Reporting Frequency and Substitutable Tasks

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Abstract

The optimal reporting frequency is an important issue in accounting. In many production settings, substitution effects across periods occur. This paper shows that the optimal reporting frequency depends on the strength of the substitution effect and on the information content of performance signals. For a subset of parameter combinations - the low-chance scenario - infrequent reporting is always efficient; for other parameter combinations - the high-chance scenario - infrequent reporting is efficient as long as first-period signals show high informativeness (and substitution effects are strong). Limited commitment by the principal does not influence results.

Keywords: dynamic agency; intertemporal aggregation; reporting frequency, performance measurement; substitutable tasks; commitment

JEL Code: D86; M12; M41; M52

1 Introduction

The optimal reporting frequency or frequency of performance evaluation is an important issue in accounting. Stated differently, one could ask whether short-term or long-term performance measures should be relevant for variable compensation. Especially in light of the recent financial crisis where short-termism was often considered to be one of its causes, the question appears highly relevant. Corporate governance codes also address the question.¹ While short-termism may not be unambiguously detrimental to a firm’s development (Dobbs 2009; Repenning and Henderson 2010), the potential problem with short-term evaluations arises if outcomes in different accounting periods are not independent from each other: Meeting a performance target in one period (quarter or half-year etc.) may make it harder to meet the target in the next period because demand is limited or there exists only a certain success potential for the whole time under consideration. For example, consider a sales manager who faces a certain annual demand

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¹The German corporate governance code stipulates long-term performance measures for purposes of variable compensation of the board. See http://www.corporate-governance-code.de/index-e.html
in the industry. Now closing a deal in the first quarter which helps to meet the quarterly performance target may reduce the chances to meet the target in the next quarter because a fraction of demand has already realized. Or, think of cost-reduction effort. If the manager succeeds in cutting costs in say the first half of the year it becomes more difficult to further reduce costs in the second half of the year (for given production levels of course). Technically, the tasks in different periods represent substitutes – increasing the chances to succeed in a given period comes at the cost of decreasing those for a subsequent period. It is the objective of this paper to analyze the optimal frequency of performance evaluation if substitution effects between periods exist.

Intertemporal aggregation of performance measures, frequency of feedback or the benefits (and costs) of withholding performance information constitutes a growing literature. Rewarding aggregate performance is optimal if no interaction effects between periods exist regardless of whether the agent observes interim outcomes (Holmstrom and Milgrom 1987) or not (Arya et al. 2004). Here aggregation does not entail a loss of information. As soon as interaction effects are present the loss of information cannot be avoided but intertemporal aggregation may still have its merits. Possible benefits include avoidance of information overload given bounded rationality, a better sorting of employees (Ray 2007a), retaining employees (Ray 2007b) or preventing sabotage in tournaments (Gürtler et al. 2010). The frequency of evaluation also influences attitudes of those who are evaluated (Cook 1967) and the steepness of the incentive scheme (Arnaiz and Fumás 2008). What unites theoretical arguments in favor of less frequent or aggregate evaluations is restraining the agent’s opportunism. If less information is available to the agent when selecting effort at different points in time, simply less opportunities for exploiting information exist. That seems to be of special importance if production and performance measurement interact. Guymon et al. (2008) and Demski et al. (2008) study multi-agent, single-period settings with such interactions, while Dikolli et al. (2009) address substitutability and complementarity in tasks of a single agent in a one-period agency. The latter find that changes in performance measure interrelations and eventually profits depend on the type of interaction between tasks. Given that interactions matter in single-period problems they should likewise matter in multi-period problems.

In this paper I analyze how substitutability between tasks across periods affects the optimal frequency of performance evaluation. For that purpose I consider a two-period agency model with risk-neutral contracting parties but the agent is protected by limited liability. Higher effort

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2The literature on intra-period aggregation is growing, too. See, for example, Arya et al. (1997); Arya and Mittendorf (2008); Cremer (1995); Christensen et al. (2002); Datar and Gupta (1994); Feltham et al. (2006); or Indjejikian and Nanda (1999).

3There are cases where aggregation actually leads to better information because it offsets possible errors in individual accounts (Datar and Gupta 1994); or because it prevents information cascades (Arya et al. 2006). Sunder (1997) points out that the aggregation process itself may add information.

4In some contrast to that the management literature advocates prompt and timely feedback to motivate employees (Milkovich and Newman 2002; Wright and Snell 1998).

5See Iyer et al. (2005) or Siggelkow (2002) for an analysis of such task interaction in a product specification setting or with regard to organizational consequences.
increases the chance to succeed in each period but effort in period 1 affects period 2 in the following way: The higher the effort in period 1 the lower the probability to succeed in period 2. This is the characterizing feature of substitutable tasks. In light of possibly very short evaluation periods due to computerized accounting systems, the assumption of substitutability appears relevant in many cases. Meeting a potentially ambitious performance target may come at the cost of lower chances to meet it in a subsequent period. The principal can choose between two evaluation systems. Under frequent evaluation, the accounting system reports performance at the end of each single period; under infrequent evaluation, a single report at the end of the second period covering both periods is prepared. I initially assume that neither the principal nor the agent observes the outcome of period 1 under infrequent evaluation. As it turns out, the principal cannot benefit from renegotiations after period 1 and therefore the setting in this paper is effectively one where the principal has the option to withhold performance information from the agent. According to the management and compensation literature in many cases firms carry out evaluations but do not inform employees (truthfully). A possible scenario would be a two-tier hierarchy with sales personnel, regional heads and headquarters where headquarters is better informed about cost allocations than regional heads. While Ray (2007b) shows that the firms’ desire to retain workers helps explain this practice, I argue that lower incentive costs can provide another rationale for it. With substitution effects between periods, a lower reporting frequency or infrequent evaluation is either unconditionally efficient (in the low-chance scenario) or conditionally efficient if substitution effects are sufficiently strong (in the high-chance scenario).

My paper is most closely related to research on intertemporal aggregation in dynamic agency relationships by Nikias et al. (2005) and Lukas (2010) who employ a probability structure similar to the one in this paper. Both consider complementary tasks across periods and find that infrequent evaluation can be efficient. A notable difference is that the principal weakly prefers infrequent evaluation in Nikias et al. (2005), while it takes a sufficiently low informativeness of the first-period outcome to make it efficient in Lukas (2010). Different long-term effects of first-period effort explain the difference. While only high effort entails a long-term effect in Nikias et al. (2005), both high effort and low effort in period 1 affect period 2 outcome probabilities in Lukas (2010). Nikias et al. (2005) also consider substitutable tasks (or negative complements in their terminology). Only a weak substitution effect leads to infrequent evaluation being optimal. In my model, in contrast, a strong substitution effect represents a sufficient condition for the efficiency of infrequent evaluation. Again the difference in the long-term effects of first-period effort accounts for the contrary findings. In sum, the work of Nikias et al. (2005), Lukas (2010) and this paper demonstrate that infrequent evaluation can be optimal even if interaction effects between periods exist. The conditions under which that optimality holds are, however, sensible to these specific interaction effects.

The remainder of the paper is organized as follows. In Section 2 I introduce the model and Section 3 presents the benchmark analysis of independent periods. Section 4 represents the

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See Ray (2007b) and the references therein.
main part of the paper investigating substitutable tasks and their implication for the optimal reporting frequency. As an extension, Section 5 considers the setting with (partly) observable interim outcomes. The final section concludes.

2 The model

I analyze a dynamic principal-agent relationship that lasts for two periods. The principal hires the agent to perform a certain task in each period: Both can commit to stay in the agency that long. By assumption, effort is binary in each period, \( e_t \in \{0,1\} \), at costs \( C(e_t) = ce_t \), \( c > 0 \). Due to the unobservability of the agent’s effort the principal relies on output-contingent compensation to motivate the agent. The verifiable outcome in each period can be either high, \( x^H \), or low, \( x^L \). Effort influences the probability distribution of outcomes in the following way:

\[
P(x_1 = x^H) = p_{e_1} \tag{1}
\]
\[
P(x_2 = x^H) = (1 - p_{e_1}) \cdot q_{e_2} \tag{2}
\]

In any given period, higher effort increases the probability of the high outcome, i.e., \( 0 < \omega_{e_1=0} < \omega_{e_1=1} < 1, \omega = p,q \). However, the higher the effort in period 1 the lower the probability of the high outcome in period 2. That means a substitution effect exists. Its extent is jointly determined by the probability of the high outcome in period 1, \( p_{e_1} \), and by the dependency parameter \( \delta, 0 \leq \delta \leq 1 \). Clearly, \( \delta = 0 \) leads to no dependency, i.e., periods are independent. Maximum dependence then obtains if \( \delta = 1 \). The analysis of the former case delivers the benchmark result.

The agent receives output-contingent compensation. Provided that the output sequence \( (x_1 = x^i, x_2 = x^j) \), \( i,j \in \{L,H\} \), is achieved, she is eligible for payment \( s^{ij} \). The corresponding probabilities \( P(x_1 = x^i, x_2 = x^j) = \pi^{ij}_{e_1,e_2} \equiv \pi^{ij}_{e_1,e_2} \) contingent on the agent’s effort in both periods are given in Table 1.

<table>
<thead>
<tr>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( \pi^{LL}_{e_1,e_2} )</th>
<th>( \pi^{HL}_{e_1,e_2} )</th>
<th>( \pi^{LH}_{e_1,e_2} )</th>
<th>( \pi^{HH}_{e_1,e_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( (1 - p_1) [1 - (1 - p^H_1)] q_1 )</td>
<td>( p_1 [1 - (1 - p^H_1)] q_1 )</td>
<td>( (1 - p_1) (1 - p^H_1) q_1 )</td>
<td>( p_1 (1 - p^H_1) q_1 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( (1 - p_1) [1 - (1 - p^L_1)] q_0 )</td>
<td>( p_1 [1 - (1 - p^L_1)] q_0 )</td>
<td>( (1 - p_1) (1 - p^L_1) q_0 )</td>
<td>( p_1 (1 - p^L_1) q_0 )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( (1 - p_0) [1 - (1 - p^H_0)] q_1 )</td>
<td>( p_0 [1 - (1 - p^H_0)] q_1 )</td>
<td>( (1 - p_0) (1 - p^H_0) q_1 )</td>
<td>( p_0 (1 - p^H_0) q_1 )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( (1 - p_0) [1 - (1 - p^L_0)] q_0 )</td>
<td>( p_0 [1 - (1 - p^L_0)] q_0 )</td>
<td>( (1 - p_0) (1 - p^L_0) q_0 )</td>
<td>( p_0 (1 - p^L_0) q_0 )</td>
</tr>
</tbody>
</table>

Table 1: Probabilities of output sequences \( \pi^{ij}_{e_1,e_2} \) contingent on agent effort

Both parties are assumed to be risk-neutral. Contractual frictions result from the agent’s limited liability. It requires that all payments to the agent have to be nonnegative, \( s^{ij} \geq 0, i,j = L,H \). Furthermore, the agent’s utility from compensation and effort is separable\(^7\) with

\(^7\)For a characterization and justification of the form \( u(s_1,s_2) = u(s_1 + s_2) \), where \( s_t, t = 1,2 \), is the payment in period \( t \), see Fishburn (1965).
\(U(s^{ij}, e_1, e_2) = s^{ij} - c(e_1 + e_2)\). To concentrate on the accounting effects of aggregation, I assume zero discounting and no time preference so that timing of payments leaves utility unaffected. To establish an incentive problem, the agent’s effort generates enough value and the principal wants to induce high effort in every period.\(^8\)

The principal may decide between two different evaluation regimes\(^9\): the frequent evaluation regime (disaggregate performance evaluation) where performance is measured in every period; or the infrequent evaluation regime (aggregate performance evaluation) where performance is measured once at the end of period 2.

Let the agent’s expected utility when selecting effort levels \(e_1(e_2)\) in period 1(2) be denoted
\[
E(S_{e_1,e_2}) = \sum_{i,j} \pi_{e_1,e_2}^{ij} \cdot s^{ij} - c \cdot (e_1 + e_2),
\]
under disaggregate evaluation, and correspondingly
\[
E(S_{e_1,e_2}) = \sum_{i,j} \pi_{e_1,e_2}^{+ij} \cdot s^{+ij} - c \cdot (e_1 + e_2)
\]
given aggregate evaluation. Notice, \((i + j)\) refers to aggregate output resulting from period 1 outcome \(x^i\) and period 2 outcome \(x^j, i, j = L, H\).

Infrequent evaluation does not allow the principal to differentiate between outcome sequences \((x^L, x^H)\) and \((x^H, x^L)\) for compensation purposes. At the same time, however, the agent chooses her second-period action without knowing first-period outcome, i.e., \(e_2\) is not conditioned on \(x^1\) as in the frequent evaluation regime. The principal’s program under either regime obtains as follows.

**Frequent performance evaluation (FPE):**

\[
\min_{s^{ij}} \sum_{i,j} \pi_{1,1}^{ij} s^{ij}
\]
subject to
\[
E(S_{1,1}) \geq 0 \quad (4)
\]
\[
E(S_{1,1}) \geq E(S_{0,0}) \quad (5)
\]
\[
E(S_{1,1}) \geq E(S_{0,1}) \quad (6)
\]
\[
[1 - p_1^\delta] q_1 s^{HH} + \left(1 - [1 - p_1^\delta] q_1\right) s^{HL} - c \geq \left[1 - p_1^\delta\right] q_0 s^{HH} + \left(1 - [1 - p_1^\delta] q_0\right) s^{HL} \quad (7)
\]
\[
[1 - p_1^\delta] q_1 s^{LH} + \left(1 - [1 - p_1^\delta] q_1\right) s^{LL} - c \geq \left[1 - p_1^\delta\right] q_0 s^{LH} + \left(1 - [1 - p_1^\delta] q_0\right) s^{LL} \quad (8)
\]
\[
s^{ij} \geq 0, \ i, j \in \{L, H\}. \quad (9)
\]

The principal needs to ensure the agent’s participation given a reservation utility of \(u = 0\), constraint (4). Conditions (5)-(8) denote incentive compatibility constraints to make the agent prefer high effort to low effort in period 1, constraints (5) and (6); and high effort to low effort in period 2 contingent on the observation of a high first-period outcome, constraint (7), or a low first-period outcome, constraint (8), respectively. (9) denotes the liability constraint, i.e., payments to the agent must be non-negative. The program without constraint (9) will be referred to as the unconstrained program.

\(^8\)See Schöndube (2008) for an analysis where the principal trades-off high period 1 effort against high period 2 effort.

\(^9\)The labels are adapted from Arya et al. (2004).
Infrequent performance evaluation (IPE):

$$\min_{s_{ij}} \sum_{i,j} \pi_{i+1,j}^{i,j} s_{i+1,j}^{i,j}$$

subject to

$$E(S_{[1,1]}) \geq 0$$
$$E(S_{[1,1]}) \geq E(S_{[0,0]})$$
$$E(S_{[1,1]}) \geq E(S_{[1,0]})$$
$$E(S_{[1,1]}) \geq E(S_{[0,1]})$$
$$s_{i+1,j} \geq 0.$$...
given frequent performance evaluation. Accordingly, let $S^{2L}$ denote the set of payments $s^{2L}_n$ where each element is part of a payment scheme $P^{IPE}_n = \{s^{2L}_n, s^{H+L}_n, s^{2H}_n\}$ which solves the unconstrained program (10) subject to (11)-(14). The agent’s rent under infrequent performance evaluation amounts to

$$R_{IPE} = \min \{|s^{2L}_n| \mid s^{2L}_n \in S^{2L}\}$$  \hspace{1cm} (17)

The difference $\Delta$ between the rents

$$\Delta = R_{FPE} - R_{IPE}$$  \hspace{1cm} (18)

constitutes the decision criterion and the principal optimally chooses frequent evaluation if $\Delta < 0$, and infrequent evaluation if $\Delta > 0$.

Given domain additivity and risk neutrality, the unconstrained program under FPE decomposes and the optimal two-period contract shows no memory (Amershi et al. 1985; Fellingham et al. 1985). Solving (3) and taking into account (16) leads to the agent’s rent under FPE of

$$R_{FPE} = c \left( \frac{p_1}{p_1 - p_0} + \frac{q_1}{q_1 - q_0} - 2 \right).$$

Under IPE, it can be shown that the principal offers a contract showing $s^{2H} > s^{H+L} = s^{2L} = 0$, i.e., all incentives are placed on the most desirable outcome sequence. With the help of some algebra the agent’s rent obtains as\textsuperscript{10}

$$R_{IPE(A1)} = c \left( \frac{2p_1q_1}{p_1q_1 - p_0q_0} - 2 \right) \text{ if A1 holds, and}$$

$$R_{IPE(A2)} = c \left( \frac{p_1q_1}{p_1(q_1 - q_0)} - 2 \right) \text{ if A2 holds, and}$$

$$R_{IPE(A3)} = c \left( \frac{p_1q_1}{q_1(p_1 - p_0)} - 2 \right) \text{ if A3 holds.}$$

Then the following relations can be readily verified:

$$\Delta_{A1} = R_{FPE} - R_{IPE(A1)} = c \left[ \frac{q_1q_0(p_1 - p_0)^2 + p_1p_0(q_1 - q_0)^2}{(p_1 - p_0)(q_1 - q_0)(p_1q_1 - p_0q_0)} \right] > 0,$$

$$\Delta_{A2} = R_{FPE} - R_{IPE(A2)} = c \frac{p_1}{p_1 - p_0} > 0,$$

$$\Delta_{A3} = R_{FPE} - R_{IPE(A3)} = c \frac{q_1}{q_1 - q_0} > 0,$$

leading to proposition 1.

**Proposition 1** With independent periods, infrequent performance evaluation is efficient.

\textsuperscript{10}It can be verified that if A1 holds, constraint (12) binds, and if A2 or A3 holds, constraint (13) or (14) binds, respectively.

7
We know from Arya et al. (2004) and Nikias et al. (2005) that under assumption A1 the principal is better-off with infrequent evaluation.\textsuperscript{11} By example, the result does not hold generally when A2 or A3 applies and the agent is risk averse as in Arya et al. (2004). Proposition 1 proves when contracting frictions result from limited liability, the principal prefers infrequent evaluation even if the single-period problems differ. The intuition for the result is, however, similar to the case with identical periods. Given independent periods the outcome sequence \( \{x^H, x^H\} \) is at least as informative as any other possible sequence. Although the principal cannot distinguish outcome sequences \( \{x^H, x^L\} \) and \( \{x^L, x^H\} \), which is valuable given differing single-period problems, the principals gains enough from placing all incentives on the most desirable (and informative) sequence. The relaxation of the incentive constraint for the lowest effort combination \([0, 0]\) under IPE more than offsets the effect of the additional constraint \( s^{HL} = s^{LH} \) imposed by moving from FPE to IPE.

4 Substitutable tasks

In this section, first-period effort entails a long-term effect that stretches into period 2. To have the substitution effect as strong a possible I assume \( \delta = 1 \). Probabilities in table 1 obtain accordingly.

4.1 Frequent performance evaluation

The substitutability of tasks gives rise to non-stationarity in the production technology. One may wonder whether the principal reacts to it by shifting incentives from one period into the other. Solving the principal’s program (3) leads to the following optimal payments (see Appendix A.1 for derivation):

\[
\begin{align*}
    s^{LL} &= 0; \\
    s^{HL} &= \frac{c}{p_1 - p_0} + \frac{q_1}{(1 - p_1)(q_1 - q_0)}; \\
    s^{LH} &= \frac{c}{(1 - p_1)(q_1 - q_0)}; \\
    s^{HH} &= s^{HL} + \frac{c}{(1 - p_1)(q_1 - q_0)}. 
\end{align*}
\]

Under frequent evaluation the agent’s rent amounts to

\[
R_{FPE} = c \left( \frac{p_1}{p_1 - p_0} + \frac{q_1}{(1 - p_1)(q_1 - q_0)} - 2 \right)
\]

Inspection of (19)-(22) leads to the conclusion that the principal does not shift incentives from period 1 into period 2 – note that \((s^{HH} - s^{HL}) = (s^{LH} - s^{LL}) = \frac{c}{(1 - p_1)(q_1 - q_0)} \) and second-period incentive constraints (7) and (8) bind in the optimum. In other words, second-period incentives are set at the sequentially rational level (Baron and Besanko 1987) or at the

\textsuperscript{11}See Arya et al. (2004, p. 649f) and Nikias et al. (2005, p. 59f).
renegotiation-proof level. It follows that the principal’s possible inability to fully commit to a two-period contract does not harm efficiency. In the present scenario with substitutable tasks raising bonuses in period 2 beyond the minimum required for incentive compatibility aggravates the incentive problem in period 1. Therefore, the long-term full-commitment contract is renegotiation-proof.\footnote{Introduction of risk-aversion on the agent’s side leaves this result unaffected.} Knowing that commitment issues do not play a role under infrequent evaluation, the result in Proposition 2 follows immediately.

**Proposition 2** For any given parameters of the agency, the efficient evaluation regime is the same under full commitment and limited commitment.

The above result is noteworthy as problems in strategic interactions deriving from the principal’s limited contractual commitment have been analyzed in various settings (for example, Arya et al. 1997; Christensen et al. 2002; Demski and Frimor 1999). A possible solution to these problems can be information rationing (Indjejikian and Nanda 1999; Lukas 2010), i.e., aggregation substitutes (a lack of) commitment. In this paper the principal’s choice of the efficient evaluation regime does not depend on his commitment.

### 4.2 Infrequent performance evaluation

In this section I characterize the optimal payments under infrequent evaluation and determine the agent’s rent under that regime. Infrequent evaluation comes at the cost of losing information: Instead of the sequence of performances the principal only gets to know the aggregate performance. This of course renders differentiating payments for output sequences \( \{x^L, x^H\} \) and \( \{x^H, x^L\} \) impossible. On the benefit side, less information will be available to the agent as well, curbing her opportunism when selecting effort in period 2.

Depending on the binding incentive constraint(s), optimal payments can be derived for program (10). They are summarized in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Binding incentive constraint(s)</th>
<th>Optimal nonzero payments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(12)</td>
<td>( s_{2H}^{(1)} = \frac{2c}{p_1(1-p_1)q_1-p_0(1-p_0)q_0} )</td>
</tr>
<tr>
<td>(2)</td>
<td>(13)</td>
<td>( s_{2H}^{(2)} = \frac{c}{p_1(1-p_1)(q_1-q_0)} )</td>
</tr>
<tr>
<td>(3)</td>
<td>(14)</td>
<td>( s_{2H}^{(3)} = \frac{c}{p_1(1-p_1)-p_0(1-p_0)q_1} )</td>
</tr>
<tr>
<td>(4)</td>
<td>(13) and (14)</td>
<td>( s_{2H}^{(4)} = \frac{c(p_1-p_0)(1-p_1)(q_1-q_0)q_1-p_0(1-p_0)}{(p_1-p_0)(1-p_1)(q_1-q_0)\sqrt{(1-p_0)}} )</td>
</tr>
</tbody>
</table>

Table 2: Optimal payments under infrequent evaluation
Keep in mind that the cases in Table 2 may occur subsequently in various orders. Fix second-period parameters \( q_0 \) and \( q_1 \) and first-period parameter \( p_0 \), then with \( p_1 \) increasing, for example, case (1) may follow case (3) or vice versa and eventually case (4) becomes relevant. After some tedious algebra, corresponding rents amount to

\[
\begin{align*}
R_{IPE(1)} &= c \left( \frac{2p_1(1-p_1)q_1}{p_1(1-p_1)q_1 - p_0(1-p_0)q_0} - 2 \right) \\
R_{IPE(2)} &= c \left( \frac{q_1}{q_1 - q_0} - 2 \right) \\
R_{IPE(3)} &= c \left( \frac{p_1(1-p_1)q_1}{p_1(1-p_1) - p_0(1-p_0)} q_1 - 2 \right) \\
R_{IPE(4)} &= c \left( \frac{(p_1^2 - p_1^4)q_1 - q_0 + q_1^2(p_1^2 - p_0^2) - [q_1^2(1 + p_0p_1 - q_1p_0p_1)](p_1 - p_0)}{(p_1 - p_0)(1-p_1)(q_1 - q_0)[p_1 - q_1(1-p_0)]} - 2 \right)
\end{align*}
\]

With rents under either evaluation regime being specified the comparison identifies the efficient regime. This will be done in the next section.

### 4.3 Optimal frequency of performance evaluation

Under either evaluation regime the agent receives a rent due to her limited liability. The principal’s decision in favor of one of the regimes optimally trades-off more contractible information under FPE against less agent opportunism when selecting second-period effort under IPE. One may suspect that – in the absence of commitment problems – availability of more informative signals leads to optimality of frequent evaluation. As the analysis shows, the opposite is true: The more informative the signal becomes in period 1, the more likely is the optimality of infrequent evaluation.

I define two scenarios which characterize quite different situations for performance evaluation. Scenario 1 comprises all circumstances where either incentive constraint (12) or incentive constraint (13) singly binds under IPE. It can be verified that this occurs only if \( p_0 \leq 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + (q_1 - q_0)}} \right) \) for (12) to be singly binding or \( p_0 \leq 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + (q_1 - q_0)}} \right) \) for (13) to be singly binding, respectively.\(^\text{13}\) Since all values \( p_0 \geq 0.5 \) are excluded from this scenario, it is labeled low-chance scenario (LC): Low effort in period 1 leads to less than a fair

\(^{13}\)For incentive constraint (12) to be singly binding, a threshold \( \overline{p}_1 \) for the transition to constraint (14) to be binding must exist. That threshold obtains by solving

\[
E \left( S_{[0,0]} \mid s_{(1)}^{2R} \right) - E \left( S_{[0,1]} \mid s_{(3)}^{2R} \right) = 0
\]

- where subscripts (1) and (3) indicate respective cases from Table 2 - which leads to

\[
\frac{(p_1^2q_1 - p_1q_1 - 2q_1p_0 + 2q_1p_0 - p_0q_0 + p_0^2q_0)}{(p_1 - p_1)q_1 + (p_0 - p_0^2)q_0} \cdot c = 0.
\]

The solution to (28) is \( \overline{p}_1 = 0.5 \pm \sqrt{0.25 - (1 - p_0)p_0} \left( 2 - \frac{q_1}{q_1 + (q_1 - q_0)} \right) \) and \( \overline{p}_1 \in \mathbb{R}^+ \iff p_0 \leq 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + (q_1 - q_0)}} \right) \).

A similar calculation leads to the condition for constraint (13) to be singly binding.
chance of achieving the high outcome in that period. At the same time, low effort in period 1 entails only a moderate substitution effect so that ceteris paribus chances to succeed in period 2 are higher. In contrast, the high-chance scenario (HC) consists of all cases with \( p_0 \geq 0.5 \), i.e., incentive constraint (12) or incentive constraint (13) never binds singly under IPE. Thus, the HC scenario includes all \( p_0 \)-values that offer at least a fair chance to succeed in period 1 given low effort. That higher chance comes at the cost of a strong substitution effect (even) for low effort; the probability to achieve the high outcome in period 2 decreases accordingly.

For future reference, the two scenarios are stated in separate definitions.

**Definition 1** The low-chance scenario is defined as the set of parameters

\[
\mathcal{LC} = \left\{ p_0, p_1, q_0, q_1 \mid p_0 \leq 0.5 \left( 1 - \sqrt{\frac{q_1}{q_1 + (q_1 - q_0)}} \right) \text{ or } p_0 \leq 0.5 \left( 1 - \sqrt{\frac{q_0}{q_1}} \right) \right\};
\]

low effort in period 1 implies less than a fair chance to achieve the high outcome in that period.

**Definition 2** The high-chance scenario is defined as the set of parameters

\[
\mathcal{HC} = \left\{ p_0, p_1, q_0, q_1 \mid p_0 \geq 0.5 \right\};
\]

low effort in period 1 implies at least a fair chance to achieve the high outcome in that period.

To provide more intuition for the two situations, one can also think of the LC scenario as one where the agent faces a difficult task such that it takes higher effort to possibly obtain a fair chance to deliver high performance in period 1. In this vein, the HC scenario could be identified as the easy-task scenario because low effort can already give a fair chance to succeed. Depending on the scenario, the principal decides differently on the optimal evaluation regime.

**Proposition 3** In the low-chance (LC) scenario, infrequent evaluation is efficient, unless the first-period outcome becomes non-informative, i.e., unless \( p_1 \rightarrow p_0 \).

**Proposition 4** In the high-chance (HC) scenario, infrequent evaluation is efficient if

\[
p_1 \geq \frac{q_1(1 - p_0) + p_0 - q_0}{1 - q_0}
\]

holds; otherwise frequent evaluation is efficient.

**Corollary 1** A sufficient condition for infrequent evaluation to be efficient is

\[
p_1 \geq \frac{q_1(1 - p_0) + p_0 - q_0}{1 - q_0}.
\]

\(^{14}\)To be precise, that incentive constraint (12) or incentive constraint (13) never binds singly under IPE merely implies \( p_0 > 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + (q_1 - q_0)}} \right) \) or \( p_0 > 0.5 \left( 1 - \sqrt{1 - \frac{q_0}{q_1}} \right) \), respectively. Assuming \( p_0 \geq 0.5 \) in the high-chance scenario leaves out some cases where \( p_0 < 0.5 \) and (12) or constraint (13) do not singly bind, but inclusion of these cases leads to considerably more expositional strain without adding much in qualitative results. The discussion of results, however, includes a reference to these cases.
According to Proposition 3, the principal strictly prefers infrequent evaluation in the LC scenario. The result holds for all possible levels of informativeness of the first-period outcome, i.e., for all cases ranging from low informativeness, \( p_1 \to p_0 \), to high informativeness \( p_1 \to 1 \). To gain intuition, note that low informativeness implies a weak substitution effect. Then the single-period problems partly separate (although not completely, of course) which resembles the benchmark. Therefore, efficiency of infrequent evaluation follows. (Recall that infrequent evaluation is efficient given independent periods.) With first-period outcome informativeness increasing, one might expect that eventually the principal would want to observe that outcome. However, with such increase the substitution effect becomes ever stronger, which in turn aggravates the incentive problem in period 2. Therefore, the principal is again better-off with not letting the agent know the first-period outcome.

Now consider the HC scenario. Here the substitution effect is stronger than in the LC scenario for any level of first-period informativeness. Clearly, if the first-period outcome is quite informative, infrequent evaluation minimizes the agent’s rent just as in the LC scenario. However, at low levels of informativeness the principal opts for frequent evaluation. The key to understanding why it can be efficient lies in the combination of low informativeness and a relatively strong substitution effect. From the agent’s point of view the effort strategy \( \{e_1 = 0, e_2 = 1\} \) becomes quite attractive as it entails only a slightly lower probability of success in period 1 while increasing the one in period 2. From the principal’s point of view the combination therefore makes for a quite informative observation of the second-period outcome – which is informative about first-period effort; and this informativeness decreases with an increasing informativeness of first-period outcome. Hence, the observation of the second-period outcome can be beneficial only at low levels of first-period outcome informativeness. To curtail the agent’s opportunism and benefit from the informative second-period outcome the sequence of outcomes needs to be observed and FPE is efficient.\(^\text{15}\)

Example 1 presents a numerical and graphical visualization of the results.

**Example 1** Parameters are chosen as follows: \( q_0 = 0.2; q_1 = 0.6; c = 2 \). In the LC scenario, \( p_0 = 0.05 \); in the HC scenario 1, \( p_0 = 0.5 \). Figure 1 plots the rents under either evaluation regime for a varying parameter \( p_1 \) given the LC scenario, and figure 2 does so for the HC scenario. The threshold in the high-chance scenario is calculated according to the condition in Proposition 4: \( p_1 = 0.75 \).

\(^{15}\)The point of transition from frequent evaluation to infrequent evaluation may not be unique. Given \( p_0 > 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + (1 - q_1)q_0}} \right) \) or \( p_0 > 0.5 \left( 1 - \sqrt{1 - \frac{q_1}{q_1 + q_0}} \right) \) such that (12) or (13) do not singly bind (as assumed in the high-chance scenario) but \( p_0 < 0.5 \) holds, IPE is efficient if incentive constraint (14) binds and

\[
0.5 - \sqrt{0.25 - \frac{(p_0 - p_0^2)q_1}{q_1 - p_0(q_1 - q_0)}} \leq p_1 \leq 0.5 + \sqrt{0.25 - \frac{(p_0 - p_0^2)q_1}{q_1 - p_0(q_1 - q_0)}}
\]

holds for real-valued \( p_1 \), and if \( p_1 \geq \frac{q_1(1 - p_0) + p_0 - q_0}{q_1 - p_0(q_1 - q_0)} \) (Proposition 4). Here the intuition is as follows. With \( p_1 \) increasing both informativeness of first-period outcome and the substitution effect increase. At moderately high levels then, the gain in informativeness offsets the stronger substitution effect and frequent evaluation becomes efficient.
In the LC scenario, infrequent evaluation is always efficient. Think of the agent being responsible for cost reductions. If low effort, or work-to-rule, is considered to have only a small chance of achieving the reductions in period 1, one could think that frequent evaluation should take place to check on the progress as early as possible. Yet choosing a longer evaluation horizon—i.e., infrequent evaluation—curtails the agent’s rent. The possibility of giving the agent another chance to reduce costs before she is called to account for the reductions is valuable.

In the HC scenario, frequent evaluation is efficient as long as $p_1 \leq 0.75$. Again think of the agent as being in charge for cost reductions. Now low effort in period 1 already offers an appreciable chance to succeed and high effort cannot increase that chance very much. One could be tempted to think that infrequent evaluation is efficient as cost reductions are quite likely under either effort and hence there is no need for an early inspection of results. However, that need exists and a shorter evaluation horizon limits the agent’s rent by restraining her opportunism—infrequent evaluation would make the action choice \( \{e_1 = 0; e_2 = 1\} \) too attractive. With increasing informativeness of first-period outcome the substitution effect becomes stronger and (additional) cost reductions in period 2 become less likely. The principal reacts and evaluates performance infrequently.

Note that, irrespective of the scenario, higher informativeness of first-period outcome leads to optimality of infrequent evaluation (corrolary 1). Stated differently, observation of highly informative performance measures is detrimental to firm profit when tasks bring about substitution effects across reporting periods. This results contrasts the finding in Lukas (2010) for complementary tasks where low informativeness of signals leads to optimality of infrequent evaluation. In addition the result should be compared with the one from Nikias et al. (2005). They find that a weak substitution effect leads to the optimality of infrequent evaluation whereas I demonstrate that it may be optimal under strong substitution effects as well. The puzzle vanishes if the differing effect of low effort in period 1 is taken into consideration. In Nikias et al. (2005) low first-period effort does not directly affect the success probability in period 2 but in my model it does. As a consequence, the outcome in period 2 becomes a less reliable indicator of period 1 effort and hence its observation is less beneficial. This is evident in the likelihood ratio for a high outcome in period 2 given high effort in both periods versus low effort in both periods: If the informativeness of period 1 outcome increases, the likelihood increases in Nikias et al. (2005), but it decreases in this paper.

5 Observability of interim outcome

The preceding analysis assumes non-observability of the interim outcome—neither principal nor agent know about first-period results before the contractual relationship continues in
5.1 Interim outcome perfectly observable

The optimal payments under IPE are given in Table 2. In cases (1) and (2) only the total $2x^H$ leads to a non-zero payment. It immediately follows that observation of $x_1 = x^L$ destroys all effort incentives for period 2. Of course, observing $x_1 = x^H$ does no harm. In case (3) the agent receives non-zero payments for achieving the total outcome $2x^H$ or $\{x^H + x^L\}$. It can be easily verified that if $(s^{2L} - s^{2H}) < (s^{2H} - s^{2L})$ holds, observation of $x_1 = x^L$ again destroys incentives for period 2. If the pay structure shows $(s^{2H} + s^{2L}) > (s^{2H} - s^{2L})$, the agent selects effort in period 2 only in response to a low outcome $x_1 = x^L$; a high outcome apparently erases effort incentives for period 2.

As expected, infrequent performance evaluation cannot be efficient (and maintain incentives in period 2) if the agent is able to perfectly observe the interim outcome at the end of period 1.

5.2 Interim outcome partly observable

Now assume at the end of period 1 the agent receives an informative signal $\sigma, \sigma \in \{L, H\}$, that has probability $\rho, \frac{1}{2} < \rho < 1$, of indicating the correct interim outcome, i.e., $\Pr(\sigma = L \mid x_1 = x^L) = \Pr(\sigma = H \mid x_1 = x^H) = \rho$. Given the efficiency of frequent evaluation with perfect observation and the possible efficiency of infrequent evaluation with no observation, one may suppose that partial observation of the interim outcome could be innocuous. As Proposition 5

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16See Solomons (1965, pp. 66ff) for examples and causes of forecast problems in this context.
and Proposition 6 show there exists a range of signal accuracy for the low-chance scenario only such that partial observation has no detrimental effect on incentive compatibility under IPE and hence it remains efficient.

**Proposition 5** In the low-chance scenario with partly observation, infrequent evaluation remains efficient if the agent observes the correct interim outcome with a probability \( \rho \leq \overline{\rho} \) where \( \overline{\rho} \) depends on the relevant incentive constraint(s) such that:

\[
\begin{array}{c|c}
\overline{\rho} & \text{binding incentive constraint(s)} \\
\hline
\frac{(1-p_1)^2 q_1 + (1-p_1)(q_1-q_0) + q_0((1-p_1)(q_1-q_0)+(1-p_0)^2)}{2(1-p_1)(q_1-q_0)} & (12) \\
\frac{(1-p_1)(q_1-q_0) - p_1(1-p_1)(1-p_0)(q_1)}{(1-p_1)(q_1-q_0)} & (14) \\
\frac{q_1(1-p_0) + p_0 - q_0}{1-q_0} & (13) \text{ and } (14)
\end{array}
\]

Otherwise frequent evaluation is efficient.

**Proposition 6** In the high-chance scenario with partly observation, frequent evaluation is efficient.

Only in the low-chance scenario, the agent may observe a noisy signal of the interim outcome at the end of period 1 and still IPE constitutes the efficient choice.\(^\text{17}\) What comes out as a necessary condition for the result is that \( s^H \) is the only non-zero payment in the optimal pay scheme without observation. As soon as the principal optimally offers a non-zero payment \( s^{H+L} \) given no observation (case (4) of Table 2), the observation of an informative signal after period 1 renders IPE infeasible; effort incentives in period 2 simply disappear following the observation of the signal. Intuitively, if for compensation purposes the principal does not need to distinguish between total outcomes \( \{2x^L\} \) and \( \{x^H + x^L\} \), the agent may without problems have access to information indicating which one is more likely. However, whenever the principal does distinguish between the two outcomes, the agent’s access to the information must be denied (if possible). In this case, when both \( s^{H+L} \) and \( s^{2H} \) are non-zero, the information whether \( \{2x^L\} \) or \( \{x^H + x^L\} \) is the more likely total outcome becomes critical.

**Example 2** Returning to Example 1, IPE is efficient in the low-chance scenario given no observation. It remains efficient under partial observation as long as \( \frac{1}{2} < \rho \leq \frac{1.25[0.6(1-p_1)^2 + 0.2095 - 0.2p_1]}{1-p_1} \) holds (the threshold given the binding incentive constraint (12) applies here). And as Figure 3 exemplifies, the probability of observing the correct interim outcome, or the signal accuracy, may be well above 0.5. If \( p_1 = 0.72 \) the threshold for signal accuracy drops to \( \rho = 0.5 \), i.e., the agent can only be allowed to observe a non-informative signal.

\(^\text{17}\)As already noted, assuming \( p_0 \geq 0.5 \) in the high-chance scenario leaves out some cases where \( p_0 < 0.5 \) and (12) or constraint (13) do not singly bind. In these cases, there may be parameter combinations such that IPE remains efficient under partial observation; they would require, however, \( p_0 \ll 0.5 \) and thus approach the low-chance scenario.
6 Summary and conclusion

This paper analyzes the optimal frequency of performance evaluations if substitution effects across periods exist. That means the performance in one period can only be increased at the cost of lower success probabilities in the future. The assumption appears reasonable for many situations where ever shorter evaluation periods with ambitious performance targets are chosen. I assume risk-neutral contracting parties and limited liability of the agent. As a first result I show that the principal always sets second-period bonuses at sequentially rational levels in the initial contract under frequent evaluation. Therefore he cannot benefit from renegotiations after the first period and can credibly commit to withhold performance information so that infrequent evaluation is feasible even if the principal observes interim outcomes. Infrequent evaluation proves to be efficient either unconditionally - in the low-chance scenario -, or if the substitution effect is sufficiently strong - in the high-chance scenario. The low-chance (high-chance) scenario refers to all parameter combinations where the agent has less than (at least a) fair chance to succeed in period 1 given low effort. Surprisingly, the result continues to hold in the low-chance scenario even if the agent observes a noisy signal of the interim outcome. Implications of this research relate to performance evaluations and the choice of the length of the evaluation period. Although computerized accounting systems would allow for rather short evaluation periods, longer evaluation periods are proven preferable from an incentive perspective. A short-term orientation of the firm manifested in a high frequency of performance evaluations and correspondingly short evaluation periods may not even pay in the short-term.

In the present analysis performance measures are exclusively used for control purposes. Alternative uses of accounting information, e.g., for learning about managerial ability or for decision making are not considered in this paper. Since more information helps to better infer managerial ability, a counterveiling effect to the substitution effect favoring less frequent evaluation would occur. However, for experienced managers with long tenure learning effects might be negligible reinforcing the argument in favor of infrequent evaluation.

Another promising way to add insights to our knowledge about the efficiency of (in)frequent evaluation might be to include psychological aspects. For instance, we know from Cook (1967) already that frequency of feedback is related to satisfaction with and interest in the job so that better performance may result from more frequent feedback. Yet there could be a critical frequency where interest and satisfaction decline leading to inferior performance. Adding additional uses of accounting information and psychological aspects to the analysis would therefore help to derive additional insightful results.
Appendix

A.1 Derivation of payments in FPE given substitutable tasks (δ = 1)

Set
\[ s^H - s^L = \frac{c}{(1 - p_1)(q_1 - q_0)} \cdot \nu^i, \nu^i \geq 1, i \in \{L, H\} \]  

(29)
such that second-period incentives constraints are satisfied. Plugging in (29) into incentive constraint (6), \( E(S_{1,1}) = E(S_{0,1}) \), and then into participation constraint (4) leads to the agent’s rent:

\[ R_{FPE} = 2c - p_1 \frac{c}{p_1 - p_0} + p_1 [(1 - (p_1 + p_0)q_1) - (1 - p_1)] \cdot q_1 \frac{c}{(1 - p_1)(q_1 - q_0)} \nu^H \]

\[ -p_1 \left[ \frac{(1 - p_0)^2 - (1 - p_1)^2}{p_1 - p_0} + (1 - p_1)^2 \right] \cdot q_1 \frac{c}{(1 - p_1)(q_1 - q_0)} \nu^L \]  

(30)

It can be shown that (30) is minimized if \( \nu^L = \nu^H = 1 \) and (29) simplifies accordingly. Plugging in into incentive constraint (6) – incentive constraint (5), \( E(S_{1,1}) \geq E(S_{0,0}) \), is slack – and given \( s^{LL} = 0 \) optimal payments in (19)-(22) obtain.

A.2 Proof of proposition 3

Rent differences between frequent evaluation and infrequent evaluation obtain by subtracting rents under IPE as given in (24)-(27) from the one under FPE as given in (23):

\[ \Delta_{(1)} = R_{FPE} - R_{IPE(1)} = c \left( \frac{p_1}{p_1 - p_0} + \frac{(1 - p_1)(q_1 - q_0)}{2p_1(1 - p_1)(q_1 - q_0)} \right) \]  

(31)

\[ \Delta_{(2)} = R_{FPE} - R_{IPE(2)} = \frac{p_1}{p_1 - p_0} + \frac{p_1 q_1}{(1 - p_1)(q_1 - q_0)} \]  

(32)

\[ \Delta_{(3)} = R_{FPE} - R_{IPE(3)} = \frac{(p_1 - p_1^2)p_1(q_1 - q_0) - [(p_1 - p_0) - (p_1^2 - p_0^2)]q_1}{(p_1 - p_0)(1 - p_1)(q_1 - q_0)(p_1 + p_0 - 1)} \]  

(33)

\[ \Delta_{(4)} = R_{FPE} - R_{IPE(4)} = c \frac{(1 - p_0)(p_1 - p_0)(q_1 - q_0) - q_1 - (p_0 - p_1)(1 - p_1)(q_1 - q_0) - p_1q_1[p_1q_1(p_1 - p_0)(1 - p_1)(q_1 - q_0) - p_1q_1(1 - p_1)]}{(p_1 - p_0)(p_1 - p_0)(1 - p_1)(1 - p_0)} \]  

(34)

where the index \( i = \{1, 2, 3, 4\} \) indicates the corresponding cases from Table 2 in section 4.2.

To ease exposition and traceability, incentive constraint \( E(S_{1,1}) \geq E(S_{0,0}) \), \( i, j \in \{0, 1\} \) will be referred to as \((i, j)\) indicating the act combination \( \{e_1 = i, e_2 = j\} \) that must not lead to higher utility for the agent than the desired act combination \( \{e_1 = 1, e_2 = 1\} \) under FPE or IPE, respectively. Under IPE, different incentive constraints may bind subsequently in different order. However, as \( p_1 \rightarrow 1 \) both \((1,0)\) and \((0,1)\) bind.

**Step 1.** Assume first, incentive constraint \((0,1)\) does not bind for all \( p_1 \). If incentive constraint \((0,0)\) or \((1,0)\) singly binds under IPE, IPE is efficient. Inspection of \( \Delta_{(2)} \) in (32) proves the latter. To prove the former, observe that IPE shows \( E(S_{1,1}) = E(S_{0,0}) \) if \((0,0)\) singly binds. FPE would show \( E(S_{1,1}) = E(S_{0,0}) \) as well if sequential rationality constraints
(7) and (8) need not be satisfied. Adding these two constraints leads to \( E(S_{[1,1]}^H) = E(S_{[1,0]}^H) = E(S_{[0,1]}^H) > E(S_{[0,0]}^H) \) under FPE making the initially binding constraint slack. Hence the solution cannot improve and IPE is efficient. Now if both \((0,1)\) and \((0,1)\) bind subsequent to either \((0,0)\) or \((1,0)\), IPE is also efficient at the level of \(p_1\) where the transition in binding constraints takes place. This implies \( \Delta_{(4)} > 0 \) in (34). Since \( \Delta_{(4)} \) is increasing in \( p_1 \), IPE continues to be efficient.

**Step 2.** Now assume incentive constraint \((0,1)\) binds for a set of \(p_1\)-values. Further assume with \(p_1\) increasing, \((0,1)\) binds prior to \((0,0)\) or \((1,0)\). For \(p_1 \rightarrow p_0\), FPE is efficient since \( \lim_{p_1 \rightarrow p_0} \Delta_{(3)} < 0 \) in (33). For \(p_1\) sufficiently close to \(p_0\) but not \(p_1 \rightarrow p_0\), IPE is efficient since \( \Delta_{(3)} > 0 \) in (33) holds. To check whether the sign of \( \Delta_{(3)} \) can change, note that the derivative of the nominator in \( \Delta_{(3)} \) with respect to \(p_1\) is
\[
\frac{\partial}{\partial p_1} \left\{ \frac{(p_1 - p_1^2)p_0(q_1 - q_0)}{-(p_1 - p_0) - (p_2^2 - p_0^2)} \right\} = (1 - 2p_1)[p_0(q_1 - q_0) - q_1] \leq 0 \iff p_1 \leq 0.5. \]
Since \(p_0 < 0.5\) in the LC scenario, the nominator in \( \Delta_{(3)} \) monotonically decreases in \(p_1\) for \(p_1 \in (p_0, 0.5)\). It can be shown that the transition from \((0,1)\) binding to \((1,0)\) or \((0,0)\) binding occurs for \(p_1 = \bar{p}_1 < 0.5\). At \(\bar{p}_1\), \( \Delta_{(3)} > 0 \) in (33) still holds and no change of sign of \( \Delta_{(3)} \) occurs for \(p_1 \in (p_0, \bar{p}_1]\). For \(p_1 \in (\bar{p}_1, 1)\) step 1 of the proof applies and IPE is efficient for \(p_1 \in (p_0, 1)\).

**Step 3.** Still assume incentive constraint \((0,1)\) binds for a set of \(p_1\)-values but assume further, with \(p_1\) increasing, \((0,1)\) binds subsequent to \((0,0)\) or \((1,0)\). It can be shown that the transition occurs for \(p_1 = \bar{p}_1 > 0.5\). At \(\bar{p}_1\), \( \Delta_{(3)} > 0 \) in (33). Since \(\frac{\partial}{\partial p_1} \left\{ \frac{(p_1 - p_1^2)p_0(q_1 - q_0)}{-(p_1 - p_0) - (p_2^2 - p_0^2)} \right\} = (1 - 2p_1)[p_0(q_1 - q_0) - q_1] > 0 \) if \(p_1 > 0.5\), the positive nominator in \( \Delta_{(3)} \) further increases. If the transition in incentive constraints from \((0,1)\) to both \((1,0)\) and \((0,1)\) occurs, \( \Delta_{(4)} > 0 \) in (34). Since \( \Delta_{(4)} \) is increasing in \(p_1\), IPE continues to be efficient. Therefore IPE is efficient for \(p_1 \in (p_0, 1)\).

If the incentive constraint \((0,1)\) binds subsequent to \((0,0)\) and prior to \((1,0)\), steps 2 and 3 apply accordingly.

**A.3 Proof of proposition 4**

In the high-chance scenario incentive constraints \((0,0)\) and \((1,0)\) never bind singly because no real-valued \(p_1\) exists for the transition to the binding \((0,1)\) constraint. (See footnote 13.) Transition from \((0,1)\) binding to both \((0,1)\) and \((1,0)\) binding occurs at \(\hat{p}_1 = \frac{q_1(1-p_0)}{q_1(1-p_0)+p_0}\). The threshold \(\hat{p}_1\) is given by
\[
E \left(S_{[0,1]} | s_{(3)}^{2H} \right) - E \left(S_{[0,1]} | s_{(4)}^{H+L}, s_{(4)}^{2H} \right) = 0
\]
which leads to
\[
\frac{[q_1(1-p_0-p_1) - p_0p_1(1-q_1)] [q_1(p_0 - p_0^2) - q_0(p_1 - p_1^2)]}{(1-p_1)[p_1 - q_1(1-p_0)](q_1 - q_0)(p_1 - p_0)(p_0 + p_1 - 1)} \cdot c = 0. \quad (35)
\]
For \(p_0 \geq 0.5\), the term in the right brackets decreases monotonically. Therefore the term in the left brackets is candidate for transition from \((0,1)\) binding to both \((0,1)\) and \((1,0)\). Then it can be shown that \(p_0 \geq 0.5\) implies \(\hat{p}_1 < 0.5\) - contradicting the assumption \(p_0 < p_1\). Hence
both (0,1) and (1,0) bind if \( p_0 \geq 0.5 \). Since \( \lim_{p_1 \to p_0} \Delta_{(4)} < 0 \) and \( \lim_{p_1 \to 1} \Delta_{(4)} > 0 \) in (34) the threshold \( p_1 \geq \frac{q_1(1-p_0)+p_0-q_0}{1-q_0} \) obtains for IPE to be efficient.\[\blacksquare\]

### A.4 Proof of proposition 5

Consider case (1) from Table 2 and incentive constraint (0,0) is binding. The incentive compatibility constraint for period 2 with observation of signal \( \sigma = L \) obtains as

\[
\rho \cdot 0 + (1 - \rho) \cdot (1 - p_1)q_1s_{(1)}^{2H} - c \geq \rho \cdot 0 + (1 - \rho) \cdot (1 - p_1)q_0s_{(1)}^{2H}
\]

so that

\[
s_{(1)}^{2H} \geq \frac{c}{(1 - p_1)(q_1 - q_0)(1 - \rho)}
\]

(36)

must hold to maintain effort incentives under partial observation. Since \( s_{(1)}^{2H} = \frac{2c}{p_1(1-p_1)(q_1-p_0)(1-p_0)/q_0} \) (see Table 2), condition (36) is fulfilled if \( \rho \leq \frac{(1-p_1)^2q_1+(1-p_1)(q_1-q_0)+q_0(1-p_1-p_0)}{2(1-p_1)(q_1-q_0)} \). Proceeding in a similar way, the condition for case (3) where (0,1) is binding obtains. In both cases, incentive constraints after observing \( \sigma = H \) are met for any \( \rho > \frac{1}{2} \).

The conditions for case (4) where both (0,1) and (1,0) are binding are given by

\[
\begin{align*}
p_1 & \leq \rho \leq 1 - p_1 & \text{if } (s_{(4)}^{2H} - s_{(4)}^{H+L}) - (s_{(4)}^{H+L} - s_{(4)}^{2L}) > 0, \text{ or } \quad (37) \\
1 - p_1 & \leq \rho \leq p_1 & \text{if } (s_{(4)}^{2H} - s_{(4)}^{H+L}) - (s_{(4)}^{H+L} - s_{(4)}^{2L}) < 0 \quad \text{or } (38)
\end{align*}
\]

holds. It can be shown that

\[
(s_{(4)}^{2H} - s_{(4)}^{H+L}) - (s_{(4)}^{H+L} - s_{(4)}^{2L}) \geq 0 \iff p_1 \geq \frac{q_1(1-p_0)+p_0-q_0}{1-q_0}.
\]

(39)

Recall that (0,1) and (1,0) jointly bind only if \( p_1 \geq \frac{1}{2} \); hence condition (37) cannot be met. Only (38) can be met if \( \frac{1}{2} < \rho \leq \frac{q_1(1-p_0)+p_0-q_0}{1-q_0} \).

In case (2) with (1,0) binding \( \rho = 0 \) or \( \rho = 1 \) must hold after observing \( \sigma = L \) or \( \sigma = H \), respectively, conditions which cannot hold at the same time.\[\blacksquare\]

### A.5 Proof of proposition 6

For case (4) condition (37) cannot be met because the high-chance scenario shows \( 0.5 \leq p_0 < p_1 \). Note also that condition (38) cannot be met either because IPE is efficient if and only if \( p_1 \geq \frac{q_1(1-p_0)+p_0-q_0}{1-q_0} \) and condition (38) requires the opposite as shown in (39).\[\blacksquare\]

### References


Figures

Figure 1: Low-chance scenario: Rents given Frequent Performance Evaluation (black) and Infrequent Performance Evaluation (gray)

Figure 2: High-chance scenario: Rents given Frequent Performance Evaluation (black) and given Infrequent Performance Evaluation (gray)
Figure 3: Maximum signal accuracy $\rho$ as a function of first-period success probability $p_1$