Job Search and the Age-Inequality Profile

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Abstract

In line with earlier literature, I document a U-shaped relationship between age and wage dispersion in the U.S.. To explain this outcome, I consider a life-cycle model of labor market search with strategic wage bargaining, heterogeneous firm-worker matches, and endogenous search effort. Three factors shape the age-inequality profile of wages in the model economy: the time until retirement, match heterogeneity, and the workers’ bargaining power. Young workers switch employers often and are gradually matched to better jobs, which leads to the initial reduction in the variance of log wages. Middle-aged and older workers switch employers less frequently and have a longer search history. As workers are differently successful in the labor market, the variance of match productivities rises in the second half of the working life. The calibrated model captures the U-shape of the age-inequality profile of wages in conjunction with the hump-shaped age profile of average wages, as well as employment-to-employment transitions that decrease with age.

JEL classification: J31; J41; J64

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1 Introduction

The objective of this paper is to explore whether job search is a driving force behind wage dispersion over the life cycle. Understanding the sources of lifetime wage inequality is necessary for the design of welfare policies and insurance programs. Furthermore, the age structure of the population might be an important factor behind differences in income inequality between countries or changes in the wage structure across time. Search frictions and on-the-job search are potentially important determinants of residual wage dispersion (Burdett and Judd, 1983; Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002; Burdett and Coles, 2003; Hornstein et al., 2011). Topel and Ward (1992) find that about one third of wage growth among young workers can be attributed to employment-to-employment transitions.

In line with several studies (Mincer, 1974; Dooley and Gottschalk, 1984; Heckman et al., 2003), I find that the variance of residual log wages across workers in the U.S. follows a U-shape with age. The variance is high for young workers who have just entered the labor market. As workers grow older, it falls at first and starts to increase again in the second half of the working life. Polachek (2003) explores the variance of log wages for nine other countries\(^1\) and finds the U-shaped relationship between the variance and age for most of them.\(^2\)

In order to explore the age-inequality profile in conjunction with on-the-job search, I develop a life-cycle model of labor market search. Wages are determined by bargaining and workers’ search intensity is endogenous. Firm-worker matches have different productivities, workers search on and off the job, and incumbent employers can counter outside wage offers. Searching for a job is costly. Older workers choose to search less than young workers as the value of a job is lower when the time horizon until retirement shortens. I calibrate the model to U.S. panel data and show that the calibrated model captures the U-shaped age-inequality profile of wages in conjunction with the observed hump-shaped age profile of average wages and the employment-to-employment transition rate which decreases with age.

A large fraction of the overall wage inequality in the model is driven by match heterogeneity. Indeed, the endogenous age-variance profile of match qualities is U-shaped. Search on and off the job leads to two opposing effects on wage dispersion. Because of employment-to-employment transitions, workers are gradually matched to better jobs and this decreases the variance of match productivities. Because of unemployment-to-employment transitions, there is a permanent flow of workers into the lower tail of the productivity distribution. The first effect

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\(^1\)The nine countries considered are Australia, Belgium, Canada, Czech Republic, France, Mexico, Taiwan, Spain, and Sweden.

\(^2\)Deaton and Paxson (1994), Storesletten et al. (2004), and Huggett et al. (2011) find that wage dispersion rises with age.
is dominant for young workers who switch employers often. The second effect dominates in the second half of the working life. Older workers have a longer search history. Workers who have obtained many good job offers are employed in high productivity matches. At the same time, some workers who became unemployed again have to accept low productivity matches. Older workers sort themselves more slowly into better matches as the optimal search effort decreases with age.

The wage formation mechanism is based on the strategic wage bargaining model of Cahuc et al. (2006). The model provides rich wage dynamics and allows for wage rises within an employment while remaining solvable. In contrast to the wage-posting model of Burdett and Mortensen (1998), firms make wage offers that depend on worker characteristics. Furthermore, firms can counter the outside offers of their workers. In Cahuc et al. (2006), a high match quality offers workers more opportunities to obtain wage rises because of possible outside job offers. This option value effect lowers the wage that workers are willing to accept. There are additional implications in the present model with a finite time horizon. The shorter the remaining time horizon before retirement, the lower is the option value of on-the-job search. Hence, workers who accepted a low starting wage when young might have a credible threat to quit into unemployment when growing older as their option value of on-the-job search is lower. In that case, they negotiate wage rises from the current employer without any outside job offer.

The U-shaped age-variance profile of match qualities only translates into a U-shaped age-variance profile of wages if workers’ bargaining power is sufficiently high. If the workers’ bargaining power is too low, the option value effect is very high for young workers and older workers’ reservation wages increase strongly. The standard deviation of wages for workers close to retirement then falls sharply. If the bargaining power of workers is sufficiently high, there is a modest increase in the reservation wage only for low quality matches prior to retirement. Apart from that, the reservation wage decreases for older workers since the probability of obtaining a better job offer by waiting decreases. For the same reason, also the observed hump-shaped age profile of the average wage is better matched if the bargaining power of workers is sufficiently high.

Related models with a finite working life and on-the-job search are Jung and Kuhn (2012) and Menzio et al. (2012). Jung and Kuhn (2012) explore earnings losses after displacement for workers with high tenure in conjunction with worker flows. Menzio et al. (2012) develop a life-cycle model with directed search and human capital accumulation. Their objective is to explain the age profile of worker transitions across employment states, while I focus on the wage distribution for different age groups. The random search model of the present paper features similar life-cycle profiles of transitions from unemployment to employment and between
employers as Menzio et al. (2012). While their channel is directed search, the channel in the present model is endogenous search intensity. The main advantage of a directed search model is its solvability not only in steady state but also when the economy is not in steady state (Menzio and Shi, 2011). Here, I only consider the steady state, which is tractable since one can use the value of retirement as a terminal condition.

Other authors have explored the effects of a finite working life on labor market outcomes within search-theoretic models in which workers can only search when unemployed (Hairault et al., 2010; Hahn, 2009; De la Croix et al., 2009; Chéron et al., 2008). These models can explain the hump-shaped age profile of employment, but without additional assumptions, they imply a decreasing age-wage profile. In order to obtain the empirically observed increasing and concave age-wage profile, Hairault et al. (2010) calibrate age-specific wage offer distributions. De la Croix et al. (2009) assume that workers’ productivities increase with age and then decrease as workers approach retirement. Chéron et al. (2008) introduce human capital accumulation into their model.

This paper also relates to Bagger et al. (2011) and Yamaguchi (2010), who also explore the driving forces of wage dynamics over the life cycle in a bargaining model with counteroffers. They focus on the importance of job search and human capital accumulation for individual wage growth in a model with an infinite time horizon, while I focus on the importance of job search and a finite working life for shaping the age-inequality profile of wages.

There are different alternative approaches to the U-shape of the age-inequality profile. Heterogeneous age-tenure profiles are one potential source. Another approach attributes the high residual wage dispersion of young and older workers to investment in human capital accumulation (Mincer, 1974). Rubinstein and Weiss (2006) explore the implications of the human capital investment model and a search model of the labor market for life-cycle wages. They find empirical support for both theories. While they argue that a search model cannot give rise to a U-shaped age-inequality profile, the present paper shows that search theory is sufficient to explain the U-shape. Rubinstein and Weiss (2006) argue that in a search model workers become increasingly heterogeneous at first as they can search on-the-job and are differently successful in finding good job offers. Since workers move up the wage ladder and since the probability of obtaining a higher wage decreases in the current wage, wage dispersion finally falls. In the model developed in the present paper some unemployed older workers accept low wages and then upgrade their wages only slowly. It is shown that this channel is one potentially important reason for the rise in wage dispersion among middle-aged and older workers.

The paper is organized as follows: Section 2 describes the data and derives the empirical age profile of wage inequality to be explained using the model framework set out in section 3. In
section 4, I calibrate the model economy and quantitatively investigate the performance of the model in capturing the age-inequality profile of wages as well as age profiles of transition rates and average wages. Section 5 discusses the mechanisms that shape the age-inequality profile of wages. Section 6 concludes.

2 The empirical age profile of wage inequality

This section discusses the empirical age profile of wage inequality. The finding that the variance of the residuals of a wage regression follows a U-shape with age has its origin in the work of Mincer (1974). Mincer’s log earnings function is estimated by a regression of log earnings on years of experience, years of experience squared, and years of schooling. It has been estimated in several studies interested in the returns to schooling or post-school human capital investment. The theory states that human capital investments mostly take place when workers are young. Workers who invest in human capital on-the-job early in their career earn initially a low wage but have higher wage growth than non-investors. The standard deviation of residual log wages is then the lowest for middle-aged workers when the wage profiles of investors and non-investors cross. This implies a U-shaped age profile of wage inequality but also a negative correlation of the current wage with wage growth for young workers and a positive correlation of the current wage with wage growth for older workers. Rubinstein and Weiss (2006) find a negative correlation of the current wage with wage growth for all age groups, which is a feature of many search models in which better wage offers become less likely when the current wage is already high.

In the next section I develop a model of labor market search to explore the role of search frictions for age-specific wage dispersion more closely. In order to calibrate the model, I use data from the 1996 panel of the U.S. Census’ Survey of Income and Program Participation (SIPP), which spans the time period from December 1995 to February 2000. The SIPP contains monthly data on the worker’s employment status, earnings, weekly hours, primary job, and information on whether the worker has changed the employer. I restrict the analysis to a subsample of non-unionized men between the ages of 18 and 66, whose highest educational attainment is a high school degree, and who do not have any income from self-employment. Furthermore, I do not consider any workers in the armed forces and workers who stop working for school or training reasons. The data set comprises 10,340 individuals and 242,159 observations.

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Residual log wages are derived from a fixed-effects regression of monthly log-wages on occupational dummies, a dummy for disabled workers, regional dummies, a dummy for marital status, and weekly hours. Time fixed effects are included. The estimated model is

$$\ln w_{it} = \alpha_i + \beta X_{it} + \epsilon_{it},$$

where $w_{it}$ is monthly earnings of worker $i$ in period $t$, $\alpha_i$ is the unknown intercept for worker $i$, $\beta$ is a vector of coefficients, $X_{it}$ is a vector of regressors, and $\epsilon_{it}$ is the error term. A description of the regressors and estimation results are presented in Appendix A.

The age-inequality profile is determined by the standard deviations of the residual, $\hat{\epsilon}_{it}$ given age (in years). The residual is given by

$$\hat{\epsilon}_{it} = \ln w_{it} - \ln \hat{w}_{it},$$

where $\ln \hat{w}_{it}$ denotes the prediction of $\alpha_i + \beta X_{it}$. Figure 1 shows that the age-inequality profile is U-shaped. This result is robust to several alternative model specifications. Table 1 contains the standard deviation of residual wages for larger age groups. It is 22 percent higher for young workers aged 18 to 27 than for middle-aged workers aged 38 to 47. The standard deviation for older workers aged 58 to 66 is 27 percent higher than for middle-aged workers.

<table>
<thead>
<tr>
<th>age group</th>
<th>st.d. of wages</th>
<th>number of residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 – 27</td>
<td>0.4835</td>
<td>50,143</td>
</tr>
<tr>
<td>28 – 37</td>
<td>0.4198</td>
<td>53,509</td>
</tr>
<tr>
<td>38 – 47</td>
<td>0.3964</td>
<td>42,987</td>
</tr>
<tr>
<td>48 – 57</td>
<td>0.3971</td>
<td>24,141</td>
</tr>
<tr>
<td>58 – 66</td>
<td>0.5033</td>
<td>10,068</td>
</tr>
</tbody>
</table>

A very similar age-inequality profile is obtained if number of kids, age, age squared, and/or interaction between occupation and age are included, and if weekly hours is excluded. Also if only full-time workers are considered, the age-inequality profile is U-shaped. The corresponding figures are shown in Appendix A.
3 A life-cycle model with on-the-job search

In this section, I develop a basic life-cycle model of labor market search. The labor market is populated by a continuum of competitive firms and a unit mass of risk-neutral workers of different ages \( k = 1, 2, ..., K \). Time is discrete and the economy is in steady-state. Firms produce a unique multipurpose good, maximize profits, and live forever. Each worker lives a finite life of \( K \) periods. In steady state, all workers that leave the labor market at age \( K + 1 \) are replaced by unemployed workers of age 1. Hence, the fraction of the population aged \( k \) is given by \( l \) for all \( k < K \).

Firm-worker matches differ in their productivities denoted as \( a_i \) with \( i = 1, ..., n \) and \( a_{j-1} < a_j, j = 2, ..., n \). The probability that a potential match has productivity \( a_i \) is given by \( p_i \). The cumulative distribution of potential match qualities is denoted by \( P_i \). When a firm and a worker meet, the quality of the potential match is revealed. For convenience, I describe a firm that offers a worker a match of quality \( a_i \) as a type \( i \) firm. Output per period in a firm-worker match does not depend on the worker’s age and equals the marginal productivity of labor \( a_i \). Unemployed workers receive an income flow of \( b_U \). Workers derive utility from consumption and discount future utility at the factor \( \beta \in (0, 1) \). I am interested in the importance of search frictions and a finite planning horizon for life-cycle wage inequality. The model therefore abstracts from experience effects and does not contain accumulation of human capital.
Workers search on and off the job. Searching for a job is costly for the worker. The cost of spending an effort \( e \) on searching is given by a cost function \( c(e) \), with \( c(0) = 0 \). The cost function is increasing and strictly convex. The offer arrival rate per search effort is \( \lambda > 0 \). The search effort is derived endogenously by the worker’s optimizing behavior. The timing of events is as follows. In the beginning of a period, \( g(k, a_i) \) workers aged \( k \) are employed at a match \( a_i \). Each of these firm-worker matches is hit by an exogenous separation shock with probability \( \delta \in [0, 1] \). Workers who become unemployed can immediately search for a new job that starts in the next period. The mass of unemployed workers of age \( k \) is then

\[
    u(k) = l - (1 - \delta) \sum_{j=1}^{n} g(k, a_j).
\]

All workers that enter the labor market are unemployed, hence \( u(1) = l \).

For the quantitative analysis of section 4, I apply a richer model taking into account that the rate at which workers become unemployed is age-dependent and that not all workers enter the labor market at the same age. I further account for age-dependent flows in and out of the labor force. For reasons of clarity I initially abstract from these details.

### 3.1 Wage bargaining

The wage formation rules are based on the bargaining model of Cahuc et al. (2006). If an employed worker obtains an outside wage offer, the incumbent employer can counter the outside offer. Workers and employers have complete information over each other’s type and over the worker’s wage and job offers. Wage contracts specify a wage that can only be renegotiated by mutual agreement. A renegotiation can occur if the worker has a credible threat to quit. Wage cuts within an employment do not take place since the productivity remains constant throughout the duration of the match. Consider a worker of age \( k \) employed at a type \( i \) firm earning wage \( w \). When the worker contacts a type \( h \) firm, the incumbent and the poaching employer compete for the worker. The maximum wage a firm is able to offer equals the match productivity. The worker chooses the firm that offers the highest lifetime utility. The outcome of the bargaining process depends on the productivity of both firms and on the current wage. Three cases can occur. If \( h > i \), the worker switches to the poaching employer since the type \( h \) firm will offer the worker a wage that has a higher value than the highest wage the type \( i \) firm can offer. Note that the wage from the new employer can be smaller than \( w \) as the worker takes into account possible future wage rises. Such a wage cut is possible because of the option value of on-the-job search. An employment within a high productivity match gives the worker a better position for future
wage negotiations. If \( h < i \), the worker stays with the incumbent employer. The worker obtains a wage rise from the incumbent employer if and only if \( i \geq h \geq q(k, w, a_i) \). If \( h \) is smaller than the threshold marginal productivity index \( q(k, w, a_i) \), nothing changes for the worker. Table 2 gives an overview of the bargaining game. \( \phi(k, a_i, a_h) \) denotes the wage that is the outcome of a bargaining game between a type \( i \) firm and a type \( h \) firm, with \( h > i \), and a worker of age \( k \).

Table 2: Outcome of the wage bargaining game between a worker earning wage \( w \), the incumbent employer of type \( i \), and a poaching employer of type \( h \)

<table>
<thead>
<tr>
<th>negotiation outcome</th>
<th>( h &gt; i )</th>
<th>new employer ( h ) and a wage ( \phi(k, a_i, a_h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i \geq h \geq q(k, w, a_i) )</td>
<td>wage rise ( \phi(k, a_h, a_i) - w ) from current employer</td>
<td></td>
</tr>
<tr>
<td>( h &lt; q(k, w, a_i) )</td>
<td>no change</td>
<td></td>
</tr>
</tbody>
</table>

The mechanisms of wage bargaining discussed so far are the same as in Cahuc et al. (2006). However, while they assume that workers have an infinite life, workers leave the labor market at a given age in the present model. A young worker’s wage bargain outcome is different than that of a worker close to retirement. The option value of on-the-job search makes workers accept a low starting wage. The shorter the time horizon before retirement, the lower is the option value of on-the-job search. Hence, it can occur that workers negotiate wage rises from the current employer without any outside job offer when they have a credible threat to quit into unemployment.

Let \( \mathcal{W}(k, w, a_i) \) denote the value of a job to a worker of age \( k \) earning wage \( w \) in a match with productivity \( a_i \). When the two competing firms have productivities \( i \) and \( h \) with \( i < h \), type \( h \) firm wins the bargain by offering a wage \( \phi(k, a_i, a_h) \) that is determined by

\[
\mathcal{W}(k, \phi(k, a_i, a_h), a_h) = \mathcal{W}(k, a_i, a_i) + \gamma [\mathcal{W}(k, a_h, a_h) - \mathcal{W}(k, a_i, a_i)],
\]

where the parameter \( \gamma \in [0, 1] \) is the worker’s bargaining power. The worker obtains a value \( \mathcal{W}(k, \phi(k, a_i, a_h), a_h) \) that equals his outside option \( \mathcal{W}(k, a_i, a_i) \) - the highest value the lower productivity firm can offer - plus a share \( \gamma \) of the match surplus.\(^5\)

Consider a worker of age \( k \) earning wage \( w \) in a type \( i \) firm. The productivity index of the poaching firm must be at least equal to \( q(k, w, a_i) \) such that the worker obtains a higher lifetime

\(^5\)Here, I assume that wages are determined by a linear sharing rule. For a foundation of this wage equation by a bargaining game of alternating offers see Cahuc et al. (2006).
utility in the bargaining game. Hence, the threshold productivity index \( q(k, w, a_i) \) is the lowest index for which

\[
\mathcal{W}(k, w, a_i) < \mathcal{W}(k, a_q(k, w, a_i), a_q(k, w, a_i)) + \gamma [\mathcal{W}(k, a_i, a_i) - \mathcal{W}(k, a_q(k, w, a_i), a_q(k, w, a_i))]
\]  

(3)

is fulfilled. It follows that \( q(k, a_i, a_i) = i + 1 \). If the poaching employer has productivity \( h \) and \( i \geq h \geq q(k, w, a_i) \), the negotiation outcome is a wage \( \phi(k, a_h, a_i) \) at the incumbent firm that is determined by

\[
\mathcal{W}(k, \phi(k, a_h, a_i), a_i) = \mathcal{W}(k, a_h, a_h) + \gamma [\mathcal{W}(k, a_i, a_i) - \mathcal{W}(k, a_h, a_h)].
\]

The outside option of an unemployed worker aged \( k \) is the value of unemployment denoted by \( \mathcal{W}(k) \). A match between an unemployed worker and a type \( i \) firm is formed if and only if \( \mathcal{W}(k, a_i, a_i) \geq \mathcal{W}(k) \). Provided this condition is satisfied, the firm offers a wage \( \phi_0(k, a_i) \) that solves

\[
\mathcal{W}(k, \phi_0(k, a_i), a_i) = \mathcal{W}(k) + \gamma [\mathcal{W}(k, a_i, a_i) - \mathcal{W}(k)].
\]

(4)

A higher match quality offers the worker more opportunities to obtain wage rises because of possible outside job offers. This option value effect makes wages decrease in match quality. However, the higher the productivity of the firm that wins the bargain, the higher is the match surplus. The higher the worker’s bargaining power, the more the worker captures of the match surplus. The bargaining power effect makes wages increase in match quality. In Cahuc et al. (2006), wages decrease in the productivity of the firm that wins the bargain if \( \gamma \) is sufficiently small such that the option value effect dominates. If \( \gamma \) is large enough, the bargaining power effect dominates and wages increase in productivity. There are additional implications in a model with a finite time horizon. The shorter the remaining time horizon before retirement, the lower is the option value of on-the-job search.

### 3.2 Value functions

Each period, a worker decides how much effort \( e \) to spend on job search. The problem of an unemployed worker of age \( k < K - 1 \) is summarized by

\[
\mathcal{U}(k) = \max_{e \geq 0} \left\{ b_u - c(e) + \beta \left[ \mathcal{U}(k') + (1 - \delta) e \lambda \sum_{j=r(k')}^{n} \mathcal{W}(k', \phi_0(k', a_j), a_j) - \mathcal{U}(k') \right] p_j \right\},
\]

where \( k' = k + 1 \) and \( r(k') \) is the minimum productivity index of a match that a worker of age \( k' \) accepts. The unemployed worker’s value is the flow income of unemployment \( b_u \) minus search
costs plus the discounted continuation value. In the next period the worker obtains at least the value of unemployment. With probability $e\lambda$ he receives a job offer. The expected gain in value of an offer to the worker is $\sum_{j=r(k')}^{n} \left[ W(k', \Phi_0(k', a_j), a_j) - W(k') \right] p_j$. With probability $\delta$ the newly formed match is hit by a separation shock.

The reservation productivity $a_{r(k')}$ is the lowest productivity level for which

$$W(k', a_{r(k')}, a_{r(k')}) \geq W(k')$$

holds. Since unemployed and employed workers face the same search cost function and the same offer arrival rate per search effort, the lowest acceptable match productivity for a worker equals the flow income when unemployed, $b_U$. In the remainder of the paper, I set

$$a_1 = a_{r(k)} = b_U,$$

such that all matches have a positive surplus. Using equation (4), the value of unemployment becomes

$$U(k) = \max_{e \geq 0} \left\{ b_U - c(e) + \beta \left[ U(k') + (1 - \delta) e\lambda \gamma \sum_{j=1}^{n} \left[ W(k', a_j, a_j) - W(k') \right] p_j \right] \right\}. \tag{5}$$

The optimal search effort of an unemployed worker aged $k$, $e_U(k)$, is the solution to the first order condition (FOC) of the maximization problem

$$c'[e_U(k)] = \beta (1 - \delta) \lambda \gamma \sum_{j=1}^{n} \left[ W(k', a_j, a_j) - W(k') \right] p_j. \tag{6}$$

The value of a job to a worker of age $k < K - 1$ earning wage $w$ in a match with productivity $a_i$ is derived as follows:

$$W(k, w, a_i) = \max_{e \geq 0} \left\{ w - c(e) + \beta \left[ 1 - e\lambda [1 - P_{q(k', w, a_i) - 1}] \right] \max \left\{ W(k', w, a_i), W(k') + \gamma [W(k', a_i, a_i) - W(k')] \right\} \right. \right.$$ 

$$\left. + e\lambda \sum_{j=q(k', w, a_i)}^{i} \left( W(k', a_j, a_j) + \gamma [W(k', a_i, a_i) - W(k', a_j, a_j)] \right) p_j \right.$$ 

$$\left. + e\lambda \sum_{j=i+1}^{n} \left( W(k', a_i, a_i) + \gamma [W(k', a_j, a_j) - W(k', a_i, a_i)] \right) p_j \right\}. \tag{7}$$
The worker’s value is the current wage minus search costs plus the discounted continuation value. The worker becomes unemployed and earns a value \( \mathcal{V}(k') \) with probability \( \delta \). The employed worker does not meet an outside firm that has a productivity larger than \( a_{q(k',w,a_i)} \) with probability \( 1 - e^{\lambda}(1 - P_{q(k',w,a_i)} - 1) \). In this case the worker stays in his current match. As the option value of on-the-job search decreases with age, the worker renegotiates the wage if \( \mathcal{W}(k',w,a_i) \) becomes smaller than \( \mathcal{V}(k') + \gamma[\mathcal{W}(k',a_i,a_i) - \mathcal{V}(k')] \). If the worker meets an outside firm with lower productivity than \( a_i \) but above \( a_{q(k',w,a_i)} \), he expects a wage rise from the incumbent employer and a bargain outcome with value \( \sum_{j=q(k',w,a_i)}^i \left( \mathcal{W}(k',a_j,a_j) + \gamma[\mathcal{W}(k',a_j,a_j) - \mathcal{W}(k',a_i,a_i)] \right) p_j \). If the worker meets an outside firm with match productivity larger than \( a_i \), he switches to the poaching firm and expects a value \( \sum_{j=i+1}^n \left( \mathcal{W}(k',a_j,a_i) + \gamma[\mathcal{W}(k',a_j,a_j) - \mathcal{W}(k',a_i,a_i)] \right) p_j \). Let \( e_W(k,w,a_i) \) denote the optimal search effort of a worker earning wage \( w \) at a type \( i \) firm. The optimal search effort is the solution to the FOC

\[
\begin{align*}
\gamma' e_W(k,w,a_i) &= (1 - \delta) \lambda \left( \sum_{j=q(k',w,a_i)}^i (1 - \gamma) \mathcal{W}(k',a_j,a_j) + \gamma \mathcal{W}(k',a_i,a_i) \right) p_j \\
&\quad + \sum_{j=i+1}^n (1 - \gamma) \mathcal{W}(k',a_j,a_j) + \gamma \mathcal{W}(k',a_j,a_j) \right) p_j \\
&\quad - (1 - P_{q(k',w,a_i)} - 1) \max \left\{ \mathcal{W}(k',w,a_i), \mathcal{V}(k') + \gamma[\mathcal{W}(k',a_i,a_i) - \mathcal{V}(k')] \right\}.
\end{align*}
\]

(8)

Alternatively, an employed worker’s search effort could be chosen such that it is jointly efficient as in Lentz (2010). In this case, the corresponding employment contract would specify not only a wage but also the search effort that maximizes the joint surplus of the firm-worker match. However, since the search effort is private choice of the worker, the implementability of jointly setting the search effort via a wage contract is unclear. Furthermore, the search effort that maximizes the joint surplus of a match of highest quality \( (a_n) \) is zero even if the worker earns a low wage. As a worker in a type \( n \) match cannot upgrade his wage under such a contract, an unemployed worker who obtains a job offer at a type \( n \) match earns always a lower wage than a worker who switches from a type \( i \) match with \( 1 < i < n \) to a type \( n \) match. It is hard to justify the enforceability of zero search effort especially in this case. I therefore assume that a worker chooses the search effort to maximize his own value of the match.
Let $\mathcal{R}(K)$ be the value of retirement. A worker aged $K - 1$ faces the following values:

\begin{align}
\mathcal{W}(K - 1) &= b_U + \beta \mathcal{R}(K), \\
\mathcal{W}(K - 1, w, a_i) &= w + \beta \mathcal{R}(K).
\end{align}

### 3.3 Steady-state labor market flows

The labor market dynamics lead to the following stationary distribution of workers across employment states. Let $g(k, w, a_i)$ be the fraction of the population aged $k$, earning wage $w$, and being employed at a type $i$ firm. The fraction of the population aged $k$ being employed at a type $i$ firm is given by $g(k, a_i) = \int g(k, w, a_i) dw$. The fraction of the population aged $k'$ being employed at a type $i$ firm is made up of the pool of unemployed workers that form a match with a type $i$ firm, the workers that are recruited out of lower productivity jobs, and the workers that have stayed in a type $i$ match:

\begin{align}
g(k', a_i) &= e_U(k) \lambda u(k)p_i + (1 - \delta)\lambda p_i \sum_{j=1}^{i-1} \int e_W(k, w, a_j)g(k, w, a_j)dw \\
&\quad + (1 - \delta) \int \left[ 1 - e_W(k, w, a_i)\lambda (1 - P_i) \right] g(k, w, a_i)dw.
\end{align}

### 3.4 Wage distribution

Let $G(w|k, a_i)$ be the cumulative distribution of wages conditional on age and productivity. The maximum wage a type $i$ firm can offer is $a_i$. Hence

\begin{equation}
G(a_i|k, a_i) = 1.
\end{equation}

All newborns are unemployed. Employed workers aged $k = 2$ earn a wage $\phi_0(2, a_i)$ since they were hired out of unemployment and have not yet searched on the job. The cumulative distribution of wages conditional on age and productivity for workers of age $k' \geq 3$ is determined by

\begin{align}
G(w|k', a_i) &= I_{w \geq \phi_0(k', a_i)} \left\{ e_U(k) \lambda u(k)p_i + (1 - \delta)\lambda p_i \sum_{j=1}^{q(k', w, a_i) - 2} \int e_W(k, \tilde{w}, a_j)g(k, \tilde{w}, a_j)d\tilde{w} \\
&\quad + (1 - \delta) \int^w \left[ 1 - e_W(k, \tilde{w}, a_i)\lambda (1 - P_{q(k', w, a_i) - 1}) \right] g(k, \tilde{w}, a_i)d\tilde{w} \right\} / g(k', a_i),
\end{align}

---

6The value of $\mathcal{R}(K)$ has an effect only on the scale of the value functions but not on equilibrium wages or search efforts.
where \( I_{w \geq \phi_0(k', a_i)} \) is a dummy variable equal to 1 if \( w \geq \phi_0(k', a_i) \) and 0 otherwise. The conditional cumulative distribution of wages \( G(w|k', a_i) \) is the sum of unemployed workers with reservation wage \( \phi_0(k', a_i) \leq w \) who meet a type \( i \) firm, workers that switch from a lower productivity firm to a type \( i \) firm for a wage \( \leq w \), and workers that stay in their current match of type \( i \) who do not earn a wage larger than \( w \). Workers are only willing to switch to a type \( i \) firm for a wage \( \leq w \) if the match productivity of the current employment is smaller than \( q(k', w, a_i) \).

The cumulative distribution of wages conditional on age is determined by

\[
G(w|k) = \frac{\sum_{j=1}^{n} G(w|k, a_j)g(k, a_j)}{g(k)}.
\] (13)

### 3.5 Equilibrium

A stationary equilibrium consists of

- the optimal search efforts \( e_U(k) \) and \( e_W(k, w, a_i) \) given by the first-order conditions (6) and (8),
- the reservation wages \( \phi_0(k, a_i) \) and \( \phi(k, a_i, a_h) \) derived from equations (4) and (2),
- the threshold productivities \( q(k, w, a_i) \) of employed workers derived from condition (3),
- a stationary employment distribution of unemployed workers \( u(k) \), of employed workers \( g(k, a_i) \), and the cumulative distribution of wages \( G(w|k, a_i) \), given by equations (1), (11), and (12)

for all combinations of age \( k < K \), wages \( w \), and match productivities \( a_i \), given an exogenous productivity distribution, a constant mass of new workers of age \( k = 1 \), and the value of retirement \( R(K) \).

**Proposition.** The stationary equilibrium exists and is unique.

**Proof.** Given a value of retirement \( R(K) \), the optimal search efforts \( (e_U(k) \) and \( e_W(k, w, a_i)) \), reservation wages \( (\phi_0(k, a_i) \) and \( \phi(k, a_i, a_h)) \), and threshold productivities \( (q(k, w, a_i)) \) can be computed starting with age \( k = K - 1 \) and continuing backwards in age. Using this and the condition that all newborns are unemployed \( (u(1) = l) \), the stationary employment distribution \( (u(k) \) and \( g(k, a_i)) \) and wage distribution \( (G(w|k, a_i)) \) can be computed for all combinations of

---

7The index above the summation sign in equation (12) is set equal to \( q(k', w, a_i) - 2 \) such that the equation fulfills \( G(a_i|k', a_i) = 1 \) and equation (11).
age \( k < K \), wages \( w \), and match productivities \( a_i \) starting with age \( k = 2 \) and continuing upwards in age.

4 A quantitative analysis

In this section, I calibrate the model and derive the equilibrium life-cycle profiles of the unemployment-to-employment transition rate, the employment-to-employment transition rate and the wage distribution.

The definition of employment states and transition rates derived from the SIPP data follows Menzio et al. (2012). A worker is assigned an employer based on his primary job where he worked the most hours. A worker is not in the labor force (N) if he reports having no job, not looking for work, and not being on layoff. A worker is unemployed (U) if he reports having no job and looking for work or being on layoff. A worker is employed (E) if he reports having a job and being either on layoff or not and absent without pay or not. A worker is in the labor force (L) if he is eitheremployed or unemployed. The unemployment-to-employment transition (UE) rate is defined as the number of workers that experience a transition from unemployment to employment in a given month divided by the number of unemployed workers at the beginning of the month. The other transition rates are defined analogously.

The data shows that there are workers who flow in and out of the labor force across all age groups (see Figure 2). Furthermore, the empirical rate at which employed workers become unemployed is decreasing with age (see Figure 3). These transitions influence the wage distribution and are therefore taken into account when calibrating the model. They are directly calibrated. Figure 4 shows how the labor market dynamics enter the model calibration. The complete model is described in Appendix B.
Figure 2: Flows in and out of the labor force
Figure 3: Life-cycle profile of the EU rate

Figure 4: Labor market transitions


4.1 Calibration

The model is calibrated to monthly data. I therefore set the discount factor to $\beta = 0.9967$, which implies an annual real interest rate of 4 percent. Workers retire after 49 years in the labor market, i.e. $K = 588$. The distribution of match qualities is Weibull with scale parameter $\phi = 5$, shape parameter $\tau$, and a location parameter that equals $b_U (= a_1)$. The number of grid points is $n = 40$. The cost of spending an effort $e$ on searching is given by the quadratic cost function

$$c(e) = ce^2.$$

I set $c = 0.5$. The estimate of the vector of structural parameters $\theta = (\gamma, \lambda, \tau, b_U)$ minimizes the distance between simulated moments and corresponding moments obtained from the SIPP 1996 panel. I use 53 estimation targets: Average wages within each age group (49 targets), the standard deviation of residual wages, the EE rate, the UE rate, and the skewness of the wage distribution.

The simulated moments depend on the four structural parameters to be estimated. The transition rates are mainly influenced by the offer arrival rate per search effort $\lambda$. In addition, a higher bargaining power of workers $\gamma$ has a positive effect on the UE rate as it rises the expected value of a job. As workers obtain higher wages, $\gamma$ has a negative effect on the EE rate. The EE rate contains also information on the parameters of the distribution of match qualities. A more dispersed productivity distribution induces more EE transitions. The standard deviation of residual wages and the skewness contain mainly information on the productivity distribution. The age-wage profile contains information on the life-cycle profile of reservation wages and thereby on the bargaining power parameter $\gamma$. The influence of $\gamma$ on the life-cycle profile of wages is discussed in more detail in section 5.

Table 3 contains the calibration targets and Table 4 the estimated parameters. The model captures the hump shape of the age-wage profile. It furthermore matches well the standard deviation of wages, the average UE rate, and the average EE rate. It captures the negative sign of the skewness of the distribution of log wages, though underestimates it.

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8Christensen et al. (2005) estimate a model with on-the-job search in which the search effort is endogenous and the offer arrival rate per search effort is the same for employed and unemployed workers. Their results support a quadratic cost of search function.

9The FOCs (6) and (8) show that $\lambda$ and $c$ cannot be identified separately but only the ratio $\lambda/c$. One can therefore normalize arbitrarily $c$ and then calibrate the parameter $\lambda$. 

### Table 3: Calibration Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>Age-wage profile</td>
<td>see Figure 5</td>
<td></td>
</tr>
<tr>
<td>Std. of residual wages</td>
<td>0.4379</td>
<td>0.4377</td>
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<tr>
<td>Average UE rate</td>
<td>0.1685</td>
<td>0.1793</td>
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<tr>
<td>Average EE rate</td>
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<td>0.0151</td>
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<tr>
<td>Skewness</td>
<td>-2.5406</td>
<td>-1.7190</td>
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### Table 4: Point Estimates

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Estimate</th>
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</thead>
<tbody>
<tr>
<td>Workers’ bargaining power</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Offer arrival rate per search effort</td>
<td>$\lambda$</td>
<td>0.0600</td>
</tr>
<tr>
<td>Shape parameter</td>
<td>$\tau$</td>
<td>1.8500</td>
</tr>
<tr>
<td>Flow income of unemployment</td>
<td>$b_U$</td>
<td>0.0501</td>
</tr>
</tbody>
</table>
Figure 5: The age profiles of average log wages are standardized for comparability between model and data. The standardized age-specific average wage is derived by dividing the difference between the age-specific average wage and its mean by its standard deviation.

4.2 Life-cycle profiles

Let us turn to the standard deviation of wages illustrated in Figure 6. The age-inequality profile of wages is U-shaped in the data and in the model. The age-inequality profile falls for young workers because the EE rate is high for this age group and workers are gradually matched to better jobs. However, better job offers become less frequent for workers in a high quality match. For middle-aged and older workers the longer search history plays a dominant role. The standard deviation of match qualities increases. This occurs because workers are differently successful in finding good job offers and career paths diverge. Some workers have obtained many good job offers and are employed in a high productivity match at a high wage. At the same time, there are workers who flow from unemployment to employment for a low productivity match and a low wage. Workers reduce their search effort when they approach the retirement age and move therefore more slowly to higher productivity matches. Hence, the effect that led to the reduction in inequality among young workers is too weak for middle-aged and older workers. This is reflected in the increase in the standard deviation of this age group.
Figure 6: Wage dispersion

Figure 7 displays the life-cycle profiles of the empirical and of the model EE rate. The life-cycle profile of the EE rate decreases with age. The decreasing EE age profile is the result of two effects. A worker’s search history increases with age and so does the average match quality. The probability of obtaining a better match decreases in the quality of the current match. Furthermore, older workers reduce their search effort since the remaining time horizon in the labor market shortens. The EE rate approaches zero for workers close to retirement because these workers reduce the search effort substantially. The simulated EE rate matches the empirical one well. Compared with the data, the EE rate obtained from the simulation declines sharply for older workers as all workers retire at the same age in the model economy.

The life-cycle profile of the UE rate remains relatively constant until a few years before retirement, then declines dramatically as workers reduce their search effort substantially when they approach the retirement age (see Figure 8). Searching for a job is costly and the expected value of a job offer is small for workers close to retirement. The model UE rate slightly increases until age 48 because the rate at which workers quit employment decreases until this age group. This reduction in the quit rate has a positive effect on the value of a job and thereby also on the search effort of unemployed workers.

Figure 9 shows the age profile of the unemployment rate. It is decreasing strongly for young workers, since workers are initially unemployed and gradually matched to their first jobs.
Figure 7: Average employment-to-employment transition rates

Figure 8: Average unemployment-to-employment transition rates
The endogenous life-cycle profiles of EE and UE transitions in the present model of random search are similar to those in the directed search model of Menzio et al. (2012). Endogenous search effort is the main channel at work in the present model. Workers search less when the time until retirement is short. Workers also search less when they are well matched or earn a high wage as the expected additional value of a job offer is lower.\(^\text{10}\) The second channel that is responsible for the strong decrease in the EE rate for young workers is sorting into better matches. Workers are gradually matched to better jobs and this lowers their probability of obtaining an even better job offer. Figure 10 illustrates that the reduction in search effort and the decreasing probability that the poaching employer has a higher productivity are responsible for the decrease in the EE rate until the age of 53. Afterwards, the EE rate only falls because of the reduction in search effort while the probability that a job offer leads to an EE transition rises again. Menzio et al.’s model does not contain search effort. The mechanism in their model is directed search. There is a continuum of submarkets. Each submarket is targeted at workers of a specific age and productivity and offers workers an employment contract with a specified value. Workers face a tradeoff between a high offer arrival rate and a high value of a

\(^{10}\)Better matched workers search less in most cases. However, when two workers earn the same wage but have different match qualities, the worker with higher match quality searches more because he has a better position in wage negotiations and therefore a higher probability of obtaining a wage rise. The same applies if the worker in the higher quality match earns a lower wage.
Figure 10: Decomposition of the EE rate into the average offer arrival rate \((e \lambda)\) and the average probability \((1 - P_i)\) that the poaching employer has higher productivity than the incumbent employer (of type \(i\)).

job when choosing in which submarket to search. Workers choose to search in a submarket that offers a high value but has a low vacancy to applicant ratio and therefore low offer arrival rate if the value of their current position is high. Firms choose in which submarkets to create how many vacancies. In the calibrated model, workers search in submarkets with a lower vacancy to applicant ratio when they grow older, are employed in a good match or are less experienced and therefore less productive.  

5 Discussion

The endogenous age-inequality profile of match qualities is U-shaped. This translates into a U-shaped age-inequality profile of wages if the bargaining power of workers is sufficiently high (see Figure 11). If the bargaining power of workers is much lower than the calibrated value, the standard deviation of wages sharply decreases for older workers. Figure 12 illustrates this by showing the age-inequality profile of wages derived from a model calibration in which \(\gamma\) is set equal to 0.5. The vector of the remaining structural parameters \(\theta_{\gamma=0.5} = (\lambda, \tau, b_U)\) was

\(^{11}\)In Menzio et al. (2012) more experienced workers are more productive as learning-by-doing increases their stock of human capital.
Figure 11: Dispersion of wages and match qualities; estimated $\gamma = 0.7172$

estimated using the same targets as before.

The bargaining power parameter has an effect on the age-inequality profile of wages through the worker’s reservation wage. When $\gamma$ is too low, young workers accept wages far below the match productivity as their option value of on-the-job search is high. The shorter the time until retirement, the lower is the option value of on-the-job search. Older workers therefore demand higher wages. This increases the lowest bound of the conditional wage distribution given the match productivity. Hence, the standard deviation of wages decreases for workers close to retirement. When the bargaining power parameter is sufficiently large and the match quality is relatively high, the decreasing time horizon has the opposite effect on wages. Older workers accept lower wages since the probability of obtaining a better job offer by waiting decreases.

Figures 5 and 13 compare the age-wage profiles of the model economy with the empirical one. The age-wage profile in the U.S. economy is hump-shaped.\footnote{A concave age-wage profile in the U.S. can be found in several empirical studies including Kambourov and Manovskii (2009) and Mincer (1974).} Average wages increase with age for young and middle-aged workers. They decrease with age a few years before retirement. The worker’s bargaining power must be sufficiently high such that the present model reproduces a hump-shaped age-wage profile. Because workers are gradually matched to better jobs, the average match quality and the average wage in the model economy increase with age. It increases at a decreasing rate because job offers from higher quality matches become less probable the
Figure 12: Calibrated model if $\gamma = 0.5$

Figure 13: Calibrated model if $\gamma = 0.5$
higher the productivity of a match. Although the UE rate decreases when workers approach the retirement age, there are permanent flows from unemployment to employment until one period before retirement. All workers recruited out of unemployment who have not obtained any outside offer, have the same distribution of match productivities with a low average match quality independent of age. Because also the search effort of employed workers decreases with age, an increasing fraction of the workers in low quality matches does not move to higher quality matches. As a result, the average match quality decreases some years prior to retirement. When the worker’s bargaining power \( \gamma \) is high, the age-wage profile is similar to the age-match productivity profile and depicts the empirically observed hump-shaped age-wage profile. When \( \gamma \) is low, the above explained increase of the reservation wage has a positive effect on the average wage prior to retirement.

When the bargaining power parameter \( \gamma \) is chosen to match the empirically supported hump-shaped age-wage profile and the U-shape of the age-inequality profile of wages, it must be rather high (roughly 0.7) in this model. This is in contrast to Cahuc et al. (2006) who find for French data that \( \gamma \) lies between 0 and 0.35. An exception is the high value of \( \gamma = 0.98 \) for high skilled workers in the construction sector. Bagger et al. (2011) explore the importance of human capital accumulation and labor market competition for life-cycle wage dynamics in a bargaining framework similar to Cahuc et al. (2006). They find in their analysis of Danish data that the bargaining power \( \gamma \) lies between 0.2475 and 0.4141 and declines with education. In both papers, workers have an infinite working life. The present paper provides a different interpretation of the bargaining power parameter. It contains information on the relative importance of the option value of on-the-job search over the life cycle.

### 6 Conclusions

I consider a life-cycle model of labor market search with strategic wage bargaining, counteroffers, match heterogeneity and endogenous search effort. I show that the model can reproduce the U-shape of the age-inequality profile of wages if the bargaining power of workers is sufficiently high. Furthermore, the present model captures the shapes of the empirically observed age profiles of average wages, the unemployment-to-employment transition rate, and the employment-to-employment transition rate. The shape of the age-inequality profile of wages is mainly driven by the age profile of reservation wages, by transitions into employment, and transitions between employers. The optimal search effort of employed workers depends on the worker’s time horizon before retirement, the current wage, and the quality of the firm-worker match. Furthermore, the probability of meeting an outside firm with a higher match quality decreases in the quality
of the current match. This leads to frequent employment-to-employment transitions of young workers, a moderate employment-to-employment transition rate of middle aged workers, and a sharp decrease in the employment-to-employment transition rate of workers close to retirement. The bargaining power parameter plays an important role in the model because the option value of on-the-job search decreases when the time horizon before retirement shortens. A low bargaining power makes young workers accept a wage far below the productivity of the firm-worker match. Since the option value of on-the-job search is low for workers close to retirement, the reservation wage increases for older workers. This leads to a decline in the standard deviation of wages for older workers when the workers’ bargaining power is too low.

This paper focuses on job search as an important factor for the shape of the age-inequality profile of wages. There is evidence that residual wage dispersion is well explained by both human capital and search on-the-job (Burdett et al., 2011; Tjaden and Wellschmied, 2012). An obvious extension would therefore be the introduction of human capital accumulation through learning-by-doing. Furthermore, it would be interesting to introduce optimal human capital investments into the model in order to assess the relative contribution of job search and post-school human capital investments to the life-cycle wage inequality. The literature that combines search theory and the theory of on-the-job training (Acemoglu and Pischke, 1998; Moen and Rosén, 2004; Wasmer, 2006; Stevens, 2012) shows that there are important interactions between labor turnover and endogenous human capital investments. Exploring these interactions in a life-cycle model is the focus of a companion paper (Marotzke, 2013).
References


## APPENDIX

### A Estimation

Table 5: Regressors

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major occupations (occ14)</td>
<td>1 Executive, Administrative, and Managerial</td>
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<tr>
<td></td>
<td>2 Professional Speciality</td>
</tr>
<tr>
<td></td>
<td>3 Technicians and Related Support</td>
</tr>
<tr>
<td></td>
<td>4 Sales</td>
</tr>
<tr>
<td></td>
<td>5 Administrative Support, Including Clerical</td>
</tr>
<tr>
<td></td>
<td>6 Private Household Services</td>
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<tr>
<td></td>
<td>7 Protective Services</td>
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<tr>
<td></td>
<td>8 Services, except Household and Protective</td>
</tr>
<tr>
<td></td>
<td>9 Farming, Forestry, and Fishing</td>
</tr>
<tr>
<td></td>
<td>10 Precision Production, Craft, and Repair</td>
</tr>
<tr>
<td></td>
<td>11 Machine Operators, Assemblers, and Inspectors</td>
</tr>
<tr>
<td></td>
<td>12 Transportation and Material Moving</td>
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<tr>
<td></td>
<td>13 Handlers, Equipment Cleaners, Helpers, and Laborers</td>
</tr>
<tr>
<td>Disability that limits work (disabled)</td>
<td>0 not disabled</td>
</tr>
<tr>
<td></td>
<td>1 disabled</td>
</tr>
<tr>
<td>Census Region (region)</td>
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<td></td>
<td>2 Middle Atlantic</td>
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<td>8 Mountain</td>
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<td></td>
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<td>Weekly hours (hours)</td>
<td>Weekly hours in dominant job</td>
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Table 6: Estimation results

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<th>Variable</th>
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<th>(Robust Std. Err.)</th>
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<td>(0.000)</td>
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<tr>
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<td>4.occ14</td>
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<td>(0.031)</td>
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<td>5.occ14</td>
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<td>(0.032)</td>
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<td>6.occ14</td>
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<td>9.region</td>
<td>0.213</td>
<td>(0.133)</td>
</tr>
<tr>
<td>0b.ms2</td>
<td>0.000</td>
<td>(0.000)</td>
</tr>
<tr>
<td>1.ms2</td>
<td>0.094**</td>
<td>(0.024)</td>
</tr>
<tr>
<td>hours</td>
<td>0.017**</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Intercept</td>
<td>6.617**</td>
<td>(0.102)</td>
</tr>
</tbody>
</table>

Observations: 180,848
Groups: 8,422
R² within: 0.077
R² between: 0.356
R² overall: 0.244
F (72,8421): 46.028

A †/*/** next to the coefficient indicates significance at the 10/5/1% level.
**Robustness**

The U-shaped relationship between the wage dispersion and age remains if number of kids, age, age squared, and/or interaction between occupation and age are included, and if weekly hours is excluded in the regression (see Figure 14). The age-inequality profile is also U-shaped if only full-time workers (working at least 30 hours per week) are considered (see Figure 15).

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**Figure 14: Age-inequality profile of residual log wages**

(a) Weekly hours is excluded.

(b) Interaction between occupation and age is included.

(c) Number of kids, age, age squared, and interaction between occupation and age are included.
B  Complete model

This section introduces the complete model which is used for the calibration. Let $\delta_k$ denote the age-dependent job destruction rate which is given by the empirical smoothed EU rate. Unemployed workers in the model experience a transition out of the labor market at rate $\eta_k$ given by the empirical smoothed UN rate. Employed workers leave the labor market at rate $\zeta_k$ given by the empirical smoothed EN rate. Transitions from non-participation to unemployment (employment) occur at rate $\mu_k$ ($\nu_k$) given by the empirical smoothed NU (NE) rate. Let $l(k)$ be the mass of workers aged $k$ who participate and $n(k)$ be the mass of workers who do not participate in the labor market.

B.1 Value functions

A worker who does not participate in the labor market obtains at least his value of unemployment. Otherwise he would search for a job. I assume that the value of being not in the labor market makes the worker indifferent between searching for a job and not searching. I therefore set the value of non-participation equal to the value of unemployment.\textsuperscript{13} The value of $\textsuperscript{13}$Setting the value of non-participation equal to the value of unemployment plus a constant does not impact the results.
unemployment becomes

\[ \mathcal{U}(k) = \max_{e \geq 0} \left\{ b_U - c(e) + \beta \left[ \mathcal{U}(k') + (1 - \delta_k)(1 - \zeta_k) e \lambda \gamma \sum_{j=1}^{n} \left[ \mathcal{W}(k', a_j, a_j) - \mathcal{U}(k') \right] p_j \right\} \right. \]

(B.1)

The optimal search effort of an unemployed worker aged \( k \), \( e_U(k) \), is the solution to the first order condition (FOC) of the maximization problem

\[ c'[e_U(k)] = \beta (1 - \delta_k)(1 - \zeta_k) \lambda \gamma \sum_{j=1}^{n} \left[ \mathcal{W}(k', a_j, a_j) - \mathcal{U}(k') \right] p_j. \]

(B.2)

The value of a job to a worker of age \( k < K - 1 \) earning wage \( w \) in a match with productivity \( a_i \) is derived as:

\[ \mathcal{W}(k, w, a_i) = \max_{e \geq 0} \left\{ w - c(e) + \beta (1 - \zeta_k) \left[ \delta_k \mathcal{U}(k') + (1 - \delta_k) \left( 1 - e \lambda [1 - P_{q(k', w, a_i)} - 1] \right) \right. \right. \]

\[ \left. \left. + e \lambda \sum_{j=q(k', w, a_i)}^{i} \left( \mathcal{W}(k', a_j, a_j) + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{W}(k', a_j, a_j)] \right) p_j \right. \right. \]

\[ + e \lambda \sum_{j=i+1}^{n} \left( \mathcal{W}(k', a_j, a_i) + \gamma [\mathcal{W}(k', a_j, a_j) - \mathcal{W}(k', a_i, a_i)] \right) p_j \]

\[ + \beta \zeta_k \mathcal{U}(k') \} \right. \right. \}

(B.3)

The FOC that determines an employed worker’s search effort is

\[ c'[e_W(k, w, a)] = \beta (1 - \delta_k)(1 - \zeta_k) \lambda \left( \sum_{j=q(k', w, a_i)}^{i} \left[ (1 - \gamma) \mathcal{W}(k', a_j, a_j) + \gamma \mathcal{W}(k', a_i, a_i) \right] p_j \right. \]

\[ + \sum_{j=i+1}^{n} \left[ (1 - \gamma) \mathcal{W}(k', a_i, a_i) + \gamma \mathcal{W}(k', a_j, a_j) \right] p_j \]

\[ - (1 - P_{q(k', w, a_i)} - 1) \max \left\{ \mathcal{W}(k', w, a_i), \mathcal{U}(k') + \gamma [\mathcal{W}(k', a_i, a_i) - \mathcal{U}(k')] \right\} \right. \}

(B.4)
B.2 Transition rates and wage distribution

The wage distribution is discretized. $u(k), n(k), g(k, w_s, a_i), g(k, a_i),$ and $G(w_s | k, a_i)$ with $s = 1, \ldots, S$ and $w_{s-1} < w_s$ are determined as follows:

- The initial mass of unemployed workers $u(1)$ and of workers who do not participate in the labor force $n(1)$ is given.
- The mass of workers aged $k = 2$ who are employed at a type $i$ firm is determined as the mass of workers aged $k = 1$ who obtain a job offer from a type $i$ firm:
  \[ g(2, a_i) = [eU(1)\lambda u(1) + \nu_1 n(1)] p_i. \]
- Workers aged $k = 2$ earn their reservation wages $\phi_0(2, a_i)$. The cumulative distribution of wages for workers aged $k = 2$ in a type $i$ match is therefore given by
  \[ G(w_s | k = 2, a_i) = \begin{cases} 1 & \text{if } w \geq \phi_0(2, a_i) \\ 0 & \text{if } w < \phi_0(2, a_i) \end{cases}. \]
- The mass of workers aged $k = 2$ earning wage $w$ in a type $i$ match is given by
  \[ g(2, w_s, a_i) = \begin{cases} g(2, a_i)G(w_1 | 2, a_i) & \text{if } s = 1 \\ g(2, a_i) [G(w_s | 2, a_i) - G(w_{s-1} | 2, a_i)] & \text{if } s > 1 \end{cases}. \]
- The mass of unemployed workers aged $k = 2$ consists of the unemployed workers who did not find a job and did not leave the labor market and the workers who enter the labor market and do not immediately obtain a job:
  \[ u(2) = (1 - \lambda eU(1) - \eta_1)u(1) + \mu_1 n(1). \]
- The mass of workers who do not participate in the labor market is given by
  \[ n(2) = (1 - \mu_1 - \nu_1)n(1) + \eta_1 u(1). \]

For $k \geq 2$ the following steps are repeated:
1. The steady-state mass of workers aged $k+1$ in a type $i$ match is given by

$$g(k+1, a_i) = \left[ e_U(k) \lambda u(k) + \nu_k n(k) \right] p_i$$

$$+ (1 - \delta_k - \zeta_k) \lambda p_i \sum_{j=1}^{i-1} \sum_{s=1}^{\mathcal{S}} e_w(k, w_s, a_j) g(k, w_s, a_j)$$

$$+ (1 - \delta_k - \zeta_k) \sum_{s=1}^{\mathcal{S}} [1 - e_w(k, w_s, a_i) \lambda (1 - P_i)] g(k, w_s, a_i).$$

2. The cumulative distribution of wages in steady state for workers aged $k+1$ who are employed in a type $i$ match is derived as

$$G(w_s | k+1, a_i) = I_{w_s \geq \phi_0(k+1, a_i)} \left\{ \left[ e_U(k) \lambda u(k) + \nu_k n(k) \right] p_iight.$$
at the same age.