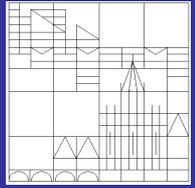




University of Konstanz  
Department of Economics



# **Risk Sharing and Employee Motivation in Competitive Search Equilibrium**

*Anna Zaharieva*

Working Paper Series  
2010-7

# Risk Sharing and Employee Motivation in Competitive Search Equilibrium

Anna Zaharieva \*

October 12, 2010

## Abstract

This paper incorporates a classical moral hazard problem with unobserved worker effort and bonus payments into a competitive search equilibrium environment with risk averse workers. The resulting framework permits an analysis of the effects of labour market competition and search frictions on individual contract setting. The paper demonstrates that the classical model of moral hazard with an ex-post wage setting regime may underestimate the optimal values of wages and bonus payments in competitive labour markets. The baseline model is extended to account for employer heterogeneity with respect to capital endowments. In the extended model, wage competition between employers serves as a source of positive correlation between wages and bonus payments reported in a number of empirical studies.

**JEL classification:** J33, J64, M52

**Key words:** Effort, bonus, risk aversion, competitive search, equilibrium efficiency

---

\*Institute of Mathematical Economics, University of Bielefeld, 33501 Bielefeld, Germany, E-mail: zaharieva.anna@gmail.com. I would like to thank Leo Kaas for his guidance and encouragement as well as seminar participants at the University of Konstanz, and session participants at the 2009 meetings of European Society for Population Economics, European Economic Association and European Association of Labour Economists.

# 1 Introduction

Labour contracts with bonus payments and profit shares are widely used to address the issues of employee motivation and asymmetric information, such as the moral hazard problem. A large branch of the literature considers these issues in the context of a single match between a firm and a worker independently of labour market conditions. The general framework for models with bonus payments in the presence of moral hazard is partial equilibrium with an ex-post wage setting mechanism where firms make take-it-or-leave wage offers after they meet a potential employee (see section IV in Laffont and Martimort (2002) and section I in Bolton and Dewatripont (2005)).

The classical contract theory approach provides foundations for the analysis of risk-sharing and employee motivation in a match with stochastic output and unobserved worker behavior. Yet, a deeper analysis of incentive contracts under different labour market regimes is required. This necessity is also supported by a number of empirical studies providing mixed evidence on the correlation between bonus payments and wages under different labour market conditions (see table 1). In particular, studies by Hart and Hübler (1991) and Cahuc and Dormont (1997) find significant positive correlation between wages and bonus payments in Germany and France respectively indicating complementarity between these two variables in Continental Europe. In contrast, a study by Wadhvani and Wall (1990) reports independence between these two types of labour compensation in the UK, while Kaufman (1998) finds evidence of a negative correlation, indicating substitution, for the set of companies in the US.

In this paper, first, the classical model on performance related pay in the presence of moral hazard is extended to the case of heterogeneous jobs in a dynamic labour market equilibrium framework with search frictions. However, the core of the moral hazard model is unchanged, the model characterizes a situation where workers possess private information about their effort choices affecting the probability distribution of output. Based on this extension, it is illustrated that the classical contract theory model with the ex-post wage setting mechanism fails to explain the complementarity effect between wages and bonus payments observed in a number of empirical studies. Motivated by this result the current study develops a new model of moral hazard in competitive search equilibrium with risk averse workers and improves the existing approach.

Study	Result	Details	No. of observations
Mitchell D.J.B., Lewin D., Lawler, E.E. (1990)	Complements US, 1974	Bonus payments are positively associated with total compensation and base wage	N=3428 Cross-section Individual level
Hart R.A., Hübler O. (1991)	Complements Germany, 1984	Wages are positively associated with probability and amount of profit shares	N=3628 Cross-section Individual level
Wadhvani S., Wall M. (1990)	Independent UK, 1974-1982	Profit shares are positively associated with total compensation	NT=900 Panel data Firm level
Kaufman R. (1998)	Substitutes US, 1978-1988	Bonus payments are positively associated with total compensation but negatively with base wage	NT=550 Panel data Firm level
Cahuc P., Dormont B. (1997)	Complements France, 1986-1989	Profit shares are positively associated with base wage	NT=688 Panel data Firm level

Table 1: Empirical research: wages and bonus payments.

The concept of competitive search has been originally introduced in a study by Moen (1997) and is based on the dynamic search and matching modeling setup of Mortensen and Pissarides (1994) and Pissarides (2000). The principal difference of competitive search is the ex-ante wage setting mechanism where firms post wages for open vacancies and unemployed workers direct their job search towards the better offers. This mechanism provides foundations for the wage competition between employers: firms offering higher wages are more likely to fill their open vacancies as opposed to the firms with low wage offers. Competition between firms provides motives for the rent-sharing between workers and firms and explains worker rents as a hiring premium left by firms in order to attract a potential employee.

This paper considers competitive search equilibrium with incentive contracts where jobs are characterized by stochastic output and workers' unobserved job performance giving rise to a moral hazard problem. The combination of incentive contracts within a match and competitive search as the match formation process describe a situation where firms pay both the motivation and the hiring premia and the equilibrium is a counteraction of the risk-sharing and the rent-sharing employer considerations. In the equilibrium, the risk-sharing curve defines the optimal proportions of risk-sharing between the firm and the worker necessary to provide workers with the correct effort incentives. While the rent-sharing curve defines the optimal rent split between the two

contracting parties necessary to achieve optimal hiring probabilities for the firm. Extended for the case of heterogeneous jobs this model defines the major contribution of this paper – to show that competitive search equilibrium with performance pay is capable of explaining complementarity between bonus payments and wages reported in a number of empirical studies. This complementarity effect is obtained due to the rent-sharing employer motive absent in the models with an ex-post wage setting.

Another contribution of this paper is comparison of the optimal labour compensation packages in terms of wages and bonus payments in labour markets with different institutional setups. This paper demonstrates that the classical model on moral hazard with risk averse workers predicts lower optimal values of wages and bonus payments than the labour market model with wage competition between employers.

Finally, this paper extends the constrained efficiency result of Hosios (1990) and Moen and Rosen (2008) for the case of risk averse employees in the presence of asymmetric information. Competitive search equilibrium with risk averse workers and unobserved worker effort is demonstrated to be constrained efficient, where the rent-sharing rule takes form of the risk adjusted Hosios (1990) condition. Here the risk adjustment is represented by a "shadow price" of a unit output. Nevertheless, the equilibrium is different from the first best social planner solution obtained in the absence of information asymmetries, indicating that the first best solution is not incentive compatible. The principal difference of the unconstrained social planner solution is full income insurance of the employed individuals implying a zero optimal value of bonus pay and a trivial risk-sharing outcome. In addition, effort distortions are fully attributed to the risk-aversion of workers, where the downward (upward) effort distortions are reported for the low (high) shadow price of a unit output.

The paper is organized as follows. Section 2 contains an overview of the related literature while section 3 presents notation and the model setup. Section 4 considers competitive search equilibrium with risk averse workers and incentive contracts, which is a baseline model of the study. Section 5 compares optimal labour compensation packages under different wage setting regimes. Section 6 presents an extension of the baseline model to account for firm heterogeneity. Section 7 contains analysis of the equilibrium efficiency and section 8 concludes the paper.

## 2 Overview of the related literature

There are a number of research directions relating this paper to the existing literature on asymmetric information in a search equilibrium framework. Guerrieri, Shimer and Wright (2010) consider the problem of adverse selection in search equilibrium with risk averse agents. Principals in their model are uninformed and compete to attract agents who are ex-ante heterogeneous and have private information about their productivity and preferences. Guerrieri, Shimer and Wright (2010) prove that an equilibrium exists where principals offer separating contracts: each contract posted attracts only one type of agent, and different types direct their search to different wages. In contrast to their study this paper investigates the problem of moral hazard in competitive search equilibrium with heterogeneous risk neutral firms and homogenous risk averse agents and therefore further extends search literature with a focus on asymmetric information.

Risk aversion in a dynamic labour market with search frictions is further studied in a research paper by Rudanko (2009). The author develops a model of competitive search equilibrium with limited commitment contracting, where firms face aggregate and idiosyncratic productivity shocks and adjust wages if either of the participation constraints of the two contracting parties is binding. Based on this model Rudanko (2009) shows that both risk aversion and limited commitment increase volatility of the market tightness.<sup>1</sup> The limited commitment mechanism in this model is similar to the state-dependent labour contract in the current study as it provides firms with an additional labour costs flexibility in the face of idiosyncratic productivity shocks. However, the second aspect of bonus payments as a discipline device is not considered in Rudanko (2009), while it serves as a major source of effort provision for workers in the current study.

The group of research papers by Moen and Rosen (2006, 2008) explicitly considers the question of efficiency wages in a dynamic search equilibrium, where efficiency wages comprise of the base wage payment and the profit-sharing bonus pay, so that the type of incentive contract is similar to the one analyzed in the current study. Both effort and the match-specific productivity (type) are private information of the worker, so that the model is characterized by a combination of moral hazard and adverse selection problems. Yet, as noted in Laffont and Rochet (1996), with an ex-ante risk, that materializes before the effort decision was made, there is a possibility to eliminate the

---

<sup>1</sup>The low market tightness volatility in a dynamic search and matching model has been first criticized in Shimer (2005).

moral hazard variable (effort) and to reduce technically the problem to the issue of pure adverse selection. This is in contrast to the current study where the moral hazard problem is not eliminated from the model and serves as a key motive for the risk-sharing between workers and firms. Summarizing the results, Moen and Rosen (2006) find that higher powered incentive contracts increase equilibrium unemployment, while Moen and Rosen (2008) prove that wages are more rigid and the unemployment rate is more volatile than in the standard model without private information.

Another crucial result obtained in Moen and Rosen (2008) is the modified Hosios condition for the equilibrium efficiency in a labour market with private information and search frictions. Originally the constrained efficiency result of competitive search equilibrium is due to Moen (1997), this study demonstrates the way search externality can be internalized via a wage posting mechanism, implying that firms set wages before they meet a potential employee and therefore consider the effect of their wage choice on unemployed workers. Moen (1997) proves that competitive search equilibrium with risk neutral workers fulfills the efficiency condition introduced in Hosios (1990) and maximizes the social welfare. Moen and Rosen (2008) extend this efficiency result to account for asymmetric information of market participants. They prove that incentive power of the equilibrium wage contract is constrained efficient in the absence of taxes and unemployment benefits.

The common constrained efficiency property of competitive search is questioned in Guerrieri (2008). The author considers a dynamic version of competitive search with private information on the side of the worker and finds that the equilibrium is different from the full information allocation and inefficient whenever the unemployment rate is away from the steady state level. Further Guerrieri (2008) finds that the full information allocation may be restored by lump-sum transfers from unemployed to employed individuals – the generalization of the money-burning effect. Overall, this paper highlights importance of money transfers and unemployment benefits in the context of search equilibrium efficiency. In this respect, Acemoglu and Shimer (1999) are the first authors to provide foundations for the analysis of an optimal unemployment insurance in a search equilibrium framework with risk averse workers. They show that an economy with risk neutral workers achieves the maximum output without an unemployment insurance, while an economy with risk averse workers requires a positive level of unemployment insurance to maximize output. Their result is extended in Coles and Masters (2006) to account for strategic bargaining and employment subsidies.

The issue of income taxation in a search equilibrium framework is considered in Boone and Bovenberg (2002), where a non lump-sum income taxation is claimed to restore efficiency in a search equilibrium when the bargaining power of workers does not fulfill the Hosios condition. Finally, the question of optimal income taxation in a search equilibrium framework with risk averse workers and unobserved search effort is analyzed in Lehmann and Linden (2004). They show that a non-linear income taxation in a combination with optimal unemployment insurance are sufficient to decentralize the optimal social planner solution.

### **3 Labour market modeling framework**

In section 4 the baseline model of the study is first introduced in a competitive search equilibrium framework where properties of the labour contract are set ex-ante and firms compete with each other in terms of a more attractive labour compensation package. Further in section 5 the model is compared to a classical model of moral hazard introduced in a search equilibrium framework without competition. Comparison of the two equilibria allows to study the transformation of an optimal labour compensation package under the different wage setting regimes.

The labour market is characterized by the following properties. There is a unit mass of infinitely lived workers and an endogenous number of firms. Workers and firms are ex-ante identical in the baseline model; the question of firm heterogeneity is addressed in section 6. Each firm has a job position which can be either filled with a worker or vacant and searching for a worker. Similarly, each worker can be either employed and producing output or unemployed and searching for a job. Unemployed workers receive a non-transferable flow value of leisure  $z$ .

Employed workers choose an optimal effort level  $e \geq 0$ . Effort is measured on a continuous scale and is not observable to the firm. All workers are risk averse and have an additively separable instantaneous utility function with respect to income and effort:  $v(x) - C(e)$ , where  $v(x)$  is an increasing and concave function of flow income  $x$  and  $C(e)$  is an increasing and convex function of effort  $e$ , such that  $C'(0) = 0$ . Here  $C(e)$  is an effort cost function expressed in worker utility units. Firms are risk neutral.

Matching between open vacancies and unemployed workers is modeled in the following way. After incurring a vacancy creation cost  $c$  the firm is entitled to post an employment contract. Each contract contains information on the labour compensation package associated with a specific surplus value for a worker conditional on being employed. Unemployed workers observe all posted contracts and direct their search to particular jobs. Both unemployed workers and firms correctly anticipate the number of job matches  $m(u, v)$  and the market tightness  $\theta = u/v$  associated with a specific surplus value. Here  $u$  is the number of unemployed individuals applying for jobs promising this surplus value and  $v$  is the number of vacancies offering it. The matching function  $m(u, v)$  is assumed to be increasing in both arguments, concave, and homogeneous of degree 1. Then the job finding rate  $\lambda(\theta)$  and the vacancy filling rate  $q(\theta)$  associated with a specific worker surplus are defined in the following way:

$$\begin{aligned}\lambda(\theta) &= \frac{m(u, v)}{u} = m(1, \theta) \\ q(\theta) &= \frac{m(u, v)}{v} = m\left(\frac{1}{\theta}, 1\right)\end{aligned}$$

Also denote  $\eta_q = -q'(\theta)\theta/q(\theta)$  – elasticity of the job filling rate  $q(\theta)$ , in the following it is assumed that  $\eta_q$  is nondecreasing in  $\theta$ .

Once employed workers choose an optimal level of effort  $e$  and start producing with an initial flow productivity  $y = y^H$ . Productivity  $y$  is stochastic for every employment relationship and the productivity shocks arrive with a Poisson arrival rate  $\delta$ . Upon the shock, the productivity variable  $y$  can take one of the two possible realizations  $\{y^H, y^L\}$  so that the following productivity switching rule applies:

$$y = \begin{cases} y^H & \text{with probability } p(e) \\ y^L & \text{with probability } 1 - p(e) \end{cases}$$

where  $p(e)$  is an increasing concave function of effort. This means that jobs where workers exert more effort  $e$  are characterized by longer expected durations of a high productivity realization  $y^H$ . Productivity realizations are observable and contractible, in addition firms have an option to dismiss or retain the worker upon the low productivity realization  $y^L$ . However, it is assumed that  $y^H$  is high enough to start the employment relationship so that it is never optimal to dismiss a worker in the high productivity state. Jobs may additionally be destroyed for exogenous reasons which happens with a Poisson arrival rate  $\gamma$ .

The paper explicitly addresses a variable wage contract with a state dependent worker remuneration. In the case of two output states  $\{y^H, y^L\}$  the labour compensation package takes form of  $(w^H, w^L)$ , where  $w^H$  is paid to the worker if  $y = y^H$  and  $w^L$  is paid if  $y = y^L$ . The vector of labour compensation  $(w^H, w^L)$  can be equivalently represented as a bonus pay contract of the type  $(w, b)$  where  $w = w^L$  – unconditionally paid base wage and  $b = w^H - w^L$  is a conditionally paid bonus pay. As a result, the following rule applies:

$$\text{Payment} = \begin{cases} w^H = w + b & \text{if } y = y^H \\ w^L = w & \text{if } y = y^L \end{cases}$$

The bonus pay labour contract described above can be viewed as an alternative to the fixed wage labour contract investigated in a complementary study by Chizhova (2007). In a fixed wage contract regime in Chizhova (2007) firms use dismissals as a discipline device and it is the income risk between employment and unemployment that creates incentives for workers to exert effort.

In a variable wage contract regime explored in this study firms use both base wages and bonus payments in order to provide workers with correct incentives. Bonus payments are paid conditionally on output realizations so that it is the income risk during employment that motivates workers to exert effort. Moreover, the variable wage contract provides firms with an additional flexibility in their choice of the labour compensation package. In particular, firms possess a valuable option to adjust the base wage  $w$  in order to avoid unprofitable worker dismissals in the low productivity state. In the equilibrium firms pay a "hiring premium" as a result of the labour market competition forcing firms to share the rents, and a "motivation premium" in order to account for the internal moral hazard problem within the firm. The primary focus of this paper is on an interaction between these two types of wage premia. Originally, a model with the simultaneous rent-sharing and the problem of moral hazard has been investigated in Demougin and Helm (2006), and Bental and Demougin (2006).

## **4 Bonus pay in competitive search equilibrium**

This section explores a model of competitive search, where workers are risk averse, while the firms are risk neutral. Firms set the terms of employment contract before they meet a potential employee. This wage setting regime creates competition between firms with respect to the value of worker remuneration and permits an analysis

of the interaction between the risk sharing and the rent sharing motives in a search equilibrium framework. The model is set in continuous time.

## 4.1 Decentralized equilibrium

### 4.1.1 Workers: optimal effort choice

Suppose first that in the low productivity state dismissals are not profitable for the firm, so that there is no dismissal threat for workers. The corresponding sufficient condition for this strategy is derived later in the paper. Denote  $W^L$  and  $W^H$  – worker surplus values in the low and high productivity states  $y^L$  and  $y^H$  respectively. Similarly let  $e^L$  and  $e^H$  denote worker effort choices. Bellman equations for employed and unemployed individuals can be written as:

$$\begin{aligned} rW^L &= \max_{e^L \geq 0} \{v(w^L) - C(e^L) + \delta p(e^L)(W^H - W^L) + \gamma(U - W^L)\} \\ rW^H &= \max_{e^H \geq 0} \{v(w^H) - C(e^H) + \delta(1 - p(e^H))(W^L - W^H) + \gamma(U - W^H)\} \\ rU &= v(z) + \lambda(\theta)(W^H - U) \end{aligned} \quad (4.1)$$

where notation  $W^H = W^H(w^H, w^L)$  and  $W^L = W^L(w^H, w^L)$  is used to simplify the representation. In each of the two productivity states workers set effort so as to balance the gain, reflected in  $\delta p(e)(W^H - W^L)$ , and the cost of an additional unit of effort, reflected in  $C(e)$ . The optimal effort choice for workers is summarized in the following lemma:

**Lemma 1:** *Optimal effort choice is constant across productivity states  $e^L = e^H = e(w^L, w^H)$  and is implicitly given by the following equation:*

$$d(v(w^H) - v(w^L)) = \frac{C'(e)}{p'(e)} \equiv \pi(e) \quad (4.2)$$

where  $d = \delta/(r + \gamma + \delta)$ . *Optimal effort  $e(w^L, w^H)$  is an increasing function of  $w^H$  (motivation effect) but a decreasing function of base wages  $w^L$  (discouragement effect). If  $\pi''(e) > 0$  for  $\forall e > 0$  then  $e(w^L, w^H)$  is a concave function of  $w^H$  and a convex function of  $w^L$ . Assumption  $\pi''(e) > 0$  also implies  $e''_{w^H w^L} > 0$ .*

**Proof:** Appendix I.

The above assumptions about the effort cost function  $C(e)$  and the output function  $p(e)$  imply that  $\pi(e)$  is an increasing function of effort, so that a higher wage  $w^H$  is creating additional incentives for workers to exert effort. A higher wage  $w^L$  produces the opposite effect: the relative income risk  $w^H - w^L$  is reduced with a higher base wage. In addition if  $\pi''(e) > 0$  for  $\forall e > 0$  optimal effort is increasing at a declining rate in  $w^H$  and decreasing at a declining rate in  $w^L$ . In the bonus payment interpretation of the labour contract lemma 1 implies  $e'_b > 0$  and  $e'_w = e'_{w^H} + e'_{w^L} < 0$ . This means that optimal effort is increasing in the bonus pay  $b$  and decreasing in the base wage  $w$ .

Denote  $R^H(w^H, w^L, U)$  – worker rents from employment defined as  $R^H(w^H, w^L, U) = W^H(w^H, w^L) - U$ . The following lemma describes the properties of  $R^H(w^H, w^L, U)$ .

**Lemma 2:** *Worker rents  $R^H = R^H(w^H, w^L, U)$  can be expressed as follows:*

$$(r + \gamma)R^H = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e) - rU,$$

where  $e = e(w^H, w^L)$  and  $\hat{p}(e) = (r + \gamma + \delta p(e))/(r + \gamma + \delta)$ .  
 $R^H(w^H, w^L, U)$  is increasing in both arguments  $w^H$  and  $w^L$ .

**Proof:** Appendix I.

Expression  $\hat{p}(e)$  stands for the effective probability of the high output realization  $y^H$ . In this state workers obtain the high wage flow  $w^H$  with a corresponding utility  $v(w^H)$ . Similarly,  $1 - \hat{p}(e)$  stands for the probability of the wage flow  $w^L$ , so that the worker rent can be expressed as a linear combination of  $v(w^H)$  and  $v(w^L)$  with the weights being equal to the probabilities of the respective utility flows. Also notice that from Lemma 2 it follows that an increase in either  $w^H$  or  $w^L$  is strictly beneficial for the worker, even if effort does not adjust. In addition, workers adjust their effort in order to maximize these gains: increase effort in response to a higher wage value  $w^H$  and decrease effort in response to a higher wage  $w^L$ . This is in line with the result from lemma 1.

Firms with an open vacancy anticipate a relationship between the posted contract  $(w^H, w^L)$  and the arrival rate of workers. In order to characterize this relationship, rewrite (4.1) in terms of the worker job-finding rate  $\lambda(\theta)$ , implicitly defining the mar-

ket tightness function  $\theta = \theta(w^H, w^L, U)$ :

$$\lambda(\theta) = \frac{(rU - v(z))}{R^H(w^H, w^L, U)} \quad (4.3)$$

Equation (4.3) describes a functional dependence between the worker rents  $R^H = R^H(w^H, w^L, U)$  and the market tightness  $\theta$ : an increase in either of the labour compensation components  $w^H$  and  $w^L$  attracts more job applicants and has a negative effect on the job-finding rate  $\lambda(\theta)$ .

#### 4.1.2 Firms: optimal contract

Consider the labour demand side of the market. Denote  $J^H$  – firm surplus from a filled job position in the high output state  $y = y^H$ , similarly denote  $J^L$  – firm surplus from a filled job position in the low output state  $y = y^L$ . Both surplus values  $J^H$  and  $J^L$  are defined conditionally on retaining the worker in the low productivity state. Bellman equations for an open vacancy and a filled job can be written as follows:

$$\begin{aligned} rJ^L &= y^L - w^L + \delta p(e)(J^H - J^L) - \gamma J^L \\ rJ^H &= y^H - w^H + \delta(1 - p(e))(J^L - J^H) - \gamma J^H \\ rV &= -c + q(\theta)(J^H - V) \end{aligned} \quad (4.4)$$

where  $J^H = J^H(w^H, w^L)$ ,  $J^L = J^L(w^H, w^L)$  and  $V$  is the firm surplus from an open vacancy. The filled job surplus  $J^H(w^H, w^L)$  can be expressed in the following way:

$$(r + \gamma)J^H(w^H, w^L) = \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L),$$

meaning that the net productivity flow  $y^H - w^H$  accrues to firms with an effective probability  $\hat{p}(e)$ , while the net productivity flow  $y^L - w^L$  accrues with an effective probability  $(1 - \hat{p}(e))$ .

Firms choose the vector of labour compensation  $(w^H, w^L)$  in order to maximize the vacancy surplus  $V$  given the effort response function  $e(w^H, w^L)$  and the market tightness response function  $\theta(w^H, w^L, U)$  (see equations (4.2) and (4.3)):

$$V(\theta(U)) \equiv \max_{w^H, w^L} V(w^H, w^L, e(w^H, w^L), \theta(w^H, w^L, U)) \quad (4.5)$$

$$\text{s.t. } R^H(w^H, w^L, U) \geq 0 \quad (4.6)$$

Condition (4.6) is the worker participation condition, it means that workers reject job offers with negative surplus values. Firms face the following trade-off. On the one hand, increasing  $w^H$  and  $w^L$  by one unit respectively results in lower net profit flows  $(y^H - w^H)$  and  $(y^L - w^L)$ , but on the other hand, the job filling rate  $\theta(w^H, w^L, U)$  will be higher, while the optimal effort choice of workers  $e(w^H, w^L)$  will be lower. Solution to the firm optimization problem is summarized in proposition 1.

**Proposition 1:** *Competitive search equilibrium with bonus payments is characterized by a tuple of variables  $\{e, w^H, w^L, U, \theta\}$  satisfying conditions (4.2), (4.3), as well as equations (a) and (b) below and the free entry condition  $V(\theta(U)) = 0$ . The necessary condition for the equilibrium existence is:*

$$y^L - w^L + d(\Delta y - \Delta w)p(e) \geq 0 \quad (4.7)$$

where  $e = e(w^H, w^L)$ ,  $\Delta y = y^H - y^L$  and  $\Delta w = w^H - w^L$ .

(a) *The optimal value of  $w^H$  is obtained from equation:*

$$\eta_{\hat{p}} = (1 - \hat{p}(e)) \left[ 1 - \frac{v'(w^H)}{v'(w^L)} \right] \quad (4.8)$$

where  $\eta_{\hat{p}} \equiv -\partial \ln \hat{p}(e) / \partial \ln(\Delta y - \Delta w)$  – negative of the elasticity of the effective probability  $\hat{p}(e)$  with respect to the flow profit difference  $\Delta y - \Delta w$ .

(b) *The optimal value of  $w^L$  is obtained from the modified Hosios condition:*

$$J^H = [1 - \eta_{\hat{p}}] \frac{1 - \eta_q}{\eta_q} \frac{R^H}{v'(w^H)} \quad (4.9)$$

where  $J^H = J^H(w^H, w^L)$  and  $R^H = R^H(w^H, w^L, U)$ .

**Proof:** Appendix II.

In the following subindex "C" is attached to the tuple  $\{e, w^H, w^L, U, \theta\}$  corresponding to the unrestricted competitive search equilibrium with bonus payments described in proposition 1. Equation (4.7) is a necessary condition for firms to retain workers in case when output is low. This equation is obtained from the requirement  $J^L > V = 0$ , so that firms do not dismiss workers and continue employment relationship in the low productivity state  $y = y^L$ . This requirement is also sufficient to guarantee the participation of firms as  $J^H \geq J^L \geq 0$ .

Equation (4.8) can be interpreted as a risk sharing curve (RSS). Notice that for the risk neutral workers with  $v(w) = w$ , the right-hand side of equation (4.8) is zero, so that in the equilibrium  $\eta_{\hat{p}} = 0$  and  $\Delta w = \Delta y$  since the elasticity variable  $\eta_{\hat{p}}$  becomes:

$$\eta_{\hat{p}} \equiv -\frac{\partial \ln \hat{p}(e)}{\partial \ln(\Delta y - \Delta w)} = \frac{(\Delta y - \Delta w)}{\hat{p}(e)} \hat{p}'(e) e'_{w^H}$$

Variable  $\Delta w$  can be interpreted as an additional bonus payment in access of the base wage  $w^L$ , so that the risk neutral case corresponds to a situation where workers are not sensitive to risk and firms set the maximum value of the bonus payment  $b = \Delta y$  in order to achieve the maximum effort. When workers are risk averse with an increasing and concave utility function, the right hand side of equation (4.8) is positive as  $v'(w^L) > v'(w^H)$ , so that  $0 < \eta_{\hat{p}} < 1$  and  $b = \Delta w < \Delta y$ . This means that when workers are risk averse the total productivity risk reflected in  $\Delta y$  is split in a proportion  $[\Delta w, \Delta y - \Delta w]$  between workers and firms respectively.

The risk sharing curve (RSS) is obtained from the following condition:

$$\frac{\partial J^H / \partial w^H}{\partial R^H / \partial w^H} = \frac{\partial J^H / \partial w^L}{\partial R^H / \partial w^L} \quad (4.10)$$

implying that in the equilibrium the firm's and the worker's indifference curves  $J^H = \text{const}$  and  $R^H = \text{const}$  should be tangent to each other in the space  $(w^H, w^L)$ .

Consider risk averse workers with a logarithmic utility function  $v(x) = \ln(x)$ , the risk sharing curve (RSS) can be rewritten as:

$$d(\Delta y - b) \hat{p}'(e) = \hat{p}(e)(1 - \hat{p}(e)) b \pi'(e) \quad (4.11)$$

It can be shown that for sufficiently low success probability  $\hat{p}(e) \leq 1/2$  the risk sharing curve (RSS) defines a positive relationship between wages and bonus payments implying complementarity between these two variables. The probability assumption  $\hat{p}(e) \leq 1/2$  is sufficient but not a necessary condition here. The complementarity effect can be explained by the fact that effort is decreasing in the base wage  $w$  so that the optimal bonus should increase in order to mild the effort reduction. This effect is illustrated in figure 1. Also notice that as the base wage is increasing, the bonus payment  $b$  is approaching the maximum level of  $\Delta y$ .

Equation (4.9) can be interpreted as a rent sharing curve (RNS). It defines the share of total surplus retained by the firms  $J^H$ . The rent sharing equation is obtained from the following condition:

$$J^H = \left[ -\frac{\partial J^H / \partial w^H}{\partial R^H / \partial w^H} \right] \frac{1 - \eta_q}{\eta_q} R^H \quad (4.12)$$

implying that in the equilibrium indifference curves  $U = const$  and  $V = 0$  should be tangent to each other in the space  $(w^L, \theta)$ . As follows from the above equation the rent sharing curve is defined for wage values  $w^H$  such that  $\partial J^H / \partial w^H < 0$  for the set of feasible contracts. This implies that the firm surplus  $J^H$  is strictly decreasing in both arguments  $w = w^L$  and  $b = \Delta w$ . In order to interpret the right hand side of equation (4.9) rewrite it using the risk sharing curve:

$$J^H = \left[ \frac{\hat{p}(e)}{v'(w+b)} + \frac{1 - \hat{p}(e)}{v'(w)} \right] \frac{1 - \eta_q}{\eta_q} R^H \quad (4.13)$$

This means that in the equilibrium with bonus payments the modified Hosios condition simplifies to the risk-adjusted Hosios condition. It follows from the fact that the term in brackets on the right hand side of equation (4.13) can be interpreted as an inverse of the shadow price of a unit output for the worker. The price of a single output unit is state-dependent, meaning that, when productivity is high and workers obtain the income flow  $w + b$  a unit transfer from firms to the workers results in a utility increase  $v'(w + b)$  which is lower than  $v'(w)$  – utility gain for a worker in the low productivity case. Overall the price  $1/v'(w + b)$  applies with a probability  $\hat{p}(e)$ , while the price  $1/v'(w)$  applies with an opposite probability.

Consider the case of risk averse workers with a logarithmic utility function described above, the rent sharing curve then becomes:

$$J^H = [w + \hat{p}(e)b] \frac{1 - \eta_q}{\eta_q} R^H \quad (4.14)$$

As follows from lemma 2 the worker rent  $R^H(w^H, w^L)$  is increasing in both arguments. In contrast, the inverse of the shadow price  $w + \hat{p}(e)b$  is increasing in the bonus pay  $b$  but the effect of wage  $w$  is generally ambiguous. Nevertheless, it can be shown that for sufficiently low output risk  $\Delta y$  (such that  $w + \hat{p}(e)\Delta y$  is an increasing function of  $w$ ) the worker utility gain from a higher wage expressed in terms of the firm surplus units is increasing in  $w$ . This in turn means that the rent sharing curve is describing a substitution effect between the wage and the bonus payments. This effect is also illus-

trated in figure 1. The substitution effect can be explained by the fact that the market tightness  $\theta$  is decreasing in both arguments  $w$  and  $b$  so that the optimal bonus should decrease in order to mild the effect of a lower  $\theta$  in response to a higher value of  $w$ .

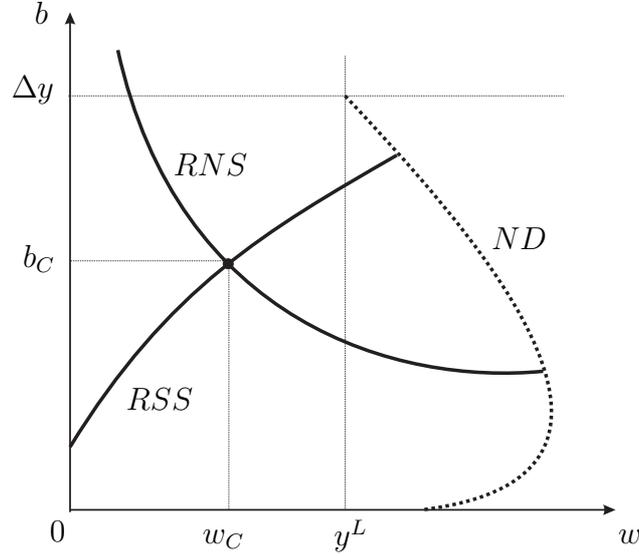


Figure 1: Optimal labour compensation package

The dashed curve  $ND$  on figure 1 stands for the no-dismissal condition and corresponds to equation  $J^L = 0$ . This means that the labour compensation packages  $[w, b]$  outside the area given by the curve  $ND$  do not satisfy the no-dismissal condition  $J^L > V = 0$ . The equilibrium labour contract  $[b_C, w_C]$  obtains at the intersection of the risk sharing curve ( $RSS$ ) and the rent sharing curve ( $RNS$ ) and implies risk sharing between a firm and a worker since  $b_C < \Delta y$ . This is due to the fact that if workers are risk averse firms face a trade-off between incentives provision and income insurance. As a result the optimal bonus payment is lower than in the case of risk neutral workers since firms provide partial income insurance to the workers. Also note that as follows from equations (4.10), (4.12)  $\partial J^H / \partial w^H < 0$  along both curves ( $RSS$  and  $RNS$ ) as well as in the equilibrium. This means that firms have incentives to reduce the amount of the bonus pay ex-post after the vacancy is filled with a worker. The same is true for the base wage  $w_C$ , so that the firm commitment to the ex-ante labour contract is a necessary condition for the equilibrium existence.

## 4.2 Limited liability constraint

Introducing a contract with state dependent wage payments  $w^H = w + b$  and  $w^L = w$  in competitive search equilibrium is compatible with a situation, where the base wage  $w$  is below the value of leisure  $z$ . In extreme cases when workers possess over sufficient exogenous income flows the base wage  $w$  may even take on negative values, so that it is also instructive to consider a restricted firm optimization problem with a wage restriction of the type  $w = w^L \geq \bar{w}$ . The wage restriction may have different origins. One possibility to interpret a wage restriction is to view it as a limited liability constraint of the worker. In case when workers face exogenous financial constraints their job acceptance decision may additionally depend on the restriction  $w \geq \bar{w}$ , where the  $\bar{w}$  may stand for a continuous outflow from the worker income corresponding to his exogenous financial obligation. The limited liability constraint is particularly important in situations of moral hazard, where effort has random effect on the outcome, which may deter economic agents from entering the contract with an unlimited liability.

Another possibility to view the wage restriction may be explained by a minimum wage requirement on the government level or on the industry level resulting from a bargaining process with trade unions. Solution to the firm optimization problem with a wage restriction of the type  $w^L \geq \bar{w}$  is summarized in proposition 2.

**Proposition 2.** *Consider a binding wage restriction of the type  $w^L \geq \bar{w}$ . The restricted competitive search equilibrium with bonus payments is characterized by a tuple  $\{e, w^H, w^L, U, \theta\}$  satisfying  $w^L = \bar{w}$ , equations (4.2), (4.3), as well as the rent sharing equation (4.9) above and the free entry condition  $V(\theta(U)) = 0$ . The necessary condition for the equilibrium existence is:*

$$y^L - \bar{w} + d(\Delta y - \Delta w)p(e) > 0$$

where  $e = e(w^H, \bar{w})$ ,  $\Delta y = y^H - y^L$  and  $\Delta w = w^H - \bar{w}$ .

**Proof:** Differentiate equations (4.3), (4.4) with respect to  $w^H$  and use the fact that  $V = 0$  in the equilibrium, this yields:

$$\frac{\theta}{\eta_q J} \cdot \frac{\partial J}{\partial w^H} = \frac{\partial \theta}{\partial w^H} = -\frac{\theta}{(1 - \eta_q)R} \cdot \frac{\partial R}{\partial w^H} \quad (4.15)$$

Equation (4.15) is equivalent to the rent sharing equation (4.13). ◇

In the following subindex "CR" is attached to the tuple  $\{e, w^H, \bar{w}, U, \theta\}$  characterizing competitive search equilibrium with a binding limited liability constraint  $w \geq \bar{w}$ .

The restricted competitive search equilibrium is obtained at the intersection of the rent sharing curve and the wage restriction  $w = \bar{w}$ . Consider the case of risk neutral workers, so that the optimal initial contract is given by  $[b_C = \Delta y, w_C]$  and the corresponding surplus of the unemployed workers is  $U_C$  (see figure 2). In the short term perspective, corresponding to the partial equilibrium effect, with a fixed surplus value  $U_C$  restricting the base wage  $w = \bar{w}$  implies a lower value of the bonus pay  $b$  so that  $b(U_C, \bar{w}) < b(U_C, w_C) = \Delta y$ .

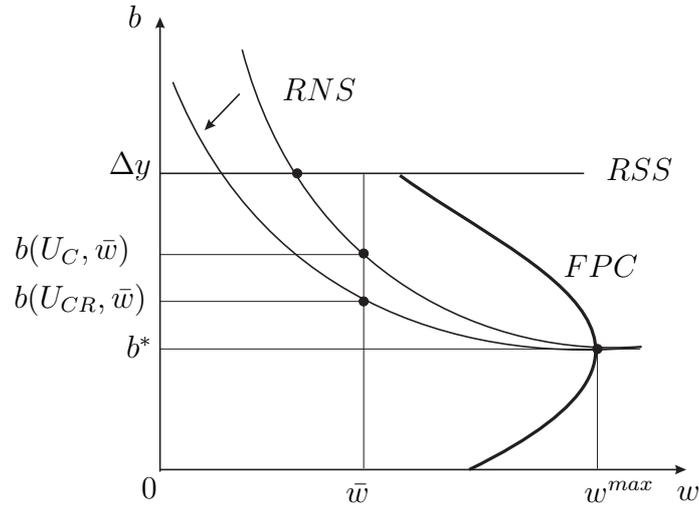


Figure 2: Limited liability constraint in CSE: risk neutral workers

In the long run perspective corresponding to the general equilibrium effect surplus of the unemployed workers  $U$  is decreasing, this is due to the fact that firms earn lower profits and hence less jobs are created. A lower value of  $U$  implies a downward rotation of the rent sharing curve resulting in a further reduction of the incentive pay:  $b(U_{CR}, \bar{w}) < b(U_C, w_C)$ . Furthermore, a lower bonus value results in a lower value of the equilibrium effort level, so that the total match surplus is lower and workers face lower motivation incentives. Unemployed workers loose from a binding wage restriction so that  $U_C > U_{CR}$ .

Note also that a continuum of the rent sharing curves corresponding to different values of  $U$  intersect at the contract  $[b^*, w^{max}]$ . This labour contract is obtained from a system of equations:

$$J^H(w, b) = 0, \quad \frac{\partial J^H(w, b)}{\partial b} = 0$$

Here the first curve  $J^H(w, b) = 0$  corresponds to the binding firm participation constraint denoted  $FPC$ , while the second curve  $\partial J^H / \partial b = 0$  is a restricted risk sharing condition and is investigated in more details in section 5.2. It can be shown that risk neutrality implies a constant optimal value  $b^*(w) = b^* \forall w$  as illustrated in figure (2). This result implies that the optimal bonus value  $b_{CR}$  is bounded in the following way:  $\Delta y = b_C \geq b_{CR} \geq b^*$  meaning that a binding wage restriction induces risk sharing between workers and firms even if workers are risk neutral.

## 5 Search equilibrium with ex-post wage setting

### 5.1 Decentralized equilibrium: comparison

In order to illustrate the effect of the ex-ante wage setting mechanism on the equilibrium labour contract and to be able to decompose wages into the motivation and the hiring premia, consider a labour market with an ex-post wage setting regime. The ex-post wage setting regime arises in labour markets where job advertisements are not informative about the size of the labour compensation. In the presence of labour market uncertainty unemployed workers can not direct their search towards the better paid jobs, while firms don't need to account for the effect of wages on the number of job applications and market tightness  $\theta$ . The optimal strategy of the firm is then to maximize the job surplus  $J^H$  with respect to  $w^H$  and  $w^L$  subject to the worker incentive compatibility constraint (4.2) and the worker participation constraint  $R(w^H, w^L, U) \geq 0$ . The firm optimization problem in the ex-post wage setting regime can be stated as follows:

$$\begin{aligned} (r + \gamma)J^H &= \max_{w^H, w^L} \{ \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \} \\ \text{s.t.} \quad & R^H(w^H, w^L, U) \geq 0 \quad \text{and} \quad e = e(w^H, w^L). \end{aligned} \quad (5.1)$$

In the ex-post wage setting regime there are no incentives for firms to leave rents to the workers, as any wage offer delivering a non-negative rent to the worker will be accepted. This means that in the equilibrium, it should be true that  $R^H(w^H, w^L, U) = 0$ . Zero worker rents in the equilibrium imply a monopsonistic type of the market, where firms obtain the full match surplus, and therefore the equilibrium with an ex-post wage

setting resembles properties of the Diamond (1971) equilibrium. The paradox of the Diamond equilibrium is that the monopsony outcome obtains as long as the search costs of workers are positive. Competitive equilibrium outcome in the Diamond model does not arise even when the search costs of workers are arbitrarily small and the number of firms is sufficiently large.

In addition notice, that the search equilibrium with an ex-post wage setting is a direct extension of a classical contract theory solution for the optimal bonus pay in the presence of moral hazard (as a reference see Laffont and Martimort (2002) and Bolton and Dewatripont (2005)). The extension involves introducing the classical contract theory model with bonus payments in a general equilibrium framework where the labour market is characterized by search frictions. However, the (ex-post) wage setting mechanism is preserved unchanged. Solution to the firm optimization problem in the ex-post wage setting regime is summarized in proposition 3.

**Proposition 3:** *The search equilibrium with bonus payments and ex-post wage setting is characterized by a tuple  $\{e, w^H, w^L, U, \theta\}$  satisfying equations (4.2),  $rU = v(z)$ , the risk sharing equation (4.8) as well as equation (a) below and the free entry condition  $V(\theta) = 0$ .*

(a) *The optimal value of  $w^L$  is obtained from the worker participation constraint:*

$$R^H(w^H, w^L, U) = W^H(w^H, w^L) - U = 0 \quad (5.2)$$

**Proof:** Appendix III.

In the following subindex "P" is attached to the tuple  $\{e, w^H, w^L, U, \theta\}$  characterizing the search equilibrium with bonus payments under the ex-post wage setting regime.

The search equilibrium with the ex-post wage setting is obtained at the intersection of the risk sharing curve (4.8) and the worker participation constraint  $R^H(w^H, w^L, U) = 0$  denoted  $WPC$ . In case when workers are risk neutral the risk sharing condition again implies  $b = \Delta y$ . The base wage  $w$  is then set according to the worker participation constraint  $R^H(w^H, w^L, U) = 0$ . When the value of the bonus payment is zero there are no incentives for firms to set the base wage  $w$  above the reservation value  $z$ . This is illustrated in figure 3. Notice also that the equilibrium implies  $w_P < z$  meaning that companies use both the award based motivation reflected in  $b_P > 0$  and the

punishment based motivation reflected in  $w_P - z < 0$ . The motivation premium in this case can be expressed as  $b_P$ , while the motivation penalty is  $w_P - z$ .

In addition the rent sharing equation (4.9) implies  $R^H(w_C^H, w_C^L, U_C) > 0$  in the competitive search equilibrium with bonus payments. This is different under the ex-post wage setting regime, where in the equilibrium it is true that  $R^H(w_P^H, w_P^L, U_P) = 0$ . This means that the rent sharing curve in competitive search equilibrium is situated above the participation constraint in the space  $(b, w)$ . This result also takes account of the fact that  $U_C > U_P$ , which follows from the inequality  $rU_C = v(z) + \lambda(\theta_C)R^H(w_C^H, w_C^L, U_C) > v(z) = rU_P$ . This result implies that unemployed workers are strictly better off in competitive search equilibrium where wages are set ex-ante as opposed to the ex-post wage setting regime.

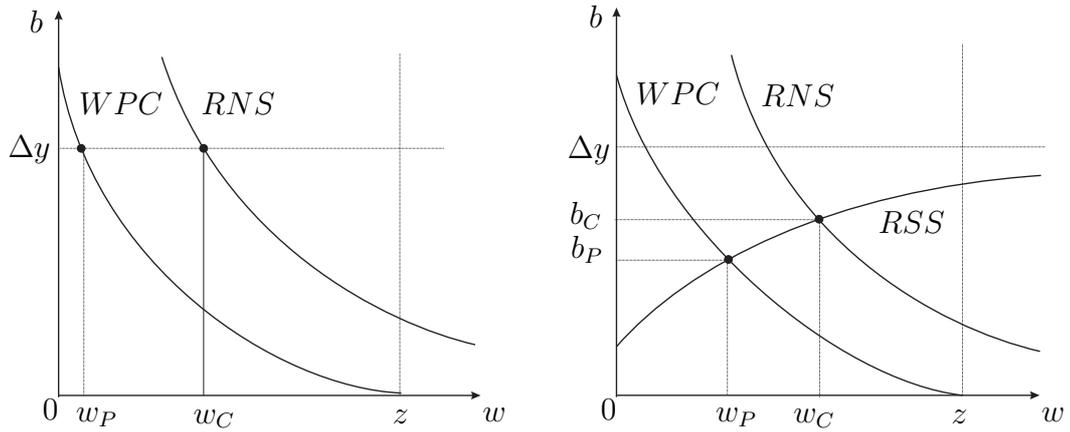


Figure 3: Optimal contracts under ex-post vs. ex-ante wage setting. Left: risk neutral workers. Right: risk averse workers

Now compare the equilibrium labour contracts  $w_C, b_C$  vs.  $w_P, b_P$  under respectively the ax-ante and the ex-post wage setting. This comparison is also illustrated in figure 3. When workers are risk neutral the optimal bonus payment is equal in both types of the labour market, namely  $b_C = b_P = \Delta y$ , so that the motivation premia are also the same. However, the optimal wages are different, in particular it is true that  $w_C > w_P$ , implying that firms in competitive search equilibrium pay an additional hiring premium to their employees. If workers are risk averse with a logarithmic utility function described above it is true that  $b_C > b_P$  and  $w_C > w_P$ , so that both expressions  $b_C - b_P > 0$  and  $w_C - w_P > 0$  stand for the hiring premia in competitive search equilibrium.

## 5.2 Limited liability constraint

As follows from the above analysis, unrestricted search equilibria with risk neutral workers always yield the maximum value of the bonus pay  $b = \Delta y$  so that in the equilibrium there is no risk sharing between workers and firms. However, this is not the case if a wage restriction, explained by the limited liability or the minimum wage requirement is binding. This section considers properties of the search equilibrium with an ex-post wage setting and a binding wage constraint. In the presence of a wage restriction the optimal strategy of the firm is then to maximize the job surplus  $J^H$  with respect to  $w^H$  and  $w^L$  subject to the wage constraint, the worker incentive compatibility constraint (4.2) and the worker participation constraint. The firm optimization problem in the ex-post wage setting regime can be stated as follows:

$$\begin{aligned} (r + \gamma)J^H &= \max_{w^H, w^L} \{ \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \} \\ \text{s.t.} \quad &w \geq \bar{w}, \quad R^H(w^H, w^L, U) \geq 0 \quad \text{and} \quad e = e(w^H, w^L) \end{aligned}$$

Solution to this optimization problem is presented in proposition 4.

**Proposition 4:** *Consider a binding wage restriction of the type  $w \geq \bar{w}$ . The restricted search equilibrium with bonus payments and ex-post wage setting is characterized by a tuple  $\{e, b, w, U, \theta\}$  satisfying requirements  $w = \bar{w}$ , (4.1), (4.2) as well as the free entry condition  $V(\theta) = 0$ ; the optimal bonus payment is obtained as  $b = \max(b^*, b^{**})$ , where  $b^*$  is solution to  $\eta_{\hat{p}} = 1$ , which can be written as:*

$$(\Delta y - b)\hat{p}'_e e'_b = \hat{p}(e), \quad \text{where} \quad e = e(b, \bar{w}) \quad (5.3)$$

and  $b^{**}$  is obtained from the worker participation constraint  $R(\bar{w}, b^{**}, U) = 0$ .

**Proof:** Appendix III.

In the following subindex "PR" is attached to the tuple  $\{e, w^H, w^L, U, \theta\}$  characterizing the restricted search equilibrium with bonus payments under the ex-post wage setting regime.

Equation (5.3) can be interpreted as a restricted risk sharing condition (see figure 4). It comes from the firms first order condition  $\partial J^H / \partial b = 0$  and defines the risk sharing proportions between a firm and a worker. Note that, if workers are risk neutral the

optimal effort is independent of the wage  $\bar{w}$ , so that equation (5.3) produces a fixed value of the bonus pay  $b^* < \Delta y$ . However, if  $\bar{w}$  and the corresponding value  $b^*(\bar{w})$  are not sufficient to fulfill the worker participation constraint denoted  $WPC$  and to provide workers with a necessary job rent, the firm will increase the optimal bonus pay to the point where workers are just indifferent between working and staying unemployed, this value of the bonus pay is denoted by  $b^{**}$ , so that  $b_{PR} = \max(b^*, b^{**})$ .

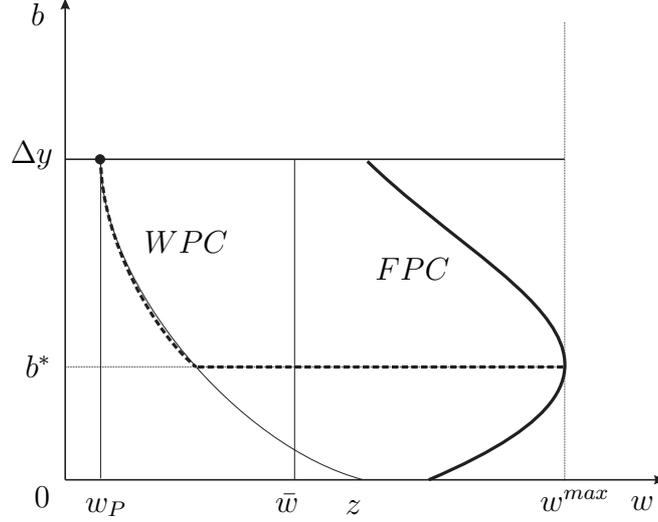


Figure 4: Limited liability in SE with ex-post wage setting; risk neutral workers

## 6 Heterogeneous capital intensity

This part of the paper presents extensions of the models in sections 4 and 5 for the case of heterogeneous jobs. Suppose that the firm entry mechanism is as follows. Firms pay an ex-ante capital investment  $K$  in order to enter the market, the capital investment is irreversible. Upon entry each firm draws a firm-specific capital intensity  $k$  from distribution  $F(k)$  with the range of capital intensity values  $[\underline{k}, \bar{k}]$ . Assume that the minimal capital intensity value  $\underline{k}$  is sufficient for the firm to stay in the market, so that  $V(\underline{k}) = V(w^H(k), w^L(k)) > 0$ . This means that the free entry condition becomes:

$$K = \int_{\underline{k}}^{\bar{k}} V(k) dF(k)$$

The capital intensity distribution creates ex-post productivity diversity in the economy. Capital is included into the model in a multiplicative way, so that the worker produc-

tivity is defined according to the following rule:

$$\begin{cases} y^H = a^H f(k) \\ y^L = a^L f(k) \end{cases}$$

where  $\Delta a = a^H - a^L > 0$  and  $f(k)$  is a standard production function in the intensive form, increasing and concave in  $k$ . This approach creates productive heterogeneity among jobs, where *ceteris paribus* jobs with a higher capital stock intensity are characterized by a higher expected output flow  $m(k)$ , but also face a higher variance of the output  $\sigma^2(k)$  and a higher risk  $\Delta y = \Delta a f(k)$ :

$$\begin{cases} m(k) = (a^L + \Delta a \hat{p}(e)) f(k) \\ \sigma^2(k) = \hat{p}^2(e) (1 - \hat{p}(e))^2 f^2(k) \Delta a^2 \end{cases}$$

In order to make a reference about the correlation between wages and bonus payments in this economy consider two jobs with capital intensities  $k_2 > k_1 \in [\underline{k}, \bar{k}]$ . Both firms face the same worker participation constraint  $R^H(k) = R^H(w^L(k), w^H(k), U) \geq 0$ , where the unemployed worker surplus value  $U = U(k) \forall k$  is now obtained from:

$$rU = v(z) + \lambda(\theta(k)) R^H(k) \quad (6.1)$$

However, the two risk-sharing curves faced by firms are different due to the fact that  $\Delta y(k_2) > \Delta y(k_1)$ . As follows from the risk sharing equation (4.8) the optimal bonus payment  $b(k) = \Delta w(k)$  is an increasing function of  $\Delta y$ , so that the risk sharing curve of the more capital intensive firm is situated above the corresponding curve of the less capital intensive firm. This result is illustrated in figure 5. Notice that equilibrium contracts in the search equilibrium framework with an ex-post wage setting regime are obtained at the intersection between the worker participation constraint and the risk sharing curve, so that it can be concluded that  $w^L(k_2) < w^L(k_1)$  and  $b(k_2) > b(k_1)$ . The more capital intensive firm is more productive in expectation so that the marginal gain of a unit effort increase is larger in this firm compared to the less productive firm. In order to achieve a higher effort level the firm sets optimally a higher value of bonus pay  $b(k_2)$  and a lower value of the base wage  $w^L(k_2)$ . Note, that both actions lead to an increase in the worker effort. The lower value of the base wage also guarantees that the worker participation constraint is binding. Overall, the search equilibrium with an ex-post wage setting regime exhibits the substitution effect between wages and bonus payments and fails to account for the complementarity effect observed in a number of empirical studies (see table 1).

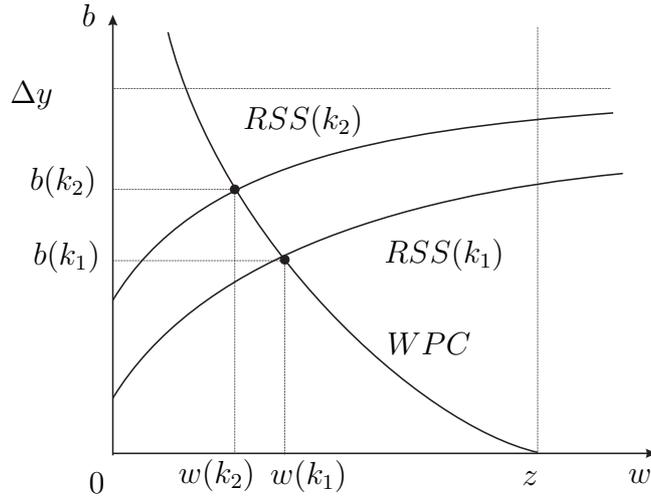


Figure 5: SE with ex-post wage setting and heterogeneous jobs

In competitive search equilibrium both firms face the same labour supply equation (6.1) in the space  $[\theta, w^L]$  for a given bonus pay value  $\Delta w$ . It can also be interpreted as workers' indifference curve. This follows from worker homogeneity in the economy and is represented by a convex decreasing curve in figure 6. The curve is decreasing since workers prefer both high wages and high market tightness. However, each firm is maximizing an individual vacancy surplus expression  $V(k) = -c + q(\theta)(J^H(k) - V(k))$ , where the job surplus  $J^H(k)$  is obtained in the following way:

$$J^H(k) = J^H(y^L(k), \Delta y(k)) = y^L(k) - w^L + \hat{p}(e)(\Delta y(k) - \Delta w)$$

Job surplus  $J^H(y^L(k), \Delta y(k))$  is increasing in both arguments  $y^L(k)$  and  $\Delta y(k)$ , meaning that the more capital intensive firm produces more output in the low productivity state  $y^L(k)$  and also enjoys a larger output increase  $\Delta y(k)$  if the high productivity state is realized. Both vacancy surplus equations for the two firms are represented by concave decreasing curves in the space  $[\theta, w^L]$  and are illustrated in figure 6. These curves can also be interpreted as iso-profits curves. Both curves are decreasing since firms prefer both low wages and low market tightness.

Concavity of the iso-profit curve and convexity of the worker's indifference curve are guaranteed by the assumption of the increasing elasticity of the job filling rate  $\eta_q$  with respect to the market tightness  $\theta$ . For a fixed value of  $\Delta y(k)$ , the more capital intensive firm faces a flatter indifference curve  $V(k_2) = const$ , this means that the optimal vector of variables  $[\theta, w^L]$  is such that  $\theta(k_2) < \theta(k_1)$  and  $w^L(k_2) > w^L(k_1)$ . A further

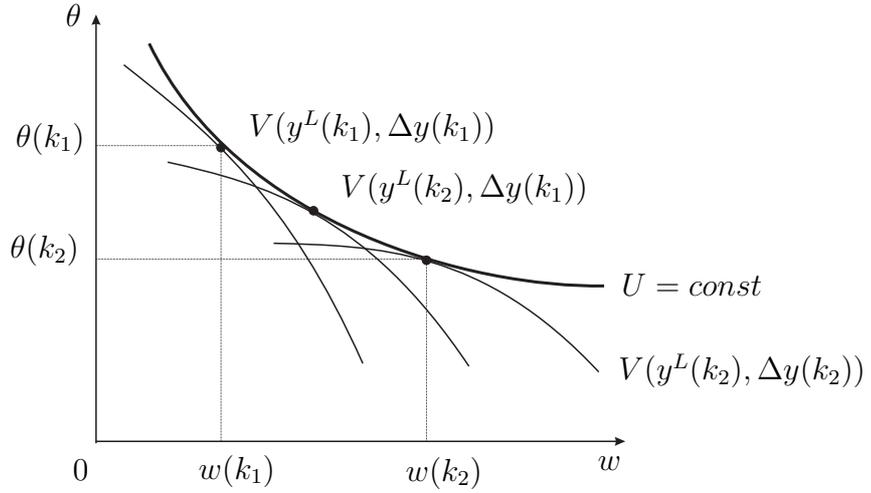


Figure 6: Market tightness in CSE with heterogeneous jobs

difference in risk variables  $\Delta y(k)$  between the two firms implies a further rotation of the firm indifference curve and strengthens the preceding result. The intuition behind this result is such that the more capital intensive firm faces larger search costs in terms of forgone output value and so the firm is more willing to trade off the low wages for low labour market tightness.

For the rent-sharing curve (RNS) the fact that  $w^L(k_2) > w^L(k_1)$  for every value of  $b = \Delta w$  implies an upward shift in the space  $[b, w]$ , so that the rent-sharing curve of the more capital intensive firm is situated above the corresponding curve of the less capital intensive firm. This is illustrated in figure 7. The reason for this shift is twofold: due to the larger values of  $y^L(k_2)$  and  $\Delta y(k_2)$ . More capital intensive firms are more productive, obtain higher rents  $J^H(k_2) - V(k_2)$  and share these rents with their employees. Firms lose from higher labour costs, both in terms of wages and bonus payments, but gain from a higher job filling rate  $q(\theta)$ . Note that for the fixed risk-sharing curve the rent-sharing motive implies complementarity between bonus payments and wages in the case of risk averse workers.

Consider the difference in the risk sharing curves. As already described above in the case of ex-post wage setting, both firms face different risk-sharing curves, where the RSS curve for  $k_2$  is situated above the corresponding curve for  $k_1$ . Here the more productive firm substitutes wages for bonus payments and gains from an unambiguously higher worker effort. Overall, optimal contract comparison of the two firms with different values of capital intensity highlights the fact, that the more capital intensive

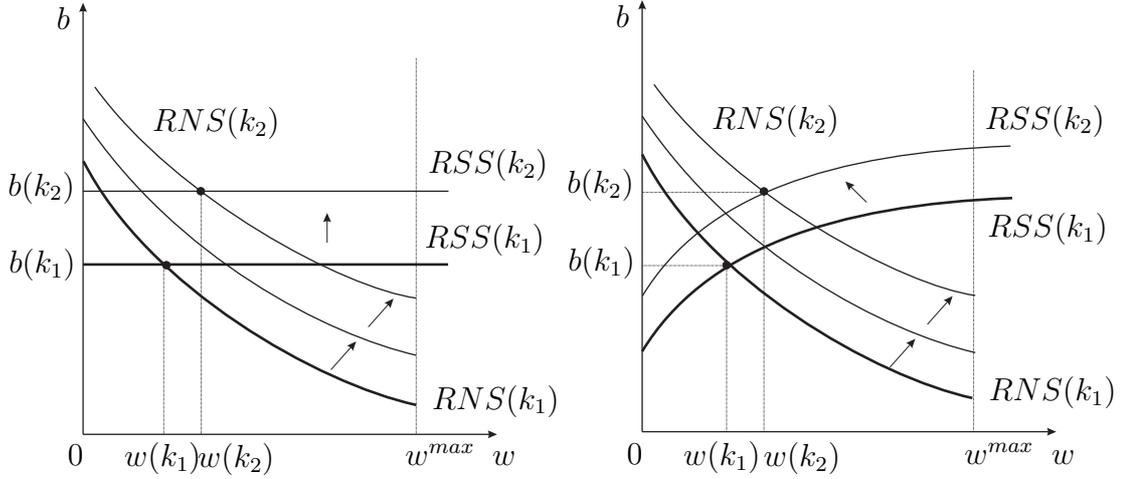


Figure 7: Competitive search equilibrium with heterogeneous jobs. Left: risk neutral workers. Right: risk averse workers

firm will unambiguously offer a larger value of the bonus payment  $b(k_2)$ , which has a positive effect both on the optimal worker effort and on the firm hiring rate. The effect of capital differences on base wages is however ambiguous, it is more likely to be positive if the slope of the risk sharing curve is close to 1 in the relevant range of capital intensities  $[\underline{k}, \bar{k}]$ . Also notice that in the case of homogeneous variation in output ( $\Delta y(k) = const \Rightarrow \partial \Delta y / \partial k = 0$ ) the baseline model of the paper with risk averse workers unambiguously predicts positive cross-sectional correlation between bonus payments and wages.

Summarizing, in the presence of jobs heterogeneity competitive search equilibrium with bonus payments extends the classical contract theory approach with ex-post wage setting by explaining the sources of cross-sectional complementarity between bonus payments and wages. This complementarity effect is based on the rent sharing mechanism between the firm and the worker inherent in the ex-ante wage setting regime.

## 7 Social welfare and constrained efficiency

This section considers welfare properties of competitive search equilibrium with risk averse workers and bonus payments. As mentioned in the introduction, with respect to the social planner solution this paper can be seen as a generalization of the equilibrium efficiency result presented in Moen and Rosen (2008) in competitive search equilibrium with risk averse workers in the presence of asymmetric information. However, the social planner optimization problem is investigated in the absence of income taxes

and unemployment benefits, so that the main research question raised in this section is whether the wage contracts chosen by firms are socially optimal given the optimal worker behavior and the free entry of firms. Two informational settings are considered: first, the unconstrained social planner optimization problem (first best) is analyzed, where the effort choice of workers is assumed to be observable by the social planner, then the constrained social planner solution is compared to the first best solution.

Consider a social planner implementing a state dependent wage contract with wages  $w^L$  and  $w^H$  in respective productivity states  $y^L$  and  $y^H$ . The utilitarian social planner is maximizing the present discounted value of a sum of utility flows of the unemployed and employed individuals. In the first best case the choice variables of the social planner can be represented as a tuple of variables  $\{e, w^H, w^L, \theta\}$ , so that the planner's objective function can be expressed in the following way:

$$\max_{e, w^L, w^H, \theta} \int_0^{\infty} e^{-rt} \left[ uv(z) + (1 - u)\hat{v}(e, w^L, w^H) \right] dt, \quad \text{where}$$

$$\hat{v}(e, w^L, w^H) = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e)$$

Variable  $\hat{v}(e, w^L, w^H)$  denotes utility flow of the employed individual working under the wage contract  $w^L, w^H$  and exerting  $e$  units of effort. The unemployment rate evolves according to the following differential equation:

$$\dot{u} = (1 - u)\gamma - u\lambda(\theta)$$

The planner's resource constraint can be summarized as follows:

$$cu\theta = (1 - u) \left( \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \right) \quad (7.1)$$

Equation (7.1) implies that the planner's budget is balanced and the monetary outflow for maintaining the vacancies on the left hand side equals the monetary inflow from the filled jobs on the right hand side of this equation. The social planner optimization programme is solved using a current-value Hamiltonian approach. Solution to this optimization programme is presented in proposition 5.

**Proposition 5:** Consider a social planner implementing a variable wage contract. The unconstrained (first best) social planner solution is characterized by a tuple of variables  $\{e, w^H, w^L, \theta, U\}$  satisfying condition  $w^H = w^L$ , the reservation utility equation (4.1), the job creation condition  $c = q(\theta)J^H(w^H, w^L)$ , as well as equations (a) and (b) below.

(a). The planner's effort choice is given by:

$$dp'(e)\Delta yv'(w) = C'(e) \quad (7.2)$$

(b). The risk adjusted Hosios surplus split:

$$J^H(w^H, w^L) = \frac{1 - \eta_q}{\eta_q} \frac{R(w^H, w^L, U)}{v'_w} \quad (7.3)$$

**Proof:** Appendix IV.

As follows from proposition 5 the unconstrained social planner optimally sets  $w^H = w^L$ . This means that the optimal bonus payment  $b$  is set to zero, implying income insurance for workers against productivity shocks. Having guaranteed income stability for the employed population the social planner chooses an optimal effort level by maximizing the total surplus of a filled job. This is given by equation (7.2), where the left hand side of equation stands for the social gain of increasing the effort, while the right hand side can be interpreted as a marginal loss. The social loss  $C'(e)$  is directly estimated in worker utility units, while the social gain is estimated as an increase in the expected productivity flow  $dp'(e)\Delta y$  multiplied by the respective shadow price of an output unit, represented by the term  $v'(w)$ .

Notice that the optimal effort equation of a social planner (eq. 7.2) is different from the worker incentive compatibility constraint (4.2). Here the social cost of increasing effort  $C'(e)$  coincides with a private cost of the employee, however the social gain  $dp'(e)\Delta yv'(w)$  is generally different from a private gain of the employee, which can be expressed as  $dp'(e)(v(w^H) - v(w^L))$ . Denote  $x_0$  – solution to the following equality:

$$\Delta yv'(x_0) = v(w^H) - v(w^L), \quad x_0 > w^L$$

Then competitive search equilibrium with risk averse workers, unobserved effort and bonus payments entails a downward effort distortion with respect to the first best out-

come if  $w < x_0$ , otherwise if  $w > x_0$  effort is biased upward. Intuitively, if  $w$  is less than  $x_0$ , the shadow price of a single output unit is high, so that the social gain expressed in worker utility units is higher than the private gain of a worker, as a result the social planner will demand more effort from workers compared to the decentralized equilibrium with risk averse workers and unobserved effort values. The opposite holds when  $w > x_0$ , in this case the social gain converted into worker utility units is lower than the private gain of a worker and therefore the first best effort level is lower than effort in a decentralized equilibrium.

In addition, it should also be noted that effort distortions are purely attributed to the risk aversion of workers. If workers are risk neutral, the social gain of exerting effort expressed as  $dp'(e)\Delta y$  coincides with a private gain of a worker  $dp'(e)b$ . This is the case because firms in a decentralized equilibrium optimally choose the maximum bonus payment value  $b = \Delta y$  (see proposition 1). This, however, does not imply that the first best social planner solution with risk neutral workers may be decentralized by the market. The reason is that the maximum effort value is not compatible with a zero bonus payment when effort is unobserved.

Now consider the second best solution where the social planner is constrained by the information asymmetries arising from the unobserved worker effort choice. In this case the social planner is maximizing the present discounted value of a sum of utility flows of the unemployed and employed individuals with respect to the choice variables represented by a tuple  $\{w^H, w^L, \theta\}$ . The objective function of a constrained social planner becomes:

$$\max_{w^L, w^H, \theta} \int_0^{\infty} e^{-rt} \left[ uv(z) + (1 - u)\hat{v}(w^L, w^H) \right] dt$$

where  $\tilde{v}$  is given above. In addition, the worker incentive compatibility constraint (equation 4.2), as well as the budget constraint of the social planner and the unemployment dynamics equation should be fulfilled. The result of this optimization problem is summarized in proposition 6.

**Proposition 6:** *Consider a social planner implementing a variable wage contract. The constrained (second best) social planner solution is characterized by a tuple of variables  $\{e, w^H, w^L, \theta, U\}$  satisfying the reservation utility equation (4.1), the job creation condition  $c = q(\theta)J^H(w^H, w^L)$ , the worker incentive compatibility constraint*

(4.2), as well as the risk-sharing equation (4.8) and the rent-sharing equation (4.9). Therefore, competitive search equilibrium with bonus payments and unobserved effort is constrained efficient.

**Proof:** Appendix IV.

Proposition 6 characterizes the major properties of the constrained social planner solution. It follows that the set of five equations describing the optimal solution of a social planner coincides with the set of equations in a decentralized competitive search equilibrium with unobserved effort and bonus payments. This means that the social planner will choose exactly the same optimal package of labour compensation  $(w^L, w^H)$  resulting in the same effort level of the employed and the same market tightness variable  $\theta$ . Therefore, it can be concluded that competitive search equilibrium with risk averse workers, bonus payments and unobserved effort is constrained efficient.

## 8 Conclusions

This paper develops a model of competitive search with risk averse workers in the presence of asymmetric information. Information asymmetries arise from the fact that workers possess private information about their effort choice on the job. The moral hazard problem within a match forces firms to use motivation devices such as the bonus pay in order to provide workers with the correct working incentives. This setup creates in a situation where the equilibrium labour contract entails both a hiring and a motivation wage premia. The hiring premium results from the rent-sharing incentive of firms ensuring them a sufficient job-filling rate, while the motivation premium results from the firm's risk-sharing incentive necessary to guarantee a sufficient effort level.

The baseline model of the paper is compared to the classical model of moral hazard extended to account for labour market search frictions but preserving the essence of the ex-post wage setting mechanism. This benchmark model is proved to predict a lower amount of the bonus pay than the baseline model with wage competition between employers. Similarly, both models are compared in the presence of a wage restriction imposed to reflect a binding limited liability constraint or a minimum wage requirement.

Furthermore, the paper presents an extension of the competitive search model with bonus payments to account for jobs heterogeneity. In particular, jobs are allowed to differ with respect to their capital endowments affecting both the expectation and the variation of output. The rent-sharing motive forces more capital intensive firms to leave higher rents to their employees. The higher rent comes in the form of a higher base wage as well as a higher bonus pay values. This complementarity effect provides rationale for the positive cross-sectional correlation between bonus payments and wages reported in a number of empirical studies. The rent-sharing motive is absent in the model with an ex-post wage setting so that bonus payments and wages act as substitutes in a cross-section of firms. Based on the above theoretical analysis this paper concludes that the correlation between bonus payments and wages is specific to the type and the structure of the labour market. This is also in line with the observed empirical evidence.

Finally, this paper considers efficiency implications of incentive contracts in a competitive search equilibrium. The equilibrium is proved to be constrained efficient in the absence of tax payments and unemployment benefits. Nevertheless, competitive search equilibrium with bonus payments does not coincide with the full information allocation of the social planner. This is due to the fact that the private gain from exerting effort is different from the social gain, so that in the full information allocation the social planner will demand a different effort level from workers compared to the decentralized equilibrium.

## 9 Appendix

### APPENDIX I: Proof of lemmas 1-2.

Differentiate  $W^L$  and  $W^H$  with respect to  $e^L$  and  $e^H$  respectively to obtain:

$$-C'(e^L) + \delta p'(e^L)(W^H - W^L) = 0 \quad (9.1)$$

$$-C'(e^H) + \delta p'(e^H)(W^H - W^L) = 0 \quad (9.2)$$

Equations (9.1)-(9.2) imply  $e^L = e^H = e(w^L, w^H)$ , so that:

$$\delta(W^H - W^L) = \frac{C'(e)}{p'(e)} \equiv \pi(e) \quad (9.3)$$

Subtracting  $W^L$  from  $W^H$  yields:

$$\delta(W^H - W^L) = d(v(w^H) - v(w^L)) \quad (9.4)$$

which proves equation (4.2). Differentiate equation (4.2) with respect to  $w^H$  to obtain  $e'_{w^H} > 0$ . Similarly, differentiate equation (4.2) with respect to  $w^L$  to obtain  $e'_{w^L} < 0$ . The second order derivatives  $e''_{w^L w^H}$ ,  $e''_{w^H w^L}$  and  $e''_{w^H w^H}$  can be found as follows:

$$\begin{aligned} e''_{w^L} &= -d \frac{(v''_{w^L} \pi'_e - v'_{w^L} \pi''_e)}{(\pi'_e)^2} > 0 \quad \text{if } \pi''_e > 0 \\ e''_{w^H} &= d \frac{(v''_{w^H} \pi'_e - v'_{w^H} \pi''_e)}{(\pi'_e)^2} < 0 \quad \text{if } \pi''_e > 0 \\ e''_{w^L w^H} &= d^2 \frac{(v'_{w^H} \pi''_e v'_{w^L})}{(\pi'_e)^3} > 0 \quad \text{if } \pi''_e > 0 \end{aligned}$$

Using equation (9.4) rewrite  $R(w^H, w^L)$  in the following way:

$$(r + \gamma)R^H(w^H, w^L) = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e) - rU$$

where  $\hat{p}(e) = (r + \gamma + \delta p(e))/(r + \gamma + \delta)$ . Differentiate  $R^H(w^H, w^L)$  with respect to  $w^H$  and  $w^L$  and apply the envelope theorem to obtain:  $\partial R^H(w^H, w^L)/\partial w^H > 0$  and  $\partial R^H(w^H, w^L)/\partial w^L > 0$ .

## Appendix II: Proof of proposition 1.

Differentiate equations (4.3), (4.4) with respect to  $w^H$  and  $w^L$  and use the fact that  $V = 0$  in the equilibrium, this yields:

$$\frac{\partial R^H / \partial w^H}{\partial R^H / \partial w^L} = \frac{\partial \theta / \partial w^H}{\partial \theta / \partial w^L} = \frac{\partial J^H / \partial w^H}{\partial J^H / \partial w^L} \quad (9.5)$$

where  $R^H = R^H(w^H, w^L, U)$  and  $J^H(w^H, w^L)$ . Differentiate  $J^H(w^H, w^L)$  with respect to both arguments to obtain:

$$\begin{aligned} (r + \gamma) \frac{\partial J^H}{\partial w^H} &= \hat{p}'(e)(\Delta y - \Delta w)e'_{w^H} - \hat{p}(e) \\ (r + \gamma) \frac{\partial J^H}{\partial w^L} &= \hat{p}'(e)(\Delta y - \Delta w)e'_{w^L} - (1 - \hat{p}(e)) \end{aligned}$$

Insert expressions for  $\partial R^H / \partial w^H$ ,  $\partial R^H / \partial w^L$ ,  $\partial J^H / \partial w^H$  and  $\partial J^H / \partial w^L$  into (9.5):

$$\frac{\hat{p}(e)v'(w^H)}{(1 - \hat{p}(e))v'(w^L)} = \frac{\hat{p}'(e)(\Delta y - \Delta w)e'_{w^H} - \hat{p}(e)}{\hat{p}'(e)(\Delta y - \Delta w)e'_{w^L} - (1 - \hat{p}(e))}$$

Insert expressions  $e'_{w^H} = dv'(w^H)/\pi'(e)$  and  $e'_{w^L} = -dv'(w^L)/\pi'(e)$  to obtain equation (4.8). Differentiate equation (4.4) with respect to  $w^H$  to obtain:

$$J^H(w^H, w^L) = \left[ -\frac{\partial J^H / \partial w^H}{\partial R^H / \partial w^H} \right] \frac{1 - \eta_q}{\eta_q} R^H(w^H, w^L, U)$$

Insert expressions for  $\partial J^H / \partial w^L$ ,  $\partial R^H / \partial w^L$  to obtain the risk-adjusted Hosios condition (4.9).

### Appendix III: Proof of propositions 3-4.

In the search equilibrium with ex-post wage setting the firm is maximizing its surplus with respect to the wage value  $w^H$  given that the wage  $w^L$  is adjusting according to the worker participation constraint. This gives rise to the following optimization problem:

$$\max_{w^H, w^L} J^H(w^H, w^L) + \lambda_u R^H(w^H, w^L, U)$$

where  $\lambda_u$  stands for the Lagrange multiplier. The first order conditions for this optimization problem are given by:

$$\frac{\partial J^H}{\partial w^H} + \lambda_u \frac{\partial R^H}{\partial w^H} = 0 \quad (9.6)$$

$$\frac{\partial J^H}{\partial w^L} + \lambda_u \frac{\partial R^H}{\partial w^L} = 0 \quad (9.7)$$

$$\lambda_u R(w^H, w^L, U) = 0 \quad (9.8)$$

where the last equation represents a complementary slackness condition. In the unrestricted firm optimization problem with the ex-post wage setting the worker participation constraint is binding, which means that  $\lambda_u \neq 0$ , while  $R(w^H, w^L, U) = 0$ . Then equations (9.6) - (9.7) can be rearranged to produce the risk sharing curve given by equation (4.10).

Now consider the restricted firm optimization problem with a binding limited liability constraint of the type  $w \geq \bar{w}$ . The firm optimization problem can be written as follows:

$$\max_{w^H} J^H(w^H, \bar{w}) + \lambda_R R^H(w^H, \bar{w}, U)$$

where  $\lambda_R$  stands for the Lagrange multiplier. The first order condition for this optimization problem is given by:

$$\frac{\partial J^H}{\partial w^H} + \lambda_R \frac{\partial R^H}{\partial w^H} = 0 \quad (9.9)$$

$$\lambda_R R(w^H, w^L, U) = 0 \quad (9.10)$$

where the last equation stands for the complementary slackness condition. If the worker participation constraint is not binding then  $\lambda_R = 0$  and the optimal bonus payment is given by  $\partial J^H / \partial w^H = 0$  implying that:

$$(\Delta y - b) \hat{p}'_e e'_b = \hat{p}(e), \quad \text{where } e = e(b, \bar{w})$$

which is the restricted risk sharing condition (5.3). Solution to this equation is denoted by  $b^*$ . If the worker participation constraint is binding meaning that  $b^*$  is too low, then  $\lambda_R \neq 0$ , so that the optimal bonus payment  $b^{**}$  is given by equation  $R(b^{**}, \bar{w}, U) = 0$ . Overall, the optimal bonus payment is given by:  $b_{PR} = \max(b^*, b^{**})$ .

#### Appendix IV: Proof of propositions 5-6.

The current value Hamiltonian for the unconstrained planner problem (first best) is:

$$\begin{aligned} H &= uv(z) + (1-u)\hat{v}(w^L, w^H, e) - \gamma_u[(1-u)\gamma - u\lambda(\theta)] \\ &+ \alpha[cu\theta - (1-u)(\hat{p}(e)(y^H - w^H) + (1-p(e))(y^L - w^L))] \end{aligned}$$

where  $\alpha$  is a Lagrange multiplier and  $\gamma_u$  is a costate variable corresponding to  $u$ . The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma_u \Rightarrow \alpha J^H + R^H = \gamma_u \quad (9.11)$$

Maximizing  $H$  with respect to  $e$ ,  $w^H$ ,  $w^L$  and  $\theta$  yields:

$$\frac{\partial H}{\partial e} = 0 \Rightarrow d[(\Delta y - \Delta w)\alpha + \Delta v] = \pi'(e) \quad (9.12)$$

$$\frac{\partial H}{\partial w^H} = 0 \Rightarrow \alpha = v'(w^H) \quad (9.13)$$

$$\frac{\partial H}{\partial w^L} = 0 \Rightarrow \alpha = v'(w^L) \quad (9.14)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma_u(1 - \eta_q)q(\theta) = c\alpha \quad (9.15)$$

where  $\Delta w = w^H - w^L$  and  $\Delta v = v(w^H) - v(w^L)$ . Equations (9.13)-(9.14) imply  $w^H = w^L$  and  $\alpha = v'(w)$  – shadow price of an output unit. These results transform equations (9.12), (9.15) and (9.11) into:

$$d\Delta y v'(w) = \pi'(e) \quad \text{and} \quad J^H = \frac{1 - \eta_q}{\eta_q} \frac{R^H}{v'(w)}$$

Consider the current value Hamiltonian function for the constrained social planner problem (second best). Denote  $\mu$  – Lagrange multiplier corresponding to the budget constraint of the social planner, and  $\gamma_c$  – a costate variable corresponding to  $u$ . The optimal social planner solution in this case must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma_c \Rightarrow \mu J^H + R^H = \gamma_c \quad (9.16)$$

$$\frac{\partial H}{\partial w^H} = 0 \Rightarrow \hat{p}(e) - dp'(e)e'_{w^H}(\Delta y - \Delta w) = \hat{p}(e) \frac{v'(w^H)}{\mu} \quad (9.17)$$

$$\frac{\partial H}{\partial w^L} = 0 \Rightarrow (1 - \hat{p}(e)) \frac{v'(w^L)}{\mu} = 1 - p(e) - dp'(e)e'_{w^L}(\Delta y - \Delta w) \quad (9.18)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma_c(1 - \eta_q)q(\theta) = c\mu \quad (9.19)$$

The ratio of equations (9.17),(9.18) can be rearranged to produce equation (4.8). In addition equations (9.16) and (9.19) imply the following optimal rent split:

$$J^H = \frac{1 - \eta_q}{\eta_q} \frac{R^H}{\mu}$$

## 10 References

- ACEMOGLU D., R. SHIMER (1999): "Efficient Unemployment Insurance", 107(5): 893-928.
- BENTAL B., D. DEMOUGIN (2006): "Institutions, Bargaining Power and Labour Shares", *SFB 649 Discussion paper*, No. 09.
- BOLTON P., M. DEWATRIPONT (2005): "Contract theory", MIT Press.
- BOONE J., L.A. BOVENBERG (2002): "Optimal Labour Taxation and Search", *Journal of Public Economics*, 85(1):53-97.
- CAHUC P., B. DORMONT (1997): "Profit-sharing: Does it increase productivity and employment?", *Labour economics* 4: 293-319.
- CHIZHOVA A.S. (2007): "Working Effort and Endogenous Job Separations in Search Equilibrium", Working paper, University of Konstanz.
- COLES M., A. MASTERS (2006): "Optimal Unemployment Insurance in a Matching Equilibrium", *Journal of Labour Economics*, 24(1): 109-138.
- DEMOUGIN D., C. HELM (2006): "Moral Hazard and Bargaining Power", *German Economic Review*, 7(4): 463-470.
- DIAMOND P.A. (1971): "A Model of Price Adjustment", *Journal of Economic theory* 3: 156-168.
- GUERRIERI V. (2008): "Inefficient Unemployment Dynamics under Asymmetric Information", *Journal of Political Economy* 116(4): 667-708.
- GUERRIERI V., R. SHIMER, R. WRIGHT (2010): "Adverse Selection in Competitive Search Equilibrium", *Econometrica*, forthcoming.
- HART R. A., O. HUEBLER (1991): "Are Profit Shares and Wages Substitute or Complementary Forms of Compensation?", *Kyklos*, 44(2): 221-231.
- HOSIOS A.J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies* 57: 279-298.
- KAUFMAN R.T. (1998): "The Effects of Gain Sharing on the Basic Wage: The case of IM-

PROSHARE", *Labour*, 12(3): 595-610.

LAFFONT J-J., D. MARTIMORT (2002): *The Theory of Incentives*, Princeton University Press, New-York.

LAFFONT J-J., J-C. ROCHET (1998): "Regulation of a Risk Averse Firm", *Games and Economic Behavior*, 25: 149-173.

LEHMANN E., B.V. LINDEN (2004): "On the Optimality of Search Matching Equilibrium when Workers are Risk Averse", IZA Discussion Paper No. 1172.

MITCHELL D.J.B., D. LEWIN, E.E. LAWLER III (1990): "Alternative Pay Systems, Firm Performance, and Productivity", in *Paying for Productivity: A Look at the Evidence*, ed. by A.S. Blinder, *The Brookings Institution*, Washington, D.C.

MOEN E.R. (1997): "Competitive Search Equilibrium", *Journal of Political Economy*, 105(2): 385-411.

MOEN E.R., A. ROSEN (2006): "Equilibrium Incentive Contracts and Efficiency Wages", *Journal of European Economic Association*, 4(6): 1165-1192.

MOEN E.R., A. ROSEN (2008): "Incentives in Competitive Search Equilibrium", *Review of Economic Studies*, forthcoming.

MORTENSEN D.T., C.A. PISSARIDES (1994): "Job Creation and Job Destruction in the theory of unemployment", *Review of Economic Studies*, 61: 397-415.

PISSARIDES C. A. (2000): *Equilibrium Unemployment Theory*. The MIT Press. Cambridge.

RUDANKO L. (2009): "Labour Market Dynamics under Long Term Wage Contracting", *Journal of Monetary Economics*, 56(2): 170-183.

SHAPIRO C., J.E. STIGLITZ (1984): "Equilibrium Unemployment as a Worker Discipline Device", *The American Economic Review*, 74(3): 433-444.

SHIMER R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", *American Economic Review*, 95: 25-49.

WADHWANI S., M. WALL (1990): "The Effects of Profit-sharing on Employment, Wages, Stock Returns and Productivity", *The Economic Journal*, 100: 1-17.