 < \tau < 1. \end{aligned}$$

Additionally, the following restrictions have to be fulfilled to ensure the existence of steady states:

$$\frac{\gamma}{4-\gamma} < \beta < \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}.$$

When choosing parameter values, I aim to fit the steady state with low fertility to US data. I choose parameters as described in the following:

1. I start with choosing γ. To my knowledge, there is no consensus over the level of γ in the literature. I decided to take the average level of 0.5 and two other levels as robustness check: $\gamma \in \{0.1, 0.5, 0.9\}$.

2. Parameter β has to be chosen such that the condition for the existence of steady states is fulfilled. I therefore set the level of this parameter to the average of the lower and upper boundary defined by the inequality shown above:

$$\beta = \frac{\gamma}{4-\gamma} + \frac{1}{2}\frac{\sqrt{8}-2}{4-\gamma}.$$

3. I choose τ such that n_{low} fits the fertility rate of the US in 2010 ($n_{US} = 2.1$). Using equation (1.15), I find that $\tau = 0.101467$ fulfills this condition for any γ. Some literature contributions find that the proportion of woman's time endowment which is needed to bear and raise one child in the range of 0.05 to 0.1 and provide some empirical evidence for their choice.[17] The value derived here is very close to this

[17] See Echevarria and Merlo (1999) and Erosa, Fuster and Restuccia (2002).

		SS with low fertility			SS with high fertility			
γ	β	n	Δe	y	n	Δe	y	$\frac{(7)}{(12)}$
0.1	0.13	2.1	0.62	9,403	8.7	0.014	547	17.2
0.5	**0.26**	**2.1**	**0.62**	**47,016**	**8.7**	**0.014**	**2,735**	**17.2**
0.9	0.42	2.1	0.62	84,629	8.7	0.014	4,923	17.2
Other parameters: $A = 1$, $w = 1,000$ and $\tau = 0.101467$								

Table 1.1: Numerical illustration of the two steady states

range and thus can be deemed as appropriate.

4. Parameters A and w are pure scale parameters affecting aggregated human capital and income per capita. Both parameters do not affect the variables of interest, i.e., neither the fertility rate nor the gender gap in education in the steady state. I normalize the value of A to one and choose the value of w so as to bring income per capita in the steady state with low fertility close to the data of the US in 2010 ($y_{US} = 46,500$ USD). This is approximately achieved when $w = 1,000$.

Having fixed the parameters of the model, I use equations (1.13), (1.15), (1.16), (1.18), (1.19) and (1.20) to calculate the fertility rate, the gender gap in education and the income per capita for both steady states. The results are presented in Table 1.1. The first column of the table presents the choice of γ; the second column presents the corresponding value of β; columns 3 to 5 contain the data of the steady state with low fertility while columns 6 to 8 describe the second steady state with high fertility; in the last column, I calculate the ratio between the income per capita of both steady states.

The table shows that the actual choice of β and γ does not affect the most interesting variables which are the fertility rates, the gender gaps in education and the ratio of income per capita between the two steady states. Hence, the results are robust to variations in β and γ with respect to these values. In the following, I focus on the highlighted case with $\gamma = 0.5$ and $\beta = 0.26$.

In the highlighted numerical example, the model generates a difference in fertility of 2.1 to 8.7 and in income per capita of 17.2 to 1 between the two steady states. As for the gender gap in education, the model generates a value of 0.62 for the steady state with low fertility and of 0.014 for the steady state with high fertility. These numbers mean that investments in girls' education correspond to 62% or 1.4% of the investments in boys' education respectively. Hence, countries differ substantially when being in different steady states though they are assumed to be similar in their fundamental characteristics.

Taking the US as an example for a country in the steady state with low fertility, countries in the other steady state have a fertility rate of 8.7 children per woman and an income per capita of approx. 2,735 USD. In the data, I find that the highest fertility rates of 6 to 7 (children per woman) are observed in countries with income per capita below 1,000 USD. I conclude from this result that the spread in fertility and income per capita generated by the model between the two steady states fits real data well.

As for the gender gap in education, the model generates substantial differences between the two steady states (0.62 to 0.014). Taking the gender parity index (GPI) as a measure for gender equality in reality, the US achieves a value of 1.11 while countries with the highest fertility rates achieve values of around 0.65.[18] These numbers show also a sizable difference in the education gender gap between countries with low and with high fertility in reality. While both sexes receive similar education (or even slightly better education for girls)[19] in the US, countries with the highest fertility rates offer much better education for their boys than for their girls.

However, the absolute values of Δe are much lower in the model than in reality; in other words, the numerical example generates too much inequality in education investments. The reason is that the difference in education investments is determined by the earning opportunities of men and women alone. More precisely, the next period's fertility rate determines the expected earning ratio and thus the targeted human capital ratio for the next period (see equation (1.22)):

$$\frac{h_{t+1}^f}{h_{t+1}^m} = \frac{1 - \tau n_{t+1}}{1}.$$

If a couple is expected to have 2.1 children, parents in period t invest in children's education in such a way that women accumulate only 79% of men's human capital in the next period. So even a relatively small fertility rate generates already a sizable difference between genders in the model. Note that this difference is even amplified when taking a look at education investments. The reason is the assumption of decreasing marginal returns in the form of a root-function in human capital production, which leads to

$$\Delta e_t = \left[\frac{h_{t+1}^f}{h_{t+1}^m} \right]^2.$$

[18]Note that the GPI measures the ratio between females to males in primary, secondary and tertiary education. One should correct this number by the females to males ratio in the relevant age group. As long as parents do not interfere the natural distribution of sexes, this ratio (also referred to as secondary sex ratio) is around $\frac{100}{105} \approx 0.95$.

[19]The high value of 1.11 for the US could be due to the female advantage in tertiary education: around 60% of the Bachelor's Degrees are awarded to women. Only recently has the literature started to discuss this phenomenon (Buchmann and DiPetre, 2006).

To achieve a human capital ratio of 79%, investments in girls' education are about 62% of boys' education. In reality, the effect of the fertility rate might be weaker than in the model and not the only factor influencing the education decision. For instance, parents may simply value equity among their children which is not included in the model of this chapter. Parents may also wish that their daughters do not depend so much on their future husbands' income and educate girls better. Furthermore, the effect of decreasing marginal returns in human capital production might be weaker in reality than in the model.

1.7 Conclusion

Empirical data show evidence of significant cross-country variations in gender difference in education. Moreover, they suggest a negative relationship between fertility and gender equality and a positive relationship between per capita income and gender equality. The objective of this chapter is to explain these phenomena by developing a new and tractable theoretical framework.

Taking the Lagerlöf (2003) framework as the starting point, I propose a two-sex overlapping-generations model with endogenous fertility and education decisions, where women are assumed to bear and raise children alone. Additionally, I reformulate the utility function in order to make the new framework tractable. In this framework, gender difference in education arises because marginal benefits of education differ between sons and daughters. Since women bear children and spend more time than men in raising them, they can only spend less time in the labor market compared to men; hence, investing in daughters' education is less beneficial than in sons'.

The model involves two steady states, an outcome which reflects actual variations in gender equality across countries. One steady state comprises relatively equal education between boys and girls at low level of fertility and high income per capita. In the other steady state, education is less equal at high level of fertility and low income per capita. The analysis shows that a small difference between men and women in time spent on child bearing is enough to cause significant inequality in education. Additionally, the chapter demonstrates that gender difference in education can vary even across *a priori* identical countries. Using a numerical example, I show that the results of this chapter qualitatively fit empirical data. Yet, the degree of the schooling difference between sons and daughters is larger in the numerical example than in reality.

The chapter highlights the impact of fertility and the role of dividing the child raising task between parents on gender difference in education. The findings in this chapter suggest that any policy instrument which affects the fertility decision of a couple will also have an impact, although indirectly, on gender equality in education. The relationship between the fertility rate and the gender equality may also be exploited in the opposite direction: any policy measure which aims to make schooling more equal between sexes will lead to lower fertility and higher income levels.

The analysis in this chapter leaves space for future research. First, an empirical test of the links between gender difference in schooling, fertility and income per capita could be considered. In order to obtain reliable and generalized results, such an empirical study should work with a representative sample of developed and developing countries. However, the scarcity of observations on gender difference in schooling over all levels of education (i.e., primary, secondary and tertiary) can be an issue.

Second, other factors which potentially impact gender difference in schooling could be considered in both theoretical and empirical studies. For instance, *a priori* heterogeneities across countries may play a role in explaining variations in the gender difference. Some of these heterogeneities are, for instance, divergent quality of education system (which leads to country-specific production-functions for human capital), or differences in organizing child care (implying that the time cost of raising one child differs between countries).

Third, the evolution of gender equality over time and its impact on economic growth have not received much attention: dynamic two-sex models have been recently introduced in the endogenous growth literature.[20] In order to contribute to this literature strand, the framework applied in this chapter could be extended to the unified growth theory[21] which can be achieved by modeling the production sector in more detail and including endogenous technological progress. Such an analysis sheds light on how gender equality is related to economic growth and how it is related to the demographic transition. I leave these extensions for future research.

[20]Echevarria and Moe (2000) provide an overview of dynamic two-sex models and provide arguments why such models are useful.

[21]Galor and Weil (2000) and Galor (2005) present a unified framework which captures Malthusian stagnation, sustained economic growth and the transition between the two regimes. Lagerlöf (2006) provides a quantitative exercise to the theoretical model of Galor and Weil (2000). The driving force is the interaction between technological progress and the size and composition of the population.

Appendix

1.8.1 Data

The data used in this chapter were collected from the *Gender Info 2010* database, which is provided by the United Nations Statistics Division in collaboration with the United Nations Children's Fund (UNICEF) and the United Nations Population Fund (UNFPA). The collected data cover the gender parity index (GPI), the total fertility rate (TFR) and income per capita. Note that the GPI is defined as the ratio between female to male students in primary, secondary and tertiary education within this database; the TFR follows the standard definition and denotes the average number of children that would be born to a woman over her lifetime; income per capita is denoted in USD and controlled for the purchasing power parity (PPP). All data are for the year 2010 and 62 countries because the number of countries where all three variables are available is the highest in that year.[22] An overview of the countries and the data is provided in Table 1.2.

1.8.2 Sharing the Child Raising Work

I assume in the chapter that women give birth and raise the couple's children alone. In reality, however, the child raising task can be shared between husband and wife. This Appendix analyzes in an adapted setting where partners can share the child raising work who raises the couple's children. It also proves whether the assumption that women raise children alone is critical or not.

So far, τ units of the wife's time endowment have been necessary to bring up one child. In the following, I assume that child bearing needs $\hat{\delta}$ units and child raising needs δ units of time. The two setting are related to each other by the equation $\tau = \hat{\delta}+\delta$. Since only women can give birth, the time cost of $\hat{\delta}$ is borne by women alone. Child raising, in contrast, can be shared between partners such that $\delta_t^m + \delta_t^f = \delta$ holds where δ_t^m denotes the husband's and δ_t^f the wife's part in raising one child and where $\delta_t^m, \delta_t^f \geq 0$. Both variables are indexed by t because each generation may choose a different sharing rule. It is plausible to assume that $\hat{\delta} < \delta$ because only a small proportion of the overall time needed to bring up a child (i.e. $\hat{\delta} + \delta$) is assigned to the wife for biological reasons.

[22]The availability of the GPI has been the most limiting factor. Alternatively, one could find a measure for gender difference in education in other databases. However, all other databases provide such data separately for primary, secondary and tertiary education. The advantage of the *Gender Info 2010* database is that it provides the GPI cumulatively over all three education levels which provides a much better and more complete figure of gender inequality in a country.

Country	TFR	GPI	$\log[y]$	Country	TFR	GPI	$\log[y]$
Angola	5.4	0.78	3.64	Kazakhstan	2.6	1.05	3.97
Armenia	1.7	1.07	3.48	Laos	2.7	0.88	3.04
Australia	1.9	1.04	4.76	Latvia	1.5	1.12	4.03
Azerbaijan	2.2	0.98	3.76	Lebanon	1.8	1.07	3.94
Barbados	1.6	1.23	4.19	Lithuania	1.5	1.10	4.04
Bhutan	2.4	1.00	3.34	Malawi	6	1.00	2.55
Bosnia and Herzegovina	1.1	1.05	3.65	Mali	6.3	0.78	2.79
Brunei Darussalam	2	1.05	4.49	Malta	1.3	0.98	4.29
Bulgaria	1.5	1.03	3.80	Mauritania	4.5	0.99	2.96
Burkina Faso	5.9	0.85	2.73	Mexico	2.3	1.02	3.96
Burundi	4.3	0.93	2.39	Mongolia	2.5	1.10	3.35
Cameroon	4.5	0.85	3.08	Montenegro	1.7	1.04	3.81
Cape Verde	2.4	1.06	3.52	New Zealand	2.2	1.09	4.52
Central African Republic	4.6	0.68	2.65	Niger	7.1	0.78	2.56
Chad	6	0.63	2.90	Norway	1.9	1.08	4.93
China	1.6	1.04	3.65	Peru	2.5	1.01	3.73
Colombia	2.4	1.05	3.79	Qatar	2.3	1.17	4.86
Croatia	1.5	1.10	4.13	Republic of Korea	1.3	0.90	4.32
Cuba	1.5	1.14	3.76	Romania	1.4	1.07	3.88
Cyprus	1.5	0.98	4.45	Rwanda	5.4	1.01	2.72
Egypt	2.7	0.95	3.42	Saint Lucia	2	1.05	3.82
El Salvador	2.3	0.98	3.54	Sao Tome and Principe	3.7	1.00	3.10
Eritrea	4.5	0.78	2.61	Saudi Arabia	2.8	1.00	4.22
Ethiopia	4.2	0.86	2.51	Senegal	4.8	0.97	3.01
Finland	1.9	1.07	4.64	Serbia	1.6	1.06	3.70
Guyana	2.3	1.12	3.48	Sweden	1.9	1.10	4.69
Indonesia	2.1	1.00	3.47	Switzerland	1.5	0.99	4.86
Iran	1.7	0.94	3.76	Tajikistan	3.3	0.85	2.91
Jamaica	2.3	1.04	3.69	Ukraine	1.4	1.05	3.48
Japan	1.4	0.98	4.64	United States	2.1	1.11	4.67
Jordan	3.1	1.04	3.63	Vietnam	1.8	1.01	3.08

Table 1.2: Collected data for gender equality (GPI), fertility (TFR) and income per capita ($\log[y]$).

Couple's maximization problem reads:

$$\max_{c_t, n_t, e_t^m, e_t^f, \delta_t^f} \quad U(h_t^m, h_t^f) = \log[c_t] + \beta \log[n_t] + \gamma \frac{U(h_{t+1}^m, \tilde{h}_{t+1}^f) + U(\tilde{h}_{t+1}^m, h_{t+1}^f)}{2}.$$

$$\text{s.t.} \quad c_t + \frac{n_t}{2}(e_t^m + e_t^f) = \Big[\big(1 - (\delta - \delta_t^f)n_t\big)h_t^m + \big(1 - (\hat{\delta} + \delta_t^f)n_t\big)h_t^f\Big]w.$$

where I used $\delta_t^m + \delta_t^f = \delta$ to substitute for δ_t^m. The dynamic optimization problem can be solved using the dynamic programming approach. The system of first order conditions determines the optimal decisions.

The optimal sharing rule derives as

$$\left(\delta_t^f\right)^* \begin{cases} = 0 & \text{if } h_t^f > h_t^m, \\ \in [0, \delta] & \text{if } h_t^f = h_t^m, \\ = \delta & \text{if } h_t^f < h_t^m. \end{cases} \tag{1.24}$$

with $\left(\delta_t^m\right)^* = \delta - \left(\delta_t^f\right)^*$. The partner with higher human capital does not participate in child raising because his (or her) marginal labor income is higher than that of the partner with lower human capital. Couple t's fertility rate follows

$$\left(n_t\right)^* = G\left(\delta_{t+1}^f\right) = \frac{h_t^f + h_t^m}{(\hat{\delta} + \delta_t^f)h_t^f + (\delta - \delta_t^f)h_t^m} \times \left[\frac{\beta}{1+\beta} - \frac{\gamma}{4} \frac{\left(1 - (\delta + \delta_{t+1}^f)n_{t+1}\right)^2 + \left(1 - (\hat{\delta} - \delta_{t+1}^f)n_{t+1}\right)^2}{2 - (\delta + \hat{\delta})n_{t+1}}\right] \tag{1.25}$$

and shall be described by a function $G(.)$ depending on the expected sharing rule in the next period δ_{t+1}^f only.[23] Finally, couple t chooses education for their children according to

$$\left(e_t^m\right)^* = \frac{[1 - (\delta - \delta_{t+1}^f)n_{t+1}]^2}{(n_t)^*} B_t, \tag{1.26}$$

$$\left(e_t^f\right)^* = \frac{[1 - (\hat{\delta} + \delta_{t+1}^f)n_{t+1}]^2}{(n_t)^*} B_t \tag{1.27}$$

with $B_t = \frac{1}{2} \frac{(h_t^m + h_t^f)\gamma w_t}{2 - (\delta + \hat{\delta})n_{t+1}}$. Children's education depends crucially on two factors: (i) the

[23] In fact, optimal fertility depends also on other factors. However, I do not include these factors as arguments in the $G(.)$-function to keep notation simple and because these factors are not crucial for our analysis here.

expected sharing rule δ^f_{t+1} and (ii) the expected number of children in the next period n_{t+1} because both determine the benefit of education investments for boys and for girls. I did not replace the optimal number of children $(n_t)^*$ in the above equations to keep the two formulas short and tractable.

The next step is to combine equations (1.24) to (1.27) in order to determine explicitly the optimal education choice. Combining the equations leads to

$$e^m_t = \begin{cases} \frac{1}{n^\delta_t} B_t & \text{if } \tilde{e}^f_t < \frac{1}{n^\delta_t} B_t, \\ \frac{[1-\delta n_{t+1}]^2}{n^0_t} B_t & \text{else.} \end{cases} \tag{1.28}$$

$$e^f_t = \begin{cases} \frac{[1-\hat{\delta} n_{t+1}]^2}{n^0_t} B_t & \text{if } \tilde{e}^m_t < \frac{[1-\hat{\delta} n_{t+1}]^2}{n^0_t} B_t, \\ \frac{[1-(\hat{\delta}+\delta) n_{t+1}]^2}{n^\delta_t} B_t & \text{else.} \end{cases} \tag{1.29}$$

with $n^\delta_t = G(\delta)$ and $n^0_t = G(0)$. In (1.28) and (1.28), two cases are distinguished: (i) my children receive better education than their future partners (i.e. $e^m_t > \tilde{e}^f_t$ and $e^f_t > \tilde{e}^m_t$) or (ii) my children receive lower than or the same education as their future partners. Depending on the gender gap in education,parents form their belief over the future sharing rule. When boys are expected to receive better education than girls, women will raise the children alone in the next period ($\delta^f_{t+1} = \delta$). In this case, couple t chooses its fertility rate as $n^\delta_t = G(\delta)$. When boys are expected to receive less education than girls, men will raise the children alone in the next period ($\delta^f_{t+1} = 0$). Couple t then chooses fertility as $n^0_t = G(0)$. Note that fertility in the first case is always smaller than in the second:

$$n^0_t - n^\delta_t = \frac{\delta\hat{\delta}\gamma(h^m_t + h^f_t)(n_{t+1})^2}{2[(\delta - \delta^f_t)h^m_t + (\hat{\delta} + \delta^f_t)h^f_t][2 - (\delta + \hat{\delta})n_{t+1}]} > 0,$$

$$\Rightarrow \qquad n^\delta_t > n^0_t.$$

The reason for this result is that human capital is accumulated following a function with decreasing marginal returns. To reach a certain sum of human capital, it is better to invest rather similar amounts in children's education. In other words, the more unequal investments in boys' and girls' education, the higher must be the sum invested in children's education. It follows then that a lower budget is available for investments in the number of children. As a consequence, the more unequal education investments are, the lower the fertility rate.

Using equations (1.28) and (1.29), I can determine the optimal education choice. The unique Nash equilibrium in the game played between families when choosing education

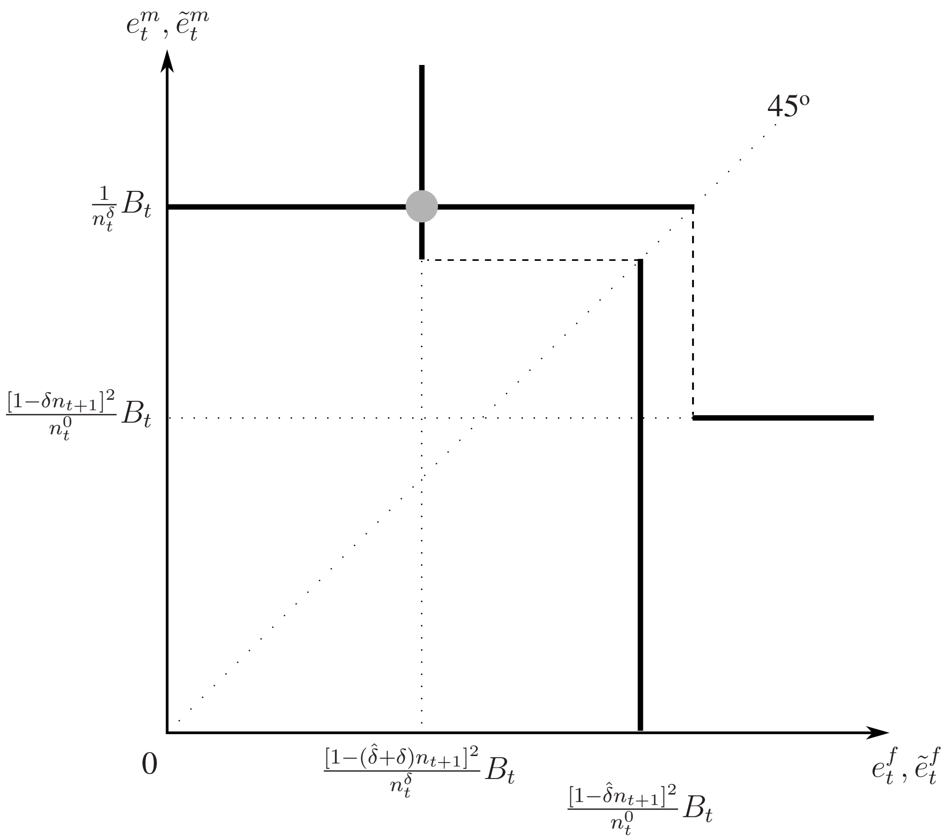


Figure 1.4: Optimal education for boys and girls in period t.

for their children derives as

$$\left(e_t^m\right)^{Nash} = \frac{1}{n_t^\delta}A_t, \tag{1.30}$$

$$\left(e_t^f\right)^{Nash} = \frac{[1-(\hat{\delta}+\delta)n_{t+1}]^2}{n_t^\delta}A_t \tag{1.31}$$

and is depicted in Figure 1.4. The Nash equilibrium is unique because, graphically speaking, the two curves cannot intersect in the triangle below the 45°-line in Figure 1.4 in which girls receive better education than boys. The reason is that inequality

$$\frac{1}{n_t^\delta}A_t > \frac{[1-\hat{\delta}n_{t+1}]^2}{n_t^0}A_t$$

is always fulfilled since $1 > [1-\hat{\delta}n_{t+1}]^2$ and $n_t^\delta < n_t^0$. On the contrary, the two curves always intersect once (and only once) in the triangle above the 45°-line in which boys receive better education than girls.

I conclude from the unique Nash equilibrium that parents always choose education for their children such that girls accumulate less human capital than boys. Due to this gender difference in education, children are raised by women alone: $\delta_t^f = \delta$ for all t. Hence, assuming that women raise children alone instead of modeling the endogenous choice of who is raising the couple's children is not critical.

1.8.3 Optimal Investments in Child Quantity and Quality

Equations (1.6) and (1.9) to (1.11) determine couple t's optimal decisions. Using these equations, I can deduce additional insights into how couple t spends their income optimally.

It follows from the budget constraint and the optimal consumption equation (1.6) that couple t consumes the proportion $\frac{1}{1+\beta}$ of their potential income and invests the rest in child quantity and child quality:

$$(c_t)^* = \frac{1}{1+\beta}(h_t^m + h_t^f)w,$$

$$\tau(n_t)^* h_t^f w + \frac{(n_t)^*}{2}\left[(e_t^m)^* + (e_t^f)^*\right] = \frac{\beta}{1+\beta}(h_t^m + h_t^f)w. \tag{1.32}$$

Using equations (1.9), (1.10) and (1.11), the investments in child quantity and child quality can be derived as

$$\tau(n_t)^* h_t^f w = \frac{\tau B(n_{t+1})}{2(2-\tau n_{t+1})}(h_t^m + h_t^f)w,$$

$$\frac{(n_t)^*}{2}\left[(e_t^m)^* + (e_t^f)^*\right] = \gamma\frac{1+(1-\tau n_{t+1})^2}{4(2-\tau n_{t+1})}(h_t^m + h_t^f)w.$$

Both amounts add up to the amount specified in (1.32).

All three amounts, i.e., consumption, investments in child quantity and investments in child quality, are increasing in parents human capital. However, when the father's human capital is rising, only consumption (c_t) and the number of children (n_t) are increasing. Child quality (e_t^m, e_t^f) does not change because the marginal costs and the marginal benefits are unaffected by changes in father's human capital and thus investments in each child's education stay the same. Note that the amount invested in all children's education is rising because the number of children is increasing. Mother's human capital raises all three amounts. Nevertheless, the number of children (n_t) is falling because the costs of having children are also increasing in mother's human capital: the substitution effect is at play. As a consequence and since the number of children is falling, the budget for children's education is spent on fewer heads and thus the individual investments in each child's education are increasing.

1.8.4 Proof of Proposition 1

Proposition 1. *Steady states exist when the following parameter restriction is fulfilled:*

$$\frac{\gamma}{4-\gamma} < \beta < \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}. \tag{1.33}$$

Proof. For the existence of steady states in the model, the two potential steady state fertility rates, i.e.,

$$n_{low} = \frac{2+\beta(4-\gamma)-\gamma-\sqrt{D}}{(4-\gamma)(1+\beta)\tau},$$
$$n_{high} = \frac{2+\beta(4-\gamma)-\gamma+\sqrt{D}}{(4-\gamma)(1+\beta)\tau}$$

with

$$D = 4-\beta^2(4-\gamma)^2+4\gamma-\gamma^2-2\beta(8-6\gamma+\gamma^2),$$

have to be reasonable solutions. They are reasonable solutions if and only if the following conditions hold:

$$D > 0,$$
$$0 < n_{low} < n_{high} \leq \frac{1}{\tau}.$$

The term D is strictly positive, if

$$\frac{\gamma}{4-\gamma} - \frac{\sqrt{8}+2}{4-\gamma} < \beta < \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}$$

holds. The lower fertility rate is strictly positive, if

$$\frac{\gamma}{4-\gamma} < \beta \leq \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}$$

holds. The higher fertility rate is not above the maximum number of children, if

$$\frac{\gamma}{4-\gamma} < \beta \leq \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}$$

holds. Note further that $n_{low} < n_{high}$ is always fulfilled when D is strictly positive.

Combining these results yields the condition for steady state existence:

$$\frac{\gamma}{4-\gamma} < \beta < \frac{\gamma}{4-\gamma} + \frac{\sqrt{8}-2}{4-\gamma}.$$

□

1.8.5 Effect of Fertility on Human Capital

The fertility decision affects the education decisions and hence human capital via two channels. The first is via parents joint decision on child quantity and quality. Having more children implies lower investments in children's education because the budget for investments in both children's quantity and quality is fixed.[24] The second channel is via the fertility rate in the next period which affects today's decision on children's education. Since the second channel only is present in equations (1.18) and (1.19), we cannot use these two equations to derive the effect of fertility in human capital. The reason is because we solved these equations from the optimal education decision where we implicitly assume that today's fertility is optimally chosen and thus cannot be changed. Differentiating equations (1.18) and (1.19) with respect to n does not derive the correct effect of fertility on steady-state human-capital.

In order to derive the correct effect, we use FOCs in (1.3) and (1.4) and solve for optimal education investments in period t again which yields

$$e_t^m = \frac{\gamma(h_t^m + h_t^f)w}{2n_t(2-\tau n_{t+1})},$$

$$e_t^f = \frac{\gamma(h_t^m + h_t^f)w}{2n_t(2-\tau n_{t+1})}(1-\tau n_{t+1})^2.$$

Here, education investments in period t depend on parent's human capital (h_t^m and h_t^f), today's fertility rate (n_t) and expected future fertility (n_{t+1}). Both channels, i.e., via n_t and n_{t+1}, are present. In a steady state, the equations simplify to

$$e^m = \frac{\gamma(h^m + h^f)w}{2n(2-\tau n)},$$

$$e^f = \frac{\gamma(h^m + h^f)w}{2n(2-\tau n)}(1-\tau n)^2.$$

Knowing that $h^m = A\sqrt{e^m}$ and $h^f = A\sqrt{e^f}$ holds, we can use the two equations to solve

[24] The budget for investments in child quantity and quality is given by $(h_t^m + h_t^f)w - c_t = \frac{\beta}{1+\beta}(h_t^m + h_t^f)w$.

for h^m and h^f:

$$h^m = \frac{\gamma A^2 w}{2n},$$
$$h^f = \frac{\gamma A^2 w}{2n}(1 - \tau n).$$

The effect of fertility on human capital is then clearly negative since

$$\frac{\partial h^m}{\partial n} = -\frac{\gamma A^2 w}{2n^2} < 0,$$
$$\frac{\partial h^f}{\partial n} = -\frac{\gamma A^2 w}{2n^2} < 0.$$

1.8.6 Hessian Matrix

Whether or not couple's utility is maximized in a steady state can be proved using the Hessian matrix. The Hessian derives as

$$H(n_t, e_t^m, e_t^f, h_t^m, h_t^f, n_{t+1}, w, \beta, \gamma, \tau) = \begin{pmatrix} \frac{\partial FOC_1}{\partial n_t} & \frac{\partial FOC_1}{\partial e_t^m} & \frac{\partial FOC_1}{\partial e_t^f} \\ \frac{\partial FOC_2}{\partial n_t} & \frac{\partial FOC_2}{\partial e_t^m} & \frac{\partial FOC_2}{\partial e_t^f} \\ \frac{\partial FOC_3}{\partial n_t} & \frac{\partial FOC_3}{\partial e_t^m} & \frac{\partial FOC_3}{\partial e_t^f} \end{pmatrix}.$$

It depends on the choice variables (n_t, e_t^m, e_t^f), the state variables (h_t^m, h_t^f), anticipated variable (n_{t+1}) and parameters (w, β, γ, τ).

In a steady state, the following equations hold for all t:

$$n_t = n,$$
$$A\sqrt{e_t^m} = \frac{\gamma A^2 w}{B(n)}(1 - \tau n),$$
$$A\sqrt{e_t^f} = \frac{\gamma A^2 w}{B(n)}(1 - \tau n)^2.$$

The Hessian then simplifies to

$$H^{ss}(n, w, \beta, \gamma, \tau) =$$
$$H\left(n, \left[\frac{\gamma A w}{B(n)}(1 - \tau n)\right]^2, \left[\frac{\gamma A w}{B(n)}(1 - \tau n)^2\right]^2, \frac{\gamma A^2 w}{B(n)}(1 - \tau n), \frac{\gamma A^2 w}{B(n)}(1 - \tau n)^2, n, w, \beta, \gamma, \tau\right).$$

Plugging in the two steady-state fertility-rates yields

$$H^{ss}_{low}(w,\beta,\gamma,\tau) = H^{ss}(n_{low}, w,\beta,\gamma,\tau),$$
$$H^{ss}_{high}(w,\beta,\gamma,\tau) = H^{ss}(n_{high}, w,\beta,\gamma,\tau).$$

Calculating the principal minors shows that the sign depends on β and γ only. For instance, the first principle minor for the steady state with low fertility rate derives as:

$$\det[-H^{ss}_{low}(w_t,\beta,\gamma,\tau,g_h,g_w)_1]$$
$$= -\frac{2\beta(1+\beta)(4-\gamma)^2\gamma^2\Big[-20+4\gamma-(6+\gamma+\beta(-4+\gamma))S(\beta,\gamma)-4\beta(4-\gamma)\Big]}{\Big[-2-\beta(4-\gamma)+\gamma+S(\beta,\gamma)\Big]^4}\tau^4 w^4,$$

with $\quad S(\beta,\gamma) = \sqrt{4-\beta^2(4-\gamma)^2+\gamma(4-\gamma)-2\beta(8-6\gamma+\gamma^2)}.$

Using

$$\beta = \frac{\gamma}{4-\gamma} + \alpha\frac{\sqrt{8}-2}{4-\gamma} \tag{1.34}$$

to replace β and deriving numerical solutions for[25]

$$\alpha \in (0,1), \tag{1.35}$$
$$\gamma \in (0,4), \tag{1.36}$$

shows that all six principle minors of the negative Hessians are strictly positive:

$$\det[-H^{ss}_{low}(w_t,\beta,\gamma,\tau,g_h,g_w)_1] > 0,$$
$$\det[-H^{ss}_{low}(w_t,\beta,\gamma,\tau,g_h,g_w)_2] > 0,$$
$$\det[-H^{ss}_{low}(w_t,\beta,\gamma,\tau,g_h,g_w)_3] > 0,$$

$$\det[-H^{ss}_{high}(w_t,\beta,\gamma,\tau,g_h,g_w)_1] > 0,$$
$$\det[-H^{ss}_{high}(w_t,\beta,\gamma,\tau,g_h,g_w)_2] > 0,$$
$$\det[-H^{ss}_{high}(w_t,\beta,\gamma,\tau,g_h,g_w)_3] > 0.$$

The signs of the principle minors suggest that the negative Hessian matrices are positive

[25]Equation (1.34) ensures that conditions as stated in (1.17) are fulfilled.

definite which means that the Hessian matrices themselves are negative definite. In other words, utility is concave in both steady states. I therefore conclude that utility is maximized in both steady states.

1.8.7 Proof of Proposition 2

Proposition 2. *The dynamics of the fertility rate described by*

$$n_t = \begin{cases} \eta\left(\frac{h_0^m}{h_0^f}, n\right) & \textit{for } t = 0, \\ n & \textit{for } t \geq 1, \end{cases}$$

with $n \in \{n_{low}, n_{high}\}$ *holds for all parameter sets which fulfill* (1.17).

Proof. The dynamics described by

$$n_t = \begin{cases} \eta\left(\frac{h_0^m}{h_0^f}, n\right) & \text{for } t = 0, \\ n & \text{for } t \geq 1, \end{cases}$$

with $n \in \{n_{low}, n_{high}\}$ holds for all parameter sets when (i) the chosen fertility rate n_t is between zero and $1/\tau$ and (ii) cycles do not exist. Graphically speaking, the two blue areas in Figure 1.3 are always in the range of zero to $1/\tau$ and never overlap. Using the two solutions of equation (1.23), these conditions translate into the following inequalities:

as for (i):

$$0 \leq \frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} - D}{4\tau(2 - \tau n_{t+1})},$$

$$\frac{1}{\tau} \geq \frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} + D}{4\tau(2 - \tau n_{t+1})},$$

as for (ii):

$$\frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} - D}{4\tau(2 - \tau n_{t+1})} < \frac{4 + \tau B(n_{t+1}) - 2\tau n_{t+1} + D}{4\tau(2 - \tau n_{t+1})}.$$

Note that the third inequality simplifies to $D > 0$ and that the three above presented conditions are identical with the conditions for the existence of steady states (please refer to Appendix 1.8.4). It follows that the modeled economy always follows the dynamics described above when parameters fulfill the inequalities in (1.17) and hence in every case where steady states exist. □

Chapter 2

How Does Skill-Biased Technological Change Affect Human Capital Accumulation?

2.1 Introduction

In past decades, it has been observed that wage inequality between skilled and unskilled workers[1] was rising while the relative skill supply increased in the same time.[2] One possible explanation for this phenomenon is the occurrence of skill-biased-technological change (SBTC). Autor, Katz and Krueger (1998), for instance, argue that the diffusion of computers and related technologies led to rising wage inequality in the U.S. during the 1980s which could potentially offset the effect rising relative skill supply. Other contributions to the literature paid attention to the effect of SBTC on wages/wage inequality has well.[3] Though the rise in wage inequality may induce individuals to invest more in education in order to benefit from increasing skill premium, the effect of SBTC on human capital accumulation (HCA), however, has attracted little attention in the literature. This chapter focuses on this link and aims to answer the question whether SBTC accelerates or slows down HCA? Moreover, the chapter discusses whether SBTC affects economies with different characteristics equally or not. I distinguish the cases of economies with high or low fertility and with high or low education costs.

The chapter combines two literature strands: the literature on SBTC and the literature on HCA. As briefly outlined above, the SBTC literature strand aims to explain the

[1] Following the usual definition, I refer to college graduates as skilled workers and to high school graduates as unskilled workers throughout the chapter.

[2] Empirical studies are provided by Murphy and Welch (1989) and Levy and Murnane (1992).

[3] See the SBTC literature overview below for more details.

phenomenon of rising wage inequality in the U.S. during the 1980s, though the relative skill supply increased in the same time. Acemoglu (1998) and Bound and Johnson (1992) explain this phenomenon by arguing that SBTC increased the demand for skilled labor relative to unskilled labor which led to rising wage inequality between skilled and unskilled workers.[4] However, only *extensive* SBTC leads to the observed changes in wages (see Johnson (1997)), where extensive SBTC means that skilled workers become more productive in jobs that were formerly done by unskilled workers, as for instance the introduction of robotics in manufacturing.[5] Throughout this chapter, I assume SBTC to be extensive.

The second literature strand studies the link between income distribution and growth via human capital investments. Galor and Zeira (1993), for instance, point out the importance of income inequality for human capital investments when credit markets are imperfect. The link between parents' income today and human capital of the next period is analyzed in detail by Maoz and Moav (1999). They use an overlapping-generations model with constant technology to examine how incentives for and constraints on investment in education affect HCA and vice versa. They show that next period's income inequality is the incentive for investing in education. Equally important, parents' income constrains the investment in children's education because borrowing is not possible due to imperfect capital markets. During the growth process, wages of skilled workers fall while wages of unskilled workers rise. These changes in the wage structure have two conflicting effects on future growth: On the one hand, liquidity constraints on the poor are relaxed leading to higher investments in children's education. On the other hand, wage inequality declines which reduces the incentive to invest in education. The overall effect depends then on the current development stage. Galor and Moav (2004) find a similar result when including physical capital in the model. They show that the positive impact of inequality on growth is reversed when the prime source of growth switches from physical capital accumulation to human capital accumulation.

Only few literature contributions on HCA consider technological change. Among them are Galor and Tsiddon (1997a, 1997b) who assume skilled and unskilled labor to be perfect substitutes and include skill-neutral technological progress in their studies. In the first paper, technological progress depends positively on the average human capital in a society. This assumption creates a link across families since the investment in human capital of one family affects, via technological progress, the return to education investments for all families. On the other hand, the authors assume that parents' human capital has a direct

[4]See Machin and van Reenen (1998) for a comparison of SBTC in the U.S. and seven other OECD countries.

[5]In contrast and according to Johnson (1997), SBTC is called intensive when skilled workers become more productive in jobs they already perform.

and positive effect on their children's future human capital. In this framework, income inequality falls during the growth process; however, there exists a poverty trap when poor economies try to reduce inequality by political measures. In the second paper, Galor and Tsiddon endogenize technology change by introducing an R&D sector. Here, the evolution of inequality depends on the type of technological change: inventions (i.e. major technological progress) lead to higher inequality and faster growth while innovations (i.e. improved accessibility of existing technologies) reduce inequality and harm future growth. In contrast to Galor and Tsiddon papers, Eicher and García-Peñalosa (2001) see skilled and unskilled labor as imperfect substitutes and consider skill-biased technological change where the degree of SBTC is endogenously determined. In a steady-state analysis, the authors find that an economy can be in one of (up to) three steady-states which is consistent with cross-country data on inequality and skill-premium.

This chapter contributes to the existing literature by analyzing the effect of skill-biased technological change on human capital accumulation where SBTC is modeled as a single, persistent and exogenous shock. This allows me to study the short- and long-run effects of such a shock. Moreover, I analyze whether SBTC has similar or different effects in heterogeneous countries. For this analysis, I assume that SBTC occurs in all economies at the same time.[6] In contrast to the existing literature on technology and HCA, I assume that capital markets are imperfect which generates a different inter-generational link since parents finance and decide on their children's education and hence future human capital.

To analyze the effects of SBTC on HCA, I use elements from both the SBTC-literature and the HCA-literature. The setup contains an overlapping-generations model with imperfect capital market where this assumption is driven to the extreme of no borrowing and no lending possibilities. In each period, a single homogeneous good is produced using skilled and unskilled labor as input factors. The skill supply of the next period is endogenously determined by parents' decision on their children's education. The education decision depends crucially on the incentive to invest in children's education which is given by the next period's wage inequality. Furthermore, it depends on relative education costs, i.e., the education costs relative to parent's income. SBTC is introduced into the model as a persistent shock raising productivity of skilled workers relative to productivity of unskilled workers.

I find that SBTC leads to ambiguous effects on HCA: the negative effect dominates when human capital per capita is low implying that HCA is slowing down; the positive effects dominate when human capital per capita is high leading to accelerated HCA. As

[6] The argument behind this assumption is that technologies like computers or computer aided process control can be used by all skilled workers regardless of whether the economy they are living in is developed or not (as approximated by fertility rate and education cost in this chapter). These skill-biased technologies therefore widespread quickly between economies once they are available.

for the comparison of SBTC effects in economies with high or low fertility or education costs, I find that countries with higher fertility or higher education costs accumulate less human capital per capita than countries with lower fertility or lower education costs because financing education for all children in these societies is simply more expensive. Such economies face negative overall effects on their HCA process when SBTC occurs: human capital growth slows down and the steady state level of human capital per capita shrinks. They are advised to introduce policies which diminish the negative SBTC effects. Economies with a low fertility rate and low education costs, in contrast, experience accelerated human capital growth and converge to a new steady state with higher human capital per capita.

The remaining part of the chapter is organized as follows. I describe the setup in Section 2 and solve for human capital accumulation in Section 3. Section 4 discusses the effects of skill-biased technological change on human capital accumulation while Section 5 concludes.

2.2 Model Description

I use an overlapping-generations model in discrete time where a single homogeneous good is produced in each period using skilled and unskilled labor. The skill supply (of the next period) is endogenously determined by parents' decision on children's education. I assume that the capital market is imperfect such that individuals can neither borrow nor lend. The labor market, in contrast, is characterized by perfect competition.

The setup combines elements from both literature strands: the OLG-framework, the endogenous education decision and the imperfect capital market are key elements in the HCA-literature (Galor and Zeira (1993) and Maoz and Moav (1999)), while the production function is taken from the SBTC-literature (Acemoglu (2002) and Card and DiNardo (2002)).

2.2.1 Individuals

Individuals live for two periods which are labeled as childhood and adulthood. In childhood, individuals neither work nor consume but may receive education financed by their parents.[7] Children who receive education become skilled adults in the next period while

[7]The focus of this chapter is on human capital accumulation driven by endogenous education decisions. Allowing individuals to work and consume in childhood would not affect qualitatively the results.

children who do not receive education stay unskilled.[8] In adulthood, each individual supplies labor inelastically and gives birth to n children.[9] Moreover, each adult spends his labor income on consumption for himself and on education for his children.

Each adult shares the same utility function which depends on his consumption and the sum of expected income of his children. The utility is given by

$$U_t^i = \ln[c_t^i] + \beta \ln \left[w_{t+1}^u + \lambda_t^i \Big(w_{t+1}^s - w_{t+1}^u \Big) \right] \tag{2.1}$$

where i denotes parent's skill type, with $i = u$ for an unskilled and $i = s$ for a skilled parent.[10] In the utility function, c_t^i denotes parent's consumption, β represents the degree of altruism and w_{t+1}^s (resp. w_{t+1}^u) denotes expected wage income of the parent's skilled (resp. unskilled) grown-up children in the next period. The proportion of children who receive education is denoted by λ_t^i, with $\lambda_t^i \in [0, 1]$, and is endogenously determined by the parent's decision on children's education.

Note that the utility function in (2.1) does not contain any expectation operator although expected future wages of children influence parental utility. The reason is because I suppose that there is no uncertainty in the model and that individuals form rational expectations over future variables. Individuals therefore have perfect foresight over expected future variables.

In the education decision, each parent maximizes his utility subject to his budget constraint which is given by

$$w_t^i = c_t^i + \lambda_t^i ne \tag{2.2}$$

where w_t^i denotes wage income of a type i parent and e denotes the exogenously given and fixed education costs per child.[11]

[8] Education in this model stands for tertiary education. Skilled individuals are those with college and higher degrees, while unskilled individuals are those with high school and lower degrees.

[9] Starting with Becker (1960) and Becker and Lewis (1973), some literature contributions argue that parents' fertility decision and parents' decision on children's education are interdependent. Endogenizing fertility in the framework would lead to faster human capital accumulation because parents would reduce the number of children and invest even more in the remaining children's education when human capital per capita rises. The effects of SBTC, however, would be the same as in the presented framework.

[10] Fertility rate n is not included in the utility function since it is exogenous and would not affect the results presented in this chapter. In case of endogenous fertility decision, the fertility rate should be included in the utility function. This would, as mentioned before, pronounce the effect of education investments for human capital accumulation.

[11] Education costs per child may also depend on average wage (as for instance in Maoz and Moav (1999); de la Croix and Doepke (2003, 2004)) or on the skilled wage only. What matters for the education decision are the education costs relative to parent's income. To support the chapter's results, the occurrence of SBTC has to reduce the relative education costs for skilled parents and to raise the relative education costs for unskilled parents. All mentioned specifications fulfill this condition. To simplify the model and the intuition behind the results, I choose fixed education costs per child.

2.2.2 Production and Wages

Firms produce a single consumption good by employing both skilled and unskilled labor in production. Output Y_t follows the constant returns to scale and constant elasticity of substitution production function

$$Y_t = \left[b(W_t^s)^\gamma + (1-b)(W_t^u)^\gamma\right]^{\frac{1}{\gamma}} \tag{2.3}$$

where W_t^s (resp. W_t^u) denotes the number of skilled (resp. the number of unskilled) workers employed in the production process. Input factors are weighted by the technology parameter b, with $b \in (0.5, 1)$.[12] Exponent $\gamma \in (0, 1)$ determines the elasticity of substitution between both labor inputs.[13] Note that this formulation of the production function allows for the occurrence of SBTC which is later modeled by an exogenous rise in b.[14]

In a perfectly competitive labor market, wages are determined by the marginal product of skilled and unskilled labor. In the case of CRS production function, the marginal product for each labor type is uniquely determined by the input factor ratio $\frac{W_t^s}{W_t^u}$. I therefore derive this input factor ratio in the following, in order to solve for skilled and unskilled wages.

Suppose each adult is employed in production according to his skill type. The input factor ratio is then equal to the ratio of skilled to unskilled adults in the population. If the resulting skilled wage is not below the unskilled wage, the labor market is in equilibrium. However, when many adults are skilled and thus many skilled workers are hired, the skilled wage may fall below the unskilled wage because unskilled labor is scarce in this situation. But since a skilled adult has high knowledge which allows him to work in jobs requiring either high or low skills, he chooses the type of work which offers him the highest remuneration. Unlike a skilled adult, an unskilled adult can only apply for jobs which require low skills. It follows that skilled adults have an incentive to apply for unskilled jobs in situations where the skilled wage is lower than the unskilled one. Because skilled adults then move to unskilled jobs, the skilled wage rises while the unskilled wage declines. When both skilled and unskilled wages are equalized, skilled adults are indifferent between working in skilled or unskilled jobs and the labor market is in equilibrium.

Before deriving wages and input factor ratios, let me define α_t as the proportion of

[12]The assumption on b ensures that productivity of a skilled worker is higher than that of an unskilled worker given that both are employed in the same quantity.

[13]The assumption on γ ensures that both input factors are substitutes. Though this parameter is hard to estimate, empirical evidence suggest a range of $\gamma \in (0, 0.5)$ (see Freeman (1986)). Skilled and unskilled labor inputs being gross substitutes ensures that SBTC has a positive effect on the skill premium (see Acemoglu (2002) for a detailed discussion of how SBTC can be modeled).

[14]The formulation of the production function and the modeling of SBTC follows Acemoglu (2002).

skilled adults in the population:

$$\alpha_t = \frac{L_t^s}{L_t^s + L_t^u}$$

where L_t^s (resp. L_t^u) denotes the number of skilled adults (resp. unskilled adults) in period t. For $\alpha_t \leq \widehat{\alpha}$, with

$$\widehat{\alpha} = \frac{1}{1 + \left(\frac{b}{1-b}\right)^{\frac{1}{\gamma-1}}},$$

adults work according to their skill level, wages derive as

$$w_t^s = b\left[b + (1-b)\Big(\frac{1-\alpha_t}{\alpha_t}\Big)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}} \equiv w^s(\alpha_t, b), \tag{2.4}$$

$$w_t^u = (1-b)\left[b\Big(\frac{\alpha_t}{1-\alpha_t}\Big)^{\gamma} + 1 - b\right]^{\frac{1-\gamma}{\gamma}} \equiv w^u(\alpha_t, b), \tag{2.5}$$

and the input factor ratio follows $\frac{W_t^s}{W_t^u} = \frac{\alpha_t}{1-\alpha_t}$. The ratio between skilled and unskilled wages is then given by

$$\frac{w_t^s}{w_t^u} = \frac{b}{1-b}\left(\frac{\alpha_t}{1-\alpha_t}\right)^{\gamma-1} \tag{2.6}$$

and is decreasing in the proportion of skilled adults α_t.

For $\alpha_t > \widehat{\alpha}$, in contrast, wages are equalized at

$$w_t^s = w_t^u = \left[b^{\frac{1}{1-\gamma}} + (1-b)^{\frac{1}{1-\gamma}}\right]^{\frac{1-\gamma}{\gamma}} \equiv \bar{w}$$

and the input factor ratio is equal to $\widehat{\alpha}$. Figure 2.1 provides a graphical illustration of the wage determination.

2.2.3 Parameter Choice

Throughout this chapter, I use the following parameter values for graphical representations:

Economy	γ	b	n	e	β
standard (low fertility, low education costs)	0.33	0.55	1	0.075	0.2
high fertility	0.33	0.55	**4**	0.075	0.2
high education costs	0.33	0.55	1	**0.3**	0.2

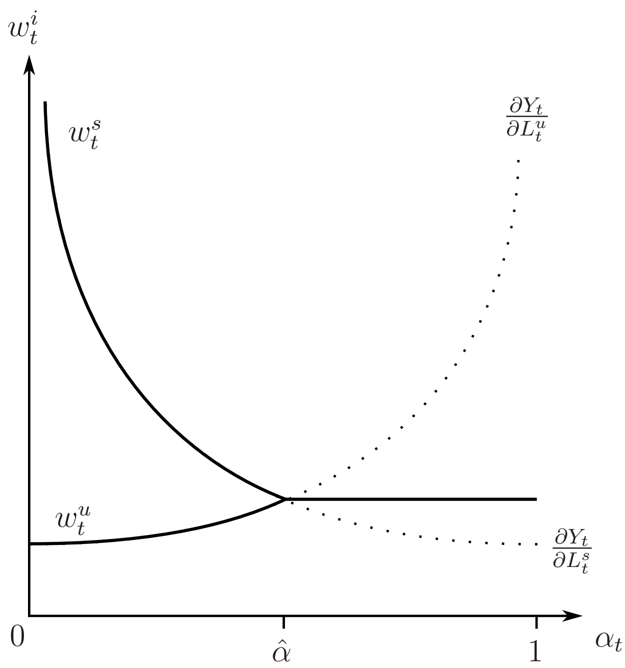


Figure 2.1: Skilled and unskilled wages for given proportion of skilled adults α_t.

In the following, I discuss briefly the parameter choice; more details are provided in Appendix 2.6.2.

Parameters for the standard economy are chosen such that they fit to U.S. data of the 1970s which is the decade before the occurrence of SBTC. For instance, parameter γ is chosen such that it fits empirical data on the elasticity of substitution between input factors.[15] I calculated parameter b by using data presented by Acemoglu (2002) for the U.S. over the period 1960-1996 on the relative skill supply $(\frac{\alpha_t}{1-\alpha_t})$ and the skill premium $(\frac{w_t^s}{w_t^u})$. Using the production function in equation (2.3), I find that b was approximately 0.55 throughout the 1970s and rises to approximately 0.7 in 1995. Therefore, I simulate SBTC later by raising b from 0.55 to 0.7.

The observed fertility rate in the U.S. during the 1970s implies a fertility rate of $n = 1$ in the model. The high fertility rate of $n = 4$, in contrast, represents the highest observed fertility rates in countries like Afghanistan, Chad and Zambia. As for the costs of education e, I calculate the expenditure on tertiary education per student relative to average income in the U.S. during the period 2000-2008. I find that the costs of studying 5 years in college is approximately 7.5% of parent's life-time income (based on an annual income of 40,000 USD). Regarding the value of β, there is no estimation available in the literature. However, having determined all other parameters, I pin down β such that the resulting skill supply in the steady state (α^{ss}) fits the empirical observation for the U.S. during the 1970s.

[15]See, for instance, Acemoglu (2002), Card and DiNardo (2002), and Katz and Murphy (1992).

2.3 Dynamics

In this section, I derive the human capital accumulation path for the modeled economy. An adequate measure for human capital is the level of human capital per capita in each period. Since skill levels are constant and exogenously given in the model, the human capital per capita is uniquely determined by the proportion of skilled adults denoted by α_t. I therefore concentrate on the dynamics of α_t in the following.

The proportion of skilled adults evolves over time following the equation

$$\alpha_{t+1} = \alpha_t \lambda_t^{s*} + (1 - \alpha_t)\lambda_t^{u*}. \tag{2.7}$$

Equation (2.7) states that next period's proportion of skilled adults α_{t+1} is determined by today's optimal education decision of skilled adults λ_t^{s*} and that of unskilled adults λ_t^{u*}, weighted by their proportion in the current population.

Optimal education decisions are the result of the following maximization problems:

$$\max_{\lambda_t^i} U_t^i \quad \text{s.t.} \quad w_t^i = c_t^i + \lambda_t^i ne \tag{2.8}$$

with $i = s, u$: an adult with skill level i maximizes utility over the proportion of his children who shall receive education subject to his budget constraint. In order to analyze the main factors influencing the optimal education decisions, I take the expected next period's wages w_{t+1}^i for a moment as given. Optimal education then derives as

$$\lambda_t^{i*} = \begin{cases} 0 & \text{if } \frac{1}{1+\beta}\left[\beta \frac{w_t^i}{ne} - \frac{w_{t+1}^u}{w_{t+1}^s - w_{t+1}^u}\right] < 0, \\ 1 & \text{if } \frac{1}{1+\beta}\left[\beta \frac{w_t^i}{ne} - \frac{w_{t+1}^u}{w_{t+1}^s - w_{t+1}^u}\right] > 1, \\ \frac{1}{1+\beta}\left[\beta \frac{w_t^i}{ne} - \frac{w_{t+1}^u}{w_{t+1}^s - w_{t+1}^u}\right] & \text{else,} \end{cases} \tag{2.9}$$

with $i = s, u$ meaning that an adult may choose to invest in the education of none, all or some of his children. Inspecting equation (2.9) reveals the following insights. First, the higher the ratio $\frac{e}{w_t^i}$, the lower the proportion of children receiving education. This ratio measures the costs of sending one child to college relative to a parent's income which I refer to as relative education costs. Relative educations costs rise when the costs for educating one child are higher or when parent's income declines. Note that the latter implies that skilled parents never send a smaller proportion of their children to college than unskilled parents do because wages always fulfill the condition $w_t^s \geq w_t^u$. It also reflects well-known empirical observations of richer parents investing more in education of their children than poorer parents. Second, the higher the skill premium in the next period, given by $\frac{w_{t+1}^s - w_{t+1}^u}{w_{t+1}^u}$, the more children receive education. The reason is that the skill premium is the incentive to invest in children's education. Note that the skill premium depends on the wage ratio and is positive for wage ratios larger than one: $\frac{w^s_{t+1}-w^u_{t+1}}{w^u_{t+1}} = \frac{w^s_{t+1}}{w^u_{t+1}} - 1$. Third, the higher fertility n, the lower the proportion of children receiving education because educating a certain proportion of children is more expensive for the parent when he has many children.

So far, I took next period's wages w^i_{t+1} as given. However, next period's wages depend on the actual skill formation in the next period, i.e., wages w^i_{t+1} are a function of α_{t+1}. Optimal education decisions in period t are therefore given by the functions

$$\lambda^{i*}(\alpha_t, \alpha_{t+1}, b) = \begin{cases} 0 & \text{if } \frac{1}{1+\beta}\,[.] < 0, \\ 1 & \text{if } \frac{1}{1+\beta}\,[.] > 1, \\ \frac{1}{1+\beta}\left[\beta\frac{w^i(\alpha_t,b)}{ne} - \frac{w^u(\alpha_{t+1},b)}{w^s(\alpha_{t+1},b)-w^u(\alpha_{t+1},b)}\right] & \text{else,} \end{cases} \tag{2.10}$$

with $i = s, u$. Plugging these functions into equation (2.7) leads to an implicit definition of α_{t+1}:

$$\alpha_{t+1} = \alpha_t\lambda^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda^{u*}(\alpha_t, \alpha_{t+1}, b). \tag{2.11}$$

In other words, α_{t+1} follows an implicitly defined function of α_t:

$$\alpha_{t+1} = \phi(\alpha_t),$$

with the properties as discussed in the following proposition.

Proposition 3. *For $\alpha_t \in [0, 1]$, there exist a unique and continuous function $\phi(\alpha_t)$ for which $\alpha_{t+1} = \phi(\alpha_t)$ holds.*

Proof. See Appendix 2.6.1. □

Knowing that the function $\phi(\alpha_t)$ exists allows me to study human capital accumulation over time. Figure 2.2 displays the dynamic behavior of α_t for an example economy.[16] As can be seen in the figure, an economy starting at α_0 accumulates human capital along the path described by the arrows. In the long run, the economy converges to a unique steady state α^{ss} given by the intersection of $\phi(\alpha_t)$ with the 45°-line. Using equation (2.11), the steady state derives implicitly as

$$\alpha^{ss} = \alpha^{ss}\lambda^{s*}(\alpha^{ss}, \alpha^{ss}, b) + (1-\alpha^{ss})\lambda^{u*}(\alpha^{ss}, \alpha^{ss}, b).$$

The shape of $\phi(\alpha_t)$ is discussed in detail in Appendix 2.6.3. Nevertheless, I discuss in

[16]I choose the following parameter values for the example in Figure 2.2: $\beta = 0.2$, $\gamma = 0.33$, $b = 0.55$, $e = 0.075$, $n = 1$. For a discussion of how parameter values are chosen, please refer to Appendix 2.6.2.

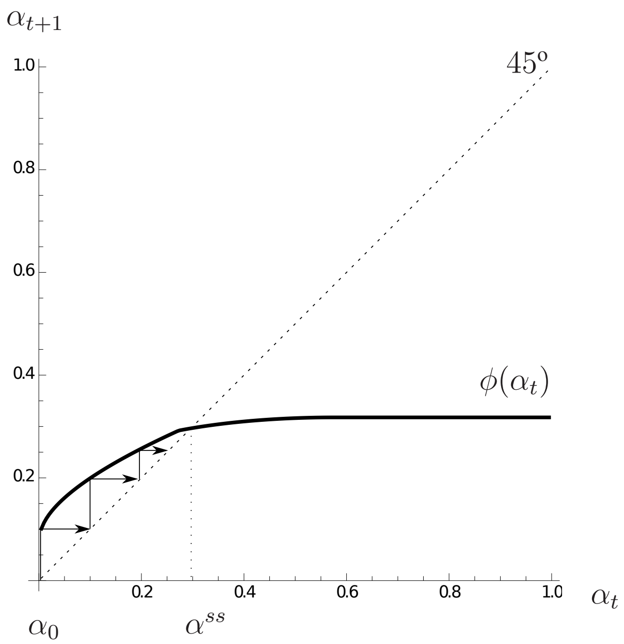


Figure 2.2: Human capital accumulation over time.

the following what drives human capital accumulation. The growth in human capital, as displayed in Figure 2.2, is driven by the evolution of wages. For α_t close to zero, skilled wages are extremely high and unskilled wages are low implying a high skill premium and a high incentive to invest in children's education. Because of their high income, skilled parents send all their children to college. Unskilled parents, in contrast, face high relative education costs due to their low income and send only some of their children to college. As a consequence, the economy starts to accumulate human capital. With the rising α_t, unskilled wages increase and unskilled parents send more of their children to college which provides additional fuel for the accumulation of human capital. However, skilled wages fall with rising average human capital. When α_t reaches a certain level, skilled parents' income is not high enough anymore to finance education for all their children, and they start to send only some of their children to college. This change in skilled parents' behavior causes the kink in $\phi(\alpha_t)$ implying a slow down in HCA. The skill premium and thus the incentive to invest in children's education declines during the growth process as well. When the incentive is so small that parents decide to keep the proportion of children sent to college constant, the steady state is reached. Note that the limiting factors for human capital growth are (i) the low income of unskilled parents for small α_t, because it causes high relative education costs, and (ii) the low skill premium due to a small wage ratio for high α_t.

2.4 Effects of Skill-Biased Technological Change on Human Capital Accumulation

In the previous section, I studied human capital accumulation of an economy with constant technology. This section first analyzes the effects of an unexpected but persistent skill-biased technological change on the human capital accumulation path and second compares these effects between two economies which differ in their fertility rate.

2.4.1 General Effects of SBTC on HCA

SBTC is modeled as exogenous shock by raising the technology parameter b. To study whether human capital accumulation accelerates or slows down after SBTC has occurred, it is sufficient to determine the sign of $\frac{\partial \phi(\alpha_t)}{\partial b}$. Proposition 4 addresses how the sign can be determined. The proposition rests on the fact that human capital accumulation is driven by parents' decision on children's education.

Proposition 4. *For* $0 \leq \alpha_t \leq 1$,

$$\operatorname{sgn}\left[\frac{\partial \phi(\alpha_t)}{\partial b}\right] = \operatorname{sgn}\left[\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}\right].$$

Proof. See Appendix 2.6.4. □

While proceeding with analyzing parents' decision on children's education, note that SBTC does not affect education decisions when they are solved by corner solutions. So focusing on inner solutions, the changes in education decisions derive as

$$\frac{\partial \lambda_t^{s*}}{\partial b} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}\frac{\partial w_t^s}{\partial b} + \frac{1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^2}\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b}\right],$$

$$\frac{\partial \lambda_t^{u*}}{\partial b} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}\frac{\partial w_t^u}{\partial b} + \frac{1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^2}\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b}\right].$$

Both derivatives depend on how wages change due to SBTC. Using the wage equations (2.4) and (2.5), wage changes derive as[17]

$$\frac{\partial w_t^s}{\partial b} > 0, \qquad \frac{\partial w_t^u}{\partial b} < 0.$$

As one may expect, SBTC leads to rising skilled wages and falling unskilled wages. These changes in wages have three different effects on parents' decision on children's education. First, the changes in wages lead to a higher skill premium[18], i.e.,

$$\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b} > 0,$$

and hence increase the incentive to invest in children's education.[19] Both skilled and unskilled parents face this positive effect of SBTC. Second, the rising skilled wage implies lower relative education costs for skilled parents while third, the falling unskilled wage leads to higher relative education costs for unskilled parents.

The three effects influence the education decision of skilled and unskilled parents in the following ways. For skilled parents, the lower relative education costs and the higher incentive to invest in children's education clearly have a positive effect:

$$\frac{\partial \lambda_t^{s*}}{\partial b} > 0.$$

In contrast, the effect of SBTC is ambiguous for unskilled parents. They face a negative cost effect and a positive incentive effect. Depending on the size of the two effects, unskilled parents may choose to send less, more or the same proportion of their children to college:

$$\frac{\partial \lambda_t^{u*}}{\partial b} \lesseqgtr 0.$$

As a consequence, the net effect of SBTC on HCA is ambiguous too. However, it is possible to determine the net effect under a mild assumption. The following proposition summarizes the impact of SBTC on human capital accumulation.

Proposition 5. *SBTC has a positive effect on human capital accumulation when all parents*

[17]The derivatives hold for interior solutions only (i.e. $\alpha_t < \hat{\alpha}$). Note that the economy will be always in this area (the only exception could be the starting period).

[18]This derivative holds for all $\alpha_t \in (0, 1)$ since the skill premium in the next period will be always in the area of interior solutions (i.e. $\alpha_t < \hat{\alpha}$), because there would be no incentive to invest in education otherwise.

[19]Maoz and Moav (1999) study HCA for constant technology and point out that wage inequality monotonically declines during the human capital accumulation process. A similar trend in wage inequality is present in this chapter as long as technology is constant. Maoz and Moav (1999) suspect that SBTC may break this monotonic trend; and indeed, SBTC raises wage inequality in the period of its occurrence: $\frac{w_t^s}{w_t^u} \uparrow$. In the following periods, wage inequality declines again due to the human capital accumulation process.

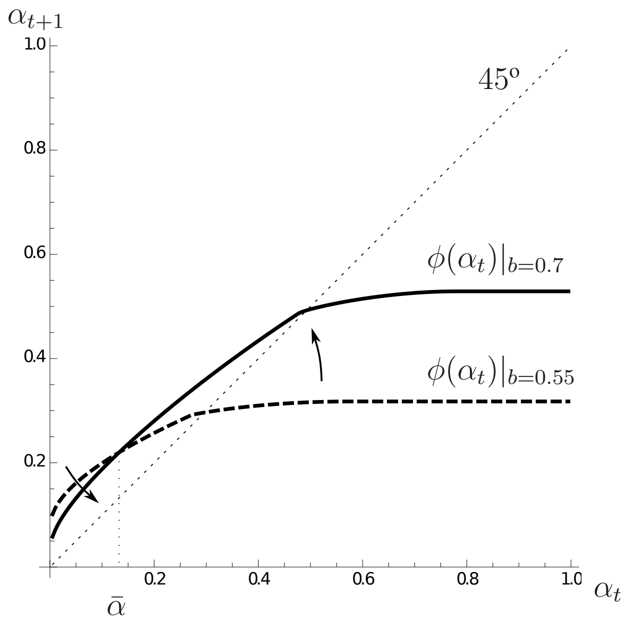


Figure 2.3: The effect of SBTC on $\phi(\alpha_t)$.

are skilled, i.e.,

$$\lim_{\alpha_t \to 1} \quad \alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b} > 0.$$

Under the assumption that education costs are not too low, SBTC has a negative effect on human capital accumulation when all parents are unskilled:

$$\lim_{\alpha_t \to 0} \quad \alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b} < 0.$$

As a consequence, function $\phi(\alpha_t)$ rotates anticlockwise in the (α_{t+1}, α_t)-space after the occurrence of SBTC (see Figure 2.3).[20]

Proof. See Appendix 2.6.5. □

The intuition for this result can be understood when focusing on the limiting factors for human capital accumulation. The direction of change in these factors will dominate all other effects and determine how SBTC affects HCA. For instance, the limiting factor for low levels of α_t is unskilled parents' low income which indicates high relative education costs for these parents. Since SBTC causes a decline in the unskilled wage and thus a rise in relative education costs for unskilled parents, fewer children are sent to college and function $\phi(\alpha_t)$ shifts downwards. Note that SBTC does not induce skilled parents to send

[20] In Figure 2.3, I raise b from 0.55 to 0.7 while keeping all other parameters of the economy introduced in Figure 2.2 constant.

more children to college for low α_t because all their children already attend college.[21] On the contrary, for high levels of α_t, the limiting factor for HCA is the expected skill premium in the next period. This factor limits both skilled and unskilled parents in their decision on children's education. Since SBTC leads to a higher skill premium, all parents decide to send more children to college which implies an upward shift of the $\phi(\alpha_t)$-function.

The rotation of $\phi(\alpha_t)$ influences the economy's process of human capital accumulation. A downward shift of $\phi(\alpha_t)$ implies a slow down in human capital growth while an upward shift of $\phi(\alpha_t)$ suggests an acceleration of human capital accumulation. When comparing the steady states before and after the occurrence of SBTC, the economy displayed in Figure 2.3 converges to a steady state with higher human capital when using the new technology. In the short run, SBTC slows down human capital growth for $\alpha_t < \bar{\alpha}$ and accelerates HCA otherwise. In the long run, however, the economy clearly benefits in terms of human capital accumulation.

2.4.2 Comparing the Effects of SBTC on HCA in Two Different Countries

In this subsection, I study whether economies with different characteristics experience similar SBTC effects on HCA or not. This comparison is motivated by the question whether SBTC has similar or different effects across developed and developing countries. Given the setup of the model, I distinguish developed and developing economies by the fertility rate. Well-known empirical data show that parents in less developed countries tend to have more children than parents in developed countries. Another possibility is to account for differences in the education costs per child because developed countries might be more efficient than less developed countries when providing education of the same quality.

To perform the analysis of SBTC effects in countries which differ either in their fertility rate or in education costs per child, it is useful to study first how fertility and education costs influence human capital accumulation when the technology is constant. Proposition 6 addresses this issue and Figures 2.4 and 2.5 provide graphical presentations.

Proposition 6. *Higher fertility and education costs per child shift $\phi(\alpha_t)$ downwards and*

[21] For $0 < \alpha_t < \widetilde{\alpha}$, skilled parents' decision on children's education is solved by corner solution; skilled parents send all their children to college, i.e., $\lambda_t^{s*} = 1$.

lead to lower human capital in the steady state:

$$\frac{\partial \phi(\alpha_t)}{\partial n} < 0, \qquad \frac{\partial \phi(\alpha_t)}{\partial e} < 0,$$

$$\frac{\partial \alpha^{ss}}{\partial n} < 0, \qquad \frac{\partial \alpha^{ss}}{\partial e} < 0.$$

Proof. See Appendix 2.6.6. □

Since the two figures are very similar, fertility and education costs seem to affect human capital accumulation in a similar way. The reason is that both n and e influence education decisions in exactly the same way, as can be seen in equation (2.9) where both enter in the denominator of the first summand. The ratio $\frac{ne}{w_t^i}$ in the equation for optimal education decision measures the cost of educating all children relative to parent's income. The ratio goes up when fertility and/or education costs per child rise. For the following analysis, it is therefore sufficient to focus on fertility differences between countries. Countries which differ in education costs per child experience exactly the same effects.

SBTC effects in an economy with low fertility have been studied in the previous subsection; Figure 2.3 displays the result. To study the effects of SBTC in a high-fertility economy, the same technology change, i.e., technology parameter b is raised from 0.55 to 0.7, is now analyzed in Figure 2.6 for an economy with high fertility.[22] As in a low-fertility economy, function $\phi(\alpha_t)$ rotates anticlockwise in the (α_{t+1}, α_t)-space. However, the SBTC effects on HCA are different. Assuming that an economy always converges from the left to the steady state, SBTC causes in the short run a slow down in human capital growth. Moreover, the high-fertility economy converges to a steady state with lower human capital than before the occurrence of SBTC. The reason for both the short and long run effects is that human capital accumulation is always in the interval where unskilled parents' income is the main limiting factor for HCA. Since SBTC causes a reduction in the unskilled wage, fewer children receive education which translates into slower growth in human capital and a decline in steady state human capital.

Comparing the results between the two modeled economies, economies with low fertility are likely to benefit (in terms of human capital per capita) from SBTC in the long run and must not fear negative effects though human capital growth may slow down in the short run. Economies with high fertility, in contrast, face slower human capital growth in the short run and lower average human capital in the steady state.

[22]For better visibility, the scale on the x-axis is reduced to $\alpha_t \in [0, 0.1]$.

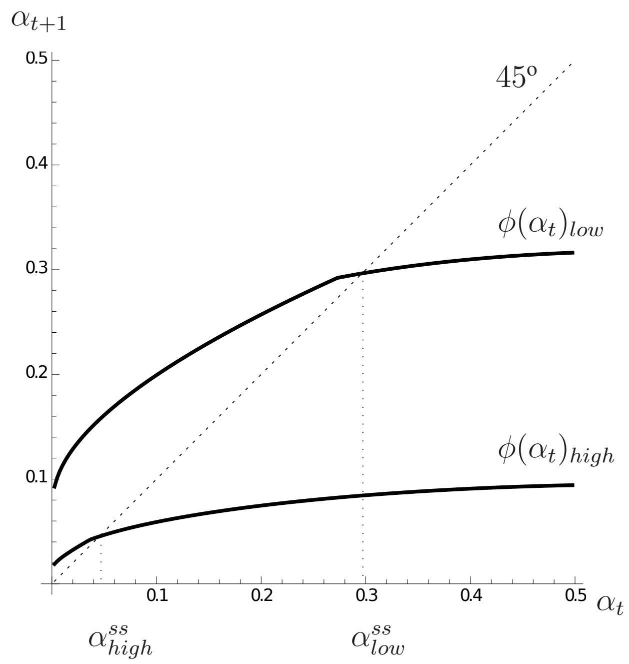


Figure 2.4: Human capital accumulation in a high-fertility country ($n = 4$) and a low-fertility country ($n = 1$).

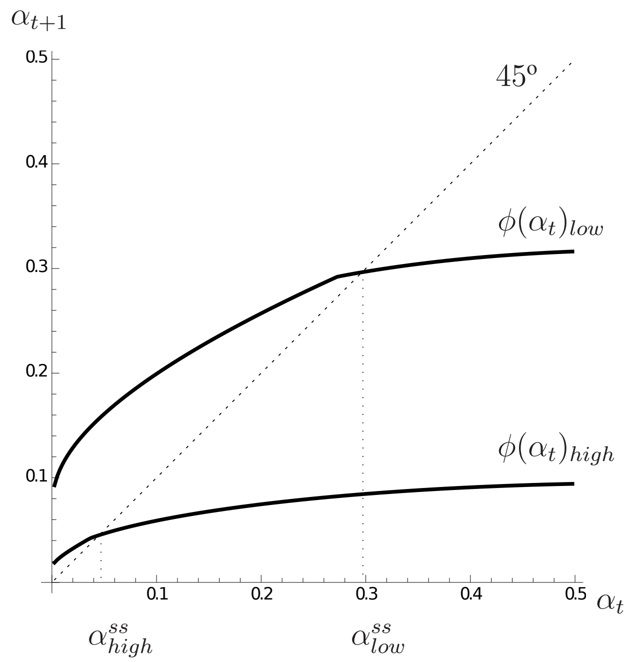


Figure 2.5: Human capital accumulation in countries with high ($e = 0.3$) and low ($e = 0.075$) education costs per child.

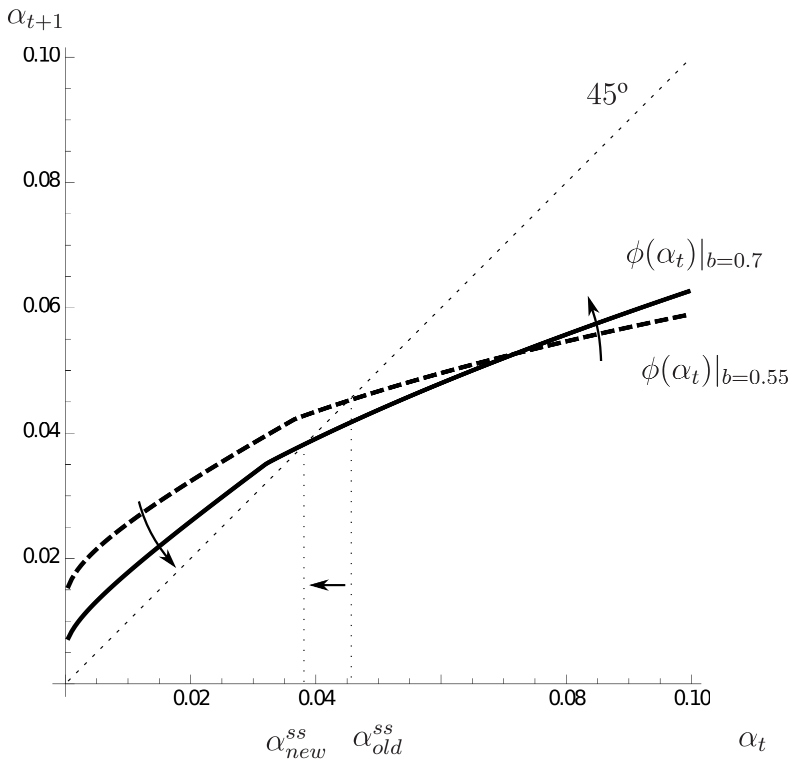


Figure 2.6: The effect of SBTC on $\phi(\alpha_t)$ for an economy with high fertility ($n = 4$).

To prevent the negative effects caused by SBTC, economies are advised to accompany the introduction of the new technology by adequate political measures. Since the negative effect is caused by rising relative educations costs for unskilled parents, any measure which diminishes or even circumvents this rise will reduce the negative SBTC effect. Such measures are (i) subsidizing unskilled parents if they send a child to college, (ii) financing education by partly public funds, and (iii) redistributive taxation. Introducing one of these measures lets the relative education costs for skilled parents rise while relative education costs for unskilled parents fall. Consider, for instance, the introduction of redistributive taxation in the model: skilled wages net of tax would fall while unskilled wages net of tax would rise; both effects lead to the changes in relative education costs discussed before.

2.5 Conclusion

I find that SBTC leads to rising skilled wages and falling unskilled wages causing ambiguous effects on HCA. First, SBTC raises unskilled parents' relative education costs inducing them to send fewer children to college. Second, skilled parents experience falling relative education costs and invest more in children's education. Third, wage inequality in the next period rises since the new skill-biased technology will still be in use. I prove that the negative effect dominates when human capital per capita is low implying HCA to slow down; whereas the positive effects dominate when human capital per capita is high leading to accelerated HCA.

To compare the effects of SBTC on HCA in economies with different characteristics, I distinguish economies by their fertility and education costs per student. What matters for HCA and the effects of SBTC on HCA are the costs when educating all children relative to a parent's income. This ratio measures how expensive children's education for one parent is. The higher the ratio, the smaller the proportion of children receiving education. Therefore, economies with higher fertility and/or higher education costs per student accumulate less human capital per capita over time, simply due to the higher costs of children's education.

SBTC effects in countries differing in their characteristics are not similar. Economies with low fertility and low education costs per student are likely to see accelerated HCA and higher human capital per capita in the steady state. Economies with high fertility and/or high education costs per student, however, are likely to experience slower human capital growth and a reduction in steady state human capital.

Societies facing negative SBTC effects are advised to introduce political measures to circumvent or diminish the negative effects. Since the negative effects stem from rising relative education costs for unskilled parents, any measure which reduces relative education costs for these parents diminishes the negative effects of SBTC. Such measures are, for instance, redistributing income via the tax system or subsidizing unskilled parents when their children attend college.

Knowing that SBTC causes negative effects on HCA in some economies, it is interesting to study when an economy introduces a new skill-biased technology. On the one hand, there may exist conflicting interests between the ones who decide on the introduction of the new technology, as for instance entrepreneurs, and society as a whole. On the other hand, HCA may induce SBTC because an economy with higher human capital per capita could be more successful in conducting research. In this case, not only does technology affect HCA, as studied in this chapter, but also HCA does affect technology. Endogenizing the technology choice is left for future research.

The effects analyzed in the chapter are caused by the changes in wages due to SBTC occurrence. Any other factor influencing wages may cause similar effects on HCA. For instance, international trade may affect domestic wage levels in open economies. According to the Stolper-Samuelson theorem, domestic input factor rewards are affected when the prices of final goods on the world market change. Wood (1995), Burtless (1995) and Wood (1998), for instance, argue that the rising wage inequality in the U.S. during the 1980s was caused by such price changes on the world market. However, Acemoglu (2002) and Winchester and Greenaway (2007) find in their empirical studies that SBTC was the major source for the observed rise in wage inequality during the 1980s.

Appendix

2.6.1 Proof of Proposition 3

Proposition 1. *For $\alpha_t \in [0,1]$, there exist a unique and continuous function $\phi(\alpha_t)$ for which $\alpha_{t+1} = \phi(\alpha_t)$ holds.*

Proof. The dynamic behavior of α_t derives from

$$\alpha_{t+1} = \alpha_t \lambda^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda^{u*}(\alpha_t, \alpha_{t+1}, b). \tag{2.12}$$

I start by proving the uniqueness of equation (2.12) which helps to prove in a second step that $\phi(\alpha_t)$ is unique and continuous.

Uniqueness of equation (2.12): Let us define the LHS of equation (2.12) as a function $L(\alpha_{t+1})$ and the RHS as a function $R(\alpha_{t+1})$. There exists a unique solution for equation (2.12) when the functions $L(\alpha_{t+1})$ and $R(\alpha_{t+1})$ intersect only once.

To prove whether the two functions fulfill this condition, I begin with calculating the slope of both functions. The slope of $L(\alpha_{t+1})$ is given by

$$\frac{\partial L(\alpha_{t+1})}{\partial \alpha_{t+1}} = 1,$$

implying that $L(\alpha_{t+1})$ is a strictly monotonic increasing function in α_{t+1}. The slope of $R(\alpha_{t+1})$ is given by

$$\frac{\partial R(\alpha_{t+1})}{\partial \alpha_{t+1}} = \alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_{t+1}} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial \alpha_{t+1}}$$

with

$$\frac{\partial \lambda_t^{i*}}{\partial \alpha_{t+1}} = \begin{cases} \frac{1}{1+\beta} \frac{1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^2} \frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}} < 0 & \text{when } 0 < \lambda_t^{i*} < 1, \\ \\ 0 & \text{else,} \end{cases}$$

where $i = s, u$ and $\alpha_t \in [0,1]$. Note that the partial derivatives are non-positive because a higher α_{t+1} lowers the wage ratio in the next period and thus reduces the incentive to invest in children's education for all parents:

$$\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}} = -\frac{1-\gamma}{\alpha_{t+1}(1-\alpha_{t+1})} \frac{w_{t+1}^s}{w_{t+1}^u} < 0.$$

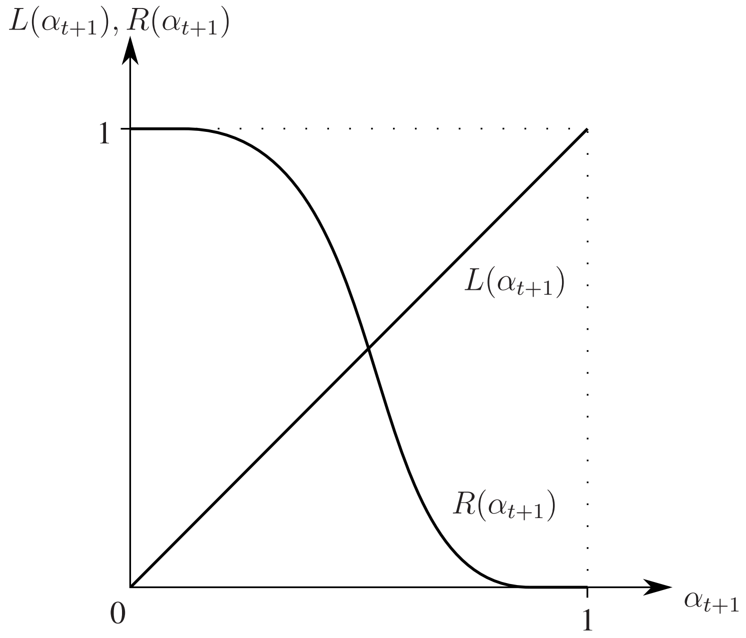


Figure 2.7: Graphical solution for α_{t+1} to prove uniqueness of $\phi(\alpha_{t+1})$.

As a consequence, $R(\alpha_{t+1})$ is a monotonically decreasing function in α_{t+1}. With $L(\alpha_{t+1})$ being strictly monotonic increasing and $R(\alpha_{t+1})$ being monotonically decreasing in α_{t+1}, both functions intersect at maximum once.

Finally, I show that this single intersection exists. The easiest way to do so is to show that inequalities $L(0) < R(0)$ and $L(1) > R(1)$ hold. The derivation of $L(0)$ and $L(1)$ is trivial. With respect to $R(0)$, note that for any $\alpha_t \in [0, 1]$ (i.e., for any wage formation in period t), parents choose education for all of their children because the wage ratio $\frac{w^s_{t+1}}{w^u_{t+1}}$ and thus the incentive to invest in children's education is infinitely large when $\alpha_{t+1} = 0$. With respect to $R(1)$, note that for any $\alpha_t \in [0, 1]$, parents choose not to invest in children's education because the wage ratio $\frac{w^s_{t+1}}{w^u_{t+1}}$ is equal to one and thus there is no incentive to invest in children's education when $\alpha_{t+1} = 1$. Formally, these considerations can be expressed as

$$L(0) = 0, \qquad L(1) = 1,$$
$$R(0) = 1, \qquad R(1) = 0.$$

As a result, I conclude that $L(\alpha_{t+1})$ and $R(\alpha_{t+1})$ intersect exactly once and thus, there exists a unique α_{t+1} solving equation (2.12). Figure 2.7 depicts qualitatively the two functions for a given value of α_t.

Properties of $\phi(\alpha_t)$: Using equation (2.12), I define the function

$$G(\alpha_t, \alpha_{t+1}) \equiv \alpha_t \lambda^{s*}(\alpha_t, \alpha_{t+1}, b) + (1 - \alpha_t)\lambda^{u*}(\alpha_t, \alpha_{t+1}, b) - \alpha_{t+1}.$$

The implicit function theorem holds because

(1) $G(\alpha_t, \alpha_{t+1})$ is continuously differentiable in both α_t and α_{t+1},

(2) $G(\alpha_t, \alpha_{t+1}) = 0$ is unique (see proof above), and

(3) $\frac{\partial G(\alpha_t, \alpha_{t+1})}{\partial \alpha_{t+1}} < 0$, i.e., the matrix of partial derivatives of $G(x, y)$ with respect to y is invertible.

Therefore, the unique and continuous function $\phi(\alpha_t)$ exists such that $\alpha_{t+1} = \phi(\alpha_t)$. □

2.6.2 Parameter choice

I use the following parameter values throughout this chapter. Parameters for the standard

Economy	γ	b	n	e	β
standard (low fertility, low education costs)	0.33	0.55	1	0.075	0.2
high fertility	0.33	0.55	**4**	0.075	0.2
high education costs	0.33	0.55	1	**0.3**	0.2

economy are chosen such that they fit to U.S. data of the 1970s (which is the decade before the occurrence of SBTC).

γ: Empirical data on the elasticity of substitution between input factors, denoted by σ, suggest that $\sigma \in [1, 2]$. Given that $\gamma = \frac{\sigma - 1}{\sigma}$ and choosing the average level $\sigma = 1.5$ leads to $\gamma = 0.33$.[23]

b: Acemoglu (2002) presents data for the U.S. over the period 1960-1996 on the relative skill supply $\frac{\alpha_t}{1-\alpha_t}$ and the skill premium $\frac{w_t^s}{w_t^u}$. Using the production function in equation (2.3), I calculate with these data the technology parameter b. As a result, b was approximately 0.55 throughout the 1970s and rises to approximately 0.7 in 1995.

n: The fertility rate in the U.S. during the 1970s is approximately 2 which implies a fertility rate of $n = 1$ in the model.[24] The high fertility rate of $n = 4$ represents the highest observed fertility rates in countries like Afghanistan, Chad and Zambia.

[23] See, for instance, Acemoglu (2002), Card and DiNardo (2002), and Katz and Murphy (1992).

[24] Note that each adult is fertile in the model while the empirically observed fertility rate is calculated for woman in birth-bearing age.

e: I calculate the expenditure on tertiary education per student relative to average income in the U.S. during the period 2000-2008. I find that the costs of studying 5 years in college is approximately 7.5% of parent's life-time income (based on annual income of 40,000 USD). The higher education costs are chosen randomly because data for less developed countries are not complete enough to calculate their education costs per child. However, data from India suggest that college education is twice as costly as in the U.S.

β: There is no estimation for this parameter in the literature. Having determined all other parameters, I choose β such that the skill supply in the steady state (α^{ss}) fits the empirical observation for the U.S. during the 1970s.

2.6.3 Shape of $\phi(\alpha_t)$

In order to determine the shape of $\phi(\alpha_t)$, I study the sign of the first and second derivative of equation (2.11) in the following. It is useful for the discussion to distinguish three different regimes:

- **Regime (1)** for $0 \leq \alpha_t \leq \widetilde{\alpha}$ with $\lambda_t^{s*} = 1$ and $0 < \lambda_t^{u*} < 1$:
  Since skilled wages are very high, skilled adults send all their children to college. In contrast, the unskilled wage is low implying that unskilled adults send only some of their children to college.

- **Regime (2)** for $\widetilde{\alpha} < \alpha_t < \widehat{\alpha}$ with $0 < \lambda_t^{s*} < 1$ and $0 < \lambda_t^{u*} < 1$:
  In this range of α_t, both skilled and unskilled parents send only some of their children to college because their income is not high enough to finance education for all their children. However, the proportion of children sent to college is higher for skilled than for unskilled parents since skilled wages are strictly higher than unskilled wages.

- **Regime (3)** for $\widehat{\alpha} \leq \alpha_t \leq 1$ with $0 < \lambda_t^{s*} = \lambda_t^{u*} < 1$:
  Skilled and unskilled wages are equally high in this range of α_t implying that both types of parents send the same proportion of children to college.

First derivative The first derivative derives implicitly as

$$\frac{\partial \phi(\alpha_t)}{\partial \alpha_t} = -\frac{\frac{\partial F}{\partial \alpha_t}}{\frac{\partial F}{\partial \alpha_{t+1}}}$$

with $F \equiv \alpha_t \lambda_t^{s*} + (1-\alpha_t)\lambda_t^{u*} - \alpha_{t+1}$. The derivative in the numerator derives as

$$\frac{\partial F}{\partial \alpha_{t+1}} = \alpha_t \frac{\partial \lambda_t^s}{\partial \alpha_{t+1}} + (1-\alpha_t)\frac{\partial \lambda_t^u}{\partial \alpha_{t+1}} - 1$$

and is always negative due to the fact that education decisions are either unaffected or negatively affected by a rising α_{t+1}:

$$\frac{\partial \lambda_t^{s*}}{\partial \alpha_{t+1}} = \begin{cases} 0 & \text{for } \alpha_t \leq \tilde{\alpha}, \\ \frac{1}{1+\beta}\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^{-2} \frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}} & \text{else.} \end{cases}$$

$$\frac{\partial \lambda_t^{u*}}{\partial \alpha_{t+1}} = \frac{1}{1+\beta}\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^{-2} \frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}}$$

$$\text{with} \quad \frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}} = \begin{cases} \frac{1}{1-b}\left[\frac{\alpha_{t+1}}{1-\alpha_{t+1}}\right]^{\gamma-2} \frac{\gamma-1}{(1-\alpha_{t+1})^2} < 0 & \text{for } \alpha_{t+1} \leq \hat{\alpha} \\ 0 & \text{else.} \end{cases}$$

$$\Rightarrow \qquad \frac{\partial F}{\partial \alpha_{t+1}} < 0 \qquad \text{for } \alpha_{t+1} \in [0,1]$$

The intuition for this result is that the more adults are skilled in the the next period (i.e. the higher α_{t+1}), the smaller is the skill premium and thus the smaller is the incentive to invest in education.

The derivative in the denominator derives as

$$\frac{\partial F}{\partial \alpha_t} = \lambda_t^{s*} - \lambda_t^{u*} + \alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_t} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial \alpha_t}.$$

The term $\lambda_t^{s*} - \lambda_t^{u*}$ is non-negative due to the wages as given by equations (2.4) to (2.6):

$$\lambda_t^{s*} - \lambda_t^{u*} = \frac{\beta}{1+\beta}\frac{1}{ne}\left[w_t^s - w_t^u\right] \begin{cases} > 0 & \text{for } \alpha_t < \hat{\alpha}, \\ = 0 & \text{else.} \end{cases}$$

Changes in education decisions of skilled and unskilled parents derive as

$$\frac{\partial \lambda_t^{s*}}{\partial \alpha_t} = \begin{cases} 0 & \text{for } \alpha_t \leq \tilde{\alpha} \\ \frac{\beta}{1-\beta}\frac{1}{ne}\frac{\partial w_t^s}{\partial \alpha_t} < 0 & \tilde{\alpha} < \alpha_t < \hat{\alpha}, \\ 0 & \text{else.} \end{cases}$$

$$\frac{\partial \lambda_t^{u*}}{\partial \alpha_t} = \begin{cases} \frac{\beta}{1-\beta} \frac{1}{ne} \frac{\partial w_t^u}{\partial \alpha_t} > 0 & \alpha_t < \hat{\alpha}, \\ 0 & \text{else.} \end{cases}$$

Due to the falling skilled wage rate, skilled parents decide to send fewer children to college when $\tilde{\alpha} < \alpha_t < \hat{\alpha}$. Unskilled parents, in contrast, educate more of their children since their income is rising. Though these two effects are working in opposite directions, the overall effect of α_t is positive because for all $\alpha_t \in]\tilde{\alpha}, \hat{\alpha}[$

$$\begin{aligned} \alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_t} + (1-\alpha_t) \frac{\partial \lambda_t^{u*}}{\partial \alpha_t} &= \frac{\beta}{1-\beta} \frac{1}{ne} \left[\alpha_t \frac{\partial w_t^s}{\partial \alpha_t} + (1-\alpha_t) \frac{\partial w_t^u}{\partial \alpha_t} \right] \\ &= \frac{\beta}{1-\beta} \frac{2(1-b)b(1-\gamma)}{(1-\alpha_t)ne} \left(\frac{\alpha_t}{1-\alpha_t} \right)^{\gamma} \left[1 - b + b \left(\frac{\alpha_t}{1-\alpha_t} \right)^{\gamma} \right]^{\frac{1-2\gamma}{\gamma}} \\ &> 0. \end{aligned}$$

This means that the derivative $\frac{\partial F}{\partial \alpha_t}$ is always non-negative:

$$\frac{\partial F}{\partial \alpha_t} = \begin{cases} 1 - \lambda_t^{u*} + (1-\alpha_t) \frac{\partial \lambda_t^{u*}}{\partial \alpha_t} > 0 & \text{for } \alpha_t \leq \tilde{\alpha}, \\ \lambda_t^{s*} - \lambda_t^{u*} + \alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_t} + (1-\alpha_t) \frac{\partial \lambda_t^{u*}}{\partial \alpha_t} > 0 & \text{for } \tilde{\alpha} < \alpha_t < \hat{\alpha}, \\ 0 & \text{else.} \end{cases}$$

As a consequence, the first derivative is positive or zero:

$$\frac{\partial \phi(\alpha_t)}{\partial \alpha_t} \begin{cases} > 0 & \text{for } 0 \leq \alpha_t < \hat{\alpha}, \\ = 0 & \text{for } \hat{\alpha} \leq \alpha_t \leq 1. \end{cases}$$

Second derivative The second derivative derives as

$$\frac{\partial^2 \phi(\alpha_t)}{\partial \alpha_t^2} = - \frac{\frac{\partial^2 F}{\partial \alpha_t^2} \frac{\partial F}{\partial \alpha_{t+1}} - \frac{\partial F}{\partial \alpha_t} \frac{\partial^2 F}{\partial \alpha_t \partial \alpha_{t+1}}}{\left(\frac{\partial F}{\partial \alpha_{t+1}} \right)^2}.$$

Since $\frac{\partial^2 F}{\partial \alpha_t \partial \alpha_{t+1}} = 0$ and since $\frac{\partial F}{\partial \alpha_{t+1}} < 0$, the sign of the second derivative depends on $\frac{\partial^2 F}{\partial \alpha_t^2}$ only:

$$\text{sgn} \left[\frac{\partial^2 \phi(\alpha_t)}{\partial \alpha_t^2} \right] = \text{sgn} \left[\frac{\partial^2 F}{\partial \alpha_t^2} \right].$$

In general, the derivative of interest derives as

$$\frac{\partial^2 F}{\partial \alpha_t^2} = 2\left[\frac{\partial \lambda_t^{s*}}{\partial \alpha_t} - \frac{\partial \lambda_t^{u*}}{\partial \alpha_t}\right] + \alpha_t \frac{\partial^2 \lambda_t^{s*}}{\partial \alpha_t^2} + (1-\alpha_t)\frac{\partial^2 \lambda_t^{u*}}{\partial \alpha_t^2}.$$

For $\alpha_t \in [0, \tilde{\alpha}]$, skilled parents send all their children to college and hence the derivative simplifies to

$$\frac{\partial^2 F}{\partial \alpha_t^2} = \frac{\beta}{1+\beta}\frac{1}{ne}\left[-2\frac{\partial w_t^u}{\partial \alpha_t} + (1-\alpha_t)\frac{\partial^2 w_t^u}{\partial \alpha_t^2}\right] = -\frac{\beta}{1+\beta}\frac{1}{ne}\times$$

$$\frac{\left(\frac{\alpha_t}{1-\alpha_t}\right)^\gamma (1-b)b\left[1-b+b\left(\frac{\alpha_t}{1-\alpha_t}\right)^\gamma\right]^{\frac{1-3\gamma}{\gamma}}(1-\gamma)\left[(1-b)(1-\gamma)+b\gamma\left(\frac{\alpha_t}{1-\alpha_t}\right)^\gamma\right]}{(1-\alpha_t)\alpha_t^2}$$

$$< 0$$

and is always negative. For $\alpha_t \in]\tilde{\alpha}, \hat{\alpha}[$, skilled parents sent only some of their children to college and two additional terms enter the derivative $\frac{\partial^2 F}{\partial \alpha_t^2}$. The two additional terms are in sum negative too:

$$2\frac{\partial \lambda_t^{s*}}{\partial \alpha_t} + \alpha_t \frac{\partial^2 \lambda_t^{s*}}{\partial \alpha_t^2} = \frac{\beta}{1+\beta}\frac{1}{ne}\left[2\frac{\partial w_t^s}{\partial \alpha_t} + \alpha_t \frac{\partial^2 w_t^s}{\partial \alpha_t^2}\right] = -\frac{\beta}{1+\beta}\frac{1}{ne}\times$$

$$\frac{\left(\frac{1-\alpha_t}{\alpha_t}\right)^\gamma (1-b)b\left[\left(\frac{1-\alpha_t}{\alpha_t}\right)^\gamma (1-b)+b\right]^{\frac{1}{\gamma}}(1-\gamma)\left[(1-b)\left(\frac{1-\alpha_t}{\alpha_t}\right)^\gamma \gamma + b(1-\gamma)\right]}{(1-\alpha_t)^2\alpha_t\left[(1-b)\left(\frac{1-\alpha_t}{\alpha_t}\right)^\gamma + b\right]^3}$$

$$2\frac{\partial \lambda_t^{s*}}{\partial \alpha_t} + \alpha_t \frac{\partial^2 \lambda_t^{s*}}{\partial \alpha_t^2} < 0.$$

For $\alpha_t \in [\hat{\alpha}, 1]$, the second derivative is zero because education decisions do not change:

$$\frac{\partial^2 F}{\partial \alpha_t^2} = 0.$$

In sum, the second derivative is non-positive:

$$\frac{\partial^2 \phi(\alpha_t)}{\partial \alpha_t^2}\begin{cases} < 0 & \text{for } 0 \leq \alpha_t < \hat{\alpha}, \\ = 0 & \text{for } \hat{\alpha} \leq \alpha_t \leq 1. \end{cases}$$

Concave shape and uniqueness of the steady state From the non-negative first and non-positive second derivative, I conclude that $\phi(\alpha_t)$ has a concave shape. In order to prove the existence and the uniqueness of the steady state, I show in the following that $\phi(0) > 0$ and $\phi(1) < 1$ which implies that there exist one and only one steady state. Graphically, this steady state is then given by the intersection of $\phi(\alpha_t)$ with the 45-degree-line.

When the economy consist of unskilled adults only (i.e. $\alpha_t = 0$), the proportion of skilled adults in the next period derives from the education decision of unskilled parents only:

$$\alpha_{t+1} = \lambda_t^{u*}. \tag{2.13}$$

Unskilled parents' education decision follows

$$\lambda_t^{u*} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}(1-b)^{\frac{1}{\gamma}} - \frac{1}{\frac{b}{1-b}\left(\frac{1-\alpha_{t+1}}{\alpha_{t+1}}\right)^{1-\gamma} - 1}\right]. \tag{2.14}$$

Equations (2.13) and (2.14) define implicitly the optimal proportion of children sent to college which has to be positive:

$$0 < \phi(0) < \hat{\alpha}$$

It also has to be smaller than $\hat{\alpha}$ because wages are equalized for values above this threshold which means that there is no incentive to invest in college education.

When the economy consist of skilled adults only (i.e. $\alpha_t = 1$), parents send only some of their children to college. The proportion has to be smaller than $\hat{\alpha}$ again since otherwise the incentive to invest in children's education is zero:

$$\phi(1) < \hat{\alpha}.$$

In sum, I proved that function $\phi(\alpha_t)$ is concave, starts with a positive value (i.e. there is not trivial steady state) and is strictly smaller than one at $\phi(1)$. These three characteristics imply that there exist always one and only one steady state.

Graphical presentations of the shape of $\phi(\alpha_t)$ and of the wage structure in each regime are provided in figures 2.8 and 2.9.

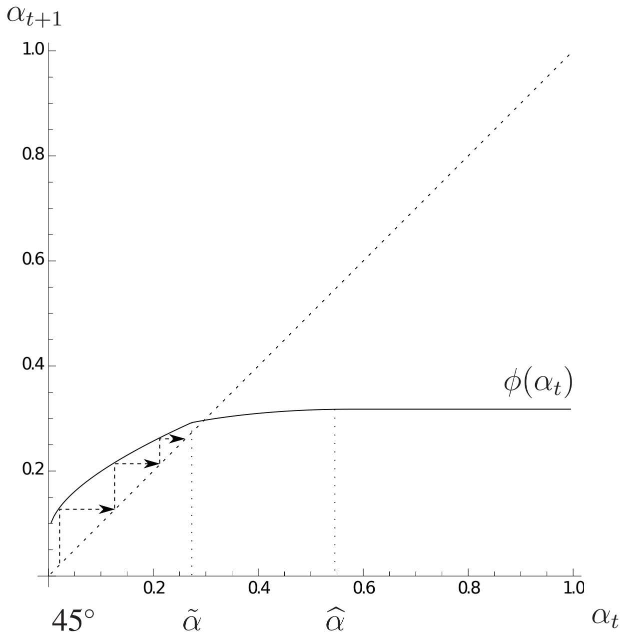


Figure 2.8: The three regimes of $\phi(\alpha_t)$.

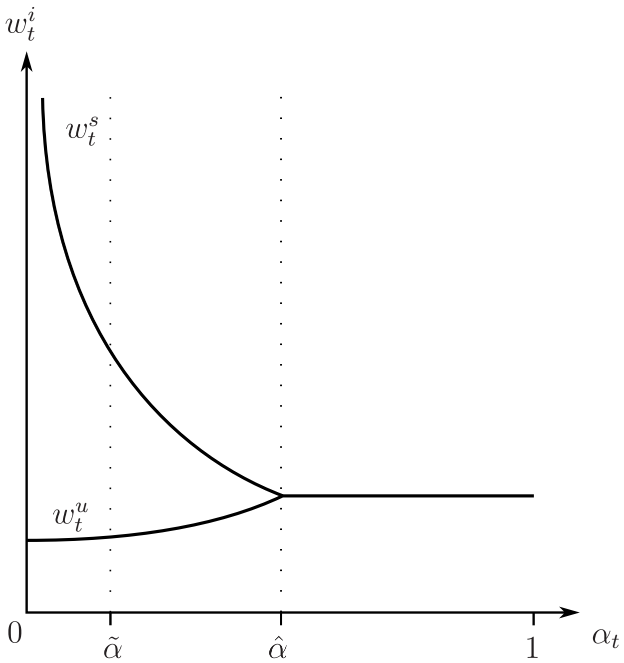


Figure 2.9: Skilled and unskilled wages in each regime.

2.6.4 Proof of Proposition 4

Proposition 2. *For* $\alpha_t \in [0,1]$,

$$\text{sgn}\left[\frac{\partial\phi(\alpha_t)}{\partial b}\right] = \text{sgn}\left[\alpha_t \frac{\partial\lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial\lambda_t^{u*}}{\partial b}\right].$$

Proof. First, I use equation (2.11) to define a function G:

$$G \equiv \alpha_t \lambda^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda^{u*}(\alpha_t, \alpha_{t+1}, b) - \alpha_{t+1}.$$

Second, I derive the implicit derivative of $\frac{\partial\alpha_{t+1}}{\partial b}$ using the G-function which leads to

$$\frac{\partial\alpha_{t+1}}{\partial b} = -\frac{\alpha_t \dfrac{\partial\lambda^{s*}(\alpha_t, \alpha_{t+1}, b)}{\partial b} + (1-\alpha_t)\dfrac{\partial\lambda^{u*}(\alpha_t, \alpha_{t+1}, b)}{\partial b}}{\alpha_t \underbrace{\dfrac{\partial\lambda^{s*}(\alpha_t, \alpha_{t+1}, b)}{\partial\alpha_{t+1}}}_{\leq 0} + (1-\alpha_t)\underbrace{\dfrac{\partial\lambda^{u*}(\alpha_t, \alpha_{t+1}, b)}{\partial\alpha_{t+1}}}_{<0} - 1}.$$

Since the denominator is always negative, the sign of $\frac{\partial\alpha_{t+1}}{\partial b}$ is uniquely determined by the sign of the numerator:

$$\text{sgn}\left[\frac{\partial\phi(\alpha_t)}{\partial b}\right] = \text{sgn}\left[\alpha_t \frac{\partial\lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial\lambda_t^{u*}}{\partial b}\right],$$

with

$$\alpha_{t+1} = \phi(\alpha_t), \qquad \lambda_t^{s*} = \lambda^{s*}(\alpha_t, \alpha_{t+1}, b), \qquad \lambda_t^{u*} = \lambda^{u*}(\alpha_t, \alpha_{t+1}, b)$$

to shorten the notation. □

2.6.5 Proof of Proposition 5

Proposition 3. *SBTC has a positive effect on human capital accumulation when all parents are skilled, i.e.,*

$$\lim_{\alpha_t \to 1} \quad \alpha_t \frac{\partial\lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial\lambda_t^{u*}}{\partial b} > 0.$$

Under the assumption that education costs are not too low, SBTC has a negative effect on human capital accumulation when all parents are unskilled:

$$\lim_{\alpha_t \to 0} \quad \alpha_t \frac{\partial\lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial\lambda_t^{u*}}{\partial b} < 0.$$

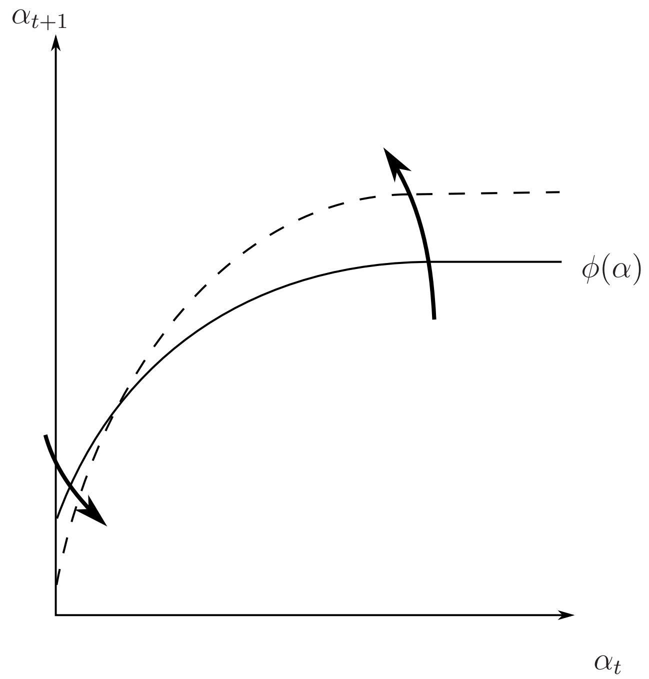


Figure 2.10: Rotation of $\phi(\alpha_t)$ when SBTC occurs.

As a consequence, function $\phi(\alpha_t)$ rotates anticlockwise in the (α_{t+1}, α_t)-space after the occurrence of SBTC (see Figure 2.3).

Proof. The rotation of $\phi(\alpha_t)$, as shown in Figure 2.10, follows from the two limits. I prove in the following paragraphs whether the two limits hold or not.

In the case of α_t approaching one, wages of skilled and unskilled parents are equalized, i.e., $w_t^s = w_t^u = \bar{w}$, and α_{t+1} derives as

$$\alpha_{t+1} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}\bar{w} - \frac{1}{\frac{w_{t+1}^s}{w_{t+1}^u} - 1}\right].$$

Using this equation to define $G \equiv \frac{1}{1+\beta}[...] - \alpha_{t+1} = 0$ and taking the implicit derivative shows that the sign of $\frac{\partial \alpha_{t+1}}{\partial b}$ is determined by the sign of $\frac{\partial G}{\partial b}$ alone:

$$\frac{\partial \alpha_{t+1}}{\partial b} = -\frac{\frac{\partial G}{\partial b}}{\frac{\partial G}{\partial \alpha_{t+1}}}$$

$$= -\frac{\dfrac{\partial G}{\partial b}}{\dfrac{1}{1+\beta}\dfrac{1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u} - 1\right]^2}\underbrace{\dfrac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial \alpha_{t+1}}}_{(-)}},$$

$$\Rightarrow \qquad \operatorname{sgn}\left[\frac{\partial \alpha_{t+1}}{\partial b}\right] = \operatorname{sgn}\left[\frac{\partial G}{\partial b}\right].$$

Deriving the partial derivative $\frac{\partial G}{\partial b}$ while keeping α_{t+1} constant results in

$$\frac{\partial G}{\partial b} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}\underbrace{\frac{\partial \bar{w}}{\partial b}}_{(+)} + \frac{1}{\left[\frac{w^s_{t+1}}{w^u_{t+1}} - 1\right]^2}\underbrace{\frac{\partial \frac{w^s_{t+1}}{w^u_{t+1}}}{\partial b}}_{(+)}\right].$$

Since the wage rate $\bar{w}$ is increasing in the technology parameter for $b \in (0.5, 1)$, the derivative $\frac{\partial G}{\partial b}$ as a whole and hence the limit I am looking for are always positive:

$$\lim_{\alpha_t \to 1} \alpha_t \frac{\partial \lambda^{s*}_t}{\partial b} + (1-\alpha_t)\frac{\partial \lambda^{u*}_t}{\partial b} > 0.$$

In the case where the number of skilled adults is close to zero (precisely, if $\alpha_t < \tilde{\alpha}_t$), skilled parents send all children to college ($\lambda^{s*}_t = 1$) and hence α_{t+1} follows

$$\alpha_{t+1} = \alpha_t + (1-\alpha_t)\lambda^{u*}_t.$$

Calculating the effect of SBTC on human capital accumulation yields the following derivative:

$$\frac{\partial \alpha_{t+1}}{\partial b} = (1-\alpha_t)\frac{\partial \lambda^{u*}_t}{\partial b}.$$

In the next step, I derive the change in unskilled parents' education decision:

$$\frac{\partial \lambda^{u*}_t}{\partial b} = \frac{1}{1+\beta}\left[\frac{\beta}{ne}\underbrace{\frac{\partial w^u_t}{\partial b}}_{(-)} + \underbrace{\frac{1}{\left[\frac{w^s_{t+1}}{w^u_{t+1}} - 1\right]^2}\frac{\partial \frac{w^s_{t+1}}{w^u_{t+1}}}{\partial b}}_{\equiv A}\right].$$

The technology change has ambiguous effects on the education decision of unskilled parents. On the one hand, it has a negative effect on unskilled parents' wage income. On the other hand, it has a positive effect on the skill premium in the future. As long as the latter is smaller than the former, the overall effect is negative. Focusing on the latter effect and deriving the derivative in the A-term yields the following equation:

$$A = \frac{1}{b(1-b)\left[\sqrt{\frac{w^s_{t+1}}{w^u_{t+1}}} - \frac{1}{\sqrt{\frac{w^s_{t+1}}{w^u_{t+1}}}}\right]^2}.$$

According to this equation, the A-term and hence the positive effect is smaller, the higher the wage ratio $\frac{w^s_{t+1}}{w^u_{t+1}}$. From equation (2.6), I know that the wage ratio is higher, the smaller α_{t+1}. So given that α_{t+1} is small enough, the positive incentive effect is small and hence, the overall effect is negative. The question arises: when is α_{t+1} small enough? It is small enough when education costs are not too low because high education costs imply low investments in kids' education which translate into a low α_{t+1}. To sum up the line of arguments: when education costs are high enough, the derivative $\frac{\partial \lambda^{u*}_t}{\partial b}$ is negative. As a consequence,

$$\lim_{\alpha_t \to 0} \alpha_t \frac{\partial \lambda^{s*}_t}{\partial b} + (1-\alpha_t)\frac{\partial \lambda^{u*}_t}{\partial b} < 0.$$

□

2.6.6 Proof of Proposition 6

Proposition 4. *Higher fertility and education costs per child shift $\phi(\alpha_t)$ downwards and lead to lower human capital in the steady state:*

$$\frac{\partial \phi(\alpha_t)}{\partial n} < 0, \qquad \frac{\partial \phi(\alpha_t)}{\partial e} < 0,$$
$$\frac{\partial \alpha^{ss}}{\partial n} < 0, \qquad \frac{\partial \alpha^{ss}}{\partial e} < 0.$$

Proof. We prove first the effects on $\phi(\alpha_t)$ and second the effects on steady state human capital.

Effects on $\phi(\alpha_t)$: Function $\phi(\alpha_t)$ is implicitly defined by equation (2.11). Using this equation to define the function

$$G \equiv \alpha_t \lambda^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda^{u*}(\alpha_t, \alpha_{t+1}, b) - \alpha_{t+1}$$

and taking implicit derivatives yields

$$\frac{d\alpha_{t+1}}{dn} = -\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial \alpha_{t+1}}} \quad \text{and} \quad \frac{d\alpha_{t+1}}{de} = -\frac{\frac{\partial G}{\partial e}}{\frac{\partial G}{\partial \alpha_{t+1}}}.$$

with the three partial derivatives

$$\frac{\partial G}{\partial n} = \alpha_t \frac{\partial \lambda^{s*}_t}{\partial n} + (1-\alpha_t)\frac{\partial \lambda^{u*}_t}{\partial n},$$

$$\frac{\partial G}{\partial e} = \alpha_t \frac{\partial \lambda_t^{s*}}{\partial e} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial e},$$

$$\frac{\partial G}{\partial \alpha_{t+1}} = \alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_{t+1}} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial \alpha_{t+1}} - 1.$$

Because optimal education decisions derive as

$$\lambda^{i*}(\alpha_t, \alpha_{t+1}, b) = \begin{cases} 0 & \text{if } \frac{1}{1+\beta}[.] < 0, \\ 1 & \text{if } \frac{1}{1+\beta}[.] > 1, \\ \frac{1}{1+\beta}\left[\beta \frac{w^i(\alpha_t,b)}{ne} - \frac{w^u(\alpha_{t+1},b)}{w^s(\alpha_{t+1},b)-w^u(\alpha_{t+1},b)}\right] & \text{else,} \end{cases}$$

with $i = s, u$, the six partial derivatives of optimal education decisions are

$$\frac{\partial \lambda_t^{s*}}{\partial n} \leq 0, \qquad \frac{\partial \lambda_t^{u*}}{\partial n} < 0,$$

$$\frac{\partial \lambda_t^{s*}}{\partial e} \leq 0, \qquad \frac{\partial \lambda_t^{u*}}{\partial e} < 0,$$

$$\frac{\partial \lambda_t^{s*}}{\partial \alpha_{t+1}} \leq 0, \qquad \frac{\partial \lambda_t^{u*}}{\partial \alpha_{t+1}} < 0.$$

These derivative reflect the following mechanisms: rising fertility and rising education costs let parents choose to invest in education of a lower proportion of their children. Also, a rise in the next period's proportion of skilled adults leads to a smaller expected wage gap and thus reduces the incentive to invest in children's education. As a consequence, education decisions are always negatively affected. There is one exception for skilled adults. Skilled adults choose to educate all their children when the population contains only few skilled adults (see Appendix 2.6.3 for further explanation of the corner solution in skilled parents' education decision). In this case, a change in fertility, education costs or next period's expected skill formation does not change their education decision.

Since all six partial derivatives of the education decision are non-positive, we can conclude

$$\frac{\partial G}{\partial n} < 0, \qquad \frac{\partial G}{\partial e} < 0, \qquad \frac{\partial G}{\partial \alpha_{t+1}} < 0,$$

and thus

$$\frac{\partial \alpha_{t+1}}{\partial n} < 0, \qquad \frac{\partial \alpha_{t+1}}{\partial e} < 0.$$

Effects on steady state human capital: The steady state is implicitly derived by $\alpha^{ss} = \phi(\alpha^{ss})$. Defining

$$G \equiv \phi(\alpha^{ss}) - \alpha^{ss}$$

and taking implicit derivatives yields

$$\frac{\partial \alpha^{ss}}{\partial n} = -\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial \alpha^{ss}}} = -\frac{\frac{\partial \phi(\alpha^{ss})}{\partial n}}{\frac{\partial \phi(\alpha^{ss})}{\partial \alpha^{ss}} - 1} = -\frac{(-)}{(-)} < 0,$$

$$\frac{\partial \alpha^{ss}}{\partial e} = -\frac{\frac{\partial G}{\partial e}}{\frac{\partial G}{\partial \alpha^{ss}}} = -\frac{\frac{\partial \phi(\alpha^{ss})}{\partial e}}{\frac{\partial \phi(\alpha^{ss})}{\partial \alpha^{ss}} - 1} = -\frac{(-)}{(-)} < 0.$$

Note that the slope of $\phi(\alpha_t)$ is less than one at the steady state (otherwise $\phi(\alpha_t)$ would not cross the 45°-line): $0 < \frac{\partial \phi(\alpha^{ss})}{\partial \alpha^{ss}} < 1$. □

Chapter 3

Towards the Explanation of Differentiated Tuition Fees Among Domestic and Foreign Students: A Positive Analysis

3.1 Introduction

Students became increasingly mobile over the recent past. The five main destination countries of mobile students faced on average an increase of 73% in the number of foreign students between 1998 and 2007 (see Table 3.1 in Appendix 3.7.1). A very similar trend was already present in the period between 1980 and 2000 (see OECD (2004)). In the two countries with the highest shares of foreign students, namely Australia and the United Kingdom, the number of foreign students more than doubled between 1998 and 2007. Interestingly, foreign students have to pay three times higher tuition fees than domestic students in these two countries (see Appendix 3.7.1 for more details) while other countries, like France or Germany with much less foreign students, do not differentiate tuition fees.

The effect of mobility on the public provision of higher education has been studied in several contributions in the literature. The mobility of workers, for instance, leads to under-provision of higher education because the benefits of higher education do not accrue where it was funded (Justman and Thisse (1997, 2000)). Mobile students 'free-ride' on the education system of the host country if they are going to leave the study location after graduation (Del Ray (2001)). But some foreign students may also stay after graduation and work in the host country.[1] In this case, student mobility opens the possibility to attract fu-

[1] Dreher and Poutvaara (2005) provide some empirical evidence.

ture high-skilled workers via attracting students.[2] Lange (2009) shows that accounting for both imperfect student and worker mobility may give rise to under- or over-provision. In this line, Demange, Fenge and Uebelmesser (2014) show that there is a sub optimal shift from taxes to fees and the number of students is too low when only skilled workers are mobile; when also students can migrate, there is a countervailing force such that maintaining the optimal financial mix becomes possible. Against this theoretical background, the change in student mobility may have affected the funding of higher education; and indeed, the share of private funds has been increasing.[3] However, the phenomenon of differentiated tuition fees has not been in the focus of these contributions.

The contribution of this chapter to the existing literature is twofold. (1) The chapter identifies the determinants of tuition fee differentials among domestic and foreign students. I define the tuition fee differential throughout this chapter as the difference between the tuition fee for foreign students minus the tuition fee for domestic students. (2) Moreover, I study the questions whether differentiating tuition fees improves a country's welfare and whether cooperation between countries can improve the welfare of each country.

I base the analysis on a two-country model with mobile students and mobile workers. Governments of both countries decide independently on tuition fees for domestic and for foreign students and on the tax rate while taking the quality of education as given. Each government's objective is to maximize local GDP which is a non-discriminating objective in the sense that no student group, neither domestic nor foreign, is per se favored by the decision maker.[4] I choose this objective in order to investigate other determinants of tuition fee differentials than pure preference of the decision maker for one student group. The results of the chapter are derived first in a framework where two similar countries compete for students - referred to as standard setup. Later, the setup is extended to the case of two heterogeneous countries where education quality differs between them.

I find that tuition fee differentials are driven by 3 factors: student mobility, worker mobility and education quality. Student mobility affects the tuition fee differential positively. The reason is that countries aim to attract students in order to attract future high skilled workers. If students are becoming more mobile, foreign students are easier to attract while

[2] That this is not just a theoretical possibility but something politicians in the real world think of is shown by the example of Germany where, since 2012, (certain) foreign students are allowed to stay longer an search for a job in Germany after having completed their studies.

[3] Kärkkäinen (2006) provides empirical evidence for the partial shift from public to private funding of higher education. Haussen and Uebelmesser (2015) find in an empirical study on 22 OECD countries for the period of 2000 to 2011 a significant positive correlation between the share of international students and the share of private funds in higher education.

[4] A counter example of a discriminating target is maximizing income of natives. In this case, natives are clearly favored by the decision maker. Such an objective function would lead to higher tuition fees for foreign students compared to the ones for domestic students.

it becomes more difficult to attract domestic students. Lowering tuition fees for domestic students makes it more attractive for students to study at home; this reduction in the tuition fee for domestic students is financed by raising the tuition fee for foreigners. Worker mobility, in contrast, does not affect tuition fee differentials when both natives and foreigners have similar migration patterns. But it has a positive effect on tuition fee differentials if migration patterns differ, as for instance when natives are more likely to stay in the study location than foreigners. The explanation rests on the following argument: when foreign students are more likely to leave after graduation, they contribute less to the future human capital stock and GDP of the host country than domestic students. Governments therefore prefer to attract natives as students while they prefer to reduce the number of foreign student by setting a lower tuition fee for domestic students compared to the one for foreign students. As for education quality, I find that the tuition fee differential in a country widens when the country improves the quality of education. At the same time, the tuition fee differential in the other country is negatively affected. The reason for this result is that better education quality raises the expected loss in future human capital associated with the fact that foreigners are more likely to leave the study location than natives. In order to attract less foreign and more domestic students, the government shifts the level of tuition fees from domestic to foreign students. The other country which did not improve the education quality just reacts to these changes in tuition fees.

Regarding welfare, I find that a country's welfare is maximized when tuition fees are not differentiated. The reason is that differentiated tuition fees distort student migration decisions and lead to either too much or too little migration depending on the sign of the tuition fee differential. This result provides an economic argumentation for the rules implemented in the European Union where member countries are in general allowed to differentiate tuition fees but not for students coming from one of the EU member countries. The finding regarding welfare is furthermore in line with Hübner (2009); he shows that differentiating tuition fees on the state-level does not increase federal welfare.

The rest of the chapter is organized as follows. Section 2 presents the standard setup. In section 3, I solve for optimal tuition fee differentials, analyze its determinants and discuss the prediction for future tuition fee differentials. In section 4, I introduce and analyze the extended setup with two non-similar countries which differ in the provided education quality. Section 5 provides the welfare analysis and section 6 concludes.

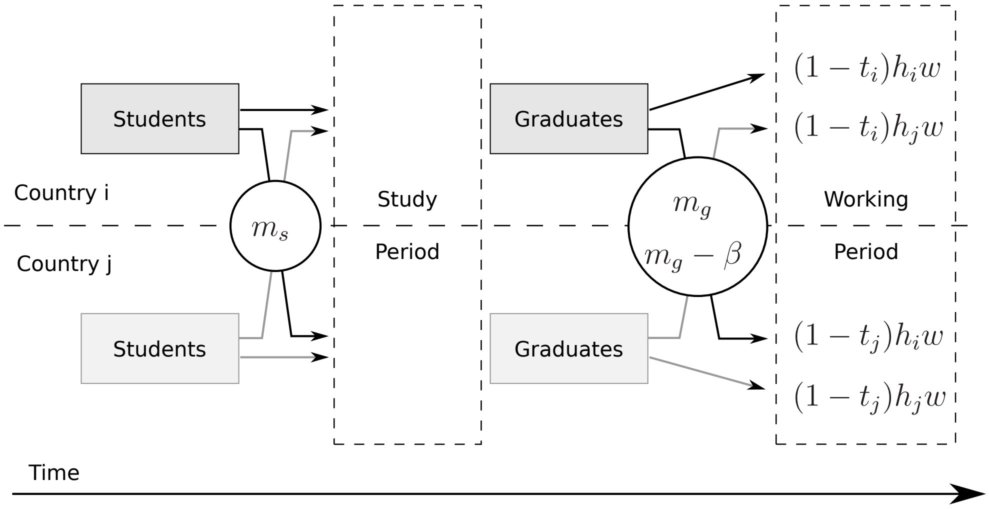


Figure 3.1: Migration decisions of students and graduates.

3.2 Model Description

The setup used in this chapter is an adopted and extended version of the model used in Krieger and Lange (2010) and comprises the following main features: I analyze government's choice of tuition fees and of the tax rate in a world consisting of two countries labeled i and j.[5] Individuals attend university[6] and have the opportunity to migrate between countries either to study or to work abroad. Facing these migration streams, governments aim to maximize GDP of their country taking into account their budget constraint. Their policy instruments are differentiated tuition fees and the tax rate. In the following, I present the time structure of the model, discuss the modeling of student and labor migration and shed some light on the role of governments in this setup.

The time structure of the model is divided into three stages: policy choice, education and working stage. In the policy choice stage, governments decide independently on tuition fees for domestic and for foreign students and on the tax rate. In the education stage, individuals decide where to study, which is either in their home country or abroad, attend university and graduate. Finally, individuals decide in the working stage where to work, which is either the country they studied in or abroad, and supply labor inelastically.

Individuals may migrate between countries to study and/or to work outside their country of origin as displayed in Figure 3.1. Regarding student migration, a student who decides to study abroad faces migration costs m_s. Migration costs reflect monetary costs, like for

[5]The setup extends Krieger and Lange (2010) in that it allows for differentiated tuition fees. Furthermore, I consider cases where the quality of education offered to students is not similar across countries, i.e., $h_i \neq h_j$.

[6]I assume an exogenous education decision where all individuals decide in favor of higher education.

instance travel and moving expenses, as well as non-monetary aspects. Non-monetary aspects are, for example, social and cultural ties: some individuals have strong ties to their family, friends and place of birth while others like to explore the big, wide world. Regarding labor migration, a graduate who studied in his home country faces migration costs of m_g while a graduate who studied outside his home country faces migration costs of $m_g - \beta$ with $\beta > 0$. Parameter β captures the migration cost advantage of a former foreign student because he has some migration experience and moves back to his home country.[7] Graduates' migration costs m_g are revealed by nature just upon graduation because some non-monetary aspects which influence the costs of migration develop during the study period: students enter a new environment, form new relationships and may find their future spouse; additionally, students studying abroad experience a new country, language and culture. I assume migration costs m_s and m_g to be uniformly distributed with a negative lower bound,[8] while the expected or average migration costs are assumed to be positive. Formally, these assumptions on migration costs read as

$$\begin{aligned} &m_s \sim U[\underline{m}_s, \overline{m}_s], &&\underline{m}_s < 0, &&E[m_s] > 0, \\ &m_g \sim U[\underline{m}_g, \overline{m}_g], &&\underline{m}_g < 0, &&E[m_g] > 0, \\ & &&\beta > 0, &&E[m_g - \beta] > 0. \end{aligned} \tag{A.1}$$

Note that individual's migration decisions determine disposable labor income in the working stage which is given by $(1 - t_a)h_b w$ with $a, b = i, j$. On the one hand, the labor migration decision determines in which country the individuals supply labor and pays taxes. The tax rate in country i (in country j) is denoted by t_i (t_j). On the other hand, the student migration decision determines the level of human capital a student achieves upon graduation: graduates who studied in country i (in country j) achieve human capital of h_i (h_j). The human capital of a graduate in turn determines his effective labor supply assuming that a worker with higher human capital supplies more efficiency units of labor. The last component in disposable labor income is the wage rate per efficiency unit of labor w which I assume to be equal across countries.[9]

Last in this section, governments are introduced to the setup. Each governments pro-

[7]Empirical evidence shows that foreign students are more likely to leave their study location than domestic students (see for instance Busch and Weigert (2010), Dreher and Poutvaara (2005) and Hein and Plesch (2009)). A positive β reflects this empirical observation because it leads to lower migration costs for graduates who studied outside their home country implying a higher probability to leave the study location.

[8]Individuals with negative migration costs have a strong desire to migrate.

[9]More technically, I assume that production follows in both countries the same constant return to scale production function with human capital as the only input factor. Under this assumption, the wage rate per efficiency unit of labor is independent of the actual human capital stock used in production and equal across countries.

vides and finances higher education in its country. Providing higher education to one student imposes costs of the amount $c_a = f(h_a)$ with $a = i, j$ which depends on the offered quality of education. To finance the expenditure for higher education, governments use two income sources, i.e., tax revenue and tuition fee revenue. The budget, or net revenue, in the case of country i derives as

$$NR_i = \underbrace{t_i w H_i}_{\text{tax revenue}} + \underbrace{s_{ii}S_{ii} + s_{ij}S_{ij}}_{\text{tuition fee revenue}} - \underbrace{c_i(S_{ii} + S_{ij})}_{\text{education expenditure}}$$

where H_i denotes the stock of human capital in country i (and is formally defined below), s_{ii} (s_{ij}) denotes the tuition fee for domestic students (tuition fee for foreign students) and S_{ii} (S_{ij}) denotes the number of domestic students (number of foreign students). Note that tuition fees and the number of students are indexed in the following way: the first index denotes the study location and the second the origin of the student.

In providing public education, the government aims to maximize regional GDP given by

$$GDP_i = wH_i$$

where GDP is simply derived by the sum of wages[10] over all workers. Note that the government is constrained by its budget since it must raise funds via taxes and/or tuition fees in order to provide public education to the students studying in its jurisdiction.

The choices of tuition fees and the tax rates are simultaneously and independently made by both governments. Throughout this chapter, I define the equilibrium as the situation where (i) each government has chosen tuition fees and the tax rate as best response to the choice of the other government and (ii) no government can benefit by changing its strategy while the other government keeps its strategy unchanged (Nash equilibrium).

3.3 The Case of Two Identical Countries

In this section, I study the case of two identical countries offering the same quality of education:

$$h_i = h_j \equiv h.$$

[10]Note that the wage rate w is a constant which reflects the implicit assumption that the marginal product of labor is endogenously given and constant. This assumption is made for convenience reasons and does not affect the qualitative results of the model. Introducing decreasing labor margins would imply that the incentive of the government to attract more students/workers is decreasing in the number of workers in that country. Note further that the education margin is endogenous (via the governments' decision on tax rates) and decreasing in the tax rates.

First, I solve the model and derive the government's choice of the policy instruments; second, I determine the equilibrium of the model. In the third subsection, I analyze the determinants of equilibrium tuition fee differentials and discuss the expected future evolution of tuition fee differentials.

3.3.1 Solving the Model

I solve the model by backwards induction: I study first labor migration and second student migration. Having solved both migration decisions, I can derive the GDP in each country and finally solve the decision problem of governments. Throughout the subsection, I focus on country i. The solution for country j can be obtained by taking the solution for country i and simply switching the indices from i to j and vice versa.

Labor Migration In general, graduates migrate and work abroad when disposable income in the other country is higher than in the study location taking into account migration costs. Focusing on graduates in country i, the migration decision of a graduate who studied in his home country follows

$$(1 - t_j)hw - m_g > (1 - t_i)hw \tag{3.1}$$

$$\Leftrightarrow \quad m_g < (t_i - t_j)hw, \tag{3.2}$$

where the LHS of inequality (3.1) reflects the graduate's disposable income when leaving the study location taking into account migration costs, while the RHS reflects his disposable income when he is staying and working in his study location. A graduate who did not study in his home country faces migration costs of $m_g - \beta$; thus, his migration decision follows the inequality

$$m_g - \beta < (t_i - t_j)hw.$$

Labor migration is driven by the individual's migration costs and the difference in disposable income which is due to tax rate differences between countries. A graduate migrates if the after-tax income differential is high enough to cover his migration costs. Note that the government of a country, say i, could attract workers by lowering the tax rate t_i because this makes working in i more attractive for both graduates from country i and from country j. Note further that the migration decision of workers in j follows similar conditions.

Student Migration When individuals decide where to study, they compare expected income when studying in their home country with expected income when studying abroad taking into account tuition fees and migration costs. Consider the case of an individual who is born in country i. He migrates and studies abroad if

$$E[\Pi_{ji}] - s_{ji} - m_s > E[\Pi_{ii}] - s_{ii} \tag{3.3}$$

where $E[\Pi_{ab}]$ denotes expected income of an individual studying in a and born in b with $a, b = \{i, j\}$. The LHS of inequality (3.3) states the expected income when studying abroad minus the tuition fee for foreign students in country j and the individual's migration costs. The RHS of the inequality states the expected income when studying in the home country minus the tuition fee for domestic students in country i. Rearranging inequality (3.3) leads to

$$m_s < E[\Pi_{ji}] - E[\Pi_{ii}] + s_{ii} - s_{ji} \equiv I_i \tag{3.4}$$

where I_i denotes the incentive to study abroad for an individual born in country i. An individual decides to study abroad if the expected income differential taking into account the tuition fees is high enough to cover his student migration costs. Note that the migration decision of individuals born in j follows similar conditions.

In order to find an explicit formulation of inequality (3.4), I derive the expected income levels $E[\Pi_{ii}]$ and $E[\Pi_{ji}]$ in the following. Expected income levels derive as

$$\begin{aligned} E[\Pi_{ii}] &= \Pr\Big[m_g > (t_i - t_j)hw\Big](1 - t_i)hw \\ &+ \Pr\Big[m_g < (t_i - t_j)hw\Big]\Big[(1 - t_j)hw - E[m_g | m_g < (t_i - t_j)hw]\Big] \qquad \text{and} \end{aligned}$$

$$\begin{aligned} E[\Pi_{ji}] &= \Pr\Big[m_g - \beta > (t_j - t_i)hw\Big](1 - t_j)hw \\ &+ \Pr\Big[m_g - \beta < (t_j - t_i)hw\Big]\Big[(1 - t_i)hw - E[m_g - \beta | m_g - \beta < (t_j - t_i)hw]\Big]. \end{aligned}$$

The first equation states expected income when a student born in i studies in his home country: after graduation, he will stay and work in country i with probability $\Pr[m_g > (t_i - t_j)hw]$ and will earn income of $(1 - t_i)hw$. With probability $\Pr[m_g < (t_i - t_j)hw]$, the student will leave country i after graduation. In this case, the student receives income of $(1 - t_j)hw$ and faces migration costs of $E[m_g | m_g < (t_i - t_j)hw]$. Migration costs as graduate enter as expected value because they are revealed upon graduation and thus unknown at the student migration decision. The second equation states expected income when the student studies abroad. It is derived in a similar way as the first equation with one difference: since the student studies abroad and benefits from the migration cost advantage, his graduate migration costs (resp. his expected graduate migration costs) are, in this case, equal to $m_g - \beta$ $(E[m_g - \beta | m_g - \beta < (t_j - t_i)hw])$.

Because of the assumption that migration costs m_g are uniformly distributed, I can calculate explicitly the expected income levels and thus the incentive to study abroad I_i. Assuming interior solutions of the expected income terms $E[\Pi_{ii}]$ and $E[\Pi_{ji}]$, inequality (3.4) can be then formulated as

$$m_s < I_i = (s_{ii} - s_{ji}) + (t_i - t_j)hw\frac{2E[m_g] - \beta}{\Delta m_g} + \frac{\beta}{2\Delta m_g}\left[\beta - 2\underline{m}_g\right]$$

with $\Delta m_g = \overline{m}_g - \underline{m}_g$ denoting the bandwidth of migration costs (see Appendix 3.7.2 for more details of the derivation). The first term in the equation for I_i means that the incentive to study abroad is higher, the lower the tuition fee abroad relative to the tuition fee in the home country. The second term reflects the fact that the higher the tax rate in the home country relative to the tax rate abroad, the more individuals decide to study abroad. The reason is because individuals anticipate a potential lock-in effect upon graduation which could make it difficult for them to avoid unfavorable high income taxation. The lower the tax rate in the other country, the higher the incentive to study abroad. Finally, the incentive to study abroad is increasing in the migration cost advantage β because a student who studies abroad faces lower labor migration costs which makes it easier for him to choose the country with the highest income. Note that the government of a country, say again i, can attract students by either lowering the tuition fee or the tax rate because both instruments make the country a more attractive place to study in.

Gross Domestic Product Having studied both migration decisions, the gross domestic product of each country can be determined. Since GDP of a country is uniquely determined by the stock of human capital, i.e., $GDP_i = wH_i$, I derive in the following the stock of human capital in country i by starting with calculating the number of domestic and of foreign students in country i.

Assuming that individuals of mass one are born in each country, the number of domestic and of foreign students in country i derive as

$$S_{ii} = 1\,\Pr[m_s > I_i] \qquad \text{and}$$
$$S_{ij} = 1\,\Pr[m_s < I_j].$$

The number of domestic students is given by the number of individuals who are born in i and who decide to stay and study in their home country; the number of foreign students depends on how many individuals born in j decide to migrate and study abroad. Calculating the migration probabilities leads to

$$S_{ii} = \begin{cases} 1 & \text{when } I_i < \underline{m}_s \\ \frac{1}{\Delta m_s}(\overline{m}_s - I_i) & \text{when } \underline{m}_s \leq I_i \leq \overline{m}_s \\ 0 & \text{when } \overline{m}_s < I_i \end{cases} \qquad \text{and}$$

$$S_{ij} = \begin{cases} 1 & \text{when } \overline{m}_s < I_j \\ \frac{1}{\Delta m_s}(I_j - \underline{m}_s) & \text{when } \underline{m}_s \leq I_j \leq \overline{m}_s \\ 0 & \text{when } I_j < \underline{m}_s \end{cases},$$

where $\Delta m_s = \overline{m}_s - \underline{m}_s$ denotes the bandwidth of student migration costs. The number of domestic and of foreign students are neither negative nor above the initial population size. For the rest of the chapter, I concentrate on cases where the above equations are solved by inner solutions.

The stock of human capital follows the equation

$$H_i = h\Big[\, S_{ii}\Pr[m_g > (t_i - t_j)hw] + S_{ij}\Pr[m_g - \beta > (t_i - t_j)hw] \\ + S_{jj}\Pr[m_g < (t_j - t_i)hw] + S_{ji}\Pr[m_g - \beta < (t_j - t_i)hw]\Big].$$

It derives from the human capital per worker h times the number of workers living in country i which is represented by the bracket term in the above equation. The number of workers in country i depends on the number of students who graduated and stay in i (the first and second summand in the bracket term) and on the number of students who graduated in country j and decide to migrate and work in i (the third and fourth summand in the bracket term). Using the assumption that individuals of mass one are born in each country, i.e., $S_{ii} + S_{ji} = S_{jj} + S_{ij} = 1$, the equation for H_i can be rearranged to

$$H_i = h\Big[(S_{ii} + S_{ij})H_1 + H_2 + \frac{2hw}{\Delta m_g}(t_j - t_i)\Big] \tag{3.5}$$

with $H_1 = \frac{1}{\Delta m_g}[2E[m_g] - \beta] > 0$ and $H_2 = \frac{1}{\Delta m_g}[\beta - 2\underline{m}_g] > 0$. The human capital stock depends on the number of students studying in the country, given by $S_{ii} + S_{ij}$, and the choice of the tax rate t_i. The government can raise human capital either by attracting more students or more workers.

Government's Decision Problem The government of a country maximizes GDP over the three policy instruments subject to the budget constraint. In doing so, it takes the policy choice of the other country, i.e., s_{jj}, s_{ji} and t_j, as given. Focusing on country i, the maximization problem of the government reads as

$$\max_{s_{ii}, s_{ij}, t_i} GDP_i \quad \text{s.t.} \quad NR_i \geq 0$$

where the side constraint means that the budget must be at least balanced.

Rewriting the maximization problem as Lagrangian function, i.e.,

$$L_i = GDP_i + \lambda_i NR_i$$

with λ_i the Lagrange-multiplier for country i, the optimal choice of the policy instruments is defined by the following first order conditions:

$$FOC_1 \quad \frac{\partial L_i}{\partial s_{ii}} = \frac{\partial GDP_i}{\partial s_{ii}} + \lambda_i \frac{\partial NR_i}{\partial s_{ii}} = 0,$$

$$FOC_2 \quad \frac{\partial L_i}{\partial s_{ij}} = \frac{\partial GDP_i}{\partial s_{ij}} + \lambda_i \frac{\partial NR_i}{\partial s_{ij}} = 0,$$

$$FOC_3 \quad \frac{\partial L_i}{\partial t_i} = \frac{\partial GDP_i}{\partial t_i} + \lambda_i \frac{\partial NR_i}{\partial t_i} = 0,$$

$$FOC_4 \quad NR_i = 0 \quad \text{if} \quad \lambda_i \neq 0.$$

The government chooses optimal tuition fees and tax rate such that the costs for a marginal increase in GDP are equalized among the policy instruments:

$$\frac{\frac{\partial NR_i}{\partial s_{ii}}}{\frac{\partial GDP_i}{\partial s_{ii}}} = \frac{\frac{\partial NR_i}{\partial s_{ij}}}{\frac{\partial GDP_i}{\partial s_{ij}}} = \frac{\frac{\partial NR_i}{\partial t_i}}{\frac{\partial GDP_i}{\partial t_i}} = -\frac{1}{\lambda_i}.$$

Furthermore, the budget is balanced meaning that the side constraint is binding in optimum. The reason is because GDP is decreasing in the tuition fees and the tax rate. For instance, deriving the effect of tuition fees on GDP leads to the following derivatives:

$$\frac{\partial GDP_i}{\partial s_{ii}} = w\frac{\partial H_i}{\partial s_{ii}} = whH_1\frac{\partial S_{ii}}{\partial s_{ii}} = -whH_1\frac{1}{\Delta m_s} < 0,$$
$$\frac{\partial GDP_i}{\partial s_{ij}} = w\frac{\partial H_i}{\partial s_{ij}} = whH_1\frac{\partial S_{ij}}{\partial s_{ij}} = -whH_1\frac{1}{\Delta m_s} < 0.$$

Higher tuition fees make studying in country i less attractive and reduce the number of students. The reduction in the number of students in turn leads to fewer workers, less human capital and thus lower GDP in country i. The effect of the tax rate on GDP derives as

$$\begin{aligned}\frac{\partial GDP_i}{\partial t_i} &= w\frac{\partial H_i}{\partial t_i} = wh\left[\Big(\frac{\partial S_{ii}}{\partial t_i} + \frac{\partial S_{ij}}{\partial t_i}\Big)H_1 - \frac{2hw}{\Delta m_g}\right] \\ &= -\frac{2(hw)^2}{\Delta m_s}\left[(H_1)^2 + \frac{\Delta m_s}{\Delta m_g}\right] < 0.\end{aligned}$$

The tax rate affects GDP in two ways: when the tax rate rises, fewer graduates decide to work in country i; this effect is reflected by the term $-\frac{2hw}{\Delta m_g}$ in the above derivative. Additionally, the tax rate reduces the number of students because students anticipate unfavorable high taxation in the student migration decision; this effect is reflected by the term $(\frac{\partial S_{ii}}{\partial t_i} + \frac{\partial S_{ij}}{\partial t_i})H_1$. As a consequence, the government reduces tuition fees and the tax rate until the budget is balanced in order to attract students and workers and to maximize GDP.

Using the four first order conditions, I can solve for the optimal choice of the policy instruments. Since the focus of the chapter is on tuition fee differentials, I present the difference between the optimal tuition fees only:

$$\Delta s_i = \frac{\beta}{2\Delta m_g}(\beta - 2\underline{m}_g) - E[m_s] - \frac{\Delta s_j}{2} \tag{3.6}$$

with $\Delta s_i \equiv s_{ij} - s_{ii}$ and $\Delta s_j \equiv s_{ji} - s_{jj}$ denoting the tuition fee differential in each country. The optimal tuition fee differential in country i depends on the tuition fee differential in country j and exogenously given variables. I use equation (3.6) in the next subsection to determine the Nash equilibrium.

3.3.2 Equilibrium Tuition Fee Differentials

Governments in both countries choose independently and simultaneously tuition fees and the tax rate in the policy choice stage. I determine the Nash equilibrium in the following focusing on tuition fee differentials.

To solve for tuition fee differentials in the Nash equilibrium, I use equation (3.6) and a similar equation for country j. The equilibrium tuition fee differentials derive as

$$\Delta s_i^* = \Delta s_j^* = \frac{\beta}{3\Delta m_g}\big(\beta - 2\underline{m}_g\big) - \frac{2}{3}E[m_s] \equiv \Delta s. \tag{3.7}$$

Governments in both countries choose exactly the same tuition fee differential. In fact, not only tuition fee differentials are the same across countries, also tuition fees and the tax rate are identical:

Proposition 5. *The Nash equilibrium is unique, symmetric and stable:*

$$s_{ii}^* = s_{jj}^*, \qquad s_{ij}^* = s_{ji}^*, \qquad t_i^* = t_j^*.$$

Proof. The four first order conditions of the governmental decision problem define the best-response tuition fees and tax rate of one country given the policy choice of the other country. Using the six best-response functions, one can solve for equilibrium tuition fees and tax rate in both countries. The solution is unique and symmetric. More details of the proof as well as the proof of stability are provided in Appendix 3.7.3. □

I discuss the main determinants of the equilibrium tuition fee differential in the next subsection. It is useful for that discussion to state the equilibrium tuition fee differential as a function of elasticities. Defining the tuition fee elasticity of the number of domestic students (resp. the tuition fee elasticity of the number of foreign students) as

$$\epsilon_{S_{ii},s_{ii}} = \frac{\partial S_{ii}}{s_{ii}} \frac{s_{ii}}{S_{ii}} \qquad \left(\epsilon_{S_{ij},s_{ij}} = \frac{\partial S_{ij}}{s_{ij}} \frac{s_{ij}}{S_{ij}}\right),$$

the difference in equilibrium tuition fees derives as[11]

$$\Delta s = \Delta m_s \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}, \tag{3.8}$$

with

$$\tilde{\epsilon} = \frac{1 + \epsilon_{S_{ij},s_{ij}}|_{equi.}}{1 + \epsilon_{S_{ii},s_{ii}}|_{equi.}}$$

denoting the equilibrium ratio of the two elasticities.

Equation (3.8) allows me to study how governments set tuition fees for domestic and foreign students in more detail. First, note that the ratio of elasticities $\tilde{\epsilon}$ is higher, the more elastic foreign students react to tuition fee changes relative to the reaction of domestic students. Second, the tuition fee differential is decreasing in the ratio of elasticities:

$$\frac{\partial \Delta s}{\partial \tilde{\epsilon}} = -\frac{2\Delta m_s}{(1+\tilde{\epsilon})^2} < 0.$$

From these two facts, I conclude that the government of a country charges a higher tuition fee to the 'less elastic' student group and charges a lower tuition fee to the 'more elastic'

[11] Appendix 3.7.4 provides the derivation of equation (3.8) in detail.

student group. In other words, the government acts similarly to the Ramsey-rule known from optimal taxation literature when differentiating tuition fees. For instance, if both student groups react in a similar way to changes in tuition fees, the ratio of the elasticities is equal to one and governments set equally high tuition fees for both domestic and foreign students:

$$\tilde{\epsilon} = 1 \qquad \Rightarrow \qquad \Delta s = 0.$$

If foreign students react more elastically than domestic students, the ratio of elasticities is above one and the tuition fee for foreign students is lower than the tuition fee for domestic students:

$$\tilde{\epsilon} > 1 \qquad \Rightarrow \qquad \Delta s < 0.$$

3.3.3 Main Determinants of Tuition Fee Differentials

Two factors mainly determine the tuition fee differential in a country: student mobility, represented by the average mobility costs of students $E[m_s]$, and the difference in labor mobility between natives and foreigners, represented by β. I discuss both factors in the following paragraphs.

Student mobility The degree of student mobility is reflected by average migration costs of students: the higher $E[m_s]$, the lower student mobility. The effect of student mobility on the equilibrium tuition fee differential is given by the following derivative:[12]

$$\frac{\partial \Delta s}{\partial E[m_s]} = -\frac{2}{3} < 0.$$

Higher average migration costs of students (i.e. lower student mobility) have a negative effect on the tuition fee differential. The reason is because foreign students react more elastically to changes in tuition fees than domestic students when $E[m_s]$ is rising:

$$\frac{\partial \tilde{\epsilon}}{\partial E[m_s]} = \frac{12\Delta m_s \Delta m_g^2}{\left[\beta^2 + \beta\Delta m_g + 3\Delta m_s \Delta m_g - 2\Delta m_g E[m_s] - 2\beta E[m_g]\right]^2} > 0.$$

As a result, governments reduce the tuition fees for foreign students, the student group which reacts more elastically, and raise the tuition fee for domestic students, the student group which reacts less elastically.

An intuitive explanation of the result is based on the fact that rising migration costs prevent more individuals from migrating and studying abroad; as a consequence, individ-

[12] I hold the variance of student migration costs Δm_s constant.

uals are more likely to stay and study in their home country. A government which aims to attract students in order to maximize the country's GDP reacts in two ways to rising student migration costs. First, it reduces the tuition fee for foreign students to make the country a more attractive study location for foreigners and to 'compensate' for higher migration costs. Second, the government increases the tuition fee for domestic students to finance the reduction in the tuition fee for foreign students. The rise in domestic students' tuition fee does not lead to a decreasing number of domestic students because natives do not want to study abroad due to the high migration costs. As a consequence, the difference in tuition fees falls.

Migration cost advantage of foreigners The effect of the migration cost advantage can be derived by the derivative

$$\frac{\partial \Delta s}{\partial \beta} = \frac{2}{3}\frac{1}{\Delta m_s}\left[\beta - \underline{m}_g\right] > 0.$$

The difference in tuition fees rises when foreigners have a higher migration cost advantage in labor migration. The reason is because foreign students react less elastically to changes in tuition fees than domestic students when the migration cost advantage for foreigners increases:

$$\frac{\partial \tilde{\epsilon}}{\partial \beta} = -\frac{12\Delta m_s \Delta m_g(\beta - \underline{m}_g)}{\left[\beta^2 + \beta\Delta m_g + 3\Delta m_s\Delta m_g - 2\Delta m_g E[m_s] - 2\beta E[m_g]\right]^2} < 0.$$

As a consequence, governments shift the tuition fee burden from domestic to foreign students by lowering the tuition fee for domestic students and raising the tuition fee for foreign students. This translates into rise in the tuition fee differential.

The intuitive explanation for this result follows from two arguments. (i) When a student, either domestic or foreign, leaves the study location after graduation, the public provision of education to this student does not pay off in terms of higher GDP for the country which provided the education. (ii) Due to the migration cost advantage of foreigners, foreign students are more likely to leave their study location after graduation than domestic students. Combining the two arguments implies that a country gains more from the provision of public education to a domestic student compared to the provision of public education to a foreign student. Note that the additional profit of providing education to a domestic student is higher, the higher the migration advantage of foreigners because the probability that a foreign student leaves the country after graduation increases with β. When β rises, governments prefer to attract more natives and less foreigners as students. In order to do so, they lower the tuition fee for domestic students and raise the tuition fee for foreign students. This change in tuition fees in turn implies a rising tuition fee differential.

Costs and benefits of attracting students The above analysis can be summed up by stating that the costs and benefits of attracting a student, either a native or a foreigner, determine the size of the tuition fee differential. The costs are represented by student migration costs or, in other words, by student mobility. The reason is because the degree of student mobility determines how likely it is that an individual studies in a certain country. In case an individual is likely to study in a country, the government of that country can charge higher tuition fee to this student. In the opposite case where an individual is unlikely to study in a country, the government of that country rather sets lower tuition fee in order to still attract the student. The benefits, on the other hand, are given by the expected contribution of the student to the country's GDP. This expected contribution to GDP crucially depends on the likelihood that the student stays and works in his study location after graduation. Any factor influencing either the costs or the benefits of attracting a student is therefore going to affect the tuition fee differential.

Labor mobility In order to determine the effect of labor mobility on the equilibrium tuition fee differential, I use $\underline{m}_g = E[m_g] - \frac{1}{2}\Delta m_g$ to replace the lower bound of labor migration cost in equation 3.7. The effect of labor mobility on the equilibrium tuition fee differential is then given by the following derivative:[13]

$$\frac{\partial \Delta s}{\partial E[m_g]} = -\frac{2\beta}{3\Delta m_g} < 0.$$

As long as there is a migration cost advantage for foreigners β, higher labor mobility (reflected by lower average mobility costs for graduates $E[m_g]$) leads to increasing tuition fee differentials in equilibrium. The reason is because foreign students are getting more mobile relative to domestic students upon graduation. This implies that governments have a lower incentive to attract foreign students.

Note that in case a migration cost advantage for foreigners does not exist (i.e. $\beta = 0$), labor mobility does not affect the tuition fee differential in equilibrium. The reason is that the relative probabilities to leave after graduation for domestic and foreign students does not change in this case and hence the governments' incentives to attract domestic and foreign students are affected in the same way by increasing labor mobility.

[13] I hold the variance of labor migration costs Δm_g constant.

3.4 The Case of Two Heterogeneous Countries

In the previous section, I analyzed two identical countries and determined the main drivers of tuition fee differentials. In this section, in contrast, I study the case of two heterogeneous countries. More precisely, I focus on countries that offer education of different quality: I assume in the following that $h_i \neq h_j$ holds and that, without loss of generality, $f(h_a) = {h_a}^2$.[14] The contribution of this section is to study the effect of education quality differences between countries on tuition fee differentials. In the following subsections, I discuss first briefly how the model is solved when countries differ in the offered education quality. Second, I employ a numerical example to analyze how education quality affects tuition fee differentials in equilibrium.

3.4.1 Solving the Model

In principal, the solution of the model with two nonidentical countries follows the same steps as the solution for the case of two identical countries: First, one solves the labor migration decision, second, the student migration decision and third, the government's decision problem. The result are best responses of one country to the policy choice of the other country. Using these best responses, one may solve for the policy choices in a Nash equilibrium. However, the model with two nonidentical countries cannot be analytically solved for equilibrium tuition fee differentials. I therefore use a numerical example to study the effect of differences in education quality on tuition fee differentials. Details of the solution are provided in Appendix 3.7.5.

3.4.2 Numerical Example

This subsection solves for tuition fee differentials in a Nash equilibrium using a numerical example. In the numerical example, I fix all variables besides the education quality in country i. I then vary the value of h_i and calculate the equilibrium tuition fee differential in country i and j. When varying h_i, changes in Δs_i^* reveal how the difference in equilibrium tuition fees is affected by the quality of education at home, while changes in Δs_j^* reveal how the difference in equilibrium tuition fees is affected by the quality of education abroad. Note that this is a comparative static analysis.

[14]In other words, I assume that human capital production follows a concave production function: $h_a = \sqrt{c_a}$ with $a = i, j$.

I choose the following values for the variables in the numerical example:

$$\begin{aligned} E[m_s] &= 0.05 & \Delta m_s &= 1 \\ E[m_g] &= 0.4 & \Delta m_g &= 1 \\ \beta &= 0.3 & w &= 1 \\ h_j &= 1 \end{aligned} \tag{3.9}$$

and vary h_i within the bandwidth of

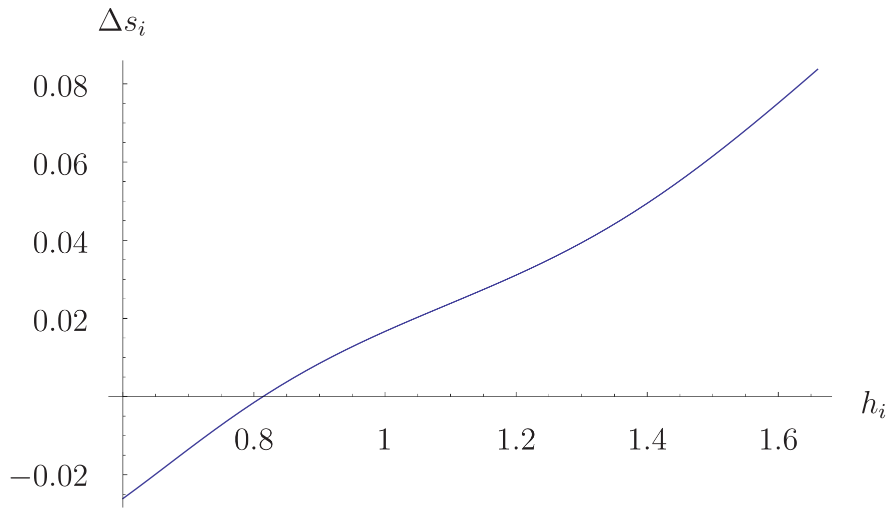
$$0.6 \leq h_i \leq 1.68. \tag{3.10}$$

I choose the variables such that the assumptions in (A.1) are fulfilled. Furthermore, the model shall be solved by inner solutions which means that student numbers in both countries are strictly between zero and one. This condition is met by imposing the bandwidth specified in (3.10).[15]

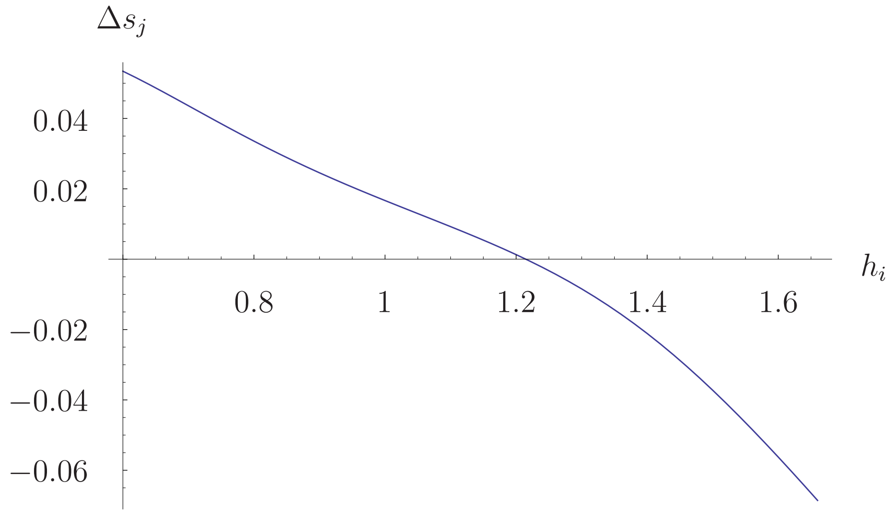
Given the variable choice presented above, the resulting differences in equilibrium tuition fees in country i and j are displayed in Figures 3.2(a) and 3.2(b). I conclude from the two figures that the equilibrium tuition fee differential is increasing in the education quality offered within the same country whereas it is decreasing in the education quality offered abroad. In other words, the country that offers better education charges relatively more to foreign students than to domestic students.

The intuition for this result is the following. Governments aim to attract students in order to maximize GDP. The expected contribution of a student to the country's GDP depends on two factors: (i) the student's human capital acquired upon graduation and (ii) the student's probability to stay in the study location after graduation. The human capital of a student is similar across domestic and foreign students of one country because all of them receive the same quality of education and have the same abilities. The stay rate, in contrast, differs between domestic and foreign students since foreign students are more likely to leave the study location after graduation than domestic students. Thus, there exists an advantage in attracting a native instead of a foreigner simply because the native is more likely to stay and to contribute to the country's GDP in the future. Governments take into account this advantage when choosing tuition fees for domestic and for foreign students. Note that the advantage of attracting a native instead of a foreigner is increasing in the human capital students acquire upon graduation. As a consequence, the government prefers

[15]If one of the countries offers a much higher education quality than the other, all students prefer to study in this country and the model is 'solved by a corner solution'. Imposing the bandwidth in (3.10) rules out such cases.



(a) in country i



(b) in country j

Figure 3.2: The difference in equilibrium tuition fees for different values of education quality h_i when h_j is constant.

to attract more domestic and less foreign students when the education quality is rising (ceteris paribus). This goal can be achieved by lowering the tuition fee for domestic students and raising the tuition fee for foreign students. Or in other words, the reduction in the tuition fee for domestic students is financed by raising the tuition fee for foreign students. Consequently, the tuition fee differential rises as shown in Figure 3.2(a). The other country with unchanged education quality, country j in the numerical example, just reacts to the changes in tuition fees in country i.

The advantage of educating a native instead of a foreigner can also be determined mathematically. Let me first derive the expected contribution to the GDP of a domestic and of a foreign student in country i:

$$\begin{aligned}\frac{\partial GDP_i}{\partial S_{ii}} &= w\frac{\partial H_i}{\partial S_{ii}} = wh_i \Pr[m_g > (t_i - t_j)h_i w],\\ \frac{\partial GDP_i}{\partial S_{ij}} &= w\frac{\partial H_i}{\partial S_{ij}} = wh_i \Pr[m_g - \beta > (t_i - t_j)h_i w].\end{aligned}$$

Since the stay rate is higher for domestic students, i.e.,

$$\Pr[m_g > (t_i - t_j)h_i w] > \Pr[m_g - \beta > (t_i - t_j)h_i w]$$

for $\beta > 0$, a domestic student contributes more to the future GDP than a foreign student. The advantage of attracting and providing education to a native derives as

$$\frac{\partial GDP_i}{\partial S_{ii}} - \frac{\partial GDP_i}{\partial S_{ij}} = wh_i \frac{\beta}{\Delta m_g}. \tag{3.11}$$

Equation (3.11) shows clearly the above discussed relationship between education quality and the advantage in attracting natives: the higher the provided quality of education h_i, the higher the advantage of attracting natives.

Equation (3.11) also provides a formal explanation for the positive effect of β on tuition fee differentials, as discussed in the previous section. The advantage of attracting natives is increasing in β. When the migration advantage of foreigners rises (ceteris paribus), governments prefer to attract more natives and less foreigners which is achieved by raising tuition fee differentials. As a consequence, a rising β leads to increasing tuition fee differentials.

3.5 Welfare Analysis

In this section, I study whether or not differentiating tuition fees improves local welfare (welfare in short in the following). In order to answer this question, I derive the tuition fee differential which maximizes equilibrium welfare and compare it to the equilibrium tuition fee differential which governments would choose in a decentralized decision process as derived in Section 3.3. In the following, I introduce the framework used for the welfare analysis, determine equilibrium welfare and derive the welfare maximizing tuition fee differential. In the last subsection, I draw conclusions regarding the welfare analysis.

3.5.1 Framework

For the welfare analysis, I use an adapted version of the standard setup with two identical countries offering the same quality of education, i.e., $h_i = h_j \equiv h$.[16] The adaptation concerns the notation regarding tuition fees where I express the tuition fee for foreign students by the tuition fee for domestic students plus the tuition fee differential:

$$\begin{aligned} s_{ij} &= s_{ii} + \Delta s_i, \\ s_{ji} &= s_{jj} + \Delta s_j. \end{aligned}$$

The social planner maximizes global welfare where welfare is defined as the sum of utility over all individuals. Individual utility shall follow a linear function in individual income net of taxes, tuition fees and migration costs. Global welfare follows the function $G(s_i, \Delta s_i, t_i, s_j, \Delta s_j, t_j)$ and derives as

$$\begin{aligned} W =& G(s_{ii}, \Delta s_i, t_i, s_{jj}, \Delta s_j, t_j) \\ =& w(H_i + H_j) \\ &- \Big[t_i w H_i + t_j w H_j \Big] \\ &- \Big[S_{ii} s_{ii} + S_{ij}(s_{ii} + \Delta s_i) + S_{jj} s_{jj} + S_{ji}(s_{jj} + \Delta s_j) \Big] \\ &- \left[S_{ji} \frac{I_i + m_s}{2} + S_{ij} \frac{I_j + m_s}{2} \right] \\ &- \left[S_{ii} \frac{(t_i - t_j)hw + m_w}{2} + S_{ij} \frac{(t_i - t_j)hw + m_w - \beta}{2} \right] \\ &- \left[S_{jj} \frac{(t_j - t_i)hw + m_w}{2} + S_{ji} \frac{(t_j - t_i)hw + m_w - \beta}{2} \right]. \end{aligned}$$

Describing each line of the above function, global welfare derives from (i) GDP in both countries minus (ii) tax revenues, (iii) tuition fee revenues, (iv) sum of migration costs for

[16]The advantage of focusing on two identical countries is that I can study analytically the welfare effects of introducing differentiated tuition fees. Analyzing welfare effects for the case of two nonidentical countries instead requires numerical examples. In both cases, however, the welfare effects of introducing differentiated tuition fees are the same.

students studying abroad, (v) sum of migrations costs of graduates who studied in i but choose to work in j and (vi) sum of migrations costs of graduates who studied in j but choose to work in i.

3.5.2 Equilibrium Welfare and the Welfare Maximizing Tuition Fee Differential

Given the framework described above, I can solve analytically for equilibrium welfare. The social planner's decision problem can be written as

$$\max_{s_{ii},t_i,s_{jj},t_j} W \qquad \text{s.t.} \qquad NR_i \geq 0 \quad \text{and} \quad NR_j \geq 0.$$

The social planner aims to maximize global welfare subject to non-negative local budget constraints. Note that I do not include the tuition fee differential in the social planner's decision problem because I want to keep it exogenous for the moment; the derived equilibrium global welfare is going to depend on the size of the tuition fee differential. Having derived equilibrium global welfare with exogenous tuition fee differential, I can turn to determine the tuition fee differential which maximizes equilibrium global welfare.

The optimal choice of the standard tuition fees and the tax rates follow the first order conditions

$$\frac{\partial W}{\partial s_{ii}} + \lambda \frac{\partial NR_i}{\partial s_{ii}} = 0,$$

$$\frac{\partial W}{\partial t_i} + \lambda \frac{\partial NR_i}{\partial t_i} = 0,$$

$$\frac{\partial W}{\partial s_{jj}} + \lambda \frac{\partial NR_j}{\partial s_{jj}} = 0,$$

$$\frac{\partial W}{\partial t_j} + \lambda \frac{\partial NR_j}{\partial t_j} = 0,$$

$$NR_i = 0 \quad \text{with} \quad \lambda \neq 0, \qquad\qquad NR_j = 0 \quad \text{with} \quad \lambda \neq 0.$$

Assuming a symmetric equilibrium and simplifying notation:

$$s^* = s^*_{ii} = s^*_{jj},$$
$$t^* = t^*_i = t^*_j,$$
$$\Delta s = \Delta s_i = \Delta s_j,$$

I can solve for the equilibrium tuition fee s^* and the equilibrium tax rate t^*:

$$s^* = F_s(\Delta s),$$
$$t^* = F_t(\Delta s).$$

To save space and keep notation simple, I denote the solutions by functions $F_s(\Delta s)$ and $F_t(\Delta s)$. Equilibrium global welfare then follows

$$W^* = G(s^*, \Delta s, t^*, s^*, \Delta s, t^*) \equiv F_W(\Delta s)$$

and depends on the tuition fee differential.

Knowing the functional form of equilibrium global welfare, I can determine the welfare maximizing tuition fee differential.

Proposition 6. *Equilibrium social welfare is highest when tuition fees are not differentiated:*

$$\Delta s = 0 \quad \arg\max \quad F_W(\Delta s).$$

Proof. Taking the first and second derivatives of equilibrium welfare with respect to the tuition fee differential leads to

$$\frac{\partial W^*}{\partial \Delta s} = -\frac{2\Delta s}{\Delta m_s},$$

$$\frac{\partial^2 W^*}{\partial (\Delta s)^2} = -\frac{2}{\Delta m_s} < 0.$$

Since the first derivative is zero at $\Delta s = 0$:

$$\left.\frac{\partial W^*}{\partial \Delta s}\right|_{\Delta s = 0} = 0,$$

and the second derivative is strictly negative for all Δs, equilibrium welfare is globally maximized when tuition fees are not differentiated. □

The intuition for the result is the following. When tuition fees are not differentiated, students face the same tuition fee wherever they would like to study. In other words, tuition fees do not play a role when students decide on migration; only student migration costs and the advantage of studying abroad determine where a student is going to study. In contrast, the student migration decision is distorted when governments differentiate tuition fees, i.e. when $\Delta s \neq 0$. When the tuition fee differential is positive, students who may benefit from studying abroad stay in their home country because studying abroad is more expensive than studying at home. Migration is in this case too little and not all benefits of student migration are exploited. When the tuition fee differential is negative, studying abroad becomes less expensive than studying at home and too many students migrate and study abroad. In this case, the society accumulates too high migration costs.

3.5.3 Welfare Effect of Differentiating Tuition Fees

It follows from Proposition 6 that differentiating tuition fees reduces welfare. Nevertheless, governments are going to differentiate tuition fees when they have the option to do so. The reason is that it is optimal for a government to set different tuition fees for domestic and for foreign students given what the other country has chosen, as shown in Sections 3.3 and 3.4. As long as there is no credible commitment or coordination between the two countries, each government differentiates tuition fees implying that welfare is lower than it could be. In other words, governments are in a "prisoners' dilemma".

The result of the welfare analysis provides a rationale for what we can observe in reality. As long as there is no supranational institution which coordinates the interests of (at least) two countries, tuition fees are going to be differentiated with respect to the students' origin. Consider the case of the European Union where member states are not allowed to differentiate tuition fees between students coming from EU member states. According to the above analysis, this ban raises welfare in each member state. With respect to students coming from non-EU member states, however, there is no such ban and we do observe that countries, as for instance the UK, differentiate tuition fees between students coming from EU member states and students coming from non-EU member states. Besides the aim of reducing discrimination within the EU, which has probably led to the ban of differentiated tuition fees within the European Union, the chapter provides also an economic argument for the implementation of such a ban due to the positive effect of (partly) abolishing differentiated tuition fees on welfare.

3.6 Conclusion

Three question are addressed in the chapter: (1) what are the driving forces for tuition fee differentials among students of different origins? (2) Assuming that student mobility is going to rise in the future, what are the likely effects on tuition fee differentials and (3) does differentiating tuition fees increase local welfare?

In a setting with two similar countries competing for students, students' migration costs (or in other words student mobility) and the higher leaving probability of foreign students after graduation lead to possibly differentiated tuition fees. But the effects caused by the two factors work in different directions. The former, students' migration costs, leads to lower a tuition fee for foreign students compared to the tuition fee for domestic students. The reason is because students' migration costs are an obstacle for migration and thus, individuals are more likely to study in their home country. It follows that domestic students are easier to attract than foreign students. A government which aims to maximize GDP, which is equivalent with maximizing the number of students, increased the attractiveness of its country by setting a lower tuition fee for foreign students relative to the tuition fee for domestic students. More technically speaking, rising average students' migration costs make foreign students react more elastically to tuition fee changes than domestic students would do. As a consequence, a government therefore shifts the tuition fee burden from the more elastic group (foreign students) to the less elastic group (domestic students). The second factor, the higher leaving probability of foreign students, implies lower returns (in terms of future GDP) for the governmental investment in the education of a foreign student than the investment in the education of a domestic student. Therefore, a government reduces its investment (its contribution to the education cost) in foreign students by setting a higher tuition fee for foreign students. So if foreign students are more likely to leave the study location after graduation than domestic students, the tuition fee for foreign students is higher than the one for domestic students.

In an extended setup, the chapter analyzes the effect of education quality on tuition fee differentials. I find that the tuition fee differential, which is defined as the tuition fee for foreign students minus the tuition fee for domestic students, is positively affected by the education quality in the focused country while it is negatively affected by the education quality abroad. The result can be explained by the following line of arguments. Rising education quality at home attracts domestic as well as foreign students. In fact, both groups are going to react less elastically to changes in their tuition fees when the country offers better education. As a consequence, tuition fees for both student groups will go up in that country. But as shown in a numerical example, the reduction in the elasticity of foreign students is larger than for domestic students. In other words, the number of foreign students reacts less elastically than the number of domestic students on changes in the tuition fees. In sum, the government also shifts the tuition fee burden from domestic to foreign students which leads to a rise in the tuition fee differential.

As for the second research question, the chapter predicts a wider gap in tuition fees as already observed in the UK and Australia if student mobility is going to increase further in the future. This conclusion can be made from the insight regarding the effect of students' migration costs on the tuition fee differential where higher student mobility can be understood as lower migration costs for students.

Finally, the chapter proves that local welfare would be maximized when countries do not differentiate tuition fees. The previous analysis, however, has proven that countries usually use the option of differentiating tuition fees in cases where this option is available. Hence, the relatively simple framework presented in this chapter provides a rationale for the ban of differentiated tuition fees within the European Union while member countries are allowed to discriminate against students coming from non-EU member countries. From the perspective of EU member countries, this combination of policies maximizes welfare in the EU with respect to the tuition fee choice.

In this chapter, I did not take into account population size and congestion effects. Both factors do not play a role here because the costs of providing public education to one student are constant and independent of the number of students studying in a country. Future research may include education costs per student which are increasing with the number of students. In such a setting, the case of a smaller and a bigger country competing for students could be analyzed. Such an analysis can produce valuable insights especially since the main destination countries for student migration flows are big developed countries.

Appendix

3.7.1 Tuition Fee Differentials and Student Mobility in Reality

The chapter is motivated by two empirical findings: (1) tuition fees are highly differentiated in the UK and Australia and (2) student mobility has been rising over the last decade. In the following, I present some details for both findings.

Differentiated tuition fees in the UK and Australia

In the UK, students with UK citizenship and, due to European anti-discrimination law, citizens from other EU member countries paid in the academic year 2009/2010 on average about 3,225 GBP.[17] But students with other citizenship faced tuition fees between 5,500 GBP and 14,000 GBP depending on the university.

In Australia, tuition fees depend on the subject, the university and the nationality of the student. The Australian education system distinguishes three types of students: Commonwealth supported, Australian fee paying and international students. Most students with Australian citizenship study in Commonwealth supported places while the rest of the students with Australian citizenship study in Australian fee places. All non-Australian citizens are sorted into the international student group.[18] As an example, I present the tuition fee for a student studying economics at the university of Melbourne in the academic year 2010/2011. Most Australian citizens face tuition fees of about 8,859 AUD (Commonwealth supported place). Other Australian citizens face tuition fees of about 23,700 AUD (Australian fee place) while non-Australian citizens have to pay 29,700 AUD (international student place).

In both cases, foreign students face substantially higher tuition fees than domestic students.

Note that the definition of foreign and domestic students differs in the two examples. In this chapter, I call a student born within the country a domestic student and somebody born abroad a foreign student.

Evolution of student mobility

The evolution of student mobility is described in Table 3.1. The table presents the absolute numbers of foreign students (defined as all non-citizen students), the share of foreign students in all students in parentheses and the growth rate in the number of foreign students in

[17] Data for the UK are taken from Reddin (2010).
[18] Exemptions are made for some citizens of New Zealand.

Country	No. of foreign students 1998		2007		Growth rate in the no. of foreign students
Australia	109,437	(13 %)	244,309	(23 %)	123 %
France	148,000	(7 %)	246,612	(11 %)	67 %
Germany	171,150	(8 %)	258,513	(11 %)	51 %
Japan	35,700	(1 %)	125,877	(3 %)	253 %
United Kingdom	209,549	(11 %)	459,987	(19 %)	120 %
United States *	430,786	(3 %)	572,509	(3 %)	33 %
in sum	1,104,622		1,907,807		73 %

Source: Author's calculation using data from OECD.Stat.

* Data for the US are for the years 1998 and 2004.

Table 3.1: Number of foreign/non-citizen students (and their shares in all students) in the main receiving countries.

the last column. These data are presented for the main receiving countries of international student flows over the last decade. The table shows that the number of foreign students increased remarkably in all countries between 1998 and 2007.[19] In our example countries, the UK and Australia, the number of foreign students more than doubled. Furthermore, the share of foreign students is the highest for these 2 countries. Both the UK and Australia seem to be a magnet for foreign students. Potentially, the provision of English-speaking courses and a high quality of education make both countries so attractive.

Note that the large shares of foreign students may have caused the introduction of differentiated tuition fees in these two countries while countries like Germany and France with a much lower proportion of foreign students do not differentiate tuition fees. Beside the large inflow of foreign students, the introduction of differentiated tuition fees may also be driven or may be prevented by political arguments. However, I do not discuss within the present chapter whether a country shall differentiate tuition fees or not. The chapter focuses instead on cases where tuition fees are allowed to be differentiated and analyzes the underlying reasons for potential tuition fee differentials within a country.

[19]A similar trend is already present in the period from 1980 to 2000 as OECD (2004) shows.

3.7.2 Derivation of I_i

For an individual born in i, the incentive to study abroad depends crucially on the expected income when studying at home or abroad. These expected income levels derive as:

$$E[\Pi_{ii}] = \Pr\Big[m_g > (t_i - t_j)hw\Big](1-t_i)hw$$
$$+ \Pr\Big[m_g < (t_i - t_j)hw\Big]\Big[(1-t_j)hw - E[m_g|m_g < (t_i - t_j)hw]\Big] \qquad \text{and}$$

$$E[\Pi_{ji}] = \Pr\Big[m_g - \beta > (t_j - t_i)hw\Big](1-t_j)hw$$
$$+ \Pr\Big[m_g - \beta < (t_j - t_i)hw\Big]\Big[(1-t_i)hw - E[m_g - \beta|m_g - \beta < (t_j - t_i)hw]\Big].$$

Assuming that the tax rate differential $t_i - t_j$ is small enough in absolute terms, the four probabilities are solved by inner solutions. This assumption is made for convenience and plausible since, as is shown later in the section on equilibrium tuition fee differential, the equilibrium is always symmetric in the case of two identical countries which means that tax rates are equalized and thus the tax rate differential is zero.

With the above assumption, the incentive to study abroad derives as:

$$\begin{aligned}
I_i =& E[\Pi_{ji}] - E[\Pi_{ii}] + (s_{ii} - s_{ji}) \\
=& \Pr\Big[m_g - \beta > (t_j - t_i)hw\Big](1-t_j)hw \\
&+ \Pr\Big[m_g - \beta < (t_j - t_i)hw\Big]\Big[(1-t_i)hw - E[m_g - \beta|m_g - \beta < (t_j - t_i)hw]\Big] \\
&- \Pr\Big[m_g > (t_i - t_j)hw\Big](1-t_i)hw \\
&- \Pr\Big[m_g < (t_i - t_j)hw\Big]\Big[(1-t_j)hw - E[m_g|m_g < (t_i - t_j)hw]\Big] \\
&+ (s_{ii} - s_{ji}) \\
=& \frac{\overline{m}_g - \beta - (t_j - t_i)hw}{\Delta m_g}(1-t_j)hw \\
&+ \frac{(t_j - t_i)hw - \underline{m}_g + \beta}{\Delta m_g}\Big[(1-t_i)hw - \frac{(t_j - t_i)hw + \underline{m}_g - \beta}{2}\Big] \\
&- \frac{\overline{m}_g - (t_i - t_j)hw}{\Delta m_g}(1-t_i)hw \\
&- \frac{(t_i - t_j)hw - \underline{m}_g}{\Delta m_g}\Big[(1-t_j)hw - \frac{(t_i - t_j)hw + \underline{m}_g}{2}\Big] \\
&+ (s_{ii} - s_{ji})
\end{aligned}$$

$$\Rightarrow I_i = (t_i - t_j)hw\frac{2E[m_g] - \beta}{\Delta m_g} + \frac{\beta}{2\Delta m_g}\left[\beta - 2\underline{m}_g\right] + (s_{ii} - s_{ji}).$$

3.7.3 Proof of Proposition 5

In order to derive tax rates and tuition fees in the Nash equilibrium, I first determine the best responses of each country given what the other country has chosen. Second, the best-response functions are used to solve for equilibrium values. Finally, I proof the stability of the Nash equilibrium.

Solving the maximization problem of government i, leads to the first order conditions

$$\frac{\partial L_i}{\partial s_{ii}} = \frac{\partial GDP_i}{\partial s_{ii}} + \lambda_i \frac{\partial NR_i}{\partial s_{ii}} = 0,$$
$$\frac{\partial L_i}{\partial s_{ij}} = \frac{\partial GDP_i}{\partial s_{ij}} + \lambda_i \frac{\partial NR_i}{\partial s_{ij}} = 0,$$
$$\frac{\partial L_i}{\partial t_i} = \frac{\partial GDP_i}{\partial t_i} + \lambda_i \frac{\partial NR_i}{\partial t_i} = 0,$$
$$NR_i = 0 \quad \text{with} \quad \lambda_i \neq 0.$$

Using the first order conditions, the best responses of i derive as

$$s_{ii}^{BR} = \frac{s_{ji}}{2} + \frac{h^2 + E[m_s] + \frac{\Delta m_s}{2}}{2} - \frac{1}{2\Delta m_g}(\beta - 2\underline{m}_g)E[m_g],$$
$$s_{ij}^{BR} = \frac{s_{jj}}{2} + \frac{h^2 - E[m_s] + \frac{\Delta m_s}{2}}{2} + \frac{1}{2\Delta m_g}(\beta - 2\underline{m}_g)(\beta - E[m_g]),$$
$$t_i^{BR} = F(t_j, s_{jj}, s_{ji}, h).$$

I state t_i^{BR} as a function $F(.)$ because the explicit expression is too large to be shown here. The best responses of country j are

$$s_{jj}^{BR} = \frac{s_{ij}}{2} + \frac{h^2 + E[m_s] + \frac{\Delta m_s}{2}}{2} - \frac{1}{2\Delta m_g}(\beta - 2\underline{m}_g)E[m_g],$$
$$s_{ji}^{BR} = \frac{s_{ii}}{2} + \frac{h^2 - E[m_s] + \frac{\Delta m_s}{2}}{2} + \frac{1}{2\Delta m_g}(\beta - 2\underline{m}_g)(\beta - E[m_g]),$$
$$t_j^{BR} = F(t_i, s_{ii}, s_{ij}, h).$$

Solving the system of six equations (three best-response functions of country i and three best-response functions of country j) for the six unknowns (s_{ii}^*, s_{ij}^*, t_i^*, s_{jj}^*, s_{ji}^* and t_j^*) yields the unique solution

$$s_{ii}^* = s_{jj}^* = \frac{1}{6\Delta m_g}\Big[2\beta^2 - 2\beta(5E[m_g] - \Delta m_g) + \Delta m_g(3\Delta m_s + 6h^2 + 2E[m_s]) + 6E[m_g](2E[m_g] - \Delta m_g)\Big],$$

$$s_{ij}^* = s_{ji}^* = \frac{1}{6\Delta m_g}\Big[4\beta^2 - 2\beta(7E[m_g] - 2\Delta m_g) + \Delta m_g(3\Delta m_s + 6h^2 - 2E[m_s]) + 6E[m_g](2E[m_g] - \Delta m_g)\Big],$$

$$t_i^* = t_j^* = \frac{1}{18\Delta m_s {\Delta m_g}^2 hw}\Bigg[-\beta^4 + 2\beta^3(2E[m_g] - \Delta m_g) + \beta^2(4\Delta m_g(E[m_s] + E[m_g]) - 9\Delta m_s\Delta m_g - {\Delta m_g}^2 - 4E[m_g]^2) + \beta\Delta m_g\Big(9\Delta m_s(4E[m_g] - \Delta m_g) + 4E[m_s](\Delta m_g - 2E[m_g])\Big) + \Delta m_g(18\Delta m_s(\Delta m_g - 2E[m_g])E[m_g] - 9{\Delta m_s}^2\Delta m_g - 4\Delta m_g E[m_s]^2)\Bigg].$$

Both countries choose the same tuition fee for domestic students, the same tuition fee for foreign students and the same tax rate: the Nash equilibrium is symmetric. The resulting tuition fee differential derives as

$$\Delta s_i^* = \Delta s_j^* = \frac{\beta}{3\Delta m_g}(\beta - 2\underline{m}_g) - \frac{2}{3}E[m_s].$$

To prove that no player, i.e., no government, has an incentive to deviate from the derived equilibrium, I derive the bordered Hessian matrix:

$$H = \begin{vmatrix} 0 & \frac{\partial^2 L}{\partial\lambda_i\partial s_{ii}} & \frac{\partial^2 L}{\partial\lambda_i\partial s_{ij}} & \frac{\partial^2 L}{\partial\lambda_i\partial t_i} \\ \frac{\partial^2 L}{\partial\lambda_i\partial s_{ii}} & \frac{\partial^2 L}{\partial {s_{ii}}^2} & \frac{\partial^2 L}{\partial s_{ij}\partial s_{ii}} & \frac{\partial^2 L}{\partial t_i\partial s_{ii}} \\ \frac{\partial^2 L}{\partial\lambda_i\partial s_{ij}} & \frac{\partial^2 L}{\partial s_{ii}\partial s_{ij}} & \frac{\partial^2 L}{\partial {s_{ij}}^2} & \frac{\partial^2 L}{\partial t_i\partial s_{ij}} \\ \frac{\partial^2 L}{\partial\lambda_i\partial t_i} & \frac{\partial^2 L}{\partial s_{ii}\partial t_i} & \frac{\partial^2 L}{\partial s_{ij}\partial t_i} & \frac{\partial^2 L}{\partial {t_i}^2} \end{vmatrix}$$

The best responses and thus the derived equilibrium values determine a global maximum if the sign of the principal minors $|H_2|$, $|H_3|$ and $|H_4|$ is alternating following the rule $(-1)^n$. Calculating the principal minors results in

$$|H_2| = -X^2\frac{(-2+a)^2E[m_w]^2}{324\Delta m_s^4\Delta m_w^6} < 0,$$

$$|H_3| = 2hwX\frac{(-2+a)^2E[m_w]^2}{9\Delta m_s^4\Delta m_w^4} > 0,$$

$$|H_4| = |H| = -16h^4w^4\frac{\Delta m_s\Delta m_w + (-2+a)^2E[m_w]^2}{\Delta m_s^3\Delta m_w^3} < 0,$$

$$\text{with} \quad X = 9\Delta m_s^2\Delta m_w^2 + 9\Delta m_s\Delta m_w\Big[\Delta m_w + (-2+a)E[m_w]\Big]^2 + \Big[(-2+a)aE[m_w]^2 + \Delta m_w(-2E[m_s] + aE[m_w])\Big]^2 > 0,$$

where I defined $\beta \equiv aE[m_w]$ with $a \in (0,1)$ to replace parameter β.[20] I conclude that no government has an incentive to deviate and that the GDP in both countries is maximized in the derived equilibrium.

The uniqueness and stability of the equilibrium can be proven using the best responses of each country. Note first that the tuition fee for domestic students in i (s_{ii}) depends only on the tuition fee for foreign students in j (s_{ji}) and vice versa. The reason is that both governments compete for the same individual, the one who is born in i. Analogously, the tuition fee for foreign students in i (s_{ij}) depends only on the tuition fee for domestic students in j (s_{jj}) and vice versa because both governments compete for individuals who are born in j. Equilibrium tuition fees can be derived independently by the best-response functions without considering the tax rates. The reason is that the tuition fee choice is driven by the competition between countries for one student. Given the tuition fees, the tax rate is then chosen such that the budget is just balanced. It is therefore sufficient to focus on tuition fees only when discussing the uniqueness and stability of the equilibrium.

To prove the uniqueness of the equilibrium, consider the choice of s_{ii} and s_{ji}. Figure 3.3 plots the best response of each country given the other country's choice when competing for individuals born in i. The intersection of the two best-response functions determines the equilibrium which is independent of s_{ij}^*, s_{jj}^*, t_i^* and t_j^*. Note that both best-response functions are linear: the function for s_{ii}^{BR} has a slope of $\frac{1}{2}$ while the rearranged function for s_{ji}^{BR} has a slope of 2. Because of these slopes, the two best-response functions have to intersect once and only once: the equilibrium is unique.

Regarding the equilibrium's stability, let us take a look at Figure 3.3 again and consider that country j chooses a tuition fee of $\tilde{s}_{ji}$. The upward arrow starting in $\tilde{s}_{ji}$ then points to the tuition fee country i would choose as best response. But the combination $(\tilde{s}_{ii}, \tilde{s}_{ji})$ is not an equilibrium because $\tilde{s}_{ji}$ is not a best response to $\tilde{s}_{ii}$; country j would rather choose a higher tuition fee. The following two arrows trace how the two countries reconsider and adapt their tuition fee choice. The two countries will end up choosing the equilibrium

[20] Parameter β is assumed to fulfill the condition $E[m_w] - \beta > 0$. The definition captures this assumption.

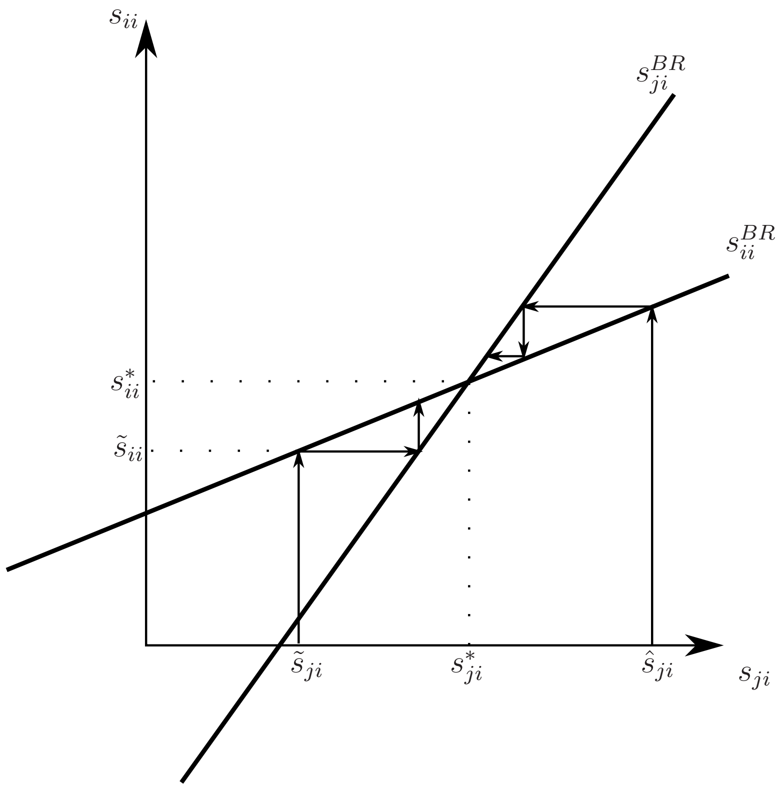


Figure 3.3: The uniqueness and stability of the equilibrium.

combination (s_{ii}^{*}, s_{ji}^{*}). You may also consider what happens when country j chooses a tuition fee which is higher than s_{ji}^{*}, as for instance $\hat{s}_{ji}$. Again, the arrows trace how the two countries adapt their tuition fee choice until they reach the equilibrium combination (s_{ii}^{*}, s_{ji}^{*}). Therefore, I conclude that the equilibrium is stable.

Though I discussed the choice of s_{ii} and s_{ji} only, I concluded that the equilibrium is unique and stable. The reason is because a similar figure to Figure 3.3 can be drawn to determine the equilibrium choice of s_{ij} and s_{jj} with analogous insights. The equilibrium combination (s_{ij}^{*}, s_{jj}^{*}) is therefore also stable. When the tuition fee choice in equilibrium is stable, also the tax rate choice is stable because taxes are uniquely determined by the balanced budget constraint.

3.7.4 Elasticity Notation

The elasticity notation of the difference in equilibrium tuition fees is based on the first order conditions and the symmetry of the Nash equilibrium. Focusing on country i, the first order conditions are

$$FOC1: \quad \frac{\partial GDP_i}{\partial s_{ii}} + \lambda_i \frac{\partial NR_i}{\partial s_{ii}} = 0,$$

$$FOC2: \quad \frac{\partial GDP_i}{\partial s_{ij}} + \lambda_i \frac{\partial NR_i}{\partial s_{ij}} = 0,$$
$$FOC3: \quad \frac{\partial GDP_i}{\partial t_i} + \lambda_i \frac{\partial NR_i}{\partial t_i} = 0,$$
$$FOC4: \quad NR_i = 0 \quad \text{with} \quad \lambda_i \neq 0.$$

Three steps are necessary to achieve the elasticity notation: in the first step, I derive the difference in equilibrium tuition fees. In steps 2 and 3, the result of step 1 is transformed to achieve the elasticity notation.

Step 1 We know from $FOC4$ that the budget is balanced in equilibrium:

$$NR_i = tGDP_i + (s_D - h^2)S_D + (s_F - h^2)S_F = 0,$$

where I use the following notation:

$$\begin{aligned} S_D &= S_{ii}|_{equi.} = S_{jj}|_{equi.}, \\ S_F &= S_{ij}|_{equi.} = S_{ji}|_{equi.}, \\ s_D &= s_{ii}|_{equi.} = s_{jj}|_{equi.}, \\ s_F &= s_{ij}|_{equi.} = s_{ji}|_{equi.}, \\ t &= t_i|_{equi.} = t_j|_{equi.}. \end{aligned}$$

Using the equations for the number of domestic and of foreign students:

$$\begin{aligned} S_D &= \frac{1}{\Delta m_s}\left[\overline{m}_s - A + s_F - s_D\right], \\ S_F &= \frac{1}{\Delta m_s}\left[A - s_F + s_D - \underline{m}_s\right], \end{aligned}$$

with $A = \frac{\beta}{2\Delta m_g}(\beta - 2\underline{m}_g)$, the equilibrium tuition fees derive as

$$\begin{aligned} s_D &= h^2 - \frac{tGDP}{S_F(1+\tilde{S})} - \frac{1}{1+\tilde{S}}(A - E[m_s]) + \frac{\Delta m_s}{2}\frac{1-\tilde{S}}{(1+\tilde{S})^2}, \\ s_F &= h^2 - \frac{tGDP}{S_F(1+\tilde{S})} + \frac{\tilde{S}}{1+\tilde{S}}(A - E[m_s]) - \frac{\Delta m_s}{2}\frac{(1-\tilde{S})\tilde{S}}{(1+\tilde{S})^2}, \end{aligned}$$

with $\tilde{S} \equiv \frac{S_D}{S_F}$ denoting the equilibrium ratio between domestic and foreign students. The equilibrium tuition fee differential then derives as

$$\Delta s = s_F - s_D = A - E[m_s] - \frac{\Delta m_s}{2} \frac{1-\tilde{S}}{1+\tilde{S}} \,. \tag{3.12}$$

In the next two steps, I replace the equilibrium ratio of domestic to foreign students $\tilde{S}$ and the term $A - E[m_s]$ in equation (3.12) to obtain the elasticity notation.

Step 2 The equilibrium ratio of domestic to foreign students $\tilde{S}$ can be replaced using the $FOC1$ and $FOC2$. Combining and rearranging these two conditions yields

$$\begin{aligned}
FOC1 &= FOC2 \\
\frac{\partial NR_i}{\partial s_{ii}} &= \frac{\partial NR_i}{\partial s_{ij}} \\
t_i \frac{\partial GDP_i}{\partial s_{ii}} + S_{ii} + (s_{ii} - h^2) \frac{\partial S_{ii}}{\partial s_{ii}} &= t_i \frac{\partial GDP_i}{\partial s_{ij}} + S_{ij} + (s_{ij} - h^2) \frac{\partial S_{ij}}{\partial s_{ij}} \\
S_{ii} + s_{ii} \frac{\partial S_{ii}}{\partial s_{ii}} &= S_{ij} + s_{ij} \frac{\partial S_{ij}}{\partial s_{ij}} \\
S_{ii}(1 + \epsilon_{S_{ii},s_{ii}}) &= S_{ij}(1 + \epsilon_{S_{ij},s_{ij}}) \\
\Rightarrow \quad \frac{S_{ii}}{S_{ij}} &= \frac{1 + \epsilon_{S_{ij},s_{ij}}}{1 + \epsilon_{S_{ii},s_{ii}}}
\end{aligned}$$

with $\epsilon_{S_{ii},s_{ii}} = \frac{\partial S_{ii}}{\partial s_{ii}} \frac{s_{ii}}{S_{ii}}$ and $\epsilon_{S_{ij},s_{ij}} = \frac{\partial S_{ij}}{\partial s_{ij}} \frac{s_{ij}}{S_{ij}}$ denoting the tuition fee elasticity of domestic students and of foreign students. Since the equilibrium is symmetric, I can write

$$\tilde{S} = \tilde{\epsilon} \tag{3.13}$$

where I define

$$\tilde{\epsilon} \equiv \frac{1 + \epsilon_{S_{ij},s_{ij}}|_{eq.}}{1 + \epsilon_{S_{ii},s_{ii}}|_{eq.}} = \frac{1 + \epsilon_{S_{ji},s_{ji}}|_{eq.}}{1 + \epsilon_{S_{jj},s_{jj}}|_{eq.}}.$$

Ratio $\tilde{\epsilon}$ denotes the equilibrium ratio between the tuition fee elasticity of foreign students and the tuition fee elasticity of domestic students. The higher $\tilde{\epsilon}$, the more elastically do foreign students react to tuition fee changes comparing to the reaction of domestic students.

With the help of equation (3.13), I can replace $\tilde{S}$ in equation (3.12):

$$\Delta s = A - E[m_s] - \frac{\Delta m_s}{2} \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}. \tag{3.14}$$

Step 3 Finally I can replace the term $A - E[m_s]$ in equation (3.14) using the equation for $\tilde{\epsilon}$ which can be calculated as

$$\tilde{\epsilon} = \frac{1 - \frac{s_F}{\Delta m_s S_F}}{1 - \frac{s_D}{\Delta m_s S_D}} = \frac{\frac{3}{2}\Delta m_s - A + E[m_s]}{\frac{3}{2}\Delta m_s + A - E[m_s]}.$$

Rearranging the above equation leads to

$$A - E[m_s] = 3\frac{\Delta m_s}{2}\frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}.$$

Thus equation (3.14) can be written as

$$\Delta s = \Delta m_s \frac{1-\tilde{\epsilon}}{1+\tilde{\epsilon}}$$

stating the difference in equilibrium tuition fees depending on tuition fee elasticities only.

3.7.5 Solution of the Extended Setup

The setup with two countries, which do not offer the same quality of education ($h_i \neq h_j$) and are thus not fully identical, can be solved in 3 steps: first, labor migration, second, student migration and third, the government's decision problem is analyzed in the following. The focus is on country i; the solution for country j can be obtained by taking the solution for country i and switching the indices.

Labor Migration A graduate who was born and studied in country i migrates to country j if the income difference is high enough to cover migration costs:

$$(1-t_j)h_i w - m_g > (1-t_i)h_i w$$
$$\Rightarrow \quad m_g < (t_i - t_j)h_i w.$$

A graduate who did not study in his home country, i.e., he was born in j and studied in i, leaves his study location if

$$m_g - \beta < (t_i - t_j)h_i w.$$

Student Migration A student who is born in country i migrates if his expected income when studying abroad is higher than his expected income when studying in his home country taking into account migration costs and tuition fees:

$$E[\Pi_{ji}] - s_{ji} - m_s > E[\Pi_{ii}] - s_{ii} \tag{3.15}$$

$$\Leftrightarrow \quad m_s < E[\Pi_{ji}] - E[\Pi_{ii}] + s_{ii} - s_{ji} \tag{3.16}$$

where $E[\Pi_{ab}]$ denotes expected income of an individual studying in a while born in b with $a, b = \{i, j\}$. Calculating the expected income terms simplifies the above inequality to

$$\begin{aligned} m_s < & \frac{1}{2\Delta m_g}(t_i - t_j)^2 w^2 ({h_j}^2 - {h_i}^2) + \frac{\beta}{\Delta m_g}(t_j - t_i)h_j w + (h_j - h_i)w \\ & + \frac{w}{\Delta m_g}\Big[t_i(h_i\overline{m}_g + h_j\underline{m}_g) - t_j(h_j\overline{m}_g + h_i\underline{m}_g)\Big] \\ & + \frac{\beta}{2\Delta m_g}\Bigg[\beta - 2\underline{m}_g\Bigg] - s_{ji} + s_{ii} \equiv I_i. \end{aligned}$$

The incentive to study abroad is higher, the higher the tax rate in the home country t_i or the higher the quality of education in the other country h_j. Note that this inequality simplifies to the version presented in the chapter for two identical countries when $h_i = h_j = h$.

Number of Students and Human Capital Stock Given the solution of labor and student migration and assuming inner solutions, the student numbers in country i derive as

$$S_{ii} = \Pr[m_s > I_i] = \frac{1}{\Delta m_s}(\overline{m}_s - I_i),$$

$$S_{ij} = \Pr[m_s < I_j] = \frac{1}{\Delta m_s}(I_j - \underline{m}_s).$$

The stock of human capital is then given by

$$\begin{aligned} H_i = & h_i\Bigg[S_{ii}\Pr[m_g > (t_i - t_j)h_i w] + S_{ij}\Pr[m_g - \beta > (t_i - t_j)h_i w]\Bigg] \\ & + h_j\Bigg[S_{jj}\Pr[m_g < (t_j - t_i)h_j w] + S_{ji}\Pr[m_g - \beta < (t_j - t_i)h_j w]\Bigg] \end{aligned}$$

which can be rearranged to

$$H_i = S_{ii}H_{i1} + S_{ij}H_{i2} + H_{i3}$$

with

$$H_{i1} = \frac{1}{\Delta m_g}\Big[h_i\overline{m}_g + h_j\underline{m}_g - (t_i - t_j)({h_i}^2 - {h_j}^2)w - h_j\beta\Big],$$
$$H_{i2} = \frac{1}{\Delta m_g}\Big[h_i\overline{m}_g + h_j\underline{m}_g - (t_i - t_j)({h_i}^2 - {h_j}^2)w - h_i\beta\Big],$$
$$H_{i3} = \frac{h_j}{\Delta m_g}\Big[2(t_j - t_i)h_j w\beta - 2\underline{m}_g\Big].$$

Governmental choice of tuition fees and Nash equilibrium Governments maximize the GDP of their country subject to the budget constraint. In the case of country i, the maximization problem is given by

$$\max_{s_{ii}, s_{ij}, t_i} GDP_i \qquad \text{s.t.} \qquad NR_i \geq 0$$

with

$$GDP_i = H_i w,$$
$$NR_i = tH_i w - c_i\big(S_{ii} + S_{ij}\big) + s_{ii}S_{ii} + s_{ij}S_{ij} \quad \text{and}$$
$$c_i = {h_i}^2.$$

In the policy choice stage, each government chooses tuition fees and the tax rate independently. When deciding on tuition fees and the tax rate, each government anticipates the behavior of the other. The first step to solve for Nash equilibria in the game played by the two governments is to derive the best-response tuition fees and tax rate in country i for given policy instruments chosen by country j. Using the Lagrangian

$$L = GDP_i + \lambda_i NR_i,$$

best responses of country i follow the FOCs

$$FOC1: \quad \frac{\partial GDP_i}{\partial s_{ii}} + \lambda_i \frac{\partial NR_i}{\partial s_{ii}} = 0,$$
$$FOC2: \quad \frac{\partial GDP_i}{\partial s_{ij}} + \lambda_i \frac{\partial NR_i}{\partial s_{ij}} = 0,$$
$$FOC3: \quad \frac{\partial GDP_i}{\partial t_i} + \lambda_i \frac{\partial NR_i}{\partial t_i} = 0,$$
$$FOC4: \quad NR_i = 0 \quad \text{if} \quad \lambda_{1i} \neq 0.$$

Note that the budget constraint is binding because the government of country i lowers tuition fees and the tax rate as much as possible in order to attract workers and students. The best response of country i may be solved using the four FOCs. However, an explicit solution cannot be obtained. As a consequence, equilibrium tuition fees and tax rate in both countries cannot be determined analytically. I therefore use a numerical example to study the effect of education quality on tuition fee differentials.

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