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BUDGETARY POLICY AND UNEMPLOYMENT DYNAMICS IN AN OLG MODEL WITH COLLECTIVE BARGAINING*

Leo Kaas and Leopold von Thadden

We consider a dynamic general equilibrium model with capital accumulation and collective wage bargaining and investigate how unemployment responds to structural shocks under two stylised budgetary policies. Under balanced budgets, tax adjustments lead to higher unemployment on impact but enable a quick recovery of employment. By contrast, unbalanced budgets mitigate unemployment effects on impact but dynamics are more persistent due to weaker capital accumulation and future fiscal consolidations. These results are consistent with empirical evidence on a positive cross-country relationship between government borrowing and unemployment persistence.

This paper starts out from the well-known fact that many European countries have suffered from persistently high unemployment over the past two decades. There is a large literature which attempts to explain this development as resulting from a combination of structural shocks and mechanisms which make the effects of any such shifts more persistent. One prominent branch of this literature stresses that adverse structural shocks tend to induce a slowdown in capital accumulation which reinforces the employment losses and adds to unemployment persistence (Bean, 1989; Benassy, 1997; Blanchard, 1997; Burda, 1988; Caballero and Hammour, 1998; Daveri and Tabellini, 2000). Similarly, these studies point out that, when the shock is over, the recovery of employment depends critically on the speed with which investments in capital respond to the new situation.

The purpose of this paper is to reconsider this 'capital-shortage hypothesis' by linking it explicitly to the stance of fiscal policy. To see the importance of fiscal policy in this context, consider a government which faces an adverse *structural* shock (such as a drop in employment due to a wage-setting shock) and decides whether it should keep its budget balanced or not. This decision will affect the overall structure of assets in the economy (i.e. the mix between capital and government bonds), and this mix will in turn be an important determinant of the speed with which the economy can create employment once the shock is over. In particular, according to the capital-shortage hypothesis, the recovery in terms of employment is likely to take longer under a policy which temporarily accepts structural budget deficits, particularly when crowding out effects have been substantial.

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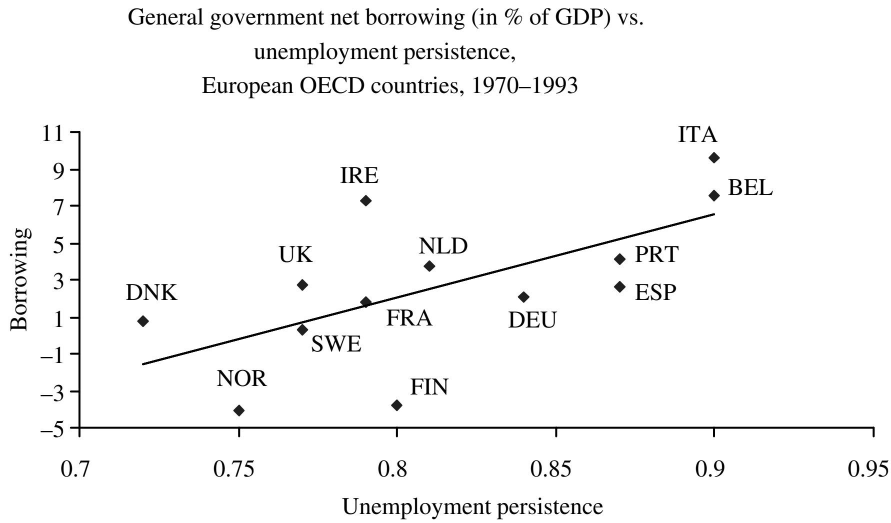

Fig. 1. *Government Net Borrowing vs. Unemployment Persistence*

To illustrate the relevance of this line of reasoning, we report in Figure 1 a positive relationship between a measure of unemployment persistence and average government borrowing in 13 European countries for the period 1970–93.[1]

As reported in detail, for example, in Blanchard (1997), the countries covered in Figure 1 experienced a similar structural shock in the 1970s and early 1980s in the sense that (excessive) wage setting was not in line with a slowdown of productivity. However, during and in the aftermath of this period the average budgetary position of the countries (measured as the average annual net borrowing position of governments as a fraction of GDP) shows marked differences and correlates positively with a country-specific estimate of unemployment persistence.[2] Interestingly, the correlation coefficient in Figure 1 drops from 0.6 to a value of 0.4 if one includes the non-European countries US, Japan, Australia, and Canada. This finding points to the potential importance of wage-bargaining systems which are typical of European countries. We consider these observations to be sufficiently suggestive, from a theoretical perspective, to think about specific

[1] Figure 1: The measure of 'unemployment persistence' is the lagged coefficient on the unemployment rate reported by Scarpetta (1996, p. 66, equation 11, table 5) who estimates adjustment dynamics for unemployment rates in 17 major OECD countries in a cross-country framework, using annual data for the period 1970–93 and controlling for shifts in the long-run equilibrium rate of unemployment due to changes in labour market policies and institutional features. 'General government net borrowing' (annual averages) according to OECD *Economic Outlook* and, whenever unavailable, complemented by the AMECO database. The positive correlation presented in Figure 1 is confirmed, when we use instead estimates from equation 12 of Scarpetta's study or the persistence measure derived in Elmeskov and MacFarlan (1993) from an ARMA$(1,x)$ specification.

[2] Adjustment dynamics of unemployment are measured relative to time-varying equilibrium rates of unemployment. Given the difficulties of estimating slow adjustment dynamics in finite samples in a robust way (in particular, if compared with the alternative of a unit root in ARMA specifications) as discussed, for example, in Roed (1996) or Bianchi and Zoega (1998), we verified that a relationship similar to Figure 1 obtains if one replaces the unemployment persistence measure simply by the average level of the unemployment rate.

ways how the stance of fiscal policy affects unemployment dynamics if labour markets are characterised by collective wage bargaining.

To this end, we develop a dynamic general equilibrium model with collective wage bargaining to investigate the interaction between budgetary policy, capital formation and unemployment. Specifically, we derive the dynamic response of unemployment to structural shocks under both balanced and unbalanced budgets and show that in the latter case unemployment persistence is higher. To keep policies comparable, we do not investigate fiscal experiments leading to permanently unbalanced budgets. Instead, our analysis of 'unbalanced budgets' is restricted to episodes of prolonged, but in any case temporary, budgetary imbalances. That is, we impose throughout that the economy ultimately returns to the pre-shock debt level (which, by definition, obtains under the balanced budget policy).

Our model has a number of distinctive features. First, to capture a key aspect of typical collective wage settlements, the labour market is described by a 'right-to-manage' model in which sector-specific trade unions and employers' federations bargain over wages, and firms decide upon employment. We also allow for turnover frictions, ensuring that any equilibrium is characterised by unemployment, irrespective of the level of the capital stock. Second, we assume a standard neoclassical production technology with a low elasticity of substitution between the two homogenous inputs labour and physical capital, implying that a lower capital stock raises equilibrium unemployment. As a consequence of this feature, an adverse shock followed by declining investments in capital induces persistent unemployment dynamics. Third, we employ an overlapping generations economy without bequest motives. This framework has the advantage that Ricardian equivalence does not hold and bonds are perceived as net wealth. Hence, budgetary policy affects capital accumulation and the dynamics of unemployment. Finally, the government runs an unemployment insurance system which is financed by a wage-income tax or, alternatively, by the issuing of government bonds. The government's choice between these two instruments has non-trivial consequences for two reasons. On the one hand, we assume that unemployment benefits are not (perfectly) indexed to net wages. Thus, similarly to Pissarides (1998), higher wage taxes lead under decentralised wage bargaining to higher gross wages, thereby reducing employment in the short run. On the other hand, since Ricardian equivalence does not hold, shifts from bonds to higher taxes lead to more investment in physical capital, thereby raising future employment. Thus, the effects of the two policies on employment and output operate in opposite directions.[3]

Given these characteristics of the model, we consider two distinct budgetary policies. First, we assume the government maintains *balanced budgets* by adjusting the wage-income tax rate in response to a structural shock. Under such a policy, stability features of steady states coincide exactly with those of the perfectly competitive benchmark model with full employment. A dynamically efficient steady

[3] The adverse effect of loose budgetary policy on capital accumulation is particularly pronounced in the closed-economy framework of our paper. However, the effect would also operate in a small open economy if capital is imperfectly mobile or if capital markets require a risk premium on external debt; see van der Ploeg (1996).

state (at which the interest rate exceeds the growth rate) is a saddle in the two-dimensional capital-bonds dynamics. A dynamically inefficient steady state (at which the interest rate falls below the growth rate) is a sink. In both cases, adjustment dynamics to the steady state are stable since the balanced budget policy keeps the economy on the stable saddle path. Second, we consider a policy of *unbalanced budgets* where the government keeps the tax rate fixed and reacts to a structural shock by allowing temporarily for a deficit. Stability features of steady states are altered decisively. Now the steady state in a dynamically efficient *and* inefficient economy turns out to be a saddle, unless the interest rate falls short of a threshold level (which is below the growth rate). Importantly, the saddle is characterised by *unstable* adjustment dynamics. This implies that when the government runs a deficit in response to an adverse shock, corrective fiscal measures need to be implemented in the future since the economy would otherwise diverge on a path with explosive debt accumulation.

These results relate qualitatively to the empirical finding summarised in Figure 1. In both policy regimes, a temporary wage shock increases unemployment on impact, with a stronger reaction under balanced budgets because of the initial adjustment of the wage tax. However, the unbalanced-budget policy leads to a sharper decline in capital, inducing a slower recovery of employment in the aftermath of the shock. Furthermore, future consolidation measures (that are needed when the policy becomes unsustainable) keep unemployment high during the adjustment period. For these two reasons, unemployment dynamics tend to be more persistent in the unbalanced-budgets regime. Of course, when the interest rate is very low, the initial deficit may be self-correcting without future consolidation measures, but even then unemployment dynamics are more persistent than under the balanced-budget policy.

Following Diamond (1965), we use a standard overlapping generations economy with two-period lived agents. From a quantitative perspective, the timing assumptions of this model are not ideal to study the issue at hand, and, in any case, more empirical work is required to investigate the quantitative significance of the hypothesis advanced in this paper further. However, from the theoretical perspective of our paper, the model set-up is particularly suited to emphasise the capital-shortage hypothesis as opposed to other (equally important) explanations of unemployment persistence summarised below. Specifically, in our model the only link between unemployment rates in two subsequent periods is the capital stock which is accumulated from one period to the next. Moreover, we do not address issues of short-run stabilisation policy, i.e. we do not analyse properties of unbalanced budgets via automatic stabilisation effects in a purely cyclical context. To study such issues would certainly require different timing assumptions. Instead, our focus is solely on effects of (prolonged) structural shocks. Finally, the Diamond framework allows conveniently for analytical tractability. We conjecture, however, that qualitatively similar results obtain in a Blanchard-Weil framework which offers a more realistic time horizon for agents, while at the same time admitting deviations from Ricardian equivalence which are important for our argument.

Evidently, the mechanism identified in this paper does not replace, but complements alternative explanations in the literature. In particular, persistent unemployment has been explained by the insider-outsider hypothesis of Blanchard and Summers (1986), the loss of skills of long-term unemployed (Pissarides, 1992), declining search intensities of long-term unemployed (Layard *et al.*, 1991), the ranking of job applicants according to the duration of unemployment instead of random hiring (Blanchard and Diamond, 1994), sluggish changes in labour demand due to adjustment costs (Alogoskoufis and Manning, 1988), credit market imperfections (Acemoglu, 2001), various institutional features of labour markets (Blanchard and Wolfers, 2000), or political economy arguments (Hassler *et al.*, 2001).

The remainder of the paper is structured as follows. Section 1 introduces the model and derives the labour market equilibrium. Sections 2 and 3 analyse the equilibrium dynamics under balanced and unbalanced budgets, respectively. Section 4 shows for both policies the response pattern to a temporary wage-setting shock. In Section 5 we further discuss our results in the context of related literature, and Section 6 offers conclusions. Proofs not included in the text are contained in the Appendix.

1. The Model

Consider an overlapping generations economy which comprises a continuum [0,1] of consumers living for two periods and a continuum [0,1] of firms. Consumers supply labour when they are young and consume in both lifetime periods. Firms produce a composite consumption/investment good from inputs of capital and labour. The government pays unemployment benefits which are financed by a wage-income tax and by the emission of one-period bonds. Markets for capital and goods are perfectly competitive but wages are the outcome of a bargain at the sector level. The economy consists of a large number M of symmetric sectors, in each of which a single trade union (representing a mass $1/M$ of young consumers) and a single employers' federation (representing a mass $1/M$ of firms) bargain over the wage. After wages are negotiated, employment is decided at the level of the firm ('right-to-manage model'). There is also some turnover of workers between sectors, similarly to Layard *et al.* (1991, ch. 2). This implies that wages paid in other sectors and the aggregate unemployment rate matter for wage formation in each sector. As a consequence, there is positive unemployment in any equilibrium. In detail, the economy is described as follows.

1.1. *Consumers and Trade Unions*

Each consumer born at date t supplies one unit of indivisible labour when young and wishes to consume in periods t and $t + 1$. Consumers save part of their labour income for retirement by holding capital shares or government bonds which pay a gross real rate of return R_{t+1}. An employed person receives a (real) net wage $w_t(1 - \tau_t)$ whereas an unemployed person receives the (real) unemployment benefit a. Workers are randomly allocated to jobs. Their von Neumann-Morgenstern utility function $u(c_t, c_{t+1})$ is assumed to be linearly homogeneous, strictly quasi-concave and differentiable. Thus, each young consumer's savings behaviour is described by a savings function $s(R_{t+1})I_t$ where $I_t \in \{w_t(1 - \tau_t), a\}$

denotes the first–period income and the consumer's indirect utility is $v(R_{t+1})I_t$, where $v(R_{t+1}) \equiv u[1 - s(R_{t+1}), s(R_{t+1})R_{t+1}]$. We assume $s' \geq 0$, i.e. savings are non-decreasing in the interest rate.

We assume that all young consumers are union members. Hence, each trade union represents a mass $1/M$ of young consumers. However, not all workers are eventually employed in their home sector, but there is some turnover of workers between sectors. A fraction $0 < \pi < 1$ of the initially created work relationships turns out to be unproductive and these relationships are terminated immediately. The resulting vacancies are then filled with (unemployed) persons from other sectors. Thus, if $\ell_t \leq 1/M$ jobs are created in some sector, only $(1 - \pi)\ell_t$ union members receive the negotiated sector net wage $w_t(1 - \tau_t)$, and $1/M - (1 - \pi)\ell_t$ members are either employed in another sector or remain unemployed. Their expected income, denoted w_t^*, will be determined in equilibrium below. The trade union's objective is to maximise the expected utility of a representative member which is $V_t \equiv v(R_{t+1})\{w_t(1 - \tau_t)(1 - \pi)\ell_t + w_t^*[1/M - (1 - \pi)\ell_t]\}M$. Since the number of sectors is large, trade unions ignore their impact on the aggregate unemployment rate (and thus on w_t^*) and on aggregate investment (and thus on next period's capital return R_{t+1}).[4] When negotiations break down, the union's fallback payoff is $\overline{V}_t \equiv v(R_{t+1})w_t^*$. Therefore, the union surplus of a successful negotiation is

$$V_t - \overline{V}_t = v(R_{t+1})(1 - \pi)[w_t(1 - \tau_t) - w_t^*]\ell_t M. \tag{1}$$

1.2. *Firms and Employers' Federations*

Firms produce the output good from capital and labour using the constant returns production technology $Y_t = F(K_t, L_t) = L_t f(k_t)$ where $k_t = K_t/L_t$ is the capital intensity.[5] The intensive-form production function f is assumed to be increasing and strictly concave. Capital investment is decided a period in advance and thus before wages are negotiated. However, since the investment decision of any single firm has a negligible effect on aggregate labour demand in its sector, firms ignore the impact of their investment decision on the outcome of wage negotiations.[6] Thus firms take the (perfectly foreseen) wage and the interest rate as given, and so we obtain the usual marginality conditions

$$w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t),$$

$$R_t = R(k_t) \equiv 1 - \delta + f'(k_t),$$

where δ is the depreciation rate.

[4] In particular, compared with centralised bargaining this assumption tends to increase the equilibrium unemployment rate; see, for example, Calmfors and Drifill (1988).

[5] Since there is a mass 1 of identical firms, K_t and L_t denote the capital stock and employment both at the aggregate level and at the firm level.

[6] Thus, since employers' federations represent a large number of firms, we can abstract from the typical hold-up problem of firm-specific bargaining as discussed, for instance, by Coloma (1999) and Devereux and Lockwood (1991).

Firms in each sector are represented by an employers' federation. The federation's objective is the profit of each of its members,

$$\Pi_t \equiv F(K_t, L_t) - w_t L_t = \{f[K_t/(M\ell_t)] - w_t\}M\ell_t, \tag{2}$$

using $L_t = M\ell_t$. The fallback payoff of the employers' federation is $\overline{\Pi}_t = 0$.

1.3. *The Wage Bargain*

Given the capital stock in each sector K_t/M, the trade union and the employers' federation negotiate the sector wage taking into account that employment is chosen by firms. Sector employment $\ell_t = L_t/M$ and the sector wage are related by the labour demand schedule $w_t = w[K_t/(M\ell_t)]$. The outcome of this wage bargain is determined by the Nash bargaining solution which maximises the Nash product $(\Pi_t - \overline{\Pi}_t)^{\beta}(V_t - \bar{V}_t)^{1-\beta}$ subject to the labour demand schedule. $\beta \in [0,1)$ denotes the bargaining power of the employers' federation. We show below that, because of worker turnover, there can only be an unemployment solution in a symmetric equilibrium, and in the Appendix we show that the solution is characterised by the first-order condition

$$\beta\frac{f''(k_t)k_t}{f'(k_t)} + (1-\beta)\left[\frac{k_t w'(k_t)(1-\tau_t)}{w(k_t)(1-\tau_t) - w_t^*} - 1\right] = 0. \tag{3}$$

Intuitively, this equation balances the profit loss of a wage increase (with weight β) against the workers' benefit (with weight $1 - \beta$).

1.4. *The Government*

The government pays a fixed unemployment benefit a to each unemployed person and levies a proportional wage-income tax at rate τ_t.[7] For the sake of simplicity, we abstract from other components of government spending, as well as from other tax instruments (see Section 5 for a discussion). The deficit $d_t = a(1 - L_t) - \tau_t w_t L_t$ is financed by issuing bonds. Note that lower employment raises the deficit as expenditures increase and tax revenues fall. Let b_t denote the real value of the stock of government bonds maturing at date t. The government faces a flow budget constraint given by

$$b_{t+1} = R_t b_t + d_t,$$

assuming that in a perfect capital market the return rates on government bonds and on capital shares coincide. We do not explicitly impose an intertemporal budget constraint (i.e., a no-Ponzi game condition) by requiring that any debt must be repaid by future surpluses. Note, however, that such a constraint is satisfied at any steady state with zero debt and zero deficit to be considered below.

[7] It may be assumed alternatively that the government fixes the replacement ratio $a/[w(1-\tau)]$ rather than the level of benefits. In this case the short-run equilibrium unemployment rate is independent of the tax rate. However, as discussed in Pissarides (1998), this special result would disappear if one considers a non-linear wage tax, made up of a fixed and a proportional component.

We consider two types of policies: (BB) Balanced budgets: the government adjusts the tax rate in each period such as to balance out benefit expenditures and tax revenues, i.e. $d_t = 0$ for all t.

(UB) Unbalanced budgets: the government fixes the tax rate and runs budget surpluses or budget deficits. If government debt becomes unstable in the long run due to increasing budget deficits, tax rates are adjusted in a discretionary manner to restore sustainability and to return to the pre-shock debt level.

1.5. *The Equilibrium*

$1 - (1 - \pi)L_t$ persons do not find employment in their home sector. With some probability φ_t each of them finds a job in some other sector. Given that there are πL_t vacancies left, this probability is

$$\varphi_t = \frac{\pi L_t}{1 - (1 - \pi)L_t} = \frac{\pi(1 - u_t)}{\pi(1 - u_t) + u_t}, \tag{4}$$

where $u_t = 1 - L_t$ denotes the unemployment rate. Thus the expected income of a person who is not employed in his home sector is

$$w_t^* = \varphi_t w_t(1 - \tau_t) + (1 - \varphi_t)a. \tag{5}$$

Inserting (4) into (5) shows that the fallback wage is decreasing in the unemployment rate since higher unemployment makes it more difficult for an unemployed person to find a job in another sector.

It is now easy to see that the unemployment rate must be positive, implying that it is sufficient to consider the interior first-order condition (3). If $u_t = 0$, all persons are employed with probability one, $\varphi_t = 1$ (see (4)), and it follows from (5) that the union's fallback wage coincides with the net wage of the other sectors (this argument requires positive turnover between sectors $\pi > 0$). Since any outcome of the wage bargain must pay some markup over the fallback wage whenever unions have bargaining power ($\beta < 1$), such a wage cannot be set in equilibrium. Thus, some positive unemployment is required to restrain the wage demands of unions. Of course, when there is no worker turnover or when unions do not have market power, there may be full employment solutions (see footnote 9 below).

The short-run equilibrium in any given period t can be described by the following sequence of events that is illustrated in Figure 2. For a given aggregate capital stock K_t and a tax policy τ_t, negotiation partners meet in each sector to

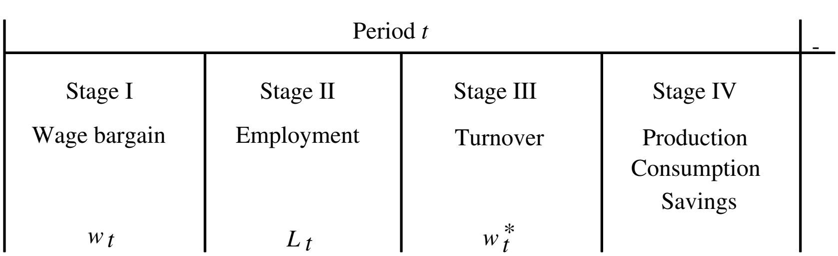

Fig. 2. *Sequence of Events in Period t*

bargain about the wage w_t (Stage I), rationally expecting firms' employment decisions and the workers' fallback wage. At Stage II, firms hire workers which determines aggregate employment L_t. At Stage III, turnover takes place, some workers lose their jobs, and some find a job in another sector. This determines the expected income w_t^* of a worker who did not find employment at Stage II. Finally at Stage IV, production, consumption and savings decisions take place.

DEFINITION *A short-run equilibrium for given* K_t *and* τ_t *is a list* (w_t, L_t, w_t^*) *such that*

(*i*) *the wage* w_t *is the outcome of a Nash bargain between unions and employers' federations who take* w_t^* *as given. Employment* L_t *is decided by firms. Thus,* $w_t = w(k_t)$, *and* $k_t = K_t/L_t$ *satisfies* (3).

(*ii*) *the fallback wage* w_t^* *is determined by* (4) *and* (5) *for given* (w_t, L_t).

To solve for a short-run equilibrium, it is useful to derive a relation between the capital intensity k_t and the unemployment rate. Combining (4) and (5) with the bargaining solution (3) and solving for the unemployment rate yields

$$u_t = u(k_t, \tau_t) \equiv \frac{\pi\mu(k_t, \tau_t)}{1 - (1-\pi)\mu(k_t, \tau_t)},$$

where

$$\mu(k, \tau) \equiv \frac{(1-\beta)\dfrac{w'(k)k}{w(k)}}{1 - \beta\left[1 + \dfrac{f''(k)k}{f'(k)}\right]} \frac{w(k)(1-\tau)}{w(k)(1-\tau) - a}.$$

Note that $0 \leq u(k, \tau) \leq 1$ if and only if $0 \leq \mu(k, \tau) \leq 1$, and that u is increasing in μ. When the production function is Cobb-Douglas, the first factor in the definition of μ is a constant and the second factor is decreasing in k. Hence, μ (and thereby u) are strictly decreasing in the capital intensity k. More generally, we impose the assumption

$$\frac{\mathrm{d}u}{\mathrm{d}k}(k, \tau) < 0. \tag{A1}$$

Assumption (A1) guarantees that a higher capital stock lowers the equilibrium unemployment rate.[8] Intuitively, there are two effects operating. First, if capital and labour are complementary inputs, a higher capital stock boosts employment (hence the first factor in the definition of μ is decreasing in k). Second, a higher capital stock shifts labour demand outwards which leads to higher equilibrium employment since benefits are not indexed to wages (hence the second factor in the definition of μ is decreasing in k). From the identity $k_t = K_t/(1 - u_t)$ we obtain

[8] It can be easily checked that assumption (A1) is satisfied if the production function has constant elasticity of substitution $\sigma \leq 1$. When $\sigma < 1$ the first factor in the expression defining μ is decreasing in k. As reported in Rowthorn (1999), most empirical studies are in favour of elasticities of substitution below unity. For a detailed discussion of the role of the elasticity of substitution in a related model, see Kaas and von Thadden (2003).

the following equation which determines the capital intensity (or equivalently the unemployment rate) in the short-run equilibrium (for given K_t and τ_t):

$$u(k_t, \tau_t) = 1 - \frac{K_t}{k_t}. \tag{6}$$

Since the left-hand side is decreasing in k_t by assumption (A1), and the right-hand side is increasing, this identity determines a unique capital intensity k_t and (positive) unemployment rate in the short-run equilibrium.[9]

For convenience, we introduce the following notation for the public deficit:

$$d(k, \tau) \equiv au(k, \tau) - \tau w(k)[1 - u(k, \tau)].$$

2. Balanced Budgets

We consider first the case in which the government adjusts tax rates in each period in order to balance its budget, i.e. we suppose that $d(k_t, \tau_t) = 0$ in every period. There are two effects of a tax rise on the public deficit: at a given employment level higher taxes reduce the deficit, but a higher tax rate also depresses employment which lowers revenues and increases benefit outlays and the deficit. It turns out that the deficit in fact behaves like an inverted Laffer curve: for low tax rates the first effect dominates and the deficit falls with higher tax rates, whereas the negative effect dominates at higher tax rates. In particular, the following lemma shows that there are two tax rates balancing the budget, provided that unemployment benefits are not too large, i.e. smaller than some upper bound $\bar{a}$.

LEMMA *Let* $k \geq \underline{k}$ and $a \leq \bar{a}$ *where* $\underline{k}$ satisfies $u(\underline{k}, 0) < 1$ *and where* $\bar{a}$ *is sufficiently small. At sufficiently low and at sufficiently high tax rates the deficit is positive. There are exactly two positive tax rates* $0 < \tau_1(k) < \tau_2(k) < 1 - a/w(k)$ *such that* $d[k, \tau_i(k)] = 0$, $i = 1, 2$. *Moreover,* $\tau_1'(k) < 0$.

Proof: Appendix.

It is natural to assume that the government chooses the smaller tax rate to balance its budget. At the larger tax rate, unemployment is higher and both the unemployment rate and the deficit could be reduced by lowering the tax rate. Such a tax rate is therefore not a likely outcome of a political decision process. Denote the unemployment rate at balanced budgets by $u_B(k) \equiv u[k, \tau_1(k)]$, $k \geq \underline{k}$. Since $u(k, \tau)$ is decreasing in k (from (A1)) and increasing in τ and because of the

[9] When there is no turnover ($\pi = 0$), the short-run equilibrium involves full employment when the capital stock is large enough. When the capital stock is low enough, however, there is an unemployment solution where $u_t = 1 - K_t/k_t > 0$ and k_t solves $\mu(k_t, \tau_t) = 1$. Similarly, when there is no bargaining power of unions ($\beta = 1$), there are full employment solutions for large capital stocks, but there can also be a (voluntary) unemployment solution where the net wage equals the reservation wage, $w(k_t)(1 - \tau_t) = a$. In both cases, the model 'degenerates' in the sense that the capital intensity in period t is independent of the aggregate capital stock K_t. Indeed, it turns out that the balanced-budgets dynamics have no stable steady state with unemployment (an analysis is available from the authors on request).

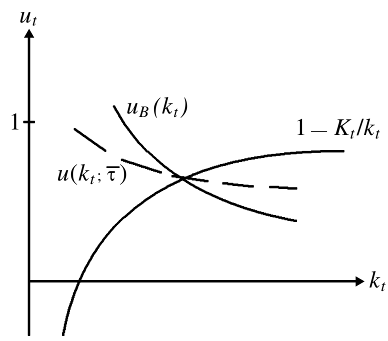

Fig. 3. *Short-run Equilibrium*

lemma, we have $u_B' < 0$. With taxes adjusting to balance the budget, the short-run equilibrium is now determined instead of (6) by

$$u_B(k_t) = 1 - \frac{K_t}{k_t}. \tag{7}$$

Equation (7) defines the unique short-run equilibrium capital intensity k_t for any level of the capital stock K_t. In fact, the left-hand side is well defined for all $k \geq \underline{k}$, is decreasing and converges to a non-negative constant less than unity for $k \to \infty$, whereas the right-hand side is increasing in k and converges to unity for $k \to \infty$. The two equilibrium curves are illustrated in Figure 3. If the capital stock falls or if the labour supply increases, the RHS shifts up, unemployment increases, and the capital intensity and the wage rate fall. On the other hand, a wage push induced by stronger union power in wage negotiations (a lower β) or by more generous benefit spending (a higher a) shifts up the LHS, raises the unemployment rate, the capital intensity and the wage rate.

We turn now to the dynamics of the capital stock and to the analysis of long-run equilibria. Since the budget is balanced by assumption, the aggregate income of the young generation in period t is

$$w_t(1-\tau)L_t + a(1-L_t) = w_t L_t. \tag{8}$$

A fraction $s(R_{t+1})$ of this income is saved in the form of capital shares or government bonds. Thus capital market equilibrium implies

$$K_{t+1} + b_{t+1} = s(R_{t+1})w_t L_t.$$

Using (7) and the definitions of R and w, this can be rewritten as

$$k_{t+1}[1 - u_B(k_{t+1})] + b_{t+1} = s[R(k_{t+1})]w(k_t)[1 - u_B(k_t)]. \tag{9}$$

The government budget constraint is

$$b_{t+1} = R(k_t)b_t. \tag{10}$$

Equations (9) and (10) define an implicit two-dimensional dynamic system in (k_t, b_t), similar to the perfectly competitive Diamond model with government bonds which obtains if we set $u_B \equiv 0$. It is easy to see that steady states $(\bar{k}, \bar{b})$ are naturally

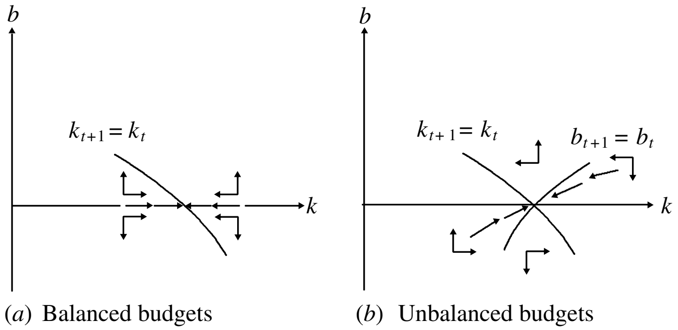

Fig. 4. *Local Dynamics at a Dynamically Efficient Steady State* $(R > 1)$

related to steady states of the perfectly competitive model and that stationary capital intensities are the same under unemployment as under perfect competition (i.e. full employment).[10] As a result, stationary interest rates and (gross) wages do not depend on labour market conditions and are the same as under perfect competition. We restrict our analysis to steady states with zero government debt:[11]

$$\bar{k} = s\left[R(\bar{k})\right]w(\bar{k}) \text{ and } \bar{b} = 0. \tag{11}$$

Not only are steady state capital intensities identical in the unemployment model and in the benchmark competitive model, but also stability features are the same. In the competitive model ($u_t = 0$), a dynamically efficient steady state (with $R(\bar{k}) > 1$) turns out to be a saddle, and a dynamically inefficient steady state (with $R(\bar{k}) < 1$) is a sink, provided that a certain stability condition is satisfied, see, for example, Azariadis (1993, ch. 7). As we show in the following Proposition, this result also extends to our economy with unemployment.

PROPOSITION 1 *Consider a steady state* $(\bar{k}, \bar{b})$ of (9) *and* (10) *satisfying* (11) *and a standard stability condition which is formally stated in the Appendix. Then, as for the perfectly competitive Diamond economy, the steady state is a saddle if it is dynamically efficient,* $R(\bar{k}) > 1$, *and it is a sink if it is dynamically inefficient,* $R(\bar{k}) < 1$.

Proof: Appendix.

Note that the benchmark case is an interest factor of unity (and hence a zero interest rate). However, once we take population growth or technological progress into account, the benchmark interest rate would be positive. In general, bond dynamics are unstable whenever the interest rate exceeds the growth rate of the economy. As long as the government does not issue bonds, even a dynamically efficient steady state is stable to perturbations of the economy, since the $b = 0$ axis is the stable manifold, as illustrated in Figure 4(a). However, once the government allows a small deficit, the debt dynamics would become unsustainable.

[10] This result depends on our assumption of homothetic utility, implying that savings are a linear function of income.

[11] Golden–rule steady states with positive government debt in dynamically inefficient economies have been analysed in a previous working paper version.

3. Unbalanced Budgets

Suppose now that the government does not adjust the tax rate to balance its budget in each period. More specifically, suppose the economy is in a steady state $(\bar{k}, \bar{b} = 0)$ and the government keeps the tax rate fixed at $\bar{\tau} = \tau_1(\bar{k})$. Now the short-run equilibrium capital intensity and unemployment rate are determined by

$$u(k_t, \bar{\tau}) = 1 - \frac{K_t}{k_t}.$$

As shown in Figure 3, the two equilibrium curves are similar to the ones under a balanced-budget rule, but the left-hand side $u(., \bar{\tau})$ is now a flatter curve than $u_B(.)$. The reason is as follows: suppose that, starting from a balanced budget, the capital stock falls, raising the unemployment rate and driving the budget into deficit. If the government balances its budget, it needs to raise the tax rate which further increases unemployment, since a higher wage tax leads to a higher gross wage in the wage bargain. Thus, a balanced-budget rule generally amplifies the shock in the short-run, whereas this effect is absent if the tax rate is not adjusted. As we show below, the opposite is the case in the long run: an unbalanced budget rule amplifies the persistence of unemployment and may even destabilise the economy.

Consider now the dynamics under unbalanced budgets. Instead of (8), the aggregate income of the young generation in period t is now $w_t L_t + d_t$. Thus, the dynamic equations (9) and (10) are replaced by

$$k_{t+1}[1 - u(k_{t+1}, \bar{\tau})] + b_{t+1} = s[R(k_{t+1})]\{w(k_t)[1 - u(k_t, \bar{\tau})] + d(k_t, \bar{\tau})\}, \tag{12}$$

and

$$b_{t+1} = R(k_t) b_t + d(k_t, \bar{\tau}). \tag{13}$$

Obviously, the steady state $(\bar{k}, \bar{b} = 0)$ of the dynamical system (9) and (10) discussed in Proposition 1 is also a steady state of (12) and (13).[12] As the following proposition shows, steady states as discussed in Proposition 1 are again either a saddle or a sink, but now the condition for a sink is stronger than with balanced budgets and deviates from the perfectly competitive case. Hence, dynamics are more likely to be unstable.

PROPOSITION 2 *Consider a steady state* $(\bar{k}, \bar{b})$ of (12) *and* (13) *satisfying* (11) *and the stability condition of Proposition* 1. *Then stability features correspond no longer to the benchmark case of a perfectly competitive economy. The steady state is a saddle if* $R(\bar{k}) > \bar{R}$ *where* $\bar{R} < 1$ *is defined in the Appendix. The steady state is a sink if* $R(\bar{k}) < \bar{R}$. *Hence, a dynamically efficient steady state is a saddle, whereas a dynamically inefficient steady state is*

[12] Other steady states of the balanced budget dynamics (i.e. the golden-rule steady state or other steady states with zero government debt) are also steady states of the dynamics with unbalanced budgets, but obviously for different tax rates. Moreover, for the fixed tax rate $\bar{\tau}$, (12) and (13) typically also have a steady state with a permanent fiscal deficit or surplus. However, such steady states will not be considered in this paper, see Chalk (2000) for an elaborate analysis of steady states with permanent deficits in the competitive Diamond model.

a saddle or a sink. In all these cases, off-steady state dynamics are monotone. If the steady state is a saddle, the stable manifold is upward sloping in (k, b) *space.*

Proof: Appendix.

To understand intuitively why the stable manifold is an upward-sloping curve, consider the following argument in the dynamically efficient case ($R(\bar{k}) > 1$): when k falls below $\bar{k}$, the deficit $d(k, \bar{\tau})$ becomes positive at the fixed tax rate since employment and the wage rate are falling. Correspondingly, the interest factor rises: $R(k) > R(\bar{k}) > 1$. If some (k, b) with $b \geq 0$ was on the stable manifold, bond dynamics would be unstable since $R(k)b + d(k, \bar{\tau}) > b \geq 0$. Hence, $b < 0$ is needed for stable bond dynamics, and therefore the stable manifold is upward–sloping. Figure 4(b) illustrates the stable manifold and the curves $k_{t+1} = k_t$ and $b_{t+1} = b_t$. By similar reasoning, $R(\bar{k}) < 1$ is not sufficient to stabilise the economy, since the issued bonds crowd out capital, thereby raising its marginal product in a possibly destabilising way. Overall dynamics will remain stable, only if, at the outset, $R(\bar{k})$ is sufficiently small ($R(\bar{k}) < \bar{R} < 1$).[13] Note that the real value of bonds cannot adjust by changes in the price level to restore stability along the stable manifold, i.e. starting out from $b = 0$, a jump of the price level cannot bring about a positive value of government asset holdings which would be needed to counteract the budget deficit after a fall in the capital intensity. Thus, saddle-path stability in Proposition 2 implies that perturbations which lead on impact to a government deficit can only be stabilised by appropriate fiscal adjustments at some point in the future.

4. Adjustment After a Temporary Wage Pressure

For the sake of a simple illustration of our results, consider temporary wage pressure which is induced, for instance, by stronger union power in wage negotiations (a fall in β). Specifically, suppose the economy is in a steady state of the type discussed in Propositions 1 and 2 and there is an unanticipated wage pressure in period t only, while from period $t + 1$ onwards labour market conditions are as before. Note that under our assumptions this experiment is qualitatively equivalent to a temporary rise in unemployment benefits a or a temporary productivity slowdown (not recognised by agents), which is widely reported for OECD countries in the 1970s. In terms of Figure 3, this means that the curves u_B and $u(.,\bar{\tau})$ shift up in period t, and are back at their original position afterwards. Thus, under both budgetary policies, the wage shock raises the capital intensity and the unemployment rate on impact, but unemployment increases more under balanced budgets. Since the stock of bonds maturing in t is predetermined at $b_t = 0$, the wage pressure leads under both budgetary rules to a state $(k_t, 0)$ with $k_t > \bar{k}$.

Under *balanced budgets* (BB), the government adjusts the tax rate such that the deficit in period t (and thereby the stock of bonds in period $t + 1$) is zero. Assuming that $w'(k)k/w(k) < 1$, higher unemployment depresses the income of the

[13] Chalk (2000) argues similarly that permanent fiscal deficits need not be sustainable, even if the interest rate falls short of the growth rate.

young generation,[14] savings are declining, and the capital stock in the next period falls. Thus in period $t+1$, when the wage pressure is over, the upward-sloping curve in Figure 3 shifts upwards, and the capital intensity falls below its steady state level. Irrespective of whether the steady state is a saddle or a sink, the capital intensity converges back to the steady state since no bonds are issued. Unemployment remains above its 'natural' level along the adjustment path. Thus, unemployment dynamics exhibit 'persistence' in the sense that temporary shocks have lasting effects on unemployment.

Consider now an *unbalanced budgets* policy (UB). Assuming again $w'(k)k/w(k) < 1$, the budget runs into deficit on impact, and the stock of bonds in period $t+1$ becomes positive. Savings are declining and are now partly absorbed by government bonds. Hence, the upward-sloping curve in Figure 3 shifts further up under (UB), and in period $t+1$ the capital intensity undershoots its stationary level by more than under (BB). Whether unemployment in period $t+1$ is higher or lower under (UB) is ambiguous and depends on the shape of the curves in Figure 3: on the one hand, the fixed tax rate under (UB) tends to keep unemployment low but, on the other hand, the lower capital stock depresses employment more. With a sufficiently low elasticity of substitution between labour and capital, the second effect dominates, and unemployment in period $t+1$ is higher under (UB) than under (BB). In most of our simulation studies this result has been confirmed indeed.

To discuss the long–run dynamics in more detail, suppose first that the steady state under (UB) is a sink , i.e. $R < \overline{R}$. According to Proposition 2, the economy converges monotonically back to the steady state, similar to the adjustment under (BB). The low interest rate guarantees that the deficit and government debt are reduced automatically, without any need of future tax adjustments. However, the speed of adjustment to the new steady state tends to be lower under (UB) than under (BB): If the interest factor is close to $\overline{R}$, adjustment to the steady state follows an eigenvalue close to one (this follows from the proof of Proposition 2 which shows that the larger eigenvalue equals unity at $R = \overline{R}$). In contrast, the relevant eigenvalue under (BB) is strictly smaller than one.

Figure 5 illustrates the result of a stylised simulation experiment. We assume a CES production function

$$F(K, L) = \left[\alpha(A_K K)^{(\sigma-1)/\sigma} + (1-\alpha)(A_L L)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

with elasticity of substitution $\sigma = 0.6$ and $A_K = A_L = 100$, $\alpha = 0.5$. Furthermore, the depreciation rate is $\delta = 0.6$, labour market parameters are $\beta = 0.1$, $\pi = 0.2$, $a = 173.37$, and the savings rate is $s = 0.3$ (resulting from a Cobb-Douglas intertemporal utility function). These parameters imply a steady state unemployment rate $u = 8.06\%$, tax rate $\tau = 6.17\%$ and wage rate $w(\overline{k}) = 246.37$, such that the steady state replacement ratio is 75%. The interest factor is $R = 0.589 < \overline{R} = 0.713$.

[14] In fact, higher wages and higher unemployment are counteracting effects on the income of the young generation. However, since $w(k)[1 - u_B(k)] = w(k)K/k$, the young generation's income falls with the increase of k provided that $w'(k)k/w(k) < 1$.

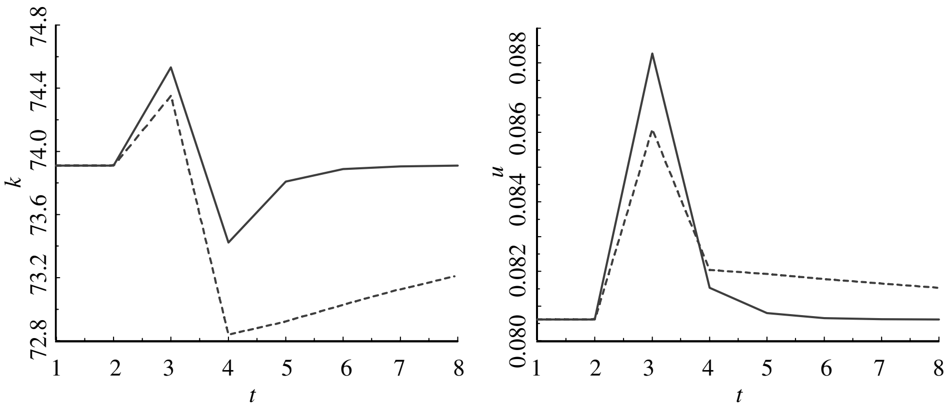

Fig. 5. *Adjustment Dynamics after Temporary Wage Pressure in the Case $R < \overline{R}$ with Balanced (solid) and Unbalanced (dashed) Budgets.*

In period 3, β falls to 0.05 and from period 4 onwards it is back at $\beta = 0.1$. The two curves show the adjustment dynamics of the capital intensity and of the unemployment rate. The solid curves are the time series under (BB) and the dashed curves, under the (UB) policy. Unemployment increases less in the period of the shock under (UB) but exceeds the level under the (BB) policy in all following periods. Moreover, under (UB) adjustment to the steady state is slower and unemployment dynamics exhibit more persistence than under (BB).

These results become even more pronounced if the interest factor exceeds the critical level $R > \overline{R}$, i.e. if the steady state is a saddle under the (UB) dynamics, in which case adjustment dynamics are unstable. Since $k_{t+1} < \overline{k}$ and $b_{t+1} > 0$, (k_{t+1}, b_{t+1}) is not on the upward-sloping saddle path, and the dynamics are diverging on a path with falling capital stock, rising interest rates and ever increasing government debt, see Figure 4(*b*). To achieve stability and to guarantee a sustainable fiscal stance in the long run, the government has to resort to a fiscal consolidation in later periods. In our model, this means that future tax increases are needed to pay back the accumulated debt of the previous periods. Such tax adjustments affect unemployment adversely during the consolidation phase. The later the fiscal consolidation is carried out, the more the tax rates need to be raised, and the stronger are the effects on future unemployment. While the exact time path of unemployment depends on the timing of consolidation efforts, the persistence of unemployment dynamics will be exacerbated, compared with a constellation of a 'low' interest rate and self-correcting deficits.

We illustrate such adjustments with another simulation study in Figure 6.[15] The solid lines show the adjustment of k, u, b and τ under (BB) and the dashed lines the (UB) dynamics, which are now unstable. Note that unemployment in period 4

[15] All model parameters are as before, apart from $\alpha = 0.6$, $\delta = 0.2$, and $a = 230.38$, leading to steady state values $u = 10.48\%$, $\tau = 8.07\%$, and $w(\bar{k}) = 334.17$, such that the steady state replacement ratio is again 75%. The interest factor is now $R = 1.032 > \bar{R} = 0.573$, and we consider the same β-shock as before.

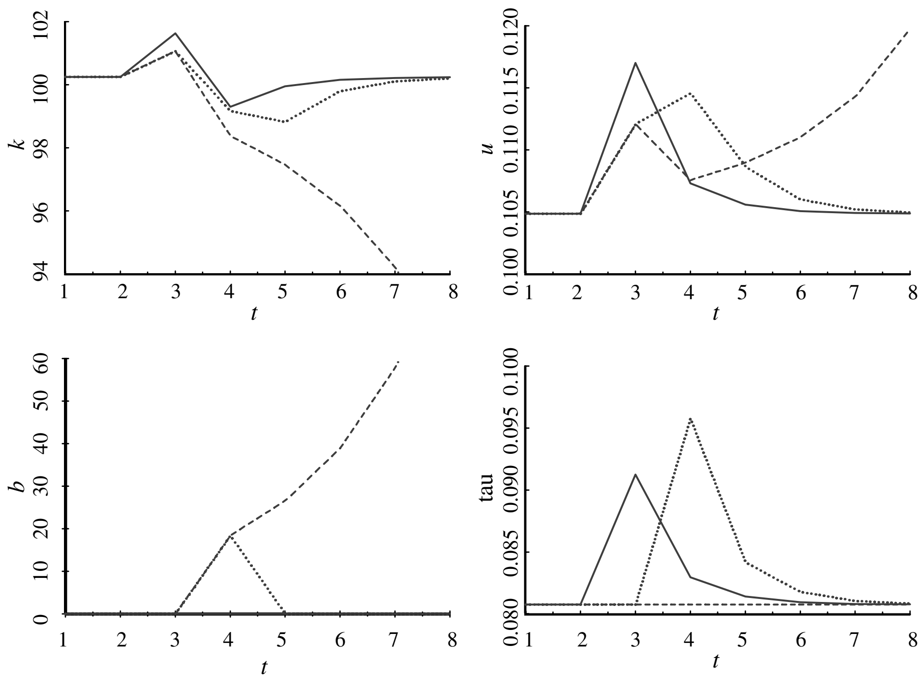

Fig. 6. *Adjustment Dynamics of k, u, b and τ after Temporary Wage Pressure in the case* $R > \overline{R}$ *with Balanced Budgets* (solid)*, Unbalanced Budgets* (dashed)*, and Initially Unbalanced Budgets in Period 3 with Complete Consolidation in Period 4* (dotted).

after the shock is just slightly higher under (UB), but increases later because of further increases in government debt and a declining capital stock. The dotted line shows the time paths induced by a policy which relies on (UB) in period 3 and switches in period 4 to a complete fiscal consolidation, i.e. the government raises the tax rate so as to pay back all debt issued in the period of the shock. After that, the government returns to a (BB) policy, i.e. it maintains a zero deficit from period 5 onwards. Note that the consolidation not only strengthens persistence of the unemployment path, but it also raises the mean rate of unemployment during the transition in comparison with the (UB) dynamics in Figure 5.

5. Discussion

Broadly speaking, we believe there is little disagreement between economists on the merits and the risks associated with a policy of unbalanced government budgets. In particular, unbalanced budgets which operate symmetrically over the business cycle tend to act as automatic stabilisers, inasmuch as demand is reduced in booms and stimulated in recessions. If used in this way, unbalanced budgets will be offsetting over the cycle and they offer a convenient 'rule' which tends to be more effective than the discretionary fine-tuning of tax and expenditure adjustments in the political decision process. Moreover, facing a lower risk premium, the government should be in a better position than private households to facilitate intertemporal consumption smoothing. Further stressing the advantages of the use of automatic stabilisers, Schmitt-Grohé and Uribe (1997) have recently shown within a neoclassical growth model that expectations of tax adjustments aimed at balancing the budget may well be self–fulfilling, leading to indeterminate rational expectations equilibria even in the absence of fundamental shocks. Using a similar argument, Rocheteau (1999) shows how balanced budgets can render the equilibrium rate of unemployment indeterminate in a matching model of the labour market.

However, as recently summarised by the OECD (1999) and investigated in this paper, relying on a policy of unbalanced budgets invariably involves the risk of accommodating (potentially long lasting) structural shocks. Thus, unbalanced budgets may well mask a deterioration of the underlying structural deficit, and required adjustments may therefore be postponed. Evidently, such a policy, though rationalisable in a political economy context, may be costly in the long run, particularly when deficits go hand in hand with adverse bond dynamics.[16]

In order to keep the comparison between the regimes of balanced and unbalanced budget policies analytically tractable, our treatment of the role of the government is deliberately simple. On the one hand, unemployment benefits are the only item on the expenditure side of the government budget. Adding an exogenous component of government consumption would not alter our main conclusions, provided that budgetary adjustments are mainly conducted on the revenue side rather than on the expenditure side. Further, we deliberately abstract from any productive (growth-enhancing) forms of government spending. On the other hand, a proportional tax on labour income is the only tax instrument in our model. Abstracting from unemployment, Uhlig and Yanagawa (1996) argue that under the special life-cycle assumptions of overlapping generations economies, revenue-neutral shifts from labour to capital taxation are likely to raise output since any such shift implies a redistribution of disposable income towards agents with a higher savings propensity. Yet, this bias of overlapping generations models towards capital taxation is less clear-cut in multiperiod settings.[17]

Closely related to our analysis, Daveri and Tabellini (2000) study the impact of proportional wage taxes on unemployment in an overlapping generations economy with collective wage bargaining, assuming balanced budgets throughout their analysis. They argue that a lasting increase of the wage tax (and thus labour costs) increases unemployment, slows down investment and reduces growth and conclude that taxation should be shifted more strongly on consumption or capital. In our analysis wage taxes are increased, if at all, temporarily to balance the budget. However, our model is perfectly compatible with the view that *permanently* higher wage taxes raise unemployment which is supported by the empirical findings of

[16] In its overview, the OECD broadly concludes that '…adverse debt dynamics have been very prominent in most OECD countries during the 1990s, especially in countries that had high debt levels from the outset…Such poor starting positions stemmed from the earlier failure to use fiscal automatic stabilizers symmetrically during previous business cycles…Most countries have succeeded in offsetting the resulting adverse debt dynamics in the 1990s by strong fiscal consolidation.' OECD (1999), p. 144.

[17] For a related discussion in a continuous-time overlapping generations economy see Bertola (1996).

Daveri and Tabellini. Our results are thus complementary to their analysis by pointing at the importance of the stance of the budgetary balance.

Finally, studying optimal tax schedules in a small open economy, Koskela and Schöb (2002) point out that the conventional wisdom to tax labour (as the less mobile factor) more strongly than capital may well be modified if reconsidered in a bargaining framework. In particular, under unemployment the supply of labour tends to be rather elastic, and according to the inverse elasticity rule this suggests that labour should not necessarily be taxed at a higher rate than capital.

Drawing on these studies, one may well argue that our treatment of a proportional wage tax as the only tax instrument makes the short-run employment effects of a balanced-budget policy particularly strong. However, a comprehensive analysis of optimal taxation in our dynamic framework, in particular with respect to the effects of capital taxation, is beyond the scope of this paper and left for future research.

6. Conclusions

We argue in this paper that the disappointing employment record of many European countries beginning in the 1970s may have been affected by significant changes in the fiscal stance. In particular, we present some evidence that a lasting weakening of the governments structural balance leads to higher unemployment persistence. To account for this evidence, we present a dynamic general equilibrium model which explores the dynamic interaction between capital formation, fiscal policies, and unemployment resulting from collective wage bargaining.

We consider the effects of two distinct budgetary rules of balanced and unbalanced budgets. Under a strict balanced-budget rule, we show that steady states and their stability features correspond naturally to the perfectly competitive benchmark case, whereas dynamics can become unstable under unbalanced budgets. We analyse how the economy reacts to adverse structural shocks (such as excessive wage setting) under either policy. Under balanced budgets, the government needs to raise taxes which leads on impact to higher unemployment. However, when the shock is over, the recovery of employment is relatively fast due to a more productive asset structure. By contrast, under unbalanced budgets the government leaves taxes unchanged and thereby mitigates adverse employment effects on impact. However, unemployment dynamics are now likely to exhibit more persistence, depending on the strength of crowding out effects. Moreover, when bond dynamics are unstable, low unemployment today will be bought at the expense of higher unemployment at some point in the future. In short, by stressing the effects on the labour market, our paper gives a rigorous account of the dangers of unbalanced budgets when used inappropriately to postpone adjustments to structural shocks.

Appendix

Solution of the Wage Bargain:

Consider the solution of the wage bargaining problem to maximise $(\Pi_t - \overline{\Pi}_t)^{\beta}(V_t - \overline{V}_t)^{1-\beta}$ subject to $w_t = w[K_t/(M\ell_t)]$. Instead of maximising over wage and employment, we use the identities $w_t = w(k_t)$ and $k_t = K_t/(M\ell_t)$ to reformulate the Nash program in terms of the sector's capital intensity. Using (1), (2), and ignoring constants, the Nash program becomes

$$\max_{k_t \geq K_t}[k_t f'(k_t)]^{\beta}[w(k_t)(1-\tau_t) - w_t^*]^{1-\beta}k_t^{-1}.$$

The Nash product is zero at k^* such that $w(k^*)(1-\tau_t) = w_t^*$ and it is zero at $k = \infty$ since $w(k)/k$ tends to zero as $k \to \infty$. Hence a maximum necessarily exists. We assume that the Nash product is strictly quasi-concave (which is the case, for example, when the production function has constant elasticity of substitution). Hence any interior (or unemployment) solution of the Nash program is characterised by the first-order condition as given in (3).

Proof of the Lemma:

Let $\underline{k}$ be such that $0 < \mu(k,0) < 1$ (and thus $0 < u(k,0) < 1$) for all $k \geq \underline{k}$ (such a $\underline{k}$ exists by assumption (A1)). Thus for each $k \geq \underline{k}$ and at a zero tax rate, the public deficit is well defined and positive: $d(k,0) > 0$. Note that $\mu_\tau > 0$. Since μ tends to infinity as τ tends to $1 - a/w(k)$, the unemployment rate equals unity (and so employment is zero) at some sufficiently high tax rate $\bar{\tau} \in (0,1 - a/w(k))$ which solves $\mu(k,\bar{\tau}) = 1$. Solving for this value yields $\bar{\tau} = 1 - a/\{w(k)[1 - \mu_1(k)]\}$, where $0 < \mu_1(k) < 1$ is defined as the first factor in the definition of μ and $w(k)[1 - \mu_1(k)] > a$ being satisfied because of $\mu(k,0) < 1$. Clearly, at $\bar{\tau}$ the deficit $d(k,\bar{\tau})$ is also positive. Using the definition of u, the deficit is positive iff

$$a\pi\mu(k,\tau) > \tau w(k)[1 - \mu(k,\tau)].$$

Using the definition of μ and after some manipulation, this becomes

$$\tau^2 w(k)[1-\mu_1(k)] + \tau\{a[1-\pi\mu_1(k)] - w(k)[1-\mu_1(k)]\} + a\pi\mu_1(k) > 0. \quad (14)$$

The LHS of (14) has a minimum

$$\tau^*(k) = \frac{w(k)[1-\mu_1(k)] - a[1-\pi\mu_1(k)]}{2w(k)[1-\mu_1(k)]},$$

with $\tau^*(k) > 0$. The deficit at $\tau^*(k)$ is negative if (14) is satisfied with '<' at $\tau^*(k)$. After some manipulations, this amounts to show that

$$4a^2\pi\mu_1(k) < \{w(k)[1-\mu_1(k)] - a[1+\pi\mu_1(k)]\}^2. \quad (15)$$

It is obvious that (15) is fulfilled if a is sufficiently small. We choose some $\bar{a}$ so that (15) is satisfied at $k = \underline{k}$ for all $a \leq \bar{a}$ (this is feasible because of $\mu_1(\underline{k}) < 1$). Since w is increasing and μ_1 is non-increasing in k, (15) is also satisfied at any $k \geq \underline{k}$. Hence, the budget is balanced by two tax rates $0 < \tau_1(k) < \tau_2(k) < \bar{\tau}(k)$ whenever $a \leq \bar{a}$ and $k \geq \underline{k}$. Moreover, since $d_k < 0$ and since $d_\tau < 0$ at $\tau = \tau_1(k)$, we have $\tau_1'(k) < 0$.

Proof of Proposition 1:

It can be easily verified that the eigenvalues of the Jacobian of the dynamic system (9) and (10) evaluated at a steady state (11) are

$$\lambda_1 = R(\bar{k}), \lambda_2 = \frac{[1-u_B(\bar{k})]\eta_w(\bar{k}) - u_B(\bar{k})\eta_{u_B}(\bar{k})}{[1-u_B(\bar{k})]\{1-\eta_s[R(\bar{k})]\eta_R(\bar{k})\} - u_B(\bar{k})\eta_{u_B}(\bar{k})}, \quad (16)$$

where $\eta_h(x) \equiv h'(x)x/h(x)$ denotes the elasticity of some function h at x. λ_2 is also the eigenvalue of the one-dimensional model with zero government bonds. Since under our assumptions $\eta_w > 0$, $\eta_s \geq 0$, $\eta_R < 0$ and $\eta_{u_B} < 0$, λ_2 is positive. Moreover, λ_2 is less than unity if and only if

$$\eta_w(\bar{k}) < 1 - \eta_s\left[R(\bar{k})\right]\eta_R(\bar{k}). \tag{17}$$

This condition is independent of labour market parameters and coincides with the usual stability condition of the competitive Diamond model. For instance, if the production function has constant elasticity of substitution $\sigma < 1$ and if the interest elasticity of savings is zero (which is the case if the intertemporal utility function is Cobb Douglas), there are typically two positive steady states with zero government bonds of which only the larger one satisfies (17) and is therefore stable. As in the competitive model, such a steady state is either a saddle or a sink.

Proof of Proposition 2:

The Jacobian matrix $\mathbf{J}$ of (9) and (10) or of (12) and (13) evaluated at a steady state $(\bar{k},\bar{b})$ with $\bar{b} = 0$ and $d = 0$ is given by:

$$\mathbf{J} = \begin{bmatrix} \frac{1}{x}\{s[w_k(1-u) - wu_k] - (1-s)d_k\} & -\dfrac{R}{x} \\ d_k & R \end{bmatrix}$$

where $x = 1 - u - ku_k - s_R R_k w(1-u) > 0$ and where all functions are evaluated at the steady state. Under (BB), we have $d_k = 0$, whereas $d_k < 0$ under (UB). Moreover, u_k is replaced by $u_{Bk} = du_B/(dk)$ in case of (BB). Calculating the determinant and trace of $\mathbf{J}$ yields

$$\begin{aligned}
\det\mathbf{J} &= \frac{sR}{x}[w_k(1-u) - wu_k + d_k], \\
\operatorname{Tr}\mathbf{J} &= R + \frac{1}{x}\{s[w_k(1-u) - wu_k + d_k] - d_k\} \\
&= R + \frac{\det\mathbf{J}}{R} - \frac{d_k}{x}.
\end{aligned} \tag{18}$$

Clearly, under (BB) $\det\mathbf{J} > 0$ and $\operatorname{Tr}\mathbf{J} > 0$ because of $d_k = 0, w_k > 0, u_{Bk} < 0, R_k < 0$. Under (UB), substituting $d_k = (a + \bar{\tau}w)u_k - \bar{\tau}w_k(1-u)$ reveals that $\det\mathbf{J} = (sR/x)[w_k(1-u)(1-\tau) - u_k w(1 - \tau - a/w)] > 0$ since $1 - \tau - a/w > 0$ from the Lemma. Moreover, $\operatorname{Tr}\mathbf{J} > 0$. The steady state is a saddle iff $\det\mathbf{J} < -1 + \operatorname{Tr}\mathbf{J}$ which means that

$$\det\mathbf{J} < -1 + R + \frac{\det\mathbf{J}}{R} - \frac{d_k}{x}.$$

Substituting $\det\mathbf{J}$ and x and rearranging yields

$$\begin{aligned}
d_k &< (R-1)[(1-u)(1 - s_R R_k w - sw_k) + u_k(sw - k) - sd_k] \\
&= (R-1)[(1-u)(1 - \eta_s\eta_R - \eta_w) - sd_k],
\end{aligned}$$

where the last line uses $s = k/w$. Under the stability assumption (17), this condition is equivalent to

$$R > 1 + \frac{d_k}{(1-u)(1 - \eta_s\eta_R - \eta_w) - sd_k} \equiv \bar{R}.$$

Since $d_k < 0$, $\bar{R} < 1$. Hence, if $R > \bar{R}$ the steady state is a saddle and adjustment dynamics are monotone. On the other hand, if $R < \bar{R}$, the steady state is a sink: $\det\mathbf{J} > -1 - \operatorname{Tr}\mathbf{J}$ is clearly satisfied since $\det\mathbf{J}$ and $\operatorname{Tr}\mathbf{J}$ are positive. Moreover, $\det\mathbf{J} > -1 + \operatorname{Tr}\mathbf{J} = -1 + R + \det\mathbf{J}/R - d_k/x$ yields

$$\frac{Rd_k}{x} > (R-1)(R - \det\mathbf{J}),$$

which implies $\det \mathbf{J} < 1$ since $d_k < 0$ and $R < \overline{R} < 1$. Furthermore, it is easy to check that adjustment dynamics to a sink is monotone, i.e. that $4 \det \mathbf{J} < (\mathrm{Tr}\, \mathbf{J})^2$ is satisfied.

It remains to be shown that the stable manifold is upward sloping in (k,b) space whenever the steady state is a saddle. Denote by α the upper left entry of the Jacobian. Then the stable manifold is upward sloping iff $\lambda_1 < \alpha$. Using the inequality $(\mathrm{Tr}\mathbf{J})^2 - 4 \det \mathbf{J} \geq (R - \det \mathbf{J}/R)^2$, we have

$$\lambda_1 = \frac{1}{2}\left[\mathrm{Tr}\,\mathbf{J} - \sqrt{(\mathrm{Tr}\,\mathbf{J})^2 - 4\det\mathbf{J}}\right] \leq (\alpha + R - R + \det\mathbf{J}/R)/2 = (\alpha + \det\mathbf{J}/R)/2.$$

Hence, $\lambda_1 < \alpha$ is satisfied if $\det \mathbf{J} < \alpha R$. But this is clearly fulfilled since $\det \mathbf{J} = \alpha R + R d_k / x < \alpha R$.

University of Vienna
Deutsche Bundesbank

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