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Development of mathematical competencies in adolescence
The PALMA longitudinal study

The *Project for the Analysis of Learning and Achievement in Mathematics* (PALMA)[1] analyses adolescents' development in the domain of mathematics during secondary school (grades five to ten). Using a longitudinal design involving annual assessments, the main study of the project aims at investigating the development of mathematical competencies over school years. Individual student characteristics in mathematics, classroom instruction in this domain, and variables of classroom and family contexts are also assessed. The comprehensive approach of the project enables us to address some of the deficits of previous research on adolescents' development in mathematics. Specifically, few of the existing studies used longitudinal designs. Also, the extant research focussed on specific competencies or school years, or only used qualitative methodology (Baumert, Gruehn, Heyn, Köller, & Schnabel, 1997; Blum, Kaiser, Burghes, & Green, 1994; Köller, Watermann, Trautwein, & Lüdtke, 2003; Kuechemann & Hoyles, 2003; Lehmann, Husfeldt, & Peek, 2001; Rost, 2000; Watson & Kelley, 2004). By implication, few generalisable conclusions can be drawn to date.

The theoretical framework of the project employs a conception of mathematical competencies that is consistent with constructs of *mathematical literacy* as used in the OECD's *Programme for International Student Assessment* (PISA). Specifically, the differentiation between competencies in mathematical modelling, on the one hand, and in performing algorithmic operations, on the other, is of fundamental importance for this project. Also, the framework emphasises the role which basic mathematical conceptual ideas (*Grundvorstellungen*) play in mathematical modelling and problem solving (Blum, 1998; vom Hofe, 1995, 1998; vom Hofe, Kleine, Blum, & Pekrun, 2005). Competencies in mathematical modelling include abilities to convert real-world problems into mathematical models, to solve these problems within the context of the mathematical models, and to transfer the solutions back into reality (Blum et al., 2004; an example can be seen in the task "chocolate", see below 2.1).

Regarding student characteristics, the project involves an analysis of the *emotions* that students experience in the domain of mathematics. Emotions have been largely neglected by educational research up until now, with only a few exceptions like studies on test anxiety (Zeidner, 1998). This also pertains to students' emotions in mathematics, research on mathematics anxiety being an exception. Basic research in psychology and the neurosciences, however, has shown that emotions are of fundamental importance for learning, memory, and competence development (Lewis & Haviland-Jones, 2004). Emotions can impact on students' academic learning by changing brain

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dopamine levels affecting long-term memory (Ashby, Isen, & Turken, 1999), by directing the use of cognitive resources (Meinhardt & Pekrun, 2003), by inducing and sustaining student interest in learning material (Ainley, Corrigan & Richardson, 2005; Krapp, 2005), by triggering different modes of information processing and problem solving (Isen, 1999), and by facilitating or impeding students' self-regulation of learning (Pekrun, Goetz, Titz, & Perry, 2002). For explaining the development of students' emotions in mathematics, and for analysing their effects on the development of students' interest, motivation, learning strategies, self-regulation, and cognitive competencies in this domain, Pekrun's control-value theory of achievement emotions is used as a theoretical framework (Pekrun, 2000, 2006; Pekrun, Goetz, Titz, & Perry, 2002; Pekrun, Frenzel, Goetz, & Perry, 2007).

The project comprises a preliminary cross-sectional pilot study, a representative cross-sectional field study, and a longitudinal main study. The pilot study and the field study mainly served to develop the assessment instruments. Based on the findings of the studies, the project also aims at developing products that can be used in educational practice, including material for mathematics instruction (such as textbooks and collections of mathematical tasks), recommendations for the development of curricula, material to be used in teacher education, instruments for assessing students' competencies and emotions in mathematics, and instruments for evaluating mathematics classroom instruction.

In this chapter, the method (section 1) and exemplary first findings (section 2) of the longitudinal study are addressed. At present, the study has not yet been completed. Data are available for the first four annual assessments, implying that findings reported here pertain to grade levels five to eight. At the end of the chapter, a short overview of our products for educational practice (section 3) and a summary of conclusions (section 4) are presented.

1 Method

1.1 Participants and procedure

The PALMA longitudinal study includes annual assessments from grades five to ten. Samples consist of students, their mathematics teachers, and their parents. To make it possible to analyse classroom instruction and the classroom context, the student samples comprise students from intact classrooms. Samples are drawn from Bavarian schools, and are drawn so that they are sufficiently representative of the student population of Bavaria, and include students from all three school types of the Bavarian school system. The three school types differ in academic demands and students' entry level of academic ability. As in most German federal states, these school types include a lower-track school (*Hauptschule*), an intermediate-track school (*Realschule*), and a higher-track school (*Gymnasium*).

In order to avoid selection effects, low-achieving students who repeated a grade were kept in the longitudinal sample. Also, across annual assessments, we used a strategy of continuously monitoring the development of intact classrooms. At each time of assessment, one important part of this strategy was to include students who had not yet participated in the study, but had become members of PALMA classrooms at the time of assessment (for more details on sampling procedures, see Pekrun et al., 2006).

At time one (grade 5), the student sample comprised 2,070 students (1,043 male and 1,027 female students; mean age 11.7 years) from 83 classrooms and 42 schools. The sample of participating parents included 1,977 parents, and the teacher sample all 83 mathematics teachers from the participating classrooms. At time two (grade 6), the samples consisted of 2,059 students (1,029 male and 1,030 female students; mean age 12.7 years), 1,883 parents, and 76 teachers from 81 classrooms of the same 42 schools. As expected, a sizable subsample of previous Hauptschule students had changed to Realschule at time three (grade 7). Keeping most of these students and adding students from the classrooms they had entered, the samples of the third assessment included 2,397 students (1,195 male and 1,202 female students; mean age 13.7 years), 2,062 parents, and 71 mathematics teachers from 74 intact classrooms and 44 schools. At time four (grade 8), the samples comprised 2,410 students from the same classrooms and schools (1,193 male and 1,217 female students; mean age 14.8 years), 1,938 parents, and 73 teachers. Participation rates were high for all three groups of participants (rates of more than 91%, 80%, and 93% for students, parents, and teachers, respectively). The longitudinal sample of students who participated in all four assessments included 1,421 students (49.6% female), thus comprising 69% of the initial student sample. Longitudinal attrition of students was 12%, 15%, and 8% from grades five to six, six to seven, and seven to eight, respectively.

At each grade level, the assessments took place towards the end of the school year (May and June). The student assessments comprised a mathematical achievement test, a test measuring basic cognitive abilities, and a student questionnaire. These instruments were administered by trained external test administrators in the students' classrooms. The total testing time was 180 minutes at each grade level. The parent and teacher assessments comprised questionnaires that were administered individually. The German Data Processing Center (DPC) of the International Association for the Evaluation of Educational Achievement (IEA) was responsible for drawing the student, parent, and teacher samples, and for organising the annual assessments.

In addition to the quantitative annual assessments, we also conducted qualitative interviews in subsamples of students. The interviews aimed at analysing the cognitive strategies students use to solve mathematical problems in more detail, as well as their mathematics emotions and their perceptions of the social context of development in mathematics. The interviews also included think-aloud procedures of assessing problem-solving strategies. They were videotaped and transcribed before being analysed (Pekrun et al., 2006).

1.2 Variables and instruments

In each of the annual assessments of the longitudinal study, we assessed students' mathematical competencies; their emotions, motivation, and learning-related behaviour in mathematics; as well as variables of mathematics classroom instruction, the classroom context, teachers' professional careers, and the students' family context. The main variables and instruments are described in the following sections.

1.2.1 Mathematical competencies

For assessing students' mathematical competencies, we developed the *Regensburg Mathematical Achievement Test* (vom Hofe et al., 2002; vom Hofe, Kleine, Pekrun, & Blum, 2005)[2]. Using Rasch-scaled scores, this test measures students' modelling competencies and algorithmic competencies in arithmetics, algebra, and geometry. It also comprises subscales pertaining to more specific contents (e.g. fractions; vom Hofe, Kleine, Pekrun et al., 2005). The test was constructed in such a way that it allows to assess mathematical competencies across all grade levels of secondary school, and for students of different ability within grade levels.

1.2.2 Student characteristics

Students' basic cognitive abilities (nonverbal reasoning abilities) were assessed by scales of the German version of the Cognitive Abilities Test (Heller & Perleth, 2000). Student characteristics in the domain of mathematics were assessed by a student questionnaire. The scales of this questionnaire measure students' self-related cognitions (such as self-concept of ability, self-efficacy, and value appraisals), emotions, interest, motivation, learning strategies, and self- vs. external regulation of learning in the domain of mathematics, as well as more general variables of students' educational careers. Students' test motivation and the emotions they experienced while working on the mathematical achievement test were also measured.

For a number of these characteristics, scales were developed in the PALMA project. Specifically, in research up until now, there is a lack of instruments which assess students' mathematics emotions other than anxiety. Therefore, we developed the *Achievement Emotions Questionnaire – Mathematics*[2] (AEQ-M; Pekrun, Goetz, & Frenzel, 2005) which measures various emotions experienced in mathematics. The AEQ-M comprises scales for mathematics-related enjoyment, pride, anger, anxiety, shame, hopelessness, and boredom. In addition, we developed scales that assess the enjoyment, anxiety, and boredom experienced by students when dealing with specific mathematical contents, including emotions experienced during modelling and algorithmic tasks (Frenzel, Jullien, & Pekrun, 2006; Zirngibl, Goetz, Pekrun, vom Hofe, & Kleine, 2005).

1.2.3 Classroom instruction, classroom context, and teachers' professional careers

Taking the perspective of both students and teachers into account, variables of mathematics classroom instruction and of the social composition of the classroom context were assessed by student and teacher questionnaires. Regarding instruction, scales from these questionnaires focus on measuring the cognitive quality of instruction (e.g. variables of instruction oriented towards mathematical modelling), its motivational quality (e.g. teacher enthusiasm), and the quality of classroom management (structure, use of time etc.). In addition, the teacher questionnaire assesses job-related teacher

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2 The *Regensburg Mathematical Achievement Test* will be available after completion of the longitudinal study in 2008. The *Achievement Emotions Questionnaire – Mathematics* is available upon request from the first author.

characteristics such as teachers' emotions and burnout in the domain of mathematics, as well as aspects of their professional careers.

1.2.4 Family context

Using student and parent questionnaires, we assessed the socio-economic status of the family and its social and cultural capital. Furthermore, scales in the questionnaires assessed students' and parents' perceptions of the co-operation between family and school, the social climate in the family, parental rearing styles, parents' general involvement in the educational career of their children, and parents' engagement in their children's competence development in mathematics. Regarding parental engagement in mathematics, the questionnaire scales assessed parents' self-concept of their ability in mathematics, their values in this domain, as well as aspirations and support relating to their children's development in mathematics.

For each of the annual assessments, findings on item and scale characteristics confirmed the psychometric quality of the instruments (Pekrun, vom Hofe, & Blum, 2003, 2006). All of the scales, as well as the results of item and scale analyses, are documented in the codebooks for the longitudinal study (Pekrun, Goetz, Jullien et al., 2002, 2003, 2004; Pekrun, Jullien, Lichtenfeld et al., 2005).

2 Results and discussion

At present, data for the development from grades five to eight are available, as noted above. In the following sections, we give an overview of exemplary findings for this time period. These findings pertain to (1) the development of mathematical competencies; (2) student characteristics, including students' emotions in mathematics; (3) classroom instruction in mathematics; and (4) the role of the family.

2.1 Development of mathematical competencies

Using our longitudinal data, we aimed at investigating how students' mathematical competencies develop over school years. Also, we wanted to analyse whether there are differential developments for different types of competencies, and for different groups of students. Our analysis pertained to the main topics of the German mathematics curriculum for the grade levels considered, with a special emphasis on fractions. One primary question was whether students' difficulties with fractions are due to inadequate mathematical conceptual ideas.

2.1.1 Development of mathematical competencies: Quantitative findings

The development of competence scores was analysed using the longitudinal sample of students who had participated in all four annual assessments from grades five to eight, as described in section 1.1. The scaling of competence scores was also based on this sample (with M = 1,000, SD = 100). As can be seen in Figure 1, there was a substantial increase in overall competence scores (time 1: M = 919.32, SD = 72.76; time 2:

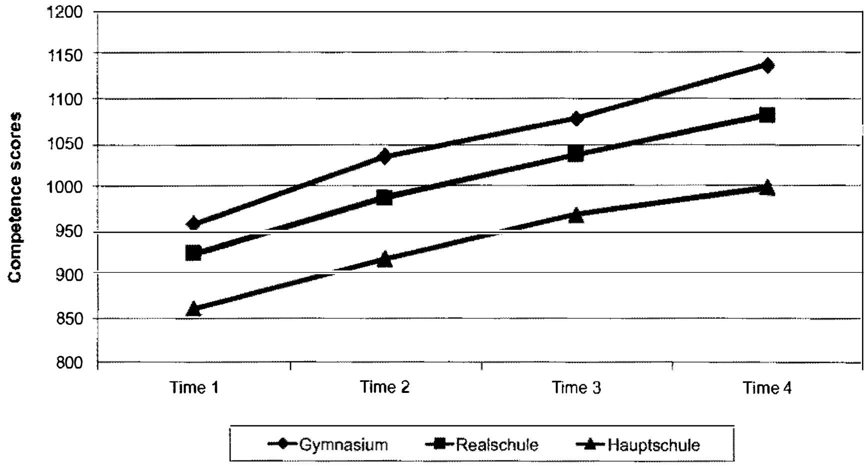
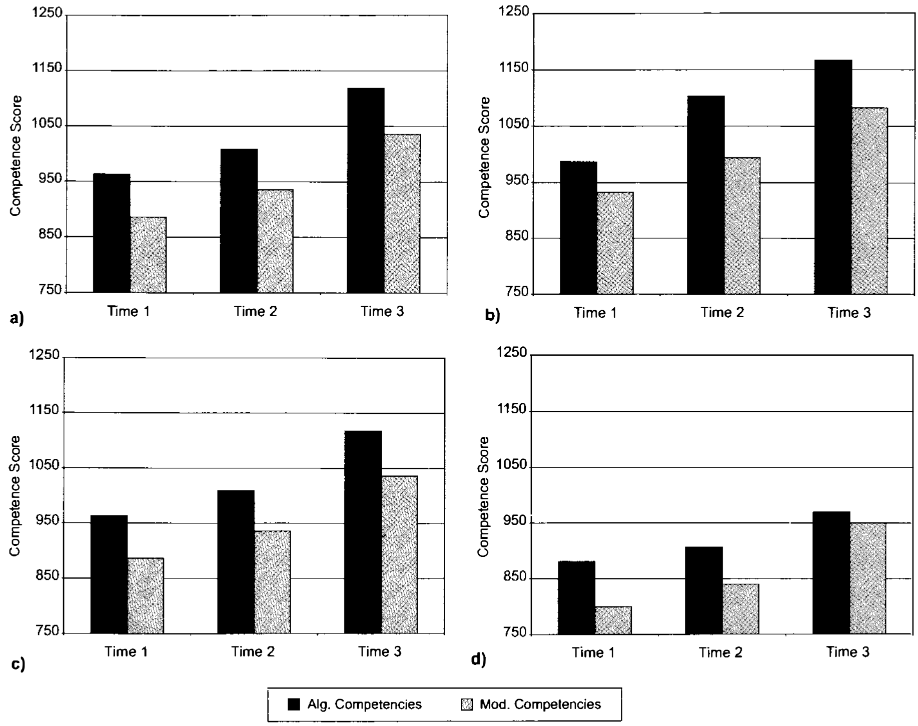


Figure 1: Development of mathematical competence from grades 5 to 8

M = 985.95, SD = 76.33; time 3: M = 1,033.16, SD = 76.49; time 4: M = 1,080.23, SD = 91.06). The results of a one-factorial ANOVA with time as the within-subjects independent variable showed that this increase was highly significant (F [3, 1411] = 3,122, $p < .001$). Post-hoc tests revealed that the increase was significant for each of the three time intervals (time 1-2: t (1,411) = 49.15, $p < .001$; time 2-3: t (1,411) = 36.29, $p < .001$; time 3-4: t (1,411) = 29.25, $p < .001$; vom Hofe, Kleine, Pekrun et al., 2005). Compared to the change in students' competence scores over one school year in comparable data sets (e.g. the German TIMSS data; Baumert et al., 1997), the annual increase in scores was relatively high in the present study.

Also, the increase in mean scores was continuous and substantial for each of the three school types (Figure 1). However, across the four assessments, the increase was somewhat higher than average for students from the Gymnasium, and lower than average for students from the Hauptschule. In a two-factorial ANOVA with time and school type as independent variables, the interaction of time of assessment and school type reached significance (F [1418, 2] = 37, $p < .01$). This finding is similar to the findings of other German studies (e.g. the Kassel-Exeter study; Blum, Burghes, Kaiser, & Green, 1994; Kaiser, Blum, & Wiegand, 2000). Since effects of regression to the mean (Marsh & Hau, 2002) were not controlled in the present analysis, it may be that the developmental differences for students from the Gymnasium and the Hauptschule are, in fact, even more pronounced than suggested by this interaction.



Figures 2a to 2d: Mean scores for modelling and algorithmic competencies; a) total sample; b) Gymnasium; c) Realschule; d) Hauptschule

Total competence scores in mathematics can be composed of different scores for more specific competencies. Data for students' mathematical modelling competencies and algorithmic competencies were available for grades five to seven. Findings showed that students' average scores for algorithmic competencies were consistently higher than their scores for modeling competencies, across all grade levels and school types included (Figure 2). However, score differences decreased over time for students from the Hauptschule.

The Regensburg Mathematical Achievement Test was constructed so that cognitive item complexity is equivalent across the subscales for modelling and algorithmic competencies. By implication, the findings suggest the following two conclusions. (1) At grade levels five to seven, German students' algorithmic competencies are higher than their competencies in mathematical modelling. (2) Contrary to our expectations, however, the differences between these competencies do not increase across grade levels.

The findings of the PISA 2000 and 2003 assessments suggest that German 15-year-old adolescents have deficits in competencies for mathematical modelling (Blum et al., 2004). The present findings imply that these deficits are not specific to 15-year-olds. Rather, they are observable at the beginning of secondary school as well. The findings thus suggest a long-term development of deficits that extends over many school years, in line with the assumption that mathematics classroom instruction in German secondary schools does not sufficiently support students' development of competencies for mathematical modelling.

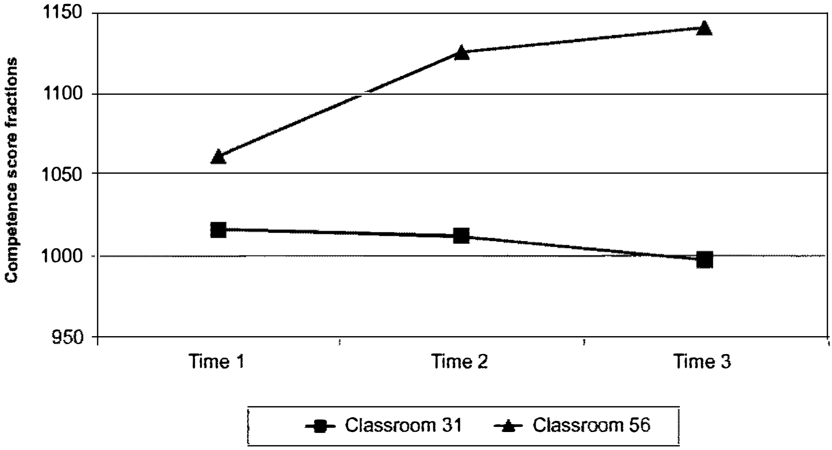


Figure 3: Development of scores for subscale fractions in classrooms 56 ($n = 26$) and 31 ($n = 25$)

However, we also found substantial differences between schools, and between classrooms within schools. An example of two classrooms showing a differential development of scores for the fractions subtest of the Regensburg test is shown in Figure 3. Whereas students of classroom 56 showed a substantial increase in average scores from grades five to seven, scores for classroom 31 did not increase at all.

Classroom instruction in mathematics is among the factors likely to contribute to such differences. As perceived by the students, instruction was less oriented towards mathematical modelling and less clearly structured in classroom 31 than in classroom 56. This lack of cognitive stimulation and structure in mathematics instruction may help to explain the unfavourable development of students' average competence scores in this classroom.

2.1.2 Students with achievement deficits ("at-risk students")

In the PISA 2000 and 2003 assessments of mathematical competence, sizable groups of German students were found to be at risk for leaving school without a level of mathematical literacy sufficient to enable them to start a vocational career and to participate in modern society (Baumert et al., 2001; Blum et al., 2004). In the German PISA assessments, a student was defined as being at risk in mathematics when his or her competence score in this domain was equal to or lower than 420 (Blum et al., 2004). Since this score was defined in more or less arbitrary ways, we preferred to employ an empirically based rationale for defining a cut-off score in the present investigation.

The first step was to evaluate all of the items of the Regensburg test as being risk-relevant or not. Relevance was rated separately for the different grade levels involved and defined by the following three criteria. (1) In order to solve the item, no more than elementary knowledge of the curriculum of the respective grade level at the Hauptschule is needed. (2) At the next grade level, the knowledge tested by the item is presumed to have been acquired, but is no longer taught as part of the curriculum. (3) The knowledge needed for solving the item is relevant for everyday life and vocational careers, as defined by the entrance exams of the German Boards of Trade. The item threshold of the most difficult item judged to be relevant was used to define the upper limit of the scores of students at risk. Students who had lower ability scores were classified as being at risk.

Using this definition, we analysed how many students were to be considered as being at risk in mathematics (Figure 4). There was an increase in the number of students fitting the criteria over grade levels (from 15% at grade 5 to almost 19% at grade 7). At each grade level, most of the students, but not all of them, were from the Hauptschule. At grade seven, more than half of the students from these schools were to be defined as being at risk, since they were not even able to solve the most basic problems provided by their curriculum.

When analysing profiles of student characteristics, we found that at-risk students had low grades in mathematics, developed unfavourable mathematics self-concepts, and reported more anxiety in mathematics than the average student. Furthermore, findings suggested that some classrooms were composed of significantly more at-risk students than others, and that classroom context factors may be important for defining the likelihood of becoming a member of the at-risk group. It seems that some classrooms promote an unfavourable development in the domain of mathematics, whereas others are able to support students' competence development. In subsequent analyses, we plan to identify factors that can help teachers and administrators to shape classroom instruction and the composition of classrooms in such a way that an unfavourable development of mathematical competencies is prevented.

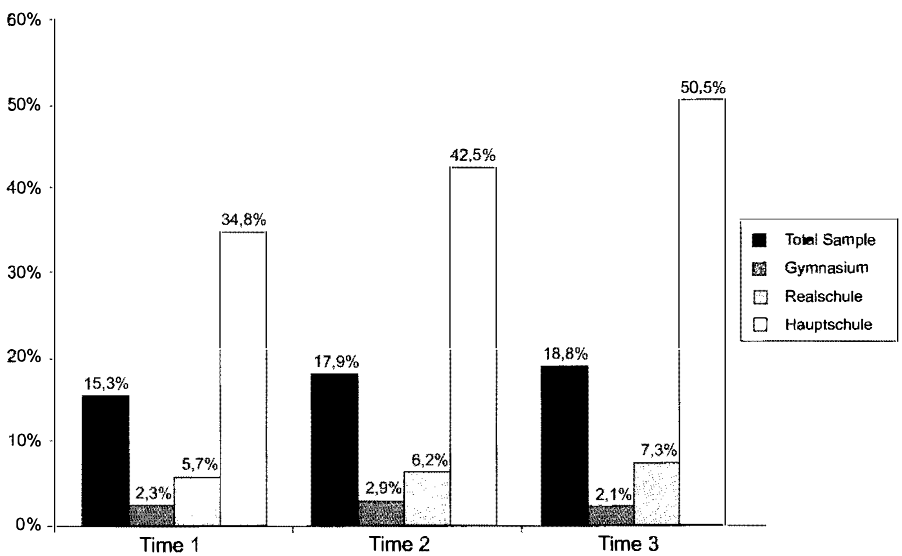


Figure 4: Percentage of at-risk students in different school types over time

2.1.3 Findings from the qualitative interviews: The role of basic mathematical conceptual ideas

By conducting qualitative interviews with subgroups of students, we aimed to analyse the strategies they use to solve mathematical problems. One primary focus was on analysing the role of basic mathematical conceptual ideas (*Grundvorstellungen*). The interviews referred to the main topics of the curriculum of grades five to eight (grades 5 and 6: fractions and proportionality; grade 7: fractions and negative numbers; grade 8: equations and functions). One main finding is the importance of inadequate conceptual ideas for students' errors when trying to solve fraction problems. The following example is taken from an interview with a 6th grade student.

Task "Chocolate"

Lily takes half of the bar of chocolate depicted at the right. She eats 3/5 of what she took. How many pieces did she eat?

Contrary to expectations, the student did not use a part-whole operation to solve this task, but made an attempt to solve it by performing a division. Her answers, given in the interview, clarified why she did this:

I: Why divide by zero point six?
S: Yes, um, yeah.
Because ten is equal to zero point five, and this is half of all of it.
And the zero point six is the three-fifths.
And that's what you need of the half, since it's only a half, and not all of it.
I: My question related more to the arithmetic operation. Why divided by?
And not times or plus or minus?
S: Because, um, by doing that, it would become more, but it has to become less
and less, because she doesn't eat more than the bar, but less than the bar.
S [performs the division 10 : 0,6 (26 seconds)]

As illustrated by this example, the primary reason given by students for performing this kind of arithmetic operation was that they could not conceive of any other way to obtain a diminution of the starting value. This erroneous strategy is based on a conceptual idea of multiplication that has not been adequately been enlarged (Wartha, 2005): while a multiplication implying natural numbers always leads to an augmentation, it can lead to a diminution when using fractions.

The findings of our interviews thus corroborated the importance of basic mathematical conceptual ideas for solving mathematical problems, and of inadequate ideas for errors (see also Fischbein, Tirosh, Stavy, & Oster, 1990; vom Hofe & Wartha, 2005). Overall, approximately half of the students' errors that were analysed in our interviews were found to be due to inadequate basic conceptual ideas.

2.1.4 Conclusions

Students' average competence scores in the domain of mathematics showed a substantial increase from grades five to eight. However, there were differential developments for different types of competencies and students. Across grade levels, scores for algorithmic competencies were higher than scores for modelling competencies. A sizable number of students were judged to be at risk, being unable to solve the most elementary mathematical problems defined by the curriculum for the respective grade level. Findings from the qualitative interviews corroborated that inadequate mathematical conceptual ideas play a major role in the difficulties many students have with mathematical tasks. By implication, supporting students to develop adequate conceptual ideas could help prevent these problems. Finally, findings suggested that school type, classroom instruction, and classroom composition may also be relevant for an at-risk development in mathematics.

2.2 Emotions and student characteristics in mathematics

2.2.1 Development over school years

How do students' emotions, motivation, and learning-related behaviour in mathematics change from grade five to eight? Our findings suggest that there is considerable developmental dynamics not only for students' competencies, but for their affective and behavioral characteristics as well (Pekrun et al., 2006). Dynamics can be inferred for average scores, but also for individual scores, as can be seen from coefficients of interindividual stability (e.g. $.31 < r < .40$ for mathematics emotion scores from grades 5 to 8).

Mean scores did not change much for students' anxiety, shame, and hopelessness in mathematics. However, scores for enjoyment and pride decreased substantially, while scores for boredom increased (Pekrun et al., 2006). For the time from grade five to eight, the decline in mean scores for enjoyment amounted to more than 70% of the standard deviation within grade levels. The change in scores decreased across the three one-year time intervals between assessments, suggesting an asymptotic development reaching its lowest level at grade eight, as far as the present data set is concerned. Consistent with the development of emotion scores, mean scores for mathematics-related self-efficacy, interest, self-reported effort, and perceived self-regulation of learning decreased as well. One possible reason for this unfavourable development of students' emotions and engagement is the perceived decline in cognitive activation and autonomy support provided by teachers in mathematics classroom instruction (see below 2.3; Pekrun et al., 2003).

2.2.2 Gender and appraisals as determinants of emotions

Based on Pekrun's (2006) control-value theory of achievement emotions, we assumed that students' control and value appraisals in mathematics are important determinants of their mathematics emotions, and that the influence of individual variables such as students' gender is mediated by these appraisals. Testing these assumptions with data from the first assessment (grade 5), we used students' self-concept of ability in mathematics as an indicator of control-related appraisals, and the perceived values of the domain of mathematics as well as of achievement in this domain, as indicators of value appraisals (Frenzel, Pekrun, & Goetz, in press). As expected, the results of regression analyses showed that self-concept related significantly positively to students' enjoyment of mathematics, and negatively to their anxiety and shame. The domain value was a positive predictor of enjoyment, and the value of achievement positively predicted anxiety and shame, corroborating that value appraisals also are critical determinants of students' achievement-related emotions.

Female students reported significantly less enjoyment and more anxiety and shame than male students, even when controlling for individual achievement scores. In mediational analyses, we found that these differences were mediated by gender differences in control and value appraisals, in line with assumptions. As compared to boys, girls had similar achievement values in mathematics. However, they had lower self-concepts of ability and valued the domain of mathematics less, which helps to explain why they reported less enjoyment and more negative emotions. In contrast to mean level differences, however, the structural relationships between achievement, appraisals, and emotions were equivalent across genders, as shown by multi-group structural equation modelling. This finding corroborates assumptions on the universality of functional relationships of emotions across genders (Pekrun, 2006; for a related analysis comparing mathematics emotions across different cultures by using German and Chinese student samples, see Frenzel, Thrash, Pekrun, & Goetz, 2007). From an educational perspective, the findings imply that educators should make an attempt to change students' patterns of appraisals when aiming to foster their affective development in mathematics.

2.2.3 Task determinants of emotions: Modelling versus algorithmic tasks

Teachers often assume that students do not like problems involving mathematical modelling, such as word problems. Is this assumption justified? Using our scales for task-specific emotions, we analysed its validity (Frenzel et al., 2006; Zirngibl et al., 2005). The findings implied that emotions experienced during modelling versus algorithmic tasks showed substantial interindividual correlations ($r = .59$, .65, and .71 for enjoyment, anxiety, and boredom, respectively). Nevertheless, the mean scores of emotions differed between the two types of tasks. Students reported significantly more enjoyment and less anxiety for modelling tasks, as compared to algorithmic tasks (with task difficulty being equivalent across types of tasks). For low-ability students, these differences were even more pronounced than for average-ability students. These findings may be of considerable relevance for mathematics education, since modelling tasks are likely better suited to support students' development of mathematical literacy than algorithmic tasks involving no more than decontextualised, rule-defined operations. The present findings suggest that students' affective reactions are not a good reason for not using modelling tasks in mathematics instruction, but, on the contrary, rather support the use of these tasks.

2.2.4 Contextual determinants of emotions: Multi-level analysis of the effects of achievement on students' mathematics emotions

The control-value theory of achievement emotions (Pekrun, 2006) implies that feedback on achievement is among the most important determinants of students' achievement-related emotions. However, with the exception of studies on test anxiety (Pekrun, 1992; Zeidner, 1998), related empirical evidence is scarce. More specifically, there is a lack of studies which adequately take the multi-level structure of students' achievement experiences into account.

In a series of longitudinal multi-level analyses, we investigated the effects of individual achievement and the classroom level of achievement on students' emotions in mathematics (Goetz et al., 2004; Pekrun & Goetz, 2005). These analyses were based on the assumption that the *Big-Fish-Little-Pond effect* (BFLP effect; Marsh, 1987) of students' achievement on their self-concepts of ability can be found for students' emotions as well. Generally, it can be assumed that success and failure determine the development of self-concepts of ability which, in turn, influence the development of achievement-related emotions. More specifically, the BFLP effect implies that individual achievement has positive effects on students' self-concepts, whereas the classroom level of achievement has negative effects (when controlling for individual achievement). The reason for such classroom-level effects is that the chances of experiencing success are reduced in high-achieving classrooms, as compared to low-achieving classrooms. Taking these differential effects of individual versus classroom-level achievement on mediating self-concepts into account, we expected that achievement would have positive effects on positive emotions and negative effects on negative emotions, whereas the reverse was expected for the classroom level of achievement (negative effects on positive emotions, and positive effects on negative emotions).

In the multi-level analyses used to test these assumptions, students' mathematics enjoyment and anxiety in grade six were predicted by grade five emotion scores, individual competence scores, and aggregated classroom competence scores. In line with expectations, the findings showed that individual achievement had a positive effect on enjoyment and a negative effect on anxiety, whereas the classroom level of achievement had a negative effect on enjoyment and a positive effect on anxiety. Since autoregressive effects of emotions were included, the effects of achievement can be interpreted as effects on the change of emotions from grade five to six. In subsequent analyses, a similar pattern of effects was found for students' pride, shame, and hopelessness in mathematics (Pekrun & Goetz, 2005). Findings thus corroborate the assumption that mathematics achievement is to be regarded as an important determinant of students' emotions. More generally, the findings suggest that an analysis of students' affective development can profit from taking the multi-level structure of the classroom context into account.

2.2.5 Effects of emotions on competence development in mathematics

While success and failure are important determinants of students' emotional development, emotions in turn influence academic learning and performance (Pekrun, 2006), implying reciprocal causation in the relationship between emotions and achievement. In cross-lagged structural equation modelling, we tested assumptions on reciprocal effects for mathematics enjoyment, anxiety, and hopelessness (grades 5 to 7; Pekrun, Jullien, Zirngibl, vom Hofe, & Perry, 2004). For each of the two time intervals included (grades 5 to 6 and 6 to 7), there were significant time-lagged effects of achievement on emotions, and of emotions on achievement. Specifically, achievement had positive effects on subsequent enjoyment, and enjoyment had positive effects on subsequent achievement. Also, achievement had negative effects on anxiety and hopelessness, and these emotions in turn had negative effects on achievement. This pattern of findings corroborates that emotions and achievement reciprocally influence each other over time. As one important implication, reciprocal causality of student affect and achievement over the school years suggests that unidirectional production models of achievement fall short of adequately representing the complexity and multi-directionality of students' academic development.

2.2.6 Effects of emotions on educational career decisions

The assumptions of the control-value theory also imply that students' emotions affect their educational and occupational career decisions. We tested this hypothesis for the decisions which students at the Realschule had to make between different school programmes after grade six (decision between programmes focussing on mathematics/science, economics, and social sciences/arts). Logistic regression was used to analyse the effects of competence, enjoyment, and boredom in mathematics on the decision between programmes. Findings showed that students' mathematics emotions were important predictors. Deciding on the mathematics/science programme instead of the social sciences/arts programme was significantly predicted by students' enjoyment and competence in mathematics ($B_{Enjoyment} = 1.10$; $B_{Competence} = .39$; $p < .001$ for each coefficient), and deciding on economics instead of the social sciences/arts programme was significantly predicted by enjoyment ($B_{Enjoyment} = .40$; $p < .001$). These results corroborate the importance of domain-related emotions for students' future-oriented decisions.

2.2.7 Conclusions

Similar to the development of students' mathematical competencies, their emotions, motivation, and learning-related behaviour were also found to show considerable developmental dynamics. For grades five to eight, there was a substantial decline in average scores for positive emotions, and for a number of motivational variables. Furthermore, findings on mathematics emotions corroborated that control and value appraisals, gender, individual achievement, and classroom-level achievement strongly influence students' emotional development in mathematics, and that emotions in turn influence students' competence development and educational career decisions.

2.3 Classroom instruction in mathematics

Using scores for students' perceptions of classroom instruction in mathematics, we analysed changes in instruction from grades five to eight. Average scores for variables of perceived classroom management did not change much over school years. Similarly, scores for instruction focussing on algorithmic tasks did not show any substantial change. In contrast, the mean values for cognitively activating instruction using mathematical modelling declined substantially. As noted above, the resulting lack of modelling-oriented instruction likely is one reason for students' deficits in mathematical modelling competencies found in this study.

One primary aim of our longitudinal study is to analyse cumulative effects of instruction on students' development in mathematics. In preliminary analyses, we found substantial cross-sectional relationships between variables of instruction, on the one hand, and students' competence and affect in mathematics, on the other (Pekrun et al., 2003). At present, longitudinal analyses have not yet been completed. However, in order to lay the foundations for longitudinal analysis, we started to analyse the dimensional structure of teachers' and students' perceptions of mathematics instruction. In an analysis of data from 168 mathematics teachers of grades five to seven, we used exploratory factor analysis to analyse the dimensionality of teachers' perceptions, and subsequently tested the validity of the findings by confirmatory factor analysis. Using this procedure, the following five dimensions of teachers' perceptions of their instruction were identified (Pekrun et al., 2006; see Baumert et al.,
analysis of data from the German PISA 2003 assessment): (1) *cognitively activating instruction* (mathematical modelling, clarity, and autonomy support for learning); (2) *individualised instruction* (individual reference norms, positive individual reinforcement, and support after failure); (3) *teacher engagement* (teacher's engagement and enthusiasm); (4) *rule-oriented instruction* (algorithmic tasks and punishment after failure); and (5) *ineffective classroom management* (waste of time and disturbances to instruction). The findings of confirmatory factor analysis (LISREL 8.72, Jöreskog & Sörbom, 2002) confirmed this structure (χ^2 = 66.82, df = 44, p = .015; CFI = .96; NNFI = .93; RMSEA = .057).

The five dimensions are largely congruent to theoretical expectations and to the dimensions found by Baumert et al. (2004). The relationships between the dimensions indicated that teachers who described their instruction as cognitively activating also received higher scores for individualised instruction, and lower scores for ineffective classroom management. Also, teachers' perceived engagement related positively to cognitively activating instruction and negatively to ineffective classroom management, suggesting that engagement is an important determinant both for the cognitive quality of instruction and for effective classroom management.

2.4 The role of the family: Socio-economic status, parental involvement, and students' development in mathematics

Besides mathematics instruction and the classroom, the family also is an important context for students' school-based development of competencies. Employing the family variables of the present data set, we have begun to analyse social disparities of students' educational opportunities and competence development over school years, as well as relationships between parental involvement and students' development. As a theoretical framework, we used social-cognitive assumptions on social status effects and the role of significant others in students' development (Jullien, 2005; Pekrun, 2006).

Regarding socio-economic status, the PISA 2000 und 2003 assessments have shown that Germany is among the countries characterised by substantial social disparities in students' education. The PISA findings pertained to 15-years-old adolescents.

The present longitudinal study makes it possible to analyse social disparities in preceding grade levels, and the development of disparities over time, as well. In line with the PISA findings, first results suggest that socio-economic status and student competencies are negatively correlated. Furthermore, results imply that the likelihood of attending a Gymnasium is significantly higher for students from high-status families, even when controlling for students' basic cognitive abilities and mathematical competence scores. These disparities were found as early as grade five and seemed to increase further across the subsequent two grade levels (Jullien, 2005).

In further analyses, we plan to assess possible reasons for the increase in disparities over school years, one of which is differential family support (Hanafin & Lynch, 2002). Preliminary analyses suggest that socio-economic status and parental involvement in mathematics do in fact correlate. Specifically, students from low-status families reported that their parents exerted high achievement pressure, but did not support them sufficiently when dealing with scholastic demands in mathematics. Consistent with this pattern of parental behaviour, students from low-status families reported more mathematics anxiety (Jullien, 2005). However, while mean values for parental behaviour and student characteristics differed, multi-group structural equation modelling showed that the relationships between parental behaviour and student variables were consistent across social status groups, thus again confirming assumptions on the relative universality of antecedents of students' development.

In addition, we also started to analyse whether parental involvement and student development in mathematics reciprocally influence each other over time, as assumed by Pekrun's (2006) control-value theory. The findings implied that there were substantial synchronous relationships between parental involvement, on the one hand, and students' mathematics emotions, on the other, in grade five. Over the subsequent two years, there were incremental, cross-lagged effects of parental involvement on students' emotions, and of students' emotions on parental involvement (Jullien, 2005; Jullien & Pekrun, 2005). However, student effects on parental involvement were relatively weak from grade five to six. These effects increased for the time interval from grade six to seven. Overall, the pattern of findings suggests that parental effects on students' development of achievement emotions may be dominant in early adolescence, before being complemented by effects of students' emotions on their parents' involvement later on.

3 Products for educational practice

The instruments developed in this project, and its empirical findings, can be used for mathematics education. Currently, we are developing the following products which may help to transfer project findings into educational practice.

1) *Instructional material.* The main topic of German mathematics curricula in grades five to seven is the extension of the number domains from natural numbers (grade 5) to fractions (grade 6), and to negative and rational numbers (grade 7). The longitudinal assessments of the project served to identify students' basic mathematical conceptual ideas and problem-solving strategies relating to these domains. Relevant findings have been used for the German textbook "*Mathematik heute*" ("Mathematics Today") and for a collection of mathematical tasks for grades five to seven.

2) *Curriculum development.* The findings of the longitudinal study will be used to develop recommendations for the development of mathematics curricula for grades five to ten.
3) *Material for teacher education.* We are developing material to be used in teacher education. This material addresses issues of how to design mathematics instruction in such a way that students' competencies in mathematical modelling are supported (in the domains of fractions and rational numbers), and issues of how to shape instruction in "emotionally sound" (Astleitner, 2000) ways (Goetz & Kleine, 2006).
4) *Instruments for assessing competencies, emotions, and classroom instruction.* As noted above, we developed the *Regensburg Mathematical Achievement Test* that measures students' modelling and algorithmic competencies in mathematics, as well as the *Achievement Emotions Questionnaire – Mathematics* (AEQ-M; Pekrun et al., 2005) that assesses students' emotions in this domain. In addition, as noted, we developed scales that assess students' and teachers' perceptions of mathematics instruction and can be used for evaluating mathematics instruction in secondary schools.

4 Conclusions

This project analyses the development of mathematical competencies and of related affective and behavioural student characteristics in adolescence (grades 5 to 10). At present, data for the first four assessments (grades 5 to 8) are available. The findings reported in this chapter showed a substantial increase in mean competence scores over school years. However, over the years, scores for students' competencies in mathematical modelling were lower than scores for their algorithmic competencies. Also, a sizable number of classrooms, and of individual students, showed an unfavourable development of mathematical competencies. The results of qualitative interviews suggest that inadequate basic mathematical conceptual ideas (*Grundvorstellungen*) play a major role in students' difficulties with mathematical contents, which suggests that addressing these ideas could help to prevent or reduce maladaptive developments.

Consistent with the growing number of students considered at risk in mathematics, average scores for students' positive emotions, self-reported effort, and perceived self-regulation of learning in mathematics declined over school years. The decrease in the cognitive and motivational quality of mathematics classroom instruction that students reported across grades may be one important reason for this development. Specifically, cognitively activating instruction oriented towards mathematical modelling became less frequent over the years, as perceived by the students. In addition, the family context was also found to be important. Specifically, there were substantial social disparities of students' educational opportunities, and of their mathematical competencies, in grade five. These disparities further increased over the subsequent two school years.

Students' control and value appraisals, gender, achievement, and the composition of the classroom were found to be important for the development of students' emotions in mathematics, in line with the assumptions of Pekrun's (2006) control-value theory of achievement emotions. Regarding the effects of students' achievement on their emotions, we found that the Big-Fish-Little-Pond effect of achievement on self-concepts seems to be operating on students' emotions as well. Specifically, whereas individual achievement had positive effects on students' subsequent positive emotions and negative effects on their negative emotions, the opposite pattern of effects was found for the classroom level of achievement. Finally, when extending unidirectional perspectives on students' development, the results of longitudinal structural equation modelling suggest that the development of mathematical competencies, affective student characteristics, and context variables reciprocally influence each other over school years.

Based on the findings of the longitudinal study, we are currently developing a number of related products that can be used for educational practice. These products include instructional materials such as mathematics textbooks and collections of mathematical tasks, recommendations for the development of mathematics curricula, material for teacher education, and diagnostic instruments for the assessment of students' competencies and emotions in mathematics, and for the evaluation of classroom instruction in this domain.

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