$\Omega(RP) \rightarrow \Omega(PO)$. A successful transition requires all the players in location 1 to play P. Hence, it is the same as the transition from $\Omega(RP)$ to $\Omega(PP)$ in Case II.1. Therefore, the minimum cost for this transition is $\lceil (2N - \lfloor d_2 N \rfloor) q^* \rceil$.

$\Omega(PO) \rightarrow \Omega(RP)$. Any transition with less than $\lceil (2N - \lfloor d_2 N \rfloor)(1 - q^*) \rceil$ mutants cannot be successful, because the minimum population in location 1 is $\lceil (2N - \lfloor d_2 N \rfloor) \rceil$, and the population share of the R-players in location 1 required for a successful transition has to be larger than $1 - q^*$. To complete the transition by $\lceil (2N - \lfloor d_2 N \rfloor)(1 - q^*) \rceil$ mutants, the population in location 1 has to be the minimum. However, without additional mutants, the population in location 1 cannot decrease. Hence, $\lceil (2N - \lfloor d_2 N \rfloor)(1 - q^*) \rceil$ mutants cannot complete the transition. To have the minimum population in location 1, one mutant is needed. Let one mutant move to location 2 and play P. With a positive probability, the dynamics will move to a state in $\Omega(PP)$ where the population in location 1 is $2N - \lfloor d_2 N \rfloor$. Then, let $\lceil (2N - \lfloor d_2 N \rfloor)(1 - q^*) \rceil$ mutants play R in location 1; the dynamics will move to $\Omega(RP)$. Hence, the minimum cost for this transition is $1 + \lceil (2N - \lfloor d_2 N \rfloor)(1 - q^*) \rceil$.

$\Omega(PO) \rightarrow \Omega(RO)$. To have a successful transition, the population share of R players in location 1 has to be larger than $1 - q^*$. Let n_P be the number of P-players who move to location 2. Then, the transition requires $n_P + \lceil (2N - n_P)(1 - q^*) \rceil$ mutants, which has the minimum when $n_P = 0$. Hence, the minimum cost for this transition is $\lceil 2N(1 - q^*) \rceil$.

$\Omega(RO) \rightarrow \Omega(PO)$. Clearly, a direct transition requires at least $\lceil 2Nq^* \rceil$ mutants. Then, we show that the direct transition from $\Omega(RO)$ to $\Omega(PO)$ costs more than an indirect transition through $\Omega(RP)$. To have a successful direct transition, the population share of the P players in location 1 has to be larger than q^*. As argued above, the most efficient way is to directly have $\lceil 2Nq^* \rceil$ P-mutants in location 1. However, an indirect transition requires $1 + \lceil (2N - \lfloor d_2 N \rfloor) q^* \rceil$, which is smaller than the cost for the direct transition.

Case II.4: $d_k = 2$ and $\lfloor d_\ell N \rfloor = 2N - 1$. Consider first $k = 1$. There are two absorbing sets, $\Omega(RO)$ and $\Omega(PO)$. One mutant is enough to complete the transition from $\Omega(RO)$ to $\Omega(PO)$. Consider one player move to location 2 and play P. Next period, all the players in location 1 will move to location 2 and play P, while the single player in location 2 will move to location 1 and play R. After this period, there is only one player in location 1 and he cannot move to location 2 because of the constraints, hence he will randomize his strategy. With a positive probability, he will play P and all the players in location 2 will move to location 1 and play P, which moves the dynamics to $\Omega(PO)$.

To complete the transition in the reverse direction, the population share of the R-players in location 1 must be at least $1 - q^*$, and the players in location 2, if any, must play R. Let n_P the number of P-players who move from location 1 to 2 and play R. The transition requires $n_P + \lceil (2N - n_P)(1 - q^*) \rceil$ mutants, which has the minimum when $n_P = 0$. Hence, the minimum cost for this transition is $\lceil 2N(1 - q^*) \rceil$. The results are similar for $k = 2$. The analysis for this case gives the results in Table 3.6.

Case II.5: $d_k = 2$ *for all* $k = 1, 2$. There are four absorbing sets, $\Omega(RO)$, $\Omega(PO)$, $\Omega(OR)$ and $\Omega(OP)$.

$\Omega(RO) \rightarrow \Omega(OP)(\Omega(OR) \rightarrow \Omega(PO))$. One mutant is enough for this transition. Let one player move to location 2 and play P. In the next period, this player will move back to location 1 and play R, while all the players in location 1 will move to location 2 and play P. After this period, this R-player in location 1 will move to location 2 and play P. Similarly, the transition from $\Omega(OR)$ to $\Omega(PO)$ also requires only one mutant.

$\Omega(RO) \leftrightarrow \Omega(OR)(\Omega(PO) \leftrightarrow \Omega(OP))$. One mutant is enough to complete the transition. Consider $\Omega(RO) \rightarrow \Omega(OR)$ first. Let one player move to location 2 and play R. In the next period, with a positive probability, all the players in location 1 will move to location 2 and play R, and the single player in location 2 will move to location 1 and play R. After this period, with a positive probability, this player will move back to location 2 and play R. The same argument holds for the transition in the reverse direction and the transition between $\Omega(PO)$ and $\Omega(OP)$.

$\Omega(PO) \rightarrow \Omega(RO)(\Omega(OP) \rightarrow \Omega(OR))$. As explained in Case II.3, the minimum cost for this transition is $\lceil 2N(1 - q^*) \rceil$.

$\Omega(RO) \to \Omega(PO)$. To have a successful transition, the proportion of the P-players in location 1 has to be at least q^*. Hence, a direct transition requires $\lceil 2Nq^* \rceil$ mutants. Consider an indirect transition through $\Omega(OP)$. As shown above, the transitions $\Omega(RO) \to \Omega(OP)$ and $\Omega(OP) \to \Omega(PO)$ require one mutant respectively. Hence, the total cost is two. As long as N is larger than 2, $\lceil 2Nq^* \rceil > 2$. The same arguments hold for the transition from $\Omega(OR)$ to $\Omega(OP)$.

$\Omega(OP) \to \Omega(RO)$. A direct transition requires $2N$ mutants. Since as long as there is at least one P-player in location 2, all the R-players will be attracted to location 2 and play P. Consider an indirect transition through $\Omega(PO)$. Then the transition $\Omega(OP) \to \Omega(PO)$ needs one mutant, and the transition $\Omega(PO) \to \Omega(RO)$ entails $\lceil 2N(1-q^*) \rceil$ mutants. Clearly, the cost for the indirect transition is less than that of the direct transition.

An important finding is that, if $\lceil d_k N \rceil \leq 2N - 1$ for both $k = 1, 2$, the minimum-cost transitions share a common characteristic that mutants only change their strategies, not their locations. Hence, in Anwar's (2002) model where $\lceil d_k N \rceil \leq 2N - 1$ for both $k = 1, 2$, the minimum-cost transitions, under the assumption that mutants randomize their strategies in their current locations, are the same as those under the assumption that mutants randomly choose their strategies and locations.

Appendix III. Other Proofs

Proof of Proposition 3.2. The elements in $\Omega(PP)$ will be selected in the long run if and only if $C(\Omega(PP))$ has the minimum cost among all the absorbing sets. One can immediately obtain the condition using Table 3.2. ■

Proof of Proposition 3.3. The elements in $\Omega(RR)$ will be the LRE if and only if $C(\Omega(RR)) \leq C(\Omega(PR))$. Rearranging the inequality, one can obtain

$$d \leq 2q^* + \frac{\Delta_A(d)}{N},$$

where $\Delta_A(d) \in (-1, 2)$ is a function of d.

To select the elements in $\Omega(PP)$ as stochastically stable requires $\lfloor dN \rfloor = 2N-1$ or $C(\Omega(PR)) = C(\Omega(PP))$. The first equation can be rearranged as

$$d = 2 - \frac{\Delta_B(d)}{N}, \tag{3.14}$$

where $\Delta_B(d) \in (0,1]$. The second requirement can be presented as

$$d = 2 + \frac{\Delta_C(d)}{N}, \tag{3.15}$$

where $\Delta_C(d)$ has a negative lower bound and a positive upper bound.

Consider the function $f(d) = 2q^* + \frac{\Delta_A(d)}{N}$, (3.14) and (3.15). Let b_1 be the maximum of the absolute values among the lower bounds and upper bounds of $\Delta_A(d)$, $\Delta_B(d)$ and $\Delta_C(d)$. Then, the following conditions hold simultaneously.

$$2q^* - \frac{b_1}{N} \leq \ 2q^* + \tfrac{\Delta_A(d)}{N} \ \leq 2q^* + \frac{b_1}{N} \tag{3.16}$$

$$2 - \frac{b_1}{N} \leq \ 2 - \tfrac{\Delta_B(d)}{N} \ \leq 2 \tag{3.17}$$

$$2 - \frac{b_1}{N} \leq \ 2 + \tfrac{\Delta_C(d)}{N} \ \leq 2 + \frac{b_1}{N} \tag{3.18}$$

Hence, for any $\eta > 0$, there exist an integer $\bar{N} > b_1/\eta$, such that for all $N > \bar{N}$, $\frac{b_1}{N} < \eta$, and hence,

$$2q^* - \eta < \ 2q^* + \tfrac{\Delta_A(d)}{N} \ < 2q^* + \eta \tag{3.19}$$

$$2 - \eta < \ 2 - \tfrac{\Delta_B(d)}{N} \ < 2 \tag{3.20}$$

$$2 - \eta < \ 2 + \tfrac{\Delta_C(d)}{N} \ < 2 + \eta \tag{3.21}$$

Therefore, for any $\eta > 0$, there exist an integer $\bar{N}$, such that for all $N > \bar{N}$,

(1) the elements in $\Omega(RR)$ will be selected if $d \leq 2q^* - \eta$;

(2) the elements in $\Omega(PR)$ and $\Omega(RP)$ will be selected if $2q^* + \eta \leq d \leq 2 - \eta$.

■

Proof of Proposition 3.4. If $c_1 \leq c_2$, TP1 has the minimum cost for the transition from $\Omega(PR)$ to $\Omega(RR)$. Hence, the elements in $\Omega(PP)$ will be selected if and only if $C(\Omega(PP)) \leq C(\Omega(PP))$ and $C(\Omega(PP)) \leq C(\Omega(RR))$, which implies $\lceil (2N - \lfloor dN \rfloor)(1-q^*) \rceil = \lceil (2N - \lfloor dN \rfloor)q^* \rceil \leq c_1$. If $c_1 > c_2$, TP2 has the minimum cost for the transition. The same argument indicates $\lceil (2N - \lfloor dN \rfloor)(1-q^*) \rceil = \lceil (2N - \lfloor dN \rfloor)q^* \rceil \leq c_2$. Combining the two cases gives the result in the statement. ∎

Proof of Proposition 3.5. Let $C_{TP1}(\Omega(RR))$ be the minimum cost of $\Omega(RR)$ through TP1, and $C_{TP2}(\Omega(RR))$ that of $\Omega(RR)$ through TP2. For $h < r$, $C_{TP1}(\Omega(RR)) \leq C_{TP2}(\Omega(RR))$ if and only if $d \leq \frac{2(1-q^*)}{1-2q^*+\hat{q}} + \frac{\Delta_D(d)}{N}$, where $\Delta_D(d) \in (-\frac{1}{1-2q^*+\hat{q}}, \frac{3}{1-2q^*+\hat{q}})$. Denote $\bar{d} = \frac{2(1-q^*)}{1-2q^*+\hat{q}}$, and $f_{\bar{d}}(d) = \bar{d} + \frac{\Delta_D(d)}{N}$. The value of d which solves

$$d = \frac{2(1-q^*)}{1-2q^*+\hat{q}} + \frac{\Delta_D(d)}{N} \tag{3.22}$$

must be in the range of $f_{\bar{d}}(d)$.

$C_{TP2}(\Omega(RR)) \geq C(\Omega(PR))$ if and only if $d \geq \frac{2(2q^*-1)}{2q^*-\hat{q}} + \frac{\Delta_E(d)}{N}$, where $\Delta_E(d)$ is bounded. Denote $\tilde{d} = \frac{2(2q^*-1)}{2q^*-\hat{q}}$, and $f_{\tilde{d}}(d) = \tilde{d} + \frac{\Delta_E(d)}{N}$. Hence, the value of d which fulfills

$$d = \frac{2(2q^*-1)}{2q^*-\hat{q}} + \frac{\Delta_E(d)}{N} \tag{3.23}$$

must be in the range of $f_{\tilde{d}}(d)$. A straightforward computation shows that if $\bar{d} \geq (<) 2q^*$, then $\tilde{d} \leq (>) 2q^*$.

Selecting the elements in $\Omega(PP)$ in the long run entails $\lfloor dN \rfloor = 2N - 1$ or $C(\Omega(PP)) = C(\Omega(PR))$, which requires (3.14) or (3.15).

Let b_2 be the maximum of the absolute value of lower bounds and upper bounds of $\Delta_X(d)$ for all $X \in \{B, C, D, E\}$. The following conditions must hold.

$$\bar{d} - \frac{b_2}{N} \leq \quad f_{\bar{d}}(d) \quad \leq \bar{d} + \frac{b_2}{N} \tag{3.24}$$

$$\tilde{d} - \frac{b_2}{N} \leq \quad f_{\tilde{d}}(d) \quad \leq \tilde{d} + \frac{b_2}{N} \tag{3.25}$$

$$2 - \frac{b_2}{N} \leq \quad 2 - \frac{\Delta_B(d)}{N} \quad \leq 2 \tag{3.26}$$

$$2 - \frac{b_2}{N} \leq \quad 2 + \frac{\Delta_C(d)}{N} \quad \leq 2 + \frac{b_2}{N} \tag{3.27}$$

Therefore, given any $\eta > 0$, there exists an $\bar{N} > b_2/\eta$, such that for all

$N > \bar{N}$ the following conditions hold.

$$\begin{aligned} \bar{d} - \eta < & \quad f_{\bar{d}}(d) & < \bar{d} + \eta \quad (3.28) \\ \tilde{d} - \eta < & \quad f_{\tilde{d}}(d) & < \tilde{d} + \eta \quad (3.29) \\ 2 - \eta < & \quad 2 - \frac{\Delta_B(d)}{N} & < 2 \quad (3.30) \\ 2 - \eta < & \quad 2 + \frac{\Delta_C(d)}{N} & < 2 + \eta \quad (3.31) \end{aligned}$$

Consequently, given any $\eta > 0$, there exists an $\bar{N}$, such that for all $N > \bar{N}$,

(1). $C_{TP1}(\Omega(RR)) < C_{TP2}(\Omega(RR))$ if $d \leq \bar{d} - \eta$, $C_{TP1}(\Omega(RR)) > C_{TP2}(\Omega(RR))$ if $d \geq \bar{d} + \eta$;

(2). $C_{TP1}(\Omega(RR)) < C(\Omega(PR))$ if $d \leq 2q^* - \eta$, $C_{TP1}(\Omega(RR)) > C(\Omega(PR))$ if $d \geq 2q^* + \eta$.

(3). $C_{TP2}(\Omega(RR)) < C(\Omega(PR))$ if $d \leq \tilde{d} - \eta$, $C_{TP2}(\Omega(RR)) > C(\Omega(PR))$ if $d \geq \tilde{d} + \eta$.

(4). $C(\Omega(PP)) > C(\Omega(PR))$, if $d \leq 2 - \eta$.

Case 1. $\bar{d} \geq 2q^*$ if and only if $\hat{q} \leq 1/q^* - 2 + 2q^*$. Then, $\tilde{d} \leq 2q^* \leq \bar{d}$. Hence, if $d \leq 2q^* - \eta$, $C_{TP1}(\Omega(RR)) < C(\Omega(PR))$, hence, the elements in $\Omega(RR)$ are selected. If $d \geq 2q^* + \eta$, $C(\Omega(PR)) < C_{TP1}(\Omega(RR))$ and $C(\Omega(PR)) < C_{TP2}(\Omega(RR))$. Hence, the elements in $\Omega(PR)$ and $\Omega(RP)$ will be selected.

Case 2. $\bar{d} < 2q^*$ if and only if $\hat{q} > 1/q^* - 2 + 2q^*$. Then, $\bar{d} < 2q^* < \tilde{d}$. if $d \leq \tilde{d} - \eta$, $C_{TP2}(\Omega(RR)) < C(\Omega(PR))$. Hence, the elements in $\Omega(RR)$ will be selected. If $d \geq \tilde{d} + \eta$, $C_{TP1}(\Omega(RR)) > C(\Omega(PR))$ and $C_{TP2}(\Omega(RR)) > C(\Omega(PR))$. Hence, the elements in $\Omega(PR)$ and $\Omega(RP)$ will be selected. ■

Proof of Lemma 3.1. Appendix II exhibits the absorbing sets in the case with asymmetric capacity and mobility constraints. The case with symmetric constraints is simply a particular situation and is contained in the asymmetric case. All the results involving the symmetric constraints still hold, which gives the statement in this lemma. ■

Proof of Theorem 3.1. *Case III.1:* $\lfloor d_k N \rfloor < 2N-1$, *for all* $k = 1, 2$. In this case, there are two absorbing sets and two non-singleton absorbing sets, and, for each of them, there are four candidates for the minimum-cost transition tree. For each class of $\Omega(\phi)$-trees ($\phi \in \Phi = \{RR, PR, RP, PP\}$), one can compare the costs of the four candidates presented in Table 3.5, which gives the following result.

(1a) The $\Omega(\phi)_1$-tree has minimum cost if and only if $d_1 \geq \Psi(d_2) + \frac{\Delta^{\phi}_{12}(d_1,d_2)}{N}$, $d_1 \geq d_2 + \frac{\Delta^{\phi}_{14}(d_1,d_2)}{N}$, and $d_1 \geq 2q^* + \frac{\Delta^{\phi}_{d_1}(d_1,d_2)}{N}$.

(1b) The $\Omega(\phi)_2$-tree has minimum cost if and only if $d_1 \leq \Psi(d_2) + \frac{\Delta^{\phi}_{12}(d_1,d_2)}{N}$, $d_1 \geq d_2 + \frac{\Delta^{\phi}_{23}(d_1,d_2)}{N}$, and $d_2 \leq 2q^* + \frac{\Delta^{\phi}_{d_2}(d_1,d_2)}{N}$.

(1c) The $\Omega(\phi)_3$-tree has minimum cost if and only if $d_2 \leq \Psi(d_1) + \frac{\Delta^{\phi}_{34}(d_1,d_2)}{N}$, $d_1 \leq d_2 + \frac{\Delta^{\phi}_{23}(d_1,d_2)}{N}$, and $d_1 \leq 2q^* + \frac{\Delta^{\phi}_{d_1}(d_1,d_2)}{N}$.

(1d) The $\Omega(\phi)_4$-tree has minimum cost if and only if $d_2 \geq \Psi(d_1) + \frac{\Delta^{\phi}_{34}(d_1,d_2)}{N}$, $d_1 \leq d_2 + \frac{\Delta^{\phi}_{14}(d_1,d_2)}{N}$, and $d_2 \geq 2q^* + \frac{\Delta^{\phi}_{d_2}(d_1,d_2)}{N}$.

One can show that $\Delta^{\phi}_{\gamma}(d_1, d_2)$ are bounded for all $\gamma \in \Gamma = \{12, 23, 34, 14, d_1, d_2\}$ and $\phi \in \Phi$. Hence, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$, $|\Delta^{\phi}_{\gamma}(d_1, d_2)/N| < \eta$, for all $\gamma \in \Gamma$, $\phi \in \Phi$ and for all (d_1, d_2) such that $d_k \geq 1$ and $\lfloor d_k N \rfloor < 2N-1$ for both $k = 1, 2$.

Hence, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$,

(2a) The $\Omega(\phi)_1$-tree has minimum cost if $d_1 \geq \Psi(d_2) + \eta$ and $d_1 \geq d_2 + \eta$, which ensures $d_1 \geq 2q^* + \frac{\Delta^{\phi}_{d_1}(d_1,d_2)}{N}$.

(2b) The $\Omega(\phi)_2$-tree has minimum cost if $d_1 \leq \Psi(d_2) - \eta$ and $d_1 \geq d_2 + \eta$, which ensures $d_2 \leq 2q^* + \frac{\Delta^{\phi}_{d_2}(d_1,d_2)}{N}$.

(2c) The $\Omega(\phi)_3$-tree has minimum cost if $d_2 \leq \Psi(d_1) - \eta$, $d_1 \leq d_2 - \eta$ and , which ensures $d_1 \leq 2q^* + \frac{\Delta^{\phi}_{d_1}(d_1,d_2)}{N}$.

(2d) The $\Omega(\phi)_4$-tree has minimum cost if $d_2 \geq \Psi(d_2) + \eta$ $d_1 \leq d_2 - \eta$ and , which ensures $d_2 \geq 2q^* + \frac{\Delta^{\phi}_{d_2}(d_1,d_2)}{N}$.

Then, for each $\xi \in \{1,2,3,4\}$, we compare the minimum costs among the $\Omega(\phi)_\xi$-trees for all $\phi \in \Phi$. The element(s) in the absorbing sets which have the lowest cost among the minimum costs of $\Omega(\phi)_\xi$-trees will be selected as stochastically stable. A straightforward computation shows that $\Omega(RP)$ has the minimum cost if the condition in (2a) holds, $\Omega(RR)$ has the minimum cost if the conditions in (2b) and (2c) hold, and $\Omega(PR)$ has the minimum cost if the condition in (2d) hold. For the area between the sets in (2b) and (2c), that is $\{(d_1,d_2)|d_1 \geq d_2-\eta, d_1 \leq d_2+\eta, d_1 \leq 2-\frac{1-q^*}{q^*}d_2-\eta$, and $d_1 \geq \frac{2q^*}{1-q^*}-\frac{q^*}{1-q^*}d_2+\eta\}$, one can see that $\Omega(RR)$ has the minimum cost.

Case III.2: $\lfloor d_kN \rfloor = 2N-1$ *and* $\lfloor d_\ell N \rfloor < 2N-1$, *for* $k,\ell = 1,2, k \neq \ell$. Consider first the case where $k=1$. According to Appendix I and II, there are two absorbing sets: $\Omega(RP)$ and $\Omega(PR,PP)$. Further, $C(\Omega(RP)) = \lceil (2N-\lfloor d_2N \rfloor)(1-q^*) \rceil$, $C(\Omega(PR,PP)) = \lceil (2N-\lfloor d_2N \rfloor)q^* \rceil$. $C(\Omega(RP)) < C(\Omega(PR,PP))$ if

$$d_2 < 2 + \frac{\Delta_C(d_2)}{N}. \tag{3.32}$$

Since $\Delta_C(d_2)$ are bounded, for any $\eta > 0$, there exists an integer $\bar{N}$ such that for all $N > \bar{N}$, $|\frac{\Delta_C(d_2)}{N}| < \eta$, and hence,

$$2-\eta < 2 + \frac{\Delta_C(d_2)}{N}.$$

Therefore, if $\lfloor d_1N \rfloor = 2N-1$ and $\lfloor d_2N \rfloor < 2N-1$, for any $\eta > 0$, there exists an integer $\bar{N}$ such that for all $N > \bar{N}$, the element in $\Omega(RP)$ is selected as stochastically stable if $d_2 \leq 2-\eta$.

The same argument holds for $k=2$. Hence, for $\lfloor d_2N \rfloor = 2N-1$ and $\lfloor d_1N \rfloor < 2N-1$, for any $\eta > 0$, there exists an integer $\bar{N}$ such that for all $N > \bar{N}$, $\Omega(PR)$ is selected as stochastically stable if $d_1 \leq 2-\eta$.

If $\lfloor d_kN \rfloor = 2N-1$ for both $k=1,2$, there is a unique absorbing set $\Omega(PR,PP,RP)$. Rearranging $\lfloor d_kN \rfloor = 2N-1$, we have

$$d_k = 2 - \frac{\Delta_B(d_k)}{N}.$$

Because $\Delta_B(d_k)$ is bounded, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$, $|\frac{\Delta_B(d_k)}{N}| < \eta$, and hence $2-\eta < 2-\frac{\Delta_B(d_k)}{N}$, for both $k=1,2$. Therefore, the elements in $\Omega(PR,PP,RP)$ can be selected only if $d_k > 2-\eta$ for both $k=1,2$. Equivalently, the elements in $\Omega(PR,PP,RP)$

cannot be selected if $d_1 \leq 2-\eta$ or $d_2 \leq 2-\eta$.

Case III.3: $d_k = 2$ and $\lfloor d_\ell N \rfloor < 2N-1$, for $k,\ell = 1,2$ and $k \neq \ell$. Consider the case where $k=1$ first. There are three absorbing sets, $\Omega(PO)$, $\Omega(RO)$ and $\Omega(RP)$. According to Table 3.6, the minimum costs of the each absorbing set are

$$\begin{aligned} C(\Omega(PO)) &= 1+\lceil (2N-\lfloor d_2 N \rfloor)q^* \rceil \\ C(\Omega(RP)) &= 1+\lceil (2N-\lfloor d_2 N \rfloor)(1-q^*) \rceil \\ C(\Omega(RO)) &= \lceil 2N(1-q^*) \rceil + \lceil \lfloor d_2 N \rfloor (1-q^*) \rceil \end{aligned}$$

A straightforward comparison shows that $C(\Omega(RO)) > C(\Omega(RP))$ and $C(\Omega(RO)) > C(\Omega(PO))$ if $N > \frac{1}{2(1-q^*)}$. Hence, if N is large enough, $\Omega(RO)$ can never be selected.

$C(\Omega(RP)) < C(\Omega(PO))$ if $d_2 \leq 2 + \frac{\Delta_C(d_2)}{N}$. Again, since $\Delta_C(d_2)$ is bounded, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$, $|\frac{\Delta_C(d_2)}{N}| < \eta$, and hence $2-\eta < 2+\frac{\Delta_C(d_2)}{N}$. Therefore, if N is large enough, the element in $\Omega(RP)$ is the unique LRE in this case. The same argument holds for the case where $k=2$, hence, $\Omega(PR)$ is the unique LRE there.

Case III.4: $d_k = 2$ and $\lfloor d_\ell N \rfloor = 2N-1$. Consider first the case $k=1$. According to Lemma 3.1, there are only two absorbing sets, $\Omega(PO)$ and $\Omega(RO)$. The analysis in Appendix II shows that the transition $\Omega(PO) \rightarrow \Omega(RO)$ only requires one mutant. The transition in the reverse direction needs $\lceil 2N(1-q^*) \rceil$ mutants. Hence, if $N > \frac{1}{2(1-q^*)}$, the element in $\Omega(PO)$ is the unique LRE. The condition $\lfloor d_2 N \rfloor = 2N-1$ can be rearranged as $d_2 = 2 - \frac{\Delta_B(d_2)}{N}$. Since $\Delta_B(d_2)$ is bounded, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$, $2-\eta < 2-\frac{\Delta_B(d_2)}{N}$, and hence, the element in $\Omega(PO)$ can be selected only if $d_2 > 2-\eta$. The same argument holds for the case where $k=2$. Therefore, for any $\eta > 0$, the element in $\Omega(OP)$ can be selected only if $d_1 > 2-\eta$.

Case III.5: $d_1 = d_2 = 2$. There are four absorbing sets, $\Omega(RO)$, $\Omega(PO)$, $\Omega(OR)$ and $\Omega(OP)$. As explained in section 3.3.2, the minimum cost of $\Omega(PO)$- or $\Omega(OP)$-tree is 3.

We have shown in Appendix II (Case II.5) that, a minimum-cost $\Omega(RO)$ ($\Omega(OR)$)-tree must involve a direct transition from $\Omega(PO)$($\Omega(OP)$) to $\Omega(RO)$ ($\Omega(OR)$), which requires $\lceil 2N(1-q^*) \rceil$ mutants.

Hence, if $N > \frac{1}{2(1-q^*)}$, the minimum transition cost of $\Omega(RO)$- or $\Omega(OR)$-tree must be larger than 3. Therefore, $\Omega(PO)$ and $\Omega(OP)$ are LRE for N large enough.

Consider all the five cases above together. For any $\eta > 0$, there exists an integer $\bar{N}'$, which is the maximum of all the $\bar{N}$s in all the cases, such that when $N > \bar{N}'$, all the results above hold simultaneously. Renaming $\bar{N}'$ as $\bar{N}$, we have the statement in the theorem. ■

Proof of Proposition 3.6. We show in the proof of Theorem 3.1 that (1) the LRE are the elements in $\Omega(PP)$ if $d_1 = d_2 = 2$, and (2) the LRE form a subset of $\Omega(RP) \cup \Omega(PR) \cup \Omega(OP) \cup \Omega(PO) \cup \Omega(PP)$, if $(d_1, d_2) \in V_c(\eta) \setminus \{(2,2)\}$.

Then we are looking for the LRE in the remaining area of $V(\eta)$. Considering a large enough N, we use a similar approach to that in the proof of Theorem 3.1. After comparing the minimum costs of different transition trees presented in Table 3.5, one can see that for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N > \bar{N}$, the following results hold.

Area $V_a(\eta)$. In this area, $\Omega(RR)_1$- or $\Omega(RR)_2$-tree has the minimum transition cost among all $\Omega(RR)$-trees. Similarly, $\Omega(RP)_1$- or $\Omega(RP)_2$-tree has the minimum transition cost among all $\Omega(RP)$-trees. Let $Z_1 = \{\Omega(RR)_1, \Omega(RR)_2, \Omega(RP)_1, \Omega(RP)_2\}$. Then, one can obtain that, for $(d_1, d_2) \in V_a(\eta)$, $C(z) < C(\Omega(PR)_\xi)$ and $C(z) < C(\Omega(PP)_\xi)$ for all $z \in Z_1$ and for all $\xi \in \{1,2,3,4\}$. Hence, the LRE in $V_a(\eta)$ form a subset of $\Omega(RR) \cup \Omega(RP)$.

Area $V_b(\eta)$. In this area, $\Omega(RR)_3$- or $\Omega(RR)_4$-tree has the minimum transition cost among all $\Omega(RR)$-trees. Meanwhile, $\Omega(PR)_3$- or $\Omega(PR)_4$-tree has the minimum cost among all $\Omega(PR)$-trees. Denote by $Z_2 = \{\Omega(RR)_3, \Omega(RR)_4, \Omega(PR)_3, \Omega(PR)_4\}$. After a series of comparisons, one can obtain that, for $(d_1, d_2) \in V_b(\eta)$, $C(z) < C(\Omega(RP)_\xi)$ and $C(z) < C(\Omega(PP)_\xi)$ for all $z \in Z_2$ and for all $\xi \in \{1,2,3,4\}$. Hence, the LRE in $V_a(\eta)$ form a subset of $\Omega(RR) \cup \{\Omega(PR)\}$.

Area $V_c(\eta)$. In this area, $\Omega(RP)_1$- or $\Omega(RP)_4$-tree has the minimum transition cost among all $\Omega(RP)$-trees. Meanwhile, $\Omega(PR)_1$- or $\Omega(PR)_4$-tree has the minimum cost among all $\Omega(PR)$-trees. Denote by $Z_3 = \{\Omega(RP)_1, \Omega(RP)_4, \Omega(PR)_1, \Omega(PR)_4\}$. After a series of comparisons, one can obtain that, for $(d_1, d_2) \in V_c(\eta)$, $C(z) < C(\Omega(RR)_\xi)$ and $C(z) < C(\Omega(PP)_\xi)$ for all $z \in Z_3$

and for all $\xi \in \{1,2,3,4\}$. Hence, the LRE in $V_a(\eta)$ form a subset of $\Omega(RP) \cup \Omega(PR)$. ■

Proof of Theorem 3.2. When $h < r$, the costs for the transition from the co-existence of conventions to global coordination on the risk-dominant equilibrium may have less cost through TP2 than those in the case where $h \geq r$ through TP1. For the transition from $\Omega(PR)$ to either $\Omega(RR)$ or $\Omega(RO)$, TP2 cost less than TP1 if

$$\lceil \lfloor d_1 N \rfloor (1-\hat{q}) \rceil + \lceil (2N - \lfloor d_2 N \rfloor)(1-q^*) \rceil \leq \lceil \lfloor d_1 N \rfloor (1-q^*) \rceil \tag{3.33}$$

Similarly, for the transition from $\Omega(RP)$ to either $\Omega(RR)$ or $\Omega(OR)$, the cost through TP2 is lower than that of TP1 if

$$\lceil \lfloor d_2 N \rfloor (1-\hat{q}) \rceil + \lceil (2N - \lfloor d_1 N \rfloor)(1-q^*) \rceil \leq \lceil \lfloor d_2 N \rfloor (1-q^*) \rceil \tag{3.34}$$

Denote $\Lambda(d_k) = 2 - \frac{\hat{q}-q^*}{1-q^*} d_k$, for $k = 1,2$. Rearranging the conditions above, we obtain respectively

$$d_2 \geq \Lambda(d_1) + \frac{\Delta_2(d_1,d_2)}{N}, \tag{3.35}$$

$$d_1 \geq \Lambda(d_2) + \frac{\Delta_1(d_1,d_2)}{N}, \tag{3.36}$$

where both $\Delta_1(d_1,d_2)$ and $\Delta_2(d_1,d_2)$ are bounded. Hence, for any $\eta > 0$, there exists an integer $\bar{N}$, such that, for all $N > \bar{N}$, $|\frac{\Delta_1(d_1,d_2)}{N}| < \eta$ and $|\frac{\Delta_2(d_1,d_2)}{N}| < \eta$.

If both (3.35) and (3.36) hold, TP2 leads to the minimum cost for both $\Omega(RP) \to \Omega(RR)$ and $\Omega(PR) \to \Omega(RR)$, hence we have to replace the costs of both transitions in the case $h \geq r$ by those through TP2. If only (3.35) or (3.36) holds, TP2 has the minimum cost only for $\Omega(RP) \to \Omega(RR)$ or $\Omega(PR) \to \Omega(RR)$ respectively. Hence, we have to use the minimum cost of $\Omega(RP)$ or $\Omega(PR)$ generated by TP2 in each of the corresponding cases. If neither of them holds, the costs are the same as in the case $h \geq r$. Hence, comparing the minimum costs of all the transition trees in each of the areas mentioned above, we can find the LRE. There are two cases involving the transitions mentioned above, $\lfloor d_k N \rfloor < 2N - 1$ for both $k = 1,2$, and $d_k = 2$ and $\lceil d_\ell \rceil < 2N - 1$ for $k, \ell = 1,2, k \neq \ell$.

Case 1: $\hat{q} \leq 1/q^* - 2 + 2q^*$.

Case1.1: $\lfloor d_k N \rfloor < 2N - 1$ *for both* $k = 1, 2$. In this case, $\Lambda(d_k) \geq \Psi(d_k)$ for both $k = 1, 2$, $d_k \in [1, 2]$. If neither (3.36) nor (3.35) holds, the LRE is the same as stated in Theorem 3.1.

If (3.36) holds, the element in $\Omega(RP)$ will be selected if and only if $d_1 \geq d_2 + \frac{\Delta_a(d_1,d_2)}{N}$ and $d_1 \geq \Upsilon(d_2) + \frac{\Delta_b(d_1,d_2)}{N}$.

Similarly, if (3.35) holds, $\Omega(PR)$ will be selected if and only if $d_1 \leq d_2 + \frac{\Delta_a(d_1,d_2)}{N}$ and $d_2 \geq \Upsilon(d_1) + \frac{\Delta_c(d_1,d_2)}{N}$.

$\Delta_y(d_1, d_2)$ are bounded for all $y \in Y = \{a, b, c\}$. Hence, for any $\eta > 0$, there exists an integer $\bar{N}$, such that, for all $N > \bar{N}$, $|\Delta_y(d_1, d_2)| < \eta$, for all $y \in Y$. Note that, if $\hat{q} \leq 1/q^* - 2 + 2q^*$, $\Upsilon(d_k) \leq \Psi(d_k) \leq \Lambda(k)$ for $d_k \in [1, 2]$ and for both $k = 1, 2$. Building on the notations introduced in the symmetric case, let $C_{TP1}(\Omega(\cdot))$ be the minimum cost of $\Omega(\cdot)$-tree where $TP1$ leads to the minimum cost for the transition from *both* $\Omega(RP)$ and $\Omega(PR)$ to $\Omega(RR)$, and $C_{TP2}(\Omega(\cdot))$ be the minimum cost of $\Omega(\cdot)$-tree where $TP2$ leads to the minimum cost either from $\Omega(RP)$ to $\Omega(RR)$, or from $\Omega(PR)$ to $\Omega(RR)$, or both. After a series of comparisons of minimum transition costs of different absorbing sets in respective areas, one can obtain that, for any $\eta > 0$, for N large enough, the following results hold.

(1.1) $C_{TP1}(\Omega(RP))$ or $C_{TP2}(\Omega(RP))$ is the minimum among the costs for all the transition trees of absorbing sets if $d_1 \geq \Psi(d_2) + \eta$ and $d_1 \geq d_2 + \eta$;

(1.2) $C_{TP1}(\Omega(PR))$ or $C_{TP2}(\Omega(PR))$ is the minimum among the costs for all the transition trees of absorbing sets if $d_2 \geq \Psi(d_1) + \eta$ and $d_1 \leq d_2 + \eta$.

(1.3) $C_{TP1}(\Omega(RR))$ is the minimum if $d_1 \leq \Psi(d_2) - \eta$ and $d_2 \leq \Psi(d_1) - \eta$.

Case 1.2: $d_k = 2$ *and* $\lceil d_\ell N \rceil < 2N - 1$ *for* $k, \ell = 1, 2, k \neq \ell$. Consider the case $k = 1$. Using Table 3.6, if TP2 lead to the minimum cost, the cost of $\Omega(RO)$-tree will change to $\lceil 2N(1-q^*) \rceil + \lceil \lfloor d_2 N \rfloor (1-\hat{q}) \rceil + \lceil (2N - \lfloor d_1 N \rfloor)(1 - q^*) \rceil$. As long as N is large enough, this cost is still larger than the minimum cost of $\Omega(RP)$-tree. Hence, the element in $\Omega(RP)$ is still selected. The same argument holds for $k = 2$. If N is large enough, $\Omega(PR)$ will still be selected for $k = 2$.

In all the other situations, the results are the same as in the case $h \geq r$. Consider all the cases altogether, we obtain the same results as in Theorem 3.1 for $\hat{q} < 1/q^* - 2 + 2q^*$.

Case 2: $\hat{q} > 1/q^* - 2 + 2q^*$**.**

Case 2.1: $\lfloor d_k N \rfloor < 2N - 1$ *for both* $k = 1, 2$. The analysis in Case 1.1 still holds. If (3.36) holds, the element in $\Omega(RP)$ will be selected if and only if $d_1 \geq \Upsilon(d_2) + \frac{\Delta_d(d_1,d_2)}{N}$ and $d_1 \geq d_2 + \frac{\Delta_e(d_1,d_2)}{N}$. If (3.35) holds, $\Omega(PR)$ will be selected if and only if $d_2 \geq \Upsilon(d_1) + \frac{\Delta_f(d_1,d_2)}{N}$ and $d_1 \leq d_2 + \frac{\Delta_g(d_1,d_2)}{N}$.

However, the difference in this case is $\Upsilon(d_k) > \Psi(d_k) > \Lambda(d_k)$ for $d_k \in [1, 2]$ and for both $k = 1, 2$, which lead to the different predictions.

The approach is the same as in Case 1. One has to compare the minimum transition costs of different absorbing sets in different areas. Then, one can obtain that, for any $\eta > 0$, there exists an integer $\bar{N}$, such that for all $N \geq \bar{N}$, the following results hold.

(2.1) $C_{TP2}(\Omega(RP))$ is the minimum if $d_1 \geq \Upsilon(d_2) + \eta$ and $d_1 \geq d_2 + \eta$;

(2.2) $C_{TP2}(\Omega(PR))$ is the minimum if $d_2 \geq \Upsilon(d_1) + \eta$ and $d_1 \leq d_2 + \eta$;

(2.3) $C_{TP1}(\Omega(RR))$ or $C_{TP2}(\Omega(RR))$ is the minimum if $d_1 \leq \Upsilon(d_2) - \eta$ and $d_2 \leq \Upsilon(d_1) - \eta$.

Case 2.2: $d_k = 2$ *and* $\lceil d_\ell \rceil < 2N - 1$ *for* $k, \ell = 1, 2, k \neq \ell$. All the results in Case 1.2 hold. That is, for N large enough, if $k = 1$, the element in $\Omega(RP)$ is the unique LRE; if $k = 2$, $\Omega(PR)$ is the unique LRE.

In all the other cases, the result is the same as in the case $h \geq r$. Combine the results in all the cases together, we have the statement in the theorem. ■

Proof of Proposition 3.7. The proof is similar to that of Proposition 3.6. The LRE in $U_c(\eta) = V_c(\eta)$ and on $(2, 2)$ are presented in the proof of Theorem 3.1. The LRE in the remaining area of $U(\eta)$ are derived by comparing the minimum costs of different transition trees. The only difference is that one has to use the costs generated by TP2 when they are proven to be the minimum. After a series of comparisons of the minimum costs of different transition trees, one can obtain the result in Proposition 3.7. ■

Proof of Theorem 3.3. Consider first the case that $h \geq r$ or $\hat{q} \leq 1/q^* - 2 - 2q^*$. The LRE in this case are presented in Theorem 3.1. $(d_1, d_2) \in D_1(\eta)$ leads to the selection of the element in $\Omega(RP)$ in the long run. Planner 2 has no incentive to change d_1 or d_2 by changing (c_2, p_2), because the individuals in location 2 are coordinating on the Pareto-efficient equilibrium. Planner 1 has no incentive to only change d_1, because changing d_1 can only move the LRE from $\Omega(RP)$ to $\Omega(RR)$ or $\Omega(RP)$ and the elements in $\Omega(RR)$. In either case, the individuals in location 1 would coordinate on the risk-dominant equilibrium. However, planner 1 has an incentive to increase d_2, because if d_2 is large enough, the LRE would become $\Omega(PR)$. Note that planner 1 cannot directly change d_2. The only possible way to change d_2 is to change p_1, because $d_2 = \min\{c_2, 2 - p_1\}$. Any intention to decrease d_1 is always feasible, because planner 1 can increase p_1 and make $2 - p_1 = d_2$. However, the effort to increase d_2 is *not* always effective. $2-p_1$ will increase by decreasing p_1, but, as long as $2 - p_1 > c_2$, $d_2 = c_2$, and decreasing p_1 cannot increase d_2 any more. Hence, let $c_2 = d_2$ and $2 - p_1 \geq d_2$. Then planner 1 has no incentive to change d_2. Therefore, any strategy profile $((c_1, p_1), (c_2, p_2))$ projected on $D_1(\eta)$ such that $d_1 = \min\{c_1, 2-p_2\}$, $d_2 = c_2$ and $2 - p_1 \geq d_2$ is a Nash equilibrium.

$(d_1, d_2) \in D_2(\eta)$ leads to the selection of $\Omega(PR)$ in the long run. The argument is the same as above. Planner 1 has no incentive to change his strategy. By setting $d_1 = c_1 < 2 - p_2$, planner 2 has no incentive to deviate either. Hence, for $(d_1, d_2) \in D_2(\eta)$, any strategy profile $((c_1, p_1), (c_2, p_2))$ projected on $D_2(\eta)$ such that $d_2 = \min\{c_2, 2 - p_1\}$, $d_1 = c_1$ and $2 - p_2 \geq d_1$ is a Nash equilibrium.

If $(d_1, d_2) \in A_1(\eta)$, the LRE are the elements in $\Omega(RR)$. Each planner k would have an incentive to increase d_ℓ $(\ell \neq k)$ by decreasing p_k. However, this effort would be ineffective if $d_\ell = c_\ell < 2-p_k$. Hence, any strategy profile $((c_1, p_1), (c_2, p_2))$ projected on $A_1(\eta)$ such that $d_k = c_k$ and $2 - p_\ell \geq d_k$ for both $k = 1, 2$, $\ell \neq k$, is a Nash equilibrium.

Lastly, we consider the area $D_3(\eta)$. If $(d_1, d_2) \in (D_3(\eta) \cap A_2(\eta))$, the LRE is the element in $\Omega(RP)$. In this case, planner 1 will have an incentive to decrease c_1 to the extent that $d_1 < d_2 - \eta$. It changes the LRE to $\Omega(PR)$ and increases the social welfare of location 1.

If $(d_1, d_2) \in (D_3(\eta) \cap A_3(\eta))$, the LRE is the element in $\Omega(RP)$. In this case, planner 2 will have an incentive to decrease c_2 to the extent that $d_2 < d_1 - \eta$. It leads to the selection of $\Omega(RP)$ in the long run, and increases the social welfare of location 2.

If $(d_1, d_2) \in (D_3(\eta) \cap V_d(\eta))$, the LRE form a subset of $\Omega(RP) \cup \Omega(PR)$. We have shown that any strategy profile leading to the element in $\Omega(RP)$ or $\Omega(PR)$ is not a NE. Hence, any strategy profile leading to the selection of the elements in both $\Omega(RP)$ and $\Omega(PR)$ in the long run is not stable either, because each social planner k will have an incentive to decrease c_k for $k = 1, 2$.

If $d_1 = d_2 = 2$, the elements in both $\Omega(PO)$ and $\Omega(OP)$ will be selected in the long run. Each of them will occur with probability 1/2. Hence, each social planner k will have an incentive to decrease c_k so that $d_k < d_\ell - \eta$ $(k \neq \ell)$. Then, the players in location k will coordinate on the efficient equilibrium with probability one, and the social welfare of location k will increase.

If $(d_1, d_2) \in V_c(\eta) \setminus \{(2, 2)\}$, the LRE form a subset of $\Omega(OP) \cup \Omega(RP) \cup \Omega(PR) \cup \Omega(PO) \cup \Omega(PP)$. Note that there are no LRE which only consist of the elements in $\Omega(PP)$. If the elements in $\Omega(PP)$ are selected, the element in either $\Omega(RP)$ or $\Omega(PR)$ (or both) will be selected as well. Then, in any possible subset of the set above, with a positive probability, at least one location will either have players coordinating on the less efficient equilibrium or have no players at all. Then, the social planner in this location k can always improve the social welfare by decreasing c_k so that $d_k < d_\ell - \eta$ $(k \neq \ell)$. Therefore, any strategy profile projected on this area is not stable.

An analogous argument holds for the case with $h < r$ and $\hat{q} > 1/q^* - 2 + 2q^*$. We only have to replace $D_1(\eta)$, $D_2(\eta)$, $D_3(\eta)$, $A_2(\eta)$, and $A_3(\eta)$ by $G_1(\eta)$, $G_2(\eta)$, $G_3(\eta)$, $B_2(\eta)$, and $B_3(\eta)$ respectively in the analysis above. Hence we obtain the result in the theorem. ■

Proof of Theorem 3.4. Consider a strategy profile $((c_1, p_1), (c_2, p_2))$ such that $c_1 = c_2 = 1$ and $p_1 = p_2 = 1$. Planner $k = 1, 2$ has no incentive to deviate from his strategy. Changing c_k has no effect, because all the individuals in location $\ell \neq k$ are immobile, hence cannot move to location k. Changing p_1 has no effect either, because the maximum capacity of location ℓ is N, hence, the mobile players in location k cannot move to location ℓ. Hence, this strategy profile is a Nash equilibrium, which corresponds to $d_1 = d_2 = 1$.

For $h \geq r$ or $\hat{q} \leq 1/q^* - 2 + 2q^*$, consider any $(d_1, d_2) \in [1, 2]^2 \setminus (\{(1, 1)\} \cup V_e(\eta))$. We first claim that any strategy profile projected on $A_1(\eta) \setminus \{(1, 1)\}$ is not a NE. In this area, the LRE are the elements in $\Omega(RR)$. The social planner of location k will always have incentive to decrease d_ℓ $(\ell \neq k)$ by increasing p_k. The reason is that the population in location k fluctuates between $2N - \lfloor d_\ell N \rfloor$ and $\lfloor d_k N \rfloor$. Decreasing d_ℓ will increase the lower bound of the population in location k, hence improving the social welfare.

For $(d_1, d_2) \in A_2(\eta)$, the LRE is the element in $\Omega(RP)$, and the population in location 1 is $2N - \lfloor d_2 N \rfloor$. The social planner in location 1 will have an incentive to decrease d_1 by setting a lower c_1, so that the LRE form a subset of $\Omega(RR) \cup \Omega(PR)$. Denote by d'_k the parameter of effective capacity of location $k = 1, 2$ after a deviation. If such a deviation leads the elements in $\Omega(RR)$ to be selected, the population in location 1 will fluctuate between $2N - \lfloor d_2 N \rfloor$ and $\lfloor d'_1 N \rfloor$, hence the social welfare will increase. If the deviation leads $\Omega(PR)$ to be selected, the players in location 1 will coordinate on P, and the population will increase to $\lfloor d'_1 N \rfloor$, which improving the social welfare of location 1. Based on the results above, any deviation that results in the selection of the elements in $\Omega(RR)$ and $\Omega(PR)$ also increase the social welfare of location 1. Hence, any strategy profile projected on $A_2(\eta)$ is not a NE.

Symmetrically, any strategy profile projected on $A_3(\eta)$ is not a NE. The argument is analogous to the case above. Here, the social planner of location 2 will always have an incentive to decrease d_2. Such a deviation can at least increase the population in location 2, hence improving the social welfare.

If $(d_1, d_2) \in V_a(\eta)$, $\Psi(d_2) - \eta < d_1 < \Psi(d_2) + \eta$. The LRE form a subset of $\Omega(RR) \cup \Omega(RP)$. We have argued that if the LRE are the elements in $\Omega(RR)$ or $\Omega(RP)$, the social planner of location 1 will always have an incentive to deviate. If the LRE are the elements in $\Omega(RR)$ and $\Omega(RP)$, with a positive probability the population in location 1 will fluctuate between $2N - \lfloor d_2 N \rfloor$ and $\lfloor d_1 N \rfloor$, and with the remaining probability the population in location 1 is $2N - \lfloor d_2 N \rfloor$. In this case, the social planner of location 1 will always have an incentive to decrease c_1 at most to the extent that $d'_1 = \Psi(d_2) - \eta$. Then, the LRE are the elements in $\Omega(RR)$ and the population in location 1 will at worst fluctuate between $2N - \lfloor d_2 N \rfloor$ and $\lfloor (d_1 - \eta) N \rfloor$ with probability one. For η small enough, this will increase the social welfare of location 1.

Symmetrically, for $(d_1, d_2) \in V_b(\eta)$, the same argument holds for the social planners of location 2. Hence, he will have incentive to decrease d_2.

If $(d_1, d_2) \in V_d(\eta)$, the LRE form a subset of $\Omega(RP) \cup \Omega(PR)$. We have shown above that the strategy profiles which lead to the selection of the element in either $\Omega(RP)$ or $\Omega(PR)$ are not NE. If the LRE are $\Omega(RP)$ and $\Omega(PR)$, the expected population in each location k should fall in the interval $]2N - \lfloor d_\ell N \rfloor, \lfloor d_k N \rfloor[$ where $d_\ell - \eta < d_k < d_\ell + \eta$, and the individuals in location k will either coordinate on R or on P. The social planner of location k will have an incentive to decrease c_k, at most to the extent that $d'_k = d_\ell - \eta$. Then, the players in location k will coordinate on P with probability one, and the population will be at at least $\lfloor (d_k - \eta)N \rfloor$. For η small enough, this deviation will increase the social welfare of location k.

If $d_1 = d_2 = 2$, the elements in both $\Omega(PO)$ and $\Omega(OP)$ will be selected in the long run. Each of them will occur with probability 1/2. Hence, the average expected payoff of location k is lower than the payoff of the Pareto-efficient equilibrium, and the expected population will fall in the interval $]0, 2N[$. The social planner of location $k = 1, 2$ will have an incentive to decrease c_k at most to the extent that $d'_k = 2 - \eta$. Then, the players in location k will coordinate on P with probability one, and the population of location k will be at least $\lfloor (2 - \eta)N \rfloor$. For η small enough, this will increase the social welfare of location k.

If $(d_1, d_2) \in (V_c(\eta) \setminus \{(2, 2)\})$, $d_k \in (2 - \eta, 2)$ for both $k = 1, 2$, and the LRE form a subset of $\Omega(OP) \cup \Omega(RP) \cup \Omega(PR) \cup \Omega(PO) \cup \Omega(PP)$. The same argument applies to show that the strategy profiles leading to the selection of the element in an singleton absorbing set ($\Omega(RP), \Omega(PR), \Omega(OP)$ or $\Omega(PO)$) are not NE. We have pointed out in the proof of Theorem 3.3 that if the elements in $\Omega(PP)$ are selected, the element(s) in either $\Omega(RP)$ or $\Omega(PR)$ (or both) must be selected as well in the long run. Hence, in any possible subset of the set above, with a positive probability, at least one location k will either have players coordinating on R or have no players at all. In such a case, the average expected payoff for the players in location k will be less than that of the Pareto-efficient equilibrium, and the expected population will be in the interval $]0, 2N[$. Hence, the social planner of location k will have an incentive to decrease c_k at most to the extent that $d'_k = 2 - \eta$. Then, the players in location k will coordinate on P with probability one, and the population will be at least $\lfloor (2 - \eta)N \rfloor$. For η small enough, this deviation will increase the social welfare of location k.

The analysis is analogous for the case with $h < r$ and $\hat{q} > 1/q^* - 2 + 2q^*$. Hence we have the result in the statement. ■

References of Chapter 3

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Erklärung

Ich versichere hiermit, dass ich die vorliegende Arbeit mit dem Thema

Asymmetric Information and Learning in Games

ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Daten und Konzept sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt. Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.

Konstanz, den 1. September 2010 Fei Shi

Abgrenzung

Ich versichere hiermit, dass ich Kapitel 1 und 3 vorliegend Arbeit ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe.

Kapitel 2 entstammt einer geimeinsamen Arbeit mit Herrn Carlos Alós-Ferrer (Universität Konstanz). Die individuelle Leistung im Rahmen dieser Arbeit gliedert sich wie folgt: Fei Shi 50%, Carlos Alós-Ferrer 50%.

Konstanz, den 1. September 2010 Fei Shi