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# Optimal cost reimbursement of health insurers to reduce risk selection

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# Optimal cost reimbursement of health insurers to reduce risk selection

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**Abstract**

In the absence of a perfect risk adjustment scheme, reimbursing health insurers' costs can reduce risk selection in community-rated health insurance markets. In this paper, we develop a model in which insurers determine the cost efficiency of health care and have incentives for risk selection. We derive the optimal cost reimbursement function which balances the incentives for cost efficiency and risk selection. For health cost data from a Swiss health insurer, we find that an optimal cost reimbursement scheme should reimburse costs only up to a limit.

*JEL*-**classification:** H42, I18.

**Keywords:** health insurance, risk selection, risk adjustment.

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# 1 Introduction

Risk selection is a major concern in community-rated health insurance markets. Insurers have an incentive to discriminate against high risks and to attract low risks in such markets since they are not allowed to charge risk-based premiums. To avoid risk selection, regulators frequently impose open enrollment, define standardized benefit packages and implement risk adjustment schemes. However, these measures may not reduce incentives for risk selection sufficiently. In this case, reimbursing health insurers' costs can be useful. Here regulators face a *selection-efficiency trade-off* (Newhouse (1996)): Lower incentives for risk selection will be accompanied by less cost efficiency.

Although the possible benefits of cost sharing are generally recognized, there has been little theoretical work on the characteristics of an optimal cost reimbursement function. Usually cost reimbursement is regarded as a "mandatory reinsurance program with regulated reinsurance premiums" (van de Ven and Ellis (2000, p. 818)). This analogy suggests that optimal cost reimbursement is similar to an optimal insurance contract. Here Arrow (1974) and Raviv (1979) have shown that full or partial coverage above a deductible is optimal. For optimal cost reimbursement this implies that costs should only be reimbursed above a threshold. This *outlier risk sharing* (van de Ven and Ellis (2000)) is used in practice. In Germany, for example, 60% of individual health care costs which exceed €20,450 are reimbursed.

In this paper, we regard cost reimbursement as an *incentive problem* and not as an *insurance problem*. We assume that insurers influence the cost of health care by organizing the delivery in a more efficient way or by negotiating lower prices with providers. Incentives for risk selection arise because two risk types differ in their expected health care costs. The regulator cannot observe the risk types. We assume, however, that he knows how health care costs of each type are distributed. To determine the optimal cost reimbursement function, we minimize the difference in expected costs between the risk types for a given increase in total costs.

We find that the optimal cost reimbursement function balances two effects. First, costs should be reimbursed where the cumulative cost reimbursement for high risk types compared to low risk types is comparatively large. Second, cost reimbursement should be avoided where there is a large concentration of individuals and therefore a high efficiency cost. It is a priori unclear which form of cost reimbursement is optimal. The precise shape of the optimal cost reimbursement function depends on how the health care costs for each type are distributed. For health cost data from a Swiss health insurer, we find that costs should generally be reimbursed only up to a limit. This is the opposite of outlier risk sharing and shows that the theory of optimal insurance does not carry over to optimal cost reimbursement to reduce risk selection.

The paper is structured as follows. In Section 2, we discuss incentives schemes to reduce risk selection and place our analysis in the context of the literature. Section 3 presents the model. The optimal cost reimbursement formula is derived and discussed in Section 4. In Section 5, we calculate optimal cost reimbursement formulas based on data from a Swiss health insurer. Section 6 summarizes the results and concludes.

## 2 Incentive schemes to reduce risk selection

Most of the literature on incentives schemes to reduce risk selection in health insurance markets has so far focused on risk adjustment. Numerous empirical studies have examined the properties of possible risk adjusters (see van de Ven and Ellis (2000) for a survey). In practice, risk adjustment cells are defined on these adjusters. The transfer payment for an individual is then determined by the average cost of all insured in the respective cell. This approach has been criticized by Glazer and McGuire (2000, 2002) and Frank et al. (2000). They show that equalizing observable differences in average costs is not optimal if observable characteristics are only imperfect signals for an individual's health status. A risk adjustment scheme which takes this into account can be much more effective in reducing risk selection. A further proposal has recently been advanced by Barros (2003). He shows that an ex-post funds can in principle avoid risk selection without comprising on cost-efficiency.

However, it remains unclear whether risk adjustment schemes or ex-post funds can sufficiently reduce risk selection. The main problem is the availability of data.

Usually, only few characteristics such as age and gender can easily be obtained. Further indicators, in particular diagnostic information, are only available at a considerable cost. Even if risk adjustment schemes are considerably improved, risk selection may still be highly profitable (Newhouse (1994)). The same problem applies to ex-post funds. As Barros (2003, p. 437) points out it must be possible to assign spending of insurers to specific diseases. Without detailed diagnostic information, ex-post funds will therefore not be able to rule out risk selection. For this reason, there is need for second-best solutions which balance the selection-efficiency trade-off. Here cost reimbursing insurers' costs can be useful.[1]

Several forms of cost reimbursement have been proposed in the literature (see van de Ven and Ellis (2000)). On the one hand, there are cost reimbursement schemes which apply to all individuals and are similar to reinsurance contracts. These include outlier risk sharing and proportional risk sharing which reimburses a fixed percentage of all costs. On the other hand, forms of cost reimbursement have been put forward which are limited to a specific group.[2] Van de Ven and van Vliet (1992) propose *risk sharing for high risks* which allows an insurer to designate a specified percentage of his insured for which all health care costs will be reimbursed. *Risk sharing for high costs* is considered by van Barneveld et al. (2001). Under this scheme, all health care costs of a predetermined number of individuals with the highest costs are paid by the regulator.

In empirical studies, van Barneveld et al. (1998, 2001) compare these selective forms of cost reimbursement to outlier risk sharing and proportional risk sharing. They find that *risk sharing for high risks* as well as *risk sharing for high costs* is superior in reducing incentives for risk selection to outlier risk sharing and proportional risk sharing. These forms of risk sharing are more effective in reimbursing only the costs of high risk types without sharing the costs of low risk types.

In our analysis we start from the assumption that the cost reimbursement scheme applies to all individuals. As opposed to the existing literature, we do not base our cost reimbursement function on reinsurance principles. Instead we formulate a model and derive the optimal cost reimbursement function. Our result can be

[1] In a recent paper, Marchand et al. (2003) show that prior expenditure can also be a useful risk adjuster to reduce risk selection.

[2] A further possibility is to make cost reimbursement dependent on a medical condition; see van de Ven and Ellis (2000, p. 822).

compared directly to other formulas which apply to all individuals, in particular to outlier risk sharing. We cannot say whether our approach is superior to the selective forms of cost reimbursement. In future studies of different cost reimbursement approaches it would be interesting to compare risk sharing for high risks or high costs to our optimal cost reimbursement approach.

# 3 The model

We analyze a health insurance market in which the regulator wants to make medical services available to all individuals at a price independent of their risk type. He imposes community rating, i.e. requires insurers to quote a uniform premium for all their insured. Furthermore, insurers must offer a standardized health insurance package. Regulation also includes open enrollment, i.e. insurers must accept all individuals applying for insurance.

Health insurers organize the delivery of medical services. By choosing higher effort $e$, health insurers can organize the delivery in a more efficient way or negotiate lower prices with providers.[3] Choosing a higher effort will therefore decrease costs to treat an illness but also the utility of an insurer. For the disutility of effort $v(e)$ we assume $v'(e) > 0$ and $v''(e) \geq 0$. Costs to treat a patient depend on the effort level $e$ and on the severity $m$ of the patient's illness where $0 \leq m \leq M$. We assume

$$C(e,m) = c(e)m, \quad c(e) > 0, c'(e) < 0, c''(e) > 0, \tag{1}$$

i.e. costs are proportional to $m$ and organizational effort is subject to decreasing returns to effort. This cost function particularly fits a situation in which insurers negotiate a baseline reimbursement factor with providers.[4] The effort level chosen by an insurer when there is no cost reimbursement is labeled $\hat{e}$. We normalize $c(\hat{e}) = 1$ so that $C = m$ in the absence of cost reimbursement.

An individual can be a high risk $h$ or a low risk $l$. Expected costs of the high risk type are larger than expected costs of the low risk type. The proportion of

[3] See Marchand et al. (2003) for a similar approach.

[4] The method we present in the following can also be applied to other cost functions. In footnote 13, we point out the implications for the optimal cost reimbursement formula.

$l$-types is $\theta$. For each risk type $i = l, h$, severity $m$ is distributed according to the distribution function $F_i(m)$. Since a substantial fraction of insured usually does not use any health services during a certain period we allow for $F_i(0) > 0$. We assume the distribution function to be continuously differentiable for all $m \geq 0$ and label the respective density function $f_i(m)$.[5] For $m > 0$ we have

$$F_i(m) = F_i(0) + \int_0^m f_i(s)\,\mathrm{d}s.$$

Expected costs of each risk type correspond to

$$\mathrm{E}_i[C(e,m)] = \int_0^M c(e) m f_i(m)\,\mathrm{d}m \tag{2}$$

with $\mathrm{E}_h[C(e,m)] > \mathrm{E}_l[C(e,m)]$.

We assume that the regulator cannot identify the risk type and therefore is not able to implement a perfect risk adjustment scheme. Neither can he observe $e$ nor $m$. However, he knows the distribution functions $F_i(m)$ for each risk type. For example, he may infer the distribution functions from a representative sample with information about the risk type.[6] His objective is to find a balance between incentives for risk selection and efficiency by sharing costs with insurers. For an individual with cost $C(e,m)$ he reimburses $r(C(e,m))$. With respect to $r(C)$, we impose two restrictions:

1. $r'(C) \leq 1$ – no incentives for cost-inflation

   This restriction guarantees that the insurer cannot increase his profits by inflating costs.

2. $r'(C) \geq 0$ – no incentives for cost-deflation

   If $r(C)$ is non-decreasing in $C$, then hiding costs cannot lead to higher profits for the insurer.

[5] To be more precise, we assume the distribution function $F_i(m)$ to be continuously differentiable for all $m > 0$ and $\lim_{m \to 0^+} F_i'(m)$ to exist; accordingly by $f_i(0)$ we mean $\lim_{m \to 0^+} f_i(m)$.

[6] See section 5 for an illustration of this procedure.

We also assume that the cost reimbursement scheme has a balanced budget, i.e.

$$\begin{aligned} r(C(e,0))[\theta F_l(0) + (1-\theta)F_h(0)] \\ + \int_0^M r(C(e,m))[\theta f_l(m) + (1-\theta) f_h(m)]\mathrm{d}m = 0. \end{aligned} \tag{3}$$

This means that the cost reimbursement scheme is self-financing, which implies that $r(C(e,0))$ must be negative if costs are reimbursed.[7]

Health insurers know to which group a particular insured belongs. Because health insurance premiums are community-rated, health insurers try to risk select if the average costs of the two groups differ. They can do so by imposing barriers for high risk individuals and by trying to attract low risk individuals. For example, they may process applications of high risks only slowly. Low risks, on the other hand, may be captured by selective advertisement.[8] Taking into account cost reimbursement by the regulator, the difference in expected costs (*DEC*) between the two risk types

$$\begin{aligned} DEC &= \mathrm{E}_h[C(e,m) - r(C(e,m))] - \mathrm{E}_l[C(e,m) - r(C(e,m))] \\ &= -r(C(e,0))F_h(0) + \int_0^M [C(e,m) - r(C(e,m))]\, f_h(m)\,\mathrm{d}m \\ &\quad -\Big(-r(C(e,0))F_l(0) + \int_0^M [C(e,m) - r(C(e,m))]\, f_l(m)\,\mathrm{d}m\Big) \end{aligned} \tag{4}$$

captures the extra profit for an insurer when he insurers a low risk type instead of a high risk type. We assume that the incentives to risk select are higher for an insurer, the larger this difference.

[7] In practice, cost reimbursement schemes are frequently financed by a uniform flat rate and define a nonnegative cost reimbursement function. In such a framework, $-r(C(e,0))$ corresponds to the uniform flat rate and $r(C(e,m)) - r(C(e,0))$ equals the cost reimbursement.

[8] We therefore focus on what Glazer and McGuire (2002, p. 154) have termed the *access problem* and do not analyze the incentives of health insurers to distort the mix of the quality of health care.

Risk selection is a zero sum game between insurers in which every insurer spends resources to attract a favorable selection of individuals. We do not explicitly model this contest and focus on a symmetric equilibrium in which each insurer ends up with a representative share of the two risk groups.[9] We assume that risk selection leads to a loss of resources which is increasing in the difference in expected costs between the two risk types.

With the design of the cost reimbursement function $r(C(e,m))$, the regulator can influence the difference in expected costs and therefore the loss of resources due to risk selection. However, cost reimbursement will also reduce effort from the first-best level $\hat{e}$ to $\tilde{e}$ and therefore lead to higher average costs

$$\begin{aligned} AC(\tilde{e}) &> AC(\hat{e}) \qquad (5) \\ \Leftrightarrow \quad \theta \mathrm{E}_l\left[C(\tilde{e},m)\right] + (1-\theta)\mathrm{E}_h\left[C(\tilde{e},m)\right] &> \theta \mathrm{E}_l\left[C(\hat{e},m)\right] + (1-\theta)\mathrm{E}_h\left[C(\hat{e},m)\right] \end{aligned}$$

The problem of the regulator is to choose the cost reimbursement function $r(C)$ such that the difference in expected costs is reduced without increasing average costs too much.

The sequence of events is as follows:

1. The regulator announces the reimbursement function $r(C)$.
2. Insurers expend resources to risk select.
3. Individuals choose insurers, each insurer ends up with a representative share of the two risk groups.
4. Insurers select organizational effort $e$.
5. The severity $m$ and costs $C(m,e)$ are determined.
6. The regulator reimburses $r(C(m,e))$.

[9] A symmetric equilibrium is often assumed in the literature on risk selection (see Glazer and McGuire (2002) and Marchand et al. (2003)). Nevertheless, asymmetric equilibria cannot a priori be ruled out. In this case, our results may need to be modified. An exploration of asymmetric equilibria is, however, beyond the scope of our paper and must be addressed in future work.

Our approach to determine the optimal function $r(C)$ is as follows. We assume that the regulator is willing to tolerate an increase in average costs by a factor $\tilde{x} > 1$. This assumption defines a level of organizational effort $\tilde{e}$ at stage 4. We solve problem (P1)

$$\min_{r(C)} DEC \qquad \text{subject to} \qquad AC(\tilde{e}) = \tilde{x} AC(\hat{e})$$
$$\text{balanced budget condition (3)} \qquad \text{(P1)}$$
$$0 \leq r'(C) \leq 1$$

This yields the optimal cost reimbursement function $r(C)$ which minimizes the difference in expected costs for insurers and therefore incentives to risk select at stage 2.

The advantage of this approach is twofold. First, we avoid assumptions about the size of loss of resources due to risk selection. In addition, we do not need to specify the trade-off the regulator is willing to make between reducing risk selection and preserving incentives for cost efficiency in detail. To characterize the main properties of the optimal cost reimbursement function, we only need to assume that the regulator finds it optimal to implement a cost reimbursement scheme.

# 4 The optimal cost reimbursement formula

To solve problem (P1) we proceed in four steps:

1. We define the effort level $\tilde{e}$ which is associated with a given increase in average costs by a factor $\tilde{x} > 1$.
2. We determine the incentive constraint which guarantees that insurers choose effort level $\tilde{e}$.
3. We reformulate problem (P1) as the optimal control problem (P2) with costs $C$ as the integration variable.
4. We solve the optimal control problem and characterize the optimal cost reimbursement function $r(C)$.

*Step 1:* Our assumption is that the regulator is willing to tolerate an increase in average cost by a factor $\tilde{x}$ compared to a situation of no cost reimbursement. The corresponding effort level $\tilde{e}$ is defined by the condition

$$\begin{aligned} AC(\tilde{e}) &= \tilde{x}AC(\hat{e}) \qquad (6)\\ \Leftrightarrow\ \theta \mathrm{E}_l\left[c(\tilde{e})m\right] + (1-\theta)\mathrm{E}_h\left[c(\tilde{e})m\right] &= \tilde{x}\Big(\theta \mathrm{E}_l\left[c(\hat{e})m\right] + (1-\theta)\mathrm{E}_h\left[c(\hat{e})m\right]\Big) \end{aligned}$$

where $\hat{e}$ is the effort level in absence of cost reimbursement. Since we normalize $c(e)$ such that $c(\hat{e}) = 1$ in a situation without cost reimbursement, condition (6) simplifies to $c(\tilde{e}) = \tilde{x}$ which implies

$$\tilde{e} = c^{-1}(\tilde{x}). \qquad (7)$$

*Step 2:* When insurers select effort $e$ at stage 4 and face a cost reimbursement function $r(C)$, their optimization problem is to minimize total costs

$$\min_e \ \theta\,\mathrm{E}_l\left[c(e)m - r(c(e)m)\right] + (1-\theta)\,\mathrm{E}_h\left[c(e)m - r(c(e)m)\right] + v(e).$$

The first-order condition is

$$-\theta \mathrm{E}_l \left[c'(e)m - r'(c(e)m)c'(e)m\right] \\ - (1-\theta)\mathrm{E}_h \left[c'(e)m - r'(c(e)m)c'(e)m\right] - v'(e) = 0$$

which corresponds to

$$-\int_0^M \left[c'(e)\,m - r'(c(e)m)c'(e)m\right]\left[\theta f_l(m) + (1-\theta) f_h(m)\right] \mathrm{d}m - v'(e) = 0. \quad (8)$$

Rearranging terms yields

$$\int_0^M r'(c(e)m)m\,g(m)\,\mathrm{d}m = k(e) \quad \text{with} \quad k(e) \equiv \int_0^M m\,g(m)\,\mathrm{d}m + \frac{v'(e)}{c'(e)} \quad (9)$$

where $g(m) \equiv \theta f_l(m) + (1-\theta) f_h(m)$ is the average density function. A sufficient condition for the corresponding effort level to yield a global profit-maximum is that the cost reimbursement function $r(C)$ is concave.[10] From condition (9), it follows that the cost reimbursement function must satisfy

$$\int_0^M r'(c(\tilde{e})m)m\,g(m)\,\mathrm{d}m = k(\tilde{e}) \quad (10)$$

if insurers are to choose effort level $\tilde{e}$. Condition (10) therefore defines the incentive constraint which guarantees that average costs increase by the factor $\tilde{x}$.

*Step 3:* To derive the optimal cost reimbursement function $r(C)$ it is convenient to express our problem with $C$ as the integration variable. We therefore transform the distribution functions $F_i(m)$ and the density functions $f_i(m)$ into functions of $C$. From $C = c(\tilde{e})m$ it follows that $m = C/c(\tilde{e})$. The distribution functions in terms of $C$ are therefore given by $\tilde{F}_i(C) = F_i(C/c(\tilde{e}))$. Differentiating with respect to $C$ yields the corresponding density functions $\tilde{f}_i(C) = f_i(C/c(\tilde{e}))/c(\tilde{e})$. The support of $C$ is given by $[0, c(\tilde{e})M] = [0, \bar{C}]$.

[10] If $r(C)$ is a concave function, then the left hand side of (9) is non-decreasing in $e$. Since the function $k(e)$ is a strictly decreasing function of $e$, the first-order condition must therefore characterize a global optimum. If $r(C)$ is not concave, then it needs to be checked whether equation (9) guarantees an optimum.

Using this transformation in the definition of the difference in expected costs (equation (4)), we obtain

$$\begin{aligned} DEC &= -r(0)\tilde{F}_h(0) + \int_0^{\bar{C}} [C - r(C)]\,\tilde{f}_h(C)\,\mathrm{d}C \\ &\quad - \Big( -r(0)\tilde{F}_l(0) + \int_0^{\bar{C}} [C - r(C)]\,\tilde{f}_l(C)\,\mathrm{d}C \Big). \end{aligned} \tag{11}$$

Noting that $\int_0^{\overline{C}} C[\tilde{f}_h(C) - \tilde{f}_l(C)]\mathrm{d}C$ is a constant, the regulator's problem (P1) therefore is equivalent to problem (P2)[11]

$$\min_{r(C)} \quad r(0)[\tilde{F}_l(0) - \tilde{F}_h(0)] + \int_0^{\overline{C}} r(C)[\tilde{f}_l(C) - \tilde{f}_h(C)]\mathrm{d}C$$

subject to

$$\int_0^{\overline{C}} r'(C)C\tilde{g}(C)\,\mathrm{d}C = c(\tilde{e})k(\tilde{e}) \tag{P2}$$

$$r(0)\tilde{G}(0) + \int_0^{\overline{C}} r(C)\tilde{g}(C)\,\mathrm{d}C = 0$$

$$0 \le r'(C) \le 1$$

$$r(0), r(\overline{C}) \text{ free}$$

where $\tilde{g}(C) = \theta\tilde{f}_l(C) + (1-\theta)\tilde{f}_h(C)$ and $\tilde{G}(C) = \theta\tilde{F}_l(C) + (1-\theta)\tilde{F}_h(C)$. The first constraint is the transformed incentive constraint (10) which ensures that average costs increase only by the factor $\tilde{x}$. The second constraint corresponds to the zero-budget constraint (3). The third constraint ensures that there are neither incentives for cost-inflation nor cost-deflation. Finally, the last constraint expresses that there are no restrictions with respect to the endpoints of $r(C)$.

[11] We assume that the difference in expected costs remains positive. If in the optimum this difference were negative, then cost reimbursement would clearly be too high and a lower value of $\tilde{x}$ should be chosen.

*Step 4:* Problem (P2) is an isoperimetric dynamic optimization problem due to the equality integral constraints.[12] It is not possible to set up the Hamiltonian and to apply the maximum principle since we allow for $F_i(0) > 0$. In the Appendix, we therefore formulate the Lagrangian function for the full problem. There, we derive the following result

**Proposition 1:** *The slope of the optimal cost reimbursement function is characterized by*

$$(1-\tilde{F}_h(C)) - (1-\tilde{F}_l(C)) - |\bar{\eta}|C\tilde{g}(C) \begin{cases} > 0 \Rightarrow & r'(C) = 1 \\ = 0 \Rightarrow & 0 \le r'(C) \le 1 \\ < 0 \Rightarrow & r'(C) = 0 \end{cases} \quad (12)$$

*where* $\bar{\eta} < 0$ *is the constant Lagrange-multiplier associated with the incentive constraint* (10). *Condition* (12) *and the zero-budget constraint*

$$r(0)\tilde{G}(0) + \int_0^{\overline{C}} r(C)\tilde{g}(C)\,dC = 0$$

*determine* $r(0)$.

To interpret this result, first note that the optimal cost reimbursement formula $r(C)$ has a slope of either zero or one unless by chance we have $(1-\tilde{F}_h(C)) - (1-\tilde{F}_l(C)) = \bar{\eta}C\tilde{g}(C)$. Furthermore, condition (12) can be decomposed into two terms with a natural interpretation:

[12] See Chiang (1992, p. 280) and Kamien and Schwartz (1991, p. 228).

1. *The anti-selection term* $(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C))$

   A large difference $(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C))$ tends to favor cost reimbursement. To explain this effect, it is important to note that $r'(C)=1$ increases cost reimbursement for all individuals with costs *above* $C$. This follows from the restriction $r'(.)\geq 0$. Since $1-\tilde{F}_i(C)$ denotes the share of $i$-types with costs higher than $C$, $(1-\tilde{F}_h(C))>(1-\tilde{F}_l(C))$ implies that there are relatively more $h$-types with costs above $C$ than $l$-types. Increasing cost reimbursement at $C$ therefore reimburses costs more for $h$-types than for $l$-types. This implies that the difference in expected costs must fall.

2. *The cost efficiency term* $|\bar{\eta}|C\tilde{g}(C)$

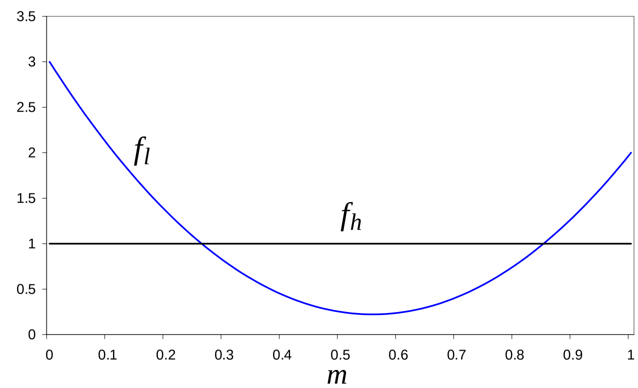
   A large value of $|\bar{\eta}|C\tilde{g}(C)$ calls for no cost reimbursement. This is because $C\tilde{g}(C)$ corresponds to the share of total costs at $C$.[13] If this share is large, then cost reimbursement at $C$ tends to have a large negative impact on the incentives for efficiency and therefore calls for no cost reimbursement. This effect is increasing in $|\bar{\eta}|$, the Lagrange-multiplier which captures the importance of incentives for cost efficiency. A lower $\tilde{x}$, i.e. higher incentives for cost efficiency increases $|\bar{\eta}|$. For a given value of the anti-selection term, a larger value of $|\bar{\eta}|$ therefore implies less cost sharing.

The optimal cost reimbursement function therefore considers for every cost level $C$ whether the reduction in incentives for risk-selection outweighs the efficiency costs of cost reimbursement. The restriction that costs are only allowed to increase by a certain percentage is reflected in the Lagrange-multiplier $\bar{\eta}$ which defines the importance of cost-efficiency.

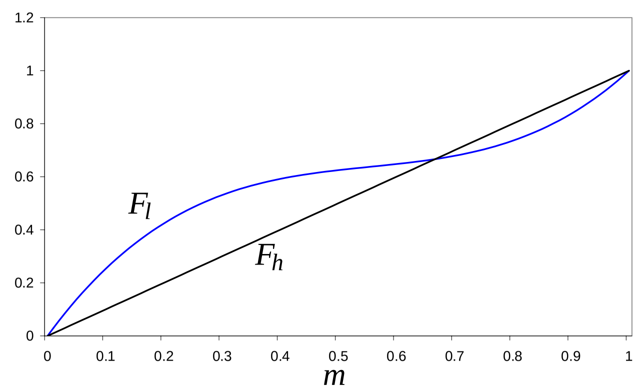
**An example**

A simple example is useful to illustrate our result. Suppose that there are two groups of equal size ($\theta=0.5$) and that $0\leq m\leq 1$. For the distribution functions we assume $f_l(m)=9m^2-10m+3$ and $f_h(m)=1$ with $F_l(0)=0$ and $F_h(0)=0$ (see Figure 1). Note that $l$-types are more likely to have low as well as high illness severities. This is not implausible. Think of $l$-types as healthy individuals who are

[13] For other cost functions than $C(e,m)=c(e)m$ this part of the cost efficiency term is different. For example, if $C(e,m)=c(e)+m$, then the cost efficiency term is $|\bar{\eta}|\tilde{g}(C)$.




(a) Density functions




(b) Distribution functions

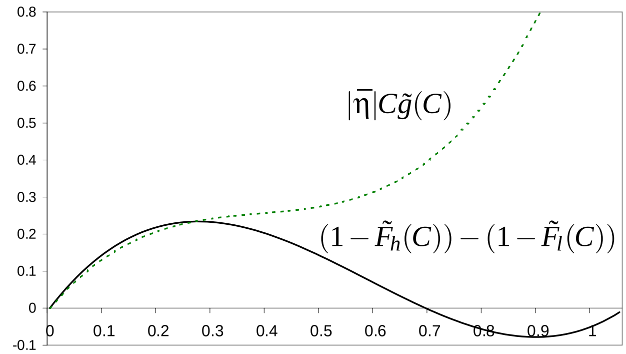
Figure 1: An example

able to practice activities which may lead to severe injuries such as motorcycling or paragliding and $h$-types as chronically ill people who rather stay at home.

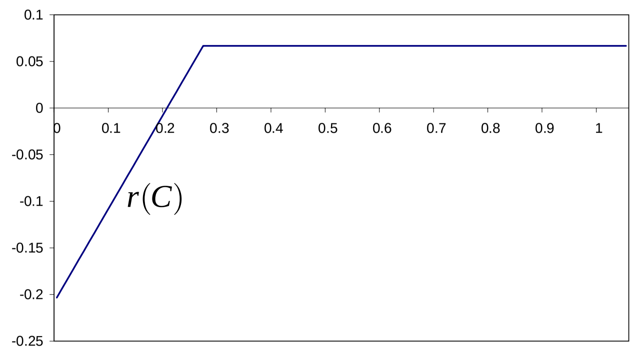
The cost and the disutility function take the form $v(e) = e$ and $c(e) = 11/24e$. Without cost reimbursement, we therefore have optimal effort $\hat{e} = 11/24$ and $c(\hat{e}) = 1$. The corresponding difference in expected costs is

$$\begin{aligned} \mathrm{E}_h[C(\hat{e})] - \mathrm{E}_l[C(\hat{e})] &= \int_0^1 c(\hat{e}) m f_h(m) dm - \int_0^1 c(\hat{e}) m f_l(m) dm \\ &= \frac{1}{2} - \frac{5}{12} = \frac{1}{12} = 0.0833. \end{aligned}$$

We assume that the regulator wants to induce a level of effort $\tilde{e}$ such that average costs increase by 5%. Thus, $\tilde{e}$ is defined by $c(\tilde{e}) = 1.05$ by condition (7) which




(a) $(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C))$ and $|\bar{\eta}|C\tilde{g}(C)$




(b) The optimal cost reimbursement function

Figure 2: Optimal cost reimbursement

implies $\tilde{e}=0.4365$ and yields

$$\begin{aligned}(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C)) &= 1.905C-4.535C^2+2.592C^3\\ \tilde{g}(C) &= 1.905-4.535C+3.887C^2.\end{aligned}$$

Solving the optimization problem, we find for the Lagrange-multiplier associated with the incentive constraint $\bar{\eta}=-0.9023$. Using condition (12), i.e.

$$(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C))-|\bar{\eta}|C\tilde{g}(C)\begin{cases} >0 & \Rightarrow \quad r'(C)=1\\ =0 & \Rightarrow \quad 0\le r'(C)\le 1\\ <0 & \Rightarrow \quad r'(C)=0\end{cases}$$

we can infer the optimal cost reimbursement function: Figure 2(a) shows the anti-selection term $(1-\tilde{F}_h(C))-(1-\tilde{F}_l(C))$ and the cost efficiency term $|\bar{\eta}|C\tilde{g}(C)$

depending on $C$. The functions intersect at $C = 0.2696$. Where the anti-selection term is larger than the cost efficiency term, marginal cost reimbursement equals one, where it is below there is no marginal cost reimbursement. Thus, we obtain

$$r'(C) = \begin{cases} 1 & \text{for} \quad C \leq 0.2696 \\ 0 & \text{for} \quad C > 0.2696 \end{cases} .$$

The balanced-budget condition requires $r(0) = -0.2033$. This yields the optimal cost reimbursement function

$$r(C) = \begin{cases} -0.2033 + C & \text{for } C \leq 0.2696 \\ 0.0663 & \text{for } C > 0.2696 \end{cases}$$

which is shown in Figure 2(b). It is worth noting that for high cost levels, $(1 - \tilde{F}_h(C)) - (1 - \tilde{F}_l(C))$ is negative since $l$-types are more likely to have high illness severities. Thus, in this range there are relatively more $l$-types with costs above $C$ than $h$-types. If costs where reimbursed at these costs levels, the difference in expected costs would therefore be increased.

Expected costs with optimal cost reimbursement are 0.4933 and 0.4488. The resulting difference in expected costs is 0.0445. Compared to the status quo of 0.0833, the difference in expected costs by is therefore reduced by 46.61%.

It is interesting to compare our result with the difference in expected costs under outlier risk sharing. We assume that 60% of all costs above a certain threshold are reimbursed. For a 5% increase in average cost, this threshold equals 0.9953. Since cost reimbursement has to be financed by insurers we also have a uniform rate for each insured. In this example this turns out to be $-0.001223$. Therefore the cost reimbursement function with outlier risk sharing is

$$r(C) = \begin{cases} -0.001223 & \text{for} \quad C < 0.9953 \\ -0.001223 + 0.6(C - 0.9953) & \text{for} \quad C \geq 0.9953 \end{cases} .$$

Expected costs are 0.5254 and 0.4371 and the difference in expected costs is 0.0883. Compared to status quo of 0.0833, the difference in expected costs therefore *increases* by 5.88%. Thus, outlier risk sharing is counter-productive in our example because it leads to efficiency losses and increases the incentives for risk selection. Two factors are responsible for this effect:

(i) In the absence of cost reimbursement, a 5% increase in total costs also increases the difference in expected costs by 5%.

(ii) Outlier risk sharing reimburses costs at high cost levels where $(1-\tilde{F}_h(C)) - (1-\tilde{F}_l(C))$ is negative. In this range, there are relatively more $l$-types, so reimbursing costs in this range mainly reimburses costs of $l$-types. This explains why outlier risk sharing increases the difference in expected costs by further 0.88%.

Our example shows that outlier risk sharing does not need to be optimal. Instead the optimal formula may be characterized by cost reimbursement up to a limit. In addition, the example demonstrates that outlier risk sharing may actually be counterproductive. In the next section, we show that these results can also arise if we apply our formula to actual health cost data.

# 5 An empirical illustration

In this section we show how our method can be applied in practice. We base our empirical analysis on administrative data provided by a Swiss health insurer. The data set includes information on individual costs, hospitalization, number of months insured, death and extent of coinsurance for the years 1997 to 1999 with 475,506 observations. We used the observations of 104,420 adult individuals being insured in the years 1998 and 1999. Their average health care expenditure was 3,250 Swiss Franks (CHF) in 1999.

We created a variable indicating to which of the 30 age-gender-cells of the Swiss risk adjustment scheme each insured belongs. Information on each group is given in Table 1 (see page 24). Since we do not have information about insurer's risk-selection activities and costs, we need to postulate hypotheses about how insurers risk select and how they can influence costs. Our results should therefore be regarded as an illustration of how our method can be applied in practice if this information is available rather than an actual policy recommendation.

Our risk selection hypothesis is that health insurers can observe whether an individual was treated in a hospital in 1998. The group $h$ is therefore given by those treated in a hospital in 1998. The $l$-types are the remaining individuals. We assume that the regulator is not able to obtain information on hospitalization.[14] To illustrate possible shapes of the optimal cost reimbursement function, we apply our method to each age-gender-cell of the Swiss risk adjustment scheme.

We proceed in two steps:

1. As in our example we assume $v(e) = e$ and $c(e) = \beta/e$ where the constant $\beta$ is chosen such that $c(\hat{e}) = 1$. The costs from our data set correspond to the costs if insurers choose this effort level since there is no cost reimbursement in Switzerland. Thus, we obtain $C^{\text{act.}} = c(\hat{e})m = m$ and use actual costs $C^{\text{act.}}$ to estimate the distribution functions $F_i(m)$ for each group.[15]

2. Second, we derive the distribution functions for an increase in costs by a factor of $\tilde{x}$ and apply our method as in the example.

We estimate the distribution function of actual costs $C^{\text{act.}}$ in 1999 for the two groups nonparametrically by a kernel density estimation.[16]

[14] In practice, regulators should be able to obtain this information. However, so far it is not used in the Swiss risk adjustment scheme. We make this assumption mainly because hospital stays are included in our data set and we can therefore use this information to illustrate our method. Nevertheless, our results may be interesting for a regulator who does not want to use information on hospital stays in a risk adjustment scheme to avoid that insurers encourage excessive hospitalization.

[15] For a real world application, our data set would need to be adjusted if it is not representative.

[16] Since there was a considerable share of observations with zero costs, we set $F_i(0)$ equal to this share and determined only $F_i(C^{\text{act.}})$ with $C^{\text{act.}} > 0$ with the kernel density estimator. When we chose a constant bandwidth for the kernel, we found that $f_i(C^{\text{act.}}) = 0$ for a number of intervals for $C^{\text{act.}} > 10,000$ CHF. This artificially improved our results because $r'(C) = 1$ does not reduce incentives for efficiency at all whenever $f_i(C) = 0$ for both groups. Therefore we transformed the data using a concave function. With the function $\ln(C^{\text{act.}})$ we did not get any intervals with $f_i(C^{\text{act.}}) = 0$. From the estimated distribution function $\hat{F}_i(\ln(C^{\text{act.}}))$ we derive the distribution function $F_i(C^{\text{act.}})$ and the density function $f_i(C^{\text{act.}})$. We also performed kernel estimation with variable bandwidths as proposed by Silverman (1986). Because we considered the differences in the bandwidth too large (the largest bandwidth was about 10,000 times as large as the smallest one) and for reasons given by Terrell and Scott (1992) we did not use this procedure.

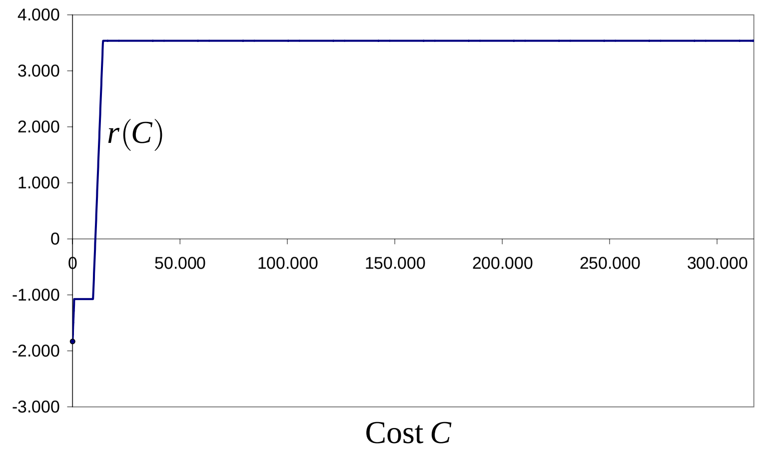
Applying our method, we find that the optimal cost reimbursement function takes one of three types displayed in Figure 3:[17]

1. The first type has marginal cost reimbursement equal to one starting at $C = 0$ up to a threshold which is between 8,000 and 18,000 CHF, with an interval of zero marginal cost sharing of about 2,000 to 5,000 CHF. There is no additional cost reimbursement above this threshold. An example is shown in Figure 3(a). We obtain $r(0) = 1,810$ CHF, i.e. each insurer must contribute 1,810 CHF per person insured to finance the cost reimbursement scheme.
2. The second type has no marginal cost reimbursement for cost below a threshold of about 20,000 and above a threshhold of about 40,000 CHF. Between 20,000 and 40,000 CHF there is marginal cost reimbursement equal to one. Above this threshold, there is no more cost reimbursement. Figure 3(b) shows a graph of this type. In this example, $r(0) = 246$ CHF.
3. The third type looks like the second but the threshold above which there is no additional cost reimbursement is much higher, mostly between 100,000 and 150,000 CHF, and with sometimes one or two intervals with no cost reimbursement. An example is shown in Figure 3(c) where $r(0) = 331$ CHF.

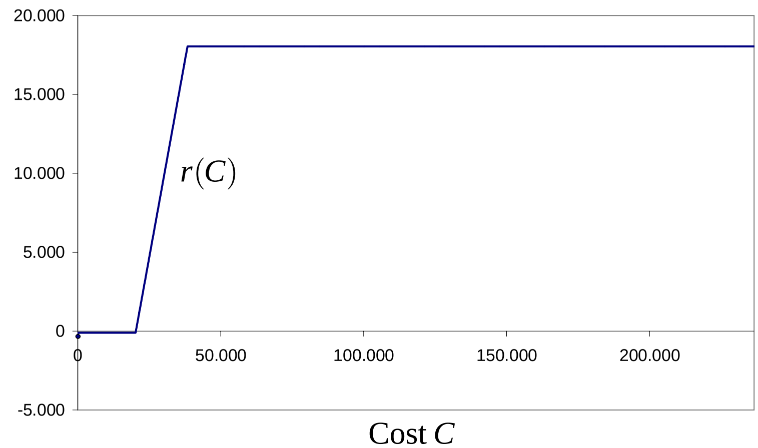
For the 30 age-gender cells, 12 of the optimal cost reimbursement functions were of type one, 10 of type two and 8 of type three. Table 1 and Figures 4 to 6 give an overview of our results and compare them to an outlier risk sharing scheme which reimburses 60% of costs above a threshold. In Table 1, we show which type arises for each age-gender cell if we allow average cost to increase by 5 %. In addition, we show the percentage reduction in the difference of average costs for both optimal cost reimbursement and outlier risk sharing.

Figure 4 illustrates the effectiveness of the optimal cost reimbursement formula for a 1 to 10% increase in total cost. On average, the difference in expected costs can be decreased by 13.18 % if we allow costs to increase by 5% and by 23.12% for a 10% increase in costs. The maximum decrease is 30.73% and 67.87% respectively.

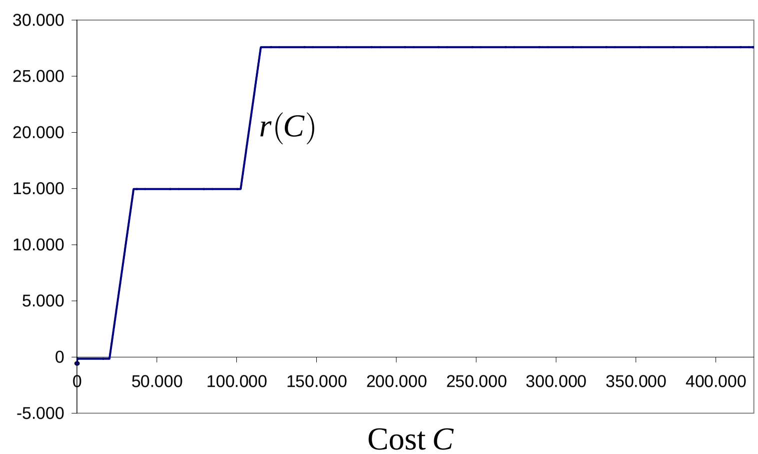
[17] The health insurers' second-order conditions were satisfied for all cost reimbursement function.




(a) Type 1, cell M13




(b) Type 2, cell F3




(c) Type 3, cell F7

Figure 3: Optimal $r(C)$-functions

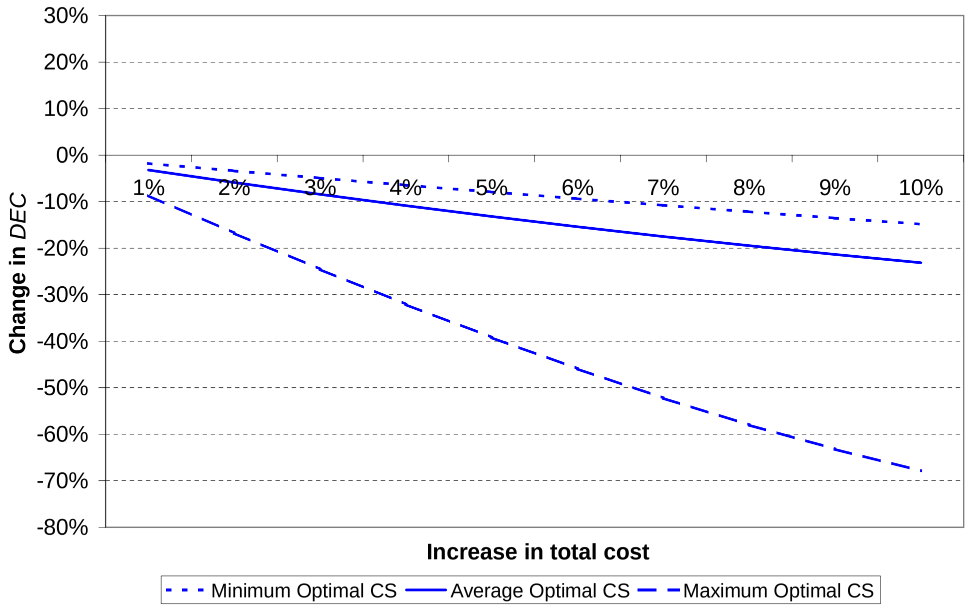



Figure 4: Reduction in the difference in expected costs with optimal cost reimbursement

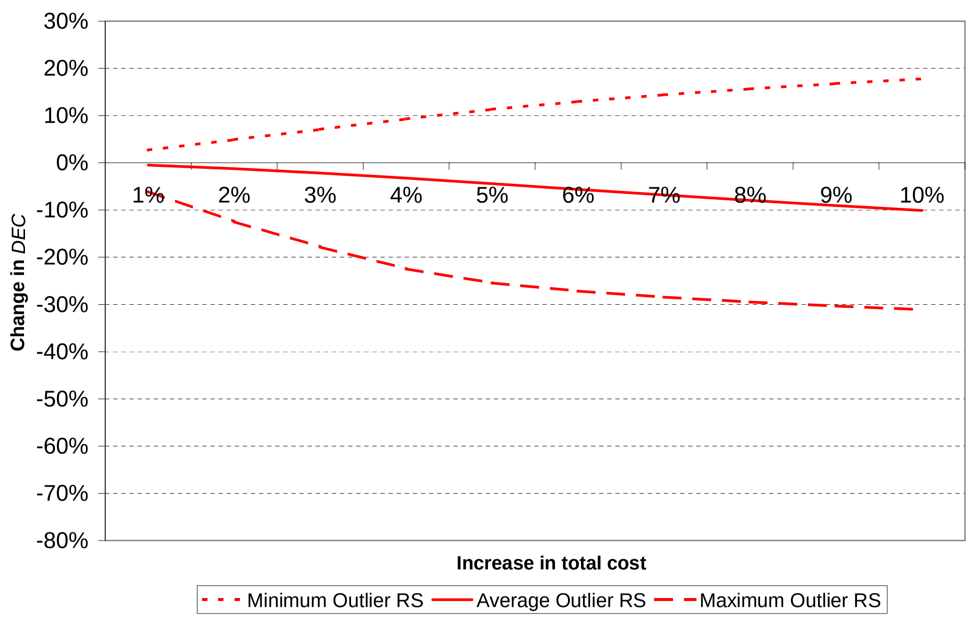



Figure 5: Reduction in the difference in expected costs with outlier risk sharing

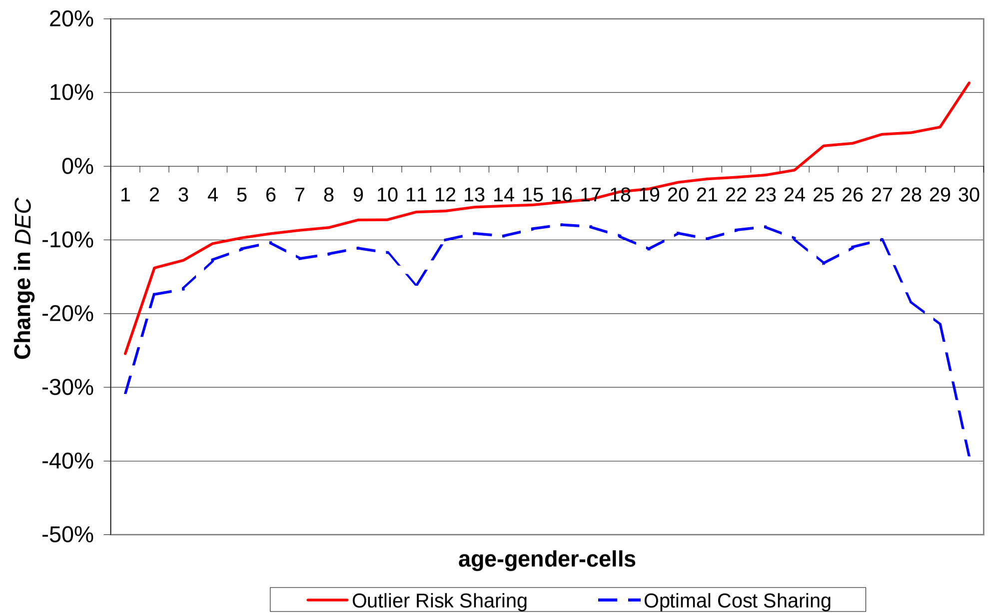



Figure 6: Optimal cost reimbursement and outlier risk sharing compared

Figure 5 shows the same graph for outlier risk sharing. We find that this type of cost reimbursement is much less effective than our method. For example, if costs are allowed to increase by 5%, then the average reduction in the difference in expected costs is only 4.43% compared to 13.18% under optimal cost reimbursement.

Finally, we show in Figure 6 the results for all 30 age-gender-cells for a 5%-increase in total costs for optimal cost reimbursement and outlier risk sharing. The cells were arranged by the magnitude of the reduction in the difference in expected costs of outlier risk sharing. We find that the difference in costs increases under outlier risk sharing in 6 of 30 age-gender-cells, in some cases by even more than total costs. This shows that the scenario in our example is also possible in practice.

Figure 6 demonstrates that optimal cost reimbursement performs well when outlier risk sharing performs well. This comes at no surprise because optimal cost reimbursement can always mimic outlier risk sharing. However, it performs also well when outlier risk sharing performs very badly. The difference in expected costs can be 45% lower if optimal cost reimbursement is used instead of outlier risk sharing. In these cases, we find that cost reimbursement of type one is optimal where the threshold above which there is no additional cost reimbursement is low. The optimal cost reimbursement formula is therefore just the opposite of outlier risk sharing.

# 6 Conclusion

In this paper, we derived the optimal cost reimbursement function based on a model in which insurers influence the cost of health care with their organizational activities. We found that the optimal cost reimbursement function balances selection and efficiency effects which are determined by the distribution of health care costs for each risk type.

When we applied our method to Swiss health cost data, we observed that costs should generally be reimbursed only up to a limit. This is opposed to outlier risk sharing which is advocated in the literature and used in Germany. Our optimal cost reimbursement formula was also much more effective than outlier risk sharing. For a five percent increase in costs, we found that the mean decrease in the average cost difference between the two groups is almost three times larger if optimal cost reimbursement is used instead of outlier risk sharing. This shows that applying principles of reinsurance may not be appropriate if the objective is to reduce risk selection.

To derive our optimal cost reimbursement function, we made a number of specific assumptions. For example, our analysis was based on two risk types. It would be interesting to extend the analysis to multiple risk types. Also other functional forms of the cost function could be considered. Furthermore, we focused on organizational effort of insurers which affects the cost of all patients. Future work could examine the implications of patient-specific effort.

On the empirical side, our method could be applied to further data sets and different risk-selection variables. Furthermore, our approach can be compared to selective forms of cost reimbursement such as risk sharing for high risks or high costs to our optimal cost reimbursement approach. In the study by van Barneveld et al. (2001), these proposals were superior to outlier risk sharing. It is interesting to see whether this result also applies to our formula.

| Cell | Age-Gender-Bracket | Number of observations | Percentage Hospitalized in 1998 | Average Cost Hospitalized | Average Cost Non-Hosp. | Average Cost Ratio | *DEC*-Change in % Optimal Cost-sharing | *DEC*-Change in % Outlier Risk-sharing | Type |
|---|---|---|---|---|---|---|---|---|---|
| F1 | Women 18-25 | 5902 | 6.2 | 4659 | 1432 | 3.25 | -30.73 | -25.43 | 3 |
| F2 | Women 26-30 | 4289 | 11.6 | 3891 | 2033 | 1.91 | -16.01 | -6.21 | 2 |
| F3 | Women 31-35 | 5372 | 14.9 | 3803 | 2180 | 1.74 | -11.93 | -8.33 | 2 |
| F4 | Women 36-40 | 5678 | 10.9 | 4255 | 2067 | 2.06 | -10.05 | -6.08 | 2 |
| F5 | Women 41-45 | 5262 | 8.7 | 6201 | 2130 | 2.91 | -12.75 | -10.50 | 2 |
| F6 | Women 46-50 | 5103 | 8.5 | 6170 | 2284 | 2.70 | -16.66 | -12.77 | 3 |
| F7 | Women 51-55 | 5049 | 8.7 | 7075 | 2642 | 2.68 | -12.56 | -8.69 | 3 |
| F8 | Women 56-60 | 4592 | 9.6 | 7249 | 2794 | 2.59 | -11.23 | -9.72 | 2 |
| F9 | Women 61-65 | 3907 | 11.5 | 7654 | 3270 | 2.34 | -8.49 | -5.25 | 2 |
| F10 | Women 66-70 | 3536 | 14.8 | 10297 | 3861 | 2.67 | -11.10 | -7.28 | 3 |
| F11 | Women 71-75 | 3224 | 18.1 | 11516 | 4634 | 2.49 | -8.21 | -4.50 | 1 |
| F12 | Women 76-80 | 2871 | 21.9 | 13543 | 5796 | 2.34 | -9.80 | -0.53 | 1 |
| F13 | Women 81-85 | 2022 | 27.5 | 16539 | 6745 | 2.45 | -13.23 | +2.77 | 1 |
| F14 | Women 86-90 | 1564 | 35.9 | 19506 | 6206 | 2.12 | -18.31 | +4.56 | 1 |
| F15 | Women 91+ | 803 | 48.9 | 22255 | 12876 | 1.73 | -39.21 | +11.32 | 1 |
| M1 | Men 18-25 | 5738 | 3.6 | 6119 | 885 | 6.91 | -17.42 | -13.80 | 3 |
| M2 | Men 26-30 | 3730 | 4.9 | 6391 | 1105 | 4.88 | -9.06 | -2.17 | 2 |
| M3 | Men 31-35 | 4609 | 4.3 | 11551 | 1213 | 9.52 | -11.73 | -7.26 | 3 |
| M4 | Men 36-40 | 4697 | 5.8 | 7630 | 1433 | 5.32 | -7.94 | -4.87 | 2 |
| M5 | Men 41-45 | 4363 | 5.6 | 6863 | 1553 | 4.42 | -9.10 | -5.55 | 2 |
| M6 | Men 46-50 | 4109 | 6.7 | 6635 | 1693 | 3.92 | -9.50 | -3.46 | 1 |
| M7 | Men 51-55 | 4240 | 7.8 | 8544 | 2003 | 4.27 | -10.33 | -9.15 | 3 |
| M8 | Men 56-60 | 3769 | 9.4 | 7495 | 2669 | 2.81 | -9.87 | -1.72 | 1 |
| M9 | Men 61-65 | 2920 | 10.8 | 10289 | 3313 | 3.11 | -11.28 | -3.06 | 2 |
| M10 | Men 66-70 | 2351 | 15.1 | 9407 | 4257 | 2.21 | -9.96 | +4.33 | 1 |
| M11 | Men 71-75 | 1852 | 19.1 | 12710 | 5612 | 2.26 | -9.53 | -5.39 | 3 |
| M12 | Men 76-80 | 1389 | 23.0 | 11615 | 6039 | 1.92 | -8.22 | -1.19 | 1 |
| M13 | Men 81-85 | 805 | 26.8 | 15037 | 7772 | 1.93 | -11.00 | +3.11 | 1 |
| M14 | Men 86-90 | 489 | 30.9 | 19596 | 9298 | 2.11 | -8.66 | -1.49 | 1 |
| M15 | Men 91+ | 185 | 37.3 | 22216 | 12853 | 1.73 | -21.51 | +5.32 | 1 |
| | Average | 3481 | 15.6 | 10190 | 4188 | 3.11 | -13.18 | -4.43 | – |

Table 1: Age-gender cells and results for a 5 % increase in costs

# Appendix

Since we allow for $F_i(0) > 0$ it is not possible to set up the Hamiltonian and apply the maximum principle to solve our isoperimetric dynamic optimization problem with free starting and end points. In the following, we therefore solve the complete problem. To save on notation, we define $\tilde{H}(m) = \tilde{F}_h(m) - \tilde{F}_l(m)$ and $\tilde{h}(m) = \tilde{f}_h(m) - \tilde{f}_l(m)$. Then problem (P2) is equivalent to the maximization problem

$$\max_{r(.)} \; r(0)\tilde{H}(0) + \int_0^{\overline{C}} r(C)\tilde{h}(C) \, \mathrm{d}C \tag{A.1}$$

s.t.

$$r(0)\tilde{G}(0) + \int_0^{\overline{C}} r(C)\tilde{g}(C) \, \mathrm{d}C = 0 \tag{A.2}$$

$$\int_0^{\overline{C}} r'(C)C\tilde{g}(C) \, \mathrm{d}C = k(\tilde{e})c(\tilde{e}) \tag{A.3}$$

$$0 \leq r'(C) \leq 1 \tag{A.4}$$

$$r(0), \; r(\overline{C}) \;\text{ free} \tag{A.5}$$

Now replace constraint (A.2) by

$$K(C) = \int_0^{C} r(s)\tilde{g}(s) \, \mathrm{d}s \;\text{ with }\; K'(C) = r(C)\tilde{g}(C), K(0) = 0 \text{ and } K(\overline{C}) = -r(0)\tilde{G}(0).$$

Furthermore set $r'(C) = u(C)$ and replace (A.3) by

$$L(C) = \int_0^{C} u(s)s\tilde{g}(s) \, \mathrm{d}s \;\text{ with }\; L'(C) = u(C)C\tilde{g}(C), \; L(0) = 0 \text{ and } L(\overline{C}) = c(\tilde{e})k(\tilde{e}).$$

Therefore the problem is

$$\max_{r(.)} \; r(0)\tilde{H}(0) + \int_0^{\overline{C}} r(C)\tilde{h}(C) \, \mathrm{d}C \tag{A.6}$$

subject to

$$\begin{aligned}
&K'(C) = r(C)\tilde{g}(C), \qquad && K(0) = 0, \quad && K(\overline{C}) = -r(0)\tilde{G}(0) \\
&L'(C) = u(C)C\tilde{g}(C), && L(0) = 0, && L(\overline{C}) = k(\tilde{e})c(\tilde{e}) \\
&r(0), r(\overline{C}) \;\text{ free} && &&
\end{aligned}$$

We can now set up the Lagrangian

$$
\begin{aligned}
L \quad = \quad & r(0)\tilde{H}(0) + \int_0^{\overline{C}} \Big\{ r(C)\tilde{h}(C) + \lambda(C)[u(C) - r'(C)] \qquad \text{(A.7)} \\
& + \mu(C)[r(C)\tilde{g}(C) - K'(C)] + \eta(C)[r'(C)C\tilde{g}(C) - L'(C)] \Big\} \mathrm{d}C \\
& + \gamma_1 K(0) + \gamma_2[r(0)\tilde{G}(0) + K(\overline{C})] + \gamma_3 L(0) + \gamma_4[k(\tilde{e})c(\tilde{e}) - L(\overline{C})].
\end{aligned}
$$

Note that $\eta(C)$ is the Lagrange multiplier associated with the incentive constraint (A.3). Integrating $\lambda(C)r'(C)$, $\mu(C)K'(C)$ and $\eta(C)L'(C)$ by parts we obtain

$$
\begin{aligned}
L \quad = \quad & r(0)\tilde{H}(0) + \int_0^{\overline{C}} \Big\{ r(C)\tilde{h}(C) + \lambda(C)u(C) + \lambda'(C)r(C) \qquad \text{(A.8)} \\
& + \mu(C)r(C)\tilde{g}(C) + \mu'(C)K(C) + \eta(C)r'(C)C\tilde{g}(C) + \eta'(C)L(C) \Big\} \mathrm{d}C \\
& - [\lambda(\overline{C})r(\overline{C}) - \lambda(0)r(0)] \\
& - [\mu(\overline{C})K(\overline{C}) - \mu(0)K(0)] - [\eta(\overline{C})L(\overline{C}) - \eta(0)L(0)] \\
& + \gamma_1 K(0) + \gamma_2[r(0)\tilde{G}(0) + K(\overline{C})] + \gamma_3 L(0) + \gamma_4[k(\tilde{e})c(\tilde{e}) - L(\overline{C})].
\end{aligned}
$$

The first differential is

$$
\begin{aligned}
\Delta L \quad = \quad & \int_0^{\overline{C}} \Big\{ [\tilde{h}(C) + \lambda'(C) + \mu(C)\tilde{g}(C)]\Delta r(C) + [\lambda(C) + \eta(C)C\tilde{g}(C)]\Delta u(C) \\
& + \mu'(C)\Delta K(C) + \eta'(C)\Delta L(C) \Big\} \mathrm{d}C \qquad \text{(A.9)} \\
& + [\tilde{H}(0) + \lambda(0) + \gamma_2\tilde{G}(0)]\Delta r(0) - \lambda(\overline{C})\Delta r(\overline{C}) + [-\mu(\overline{C}) + \gamma_2]\Delta K(\overline{C}) \\
& + K(0)\Delta\gamma_1 + [r(0)\tilde{G}(0) + K(\overline{C})]\Delta\gamma_2 + L(0)\Delta\gamma_3 + [k(\tilde{e})c(\tilde{e}) - L(\overline{C})]\Delta\gamma_4.
\end{aligned}
$$

This yields the following conditions for optimality

$$\tilde{h}(C) + \lambda'(C) + \mu(C)\tilde{g}(C) = 0 \qquad \text{(A.10)}$$

$$\lambda(C) + \eta(C)C\tilde{g}(C) \begin{cases} > 0 & \Rightarrow \quad u(C) = 1 \\ = 0 & \Rightarrow \quad 0 \le u(C) \le 1 \\ < 0 & \Rightarrow \quad u(C) = 0 \end{cases} \qquad \text{(A.11)}$$

$$\mu'(C) = 0 \quad \text{which implies} \quad \mu(C) = \overline{\mu} \qquad \text{(A.12)}$$

$$\eta'(C) = 0 \quad \text{which implies} \quad \eta(C) = \overline{\eta} \qquad \text{(A.13)}$$

$$\lambda(0) = -\tilde{H}(0) - \gamma_2\tilde{G}(0) \qquad \text{(A.14)}$$

$$\lambda(\overline{C}) = 0 \qquad \text{(A.15)}$$

$$\gamma_2 = \mu(\overline{C}) \quad \text{which implies} \quad \gamma_2 = \overline{\mu}. \tag{A.16}$$

$$K(0) = 0 \tag{A.17}$$

$$r(0)\tilde{G}(0) + K(\overline{C}) = 0 \tag{A.18}$$

$$L(0) = 0 \tag{A.19}$$

$$L(\overline{C}) = k(\tilde{e})c(\tilde{e}). \tag{A.20}$$

Integrating (A.10) yields

$$\begin{aligned}
\lambda(C) &= \lambda(0) + \int_0^C \lambda'(s)\,\mathrm{d}s \\
&= -\tilde{H}(0) - \overline{\mu}\tilde{G}(0) + \int_0^C -\tilde{h}(s) - \overline{\mu}\tilde{g}(s)\,\mathrm{d}s \\
&= -\tilde{H}(C) - \overline{\mu}\tilde{G}(C).
\end{aligned} \tag{A.21}$$

With

$$0 = \lambda(\overline{C}) = -\tilde{H}(\overline{C}) - \overline{\mu}\tilde{G}(\overline{C}) = 0$$

we get $\overline{\mu} = 0$ which simplifies (A.21) to

$$\lambda(C) = -\tilde{H}(C).$$

Inserting into (A.11) we obtain

$$-\tilde{H}(C) + \eta(C)C\tilde{g}(C) \begin{cases} > 0 & \Rightarrow \quad u(C) = r'(\overline{C}) = 1 \\ = 0 & \Rightarrow \quad 0 \le u(C) = r'(\overline{C}) \le 1 \\ < 0 & \Rightarrow \quad u(C) = r'(\overline{C}) = 0 \end{cases} . \tag{A.22}$$

which is equivalent to condition (12). Now $\overline{\eta}$ needs to be chosen such that (A.20) is satisfied. This guarantees

$$\int_0^{\overline{C}} r'(C)C\tilde{g}(C)\,\mathrm{d}C = k(\tilde{e})c(\tilde{e}).$$

Finally $r(0)$ is set such that (A.18) is satisfied which implies

$$r(0)\tilde{G}(0) + \int_0^{\overline{C}} r(C)\tilde{g}(C)\,\mathrm{d}C = 0. \tag{A.23}$$

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