

# Optimal pacing strategy for a race of two competing cyclists

T Dahmen <sup>1</sup>✉ and D Saupeand <sup>1</sup>

## Abstract

**Background:** For optimal pacing strategies in the case of two or more competing or cooperating cyclists only few approaches take slipstreaming into account. However, by incorporating the slipstream effect in the model of a race of two runners on a flat course, it has been shown, how the trailing runner can position himself at striking distance behind the other and when he should start the final sprint. (Pitcher, 2009: Optimal strategies for a two-runner model of middle-distance running. SIAM Journal on Applied Mathematics, 70(4), 1032–1046).

**Purpose:** We transfer this approach to cycling on a track of fixed length with real-world height data. In particular on descents, high speed is involved and increases the significance of the slipstreaming strategy.

**Methods:** We adopt the standard mechanical bicycling model that accounts for pedaling power, gravity, friction, inertia, and aerial drag. The nominal aerial drag force is multiplied by a slipstream factor that has its minimum when the cyclist is located closely behind his opponent (Pitcher, 2009). Our physiological model defines the remaining anaerobic capacity that de-/increases non-linearly when the pedaling power exceeds/falls below critical power (Gordon, 2005: Optimizing distribution of power during a cycling time trial. Sports Engineering, 8(2), 81–90). The mechanical and physical parameters may be different for the two cyclists. We use a state-of-the-art optimal control method for the numerical computations (Patterson et al., 2013: GPOPS-II: A MATLAB software for solving multiple-phase optimal control problems using hp-adaptive gaussian quadrature collocation methods and sparse nonlinear programming. ACM Transactions on Mathematical Software, 39(3).).

**Results:** The optimal pacing strategy for two cyclists on a competition between Saint-Gildas-des-Bois to Redon (stage 3 of the Tour de France 2013), France, is shown in Figure 1.

**Discussion and Conclusion:** Slipstreaming has a significant impact on pacing strategies in cycling. Incorporated into the mechanical bicycling model, optimal control algorithms can be used to compute the optimal tactic for the final sprint. Future work should take into account that the cyclists will often cooperate to stay ahead of the peloton before they compete in the final sprint phase.

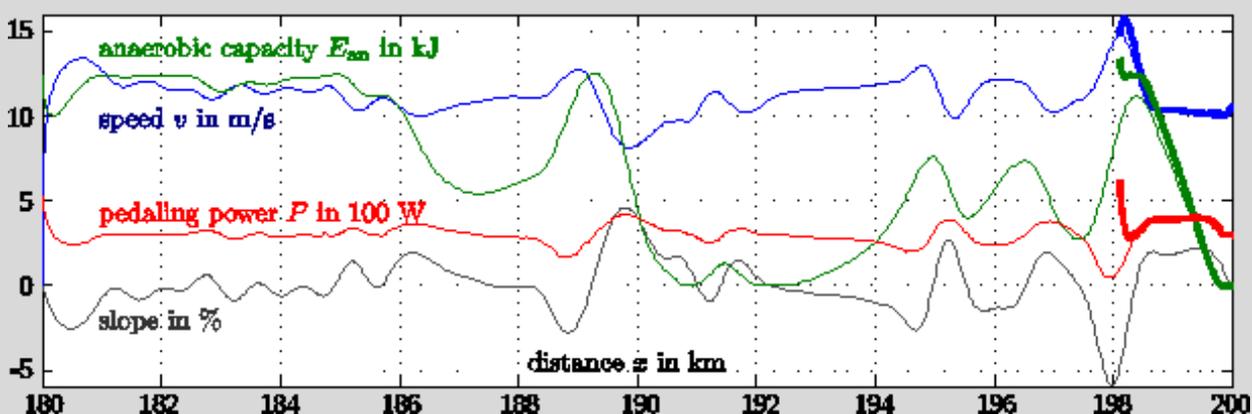


Figure 1. The thin colored lines represent the losing cyclist. Up to 198.1 km, the winner stays behind the leader (omitted). Then, he takes the lead and starts his final sprint (bold lines).

✉ **Contact email:** [thorsten.dahmen@uni-konstanz.de](mailto:thorsten.dahmen@uni-konstanz.de) (T. Dahmen)

<sup>1</sup> University of Konstanz, Konstanz, Germany.

Received: 1 May 2014. Accepted: 1 June 2014.

