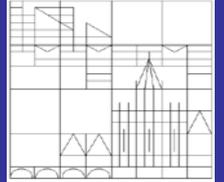




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# Technology Adoption and Demographic Change

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# Technology Adoption and Demographic Change

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## Abstract

This paper studies the effect of demographic change on the technology distribution of an economy and on aggregate productivity growth. In the quantitative dynamic model, firms decide on employment and the technology they use subject to an aging workforce. Firms with a higher share of elderly workers update their technology less often and prefer older technologies than firms with a younger workforce. The shorter expected worklife of elderly workers makes firms reluctant to train them for new technologies. I calibrate the model for the German economy and simulate the projected demographic change. The results indicate that labor force aging reduces the realized annual productivity growth rate by 0.17 percentage points between 2010–2025.

JEL Classification: J11, J21, J26, O33

Keywords: Demographic Change, TFP growth, Retirement Policies

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# 1 Introduction

As a consequence of demographic change, the share of elderly persons in the labor force will increase steeply in the industrialized countries during the first half of the 21st century. In the majority of OECD countries, the share of workers aged 55 and above in the labor force will increase by 50–100% between 2000–2030 (Carone, 2005; Toossi, 2006). Empirical studies indicate that an increase in the share of elderly workers has a substantial negative influence on aggregate productivity and productivity growth (Tang and MacLeod, 2006; Feyrer, 2007; Werding, 2008; Grönqvist, 2009). To investigate the underlying sources of this relationship, I model the effect of labor force aging on productivity growth at the firm level and for the aggregate economy.

I develop a quantitative model that explores firms' technology decisions with respect to the age composition of their workforce and explains the influence of the labor force age composition on aggregate productivity. The model allows to determine the impact of labor force aging on the technology distribution of the economy and aggregate productivity growth. In the dynamic general equilibrium model, firms decide to adjust their workforce and to adopt new technologies. The economy is populated with overlapping generations of workers who age stochastically. This aging process changes the workforce composition in each firm steadily over time. It turns out that firms that employ a higher share of elderly workers update their technology less frequently because they fear that investment in training their workers for a new technology cannot be recuperated as the expected remaining worklife is too short. Also, firms with a high share of older workers prefer to adopt non-state-of-the-art technologies at a lower training cost to reduce the investment in their elderly workers.

I calibrate the model for the German economy and use the firm policies to derive the equilibrium firm distribution of the economy. I then simulate the projected changes of the labor force age composition for the period 2003–2025 by varying the inflow of young workers into the economy. Between 2010–2025, when labor force aging is strongest, I find that demographic change lowers annual productivity growth by about 0.11 percentage points on average. This translates into a cumulated GDP loss of approximately 416bn Euro in constant prices of 2005 for this period.

Demographic change is accompanied by an increase in the average retirement age in Germany. During the simulation period, it is expected to increase from 60 years to about 63 years (Carone, 2005). Two opposing effects accompany this increase in the retirement age. At the micro level, the longer expected job duration of elderly workers leads to shorter technology updating intervals which moves the economy closer to the technological frontier and lowers the impact of demographic change. At the aggregate level, however, the increased retirement age raises the share of elderly workers further, in addition to demographic change. This amplifies the negative effect on productivity growth. The model allows to disentangle the contribution of demographic change and the increased retirement age. Taking the change of the average retirement age into account increases the average growth loss between 2010–2025 to 0.17 percentage points annually.

As a policy experiment, I analyze the consequences of an additional increase in the average retirement age of three years. It turns out that in this case the negative effect is further

amplified. The economy moves further away from the technological frontier, which adds to the negative effect of demographic change. For the period 2010–2025, the resulting annual growth loss amounts to 0.21 percentage points. As a second experiment, I reduce the firms’ cost for worker training by 10% assuming that training methods become more efficient. As expected, the economy moves closer to the technological frontier as firms update their technologies in shorter intervals and also the effect of demographic change is reduced. Between 2010–2025, the annual loss of growth now amounts to 0.09 percentage points.

The existing literature offers two alternative mechanisms to explain the negative relationship between worker aging and the adoption of modern technologies. Some authors argue that older workers are not well prepared for new technologies as economic skill obsolescence reduces the value of their knowledge over time (Rosen, 1975; de Grip and van Loo, 2002) or that they are less able to adapt to new technologies (Skirbekk, 2004; Weinberg, 2004). An alternative explanation, which I use in this model, is that the short remaining worklife duration of older workers makes the investment in training for new technologies less attractive. This idea does not depend on assumptions about the workers’ abilities and applies for all types of workers in all sectors of the economy. As Figure 1 illustrates, workers aged 55 and above receive significantly less on-the-job training than workers aged 25–54 in the European Union. On average, the amount of training that elderly workers received accumulated to only 47% of the training for prime-age workers, whereby large discrepancies between countries exist. According to Klös (2000), only 5% of citizens aged 50-55 and only 1% of citizens above 55 received on-the-job-training in Germany in 1998.

Swanson et al. (1997) also focus on the influence of the remaining worklife duration. In a life-cycle model with exogenously moving technological frontier, individuals decide whether to adopt new technologies, to work or to enjoy leisure. The authors show that individuals stop adopting new technologies in the later stages of their lives as the investment could otherwise not be recuperated. They also suggest that this mechanism leads to lower realized technological progress when the population ages. In a similar paper by Ahituv and Zeira (2000), elderly workers decide between working with an old technology, adopting a new technology and early

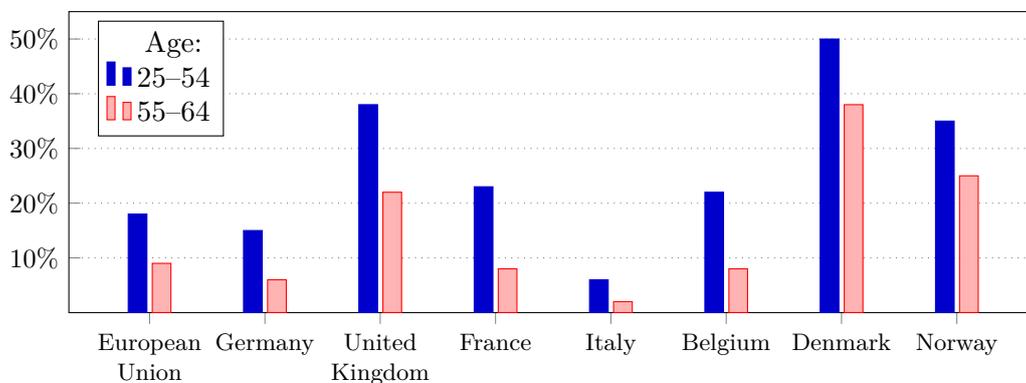


Figure 1: Annual incidence of training for young and old workers in the European Union (Source: Eurostat (2013))

retirement. Langot and Moreno-Galbis (2008) analyze the technology adoption decision from the view point of a firm that employs a single worker who ages stochastically. Their focus lies on whether technological progress is beneficial for the employment prospects of older workers or not. Nevertheless, their paper shares some results with the paper presented here: old-worker firms update their technology less often than young-worker firms, a higher training cost for new technologies decreases the updating frequency whereas a higher rate of exogenous technological progress increases it.

This model contributes to this literature by adding the following: first, the perspective is shifted towards the aggregate economy with a focus on technology diffusion. Second, the quantitative model allows to analyze the effects of demographic change on the level of total factor productivity (TFP) and realized productivity growth with a calibration to the German economy. Third, the technology decision is put in the hands of multi-worker firms which can decide when to update to a new technology and which technology to choose, and also choose the number of young and old workers they employ. Since worker aging implies that firms have heterogeneous workforces, their decision problem becomes far more complex. Firms may want to train their young workers for a new technology, however, they have to take into account their elderly employees as well, for which investment might be less profitable. This can lead firms to lay off their elderly workers but also to delay the technology upgrading decision. With this, firm heterogeneity leads to much richer dynamics compared to previous models.

The model is consistent with the findings of empirical studies on technology adoption at the firm level. For French private sector firms, Aubert et al. (2006) find that innovative firms have a lower wage-bill share of older workers. Meyer (2007, 2009) shows that an older workforce in firms leads to less technology adoption in German small and medium-sized firms, and this effect increases with worker age beginning from the age of 30. Schneider (2008) uses a linked employer-employee dataset to study the innovative potential of German firms in relation to their workforce. He finds an inverted U-shaped relationship that indicates that middle-aged workers increase the innovative activities of their employers whereas elderly workers and very young workers have a negative influence. Malmberg et al. (2008) analyze the productivity of Swedish manufacturing and mining enterprises between 1985–96 and find that firms with a higher share of employees aged 50 and above are generally less productive though more productive when technology is controlled for. This implies that firms with an older workforce use on average less productive technologies.

At the aggregate level, several empirical studies have scrutinized the relationship between the share of elderly workers and aggregate productivity and the impact of demographic change on productivity growth. Tang and MacLeod (2006) find that a one-percent increase in the share of workers aged 55 and above reduces productivity growth in Canadian provinces by 0.07%. Two studies by Feyrer (2007) and Werdinger (2008) analyze the relationship between the age composition of the labor force and TFP for a panel of 27 OECD countries and other countries for the period 1960-2000. They find a positive effect on productivity for workers aged 40–49 but negative effects for workers aged 50 and above. Grönqvist (2009) uses industry-level for Finland between 1971–2005 and finds that an increase of the share of workers aged 55 and above by 1%

lowers annual labor productivity growth by about 0.22 percentage points. A comparison of the simulation results with these estimates indicates that the model results are in a plausible range.

The remainder of the paper is organized as follows: the next section presents the model and section 3 derives the stationary equilibrium. In section 4, I show how the model is calibrated for the German economy and section 5 illustrates the results for the stationary equilibrium and discusses the effect of the model's parameters. Section 6 provides the results for the simulation of the projected demographic change followed by robustness tests for the simulation results. Section 8 concludes and discusses possible extensions of the model.

## 2 The Model

The model analyzes firm dynamics in a competitive economy in a similar framework as Hopenhayn and Rogerson (1993), where firms expand or contract, make technology choices and enter or exit the market. In their decisions, firms take into account workforce aging with overlapping generations of workers.

### Workers and Firms

The economy is populated by a continuum of firms and workers. New workers enter every period as young workers, turn into old workers and finally exit the labor market, determined by a stochastic aging process. All variables referring to young workers will be denoted by subindex  $y$  whereas  $o$  is used to indicate old workers. The exogenous probability of becoming old for a young worker is given by  $\lambda_y$  and the exogenous probability of retirement for an old worker is  $\lambda_o$ . Consequently, workers are young for  $1/\lambda_y$  periods on average whereas the expected worklife duration of an old worker is  $1/\lambda_o$  periods. Employed workers separate from firms with exogenous probabilities  $q_y$  and  $q_o$  or when exiting the labor market. Apart from the expected remaining time in the labor market and the exogenous separation probabilities, old and young workers are equal in all aspects. In particular, this means that there are no experience effects or learning on the job and also no loss of human capital or falling individual productivity with age. This assumption is made to isolate the impact of the expected remaining worklife on training and its effect on technology adoption.<sup>1</sup>

The evolution of the economy's workforce is given by:

$$P_{y,t+1} = (1 - \lambda_y) \cdot P_{y,t} + P_{y,t}^N, \quad (1)$$

$$P_{o,t+1} = \lambda_y \cdot P_{y,t} + (1 - \lambda_o) \cdot P_{o,t}, \quad (2)$$

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<sup>1</sup>The assumption that the productivity of workers remains constant over the working life is supported by recent empirical work (Börsch-Supan and Weiss, 2011; Pekkarinen and Uusitalo, 2012). Oster and Hamermesh (1998) find that publishing by economists decreases with age, however they can not discern whether this is related to a reduction of productivity or caused by other issues such as lower incentives or an increase of other duties. This latter view is supported by Rauber and Ursprung (2008), who find that the cycle pattern of economists' productivity originates from incentive effects. Besides they find very strong cohort effects at work. A similar result is found by van Ours (2009).

where  $P_{y,t}$  and  $P_{o,t}$  denote the mass of young and old workers in the economy respectively and  $P_{y,t}^N$  denotes the inflow of new young workers in a period.

Workers have a zero reservation wage and are employed by firms for which labor is the only variable input. Each single firm employs a mass of young and old workers who are hired in competitive markets at wage rates  $w_{y,t}$  and  $w_{o,t}$  and hiring cost<sup>2</sup>  $c_{N,t}$  per worker. All firms produce an identical good which is taken as the economy's numeraire and discount profits at a common, exogenous discount rate  $r$ . Firms can enter the market at a positive entry cost  $c_{E,t}$  and exit with exogenous probability  $\delta$ . The free entry condition ensures zero expected profits prior to entering.

A firm's production function is given by

$$Y_t^i = A_t^i F(y_t^i + o_t^i), \quad (3)$$

where  $y_t^i$  and  $o_t^i$  denote the total number of young and old workers employed by the respective firm at time  $t$  and  $A_t^i$  is a productivity parameter that depends on the technology that the firm currently uses. The function  $F(x)$  is increasing and convex. The latter fact ensures a large number of heterogeneous firms.

exhibits decreasing returns to scale to restrict the firm's size.

### Technological Progress

The economy features exogenous technological progress and a new technology arrives in every period. New technologies increase the productivity frontier by a constant factor  $g$  so that the productivity parameter evolves according to

$$A_{t+1} = (1 + g) \cdot A_t. \quad (4)$$

A firm has two options for adopting new technologies. It can either adopt the newest technology  $A_t$  or a technology that is  $B$  steps away from the technological frontier  $A_{t-B}$ . To adopt a production technology, a firm has to train its workers to use it. This implies that technology is embodied in the workers of a firm. All workers within a firm need to use the same technology and a firm cannot split itself into two entities that use different technologies. If a firm hires new workers, it has to train them for the technology that is currently used in the firm.

Training cost is a fixed cost per worker and depends on the type of technology that is adopted. The training cost for the newest technology  $A_t$  is denoted by  $c_{T,t}$  while adopting the older technology  $A_{t-B}$  involves the cost  $\beta \cdot c_{T,t}$  where  $0 < \beta < 1$ . If a firm hires new workers without upgrading its technology, the training cost for new workers is  $c_{T,t}$  if the firm's technology is  $A_t^i \in [A_t, A_{t-(B-1)}]$  and  $\beta \cdot c_{T,t}$  if  $A_t^i \leq A_{t-B}$ . Introducing two different technology updating choices enlarges the decision space of firms and makes the model more realistic. It also helps to fit the calibrated model as the parameter  $B$  effects the technology dispersion in the economy.

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<sup>2</sup>The hiring cost increases the value of a worker who is employed in a firm. This provides firms with an incentive to keep old workers instead of exchanging them for young workers and increases firm heterogeneity in the model.

The training cost comprises all direct and indirect costs of adopting a new technology. This includes monetary training costs, lost production for the time of training, and lower production for the time that workers need until they are experienced enough with the new technology to become more productive than with the old technology (see e.g. Helpman and Rangel (1999)). When a firm decides to use a different technology, all of its workers have to be trained for it. Worker training is firm specific, hence a worker who changes his employer has to be trained anew, irrespective of the previous trainings he received. This implies that training costs are borne by the employing firm, not by the worker (Becker, 1962).

Since technology progresses every period, all cost constants and wages are expressed in efficiency units relative to the technological frontier and written without time index, i.e. the actual costs are multiplied by  $p(t) = (1 + g)^t$ , e.g.  $c_{T,t} = (1 + g)^t c_T$ . This ensures the existence of a steady state equilibrium on the growth path.

### Timing of Events

Each period starts with production where output is determined by the firms' technology decision in the preceding period. Thereafter, the existing firms, together with new firms that enter the market, hire young and old workers or lay off part of their workforce and decide whether to upgrade to a new technology in the next period. At the end of the period, firm exit and worker aging and separation take place.

## 3 Equilibrium

In this section I focus on a stationary equilibrium along a balanced growth path.

### The firm's problem

Along the balanced growth path, the problem of an active firm that employs a mass of  $y$  young workers and a mass of  $o$  old workers at given wages in efficiency units  $w_y, w_o$ <sup>3</sup> and uses technology  $A_{(k)}$ , which has a distance of  $k$  stages to the latest technology, can be expressed recursively in

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<sup>3</sup>In a stationary equilibrium, wages in efficiency units  $w_y, w_o$  are constant over time and for each period, wages are given by  $w_{y/o,t} = (1 + g)^t w_{y/o}$

efficiency units as

$$\begin{aligned}
v(y, o, k) = & (1 + g)^{-k} F(y + o) - y \cdot w_y - o \cdot w_o \\
& + \max_{y', o', k'} \left\{ - (c_N + c_T(k)) (y^H + o^H) + \frac{1 - \delta}{1 + r} v(y', o', k + 1)(1 + g), \right. \\
& - c_T(y - y^F + o - o^F) - (c_N + c_T) (y^H + o^H) + \frac{1 - \delta}{1 + r} v(y', o', 0)(1 + g), \\
& \left. - \beta c_T(y - y^F + o - o^F) - (c_N + \beta c_T) (y^H + o^H) + \frac{1 - \delta}{1 + r} v(y', o', B)(1 + g) \right\}, \tag{5}
\end{aligned}$$

$$\text{s.t. } y' = (1 - \lambda_y)(1 - q_y) \cdot [y + y^H - y^F] \tag{6}$$

$$o' = (1 - \lambda_o)(1 - q_o) \cdot [o + o^H - o^F] + \lambda_y(1 - q_y) \cdot [y + y^H - y^F], \tag{7}$$

$$c_T(k) = \begin{cases} c_T & \text{for } k < B, \\ \beta c_T & \text{for } k \geq B, \end{cases}$$

where  $y^H, o^H$  and  $y^F, o^F$  denote hired and fired young and old workers respectively.

A firm's value is given by its instantaneous profit, that is output net of wage payments, and the discounted future value of the firm which depends on the firm's optimal policy decisions. A firm decides in every period on the optimal employment of young and old workers in the next period, given its technology decision and with respect to employment adjustment in (6) and (7). With regard to technology, a firm has three options: First, it can continue with its current production technology; second, it can update to the newest technology at cost  $c_T$  per worker; and third, it can update its technology to the level  $B$  steps behind the technological frontier at cost  $\beta \cdot c_T$  per worker. If a firm decides to continue with its actual technology and hires new workers, training cost per worker are  $c_T$  if the firm is less than  $B$  steps behind the technological frontier and  $\beta \cdot c_T$  if the firm is further behind the frontier.

The state of a firm is completely described by the mass of young and old workers it employs and the technology it uses. The state of the economy is given by the distribution of state variables for all individual firms and is expressed as a measure over triples  $\mu(y, o, k)$ , which is invariant in the stationary equilibrium.<sup>4</sup> The optimal employment decisions for young and old workers are denoted  $N_y(y, o, k)$  and  $N_o(y, o, k)$  respectively<sup>5</sup>, the optimal technology decision is denoted  $X(y, o, k) \in \{0, B, k + 1\}$ .

### Entry of new firms and wages

Entry is free; therefore entering firms expect zero profits. For entering firms, two technology choices are possible: they can either adopt the newest technology  $A_{(0)}$  or the vintage technology

<sup>4</sup>In the numerical solution where  $y$  and  $o$  are restricted to a finite number of values,  $\mu(y, o, k)$  can be represented as a three-dimensional matrix where each element gives the mass of firms in a particular state.

<sup>5</sup> $N_y(y, o, k)$  and  $N_o(y, o, k)$  denote the mass of young and old workers respectively, that a firm which is described by  $(y, o, k)$  at the beginning of the period employs after the employment decision has been made.

$A_{(B)}$ . The zero-profit conditions for the two options are given by:

$$(1+g)\frac{1-\delta}{1+r}v(y', o', 0) - c_E - (c_N + c_T)(y^H + o^H) \leq 0 \quad \forall y^H, o^H \geq 0, \quad (8)$$

$$(1+g)\frac{1-\delta}{1+r}v(y', o', B) - c_E - (c_N + \beta c_T)(y^H + o^H) \leq 0 \quad \forall y^H, o^H \geq 0, \quad (9)$$

where  $y^H, o^H$  denote the masses of young and old workers that an entrant hires and  $y'$  and  $o'$  give the labor force in the next period according to (6) and (7) respectively. Since the model features exogenous firm exit, a stationary equilibrium necessarily requires positive entry of firms. Therefore at least one of the two zero-profit conditions must be binding, that is one combination  $y^H, o^H$  exists for which a zero-profit condition is binding. For a certain range of the model's parameters, especially  $\beta$  and  $B$ , it is possible that two types of entrants exist, where one type chooses the newest technology  $A_{(0)}$  and one type chooses the vintage technology  $A_{(B)}$ . Otherwise it can turn out that only one technology choice is attractive for entering firms, so that only one entrant type exists and either all entering firms adopt the newest technology  $A_{(0)}$  or all entrants adopt the vintage technology  $A_{(B)}$ . Together with simultaneous labor market clearing for both types of workers, equations (8) and (9) determine the wage rates  $w_y, w_o$ . The fact that the entrants' technologies move with the technological frontier ensures that wages grow with the rate of technological progress and are constant in efficiency units.

Entering firms are completely described by their employment decision and the chosen technology. The distribution of entrants is therefore expressed as a measure over triples which is denoted as  $\mu^N(y^H, o^H, k)$ . With this, enough information has been collected to trace the evolution of the economy. At the beginning of period  $t$ , let the incumbents be summarized by the measure  $\mu$ . Incumbents make optimal employment decisions, given by the policy functions  $N_y(y, o, k)$  and  $N_o(y, o, k)$ , and decide on technology upgrading  $X(y, o, k)$ . At the same time, new firms summarized by the measure  $\mu^N$  enter the economy and hire the remaining workers that are not employed by incumbent firms. After workers have separated from firms with probabilities  $q_y$  and  $q_o$  and aged according to aging probabilities  $\lambda_y$  and  $\lambda_o$  and firms have exited with probability  $\delta$ , the aggregate state of the economy for period  $(t+1)$  is given by the measure  $\mu'$ . The transition from  $\mu$  to  $\mu'$  is written as  $\mu' = T(\mu, \mu^N, w_y, w_o)$ . The wage rates for young and old workers appear in the operator  $T$  as they determine the decision rules of incumbent and entering firms.

### Labor markets

In a stationary equilibrium with constant population size, the inflow of young workers must equal the outflow of old workers in every period, so  $P_y^N = \lambda_o \cdot P_o$ . The mass of the total workforce is normalized to one, so the supply of old and young workers in the labor market is given by:

$$L_y^s = P_y = \frac{\lambda_o}{\lambda_y + \lambda_o}, \quad (10)$$

$$L_o^s = P_o = \frac{\lambda_y}{\lambda_y + \lambda_o}. \quad (11)$$

Total demand for young and old workers by incumbents and entrants is given by:

$$L_y^d(\mu, \mu^N, w_y, w_o) = \int N_y(y, o, k, w_y, w_o) d\mu(y, o, k) + \int y^H d\mu^N(y^H, o^H, k), \quad (12)$$

$$L_o^d(\mu, \mu^N, w_y, w_o) = \int N_o(y, o, k, w_y, w_o) d\mu(y, o, k) + \int o^H d\mu^N(y^H, o^H, k). \quad (13)$$

### Definition of equilibrium

A stationary equilibrium is given by constant wages for young and old workers in efficiency units  $w_y^*, w_o^* \geq 0$ , a measure of entering firms  $\mu^{*N}$  and a measure of incumbent firms  $\mu^*$  such that (i)  $L_y^d(\mu^*, \mu^{*N}, w_y^*, w_o^*) = L_y^s$  and  $L_o^d(\mu^*, \mu^{*N}, w_y^*, w_o^*) = L_o^s$ , (ii)  $T(\mu^*, \mu^{*N}, w_y^*, w_o^*) = \mu^*$ , and (iii)  $(1 + g) \frac{1-\delta}{1+r} v(y', o', 0) - c_E - (c_N + c_T) (y^H + o^H) \leq 0$  and  $(1 + g) \frac{1-\delta}{1+r} v(y', o', B) - c_E - (c_N + \beta c_T) (y^H + o^H) \leq 0 \quad \forall y^H, o^H \geq 0$ , with equality for those pairs  $(y^H, o^H)$  where  $\mu^{*N}(y^H, o^H, k) > 0$ .

The conditions need not much explanation: Condition (i) demands that labor markets for young and old workers are cleared, condition (ii) demands that the state of the economy replicates itself in each period in a stationary equilibrium, given optimal decision by incumbent firms and entrants, and condition (iii) states that entry in the economy is possible with zero expected profits for entrants. The model is solved by numerical methods as described in more detail in the appendix.

### Wages, firm policies, and demographic change

A feature of the case with two types of entrants is that typically the two entrant types will pursue different hiring strategies with respect to the age distribution of the hired workforce. This happens due to the fact that the updating decisions of firms strongly depend on the age structure of their workforces, as will be shown in the next section. In this case, simultaneous labor market clearing for young and old workers is achieved by the adjustment of the masses of entrant types, which results in a block recursive equilibrium as described in Menzio and Shi (2009). This implies that when the masses of young and old workers in the economy change, the distribution of entrant types changes, but not the hiring policies and profits of the entering firms itself. Therefore wages are independent of the masses of young and old workers in the economy. From this follows that firm policies are independent of the distribution of workers. So if the relation of young and old workers changes, firm policies do not change. This feature allows to simulate demographic change by adjusting the inflow of new workers into the economy while firm policy functions remain constant as long as all other parameters including  $\lambda_y$  and  $\lambda_o$  are left unchanged.

## 4 Calibration

### Calibration

The model is calibrated to match the German economy and the projected changes in the labor force between 2003–2025 are simulated to analyze the resulting changes in the economy's average distance from the technological frontier. Figure 2 shows the projected changes in the old-age

ratio for the German population and labor force. The threshold age of 55 years that separates young workers from old workers has been chosen because the participation rate in the labor force drops dramatically after the age of 55 whereas it is rather constant between the age 20–54. That implies that the probability that a worker leaves the firm and exits the labor force is strongly increased once he has reached the age of 55. The expected time to be a young worker is therefore 35 years which gives  $\lambda_y = 0.0286$  for a period length of one year.

The change in the age composition of the labor force is not only determined by demographic change but also by changes in the average retirement age of older persons. Figure 2 illustrates the change of the labor force composition with and without the latter effect. It can be seen that the increase of the share of old workers in the economy is strongly augmented by the expected increase of the retirement age. Table 1 depicts the average retirement age and the corresponding  $\lambda_o$  over the simulation period. Different retirement ages imply different firm policies, whereby firms have to adopt their policies dynamically over time to adjust to the new exit age of old workers. In the scope of this paper, the analysis of these dynamic adjustments is not possible.

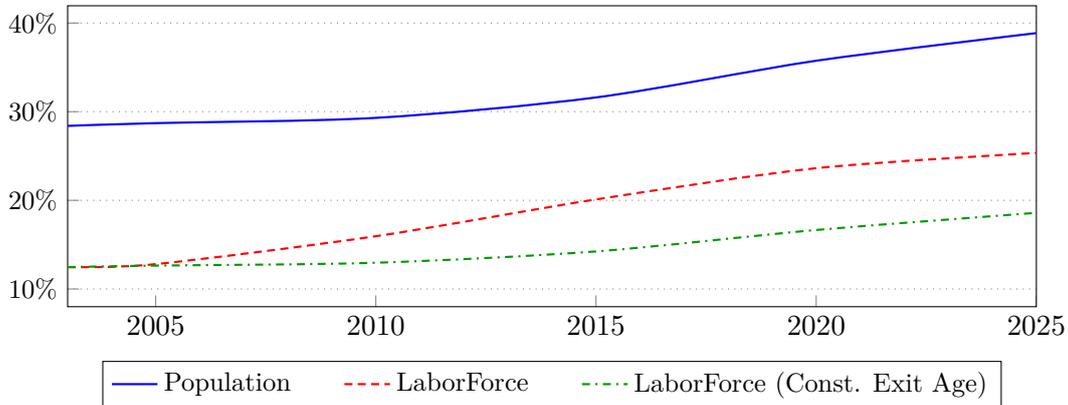


Figure 2: Projected share of elderly people in population and labor force in Germany (Source: Carone (2005); OECD (2011), Own calculations)

To take the increase of the average retirement age into account, I divide the simulation period into two periods with relatively constant average retirement age. I use the average of the retirement ages for 2003–2010 and 2015–2025, which gives an average retirement age of 60.8 years with  $\lambda_{o,2003} = 0.146$  and 62.8 years with  $\lambda_{o,2015} = 0.147$  respectively and assume that the increase of the retirement age in 2015 comes as a shock to the firms.<sup>6</sup>

Table 2 provides an overview of the calibration for the model’s parameters. The interest rate and rate of exogenous technological progress are standard values. The probability of a firm destruction shock is set to 0.058% to give an expected firm life duration of 17.25 years, which is taken from a study about firm survival and hazard rates for Germany by Fritsch et al. (2006). This number is in line with the age distribution of firms in Germany provided in Wagner (2005). The exogenous job separation probability of young workers is set to  $q_y = 0.05$ . This value is taken to match the separation rate of workers given in Zimmermann (1998). This study

<sup>6</sup>Section 6 explains further how the simulation of demographic change is carried out and how the change of the average retirement age is implemented in the simulation.

	2003	2010	2015	2020	2030	2040	2050
Average Exit Age	60.1	61.8	63.3	63.5	62.1	62.6	62.6
$\lambda_o$	0.164	0.128	0.108	0.106	0.124	0.116	0.116

Table 1: Expected retirement age and corresponding  $\lambda_o$  (Source: Carone (2005); OECD (2011), Own calculations)

Parameter	Value	Target
$r$	0.04	annual interest rate
$g$	0.02	annual rate of technological progress
$\delta$	0.058	expected firm life duration (Fritsch et al., 2006)
$q_y$	0.05	job mobility (Zimmermann, 1998)
$q_o$	0.0	job mobility (Zimmermann, 1998)
$c_E$	19.27	capital share
$c_N$	$\frac{0.7}{12}\bar{w}$	average recruitment costs (Muehlemann and Pfeifer, 2012)
$c_T$	0.4	resources for innovation (Aschhoff et al., 2005)
$\beta$	0.8	share of trained workers per period $\approx 12.7\%$ (Eurostat, 2013)
$B$	2	firm productivity dispersion (Pfeifer and Wagner, 2012)

Table 2: Calibration to German economy

distinguishes between intra-firm and inter-firm job-to-job transitions. For this paper, only the latter are of interest, yielding an annual transition probability of 4.36% for workers aged 20–55. Adding the average EU-transition rate, the annual firm-separation probability amounts to 10.5%. This gives an average job duration of 9.6 years, which is in line with the reported numbers in Bergemann and Mertens (2004) and OECD (2012). From the age of 55, the EE-transition rates go down remarkably and inter-firm transitions become nearly zero. Given the firm destruction job and the high exit probability of old workers, the quit probability is set to  $q_o = 0$ .

The recruitment cost for new workers is set to 70% of the monthly average wage, which is taken from a recent study for skilled workers in Germany by Muehlemann and Pfeifer (2012).<sup>7</sup> The entry cost  $c_E$  is calibrated to yield a capital share of 30% in the model.<sup>8</sup> The three technology parameters  $c_T$ ,  $B$  and  $\beta$  are chosen to match the total resources used for introduction of new technology as a share of firms' turnover, taken from Aschhoff et al. (2005), the productivity dispersion of firms given in Pfeifer and Wagner (2012), and the share of workers that receive training per year, taken from Eurostat (2013). The calibration procedure for the entry cost and the technology-cost parameters is explained in more detail in the appendix.

<sup>7</sup>The recruitment cost is in line with Chen and Funke (2003) and Bentolila and Bertola (1990) for Germany. It is also similar to the calibration in Mortensen and Pissarides (1999), which is supported by survey results in Hamermesh (1996).

<sup>8</sup>The production function does not include variable capital. Capital is only needed to set up the firm.

The production function takes the form

$$F(y, o) = (y + o)^\alpha,$$

where  $\alpha = 0.71$  is calibrated to give an average firm size of 12.5 employees (OECD, 2010).

## 5 Stationary Equilibrium

This section provides an overview of the firms' technology decisions and technology diffusion in a stationary equilibrium without changes in the age structure of the labor force. Figure 3 shows the firms' equilibrium technology decision with respect to their workforce and current technology. The graphic depicts the distance from the technological frontier at which firms update their technology:

$$k^*(y, o) = \min(k | X(y, o, k) \in \{0, B\}) \quad \forall y, o \geq 0.$$

It turns out that the distance from the technological frontier at which a firm decides to update its technology depends primarily on the age structure of a firm's workforce and to a lower extent on the firm's size. Adding old workers strongly increases the distance to the technological frontier at which a firm decides to update. Adding young workers, on the other hand, does not increase the updating distance, except for very small firms. For heterogeneous firms, increasing the number of young workers can even lower the updating distance as the average age of the workforce in the firm becomes lower. The reason for this is that firms with old workers prefer to delay training their workers for a new technology because they expect them to retire soon, making the investment unprofitable. A higher number of old workers increases the updating distance irrespective of the age structure of the firm. This happens because firms with many old workers wait with technology updating to give old workers a chance to drop out of the labor market first. If these firms finally update, they lay-off some of their old workers in the process, as it is unprofitable to invest training cost for all of them.

For very small firms, the distance from the technological frontier at which they decide to upgrade becomes dramatically smaller. This is caused by the fact that these firms want to increase their workforce to the optimal level. When a firm hires new workers, it has to invest in training cost for the new hires. However, if a firm has to pay training cost for the new hires anyway, it prefers to train them for the newest technology and train its few already existing employees as well, instead of training the new hires for the vintage technology that the firm currently uses and having to train them again some periods later when the firm finally updates its technology. So, hiring new workers complements technology renewal and the smaller a firm is, the greater are the incentives to hire new workers and to update the firm's technology at the same time.

Firms do not only differ in their distance to the technological frontier at which they decide to update but also choose different technologies when updating, depending on the age structure of their workforce. Figure 4 illustrates which kind of firms choose to update to the newest

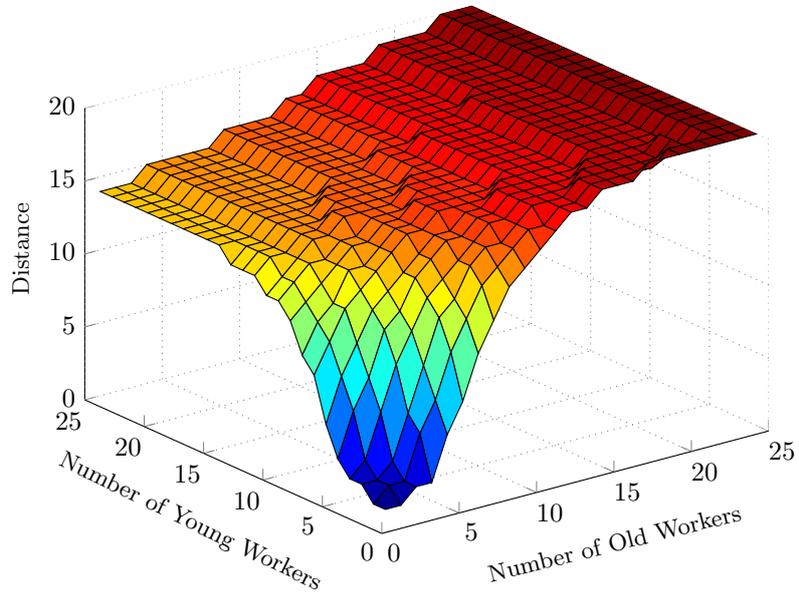


Figure 3: Distance from the technological frontier at updating

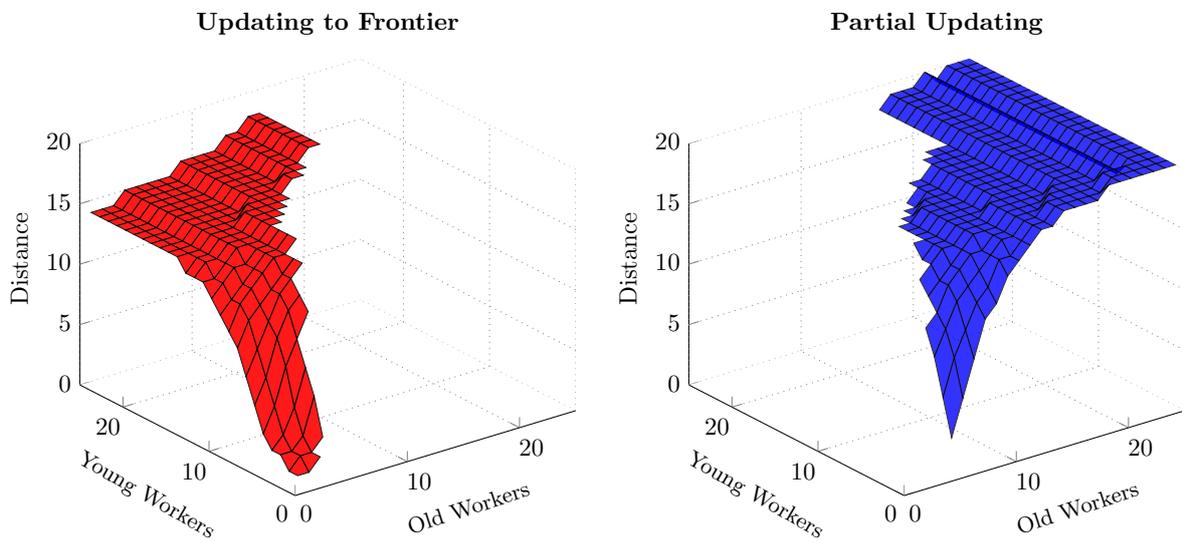


Figure 4: Technology choices of updating firms

technology  $A_{(0)}$  and which firms prefer to update only to the non-state-of-the-art technology  $A_{(B)}$  at a lower updating cost. As expected, firms with a larger share of old workers prefer to upgrade to the older technology to reduce investments in their elderly workers who may otherwise retire before the training cost for the high technology is recovered. An exception are very small firms that are close to the technological frontier. These firms that use an in-between technology  $A^i \in (A_{(0)}, A_{(B)})$  update to the highest level in order to hire new workers in the process even if they have only old workers. Nevertheless, if such small old-worker firms are further away from the frontier, they would update to the lagged technology instead. However, as the graphic shows only the first time a firm updates for a given workforce, this is not depicted in the figure.<sup>9</sup>

With regard to the aggregate level, the distribution of firms over technologies in the economy given in Figure 5 shows that firms with an older-than-average workforce lag further behind the technological frontier. This replicates the results of the empirical studies at the firm level. In addition, another well-known empirical result in terms of technology utilization by firms is evident in the firm distribution. Firms that are larger than the average use newer technologies than small firms. This may come as a surprise as the analysis of the optimal firm policy above indicated, that small firms updated their technology earlier than large firms. However, firms that update use this opportunity to hire new workers and hence firms that use the newest technology always have the largest workforce.

### Comparative Statics

In this part I analyze how firm policies and the firm distribution of the model in steady state are affected by parameter changes. The results provided here do not only apply for the calibrated version, but seem to be fairly general as the equilibrium variables behave monotonically when confronted with different parameters.

An increase in the training cost for workers  $c_T$  enlarges the distance to the technological frontier at which firms decide to update their technology. This is true for all types of firms, however,

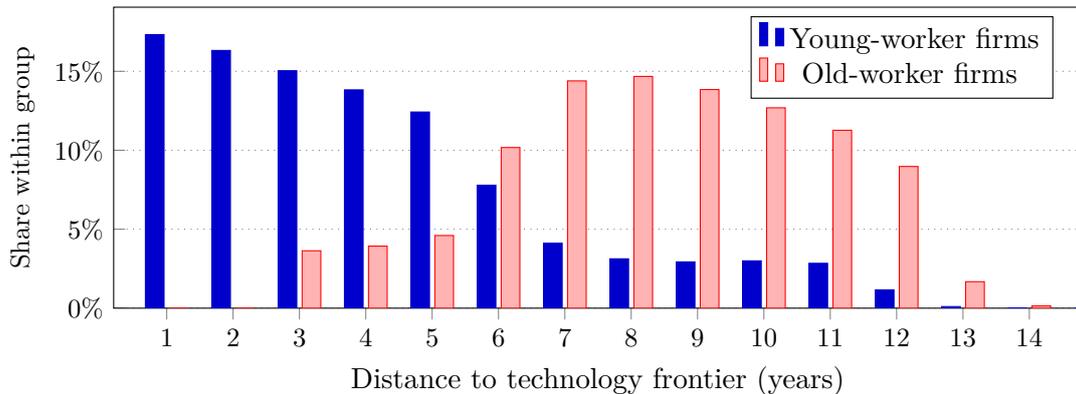


Figure 5: Technology distribution for young and old worker firms

<sup>9</sup>Of the two entrant types, those that choose the high technology mainly employ young workers whereas the entrants that start with the lower technology hire mainly old workers. This implies that old-worker firms are never close to the technological frontier.

the effect is stronger for firms with an older workforce. For smaller training costs, the updating policies converge until all firms update their technology in every period when  $c_T$  becomes very low and all firms produce at the technological frontier. Increasing the difference  $B$  between the newest technology and the non-state-of-the-art technology to which firms can update increases the distance from the technological frontier of old-worker firms and expands the dispersion of productivity in the economy. The interplay of  $c_T$ ,  $B$  and  $\beta$  together determines the share of firms that update in each period, the average distance to the technological frontier and the technology dispersion in the economy.

Increasing the exogenous rate of technological progress  $g$  reduces the distance to the technological frontier at which firms decide to update their technology since the gains from updating rise and the pressure from increasing wages is higher. Raising the entry cost of firms  $c_E$  on the other hand has little effect on the technology decision and mainly lowers wages. The hiring cost  $c_N$  reduces the firm's employment adjustment capabilities and thus leads firms with old workers to postpone the updating process, however, the effect is not very strong.

An increase of the expected worklife duration of an old worker, that is a lower  $\lambda_o$ , has two opposing effects. At the firm level, it reduces the distance to the technological frontier at which firms with older workers update. On the other hand, a lower  $\lambda_o$  increases the share of old workers in the economy. At the aggregate level, this effect moves the entire economy away from the technological frontier as more firms with old workers exist, which update later than firms that employ a younger workforce. This effect is illustrated in Figure 6. For the calibrated example, the second effect is stronger, so that an increase in the average retirement age increases the economy's average distance from the technological frontier, however, no general statement can be made here.

Changes in the exogenous separation probabilities  $q_y, q_o$  affect young workers stronger than old workers. As the expected worklife of old workers is short in any way because of their upcoming retirement, an additional increase of the separation probability does not have a great effect. For young workers on the other hand, who have a long worklife horizon, an increase in the separation probability reduces their profitability for firms and the distance at which young-

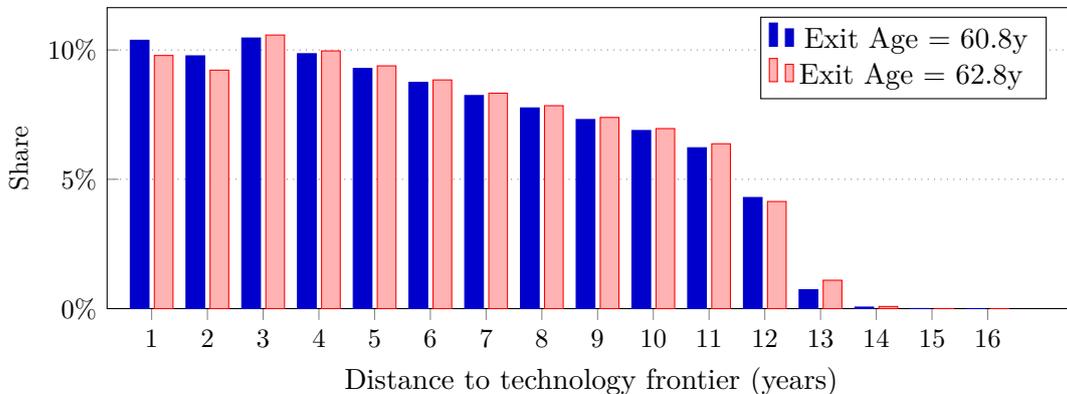


Figure 6: Technology distribution for different average retirement ages

worker firms decide to update their technology increases. If  $q_y$  becomes very large compared to  $q_o$ , the expected job duration of young workers would become shorter than that of old workers. This extreme case would reverse most results with regard to the updating decision presented before. In this case, firms with young workers would delay technology updating, since they expect their workers to leave the firm soon anyway.

Empirical studies regarding worker mobility typically find that separation rates decline with worker age. The reason for this is that young workers that entered the labor market switch their jobs often as they move to better and better jobs. Over time, this matching process slows down and workers become settled in their job. For Germany, Zimmermann (1998) finds that the EE-transition rate of workers aged 15–25 is nearly double that of workers aged 25–55 and decreases further for workers aged 55 and above. Similar results can be found for other economies: for US employees, Menzio et al. (2012) estimate a monthly job-to-job transition rate of 5% for workers aged 18, which declines dramatically until the age of 35. At this age, the estimated job-to-job transition rate is 1.8% and the further decline is only marginal. Similar results can be found in Marotzke (2014). Nevertheless, Zimmermann (1998) shows that a high share of the job-to-job transitions take place within the firm as workers are appointed to better jobs. For Germany, these intra-firm transitions make up for 60–70% of all EE transitions whereas firm separation make up only the smaller part.

These results imply, that middle-aged workers have the lowest separation rate whereas old workers and very young workers have a shorter worklife horizon. This can explain the empirical findings by Feyrer (2007); Werding (2008) and Schneider (2008), which show that the share of workers aged 35–50 has a positive influence on growth and the innovative activity within the firm. Workers aged above 50 years have a negative influence on growth, but this is also true for very young workers. The study on training in Germany by Kuwan et al. (2006) also shows that the group of employees aged 40–44 received the most on-the-job training of all age categories, whereas the group aged 60–64 received the lowest amount of training.

## 6 Simulation of Demographic Change

In the calibrated model, it turns out, that firm policies are independent of the share of young and old workers in the economy.<sup>10</sup> This allows to use the steady state firm policies for the simulation of demographic change. The simulation is undertaken by deriving the steady-state firm distribution of the economy and then adjusting the number of new workers that enter the economy in the simulation such that the labor force age composition follows exactly the projected development in Germany as depicted in Figure 2. From 2015 on, the new  $\lambda_{o,2015}$  as well as the new firm policies are used, while the simulation of demographic change in the economy is carried on. That is, I continue the simulation with the firm distribution as it has developed until 2015 given the initial firm policies and  $\lambda_{o,2003}$ . The firm distribution in the economy then

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<sup>10</sup>The calibrated model features two entrant types with different employment policies. Therefore, firm policies are independent of the age structure of the labor force (see section 5).

adjusts slowly, given the new firm policies and the ongoing change in the age-structure of the labor force under demographic change.

In the simulation I focus on the effect of demographic change on the economy's average distance from the technological frontier and on the resulting productivity growth. The distance from the technological frontier is measured as the average relative productivity lag, that is the actual output of the simulated economy is compared to an economy where all firms produce at the technological frontier. Denote the actual output of the economy, which is given by the accumulated gross output of all firms, by  $y$  and the counterfactual output of the economy where all firms use the newest technology by  $\tilde{y}$ . Then the distance  $D$  is defined by

$$D = \frac{\tilde{y} - y}{\tilde{y}} \quad (14)$$

Figure 7 illustrates how the projected demographic change affects the economy's average distance from the technological frontier. The solid line depicts the impact of demographic change alone for a constant average retirement age of 60.8 years, leaving out the increase of the average retirement age. The curve follows the pattern of demographic change given in Figure 2. As the share of old workers in the labor force increases, the economy moves away from the technological frontier. This happens because the number of firms that employ old workers and update their technologies less often increases which skews the technology distribution away from the frontier. Between 2010–2025, when the magnitude of demographic change is greatest, the economy's relative productivity gap increases by about 1.6 percentage points.

The dashed line takes the increase in the average retirement age into account. In 2015, the higher retirement age of 62.8 years together with the new firm policies is plugged into the simulation. It turns out, that the increase of the retirement age, which results in an additional increase in the share of old workers in the economy (see Figure 2), reinforces the negative effect of demographic change and the movement of the economy away from the technological frontier is further amplified. Now the relative productivity gap increases by about 2.5 percentage points between 2010–2025. This implies that the positive effect of a higher retirement age at the micro level, which induces firms with old workers to update their technology more often is smaller here than the negative effect at the macro level, a higher share of old workers.

The economy's movement away from the technological frontier implies lower productivity growth during that period.<sup>11</sup> This is illustrated in Figure 8 where the deviation of the realized productivity growth from the long-run trend is plotted. As indicated above, the effect of demographic change alone, depicted by the solid line, is much less pronounced than the case with the actual projected retirement age, illustrated by the dashed line. It turns out that demographic change has a strong negative impact on realized productivity growth. As the share of old workers in the economy increases, realized productivity growth decreases with a negative peak in 2017 where productivity growth is about 0.2 percentage points below the long-run trend for the case with

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<sup>11</sup>As long as the distance to the technological frontier remains constant, productivity is growing at the exogenous rate. If the relative productivity lag increases, growth is lower than the exogenous productivity growth and vice versa.

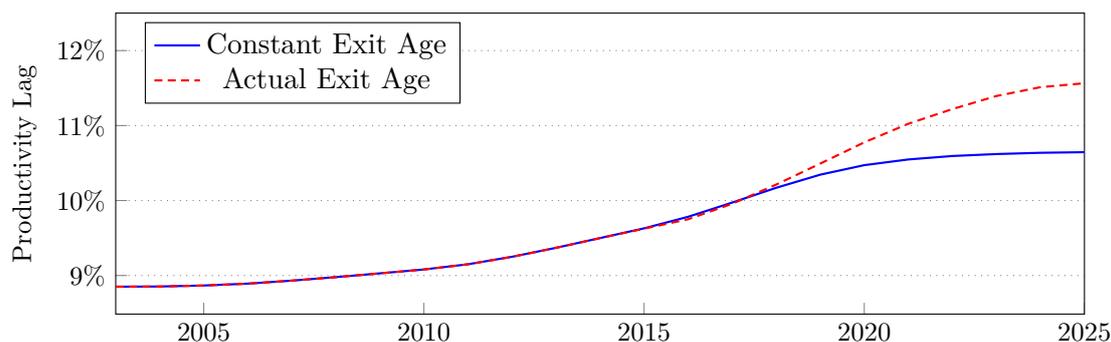


Figure 7: Effect of demographic change on the economy’s distance from the technological frontier

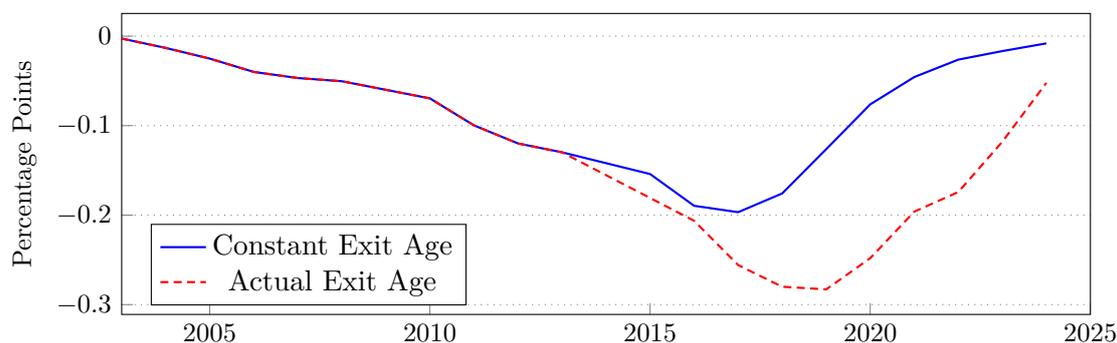


Figure 8: Productivity growth: Deviation from the trend

constant retirement age. When the projected simultaneous increase of the average retirement age is taken into account as well, this loss rises to nearly 0.3 percentage points. As demographic change slows down, productivity growth returns to its long-run trend.

Between 2010–2025, the average rate of realized productivity growth is 0.17 percentage points below the long-run trend when the projected change in the average retirement age is accounted for and 0.11 percentage points when the average retirement age of 2003–2010 is held constant. These numbers translate into a GDP loss of about 416bn Euro in constant prices of 2005 for the case of constant retirement age and 550bn Euro with the actual changes in the retirement age.<sup>12</sup> The loss of GDP under the actual retirement regime is larger than for the case with constant average retirement age for two reasons. First, the loss of productivity growth due to demographic change is higher. Second, as the average retirement age increases, the total labor force of the economy increases, which increases the economy’s output. Consequently, a given loss of productivity growth translates into a higher value of lost GDP.

As a benchmark, the quantitative results of the simulation can be compared to the results in Werding (2008) who computes forecasts for productivity and output growth for various OECD countries based on regression estimates. The evolution of productivity growth in his forecast for Germany for the same period is very similar to the results presented here, only the magnitude

<sup>12</sup>The loss of GDP is derived by accumulating the German GDP of 2010 over the period 2010–2025 given the growth rate of the simulated economy and comparing it to the cumulated GDP that grows at the exogenous rate of technological progress.

of the effect is higher. For the period 2010–2025, Werding’s estimates indicate an average loss of productivity growth of 0.4 percentage points. So the order of magnitude of the simulation results is similar although the level is a bit lower.

### Effect of an Additional Increase of the Retirement Age

As a policy experiment, the effect of an additional increase in the average retirement age by three years is simulated. Such an increase can be achieved by raising the statutory retirement age as it is done in Germany and many other European countries at the very moment, or by reducing the number of people who drop out of the labor force early. For the simulation, the three additional years are added to the average retirement age that is projected for 2015 onward, assuming that the increase in the retirement age is unexpected by the firms. This gives an average retirement age of 65.8 years.

Figure 9 illustrates how demographic change affects the economy with a higher exit age. As before, it turns out that the overall effect is negative, moving the economy even further away from the technological frontier. At the peak of demographic change in 2025, the economy’s relative productivity lag with the experimental retirement age is about 0.7 percentage points higher compared to the case with the actual projected retirement age. This shows that the additional increase of the average retirement age has a very strong negative effect, resulting in a growth reduction of about 0.43 percentage points in 2017. For the period 2010–2025, the average annual growth loss amounts to 0.21 percentage points, compared to 0.17 percentage points for the actual projected average retirement age.

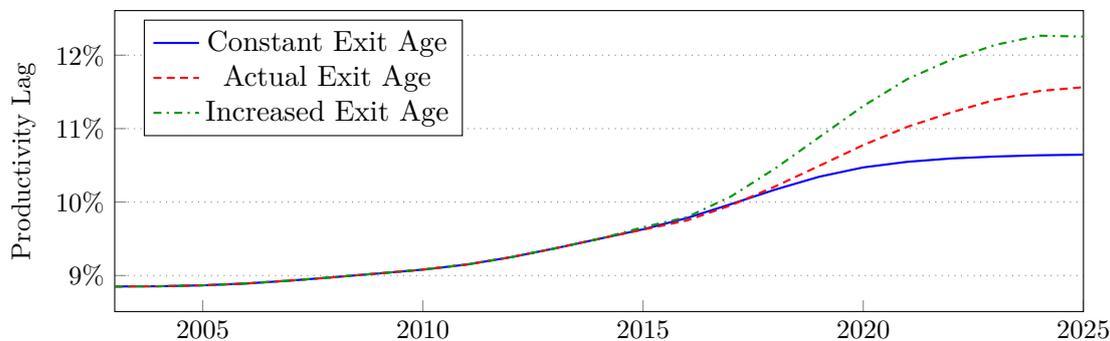


Figure 9: Demographic change and increased retirement age

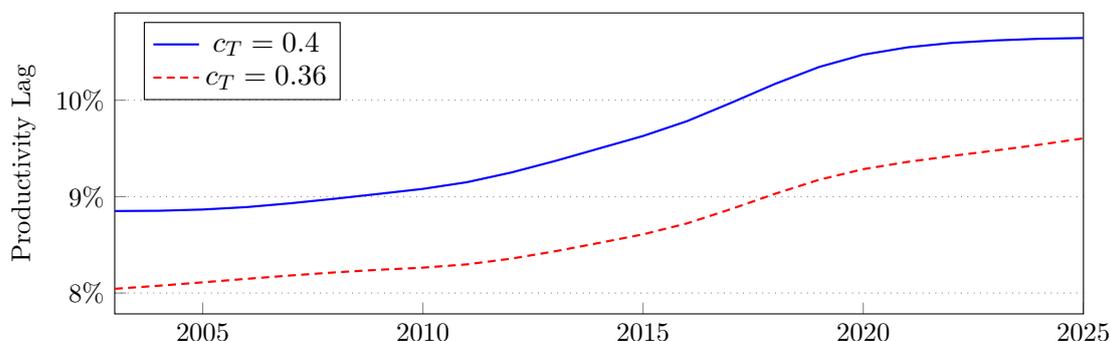


Figure 10: Demographic change and lower updating cost (constant retirement age)

### Effect of a Lower Technology Updating Cost

As a second experiment, I analyze how the economy is affected by demographic change for a lower training cost, that is updating to a new technology becomes cheaper due to better training methods. Figure 10 illustrates how the distance of the technological frontier changes, when the training cost is reduced by 10%. For both lines, the constant average retirement age of 60.8 years is used. It can be seen, that the lower updating cost moves the economy in general closer to the technological frontier as all firms tend to update more often. Furthermore, it turns out that the negative effect of demographic change is slightly reduced. As labor force aging sets in, the economy moves away from the frontier, however, the magnitude of this result is lower than for the original training cost. This is in line with the comparative static results in Section 5, which showed that firms with older workers are more sensitive towards the updating cost. Over the period 2010–2025, the average annual growth loss now amounts to 0.09 percentage points compared to 0.11 percentage points before. This implies that the growth loss during that time is reduced by about 18%.

## 7 Robustness Tests

As a first robustness check, I test whether the model is consistent in the way that the choice of the cutoff value that separates young from old workers does not affect the model's aggregate results in the steady state. For the standard calibration with early retirement, workers are young for 35 years and old for 6.8 years. I reduce the duration of the young period and add the time to the old period. Consequently, the total worklife duration remains unchanged. This implies that the total worklife horizon of young workers does not change. As a result, the updating policies of firms that employ only young workers should remain unchanged but firms that employ old workers should update their technology in a shorter time interval. Nevertheless, the economy's distance from the technological frontier for the static case (that is without demographic change) should remain the same, as the earlier updating of firms with old workers is balanced by the higher share of old workers and the economy's fundamentals remain the same.

Furthermore, total resources used for technology updating in terms of turnover should remain unchanged. The wage of young workers should remain constant as their expected worklife time does not change whereas the wage of old workers should increase, so the wage differential decreases as well. The mean wage of the economy on the other hand, together with hiring and entry cost should remain unchanged.

Figure 11 illustrates the technology updating decision of firms for different cutoff values for young and old workers. As expected, there is basically no change for firms that employ only young workers, because their expected total worklife remains unchanged. On the other hand, firms that employ old workers reduce their updating distance as the expected worklife duration of old workers increases. The longer the expected worklife duration of old workers becomes, the flatter the updating profile of the firms becomes.

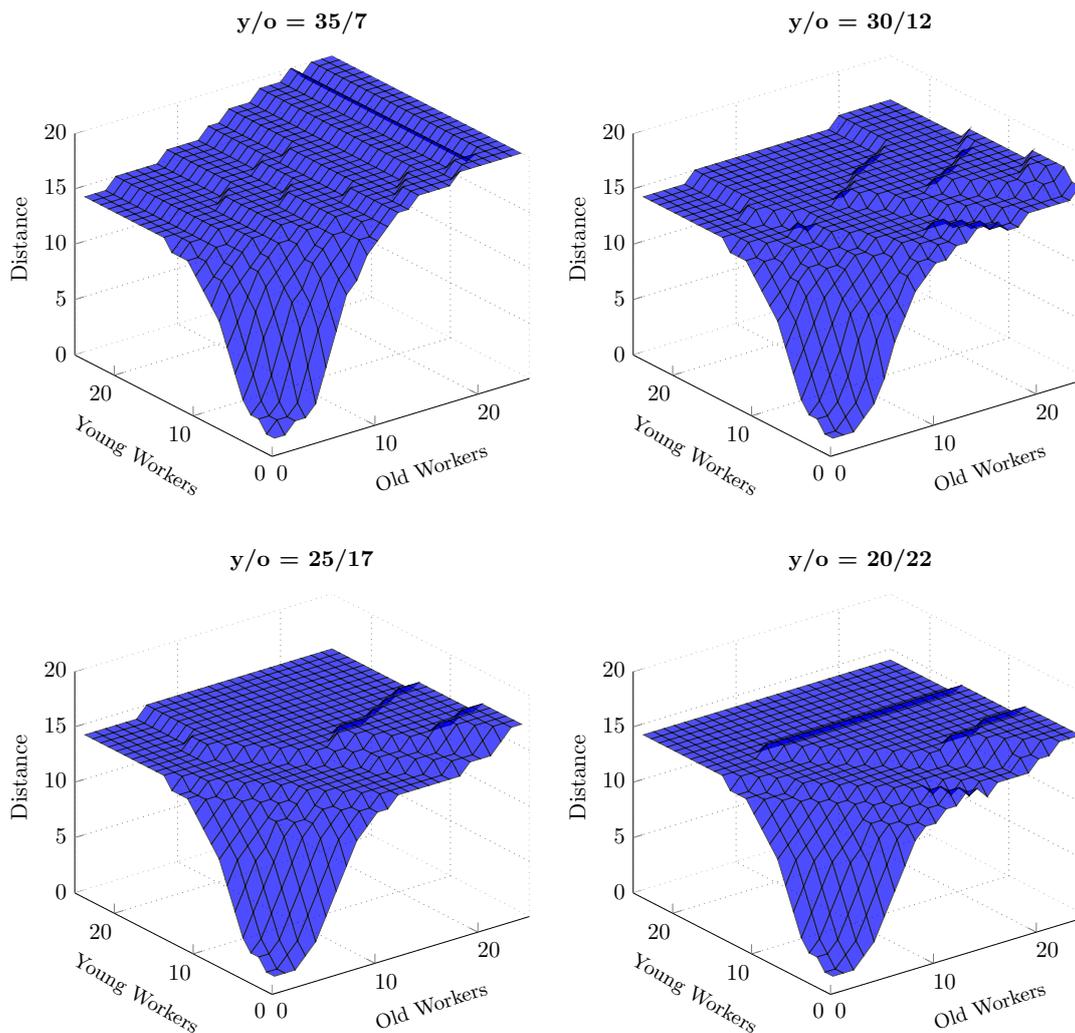


Figure 11: Updating distance for different cutoff values

Nevertheless, the aggregate variables of the economy remain unchanged, which is illustrated in Table 3. Even though the individual technology decisions of firms change, the distance of the economy from the technological frontier and resources used for innovation remain the same. Wages for young workers also remain constant whereas wages for old workers increase, as the increased expected worklife duration makes them more valuable for firms. However, the mean wage and therefore total wage payments remain constant. These results imply that also the economy's total output is unchanged.

As another robustness check, I test how sensitive the results are toward a change in the hiring cost and the entry cost. For this, I calibrate versions of the model with a 50% lower hiring cost and a 50% lower entry cost. Figure 12 shows the results. It turns out that the impact of each of these changes is very low with a small reduction of the growth loss over the period.

As a final robustness test, I analyze how the model would develop given that exogenous productivity growth would be higher by 50%, so  $g = 3\%$ . With higher exogenous growth, firms do update their technologies at a shorter interval for two reasons. First, the higher growth

	Duration Young / Duration Old			
	35/7	30/12	25/17	20/22
$w_y$	0.221	0.219	0.218	0.218
$w_o$	0.136	0.180	0.195	0.200
$\bar{w}$	0.21	0.21	0.21	0.21
Distance from Frontier	11.3%	11.2%	11.2%	11.3%
Resources for Updating	0.024	0.024	0.025	0.025

Table 3: Change of the cutoff value for young and old workers

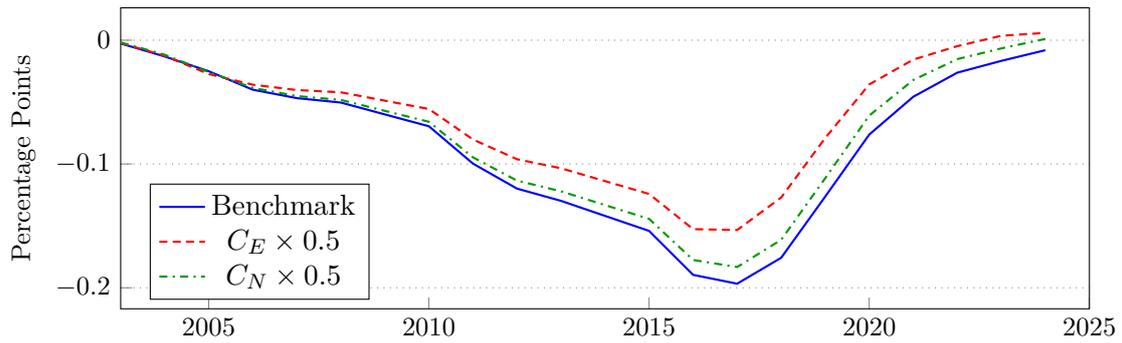


Figure 12: Demographic change with lower hiring and entry cost

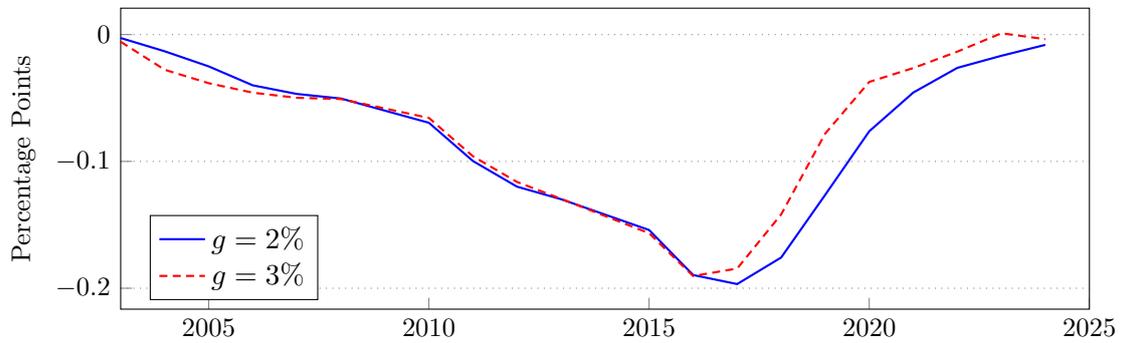


Figure 13: Productivity growth with higher exogenous productivity growth

rate makes updating more profitable, as it implies a larger productivity gain. Second, as wages increase at the rate of technological progress, firms are forced to update their technologies more often, as workers become otherwise too expensive, given their output with the current technology. The simulation results are provided in Figure 13 whereby the constant retirement age scenario is used. It turns out that the absolute loss of productivity growth in percentage points is very similar for the two rates of exogenous productivity growth. However, the relative loss of productivity growth is lower by a third for the high growth scenario.

## 8 Conclusions

Demographic change in the industrialized countries during the first half of the 21st century leads to a steep increase in the share of elderly persons in the labor force. In this paper, I develop a quantitative dynamic model that analyzes firms' technology decisions with respect to the age of their workforce and allows to determine the effect of labor force aging on the economy's technology distribution and productivity growth.

I calibrate the model to match the German economy and simulate the projected changes in the labor force age composition for the period 2003–2025. The results show that labor force aging increases the average relative productivity lag and thereby lowers aggregate productivity growth. Over the period 2010–2025, demographic change lowers the average annual rate of realized productivity growth by about 0.11 percentage points below the long-run trend. The increase in the average retirement age by about 2 years during that time further increases the negative effect, leading to an average annual growth loss of 0.17 percentage points between 2010–2025. A comparison of the simulation results to other studies indicates that the model results are in a plausible range.

For future work, the model could be applied to other countries as well to quantify the effect of demographic change on productivity growth. One problem that may arise when doing so, is that in countries with very high job mobility, the expected job duration of young and middle-aged workers could possibly be lower than that of elderly employees. Nevertheless, it is very well possible, that similar to Germany, a large share of job-to-job transitions take place within the firm, so that even with short job durations, worker-firm relationships are more stable.

Another possibly fruitful extension of the model is to use three age groups for workers. With this, the very high job-to-job transitions rates of young workers that have just entered the labor market could be taken into account explicitly. Middle-aged workers would then have the longest expected worker-firm relationship whereas it will be shorter for very young and very old workers. With this extension, the model could possibly create the hump-shaped relationship between workforce age and innovative activities of firms that have been found in the data.

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# Appendix

## Numerical Solution Procedure

The numerical solution of the stationary equilibrium is split into two steps: the derivation of firm policies and wages, and the simulation of the stable firm distribution. As explained below, depending on parameters both steps are repeated multiple times until the stationary equilibrium is found.

The first part, the derivation of firm policies and wages is an iterative procedure. First, for given wages  $w_y, w_o$ , firm policies are derived by value function iteration. Then the free entry conditions (8) and (9) have to be checked in order to adapt the wages. As pointed out in Section 3, there are two possibilities for firm entry in equilibrium: either two entrant types exist and both free entry conditions are binding for a certain pair  $(y^H, o^H)$  of hired workers, or only a single entrant type exist, i.e. only one of the free entry conditions is binding, the other is strictly negative for all hiring possibilities. If two entrant types exist and these entrant types hire workforces with different age structures, the two labor markets can be cleared by adjustment of the entering firms, resulting in a block recursive equilibrium in which firm policies do not depend on the distribution of workers and firms in the economy. The wages for young and old workers are adapted until both free entry conditions are binding. For every change in the wages, firm policies have to be derived anew until the equilibrium is found. Once the wages have been found, the firm distribution can be simulated. This is done by populating the economy with a constant flow of young workers in every period and allow firms to enter that hire these workers. This simulation runs until the firm distribution, represented by the measure  $\mu(y, o, k)$  has become stationary.

If it is not possible for both free entry conditions to be binding, then only one entrant type exist. In this case the wages for young and old workers have to be adapted to have one of the free entry conditions binding and to clear both labor markets simultaneously by the single entrant type while the other free entry condition is strictly negative. The single entrant type must hire exactly the ratio of young and old workers that becomes unemployed in a period and is not directly hired by existing firms in equilibrium. To find this solution, the firm distribution is simulated every time a new pair of wages is chosen and policy functions are derived and it is checked whether labor markets are cleared in equilibrium. In the case of a single entrant type, wages and firm policies are not independent of the share of young and old workers in the economy. This implies that a change in the relation of young and old workers (by demographic change) demands for a different hiring policy of entrants and different wages.

## Calibration of Entry Cost $c_E$ :

The model features no variable capital that is needed for production, hence capital appears only indirectly in the fixed cost for firm creation  $c_E$ . Therefore, the entry cost is interpreted as the capital share in the economy, which is set to 30%. The labor share is given by the total amount of wages that a firm expects to pay in its lifetime, calculated as present value at the time of firm entry. With a survival probability of  $(1 - \delta)$  for a firm, an average workforce of 12.5 workers,

and the average wage in the economy given by  $\bar{w} = \frac{\lambda_o w_y + \lambda_y w_o}{\lambda_y + \lambda_o}$ , the free entry cost is given by:

$$c_E = \frac{0.3}{0.7} \cdot 12.5 \cdot \sum_{t=0}^{\infty} (1 - \delta)^t \left( \frac{1 + g}{1 + r} \right)^t \cdot \bar{w}.$$

**Calibration of Technology Parameters:  $c_T$ ,  $B$ ,  $\beta$ :**

The training cost is derived by calibrating  $c_T$  to match total resources for innovative activities as a share of total turnover of German firms, which equals 2.93% for the period 2002–2004 as collected in the German Innovation Survey 2005 by the Centre for European Economic Research (ZEW), based on the harmonized methodology of the Fourth Community Innovation Survey (CIS IV) of the European Union. (Aschhoff et al., 2005) The survey comprises more than 100,000 enterprises and covers all kind of innovative activities that lead to the adoption of new technologies or processes and the introduction of new products. For this definition it does not matter, if the introduced technology is novel to the market or already established at other enterprises, it must only be new for the adopting firm.

As  $B$  defines the lag between the newest technology and the non-state-of-the-art technology that is mainly chosen by old-worker firms, it increases the technology spread over the firms and thus increases the productivity dispersion among firms in the economy. As a target for the productivity dispersion, data from Pfeifer and Wagner (2012) is used, who calculate a normalized average standard deviation of labor productivity over firms within industries over the period 2003–2006 of 0.21, which is taken as target for productivity dispersion in the model. In interplay with the other parameters,  $\beta$  determines the total updating frequency or the share of workers receiving training in each period respectively for a given average lag of the economy and a given productivity dispersion. As a target for  $\beta$ , I use data on the share of workers in the labor force that received on-the-job training over the duration of one year which is provided in Eurostat (2013) and gives an average of 12.7% for 2003.