

Three Essays on
Worker Turnover and Incentive Contracts
in Labour Market Search Equilibrium

Dissertation

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Preamble

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Chapter

Introduction

”The next stage appears to be an integration of the market frictions that characterize the DMP (Diamond-Mortensen-Pissarides) model, with efficiency wage models, which can explain wage setting within firms ...”

Olivier J. Blanchard (2008)

This dissertation thesis consists of three independent research papers and is devoted to the topic of worker turnover and incentive contracts in a labour market characterized by search frictions and matching. There exists a large body of literature dating back to the seminal studies by Diamond, Mortensen and Pissarides¹, which is analyzing effects of search frictions and productivity fluctuations on the equilibrium flows of workers between employment and unemployment. However workers and firms are typically modeled in an oversimplified way, in particular agency problems, asymmetric information between workers and firms and mutual job attachments are often ignored. Therefore, the primary aim of this dissertation is to consider the interdependence between search frictions in the labour market and agency problems created by asymmetric information within the firm. The core agency problem considered in this dissertation is hidden action on the side of the worker; this can take form of shirking or search-in-attachment, which gives rise to the use of incentive contracts by firms. Introduction of incentive contracts in the model has crucial implications for the amount of worker remuneration, the equilibrium level of unemployment and cross sectional correlation between bonus payments, wages, productivities and separation rates. Model predictions for the correlations between these variables are consistent with the empirical findings.

¹See the following studies: Diamond (1982, 1984), Mortensen (1982a, 1982b), Pissarides (1984, 1985), Mortensen and Pissarides (1994).

In the context of normative analysis, this dissertation examines the effects of incentive contracts and mutual attachment of workers and firms on the provision of unemployment insurance and social welfare in a decentralized economy. It shows that the use of incentive contracts to deal with the problem of moral hazard has primary relevance for the question of optimal unemployment insurance, while incomplete worker attachment may serve as a source of the equilibrium inefficiency.

Each of the chapters of this dissertation deals with two aspects of the employment relationship; the later can be generally summarized as consisting of hiring, motivation of the employee throughout the job as well as the final stage of the separation. The first chapter of the dissertation is mainly concerned with an issue of employee motivation by means of incentive contracts and their effect on the job separation rate in a dynamic search equilibrium framework. An alternative approach to model work intensity is presented in the second chapter, where a more general set of state-contingent employment contracts and their effect on the hiring process of firms is considered. The third chapter of the dissertation abstracts from the question of incentive contracts and has a focus on job separations and gains from worker-firm attachments in an environment with temporary productivity shocks. Thus, the exposition of the dissertation evolves logically and covers all three stages of the employment relationship, including hiring and motivation of the employee, as well as the separation process.

Chapter 1 considers job separations in a search model with risk averse workers combined with a moral hazard problem in the spirit of the original study by Shapiro and Stiglitz (1984). The starting point of this paper is to introduce a functional relationship between the unobserved work intensity of the employee and the match separation rate; this is done in order to capture the fact that worker's effort has positive impact on the expected present value of output flows. Efficiency wages result from the dynamic moral hazard problem within the job and are extended to account for the general bargaining power of the worker. In the case of ex-post heterogeneity among jobs, this paper shows that both workers and firms value productive matches and take actions to increase match stability: firms offer a share of match surplus to provide workers with correct incentives and workers take hidden actions in order to reduce the match separation rate. This mechanism creates a situation where productivity is positively correlated with wages and negatively with

separations from a cross-sectional perspective. In addition, this model shows that the described correlation structure is capable of explaining the shape of the observed earnings distribution. Neal and Rosen (2000), Bontemps, Robin, and Van den Berg (2000), Postel-Vinay and Robin (2002) as well as Mortensen (2003) report that earnings distributions are generally hump-shaped with a unique interior mode on the distribution support. The explanation provided in this paper is based on the fact that more productive jobs are more stable and therefore are more frequently observed in the equilibrium, while the opposite is true for the jobs with lower productivity. This research paper is also dealing with the question of optimal unemployment insurance. Since workers are risk averse the social planner in this economy is facing a trade-off between insurance and incentive provision. As a result the optimal unemployment insurance is partial, and the replacement ratio is an increasing function of the coefficient of risk aversion and a decreasing function of the elasticity of the separation rate.

Chapter 2 of the dissertation incorporates the classical moral hazard problem into the competitive search labour market framework. Competitive search equilibrium has been originally developed in Moen (1997); here vacant firms post wages while unemployed workers direct their search towards the better paid jobs. Consequently wage competition between firms is an inherent feature of this model. At the same time, firms choose incentive contracts in response to the problem of moral hazard. Incentive contracts considered in this paper include base wages in a combination with a profit-related bonus pay. The equilibrium remuneration package including wages and bonus payments provides workers with a hiring and a motivation premium and obtains at the intersection between the risk-sharing (RSS) and the rent-sharing (RNS) curves. This study shows that worker's effort is increasing in the profit-related bonus component and decreasing in base wages. The model is further extended to account for jobs' heterogeneity with respect to capital intensity and it is the cross-sectional complementarity of wages and bonus payments which is the main contribution of this study. Hart and Hübler (1991) find that wages in Germany are positively correlated with probability and amount of profit shares, while Cahuc and Dormont (1997) show a similar result for France. This paper proves that rent-sharing as a result of hiring competition between firms is necessary to explain the observed positive correlation between wages and bonus payments. At the same time, the classical contract theory model with an ex-post wage setting mechanism fails to explain this empirical phe-

nomenon. Wage offers of the type "take-it-or-leave-it" do not leave rents to the workers; moreover, in an economy with heterogeneous jobs firms substitute wages for bonus payments, causing negative correlation between these two variables. Overall, this research paper shows that the setup of the labour market, its institutions and internal mechanisms are crucial for the sign of the correlation between wages and bonus payments and that the partial equilibrium analysis in this case is not sufficient to explain the observed empirical facts.

The third chapter of the dissertation contains a model of job search with an attachment of labour market participants. This study relaxes the common in search literature assumption that workers and firms can not recall their previous counterparties. At the same time empirical studies show that temporary layoffs and recalls to the previous employer are an inherent feature of the European labour market. The rate of recalls to the previous employer is varying in Europe from 27 – 35% in Germany, Austria, Spain and Norway to 45 – 50% in Sweden and Denmark (see Mavromaras, Rudolph (1998), Fischer and Pichelmann (1991), Alba-Ramirez, Arranz and Munoz-Bullon (2007), Roed and Nordberg (2003), Jansson (2002), and Jensen and Svarer (2003)). Moreover, these research studies show that workers' attachment to the previous employer is incomplete, meaning that workers in the state of a temporary layoff accept jobs outside the attachment. This research paper develops a model of job search with random productivity fluctuations causing a mutually beneficial separation of workers and firms. Nevertheless, search costs incurred by firms as well as a temporary nature of productivity fluctuations mutually motivate the worker-firm attachment upon a separation. This paper suggests that the type of employment - recall or a new job respectively - is especially relevant for the wage determination mechanism and contributes to the equilibrium wage dispersion. In the described economy unemployed workers are endogenously differentiated into attached and unattached; each of the groups is characterized by a group-specific reservation wage giving rise to endogenous binary wage dispersion in the model. More specifically, attached unemployed bargain higher wages when negotiating with a new employer. These findings are confirmed on the empirical level using data from the German Social Economic Panel for the years 2003-2007. In particular, the probit regression for wage gains indicates that recalls are associated with 8% lower probability of wage improvement as opposed to the job with a new employer.

Finally, this study shows that the Hosios value of the bargaining power parameter does no longer deliver efficiency to the search equilibrium with temporary layoffs. Hosios (1990) analyzes welfare properties of a decentralized economy with search and matching where wages are determined via the concept of Nash bargaining. He proves that the equilibrium in this economy is generally constrained inefficient unless the bargaining power parameter is equal to the elasticity of the job filling rate. This study proves that the Hosios value of the bargaining power parameter is not sufficient to achieve efficiency in a job search model with incomplete worker attachment. The equilibrium inefficiency stems from the fact that workers starting a new employment impose negative externality on the previous employer, who is losing a valuable option to recall the employee. This labour market is then characterized by excessive job creation since surplus losses of previous employers are not internalized within the labour market.

Chapter

Zusammenfassung

Das vorliegende Dissertationsvorhaben besteht aus drei unabhängigen Forschungspapieren, die sich alle mit dem Thema Arbeitskräfteumlauf und Anreizverträge in einem dynamischen Search-und-Matching-Arbeitsmarktmodell beschäftigen. Das grundlegende Modell in diesem Forschungsbereich wurde in den Studien von Diamond, Mortensen und Pissarides¹ entwickelt, die den Einfluss von Suchfraktionen und Produktivitätsschwankungen auf die Mobilität von Arbeitskräften zwischen Beschäftigung und Arbeitslosigkeit untersuchen. Allerdings werden die Arbeitsverträge in diesen Studien in einer sehr vereinfachten Version dargestellt: Insbesondere die Prinzipal-Agent-Probleme sowie die asymmetrische Informationsstruktur zwischen Unternehmen und Arbeitnehmern werden nicht berücksichtigt, und auch die Arbeitgeberbindung zwischen Unternehmen und Arbeitnehmern wird oft nicht ausreichend beachtet. Aus diesem Grund widmet sich diese Dissertation der Untersuchung der Interdependenz zwischen Suchfraktionen am Arbeitsmarkt und Prinzipal-Agent-Problemen aufgrund von asymmetrischen Informationen zwischen Firmen und Arbeitnehmern. Das zentrale Prinzipal-Agent-Problem, das in dieser Dissertation behandelt wird, ist das verborgene Handeln von Arbeitnehmern, welches in Form von Drückebergerei oder der Suche nach einem anderen Arbeitsplatz vorkommt. Um diese Probleme zu beheben benutzen die Firmen Anreizverträge. Die Möglichkeit für Firmen Anreizverträge einzusetzen hat entscheidende Konsequenzen für die Löhne von Arbeitnehmern, die gleichgewichtige Arbeitslosigkeit sowie für die branchenübergreifende Korrelation zwischen den Arbeitslöhnen, Leistungsprämien, Produktivität und Entlassungsquoten. Die theoretischen Prognosen für die oben genannten Zusammenhänge stimmen mit den Ergebnissen der empirischen Studien überein.

¹Sehe die folgenden Studien: Diamond (1982, 1984), Mortensen (1982a, 1982b), Pissarides (1984, 1985), Mortensen und Pissarides (1994).

Aus der Perspektive der normativen Analyse befasst sich diese Dissertation mit den Fragen nach optimalem Arbeitslosengeld und der gesamtwirtschaftlichen Wohlfahrt im Kontext von Anreizverträgen. Die Ergebnisse der durchgeführten Untersuchung zeigen einen negativen Zusammenhang zwischen dem optimalen Niveau der Arbeitslosenversicherung und dem Umfang des Moral-Hazard-Problems in den Unternehmen. Darüber hinaus wird nachgewiesen, dass eine unvollständige Arbeitgeberbindung der Beschäftigten zu einer ineffizient niedrigeren gesamtwirtschaftlichen Wohlfahrt führen kann.

Jedes Kapitel der Dissertation beschäftigt sich mit zwei Aspekten des Beschäftigungsverhältnisses, wobei letzteres die Abfolge von Einstellung des Arbeitnehmers, die Motivation während der Beschäftigung und Entlassung des Arbeitnehmers umfasst. Das erste Kapitel des Dissertationsvorhabens befasst sich mit dem Problem der Mitarbeitermotivation anhand von Anreizverträgen, und untersucht ihren Einfluss auf die Entlassungsquoten in einem dynamischen Such-Gleichgewicht. Auf eine alternative Methode die Arbeitsintensität von Mitarbeitern zu modellieren, wird im zweiten Kapitel eingegangen. Dabei wird ein breites Set von zustandsabhängigen Verträgen berücksichtigt und ihre Effekte auf das Einstellungsverfahren in den Unternehmen analysiert. Im dritten Kapitel sehe ich vom Problem der Mitarbeitermotivation ab und gehe davon aus, dass alle Mitarbeiter die gleiche Arbeitsleistung aufweisen. Von diesen Annahmen ausgehend, werden die Auswirkungen von Arbeitsplatzverlusten, Kurzarbeit und Effizienzgewinnen durch Arbeitgeberbindung der Mitarbeiter in einem Modell mit kurzfristigen Produktivitätsschocks analysiert. Insgesamt ist die Darstellung dieser Dissertation eng mit dem Ablauf des Beschäftigungsverhältnisses verbunden, so dass die drei wichtigsten Aspekte einer Beschäftigung - die Einstellung, die Motivation und die Entlassung - in einem einheitlichen theoretischen Rahmen behandelt werden.

Das erste Kapitel des Dissertationsvorhabens befasst sich mit der Entlassung von Arbeitnehmern in einem theoretischen Suchmodell mit risikoscheuen Arbeitnehmern in der Kombination mit einem Moral-Hazard-Problem wie in der ursprünglichen Studie von Shapiro und Stiglitz (1984). Als Ausgangspunkt dieser Arbeit wird ein funktioneller Zusammenhang zwischen der Leistung eines Mitarbeiters und der Wahrscheinlichkeit seiner Entlassung postuliert. Demnach steigt der diskontierte Gegenwartswert eines Ar-

beitsplatzes mit der Arbeitsintensität des Mitarbeiters. Darüber hinaus existieren Effizienzlöhne aufgrund des dynamischen Moral-Hazard-Problems mit unbeobachteter Arbeitsanstrengung der Mitarbeiter. Das Entlohnungsschema wird zusätzlich durch einen Parameter der Verhandlungsmacht der Mitarbeiter erweitert. Im Fall von ex-post heterogenen Arbeitsplätzen wird gezeigt, dass sowohl Unternehmen als auch Arbeitnehmer produktivere Arbeitsplätze bevorzugen und die notwendigen Maßnahmen zur Sicherung dieser Stellen ergreifen: die Unternehmen sind bereit höhere Löhne zu bezahlen und die Mitarbeiter erbringen bereitwillig eine höhere Arbeitsleistung. Insgesamt beschreibt das Modell eine Situation, in der ein positiver Zusammenhang zwischen Arbeitsproduktivität und Arbeitslohn besteht, sowie ein negativer Zusammenhang zwischen Arbeitslohn und Kündigungsquote. Darüber hinaus wird im ersten Kapitel eine Erklärung zur Form der gleichgewichtigen Lohnverteilung gegeben. Es wird gezeigt, dass ein negativer Zusammenhang zwischen Löhnen und der Kündigungsquote zu einer unimodalen Lohnverteilung führen kann. Die unimodale Gestalt der Lohnverteilung wurde in den folgenden empirischen Studien festgestellt: Neal und Rosen (2000), Bontemps, Robin und Van den Berg (2000), Postel-Vinay und Robin (2002) sowie in Mortensen (2003). Die Erklärung dieses Phänomens, die in dieser Arbeit vorgeschlagen wird, basiert auf der Tatsache, dass die produktiven Arbeitsplätze als Folge der Anreizverträge von hoher Stabilität gekennzeichnet sind und daher auch öfter im Gleichgewicht zu beobachten sind, im Vergleich zu Stellen mit niedriger Produktivität.

Weiterhin wird im Kapitel 1 die Frage nach einer optimalen Arbeitslosenversicherung behandelt. Angesichts des risikoscheuen Verhaltens der Arbeitnehmer ist ein zentraler Planer mit zwei unvereinbaren Zielen konfrontiert: einerseits benötigen die risikoscheuen Arbeitnehmer eine volle Versicherung gegen die drohende Arbeitslosigkeit, andererseits vernichtet die volle Versicherung die Anreize, Leistung bei der Arbeit zu erbringen. Demzufolge ist es für den zentralen Planer optimal eine Teilversicherung anzubieten. Dabei ist die Lohnersatzrate positiv vom Koeffizient der Risikoaversion und negativ von der Elastizität der Kündigungsquote abhängig.

Im zweiten Kapitel wird das klassische Moral-Hazard-Problem in einem Such-Gleichgewicht mit Lohnwettbewerb zwischen Unternehmen betrachtet. Ein Modell zur Beschreibung des wirtschaftlichen Gleichgewichts mit Suchfraktionen und Lohnwettbewerb wurde ur-

sprünglich in der Studie von Moen (1997) entwickelt. Hierbei schreiben die Firmen offene Stellen mit Angaben zum Arbeitslohn aus, während die Arbeitssuchenden ihre Bewerbungen auf die höchstbezahlten Arbeitsplätze konzentrieren. Daher ist der Lohnwettbewerb eine wichtige Charakteristik des Gleichgewichts. Gleichzeitig benutzen die Firmen Anreizverträge, um das Moral-Hazard-Problem zu lösen. Der Anreizvertrag besteht in diesem Fall aus einem festgelegten Teil (Arbeitslohn) und einer Leistungsprämie (Bonus). Diese Struktur erlaubt es, den Zusammenhang zwischen dem Motivations- und dem Einstellungsanteil zu studieren, wobei der optimale Vertrag sich am Schnittpunkt der Risk-Sharing-Kurve (RSS) und der Rent-Sharing-Kurve (RNS) befindet. Diese Studie zeigt, dass die Arbeitsintensität der Mitarbeiter positiv von der Leistungsprämie und negativ vom Arbeitslohn abhängig ist.

Das Basismodell wird zusätzlich erweitert, um Heterogenität der Firmen bezüglich der Kapitalintensität zu berücksichtigen. Das Modell zeigt, dass Firmen mit hohem Kapitalniveau größere Verluste durch eine freie Arbeitsstelle erfahren und deshalb bereit sind, einen höheren Arbeitslohn zu bezahlen, um die offenen Stellen schneller zu besetzen. Andererseits sind kapitalintensive Firmen mit hohen Risiken konfrontiert und bereit, höhere Leistungsprämien zu zahlen, um die angestellte Mitarbeiter zu motivieren. Insgesamt ergibt sich aus dem Modell ein Gleichgewicht, in dem ein positiver Zusammenhang zwischen den Leistungssprämien und den Arbeitslöhnen besteht. Dieser Zusammenhang wurde in der Studie von Hart und Hübler (1991) für Deutschland und in der Studie von Cahuc und Dormont (1997) für Frankreich bestätigt. Gleichzeitig zeigt die Arbeit, dass die traditionelle Kontrakttheorie mit ex-post Lohnsetzung dieses empirische Phänomen nicht ausreichend erklären kann. Arbeitslöhne, die nach dem Prinzip "take-it-or-leave-it" bezahlt werden, erlauben keine Rendite für die angestellten Mitarbeiter, daher tendieren Firmen mit hohem Kapitalniveau dazu, feste Löhne gegen Leistungsgeldprämien auszutauschen. Dadurch entsteht eine negative Korrelation zwischen den beiden Variablen. Insgesamt zeigt diese Studie, dass der Aufbau eines Arbeitsmarkts, seine Institutionen und die internen Lohnsetzungsmechanismen entscheidend sind um die Zusammenhänge zwischen Löhnen und Leistungsprämien zu erläutern. Gleichzeitig macht die Arbeit deutlich, dass eine partielle Analyse des wirtschaftlichen Gleichgewichts nicht ausreichend ist, um alle empirischen Beobachtungen zu erklären.

Im dritten Kapitel des Dissertationsvorhabens wird das Problem der Kurzarbeit in einem Such-Gleichgewicht am Arbeitsmarkt behandelt. Dabei wird insbesondere davon ausgegangen, dass Firmen und ehemalige Mitarbeiter nach einer konjunkturbedingten Kündigung in Kontakt bleiben. Ergebnisse empirischer Studien zeigen, dass das Phänomen der Kurzarbeit und vorübergehender Entlassung in Europa weit verbreitet ist. Die Wiedereinstellungsrate von Arbeitgebern beträgt etwa 27 – 30% in Deutschland, Österreich, Spanien und Norwegen mit höheren Zahlen von bis zu 45–50% in Schweden und Dänemark (siehe Mavromaras und Rudolph (1998), Fischer und Pichelmann (1991), Alba-Ramirez, Arranz und Munoz-Bullon (2007), Roed und Nordberg (2003), Jansson (2002), und Jensen und Svarer (2003)). Außerdem zeigen die oben genannten Studien, dass nicht alle vorübergehend entlassenen Mitarbeiter auf eine Wiedereinstellung warten, sondern auch neuangebotene Stellen von anderen Unternehmen akzeptieren. Im Weiteren wird dies als unvollständige Arbeitgeberbindung der Arbeitnehmer bezeichnet. In dieser Studie wird ein Modell entwickelt, in dem Mitarbeiter als Folge exogener Produktivitätsschwankungen vorübergehend entlassen werden. Dabei ergeben sich aus den Suchkosten Unternehmen und dem temporären Charakter der Produktivitätsschwankungen ein Nutzen aus der Arbeitgeberbindung zwischen Unternehmen und ehemaligen Mitarbeitern während der vorübergehenden Entlassung. Darüber hinaus vergleicht diese Studie Arbeitslöhne zweier Gruppen von Mitarbeitern: Die erste Gruppe besteht aus arbeitslosen Arbeitnehmern, die mit den ehemaligen Arbeitgebern in Kontakt bleiben, während die zweite Gruppe aus nichtverbundenen Arbeitsuchenden besteht. Die Studie zeigt, dass beide Gruppen unterschiedliche Vorbehaltslöhne aufweisen, und deshalb auch unterschiedliche Löhne aushandeln, wenn sie ein Beschäftigungsverhältnis eingehen. Speziell wird bewiesen, dass die Arbeitnehmer, welche noch in Kontakt mit ihrem früheren Arbeitgeber stehen, einen höheren Lohn erzielen, wenn sie eine externe Beschäftigung annehmen. Daraus entsteht im Modell eine endogene binäre Arbeitslohnverteilung. Diese theoretischen Ergebnisse werden mit Hilfe empirischer Daten für Deutschland überprüft. Die Daten stammen aus dem German Social Economic Panel und umfassen die Zeitperiode 2003-2007. Eine Probitregression für Lohnunterschiede zeigt, dass eine Wiedereinstellung beim ehemaligen Arbeitgeber mit einem um 8% niedrigerem Arbeitslohn verbunden ist, im Vergleich zu einer externen Beschäftigung.

Schließlich zeigt diese Studie, dass das Hosioswert des Parameters der Verhandlungsmacht der Arbeitnehmer keine Garantie für die Wohlfahrteffizienz in einem dynamischen Such-Gleichgewicht mit Kurzarbeit ist. Hosios (1990) untersuchte Wohlfahrtsverluste eines dezentralisierten Gleichgewichts mit Suchfraktionen, in dem die Löhne mittels eines Nash-Verhandlungsprozesses bestimmt werden. Das resultierende Gleichgewicht ist generell ineffizient, da Arbeitslose, die eine Arbeitsstelle finden, einen negativen externen Effekt auf die verbleibenden Arbeitssuchenden ausüben. Diese Dissertation zeigt eine neue Form der Ineffizienz auf, die aus den Annahmen zur Kurzarbeit stammt. Die neue Ineffizienz des Gleichgewichts ergibt sich daraus, dass die vorübergehend entlassenen Mitarbeiter, die eine neue, externe Stelle annehmen, einen zusätzlichen negativen externen Effekt auf ihr ehemaliges Unternehmen ausüben, der aus dem Verlust der Option den Mitarbeiter zurückzurufen entspringt. Dies führt letztendlich dazu, dass der Arbeitsmarkt sich durch eine exzessive Schaffung von Arbeitsplätzen auszeichnet.

Chapter 1

Working Effort and Endogenous Job Separations in Search Equilibrium

1.1 Introduction

"Fruitful models of the employment relationship must explain the individual incidence of job attachment and job turnover ... since job turnover probabilities are not uniform across individuals and groups"

Donald O. Parsons (1986)

The objective of this study is to analyze the behavior of a model economy with search frictions, moral hazard and endogenous job separation rates. In order to achieve this goal the paper develops a model, where firms face exogenous output shocks, while workers can take hidden actions (effort) to increase stability of the output stream. Unobserved worker actions give rise to the traditional moral hazard problem, so that firms respond by paying efficiency wages. In addition, worker's control over the output stability produces endogenous job separation rates in the model. Endogenous control over the job stability is particularly relevant in a model with search frictions since job search is a time-consuming process and so the separations are costly to workers and firms. The efficiency wage determination mechanism is strongly supported by the empirical evidence. Table 1 presents statistical summary of a large European data set collected by the researchers of a Wage Dynamics Network (WDN). This data set covers more than 17000 of firms across 15 European economies, the results show that about 50% of firms prefer to dismiss workers rather than to reduce base wages in response to an output shock. At the same time one of the two major reasons for avoiding the wage reduction is to maintain high effort and working morale.

Response to demand shocks Possible strategy	Approval rate %	Reasons for avoiding base wage reductions	Approval rate %
Reduce non-labor costs	38.3	Lower work morale/less effort	86
Reduce wages	12.6	Most productive workers leave	86
Base wages	1.9	Regulations/collective bargaining	73
Flexible wage components	10.7	Difficult to attract new workers	72
Reduce amount of labour	49.4	Labor turnover costs increase	70
Permanent employees	15.7	External wages matter	68
Temporary employees	25.0	Reputation suffers	60
Hours worked per employee	8.7	Implicit contract	59

Source: Fabiani S., Galuscak K., Kwapil C., Lamo A., Room T. "Wage Rigidities and Labour Market Adjustment in Europe" (2010). Statistical data: WDN Survey

Table 1.1: Firms' adjustment strategies to demand shocks

This evidence supports a link between worker's effort and wages which is originally suggested in the study by Shapiro and Stiglitz (1984). The starting point of this paper is to introduce this link in a dynamic search and matching framework developed in Mortensen and Pissarides (1994) and Pissarides (2000). In a dynamic setting agents are forward looking and derive value from a match surplus rather than a match income flow. The difference from a static setting is that the match surplus is a function of both the net flow productivity of the match and the match separation rate, so that a lower separation rate gives rise to a higher match surplus. This paper proposes a model allowing workers to take hidden actions (effort) that have a negative impact on the match separation rate and therefore extend the expected job duration.

Model predictions can be described in the following way. First, the model incorporates the empirical evidence on efficiency wages and its implications for job stability into the search and matching labour market framework. Here firms leave positive rents to workers in order to motivate them to exert a desired level of effort and profit from an improved match stability and a higher match surplus. Workers bear the cost of effort but face a lower match separation risk. This case can be considered as a corner solution of a bargaining problem, where firms have a full bargaining power and job offers are made on the basis of "take-it-or-leave-it". The model is further generalized to characterize an equilibrium with an interior value of the bargaining power. The paper shows that wages in this case can be decomposed into the bargaining premium and the motivation premium, which would prevail in the absence of bargaining.

Second, the model is extended to the case of heterogeneous jobs. The jobs' heterogeneity is achieved ex-post on the basis of an exogenous productivity distribution. The paper shows, that firms in more productive matches offer higher wages to workers, motivate them to exert more effort and indirectly obtain lower separation rates compared to the firms with lower productivity. This mechanism creates a situation where productivity is positively correlated with wages and negatively with separations from a cross-sectional perspective. Strong empirical evidence of a negative relationship between wages and separation rates can be found in Leonard (1987), Anderson and Meyer (1994), Galizzi and Lang (1998) and Christensen et al. (2005). Capelli and Chauvin (1991) explicitly consider the effect of wages on job dismissals, their results suggest that greater wage premiums are associated with lower levels of shirking and dismissals.

Furthermore, this paper presents analysis of the interaction between the job's scarcity and its stability. In particular, it shows that the inverse relationship between the job's productivity and its separation rate is likely to produce hump-shaped equilibrium wage and productivity distributions even if the initial productivity density is downward-sloping, meaning that the more productive jobs are scarce in the economy. This offers a new explanation of an observed phenomenon of hump-shaped earnings distributions reported in Neal and Rosen (2000), Bontemps, Robin, and Van den Berg (2000), Postel-Vinay and Robin (2002) and Mortensen (2003).

This paper also considers the level of unemployment in search equilibrium with efficiency wages and shows that lower wages do not reduce the equilibrium unemployment rate. This result differs from the classical efficiency wage theory following the study by Shapiro and Stiglitz (1984). Lower wages in search equilibrium with moral hazard have two consequences: (a) firms obtain lower surplus, so the job creation is less intensive, and (b) lower effort is increasing the job separation rate, so the spells of employment are shorter. In the case of heterogeneous jobs the equilibrium unemployment rate depends on the average separation rate and the equilibrium job-finding rate. The effect of a higher reservation productivity on unemployment is traditionally positive, but its explanation is new to the literature. Here the positive effect of a lower job-finding rate is partially neutralized by a negative effect of a higher average separation rate resulting from the fact, that remaining jobs are better paid and are therefore more stable (survivorship bias).

Finally, this study investigates the question of optimal unemployment insurance (UI) in an economy with risk averse agents and moral hazard. In the absence of moral hazard Baily (1978) and later Holmlund (1998) show, that full unemployment insurance is optimal in an economy with risk averse agents. This result does not however extend to the economy with endogenous search effort among the unemployed, see Hopenhayn and Nicolini (1997) and Fredriksson and Holmlund (2001). In this paper a different aspect of the effect of UI benefits on the decisions of labour market participants is analyzed. It is the unobservable working effort of the employed that is creating a trade-off for the social planner between providing the full unemployment insurance versus the maximum effort incentives. As a result the partial unemployment insurance is optimal: it reduces expenses of the social planner for vacancies and UI benefits due to the fact that workers exert positive effort and jobs become more stable. In addition, this study shows that the optimal replacement ratio is increasing in the risk aversion of workers and is decreasing in the elasticity of the separation rate with respect to the net flow profit.

The paper is organized as follows. Section 1.2 contains an overview of the related literature and section 1.3 presents notation and the model setup. Optimal contracts and the labour market equilibrium are presented in section 1.4. Section 1.5 presents an extension of the model to account for jobs heterogeneity. Section 1.6 contains analysis of the equilibrium efficiency and the optimal unemployment insurance. Section 1.7 concludes.

1.2 Overview of the related literature

There are several major directions relating this paper to the existing literature on labour turnover. First, this paper incorporates ideas of a shirking specification of the efficiency wage theory originally developed in Shapiro and Stiglitz (1984) and explored in more details in Akerlof and Yellen (1990) and Lazear (1998). This theory hinges upon the assumption of the inability of employers to costlessly observe worker's effort. The shirking specification of the efficiency wage theory assumes a discrete choice by the worker between shirking and non-shirking strategies under the constraint that a dismissal necessarily follows if a worker is caught shirking. However, dismissals only serve as a discipline device and do not occur in the equilibrium, this is principally different in the present study where negative productivity shocks render the worker unemployed.

Another branch of this literature, namely the turnover specification of the efficiency wage theory developed in Salop (1979), assumes that the labour turnover is costly to the firm; therefore the firm may attempt to reduce separations by offering a higher wage to the worker. In the current research this idea is combined with search frictions and endogenous separation probabilities. MacLeod and Malcomson (1998) merge efficiency wages with a forward looking behavior of agents. Similarly to the current study, they use the idea of job surplus rather than a flow wage to motivate workers to perform and exert effort. The difference occurs in the treatment of separation decisions which in their study are modeled exogenously and are unrelated to worker performance.

A combination of search frictions and agency problems has been originally introduced in Moen and Rosen (2006, 2008). These authors explicitly consider the question of efficiency wages in search equilibrium and develop the setup, where both effort and the match-specific productivity (type) are private information of the worker, so that the model is characterized by a combination of moral hazard and adverse selection problems. Moen and Rosen (2006) show that more high-powered incentive contracts tend to be associated with higher equilibrium unemployment rates. Moen and Rosen (2009) combine incentive contracts and endogenous worker turnover. Their paper deals with a deferred effort compensation and on-the-job search. Allowing workers to search on-the-job creates situations, when workers quit before obtaining their performance related remuneration.

The theory of heterogeneous voluntary separations (quits) arising from search on-the-job was developed in Burdett (1978), Jovanovich (1979), Jovanovich (1984) and Burdett and Mortensen (1998) and is summarized in Mortensen (2003) and Rogerson, Shimer, and Wright (2005). The general idea of these studies is that the probability of an outside offer to exceed the worker's current wage (quit probability) is decreasing in the current wage. These models play a major role in the explanation of the hump-shaped wage and productivity density functions based on wage competition between firms and on-the-job search. The current study is complementary to this group of papers and describes an additional source of job heterogeneity resulting from internal principal-agent problems within a match and relevant for the explanation of unimodal wage and productivity distributions.

Another approach to job destruction in a search and matching general equilibrium framework was introduced in the studies by Mortensen and Pissarides (1994) and Pissarides (2000). According to this approach independent idiosyncratic productivity shocks give rise to an endogenous job destruction rate. Once the productivity falls below the reservation productivity, the firm and the worker simultaneously decide to separate. This links the reservation productivity and the job separation rate, hence making the latter endogenous. Further extensions of this approach such as Rogerson, Shimer, and Wright (2005) allow the job destruction rate to be heterogeneous across firms. It is the explanation of this result and not the result itself that is different in the current study. Mortensen and Pissarides (1994) explain job destruction on the basis of job-specific product demand fluctuations while this study attempts to extend their approach with an individual worker performance component.

1.3 Labour market modeling framework

The model is first analyzed in a homogeneous agent framework, where the focus is on individual decision making of workers and firms in the presence of asymmetric information. Further in section 1.5 the model is generalized to account for the ex-post heterogeneity of job matches. This allows to study properties of a general equilibrium in labour market characterized by search frictions and firm-specific endogenous separation rates.

In section 1.4 the labour market consists of a continuum of identical workers and firms. Each worker can be found in one of two possible states: employed and exerting nonnegative effort or unemployed and searching for a job. Similarly each firm has a job position, which can be either filled with a worker or vacant and searching for a worker. Firms and workers share a common constant discount factor r . In section 1.5 job matches are heterogeneous with respect to the productivity parameter p drawn from the productivity distribution $F(p)$. Job search is random and undirected and the productivity realization is simultaneously revealed to the worker and firm once a match has been formed. Workers reject job offers below the reservation wage, while firms reject productivity realizations below the reservation productivity.

When employed the worker chooses an optimal effort level $e \geq 0$ in response to the contract wage w . Effort is measured on a continuous scale and is not observable to the firm. In addition, workers are risk averse and have instantaneous utility functions of the form: $v(w) - C(e)$, where $v(w)$ is an increasing concave function of flow wage and $C(e)$ is an increasing and convex function of effort. Both functions are normalized to yield a zero instantaneous utility to the worker with zero wage and effort values: $v(0) = 0, C(0) = 0$. In addition, it is assumed that $C'(0) = 0$. Firms are risk-neutral.

Every employment relationship is exposed to a permanent productivity shock reducing the productivity value to zero¹. The productivity shock arrives with a Poisson arrival rate $s(e)$, which is the separation rate of a match. One of the most important features of the model is that the separation rate is modeled as a decreasing function of worker's effort, meaning that higher effort decreases the probability of a negative productivity shock, i.e. $s'(e) < 0$. Here $e = 0$ implies that the separation rate is equal to its maximum value $s(0) = \bar{s}$. Once the zero productivity value was drawn the job is destroyed and the worker becomes unemployed. One direct implication of this process is that a present discounted value of output is an increasing function of worker's effort: $\int_0^\infty p \cdot \exp(-s(e))dt$. This expression is a dynamic equivalent of a static concept of a positive relationship between expected output and workers's effort widely used in the moral hazard literature.

The concept of match separation is closely related to that of the job duration. Under the Poisson specification of separation events the expected job duration is inversely related to the separation rate of a match, i.e. $d(e) = 1/s(e)$. This offers an alternative explanation of the effect of hidden actions taken by workers: higher effort decreases the separation rate and has a positive effect on the expected job duration.

Matching between firms and unemployed workers is modeled using the matching function approach. Let u denote the unemployment rate and v - the vacancy rate (expressed as a ratio of vacant jobs to the size of the labour force). Then the number of job matches

¹Throughout the paper it is assumed, that the productivity value falls to zero upon a negative productivity shock, however it is sufficient to assume that the new productivity realization is below the worker's reservation wage.

taking place per unit time and expressed as a fraction of the labour force is given by:

$$m = m(u, v)$$

The matching function is assumed to be increasing in both arguments, concave, and homogeneous of degree 1. The homogeneity assumption is required in order to abstract from the size effects of the labour market and describe the major labour market variables in relative terms. Let θ be the labour market tightness parameter: $\theta \equiv \frac{v}{u}$ – the number of vacancies per unemployed worker. This allows to derive the job arrival rate $\lambda(\theta)$ and the vacancy filling rate $q(\theta)$ as functions of the labour market tightness parameter θ :

$$\begin{aligned}\lambda(\theta) &= \frac{m(u, v)}{u} = m(1, \theta) \\ q(\theta) &= \frac{m(u, v)}{v} = m\left(\frac{1}{\theta}, 1\right)\end{aligned}$$

Wages are determined via the generalized concept of Nash bargaining, where both workers and firms account for the expected effort response. There is no commitment, so that wages are continuously renegotiated. In the equilibrium worker rents can be decomposed into the motivation premium and the bargaining premium, where the first one implies leaving job rents to the worker in order to provide him with the correct working incentives. This reflects an essence of the efficiency wage component of the model. Once employed, the worker faces a trade-off: exerting more effort at cost $C(e)$ and decreasing the separation risk versus exerting less effort and bearing a high separation risk. Optimal effort level is obtained by equating marginal gains and marginal costs of effort in the course of the worker's surplus maximization strategy.

Employing an efficiency wage determination mechanism in addition to bargaining requires clarification of such an argument as a bonding critique. The idea of the bonding critique is that workers pay a bond or an up-front hiring fee to the firm upon taking a job which may serve as a mechanism to prevent shirking. Therefore bonds or firing fees are often viewed as a substitute for efficiency wages in the part of providing correct incentives to workers. There are several reasons why bonding is assumed to be prohibited in the model, and firms are not allowed to charge an up-front fee.

As noticed in Moen and Rosen (2006) an entrance fee would have to be paid before a worker and a firm learn their match-specific productivity. Once bond value is an interior

point in the support of the distribution of job values, a firm may adopt a strategy of leaving the most productive workers and firing the least productive workers in order to collect their bonds. This highlights an emerging moral hazard problem on the side of a firm. Therefore, allowing firms to charge an up-front fee would require extending the model to provide firms with correct incentives which is not a subject of current research. Carmichael (1990) presents a list of potential solutions, how to eliminate this moral hazard problem. The most sensible of them is to collect entrance fees into a pension fund and redistribute to the other workers if shirking occurs. Moreover, Ritter and Taylor (1994) show that bonds can be treated by workers as signals of high chances of bankruptcy. And so the safest firms will have incentives not to charge entrance fees in order to signal a high survival probability.

1.4 Moral hazard in search equilibrium

1.4.1 Workers: optimal effort choice

Let U and W denote the present discounted value of the expected income stream of respectively, an unemployed and an employed worker. When a worker accepts a job at wage w , he chooses an optimal effort level e and keeps the job until a negative productivity shock arrives and the job is destroyed. If a worker rejects the job, he receives unemployment income z and searches again next period. Bellman equations for the unemployed and employed workers are:

$$rU = v(z) + \lambda(\theta)(W - U) \quad (1.4.1)$$

$$rW = \max_{e \geq 0} \{v(w) - C(e) - s(e)(W - U)\}, \quad (1.4.2)$$

where $s(e)$ is a job separation rate. An employed worker maximizes the job surplus $(W - U)$ given a wage offer w and a value of unemployment U . The choice variable of worker's maximization problem is effort e chosen in the positive domain $[0, \infty]$ in order to balance the marginal gain of a lower separation rate $s(e)$ and the marginal cost $C(e)$. The first order condition for the worker's optimization problem takes the following form:

$$W - U = \frac{v(w) - rU - C(e)}{r + s(e)} = \left| \frac{C'(e)}{s'(e)} \right| \quad (1.4.3)$$

Equation (1.4.3) is an incentive compatibility constraint for a worker and describes the functional relationship between the optimal effort level e and the earned wage w . Optimal effort is also a function of the reservation wage of the worker denoted w_0 . Worker's

participation constraint implies $W \geq U$, so that the reservation wage w_0 can be obtained from equation $W(w_0) = U$, this means $w_0 = v^{-1}(rU)$. This is true since $e(w_0, w_0) = 0$, meaning that workers choose zero effort in response to the reservation wage w_0 . Analysis of the properties of the optimal effort function $e(w, w_0)$ gives rise to the following lemma.

Lemma 1.1: *(sufficient condition) Consider a risk averse worker with an increasing and convex effort cost function $C(e)$, such that $C(0) = 0$ and $C'(0) = 0$. Then effort $e(w, w_0)$ is an increasing function of the net flow utility $\Delta v \equiv v(w) - v(w_0)$ if $s''(e) \geq 0$. It is also true that $e(w_0, w_0) = 0$.*

Proof: Appendix 1.9.1

Let the inequality $s''(e) \geq 0$ in the following be denoted as assumption (A1).

Lemma 1.1 implies that under assumption (A1) effort $e(w, w_0)$ is an increasing function of wage w for a given reservation wage w_0 and a decreasing function of w_0 for a given wage w . This result is in accordance with the efficiency wage theory which defines efficiency wages as "high wages paid to workers to induce them to put forth more effort" (Lazear (1998, p. 70)). There is a straightforward economic explanation of this result. A higher value of wage offer w raises the present discounted value of worker's total surplus $W - U$ and therefore increases the marginal benefit of holding this position. A higher marginal benefit of the job allows the worker to increase his effort level in order to equalize the marginal cost and the marginal benefit. Put differently, a higher job surplus implies a higher value loss for the worker in case of the negative productivity shock. In this case the worker is responding by raising effort and reducing the probability of a separation. Condition (A1) also guarantees validity of the first order approach.

Further analysis of wage determination requires a statement about the curvature of the optimal effort function. For this define μ_s – absolute value of the semi-elasticity of the extended discount rate $r + s(e)$ with respect to worker's effort, formally

$$\mu_s = \left| \frac{\partial \ln(r + s(e))}{\partial e} \right| \quad (1.4.4)$$

Sufficient conditions for the concavity of the effort function are summarized in lemma 1.2.

Lemma 1.2: (*sufficient conditions*) Consider a risk-averse worker with an effort function $e(w, w_0)$ given in lemma 1.1 and an effort cost function $C(e)$ such that $C'''(e) \geq 0$ for $e \geq 0$. Then the effort function $e(w, w_0)$ is concave in the net flow utility $\Delta v \equiv v(w) - v(w_0)$ if the following conditions are satisfied:

$$\frac{\partial \tilde{\mu}_s}{\partial e} \geq 0 \quad \frac{\partial^2 \tilde{\mu}_s}{\partial e^2} \geq 0 \quad \text{where} \quad \tilde{\mu}_s = 1/\mu_s \quad (\text{A2})$$

Proof: Appendix 1.9.2

Variable $\tilde{\mu}_s$ is auxiliary, it is an inverse of the semi-elasticity variable μ_s . Assumptions (A2) require the semi-elasticity variable μ_s to be a decreasing function of effort. The semi-elasticity variable μ_s reflects the degree to which worker's actions may influence the separation rate, therefore a lower value of this variable corresponds to the situation of a lower responsiveness of the separation rate to workers' actions and forces workers to exert more effort in order to obtain the desired optimal level of job stability. This implies a positive relationship between effort e and wages w .

Example: linear job duration.

Consider the case, when job duration is a linear function of worker's effort: $d(e) = e + \delta$, where parameter δ denotes the minimum expected job duration corresponding to the case of zero effort ($d(0) = \delta$). This functional assumption gives rise to the inverse relationship between a separation rate and worker's effort taking the following form $s(e) = 1/d(e) = 1/(e + \delta)$. Here the highest separation rate \bar{s} corresponds to the case of the lowest expected job duration so that $\bar{s} = 1/d(0) = 1/\delta$. Note that $s''(e) = 2/(e + \delta)^3 > 0$ so that effort is an increasing function of wage. The inverse of the semi-elasticity variable μ_s is found as:

$$\tilde{\mu}_s = (r(e + \delta) + 1)(e + \delta) \quad (1.4.5)$$

Investigation of the properties of variable $\tilde{\mu}_s$ allows to make a reference about the curvature of the optimal effort function. As follows from expressions

$$\begin{aligned} \frac{\partial \tilde{\mu}_s}{\partial e} &= 2r(e + \delta) + 1 > 0 \\ \frac{\partial^2 \tilde{\mu}_s}{\partial e^2} &= 2r > 0 \end{aligned}$$

and lemmas 1.1-1.2, the optimal effort function is an increasing and concave function of wage. \diamond

For the subsequent analysis it is convenient to use the concept of worker rents R associated with the employment, where $R \equiv W - U$. Applying the envelope theorem to equation (1.4.3) allows to conclude that the expected worker rent R is an increasing function of the net flow utility Δv and an increasing function of wage w for a given reservation wage w_0 . There are generally three effects of w on the worker's rent. First, there is a direct positive effect on the flow utility $v(w)$. Second, there is a positive effect of w on effort. The implications of this second effect for the worker's rent are twofold: higher effort costs $C(e)$ are combined with a lower job separation rate $s(e)$. However, as effort is optimally chosen by workers, these last two effects are mutually neutralized.

1.4.2 Firms: wage determination

Let J be the present discounted value of expected profit from an occupied job and V the present-discounted value of expected profit from a vacant job. In order to maintain an open position firms incur a vacancy flow cost denoted by c . Consider Bellman equations for an open vacancy and a filled job:

$$rV = -c + q(\theta)(J - V) \quad (1.4.6)$$

$$rJ = p - w - s(e)(J - V), \quad (1.4.7)$$

where $e = e(w, w_0)$ - optimal worker effort function. Equation (1.4.7) describes a trade-off faced by a firm. For fixed values of p and w_0 a firm bargaining lower wage would enjoy a higher flow profit $p - w$ but should also expect a higher separation rate $s(w, w_0) = s(e(w, w_0))$. In contrast, a firm bargaining higher wage would bear a lower flow profit $p - w$ but should also expect a lower separation rate $s(w, w_0)$. Lower separation rate in this case implies improvement in the job stability and a longer expected job duration.

The contract wage w is determined via the concept of generalized Nash bargaining, where both bargaining parties account for the optimal effort response of the worker. Outside option of a negotiating worker is to remain unemployed and search for another job, so that the rent of such a worker is given by $R = W - U$. The rent of a firm negotiating with an unemployed worker is given by $J - V$. In addition, the free-entry condition for opening

new vacancies implies, that competition between firms drives rents from a marginal vacant job to zero: $V = 0$, so that wage w is determined in the following way:

$$\max_w \left[\frac{v(w) - v(w_0) - C(e)}{r + s(e)} \right]^\beta \left[\frac{p - w}{r + s(e)} \right]^{1-\beta} \quad \text{where } e = e(w, w_0) \quad (1.4.8)$$

Here β denotes the worker's bargaining power and the reservation wage w_0 is treated parametrically. For the interior solution of $e > 0$ the optimal wage equation is given in proposition 1.1:

Proposition 1.1: *Suppose firms and workers treat the reservation wage w_0 parametrically, then solution to the optimization problem (1.4.8) is as follows:*

(a) *For $0 < \beta < 1$ the optimal wage equation is:*

$$J = [1 - \eta_s] \frac{1 - \beta}{\beta} \frac{R}{v'_w} \quad (1.4.9)$$

where $\eta_s \equiv \partial \ln(r + s(e)) / \partial \ln(p - w)$ - elasticity of the extended discount rate $r + s(e)$ with respect to the net flow profit $p - w$.

(b) *For the case $\beta = 0$ the optimal wage equation implies $\eta_s = 1$.*

Proof: The F.O.C. of the objective function (1.4.8) with respect to w is:

$$J = \left[-\frac{\partial J / \partial w}{\partial R / \partial w} \right] \frac{1 - \beta}{\beta} R \quad \text{where}$$

$$\frac{\partial J}{\partial w} = -\frac{1 - \eta_s}{r + s(e)} \quad \text{and} \quad \frac{\partial R}{\partial w} = \frac{v'_w}{r + s(e)} \quad \diamond$$

There are a number of implications following from proposition 1.1. First, consider the interior case $0 < \beta < 1$, notice that, when workers are risk averse variable v'_w can be interpreted as a "shadow price" of an output unit for the worker. It measures the change in the worker utility value given a unit transfer of output from the firm to the worker. Therefore, equation (1.4.9) contains worker surplus value expressed in terms of the firm surplus: R/v'_w .

Second, variable η_s is an elasticity of the extended discount rate $r + s(e)$ with respect to the net flow profit $p - w$. Higher net flow profit $p - w$ implies a lower wage w , this means that workers exert less effort and the separation rate of such a match is higher. In

the equilibrium with $0 < \beta < 1$ it should be true that $\eta_s < 1$. This means that optimal wages are set above the level maximizing the firm surplus J which is obtained for $\eta_s = 1$. The situation is depicted in figure 1.1. Point A in figure 1.1 corresponds to the case of a monopsonistic labour market with $\beta = 0$, where firms maximize job surplus J with respect to wage given the optimal effort response by workers. The optimal wage in this case is denoted by $w(p, w_0)$. The bargaining power of a worker in a monopsonistic labour market is zero, and therefore, the wage takes form of a motivation premium providing incentives for the worker to exert the desired level of effort.

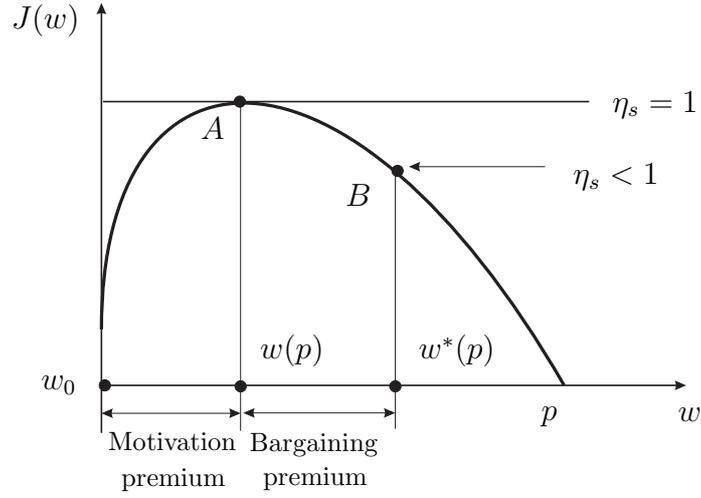


Figure 1.1: Optimal wage in search equilibrium with moral hazard

Point B in figure 1.1 corresponds to the more general case $0 < \beta < 1$, where wages are set according to (1.4.9) and $\eta_s < 1$. The optimal wage function in this case is $w^*(p, w_0)$. In the equilibrium firms pay the bargaining and the motivation premia, and therefore obtain a lower surplus value J compared to the situation with only one motivation premium in a monopsonistic labour market with search frictions. Properties of the search equilibrium with moral hazard and wage bargaining are summarized in proposition 1.2:

Proposition 1.2: *Let assumptions (A1) – (A2) be satisfied. Then search equilibrium with moral hazard and wage bargaining ($0 < \beta < 1$) is characterized by a tuple of variables $\{e, w, w_0, \theta\}$ satisfying the worker incentive compatibility constraint (1.4.3), the optimal wage equation (1.4.9), the free entry condition $V = 0$, defining variable θ , and the following reservation wage equation*

$$v(w_0) = v(z) + \lambda(\theta)R \tag{1.4.10}$$

The necessary condition for the equilibrium existence is $p \geq w_0$.

The equilibrium unemployment rate is obtained from the differential equation

$\dot{u} = s(e)(1 - u) - \lambda(\theta)u = 0$, so that

$$u = \frac{s(e)}{s(e) + \lambda(\theta)}, \quad \text{where } e = e(w, w_0) \quad (1.4.11)$$

Consider the border case $\beta = 0$ corresponding to the equilibrium with efficiency wages. Classical theory on efficiency wages (see Shapiro and Stiglitz (1984)) predicts that involuntary unemployment may appear in economies with unobservable effort, inducing firms to pay higher wages. High wages paid by firms in order to motivate their employees reduce the demand for labour and can explain the equilibrium unemployment. However, introduced in a model with search frictions, efficiency wages do not increase the number of unemployed. In contrast, paying a lower wage in the economy with search frictions and unobservable effort has two consequences: first, firms' profits fall due to a reduced output stability, so that job creation is less intensive, second, the separation rate of every match in the economy is higher. As a result, the lower job-finding rate $\lambda(\theta)$ and the higher job separation rate $s(e)$ add up to increase the equilibrium unemployment rate.

1.4.3 Comparative statics

This section considers the implications of an exogenous shift in the productivity parameter p for the optimal wage w . Results obtained in this section are consistent with the empirical findings listed below and will also prove useful for the case of heterogeneous jobs investigated in section 1.5. Consider the case $\beta = 0$, then equation $\eta_s = 1$ can be alternatively rewritten as

$$p = w + \tilde{\mu}_s/e'_w \quad (1.4.12)$$

This means that, if assumptions (A1)-(A2) are satisfied the right-hand side of this equation is an increasing function of w so that equation $\eta_s = 1$ indirectly implies a positive relationship between the wage and the productivity: $\partial w(p, w_0)/\partial p > 0$. This means that a surplus maximizing firm with a higher productivity p would offer a higher wage w to the worker and enjoy an improved output stability. In this setting the moral hazard problem forces firms to leave rents to their workers in order to induce worker's effort.

The situation is similar for the case $0 < \beta < 1$. It can be shown that the right hand side of equation (1.4.9) is an increasing function of wage, since both functions $R/v'(w)$ and $1 - \eta_s = 1 - \mu_s e'_w(p - w)$ are increasing in w if assumptions (A1)-(A2) are satisfied. The situation is depicted in figure (1.2). The left hand side of equation (1.4.9) is firm's surplus and is a decreasing function of wage in the range $w^*(p, w_0) > w(p, w_0)$. Now consider an exogenous shift in the productivity parameter from p to p' for a given value of the reservation wage w_0 . The firm surplus curve shifts outwards in the relevant range $w^*(p, w_0) > w(p, w_0)$, while the curve $R(w)(1 - \eta_s)/v'(w)$ shifts downwards, since variable η_s is increasing in p . Therefore it can be concluded, that wage is an increasing function of the productivity: $\partial w^*(p, w_0)/\partial p > 0$.

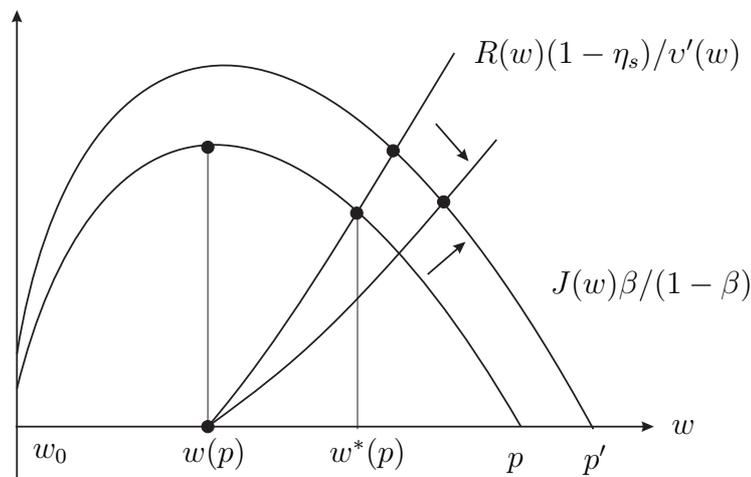


Figure 1.2: Optimal wage as a function of productivity

This result is consistent with the empirical findings. For example, Hildreth and Oswald (1997, 326) report that "the movements in the degree of firms' financial prosperity are eventually transmitted ... into movements in the pay levels of workers", which means that changes in profitability cause long-run changes in wages. Hildreth and Oswald (1997) estimated the elasticity of wages with respect to the firm's profitability to be approximately 0.02. At the same time Blanchflower, Oswald and Sanfey (1996) estimated the elasticity in the range between 0.02 to 0.05, which means that doubling profitability of a firm will result in up to a 5% increase in wages over several years.

Equation (1.4.12) also implies that the optimal wage $w(p, w_0)$ is an increasing function of its second argument (for a fixed value of p):

$$0 < \frac{\partial w(p, w_0)}{\partial w_0} < \frac{v'(w_0)}{v'(w)} \quad (1.4.13)$$

This means that a higher reservation wage w_0 forces firms to pay higher wages. Note further that if more productive firms offer higher wages to workers, meaning that workers' losses in case of a separation are higher, then (ex-ante identical) workers employed in more productive firms would exert more effort and productivity flows of those firms will be more stable on average. Formally

$$\frac{\partial e(w(p), w_0)}{\partial p} = \frac{\partial e(w(p), w_0)}{\partial w} \cdot \frac{\partial w(p)}{\partial p} > 0 \quad (1.4.14)$$

1.5 Heterogeneous productivity realizations

1.5.1 Stationary search equilibrium

Throughout this section every match of a worker and a firm is characterized by a match-specific productivity draw p from an exogenous productivity distribution $F(p)$ with the support in the range $[0, \bar{p}]$. This uncertainty about productivity is meant to reflect diversity of workers and jobs without modeling such heterogeneity explicitly. The productivity realization is simultaneously revealed to the worker and firm once the match has been formed. The matching process is random and undirected. This approach creates an ex-post productivity heterogeneity of jobs and is originally introduced in the study by Pissarides (2000). Also to simplify the representation only the case $\beta = 0$ is considered throughout this section.

In a situation when the productivity is revealed upon a match both unemployed workers and vacant jobs form expectations based on the productivity distribution $F(p)$. Bellman equations for unemployed workers and vacant jobs adjusted to account for the ex-post productivity heterogeneity can be written as:

$$rU = v(z) + \lambda(\theta) \int \max(W(p) - U, 0) dF(p) \quad (1.5.1)$$

$$rV = -c + q(\theta) \int \max(J(p) - V, 0) dF(p) \quad (1.5.2)$$

Let p_0 denote the reservation productivity, i.e. the minimum productivity level at which the firm will employ the worker. Consider a firm with a productivity draw p_0 . Offering the worker wage $w(p_0) > p_0$ will result in a negative profit flow of the firm, hence for the reservation productivity p_0 it must hold that $w(p_0) \leq p_0$ meaning that the firm surplus is nonnegative. At the same time offering the worker wage $w(p_0) < w_0$ will result in the offer rejection, hence for the reservation productivity p_0 it must also hold that $w(p_0) \geq w_0$

meaning that the worker surplus is nonnegative. In general in the equilibrium it must hold that $p_0 = w(p_0) = w_0$ guaranteeing, that at the reservation productivity participation constraints are binding for both contracting parties. Here the first part of the equality comes from the formal definition of variable p_0 , meaning that the firm surplus is zero at the reservation productivity: $J(p_0) = 0$ or $p_0 = w(p_0)$ from equation (1.4.7). The second part of the equality comes from the fact that, if a firm is offering a wage as high as the productivity draw, then wage $w(p_0)$ is the lowest wage, that unemployed workers would accept. This corresponds to the definition of the reservation wage, so that $w(p_0) = w_0$. Note also that at the wage offer $w(p_0) = w_0$ the worker would exert zero effort $e(0) = 0$ and the separation rate attains its maximum value of $s(0) = \bar{s}$.

To derive conditions characterizing an equilibrium, consider first surplus equations for a worker and a firm given, that both parties follow their optimal surplus maximizing strategies. Using the wage-setting equation (1.4.12) for the case $\beta = 0$ the firm's surplus can be rewritten as:

$$J(p, w_0) = \frac{p - w(p)}{r + s(e)} = \frac{\tilde{\mu}_s}{e'_w(r + s(e))} \quad (1.5.3)$$

where $e = e(w(p), w_0)$ and $e'_w = \partial e(w(p), w_0) / \partial w$.

Representation (1.5.3) allows to make a reference about the major properties of the firm's surplus. First of all, under assumptions (A1)-(A2) firm's surplus is an increasing function of productivity draw p meaning that firms with higher productivity draws attain higher match surplus values. This follows directly from the envelope theorem. On the one hand, a higher productivity draw implies a higher net flow profit $p - w(p)$. On the other hand, high productivity firms pay higher wages and their flow profits are more stable on average. Both effects contribute to the fact that firm surplus $J(p, w_0)$ is an increasing function of p . Additionally it can be shown that under the same set of assumptions firm's surplus $J(p, w_0)$ is an increasing function of the net flow utility Δv and therefore a decreasing function of the reservation wage w_0 since

$$\frac{\partial \Delta v}{\partial w_0} = v'(w) \frac{\partial w(p, w_0)}{\partial w_0} - v'(w_0) < 0 \quad \Rightarrow \quad \frac{\partial J(p, w_0)}{\partial w_0} < 0 \quad (1.5.4)$$

Consider an equilibrium characterized by a set of surplus equations (1.4.2), (1.4.7), (1.5.1), (1.5.2) and a free-entry condition $V = 0$. In the equilibrium it holds that $p_0 = w_0$ so that the free-entry condition can be expressed as:

$$\frac{c}{q(\theta)} = \int_{p_0} J(p, p_0) dF(p) \quad (1.5.5)$$

Equation (1.5.5) is a job creation (JC) condition and describes a decreasing relationship between the market tightness parameter θ and the reservation productivity p_0 , which means that a higher reservation productivity p_0 leads to less job creation. Intuitive explanation of this equation is, that the expected vacancy cost on the left-hand side is equated to the expected job profit on the right-hand side (if both sides are divided by $(1 - F(p_0))$). Moreover, the left-hand side of equation (1.5.5) is an increasing function of the market tightness parameter θ . This directly follows from the properties of the matching function described in section 1.3 and means that a higher value of θ makes it less probable to fill a vacancy and raises the expected vacancy cost. The right-hand side of equation (1.5.5) is a decreasing function of the reservation productivity p_0 . This follows from the fact, that a higher reservation productivity p_0 and hence a higher reservation wage w_0 , first, reduces the acceptance probability of the worker $1 - F(p_0)$ and, second, forces firms to pay higher wages. Both effects translate into a lower expected surplus of the firm.

The surplus of an employed individual can be similarly expressed as:

$$R(p, w_0) = \frac{C'(e)\tilde{\mu}_s}{r + s(e)}$$

where $e = e(w(p), w_0)$. As proved in section 1.4 surplus $R(p, w_0)$ is a decreasing function of the reservation wage w_0 . Then the equilibrium equation for the reservation productivity (RP) can be written as:

$$v(p_0) = v(z) + \lambda(\theta) \int_{p_0} R(p, p_0) dF(p) \quad (1.5.6)$$

This equation describes an increasing relationship between the market tightness parameter θ and the reservation productivity p_0 (see figure 1.3). More vacancies increase the job-finding rate $\lambda(\theta)$ and make unemployed workers more choosy: their reservation wage $w_0 = p_0$ rises. Therefore a stationary equilibrium is fully characterized by a tuple of variables (θ, p_0, w, e) where equations (1.5.5)-(1.5.6) yield unique equilibrium values of θ and p_0 and equations (1.4.12) and (1.4.3) describe the optimal values of contract wage

$w(p, p_0)$ and worker's effort $e(p, p_0)$ for every productivity draw $p \in [p_0, \bar{p}]$. The cross-sectional properties of this equilibrium are summarized in the following proposition.

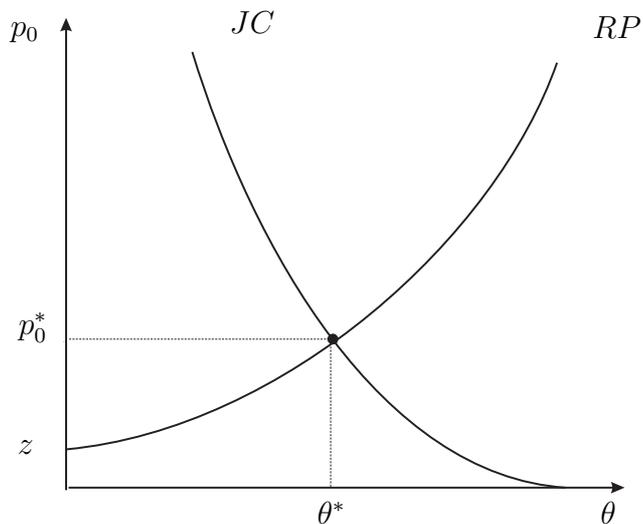


Figure 1.3: Equilibrium reservation productivity and market tightness

Proposition 1.3: *In a dynamic general equilibrium model with ex-post job heterogeneity, moral hazard and $\beta = 0$ described by a set of surplus equations (1.4.2), (1.4.7), (1.5.1), (1.5.2), a free-entry condition $V = 0$, and under a set of assumptions (A1)-(A2) there is a positive cross-sectional correlation of wages, productivity and job durations.*

Proof: A positive cross-sectional correlation of wages and productivity values is implied by equation (1.4.12). A positive correlation of wages and job durations follows from the worker incentive compatibility constraint (1.4.3). \diamond

The mutual reservation policy of workers and firms implies that the stationary productivity distribution is truncated at point p_0 and productivity draws $p < p_0$ are not observed in the equilibrium. This also has important implications for the stationary earnings distribution and the unemployment rate in the steady-state equilibrium analyzed in the following section.

1.5.2 Equilibrium earnings distribution

This section presents analysis of the equilibrium distributions of productivity and earnings in search equilibrium with heterogeneous separation rates. Let $G(p)$ denote the stationary productivity distribution, where $G(p_0) = 0$ and $G(\bar{p}) = 1$, and let $g(p)$ be the corresponding density function such that $g(p) > 0$ for $p_0 \leq p \leq \bar{p}$ and $g(p) = 0$ for $p < p_0$. Then the average job separation rate $s(p_0)$ in the general equilibrium can be written as:

$$s(p_0) = \int_{p_0}^{\bar{p}} s(p, p_0) dG(p), \quad \text{where} \quad s(p, p_0) = s(e(p, p_0)) \quad (1.5.7)$$

and $e(p, p_0) = e(w(p, p_0), p_0)$ is decreasing in p_0 .

Consider a continuum of jobs with a productivity realization p or less and denote it with $E(p)$. In the stationary equilibrium an inflow of workers into this group should be equal to the outflow of workers from this group. The inflow of workers consists of those unemployed individuals drawing the productivity value in the range $[p_0; p]$, hence the inflow of workers is equal to $u\lambda(\theta)[F(p) - F(p_0)]$. The outflow of workers from this group consists of employed individuals, who lose their jobs at rates $s(x, p_0) : x \in [p_0; p]$. Therefore, the number of jobs with a productivity realization p or less ($E(p)$) obeys the following differential equation:

$$\dot{E}(p) = u\lambda(\theta)[F(p) - F(p_0)] - (1 - u) \int_{p_0}^p s(x, p_0)g(x)dx, \quad p \in [p_0; \bar{p}] \quad (1.5.8)$$

In a stationary equilibrium $\dot{E}(p) = 0$, so that:

$$u\lambda(\theta)[F(p) - F(p_0)] = (1 - u) \int_{p_0}^p s(x, p_0)g(x)dx, \quad p \in [p_0; \bar{p}] \quad (1.5.9)$$

Setting $p = \bar{p}$ rewrite equation (1.5.9) as follows:

$$u\lambda(\theta)[1 - F(p_0)] = (1 - u)s(p_0), \quad (1.5.10)$$

which is equivalent to the differential equation $\dot{u} = 0$, so that the stationary unemployment rate u is given by:

$$u = \frac{s(p_0)}{s(p_0) + \lambda(\theta)(1 - F(p_0))} \quad (1.5.11)$$

Inserting equation (1.5.11) for the stationary unemployment rate into (1.5.9) yields the following expression:

$$s(p_0) \frac{F(p) - F(p_0)}{1 - F(p_0)} = \int_{p_0}^p s(x, p_0)g(x)dx \quad (1.5.12)$$

In order to obtain the stationary productivity density function $g(p)$ differentiate equation (1.5.12) with respect to p and use the fact that $g(p) = 0$ for $p < p_0$:

$$g(p) = \frac{s(p_0)f(p)}{s(p, p_0)[1 - F(p_0)]} \quad \text{and} \quad G(p) = \frac{s(p_0)}{[1 - F(p_0)]} \int_{p_0}^p f(x)/s(x, p_0)dx$$

for $p \in [p_0; \bar{p}]$.

There are generally two effects driving the transformation of the productivity draw distribution $F(p)$ into the stationary productivity distribution $G(p)$. See figure 1.4. Both transformations strengthen the fact, that the stationary distribution $G(w)$ dominates the initial distribution $F(w)$ ($G(w) \leq F(w)$). First of all, note that for a constant exogenous separation rate $s = s(p, p_0) = s(p_0)$ the density and the distribution functions $g(p)$ and $G(p)$ can be rewritten in the following way:

$$g(p) = \frac{f(p)}{[1 - F(p_0)]} \quad G(w) = \frac{F(p) - F(p_0)}{1 - F(p_0)} \quad (1.5.13)$$

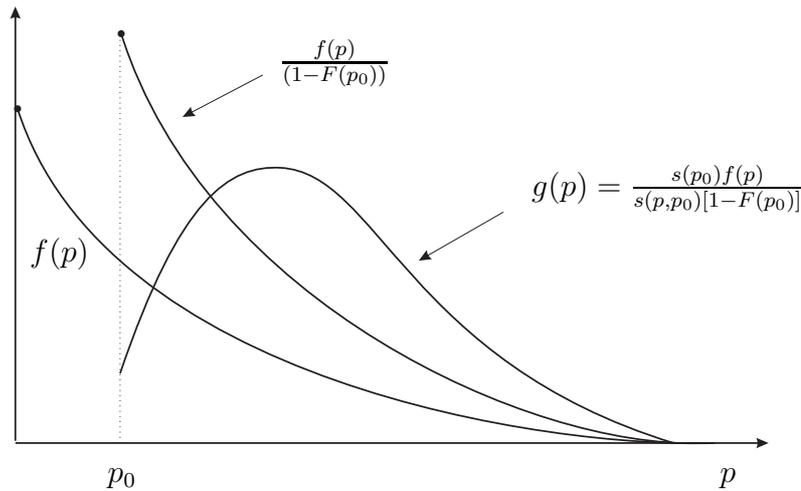


Figure 1.4: Equilibrium transformation of the productivity distribution

Hence the first transformation of $f(p)$ is explained by the reservation policy of individuals and implies that the productivity density function $g(p)$ is truncated at $p = p_0$.

The second transformation of $f(p)$ can be explained by differences in job durations $1/s(p, p_0)$ of jobs with different productivity values p . Note that the less productive jobs are less stable and are destroyed at higher intensity rates $s(p, p_0)$ than the more productive jobs. So that jobs with productivity values p such that $s(p, p_0) > s(p_0)$ are destroyed

faster than the average and jobs with productivity values p such that $s(p, p_0) < s(p_0)$ are destroyed more slowly, than the job with an average separation rate $s(p_0)$.

Now the only parameter to be defined in equations for $g(p)$ and $G(p)$ is the average separation rate in the stationary equilibrium $s(p_0)$. To obtain this parameter value recall, that $g(p)$ is a density function of the stationary productivity distribution and therefore should fulfill the following property of the density function:

$$\begin{aligned} 1 &= \int_{p_0}^{\bar{p}} g(p) dp \\ &= \frac{s(p_0)}{[1 - F(p_0)]} \int_{p_0}^{\bar{p}} f(p)/s(p, p_0) dw \end{aligned}$$

This allows to obtain expression for the average separation rate $s(p_0)$:

$$s(p_0) = \frac{[1 - F(p_0)]}{H(p_0)}, \quad (1.5.14)$$

where $H(p_0) = \int_{p_0}^{\bar{p}} f(p)/s(p, p_0) dp$ and is used to simplify the notation.

Note that because $H(p_0)$ is a strictly decreasing function of p_0 the effect of the reservation productivity p_0 on $s(p_0)$ is ambiguous. The positive part of this effect is explained by the fact, that a higher reservation productivity $p_0 = w_0$ raises the reference income point for the worker and increases thereafter the match separation rate $s(p, p_0)$. This effect translates into a lower stability of jobs and a higher separation rate for every match. The negative part of the effect corresponds to the fact that a higher p_0 reduces the number of successful matches in the economy, and therefore has a negative effect on the average separation rate.

The final expression for the stationary unemployment rate can be obtained from equation (1.5.11) by substituting the expression for the average separation rate:

$$u = \frac{s(p_0)}{s(p_0) + \lambda(\theta)(1 - F(p_0))} = \frac{1}{1 + \lambda(\theta)H(p_0)}$$

This equation is a version of the Beveridge curve describing a negative relationship between unemployment and vacancies for a given value p_0 . The structure of this equation shows, that a higher reservation productivity p_0 shifts the Beveridge curve outwards due to a lower value of $H(p_0)$. However, an increase in the reservation productivity p_0 is accompanied by a change of the market tightness θ (mutual dynamics of the two variables

is presented in figure 1.3). In general an effect of a higher p_0 on the stationary unemployment rate is ambiguous. Nevertheless, if an original shock to the economy, causing the higher reservation productivity p_0 , was such that the market tightness parameter θ decreases (it becomes relatively easier to find a job), then the labour market is characterized by an additional downward movement along the Beveridge curve which unambiguously increases the stationary unemployment rate u in the economy. This sequence of events, for example, takes place in case of a higher unemployment benefit parameter z resulting in a higher income of the unemployed, a higher reservation productivity and a higher stationary unemployment rate in the economy.

Stationary productivity distribution is an important characteristic of the model, however, one may be interested in finding an implied stationary wage (earnings) distribution, first of all for the reason, that wage is an observed variable and the model-implied theoretical distribution of wages may then be compared with its empirical counterpart.

Let $k(w)$ denote the probability density of an equilibrium wage distribution such that $k(w) > 0$ for $w \in [w_0, w(\bar{p})]$ and $k(w) = 0$ otherwise. Wages w are defined on the basis of a match-specific productivity draw p . This describes wage as a function of p : $w(p)$, which is implicitly given in equation (1.4.12) for the case $\beta = 0$. Using an expression for the probability density of a function of a random variable yields the following equation for the stationary earnings distribution $k(w)$:

$$k(w) = \frac{1}{\partial w(p)/\partial p} \cdot g\left(w + \frac{\tilde{\mu}_s}{e'_w}\right) \quad (1.5.15)$$

$$\text{where } \frac{1}{\partial w(p)/\partial p} = 1 + \tilde{\mu}'_s - \tilde{\mu}_s \frac{e''_w}{(e'_w)^2} > 1 \quad \text{for } w \in [w_0, w(\bar{p})]$$

Equation (1.5.15) shows, that the shape of the wage density function $k(w)$ is defined by the properties of the wage function $w(p)$ and the stationary productivity density function $g(p)$. As shown in section 1.5.2 the density $g(p)$ of the stationary productivity distribution is likely to have an interior mode on the support $[p_0, \bar{p}]$. In this case if wage is a concave function of productivity, so that $(\partial w/\partial p)^{-1}$ is an increasing function of wage, the wage density function $k(w)$ is likely to have a stronger right shift than the productivity density function $g(p)$.

1.6 Efficiency and unemployment insurance

1.6.1 Constrained efficiency

This section considers efficiency properties as well as the optimal unemployment insurance in a decentralized equilibrium with risk averse workers and on-the-job moral hazard. Equilibrium unemployment is an inherent component of all search models, and therefore the maximum welfare is never obtained since unemployment is a waste of labour resources. Nevertheless the welfare maximization problem of the social planner can be stated in terms of restricted efficiency, meaning that the social planner is subject to the same matching constraints as market participants.

The first question raised in this section is whether the individual decisions of market participants in a decentralized equilibrium, in particular the equilibrium wage, effort and the market tightness, maximize the social welfare. To simplify the exposition only the case of identical productivity p across jobs is considered throughout this section. The social planner is maximizing the present value of the expected utility of workers net of the effort costs. The welfare function is then given by:

$$\max_{w,\theta} \int_0^{\infty} e^{-rt} \left[uv(z) + (1-u)(v(w) - C(e)) \right] dt, \quad \text{where } e = e(w, w_0)$$

The choice of the social planner is restricted by the resource constraint, meaning that net profits obtained from production $(1-u)(p-w)$ are distributed to cover the costs of job creation $cu\theta = cv$:

$$cu\theta = (1-u)(p-w) \tag{1.6.1}$$

The unemployment rate differential equation is:

$$\dot{u} = (1-u)s(e) - u\lambda(\theta) \tag{1.6.2}$$

Note also that if workers were risk neutral the objective function of the social planner would simplify to the expected value of output net of the effort and job creation costs $uz + (1-u)(p - C(e)) - cu\theta$, which is often used in theoretical literature, see Pissarides (2000).

First order conditions of the stated optimization problem extend the result of Hosios (1990), who shows that search externalities resulting from the dependence of the transition probabilities $\lambda(\theta)$ and $q(\theta)$ on market tightness are not likely to be internalized by

the Nash surplus equation, unless a particular value of the bargaining power is assumed. A similar finding is documented in lemma 1.3 for the case of risk averse workers and on-the-job moral hazard problem:

Lemma 1.3: *Search equilibrium with risk averse workers, moral hazard and wage bargaining is constrained efficient if $\beta = \eta_q$, where*

$$\eta_q = -\frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)} \quad - \text{elasticity of the job filling rate } q(\theta) \quad (1.6.3)$$

Proof: Appendix 1.9.3.

1.6.2 Optimal unemployment insurance

This subsection considers the optimal unemployment insurance in search equilibrium with risk averse workers and moral hazard. As noted in Holmlund (1998):

*"The economics of UI has first and foremost been concerned with positive analysis of the effects of various UI policies. Much less attention has been devoted to the normative issue: what is the **optimal** level of UI benefits in an economy with risk-averse workers?"* (p.130).

Baily (1978) shows that risk aversion of workers implies optimality of the full unemployment insurance $w = z$ in the absence of informational asymmetries. To see this consider the following optimization problem of the social planner, where the unemployment insurance is now a choice variable and the moral hazard problem is omitted from the problem (so that $s(e) = s = const$):

$$\max_{w,z,\theta} \int_0^\infty e^{-rt} \left[uv(z) + (1-u)v(w) \right] dt$$

The planner's resource constraint is then modified to include the new type of expenses, namely unemployment benefits uz :

$$cu\theta + uz = (1-u)(y-w) \quad (1.6.4)$$

Solution to this optimization problem is summarized in proposition 1.4:

Proposition 1.4: *The optimal unemployment insurance policy in search equilibrium with risk averse workers implies full unemployment insurance $z = w$, so that the worker net rent R is equal to zero, the optimal wage equation is $J = K + Z$, and the optimal market tightness is given by:*

$$K = \frac{1 - \eta_q}{\eta_q} Z, \quad \text{where} \quad K \equiv \frac{c}{q(\theta)} \quad Z \equiv \frac{z}{\lambda(\theta)} \quad (1.6.5)$$

Proof: Appendix 1.9.4.

The optimal wage equation $J = K + Z$ follows directly from the planner's resource constraint and is expressed in terms of the steady state surplus values, where $K + Z$ are expected costs of maintaining one job and providing unemployment insurance to one worker. These costs are financed by firms profits with a corresponding surplus value J . The costs further are split between the firms and the workers according to the proportion $(1 - \eta_q)/\eta_q$.

Provision of full unemployment insurance is not supported by the empirical evidence, so that the basic search model has been extended in a number of relevant directions. Baily (1978) shows that unemployed workers do not have incentives to search if the full unemployment insurance is provided. This result persists even if private savings of workers are introduced into the model. The explanation for that is the fact that unemployment insurance is a sort of contingent saving, the payment obtains only if the adverse event (job loss) is realized, unlike the precautionary saving which is independent of the event occurrence. Further Shavell and Weiss (1979) in a general framework and Fredriksson and Holmlund (2001) in a search and matching framework show, that the optimal unemployment insurance should be decreasing over the unemployment spell in order to motivate unemployed workers to search. In contrast to this, Chetty (2008) shows that 60% of the increase in unemployment durations caused by UI benefits is due to a liquidity effect rather than distortions on marginal incentives to search. This is due to the fact, that increases in benefits have much larger effects on durations for liquidity-constrained households.

In this paper a different aspect of the effect of unemployment insurance on the decisions of labour market participants is analyzed. It is the on-the-job effort level workers exert, which is dependent on the unemployment insurance. To see this consider the worker's

incentive compatibility constraint (1.4.3). As shown in lemma 1.1 worker's effort is an increasing function of the net utility flow $v(w) - v(w_0)$, where w_0 is the workers reservation wage obtained as $w_0 = v^{-1}(rU)$. The reservation utility rU is an increasing function of unemployment insurance z , so that worker's effort is negatively related to z . Intuitively a lower job rent R implies a lower punishment for the worker in case of losing the job and therefore reduces worker's incentives to exert effort. The problem of the social planner in this case can be written as:

$$\max_{w,z,\theta} \int_0^\infty e^{-rt} \left[uv(z) + (1-u)(v(w) - C(e)) \right] dt, \quad \text{where } e = e(w, w_0)$$

subject to the resource constraint (1.6.4) and the differential equation for unemployment (1.6.2). Results are summarized in proposition 1.5 below.

Proposition 1.5: *The optimal unemployment insurance policy in search equilibrium with risk averse workers and unobserved effort implies partial unemployment insurance $z < w$, the optimal market tightness θ is obtained from equation $J = K + Z$ and further*

(a.) *the optimal replacement ratio z/w is implicitly given by:*

$$\frac{v'(w)}{v'(z)} = 1 - \eta_s \tag{1.6.6}$$

(b.) *the optimal surplus split is given by:*

$$Kv'(z) = \frac{1 - \eta_q}{\eta_q} (R + Zv'(z)) \tag{1.6.7}$$

Proof: Appendix 1.9.5.

Equation (1.6.6) shows, that full unemployment insurance is suboptimal if asymmetric information concerning worker's on-the-job effort is taken into account. In this setting the social planner is facing a trade-off between providing full unemployment insurance and no effort versus the absence of unemployment insurance with maximum worker's effort. As a result the partial unemployment insurance is optimal: $z < w$. This policy reduces expenses of the social planner for vacancies and unemployment benefits since workers exert positive effort and jobs become more stable. This result is supported in the theoretical literature, for example Brown, Orszag and Snower (2006) in a different framework with

taxes find that:

"Lower taxes (uncompensated costs of the employed) and lower transfers (uncompensated benefits of the unemployed) mean greater incentives for job search and work effort. The resulting rise in hiring rates and reduction in firing rates lead to a fall in unemployment. This in turn broadens the tax base and shrinks the number of people requiring support, leading to further reductions in tax rates and unemployment benefit expenditures." (p.19)

In order to obtain an approximated expression for the optimal replacement ratio I use the first order Taylor approximation of function $v'(w)$ around the point z :

$$v'(w) \simeq v'(z) + v''(z)[w - z] \quad (1.6.8)$$

so that the inverse replacement ratio w/z can be written as:

$$\frac{w}{z} \simeq 1 + \frac{\eta_s}{\rho}, \quad \text{where} \quad \rho = -\frac{v''(z)}{v'(z)}z \quad (1.6.9)$$

Here ρ is the relative risk aversion coefficient of the unemployed, so that higher risk aversion implies a higher optimal value of the replacement ratio z/w . At the same time note that the elasticity variable η_s shows the sensitivity of the separation rate with respect to the net flow profits $p - w$ and therefore also the sensitivity of the separation rate with respect to the flow wage w . If the dependence of the match separation rate on worker's effort is not recognized, then $\eta_s = 0$ and so the social planner will optimally set $z = w$, which is the case described in proposition 1.4. Otherwise a higher sensitivity of the separation rate implies a higher marginal gain of providing effort and therefore has a negative effect on the replacement ratio z/w .

1.7 Conclusions

This paper explores the question of unilateral asymmetric information and endogenous separation rates in a general equilibrium model of labour market characterized by search frictions and matching. The model proposed in the paper combines key features of the efficiency wage theory with the search and matching theory in a spirit of Mortensen and Pissarides (1994). The main (unobserved) variable in the model is worker's effort chosen in response to the contract wage. The model predicts that a higher wage yields a higher job surplus to the worker and consequently results in a higher level of worker's effort.

The key structural assumption of the model is, that the distribution of productivity shocks is linked to the worker's effort level in such a way, that higher effort raises expected duration of the productivity flow. In this situation a higher value of job surplus imposes a higher penalty for the worker in case of a separation, which necessarily follows after a negative productivity shock. Therefore, in accordance with the predictions of efficiency wage theory, workers employed at higher wages exert more effort on-the-job. In this setting, different from a shirking specification of efficiency wages, a higher effort level translates into a lower separation probability but does not prevent a separation. Additionally, this model structure guarantees a decreasing relationship between workers' performance and their dismissal probabilities, documented in the empirical literature (see Bishop (1990) and Kwon (2005)).

Wages are determined endogenously in the model using the concept of Nash bargaining generalized to include the worker's incentive compatibility constraint. Efficiency wages are then obtained as a special case for the zero bargaining power parameter. In this setup firms are facing a trade-off between the net flow profit of the job and its separation rate. Similar to the model by Mortensen and Pissarides (1994) the wage dispersion in the economy is a result of an exogenous productivity distribution which implicitly captures the firms' and workers' heterogeneity. The model predicts, that in a more productive match the firm will share the rent with the worker inducing him to exert more effort. Here the rent split resulting from bargaining is amplified by the internal agency problems within the match. This means, that the moral hazard problem and contract incompleteness force firms to share the rents even if the bargaining power of workers is equal to zero. Overall, the equilibrium is characterized by a positive correlation of wages and productivity and a negative correlation of wages and job separation rates.

Internal incentive problems between workers and firms combined with a nondegenerate productivity draw distribution create an equilibrium labour market situation with heterogeneous job separation rates. The resulting heterogeneity is such, that the more productive jobs are also more stable in expectation. The increasing job stability is interacted with an assumed declining productivity draw distribution, which serves to highlight an increasing scarcity of the more productive jobs. An interaction of job scarcity and its

stability is likely to produce the hump-shaped density functions of stationary productivity and wage distributions. This result is consistent with the reported properties of observed earnings and productivity distributions.

One of the final remarks concerns the relationship between the reservation productivity and the stationary unemployment rate. The model predicts that a higher reservation productivity (which itself may result from a higher unemployment benefit) affects unemployment in a number of ways. First, due to a lower value of the flow utility workers reduce their working effort, which results in a higher probability of negative shocks for all jobs and a higher average separation rate in the economy. Second, a higher reservation productivity in the economy translates into a lower number of successful job matches; this implies a lower job-finding rate and a lower average separation rate (survivorship bias). Nevertheless, due to a mutual neutralization of the last two effects, the model predicts, that the total effect of a higher reservation productivity on the stationary unemployment rate is unambiguously positive implying a higher stationary unemployment rate in the equilibrium.

Finally, this paper considers the question of optimal unemployment insurance in an economy with risk averse agents and on-the-job moral hazard. Partial unemployment insurance is optimal in this economy, where the social planner is facing a trade-off between incentives provision and unemployment insurance. This paper also shows, that the optimal replacement ratio is increasing in the worker's risk aversion and decreasing in the elasticity of the separation rate.

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1.9 Appendix

APPENDIX 1.9.1: Proof of lemma 1.1.

Rewrite equation (1.4.3) using a definition $\Delta v \equiv v(w) - v(w_0)$ to obtain:

$$\Delta v = C(e) - \frac{C'(e)}{s'(e)}(r + s(e)) \quad (1.9.1)$$

Differentiate equation (1.9.1) with respect to the flow utility surplus Δv to obtain:

$$\frac{1}{\partial e / \partial \Delta v} = - \left[\frac{C''(e)s'(e) - C'(e)s''(e)}{(s'(e))^2} \right] (r + s(e)) > 0 \quad \text{if } s''(e) > 0 \quad (1.9.2)$$

Therefore if $s''(e) > 0$ effort is an increasing function of Δv .

APPENDIX 1.9.2: Proof of lemma 1.2.

Using the definition of $\tilde{\mu}_s$ rewrite equation (1.9.2) in the following way:

$$\frac{1}{\partial e / \partial \Delta v} = C''(e)\tilde{\mu}_s + C'(e)[1 + \tilde{\mu}'_s] \quad (1.9.3)$$

where $\tilde{\mu}'_s = \partial \tilde{\mu}_s / \partial e$. Then from equation (1.9.3) it follows that the curvature of the optimal effort function $e(\Delta v)$, in particular the sign of the second derivative of effort

$e''(\Delta v)$ with respect to the net flow utility Δv , is defined by the sign of the following expression which is the first order derivative of the right-hand side of equation (1.9.3):

$$-[C''(e) + C'''(e)\tilde{\mu}_s + 2C''(e)\tilde{\mu}'_s + C'(e)\tilde{\mu}''_s] \quad (1.9.4)$$

Under the assumption $C'''(e) \geq 0$ expression (1.9.4) is weakly negative if $\tilde{\mu}''_s \geq 0$ and $\tilde{\mu}'_s \geq 0$. This means that conditions (A2) are sufficient for the effort function to be weakly concave in the net flow utility: $e''(\Delta v) \leq 0$.

APPENDIX 1.9.3: Proof of lemma 1.3

The current value Hamiltonian for the social planner problem is:

$$\begin{aligned} H &= uv(z) + (1-u)(v(w) - C(e)) + \gamma[u\lambda(\theta) - (1-u)s(e)] \\ &+ \alpha((1-u)(p-w) - cu\theta) \quad \text{where } e = e(w, w_0) \end{aligned}$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u . The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma \Rightarrow \alpha J + R = \gamma \quad (1.9.5)$$

since

$$R = \frac{v(w) - v(z) - C(e)}{r + s(e) + \lambda(\theta)} \quad \text{and} \quad J = \frac{p - w + c\theta}{r + s(e) + \lambda(\theta)}$$

Maximizing H with respect to w and θ yields:

$$\frac{\partial H}{\partial w} = 0 \Rightarrow v'(w) - \alpha[1 + Js'(e)e'(w)] = 0 \quad (1.9.6)$$

since from the worker incentive compatibility constraint it follows that $C'(e) = -Rs'(e)$.

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma\lambda'(\theta) = \alpha c \quad (1.9.7)$$

Expression $Js'(e)e'(w) < 0$ can be alternatively rewritten as $-\eta_s$. Then it follows from equations (1.9.5)-(1.9.7) that the optimal social planner solution is characterized by the following surplus splitting equation:

$$R = \gamma - \alpha J = \frac{\alpha J}{1 - \eta_q} - \alpha J \quad (1.9.8)$$

$$Jv'(w) = [1 - \eta_s] \frac{1 - \eta_q}{\eta_q} R \quad (1.9.9)$$

Comparing equation (1.9.9) with (1.4.9) it follows that condition $\beta = \eta_q$ guarantees efficiency of the decentralized equilibrium.

APPENDIX 1.9.4: Proof of proposition 1.4

The current value Hamiltonian for the social planner problem is:

$$\begin{aligned} H &= uv(z) + (1-u)v(w) + \gamma[u\lambda(\theta) - (1-u)s] \\ &+ \alpha((1-u)(y-w) - cu\theta - uz) \end{aligned}$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u . The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma \Rightarrow \alpha(K+Z) + R = \gamma \quad (1.9.10)$$

In the steady state the resource constraint of the social planner implies: $J = K + Z$, then equation (1.9.10) can be written as $\alpha J + R = \gamma$. Maximizing H with respect to w , z and θ yields:

$$\frac{\partial H}{\partial w} = 0 \Rightarrow v'(w) - \alpha = 0 \quad (1.9.11)$$

$$\frac{\partial H}{\partial z} = 0 \Rightarrow v'(z) - \alpha = 0 \quad (1.9.12)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma\lambda'(\theta) = \alpha c \quad (1.9.13)$$

From equations (1.9.11)-(1.9.12) it follows that $w = z$ so that $R = 0$, while the surplus splitting equation takes the following form:

$$\alpha Z = \gamma - \alpha K = \frac{\alpha K}{1 - \eta_q} - \alpha K \quad (1.9.14)$$

$$K = \frac{1 - \eta_q}{\eta_q} Z \quad (1.9.15)$$

APPENDIX 1.9.5: Proof of proposition 1.5

The current value Hamiltonian for the social planner problem is:

$$\begin{aligned} H &= uv(z) + (1-u)(v(w) - C(e)) + \gamma[u\lambda(\theta) - (1-u)s(e)] \\ &+ \alpha((1-u)(y-w) - cu\theta - uz), \quad \text{where } e = e(w, w_0) \end{aligned}$$

where α is a Lagrange multiplier and γ is a costate variable corresponding to u . The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma \Rightarrow \alpha(K+Z) + R = \gamma \quad (1.9.16)$$

In the steady state the resource constraint of the social planner implies: $J = K + Z$, then equation (1.9.16) can be written as $\alpha J + R = \gamma$. Maximizing H with respect to w , z and θ yields:

$$\frac{\partial H}{\partial w} = 0 \Rightarrow \alpha = v'(w) - e'_w(C'(e) + \gamma s'(e)) \quad (1.9.17)$$

$$\frac{\partial H}{\partial z} = 0 \Rightarrow \alpha = v'(z) \quad (1.9.18)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma \lambda'(\theta) = \alpha c \quad (1.9.19)$$

Workers incentive compatibility constraint can be written as $Rs'(e) = -C'(e)$, then equations (1.9.17)-(1.9.18) imply

$$\frac{v'(w)}{v'(z)} = 1 + Js'(e)e'(w) = 1 - \eta_s \quad (1.9.20)$$

and the surplus splitting equation (1.9.19) becomes:

$$Kv'(z) = \frac{1 - \eta_q}{\eta_q}(R + Zv'(z)) \quad (1.9.21)$$

Chapter 2

Risk Sharing and Employee Motivation in Competitive Search Equilibrium

2.1 Introduction

Labour contracts with bonus payments and profit shares are widely used to address the issues of employee motivation and asymmetric information, such as the moral hazard problem. A large branch of the literature considers these issues in the context of a single match between a firm and a worker independently of labour market conditions. The general framework for models with bonus payments in the presence of moral hazard is partial equilibrium with an ex-post wage setting mechanism, where firms make take-it-or-leave wage offers after they meet a potential employee (see section IV in Laffont and Martimort (2002) and section I in Bolton and Dewatripont (2005)).

The classical contract theory approach provides foundations for the analysis of risk-sharing and employee motivation in a match with stochastic output and unobserved worker behavior. Yet, a deeper analysis of incentive contracts under different labour market regimes is required. This necessity is also supported by a number of empirical studies providing mixed evidence on the correlation between bonus payments and wages (see table 2.1). In particular, studies by Hart and Hübler (1991) and Cahuc and Dormont (1997) find significant positive correlation between wages and bonus payments in Germany and France respectively, indicating complementarity between these two variables in Continental Europe. In contrast, a study by Wadhvani and Wall (1990) reports independence between these two types of labour compensation in the UK, while Kaufman (1998) finds evidence of a negative correlation, indicating substitution, for the set of companies in the US.

Study	Result	Details	No. of observations
Mitchell D.J.B., Lewin D., Lawler, E.E. (1990)	Complements US, 1974	Bonus payments are positively associated with total compensation and base wage	N=3428 Cross-section Individual level
Hart R.A., Hübler O. (1991)	Complements Germany, 1984	Wages are positively associated with probability and amount of profit shares	N=3628 Cross-section Individual level
Wadhvani S., Wall M. (1990)	Independent UK, 1974-1982	Profit shares are positively associated with total compensation	NT=900 Panel data Firm level
Kaufman R. (1998)	Substitutes US, 1978-1988	Bonus payments are positively associated with total compensation but negatively with base wage	NT=550 Panel data Firm level
Cahuc P., Dormont B. (1997)	Complements France, 1986-1989	Profit shares are positively associated with base wage	NT=688 Panel data Firm level

Table 2.1: Empirical research: wages and bonus payments.

In this paper, first, the classical model on performance related pay in the presence of moral hazard is extended to the case of heterogeneous jobs in a dynamic labour market equilibrium framework with search frictions. However, the core of the moral hazard model is unchanged, the model characterizes a situation, where workers possess private information about their effort choices affecting the probability distribution of output. Based on this extension, it is illustrated, that the classical contract theory model with the ex-post wage setting mechanism fails to explain the complementarity effect between wages and bonus payments observed in a number of empirical studies. Motivated by this result the current study develops a new model of moral hazard in competitive search equilibrium with risk averse workers and improves the existing approach.

The concept of competitive search has been originally introduced in a study by Moen (1997) and is based on the dynamic search and matching modeling setup of Mortensen and Pissarides (1994) and Pissarides (2000). The principal difference of competitive search is the ex-ante wage setting mechanism, where firms post wages for open vacancies and unemployed workers direct their job search towards the better offers. This mechanism provides foundations for the wage competition between employers: firms offering higher wages are more likely to fill their open vacancies as opposed to the firms with low wage offers. Wage competition between firms is an alternative to the Nash-bargaining approach,

which is traditionally used in models of job search. Empirical relevance of wage competition in addition to bargaining is provided in a study by Cahuc, Postel-Vinay and Robin (2006, 1), who write:

"We find that between-firm competition matters a lot in the determination of wages, because it is quantitatively more important than wage bargaining a la Nash in raising wages above the workers reservation wages, defined as out-of-work income." (p.323)

Competition between firms provides motives for the rent-sharing between workers and firms and explains worker rents as a hiring premium left by firms in order to attract a potential employee.

This paper considers competitive search equilibrium with incentive contracts, where jobs are characterized by stochastic output and workers' unobserved job performance, giving rise to a moral hazard problem. The combination of incentive contracts within a match and competitive search as the match formation process describe a situation, where firms pay both the motivation and the hiring premia, and the equilibrium is a counteraction of the risk-sharing and the rent-sharing employer considerations. In the equilibrium, the risk-sharing curve defines the optimal proportions of risk-sharing between the firm and the worker necessary to provide workers with the correct effort incentives. While the rent-sharing curve defines the optimal rent split between the two contracting parties necessary to achieve optimal hiring probabilities for the firm. Extended for the case of heterogeneous jobs this model defines the major contribution of this paper – to show that competitive search equilibrium with performance pay is capable of explaining complementarity between bonus payments and wages reported in a number of empirical studies. This complementarity effect is obtained due to the rent-sharing employer motive absent in the models with an ex-post wage setting.

Another contribution of this paper is comparison of the optimal labour compensation packages in terms of wages and bonus payments in labour markets with different institutional setups. This paper demonstrates that the classical model on moral hazard with risk averse workers predicts lower optimal values of wages and bonus payments than the labour market model with wage competition between employers.

Finally, this paper extends the constrained efficiency result of Hosios (1990) and Moen and Rosen (2008) for the case of risk averse employees in the presence of asymmetric information. Competitive search equilibrium with risk averse workers and unobserved worker effort is demonstrated to be constrained efficient, where the rent-sharing rule takes form of the risk adjusted Hosios condition. Here the risk adjustment is represented by a "shadow price" of a unit output. Nevertheless, the equilibrium is different from the first best social planner solution obtained in the absence of information asymmetries, indicating, that the first best solution is not incentive compatible. The principal difference of the unconstrained social planner solution is full income insurance of the employed individuals implying a zero optimal value of bonus pay and a trivial risk-sharing outcome. In addition, effort distortions are fully attributed to the risk-aversion of workers, where the downward (upward) effort distortions are reported for the low (high) shadow price of a single output unit.

The paper is organized as follows. Section 2.2 contains an overview of the related literature while section 2.3 presents notation and the model setup. Section 2.4 considers competitive search equilibrium with risk averse workers and incentive contracts, which is a baseline model of the study. Section 2.5 compares optimal labour compensation packages under different wage setting regimes. Section 2.6 presents an extension of the baseline model to account for firm heterogeneity. Section 2.7 contains analysis of the equilibrium efficiency and section 2.8 concludes the paper.

2.2 Overview of the related literature

There are a number of research directions relating this paper to the existing literature on asymmetric information in a search equilibrium framework. Guerrieri, Shimer and Wright (2010) consider the problem of adverse selection in search equilibrium with risk averse agents. Principals in their model are uninformed and compete to attract agents who are ex-ante heterogeneous and have private information about their productivity and preferences. Guerrieri, Shimer and Wright (2010) prove that an equilibrium exists where principals offer separating contracts: each contract posted attracts only one type of agent, and different types direct their search to different wages. In contrast to their study this

paper investigates the problem of moral hazard in competitive search equilibrium with heterogeneous risk neutral firms and homogenous risk averse agents and therefore further extends search literature with a focus on asymmetric information.

Risk aversion in a dynamic labour market with search frictions is further studied in a research paper by Rudanko (2009). The author develops a model of competitive search equilibrium with limited commitment contracting, where firms face aggregate and idiosyncratic productivity shocks and adjust wages, if either of the participation constraints of the two contracting parties is binding. Based on this model Rudanko (2009) shows, that both risk aversion and limited commitment increase volatility of the market tightness.¹ The limited commitment mechanism in this model is similar to the state-dependent labour contract in the current study as it provides firms with an additional labour costs flexibility in the face of idiosyncratic productivity shocks. However, the second aspect of bonus payments as a discipline device is not considered in Rudanko (2009), while it serves as a major source of effort provision for workers in the current study.

The group of research papers by Moen and Rosen (2006, 2008) explicitly considers the question of efficiency wages in a dynamic search equilibrium, where efficiency wages comprise of the base wage payment and the profit-sharing bonus pay, so that the type of incentive contract is similar to the one analyzed in the current study. Both effort and the match-specific productivity (type) are private information of the worker, so that the model is characterized by a combination of moral hazard and adverse selection problems. Yet, as noted in Laffont and Rochet (1996), with an ex-ante risk, that materializes before the effort decision was made, there is a possibility to eliminate the moral hazard variable (effort) and to reduce technically the problem to the issue of pure adverse selection. This is in contrast to the current study, where the moral hazard problem is not eliminated from the model and serves as a key motive for the risk-sharing between workers and firms. Overall, Moen and Rosen (2006) find, that higher powered incentive contracts increase equilibrium unemployment, while Moen and Rosen (2008) prove, that the unemployment rate is more volatile than in the standard model without private information.

¹The low market tightness volatility in a dynamic search and matching model has been first criticized in Shimer (2005).

Another crucial result obtained in Moen and Rosen (2008) is the modified Hosios condition for the equilibrium efficiency in a labour market with private information and search frictions. Originally the constrained efficiency result of competitive search equilibrium is due to Moen (1997), this study demonstrates the way search externality can be internalized via a wage posting mechanism, implying, that firms set wages before they meet a potential employee, and therefore, consider the effect of their wage choice on unemployed workers. Moen (1997) proves, that competitive search equilibrium with risk neutral workers fulfills the efficiency condition introduced in Hosios (1990) and maximizes the social welfare. Moen and Rosen (2008) extend this efficiency result to account for asymmetric information of market participants. They prove, that incentive power of the equilibrium wage contract is constrained efficient in the absence of taxes and unemployment benefits.

The common constrained efficiency property of competitive search is questioned in Guerrieri (2008). The author considers a dynamic version of competitive search with private information on the side of the worker and finds, that the equilibrium is different from the full information allocation and inefficient, whenever the unemployment rate is away from the steady state level. Further Guerrieri (2008) finds, that the full information allocation may be restored by lump-sum transfers from unemployed to employed individuals – the generalization of the money-burning effect. Overall, this paper highlights importance of money transfers and unemployment benefits in the context of search equilibrium efficiency. In this respect, Acemoglu and Shimer (1999) are the first authors to provide foundations for the analysis of an optimal unemployment insurance in a search equilibrium framework with risk averse workers. They show, that an economy with risk neutral workers achieves the maximum output without an unemployment insurance, while an economy with risk averse workers requires a positive level of unemployment insurance to maximize output. Their result is extended in Coles and Masters (2006) to account for strategic bargaining and employment subsidies.

The issue of income taxation in a search equilibrium framework is considered in Boone and Bovenberg (2002), where a non lump-sum income taxation is claimed to restore efficiency in a search equilibrium, when the bargaining power of workers does not fulfill the Hosios condition.

Finally, the question of optimal income taxation in a search equilibrium framework with risk averse workers and unobserved search effort is analyzed in Lehmann and Linden (2004). They show, that a non-linear income taxation in a combination with optimal unemployment insurance are sufficient to decentralize the optimal social planner solution.

2.3 Labour market modeling framework

In section 2.4 the baseline model of this study is first introduced in a competitive search equilibrium framework, where properties of the labour contract are set ex-ante, and firms compete with each other in terms of a more attractive labour compensation package. Further in section 2.5 the model is compared to a classical model of moral hazard introduced in a search equilibrium framework without competition. Comparison of the two equilibria allows to study the transformation of an optimal labour compensation package under the different wage setting regimes.

The labour market is characterized by the following properties. There is a unit mass of infinitely lived workers and an endogenous number of firms. Workers and firms are ex-ante identical in the baseline model; the question of firm heterogeneity is addressed in section 2.6. Each firm has a job position, which can be either filled with a worker or vacant and searching for a worker. Similarly, each worker can be either employed and producing output or unemployed and searching for a job. Unemployed workers receive a non-transferable flow value of leisure z .

Employed workers choose an optimal effort level $e \geq 0$. Effort is measured on a continuous scale and is not observable to the firm. All workers are risk averse and have an additively separable instantaneous utility function with respect to income and effort: $v(x) - C(e)$, where $v(x)$ is an increasing and concave function of flow income x and $C(e)$ is an increasing and convex function of effort e , such that $C'(0) = 0$. Here $C(e)$ is an effort cost function expressed in worker utility units. Firms are risk neutral.

Matching between open vacancies and unemployed workers is modeled in the following way. After incurring a vacancy creation cost c the firm is entitled to post an employment

contract. Each contract contains information on the labour compensation package associated with a specific surplus value for a worker conditional on being employed. Unemployed workers observe all posted contracts and direct their search to particular jobs. Both unemployed workers and firms correctly anticipate the number of job matches $m(u, v)$ and the market tightness $\theta = u/v$ associated with a specific surplus value. Here u is the number of unemployed individuals applying for jobs promising this surplus value, and v is the number of vacancies offering it. The matching function $m(u, v)$ is assumed to be increasing in both arguments, concave, and homogeneous of degree 1. Then the job finding rate $\lambda(\theta)$ and the vacancy filling rate $q(\theta)$ associated with a specific worker surplus are defined in the following way:

$$\begin{aligned}\lambda(\theta) &= \frac{m(u, v)}{u} = m(1, \theta) \\ q(\theta) &= \frac{m(u, v)}{v} = m(\frac{1}{\theta}, 1)\end{aligned}$$

Also denote $\eta_q = -q'(\theta)\theta/q(\theta)$ – elasticity of the job filling rate $q(\theta)$, in the following it is assumed that η_q is nondecreasing in θ .

Once employed workers choose an optimal level of effort e and start producing with an initial flow productivity $y = y^H$. Productivity y is stochastic for every employment relationship and the productivity shocks arrive with a Poisson arrival rate δ . Upon the shock, the productivity variable y can take one of the two possible realizations $\{y^H, y^L\}$, so that the following productivity switching rule applies:

$$y = \begin{cases} y^H & \text{with probability } p(e) \\ y^L & \text{with probability } 1 - p(e) \end{cases}$$

where $p(e)$ is an increasing concave function of effort. This means, that jobs, where workers exert more effort e , are characterized by longer expected durations of a high productivity realization y^H . Productivity realizations are observable and contractible, in addition firms have an option to dismiss or retain the worker upon the low productivity realization y^L . However, it is assumed that y^H is high enough to start the employment relationship, so that it is never optimal to dismiss a worker in the high productivity state. Jobs may additionally be destroyed for exogenous reasons, which happens with a Poisson arrival rate γ .

The paper explicitly addresses a variable wage contract with a state dependent worker remuneration. In the case of two output states $\{y^H, y^L\}$ the labour compensation package

takes form of (w^H, w^L) , where w^H is paid to the worker if $y = y^H$ and w^L is paid if $y = y^L$. The vector of labour compensation (w^H, w^L) can be equivalently represented as a bonus pay contract of the type (w, b) where $w = w^L$ – unconditionally paid base wage and $b = w^H - w^L$ is a conditionally paid bonus pay. As a result, the following rule applies:

$$\text{Payment} = \begin{cases} w^H = w + b & \text{if } y = y^H \\ w^L = w & \text{if } y = y^L \end{cases}$$

The bonus pay labour contract described above can be viewed as an alternative to the fixed wage labour contract investigated in a complementary study by Chizhova (2007). In a fixed wage contract regime in Chizhova (2007) firms use dismissals as a discipline device, and it is the income risk between employment and unemployment, that creates incentives for workers to exert effort.

In a variable wage contract regime explored in this study firms use both base wages and bonus payments in order to provide workers with correct incentives. Bonus payments are paid conditionally on output realizations, so that it is the income risk during employment, that motivates workers to exert effort. Moreover, the variable wage contract provides firms with an additional flexibility in their choice of the labour compensation package. In particular, firms possess a valuable option to adjust the base wage w in order to avoid unprofitable worker dismissals in the low productivity state. In the equilibrium firms pay a "hiring premium" as a result of the labour market competition forcing firms to share the rents, and a "motivation premium" in order to account for the internal moral hazard problem within the firm. The primary focus of this paper is on an interaction between these two types of wage premia. Originally, a model with the simultaneous rent-sharing and the problem of moral hazard has been investigated in Demougin and Helm (2006), and Bental and Demougin (2006).

2.4 Bonus pay in competitive search equilibrium

This section explores a model of competitive search, where workers are risk averse, while the firms are risk neutral. Firms set the terms of employment contract before they meet a potential employee. This wage setting regime creates competition between firms with respect to the value of worker remuneration and permits an analysis of the interaction

between the risk sharing and the rent sharing motives in a search equilibrium framework. The model is set in continuous time.

2.4.1 Decentralized equilibrium

Workers: optimal effort choice

Suppose first that in the low productivity state dismissals are not profitable for the firm, so that there is no dismissal threat for workers. The corresponding sufficient condition for this strategy is derived later in the paper. Denote W^L and W^H – worker surplus values in the low and high productivity states y^L and y^H respectively. Similarly let e^L and e^H denote worker effort choices. Bellman equations for employed and unemployed individuals can be written as:

$$\begin{aligned} rW^L &= \max_{e^L \geq 0} \{v(w^L) - C(e^L) + \delta p(e^L)(W^H - W^L) + \gamma(U - W^L)\} \\ rW^H &= \max_{e^H \geq 0} \{v(w^H) - C(e^H) + \delta(1 - p(e^H))(W^L - W^H) + \gamma(U - W^H)\} \\ rU &= v(z) + \lambda(\theta)(W^H - U) \end{aligned} \quad (2.4.1)$$

where notation $W^H = W^H(w^H, w^L)$ and $W^L = W^L(w^H, w^L)$ is used to simplify the representation. In each of the two productivity states workers set effort so as to balance the gain, reflected in $\delta p(e)(W^H - W^L)$, and the cost of an additional unit of effort, reflected in $C(e)$. The optimal effort choice for workers is summarized in the following lemma:

Lemma 2.1: *Optimal effort choice is constant across productivity states $e^L = e^H = e(w^L, w^H)$ and is implicitly given by the following equation:*

$$d(v(w^H) - v(w^L)) = \frac{C'(e)}{p'(e)} \equiv \pi(e) \quad (2.4.2)$$

where $d = \delta/(r + \gamma + \delta)$. *Optimal effort $e(w^L, w^H)$ is an increasing function of w^H (motivation effect) but a decreasing function of base wages w^L (discouragement effect). If $\pi''(e) > 0$ for $\forall e > 0$ then $e(w^L, w^H)$ is a concave function of w^H and a convex function of w^L . Assumption $\pi''(e) > 0$ also implies $e''_{w^H w^L} > 0$.*

Proof: Appendix 2.10.1

The above assumptions about the effort cost function $C(e)$ and the output function $p(e)$ imply that $\pi(e)$ is an increasing function of effort, so that a higher wage w^H is creating

additional incentives for workers to exert effort. A higher wage w^L produces the opposite effect: the relative income risk $w^H - w^L$ is reduced with a higher base wage. In addition if $\pi''(e) > 0$ for $\forall e > 0$ optimal effort is increasing at a declining rate in w^H and decreasing at a declining rate in w^L . In the bonus payment interpretation of the labour contract lemma 2.1 implies $e'_b > 0$ and $e'_w = e'_{w^H} + e'_{w^L} < 0$. This means that optimal effort is increasing in the bonus pay b and decreasing in the base wage w .

Denote $R^H(w^H, w^L, U)$ – worker rents from employment defined as $R^H(w^H, w^L, U) = W^H(w^H, w^L) - U$. The following lemma describes the properties of $R^H(w^H, w^L, U)$.

Lemma 2.2: *Worker rents $R^H = R^H(w^H, w^L, U)$ can be expressed as follows:*

$$(r + \gamma)R^H = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e) - rU,$$

where $e = e(w^H, w^L)$ and $\hat{p}(e) = (r + \gamma + \delta p(e))/(r + \gamma + \delta)$.

$R^H(w^H, w^L, U)$ is increasing in both arguments w^H and w^L .

Proof: Appendix 2.10.1.

Expression $\hat{p}(e)$ stands for the effective probability of the high output realization y^H . In this state workers obtain the high wage flow w^H with a corresponding utility $v(w^H)$. Similarly, $1 - \hat{p}(e)$ stands for the probability of the wage flow w^L , so that the worker rent can be expressed as a linear combination of $v(w^H)$ and $v(w^L)$ with the weights being equal to the probabilities of the respective utility flows. Also notice that from lemma 2.2 it follows, that an increase in either w^H or w^L is strictly beneficial for the worker, even if effort does not adjust. In addition, workers adjust their effort in order to maximize these gains: they increase effort in response to a higher wage value w^H and decrease effort in response to a higher wage w^L . This is in line with the result from lemma 2.1.

Firms with an open vacancy anticipate a relationship between the posted contract (w^H, w^L) and the arrival rate of workers. In order to characterize this relationship, rewrite (2.4.1) in terms of the worker job-finding rate $\lambda(\theta)$, implicitly defining the market tightness function $\theta = \theta(w^H, w^L, U)$:

$$\lambda(\theta) = \frac{(rU - v(z))}{R^H(w^H, w^L, U)} \tag{2.4.3}$$

Equation (2.4.3) describes a functional dependence between the worker rents $R^H = R^H(w^H, w^L, U)$ and the market tightness θ : an increase in either of the labour compensation components w^H and w^L attracts more job applicants and has a negative effect on the job-finding rate $\lambda(\theta)$.

Firms: optimal contract

Consider the labour demand side of the market. Denote J^H – firm surplus from a filled job position in the high output state $y = y^H$, similarly denote J^L – firm surplus from a filled job position in the low output state $y = y^L$. Both surplus values J^H and J^L are defined conditionally on retaining the worker in the low productivity state. Bellman equations for an open vacancy and a filled job can be written as follows:

$$\begin{aligned} rJ^L &= y^L - w^L + \delta p(e)(J^H - J^L) - \gamma J^L \\ rJ^H &= y^H - w^H + \delta(1 - p(e))(J^L - J^H) - \gamma J^H \\ rV &= -c + q(\theta)(J^H - V) \end{aligned} \quad (2.4.4)$$

where $J^H = J^H(w^H, w^L)$, $J^L = J^L(w^H, w^L)$ and V is the firm surplus from an open vacancy. The filled job surplus $J^H(w^H, w^L)$ can be expressed in the following way:

$$(r + \gamma)J^H(w^H, w^L) = \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L),$$

meaning that the net productivity flow $y^H - w^H$ accrues to firms with an effective probability $\hat{p}(e)$, while the net productivity flow $y^L - w^L$ accrues with an effective probability $(1 - \hat{p}(e))$.

Firms choose the vector of labour compensation (w^H, w^L) in order to maximize the vacancy surplus V given the effort response function $e(w^H, w^L)$ and the market tightness response function $\theta(w^H, w^L, U)$ (see equations (2.4.2) and (2.4.3)):

$$V(\theta(U)) \equiv \max_{w^H, w^L} V(w^H, w^L, e(w^H, w^L), \theta(w^H, w^L, U)) \quad (2.4.5)$$

$$\text{s.t. } R^H(w^H, w^L, U) \geq 0 \quad (2.4.6)$$

Condition (2.4.6) is the worker participation condition, it means that workers reject job offers with negative surplus values. Firms face the following trade-off. On the one hand, increasing w^H and w^L by one unit respectively results in lower net profit flows $(y^H - w^H)$ and $(y^L - w^L)$, but on the other hand, the job filling rate $\theta(w^H, w^L, U)$ will be higher,

while the optimal effort choice of workers $e(w^H, w^L)$ will be lower. Solution to the firm optimization problem is summarized in proposition 2.1:

Proposition 2.1: *Competitive search equilibrium with bonus payments is characterized by a tuple of variables $\{e, w^H, w^L, U, \theta\}$ satisfying conditions (2.4.2), (2.4.3), as well as equations (a) and (b) below and the free entry condition $V(\theta(U)) = 0$. The necessary condition for the equilibrium existence is:*

$$y^L - w^L + d(\Delta y - \Delta w)p(e) \geq 0 \quad (2.4.7)$$

where $e = e(w^H, w^L)$, $\Delta y = y^H - y^L$ and $\Delta w = w^H - w^L$.

(a) *The optimal value of w^H is obtained from equation:*

$$\eta_{\hat{p}} = (1 - \hat{p}(e)) \left[1 - \frac{v'(w^H)}{v'(w^L)} \right] \quad (2.4.8)$$

where $\eta_{\hat{p}} \equiv -\partial \ln \hat{p}(e) / \partial \ln(\Delta y - \Delta w)$ - negative of the elasticity of the effective probability $\hat{p}(e)$ with respect to the flow profit difference $\Delta y - \Delta w$.

(b) *The optimal value of w^L is obtained from the modified Hosios condition:*

$$J^H = [1 - \eta_{\hat{p}}] \frac{1 - \eta_q}{\eta_q} \frac{R^H}{v'(w^H)} \quad (2.4.9)$$

where $J^H = J^H(w^H, w^L)$ and $R^H = R^H(w^H, w^L, U)$.

Proof: Appendix 2.10.2

In the following subindex "C" is attached to the tuple $\{e, w^H, w^L, U, \theta\}$ corresponding to the unrestricted competitive search equilibrium with bonus payments described in proposition 2.1. Equation (2.4.7) is a necessary condition for firms to retain workers in case when output is low. This equation is obtained from the requirement $J^L > V = 0$, so that firms do not dismiss workers and continue employment relationship in the low productivity state $y = y^L$. This requirement is also sufficient to guarantee the participation of firms as $J^H \geq J^L \geq 0$.

Equation (2.4.8) can be interpreted as a risk sharing curve (RSS). Notice that for the risk neutral workers with $v(w) = w$, the right-hand side of equation (2.4.8) is zero, so that in the equilibrium $\eta_{\hat{p}} = 0$ and $\Delta w = \Delta y$ since the elasticity variable $\eta_{\hat{p}}$ becomes:

$$\eta_{\hat{p}} \equiv -\frac{\partial \ln \hat{p}(e)}{\partial \ln(\Delta y - \Delta w)} = \frac{(\Delta y - \Delta w)}{\hat{p}(e)} \hat{p}'(e) e'_{w^H}$$

Variable Δw can be interpreted as an additional bonus payment in access of the base wage w^L , so that the risk neutral case corresponds to a situation, where workers are not sensitive to risk and firms set the maximum value of the bonus payment $b = \Delta y$ in order to achieve the maximum effort. When workers are risk averse with an increasing and concave utility function, the right hand side of equation (2.4.8) is positive as $v'(w^L) > v'(w^H)$, so that $0 < \eta_{\hat{p}} < 1$ and $b = \Delta w < \Delta y$. This means that when workers are risk averse the total productivity risk reflected in Δy is split in a proportion $[\Delta w, \Delta y - \Delta w]$ between workers and firms respectively.

The risk sharing curve (RSS) is obtained from the following condition:

$$\frac{\partial J^H / \partial w^H}{\partial R^H / \partial w^H} = \frac{\partial J^H / \partial w^L}{\partial R^H / \partial w^L} \quad (2.4.10)$$

implying that in the equilibrium the firm's and the worker's indifference curves $J^H = const$ and $R^H = const$ should be tangent to each other in the space (w^H, w^L) .

Consider risk averse workers with a logarithmic utility function $v(x) = \ln(x)$, the risk sharing curve (RSS) can be rewritten as:

$$d(\Delta y - b)\hat{p}'(e) = \hat{p}(e)(1 - \hat{p}(e))b\pi'(e) \quad (2.4.11)$$

It can be shown that for sufficiently low success probability $\hat{p}(e) \leq 1/2$ the risk sharing curve (RSS) defines a positive relationship between wages and bonus payments implying complementarity between these two variables. The probability assumption $\hat{p}(e) \leq 1/2$ is sufficient but not a necessary condition here. The complementarity effect can be explained by the fact, that effort is decreasing in the base wage w so, that the optimal bonus should increase in order to mild the effort reduction. This effect is illustrated in figure 2.1. Also notice that as the base wage is increasing, the bonus payment b is approaching the maximum level of Δy .

Equation (2.4.9) can be interpreted as a rent sharing curve (RNS). It defines the share of total surplus retained by the firms J^H . The rent sharing equation is obtained from the following condition:

$$J^H = \left[-\frac{\partial J^H / \partial w^H}{\partial R^H / \partial w^H} \right] \frac{1 - \eta_q}{\eta_q} R^H \quad (2.4.12)$$

implying that in the equilibrium indifference curves $U = const$ and $V = 0$ should be tangent to each other in the space (w^L, θ) . As follows from the above equation the rent

sharing curve is defined for wage values w^H such that $\partial J^H / \partial w^H < 0$ for the set of feasible contracts. This implies that the firm surplus J^H is strictly decreasing in both arguments $w = w^L$ and $b = \Delta w$. In order to interpret the right hand side of equation (2.4.9) rewrite it using the risk sharing curve:

$$J^H = \left[\frac{\hat{p}(e)}{v'(w+b)} + \frac{1-\hat{p}(e)}{v'(w)} \right] \frac{1-\eta_q}{\eta_q} R^H \quad (2.4.13)$$

This means that in the equilibrium with bonus payments the modified Hosios condition simplifies to the risk-adjusted Hosios condition. It follows from the fact that the term in brackets on the right hand side of equation (2.4.13) can be interpreted as an inverse of the shadow price of a unit output for the worker. The price of a single output unit is state-dependent, meaning that, when productivity is high and workers obtain the income flow $w+b$ a unit transfer from firms to the workers results in a utility increase $v'(w+b)$ which is lower than $v'(w)$ – utility gain for a worker in the low productivity case. Overall the price $1/v'(w+b)$ applies with a probability $\hat{p}(e)$, while the price $1/v'(w)$ applies with an opposite probability.

Consider the case of risk averse workers with a logarithmic utility function described above, the rent sharing curve then becomes:

$$J^H = [w + \hat{p}(e)b] \frac{1-\eta_q}{\eta_q} R^H \quad (2.4.14)$$

As follows from lemma 2.2 the worker rent $R^H(w^H, w^L)$ is increasing in both arguments. In contrast, the inverse of the shadow price $w + \hat{p}(e)b$ is increasing in the bonus pay b but the effect of wage w is generally ambiguous. Nevertheless, it can be shown that for sufficiently low output risk Δy (such that $w + \hat{p}(e)\Delta y$ is an increasing function of w) the worker utility gain from a higher wage expressed in terms of the firm surplus units is increasing in w . This in turn means that the rent sharing curve is describing a substitution effect between the wage and the bonus payments. This effect is also illustrated in figure 2.1. The substitution effect can be explained by the fact, that the market tightness θ is decreasing in both arguments w and b so, that the optimal bonus should decrease in order to mild the effect of a lower θ in response to a higher value of w .

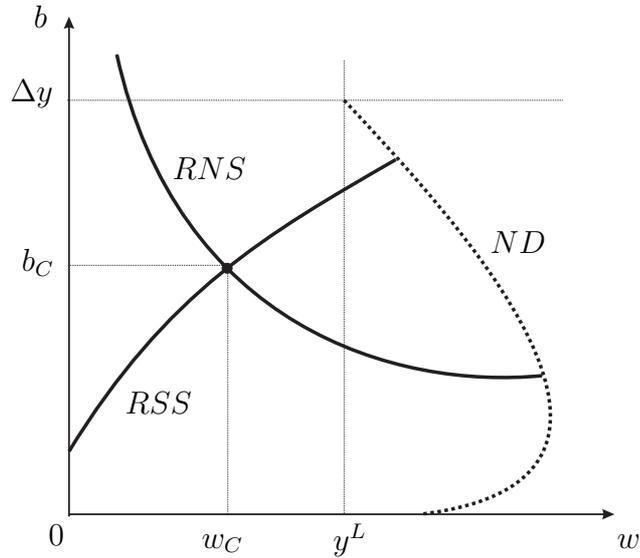


Figure 2.1: Optimal labour compensation package

The dashed curve ND on figure 2.1 stands for the no-dismissal condition and corresponds to equation $J^L = 0$. This means that the labour compensation packages $[w, b]$ outside the area given by the curve ND do not satisfy the no-dismissal condition $J^L > V = 0$. The equilibrium labour contract $[b_C, w_C]$ obtains at the intersection of the risk sharing curve (RSS) and the rent sharing curve (RNS) and implies risk sharing between a firm and a worker since $b_C < \Delta y$. This is due to the fact that if workers are risk averse firms face a trade-off between incentives provision and income insurance. As a result the optimal bonus payment is lower than in the case of risk neutral workers, since firms provide partial income insurance to workers. Also note that as follows from equations (2.4.10), (2.4.12) $\partial J^H / \partial w^H < 0$ along both curves (RSS and RNS) as well as in the equilibrium. This means that firms have incentives to reduce the amount of the bonus pay ex-post after the vacancy is filled with a worker. The same is true for the base wage w_C , so that the firm commitment to the ex-ante labour contract is a necessary condition for the equilibrium existence.

2.4.2 Limited liability constraint

Introducing a contract with state dependent wage payments $w^H = w + b$ and $w^L = w$ in competitive search equilibrium is compatible with a situation, where the base wage w is below the value of leisure z . In extreme cases when workers possess over sufficient exogenous income flows the base wage w may even take on negative values, so that it is also instructive to consider a restricted firm optimization problem with a wage restriction of

the type $w = w^L \geq \bar{w}$. The wage restriction may have different origins. One possibility to interpret a wage restriction is to view it as a limited liability constraint of the worker. In case when workers face exogenous financial constraints their job acceptance decision may additionally depend on the restriction $w \geq \bar{w}$, where the \bar{w} may stand for a continuous outflow from the worker income corresponding to his exogenous financial obligation. The limited liability constraint is particularly important in situations of moral hazard, where effort has random effect on the outcome, which may deter economic agents from entering the contract with an unlimited liability.

Another possibility to view the wage restriction may be explained by a minimum wage requirement on the government level or on the industry level resulting from a bargaining process with trade unions. Solution to the firm optimization problem with a wage restriction of the type $w^L \geq \bar{w}$ is summarized in proposition 2.2.

Proposition 2.2. *Consider a binding wage restriction of the type $w^L \geq \bar{w}$. The restricted competitive search equilibrium with bonus payments is characterized by a tuple $\{e, w^H, w^L, U, \theta\}$ satisfying $w^L = \bar{w}$, equations (2.4.2), (2.4.3), as well as the rent sharing equation (2.4.9) and the free entry condition $V(\theta(U)) = 0$. The necessary condition for the equilibrium existence is:*

$$y^L - \bar{w} + d(\Delta y - \Delta w)p(e) > 0$$

where $e = e(w^H, \bar{w})$, $\Delta y = y^H - y^L$ and $\Delta w = w^H - \bar{w}$.

Proof: Differentiate equations (2.4.3), (2.4.4) with respect to w^H and use the fact that $V = 0$ in the equilibrium, this yields:

$$\frac{\theta}{\eta_q J} \cdot \frac{\partial J}{\partial w^H} = \frac{\partial \theta}{\partial w^H} = -\frac{\theta}{(1 - \eta_q)R} \cdot \frac{\partial R}{\partial w^H} \quad (2.4.15)$$

Equation (2.4.15) is equivalent to the rent sharing equation (2.4.13). ◇

In the following subindex "CR" is attached to the tuple $\{e, w^H, \bar{w}, U, \theta\}$ characterizing competitive search equilibrium with a binding limited liability constraint $w \geq \bar{w}$.

The restricted competitive search equilibrium is obtained at the intersection of the rent sharing curve and the wage restriction $w = \bar{w}$. Consider the case of risk neutral workers, so that the optimal initial contract is given by $[b_C = \Delta y, w_C]$ and the corresponding surplus of the unemployed workers is U_C (see figure 2.2). In the short term perspective, corresponding to the partial equilibrium effect, with a fixed surplus value U_C restricting the base wage $w = \bar{w}$ implies a lower value of the bonus pay b so that $b(U_C, \bar{w}) < b(U_C, w_C) = \Delta y$.

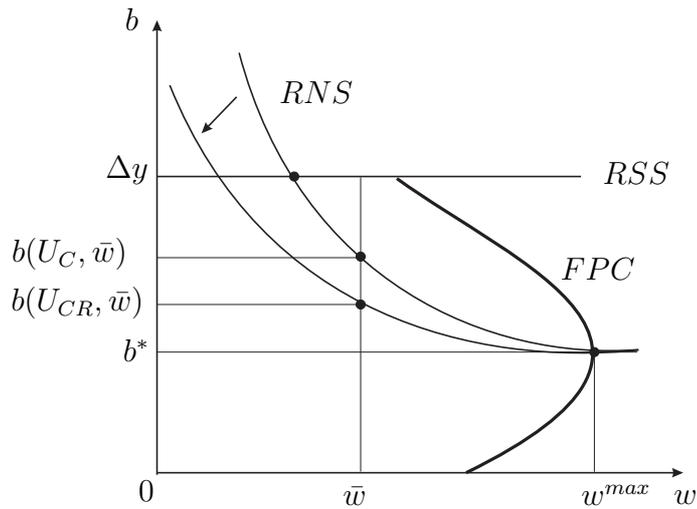


Figure 2.2: Limited liability constraint in CSE: risk neutral workers

In the long term perspective, corresponding to the general equilibrium effect, surplus of the unemployed workers U is decreasing, this is due to the fact, that firms earn lower profits and hence less jobs are created. A lower value of U implies a downward rotation of the rent sharing curve, resulting in a further reduction of the incentive pay: $b(U_{CR}, \bar{w}) < b(U_C, w_C)$. Furthermore, a lower bonus value results in a lower value of the equilibrium effort level, so that the total match surplus is lower and workers face lower motivation incentives. Unemployed workers loose from a binding wage restriction so that $U_C > U_{CR}$.

Note also that a continuum of the rent sharing curves, corresponding to different values of U , intersect at the contract $[b^*, w^{max}]$. This labour contract is obtained from a system of equations:

$$J^H(w, b) = 0, \quad \frac{\partial J^H(w, b)}{\partial b} = 0$$

Here the first curve $J^H(w, b) = 0$ corresponds to the binding firm participation constraint denoted FPC , while the second curve $\partial J^H / \partial b = 0$ is a restricted risk sharing condition and is investigated in more details in section 2.5.2. It can be shown that risk neutrality implies a constant optimal value $b^*(w) = b^* \forall w$ as illustrated in figure 2.2. This result implies that the optimal bonus value b_{CR} is bounded in the following way: $\Delta y = b_C \geq b_{CR} \geq b^*$ meaning that a binding wage restriction induces risk sharing between workers and firms even if workers are risk neutral.

2.5 Search equilibrium with ex-post wage setting

2.5.1 Decentralized equilibrium: comparison

In order to illustrate the effect of the ex-ante wage setting mechanism on the equilibrium labour contract and to be able to decompose wages into the motivation and the hiring premia, consider a labour market with an ex-post wage setting regime. The ex-post wage setting regime arises in labour markets, where job advertisements are not informative about the size of the labour compensation. In the presence of labour market uncertainty unemployed workers can not direct their search towards the better paid jobs, while firms don't need to account for the effect of wages on the number of job applications and market tightness θ . The optimal strategy of the firm is then to maximize the job surplus J^H with respect to w^H and w^L subject to the worker incentive compatibility constraint (2.4.2) and the worker participation constraint $R(w^H, w^L, U) \geq 0$. The firm optimization problem in the ex-post wage setting regime can be stated as follows:

$$\begin{aligned} (r + \gamma)J^H &= \max_{w^H, w^L} \{ \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \} \\ \text{s.t.} & \quad R^H(w^H, w^L, U) \geq 0 \quad \text{and} \quad e = e(w^H, w^L). \end{aligned} \quad (2.5.1)$$

In the ex-post wage setting regime there are no incentives for firms to leave rents to the workers, as any wage offer delivering a non-negative rent to the worker will be accepted. This means that in the equilibrium, it should be true that $R^H(w^H, w^L, U) = 0$. Zero worker rents in the equilibrium imply a monopsonistic type of the market, where firms obtain the full match surplus, and therefore the equilibrium with an ex-post wage setting resembles properties of the Diamond (1971) equilibrium. The paradox of the Diamond equilibrium is that the monopsony outcome obtains as long as the search costs of workers are positive. Competitive equilibrium outcome in the Diamond model does not arise

even, when the search costs of workers are arbitrarily small and the number of firms is sufficiently large.

In addition notice, that the search equilibrium with an ex-post wage setting is a direct extension of a classical contract theory solution for the optimal bonus pay in the presence of moral hazard (as a reference see Laffont and Martimort (2002) and Bolton and Dewatripont (2005)). The extension involves introducing the classical contract theory model with bonus payments in a general equilibrium framework, where the labour market is characterized by search frictions. However, the (ex-post) wage setting mechanism is preserved unchanged. Solution to the firm optimization problem in the ex-post wage setting regime is summarized in proposition 2.3.

Proposition 2.3: *The search equilibrium with bonus payments and ex-post wage setting is characterized by a tuple $\{e, w^H, w^L, U, \theta\}$ satisfying equations (2.4.2), $rU = v(z)$, the risk sharing equation (2.4.8) as well as equation (a) below and the free entry condition $V(\theta) = 0$.*

(a) *The optimal value of w^L is obtained from the worker participation constraint:*

$$R^H(w^H, w^L, U) = W^H(w^H, w^L) - U = 0 \quad (2.5.2)$$

Proof: Appendix 2.10.3.

In the following subindex "P" is attached to the tuple $\{e, w^H, w^L, U, \theta\}$ characterizing the search equilibrium with bonus payments under the ex-post wage setting regime.

The search equilibrium with the ex-post wage setting is obtained at the intersection of the risk sharing curve (2.4.8) and the worker participation constraint $R^H(w^H, w^L, U) = 0$ denoted *WPC*. In case when workers are risk neutral the risk sharing condition again implies $b = \Delta y$. The base wage w is then set according to the worker participation constraint $R^H(w^H, w^L, U) = 0$. When the value of the bonus payment is zero, there are no incentives for firms to set the base wage w above the reservation value z . This is illustrated in figure 2.3. Notice also that the equilibrium implies $w_P < z$ meaning, that companies use both the award based motivation reflected in $b_P > 0$ and the punishment based motivation reflected in $w_P - z < 0$. The motivation premium in this case can be expressed as b_P ,

while the motivation penalty is $w_P - z$.

In addition the rent sharing equation (2.4.9) implies $R^H(w_C^H, w_C^L, U_C) > 0$ in the competitive search equilibrium with bonus payments. This is different under the ex-post wage setting regime, where in the equilibrium it is true that $R^H(w_P^H, w_P^L, U_P) = 0$. This means that the rent sharing curve in competitive search equilibrium is situated above the participation constraint in the space (b, w) . This result also takes account of the fact that $U_C > U_P$, which follows from the inequality $rU_C = v(z) + \lambda(\theta_C)R^H(w_C^H, w_C^L, U_C) > v(z) = rU_P$. This result implies, that unemployed workers are strictly better off in competitive search equilibrium, where wages are set ex-ante as opposed to the ex-post wage setting regime.

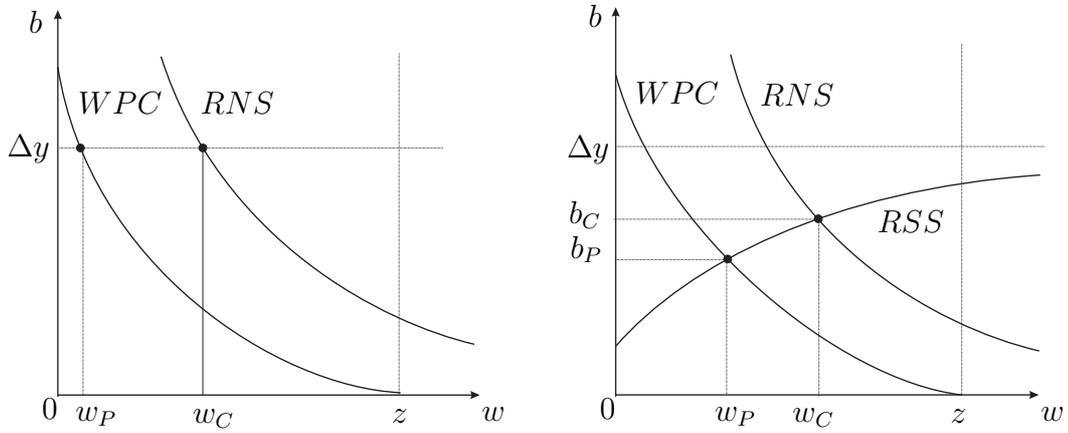


Figure 2.3: Optimal contracts under ex-post vs. ex-ante wage setting.
Left: risk neutral workers. Right: risk averse workers

Now compare the equilibrium labour contracts (w_C, b_C) vs. (w_P, b_P) under respectively the ax-ante and the ex-post wage setting. This comparison is also illustrated in figure 2.3. When workers are risk neutral the optimal bonus payment is equal in both types of the labour market, namely $b_C = b_P = \Delta y$, so that the motivation premia are also the same. However, the optimal wages are different, in particular it is true that $w_C > w_P$, implying that firms in competitive search equilibrium pay an additional hiring premium to their employees. If workers are risk averse with a logarithmic utility function described above it is true that $b_C > b_P$ and $w_C > w_P$, so that both expressions $b_C - b_P > 0$ and $w_C - w_P > 0$ stand for the hiring premia in competitive search equilibrium.

2.5.2 Limited liability constraint

As follows from the above analysis, unrestricted search equilibria with risk neutral workers always yield the maximum value of the bonus pay $b = \Delta y$, so that in the equilibrium there is no risk sharing between workers and firms. However, this is not the case if a wage restriction, explained by the limited liability or the minimum wage requirement is binding. This section considers properties of the search equilibrium with an ex-post wage setting and a binding wage constraint. In the presence of a wage restriction the optimal strategy of the firm is then to maximize the job surplus J^H with respect to w^H and w^L subject to the wage constraint, the worker incentive compatibility constraint (2.4.2) and the worker participation constraint. The firm optimization problem in the ex-post wage setting regime can be stated as follows:

$$\begin{aligned} (r + \gamma)J^H &= \max_{w^H, w^L} \{ \hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \} \\ \text{s.t.} \quad & w \geq \bar{w}, \quad R^H(w^H, w^L, U) \geq 0 \quad \text{and} \quad e = e(w^H, w^L) \end{aligned}$$

Solution to this optimization problem is presented in proposition 2.4.

Proposition 2.4: *Consider a binding wage restriction of the type $w \geq \bar{w}$. The restricted search equilibrium with bonus payments and ex-post wage setting is characterized by a tuple $\{e, b, w, U, \theta\}$ satisfying requirements $w = \bar{w}$, (2.4.1), (2.4.2) as well as the free entry condition $V(\theta) = 0$; the optimal bonus payment is obtained as $b = \max(b^*, b^{**})$, where b^* is solution to $\eta_{\hat{p}} = 1$, which can be written as:*

$$(\Delta y - b)\hat{p}'_e e'_b = \hat{p}(e), \quad \text{where} \quad e = e(b, \bar{w}) \quad (2.5.3)$$

and b^{**} is obtained from the worker participation constraint $R(\bar{w}, b^{**}, U) = 0$.

Proof: Appendix 2.10.3

In the following subindex "PR" is attached to the tuple $\{e, w^H, w^L, U, \theta\}$ characterizing the restricted search equilibrium with bonus payments under the ex-post wage setting regime.

Equation (2.5.3) can be interpreted as a restricted risk sharing condition (see figure 2.4). It comes from the firm's first order condition $\partial J^H / \partial b = 0$ and defines the risk sharing proportions between a firm and a worker. Note that, if workers are risk neutral the optimal effort is independent of the wage \bar{w} , so that equation (2.5.3) produces a fixed value of the bonus pay $b^* < \Delta y$. However, if \bar{w} and the corresponding value $b^*(\bar{w})$ are not sufficient to fulfill the worker participation constraint denoted *WPC* and to provide workers with a necessary job rent, the firm will increase the optimal bonus pay to the point, where workers are just indifferent between working and staying unemployed, this value of the bonus pay is denoted by b^{**} , so that $b_{PR} = \max(b^*, b^{**})$.

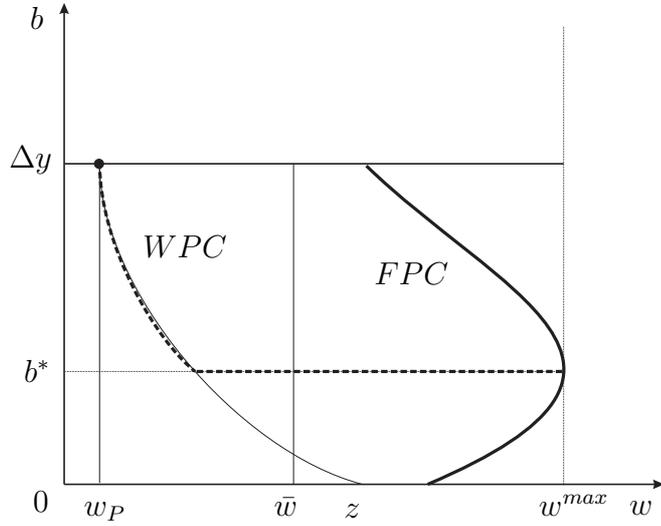


Figure 2.4: Wage restriction in SE with ex-post wage setting: risk neutral workers

2.6 Heterogeneous capital intensity

This part of the paper presents extensions of the models in sections 2.4-2.5 for the case of heterogeneous jobs. Suppose that the firm entry mechanism is as follows. Firms pay an ex-ante capital investment K in order to enter the market, the capital investment is irreversible. Upon entry each firm draws a firm-specific capital intensity k from distribution $F(k)$ with the range of capital intensity values $[\underline{k}, \bar{k}]$. Assume that the minimal capital intensity value \underline{k} is sufficient for the firm to stay in the market, so that $V(\underline{k}) = V(w^H(\underline{k}), w^L(\underline{k})) > 0$. This means that the free entry condition becomes:

$$K = \int_{\underline{k}}^{\bar{k}} V(k) dF(k)$$

The capital intensity distribution creates ex-post productivity diversity in the economy. Capital is included into the model in a multiplicative way, so that the worker productivity is defined according to the following rule:

$$\begin{cases} y^H = a^H f(k) \\ y^L = a^L f(k) \end{cases}$$

where $\Delta a = a^H - a^L > 0$ and $f(k)$ is a standard production function in the intensive form, increasing and concave in k . This approach creates productive heterogeneity among jobs, where *ceteris paribus* jobs with a higher capital stock intensity are characterized by a higher expected output flow $m(k)$, but also face a higher variance of the output $\sigma^2(k)$ and a higher risk $\Delta y = \Delta a f(k)$:

$$\begin{cases} m(k) = (a^L + \Delta a \hat{p}(e)) f(k) \\ \sigma^2(k) = \hat{p}^2(e) (1 - \hat{p}(e))^2 f^2(k) \Delta a^2 \end{cases}$$

In order to make a reference about the correlation between wages and bonus payments in this economy consider two jobs with capital intensities $k_2 > k_1 \in [\underline{k}, \bar{k}]$. Both firms face the same worker participation constraint $R^H(k) = R^H(w^L(k), w^H(k), U) \geq 0$, where the unemployed worker surplus value $U = U(k) \forall k$ is now obtained from expression:

$$rU = v(z) + \lambda(\theta(k)) R^H(k) \quad (2.6.1)$$

However, the two risk-sharing curves faced by firms are different due to the fact that $\Delta y(k_2) > \Delta y(k_1)$. As follows from the risk sharing equation (2.4.8) the optimal bonus payment $b(k) = \Delta w(k)$ is an increasing function of Δy , so that the risk sharing curve of the more capital intensive firm is situated above the corresponding curve of the less capital intensive firm. This result is illustrated in figure 2.5.

Notice that equilibrium contracts in the search equilibrium framework with an ex-post wage setting regime are obtained at the intersection between the worker participation constraint and the risk sharing curve, so that it can be concluded that $w^L(k_2) < w^L(k_1)$ and $b(k_2) > b(k_1)$. The more capital intensive firm is more productive in expectation so, that the marginal gain of a unit effort increase is larger in this firm compared to the less productive firm. In order to achieve a higher effort level the firm sets optimally a higher value of bonus pay $b(k_2)$ and a lower value of the base wage $w^L(k_2)$. Note, that both actions lead to an increase in the worker effort. The lower value of the base wage also guarantees, that the worker participation constraint is binding.

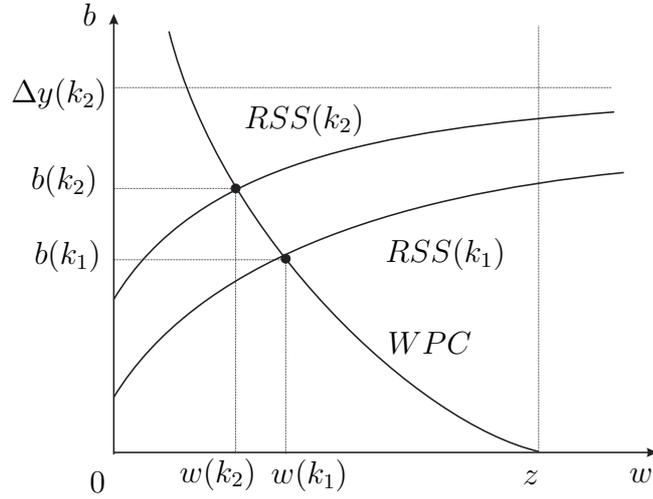


Figure 2.5: SE with ex-post wage setting and heterogeneous jobs

Overall, the search equilibrium with an ex-post wage setting regime exhibits the substitution effect between wages and bonus payments and fails to account for the complementarity effect observed in a number of empirical studies (see table 2.1).

In competitive search equilibrium both firms face the same labour supply equation (2.6.1) in the space $[\theta, w^L]$ for a given bonus pay value Δw . It can also be interpreted as workers' indifference curve. This follows from worker homogeneity in the economy and is represented by a convex decreasing curve in figure 2.6. The curve is decreasing since workers prefer both high wages and high market tightness. However, each firm is maximizing an individual vacancy surplus expression $V(k) = -c + q(\theta)(J^H(k) - V(k))$, where the job surplus $J^H(k)$ is obtained in the following way:

$$J^H(k) = J^H(y^L(k), \Delta y(k)) = y^L(k) - w^L + \hat{p}(e)(\Delta y(k) - \Delta w)$$

Job surplus function $J^H(y^L(k), \Delta y(k))$ is increasing in both arguments $y^L(k)$ and $\Delta y(k)$, meaning that the more capital intensive firm produces more output in the low productivity state $y^L(k)$ and also enjoys a larger output increase $\Delta y(k)$ if the high productivity state is realized. Both vacancy surplus equations for the two firms are represented by concave decreasing curves in the space $[\theta, w^L]$ and are illustrated in figure 2.6. These curves can also be interpreted as iso-profits curves. Both curves are decreasing since firms prefer both low wages and low market tightness.

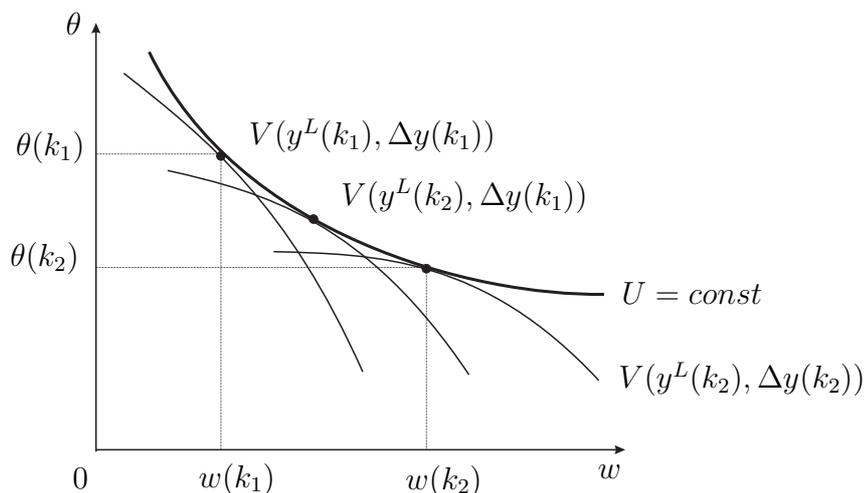


Figure 2.6: Market tightness in CSE with heterogeneous jobs

Concavity of the iso-profit curve and convexity of the worker's indifference curve are guaranteed by the assumption of the nondecreasing elasticity of the job filling rate η_q with respect to the market tightness θ . For a fixed value of $\Delta y(k)$, the more capital intensive firm faces a flatter indifference curve $V(k_2) = const$, this means that the optimal vector of variables $[\theta, w^L]$ is such that $\theta(k_2) < \theta(k_1)$ and $w^L(k_2) > w^L(k_1)$. A further difference in risk variables $\Delta y(k)$ between the two firms implies a further rotation of the firm indifference curve and strengthens the preceding result. The intuition behind this result is such, that the more capital intensive firm faces larger search costs in terms of forgone output value and so the firm is more willing to trade off the low wages for low labour market tightness.

For the rent-sharing curve (RNS) the fact that $w^L(k_2) > w^L(k_1)$ for every value of $b = \Delta w$ implies an upward shift in the space $[b, w]$, so that the rent-sharing curve of the more capital intensive firm is situated above the corresponding curve of the less capital intensive firm. This is illustrated in figure 2.7. The reason for this shift is twofold: due to the larger values of $y^L(k_2)$ and $\Delta y(k_2)$. More capital intensive firms are more productive, obtain higher rents $J^H(k_2) - V(k_2)$ and share these rents with their employees. Firms lose from higher labour costs, both in terms of wages and bonus payments, but gain from a higher job filling rate $q(\theta)$. Note that for the fixed risk-sharing curve the rent-sharing motive implies complementarity between bonus payments and wages in the case of risk averse workers.

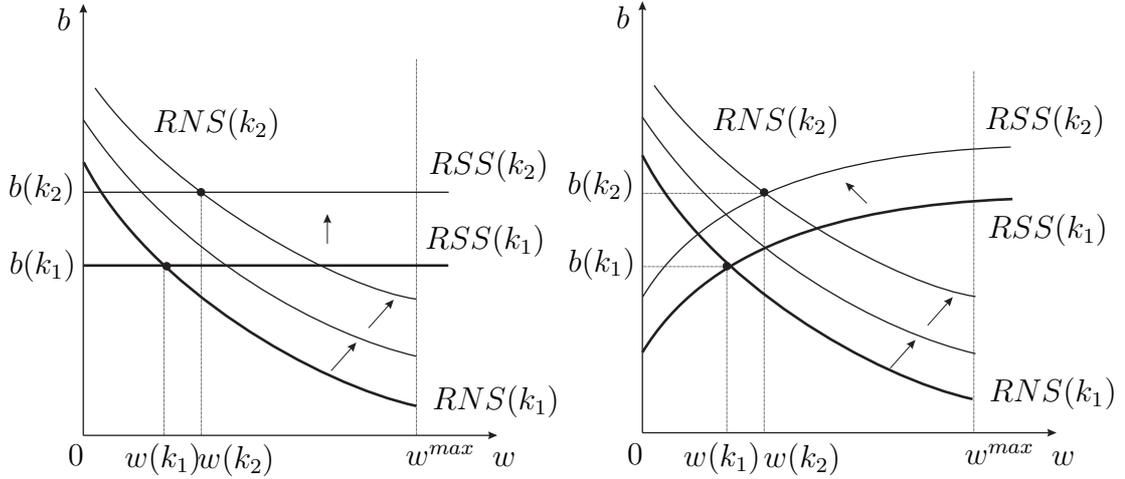


Figure 2.7: Competitive search equilibrium with heterogeneous jobs.
 Left: risk neutral workers. Right: risk averse workers

Consider difference in the risk sharing curves. As already described above in the case of ex-post wage setting, both firms face different risk-sharing curves, where the RSS curve for k_2 is situated above the corresponding curve for k_1 . Here the more productive firm substitutes wages for bonus payments and gains from an unambiguously higher worker effort. Overall, optimal contract comparison of the two firms with different values of capital intensity highlights the fact, that the more capital intensive firm will unambiguously offer a larger value of the bonus payment $b(k_2)$, which has a positive effect both on the optimal worker effort and on the firm hiring rate. The effect of capital differences on base wages is however ambiguous, it is more likely to be positive if the slope of the risk sharing curve is close to 1 in the relevant range of capital intensities $[\underline{k}, \bar{k}]$. Also notice that in the case of homogeneous variation in output ($\Delta y(k) = const \Rightarrow \partial \Delta y / \partial k = 0$) the baseline model of the paper with risk averse workers unambiguously predicts positive cross-sectional correlation between bonus payments and wages.

Summarizing, in the presence of jobs heterogeneity competitive search equilibrium with bonus payments extends the classical contract theory approach with ex-post wage setting by explaining the sources of cross-sectional complementarity between bonus payments and wages. This complementarity effect is based on the rent sharing mechanism between the firm and the worker inherent in the ex-ante wage setting regime.

2.7 Social welfare and constrained efficiency

This section considers welfare properties of competitive search equilibrium with risk averse workers and bonus payments. As mentioned in the introduction, with respect to the social planner solution this paper can be seen as a generalization of the equilibrium efficiency result presented in Moen and Rosen (2008) in competitive search equilibrium with risk averse workers in the presence of asymmetric information. However, the social planner optimization problem is investigated in the absence of income taxes and unemployment benefits, so that the main research question raised in this section is, whether the wage contracts chosen by firms are socially optimal given the optimal worker behavior and the free entry of firms. Two informational settings are considered: first, the unconstrained social planner optimization problem (first best) is analyzed, where the effort choice of workers is assumed to be observable by the social planner, then the constrained social planner solution is compared to the first best solution.

Consider a social planner implementing a state dependent wage contract with wages w^L and w^H in respective productivity states y^L and y^H . The utilitarian social planner is maximizing the present discounted value of a sum of utility flows of the unemployed and employed individuals. In the first best case the choice variables of the social planner can be represented as a tuple of variables $\{e, w^H, w^L, \theta\}$, so that the planner's objective function can be expressed in the following way:

$$\max_{e, w^L, w^H, \theta} \int_0^{\infty} e^{-rt} \left[uv(z) + (1 - u)\hat{v}(e, w^L, w^H) \right] dt, \quad \text{where}$$

$$\hat{v}(e, w^L, w^H) = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e)$$

Variable $\hat{v}(e, w^L, w^H)$ denotes utility flow of the employed individual working under the wage contract (w^L, w^H) and exerting e units of effort. The unemployment rate evolves according to the following differential equation:

$$\dot{u} = (1 - u)\gamma - u\lambda(\theta)$$

The planner's resource constraint can be summarized as follows:

$$cu\theta = (1 - u) \left(\hat{p}(e)(y^H - w^H) + (1 - \hat{p}(e))(y^L - w^L) \right) \quad (2.7.1)$$

Equation (2.7.1) implies that the planner's budget is balanced and the monetary outflow for maintaining the vacancies on the left hand side equals the monetary inflow from

the filled jobs on the right hand side of this equation. The social planner optimization programme is solved using a current-value Hamiltonian approach. Solution to this optimization programme is presented in proposition 2.5.

Proposition 2.5: *Consider a social planner implementing a variable wage contract. The unconstrained (first best) social planner solution is characterized by a tuple of variables $\{e, w^H, w^L, \theta, U\}$ satisfying condition $w^H = w^L$, the reservation utility equation (2.4.1), the job creation condition $c = q(\theta)J^H(w^H, w^L)$, as well as equations (a) and (b) below.*

(a). *The planner's effort choice is given by:*

$$dp'(e)\Delta yv'(w) = C'(e) \quad (2.7.2)$$

(b). *The risk adjusted Hosios surplus split:*

$$J^H(w^H, w^L) = \frac{1 - \eta_q}{\eta_q} \frac{R(w^H, w^L, U)}{v'_w} \quad (2.7.3)$$

Proof: Appendix 2.10.4.

As follows from proposition 2.5 the unconstrained social planner optimally sets $w^H = w^L$. This means that the optimal bonus payment b is set to zero, implying income insurance for workers against productivity shocks. Having guaranteed income stability for the employed population the social planner chooses an optimal effort level by maximizing the total surplus of a filled job. This is given by equation (2.7.2), where the left hand side of equation stands for the social gain of increasing the effort, while the right hand side can be interpreted as a marginal loss. The social loss $C'(e)$ is directly estimated in worker utility units, while the social gain is estimated as an increase in the expected productivity flow $dp'(e)\Delta y$ multiplied by the respective shadow price of an output unit, represented by the term $v'(w)$.

Notice that the optimal effort equation of a social planner (eq. 2.7.2) is different from the worker incentive compatibility constraint (2.4.2). Here the social cost of increasing effort $C'(e)$ coincides with a private cost of the employee, however the social gain $dp'(e)\Delta yv'(w)$ is generally different from a private gain of the employee, which can be expressed as $dp'(e)(v(w^H) - v(w^L))$. Denote x_0 – solution to the following equality:

$$\Delta yv'(x_0) = v(w^H) - v(w^L), \quad x_0 > w^L$$

Then competitive search equilibrium with risk averse workers, unobserved effort and bonus payments entails a downward effort distortion with respect to the first best outcome if $w < x_0$, otherwise if $w > x_0$ effort is biased upward. Intuitively, if w is less than x_0 , the shadow price of a single output unit is high, so that the social gain expressed in worker utility units is higher than the private gain of a worker, as a result the social planner will demand more effort from workers compared to the decentralized equilibrium with risk averse workers and unobserved effort values. The opposite holds when $w > x_0$, in this case the social gain converted into worker utility units is lower than the private gain of a worker and therefore the first best effort level is lower than effort in a decentralized equilibrium.

In addition, it should also be noted, that effort distortions are purely attributed to the risk aversion of workers. If workers are risk neutral, the social gain of exerting effort expressed as $dp'(e)\Delta y$ coincides with a private gain of the worker $dp'(e)b$. This is the case because firms in a decentralized equilibrium optimally choose the maximum bonus payment value $b = \Delta y$ (see proposition 2.1). This, however, does not imply, that the first best social planner solution with risk neutral workers may be decentralized by the market. The reason is that the maximum effort value is not compatible with a zero bonus payment when effort is unobserved.

Now consider the second best solution, where the social planner is constrained by the information asymmetries arising from the unobserved worker effort choice. In this case the social planner is maximizing the present discounted value of a sum of utility flows of the unemployed and employed individuals with respect to the choice variables represented by a tuple $\{w^H, w^L, \theta\}$. The objective function of a constrained social planner becomes:

$$\max_{w^L, w^H, \theta} \int_0^{\infty} e^{-rt} \left[uv(z) + (1 - u)\hat{v}(w^L, w^H) \right] dt$$

where \tilde{v} is given above. In addition, the worker incentive compatibility constraint (equation 2.4.2), as well as the budget constraint of the social planner and the unemployment dynamics equation should be fulfilled. The result of this optimization problem is summarized in proposition 2.6.

Proposition 2.6: *Consider a social planner implementing a variable wage contract. The constrained (second best) social planner solution is characterized by a tuple of variables $\{e, w^H, w^L, \theta, U\}$ satisfying the reservation utility equation (2.4.1), the job creation condition $c = q(\theta)J^H(w^H, w^L)$, the worker incentive compatibility constraint (2.4.2), as well as the risk-sharing equation (2.4.8) and the rent-sharing equation (2.4.9). Therefore, competitive search equilibrium with bonus payments and unobserved effort is constrained efficient.*

Proof: Appendix 2.10.4.

Proposition 2.6 characterizes the major properties of the constrained social planner solution. It follows that the set of five equations describing the optimal solution of a social planner coincides with the set of equations in a decentralized competitive search equilibrium with unobserved effort and bonus payments. This means that the social planner will choose exactly the same optimal package of labour compensation (w^L, w^H) resulting in the same effort level of the employed and the same market tightness variable θ . Therefore, it can be concluded that competitive search equilibrium with risk averse workers, bonus payments and unobserved effort is constrained efficient.

2.8 Conclusions

This paper develops a model of competitive search with risk averse workers in the presence of asymmetric information. Information asymmetries arise from the fact, that workers possess private information about their effort choice on the job. The moral hazard problem within a match forces firms to use motivation devices such as the bonus pay in order to provide workers with the correct working incentives. This setup creates a situation, where the equilibrium labour contract entails both a hiring and a motivation wage premia. The hiring premium results from the rent-sharing incentive of firms ensuring them a sufficient job-filling rate, while the motivation premium results from the firm's risk-sharing incentive necessary to guarantee a sufficient effort level.

The baseline model of the paper is compared to the classical model of moral hazard extended to account for labour market search frictions but preserving the essence of the

ex-post wage setting mechanism. This benchmark model is proved to predict a lower amount of the bonus pay than the baseline model with wage competition between employers. Similarly, both models are compared in the presence of a wage restriction imposed to reflect a binding limited liability constraint or a minimum wage requirement.

Furthermore, the paper presents an extension of the competitive search model with bonus payments to account for jobs' heterogeneity. In particular, jobs are allowed to differ with respect to their capital endowments affecting both the expectation and the variation of output. The rent-sharing motive forces more capital intensive firms to leave higher rents to their employees. The higher rent comes in the form of a higher base wage as well as a higher bonus pay values. This complementarity effect provides rationale for the positive cross-sectional correlation between bonus payments and wages reported in a number of empirical studies. The rent-sharing motive is absent in the model with an ex-post wage setting so, that bonus payments and wages act as substitutes in a cross-section of firms. Based on the above theoretical analysis this paper concludes, that the correlation between bonus payments and wages is specific to the type and the structure of the labour market. This is also in line with the observed empirical evidence.

Finally, this paper considers efficiency implications of incentive contracts in a competitive search equilibrium. The equilibrium is proved to be constrained efficient in the absence of tax payments and unemployment benefits. Nevertheless, competitive search equilibrium with bonus payments does not coincide with the full information allocation of the social planner. This is due to the fact that the private gain from exerting effort is different from the social gain, so that in the full information allocation the social planner will demand a different effort level from workers compared to the decentralized equilibrium.

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2.10 Appendix

APPENDIX 2.10.1: Proof of lemmas 2.1-2.2

Differentiate W^L and W^H with respect to e^L and e^H respectively to obtain:

$$-C'(e^L) + \delta p'(e^L)(W^H - W^L) = 0 \quad (2.10.1)$$

$$-C'(e^H) + \delta p'(e^H)(W^H - W^L) = 0 \quad (2.10.2)$$

Equations (2.10.1)-(2.10.2) imply $e^L = e^H = e(w^L, w^H)$, so that:

$$\delta(W^H - W^L) = \frac{C'(e)}{p'(e)} \equiv \pi(e) \quad (2.10.3)$$

Subtracting W^L from W^H yields:

$$\delta(W^H - W^L) = d(v(w^H) - v(w^L)) \quad (2.10.4)$$

which proves equation (2.4.2). Differentiate equation (2.4.2) with respect to w^H to obtain $e'_{w^H} > 0$. Similarly, differentiate equation (2.4.2) with respect to w^L to obtain $e'_{w^L} < 0$.

The second order derivatives $e''_{w^L w^H}$, $e''_{w^H w^L}$ and $e''_{w^H w^L}$ can be found as follows:

$$\begin{aligned} e''_{w^L} &= -d \frac{(v''_{w^L} \pi'_e - v'_{w^L} \pi''_e)}{(\pi'_e)^2} > 0 \quad \text{if } \pi''_e > 0 \\ e''_{w^H} &= d \frac{(v''_{w^H} \pi'_e - v'_{w^H} \pi''_e)}{(\pi'_e)^2} < 0 \quad \text{if } \pi''_e > 0 \\ e''_{w^L w^H} &= d^2 \frac{(v'_{w^H} \pi''_e v'_{w^L})}{(\pi'_e)^3} > 0 \quad \text{if } \pi''_e > 0 \end{aligned}$$

Using equation (2.10.4) rewrite $R(w^H, w^L)$ in the following way:

$$(r + \gamma)R^H(w^H, w^L) = \hat{p}(e)v(w^H) + (1 - \hat{p}(e))v(w^L) - C(e) - rU$$

where $\hat{p}(e) = (r + \gamma + \delta p(e))/(r + \gamma + \delta)$. Differentiate $R^H(w^H, w^L)$ with respect to w^H and w^L and apply the envelope theorem to obtain: $\partial R^H(w^H, w^L)/\partial w^H > 0$ and $\partial R^H(w^H, w^L)/\partial w^L > 0$.

Appendix 2.10.2: Proof of proposition 2.1.

Differentiate equations (2.4.3), (2.4.4) with respect to w^H and w^L and use the fact that $V = 0$ in the equilibrium, this yields:

$$\frac{\partial R^H/\partial w^H}{\partial R^H/\partial w^L} = \frac{\partial \theta/\partial w^H}{\partial \theta/\partial w^L} = \frac{\partial J^H/\partial w^H}{\partial J^H/\partial w^L} \quad (2.10.5)$$

where $R^H = R^H(w^H, w^L, U)$ and $J^H(w^H, w^L)$. Differentiate $J^H(w^H, w^L)$ with respect to both arguments to obtain:

$$\begin{aligned} (r + \gamma) \frac{\partial J^H}{\partial w^H} &= \hat{p}'(e)(\Delta y - \Delta w)e'_{w^H} - \hat{p}(e) \\ (r + \gamma) \frac{\partial J^H}{\partial w^L} &= \hat{p}'(e)(\Delta y - \Delta w)e'_{w^L} - (1 - \hat{p}(e)) \end{aligned}$$

Insert expressions for $\partial R^H/\partial w^H$, $\partial R^H/\partial w^L$, $\partial J^H/\partial w^H$ and $\partial J^H/\partial w^L$ into (2.10.5):

$$\frac{\hat{p}(e)v'(w^H)}{(1 - \hat{p}(e))v'(w^L)} = \frac{\hat{p}'(e)(\Delta y - \Delta w)e'_{w^H} - \hat{p}(e)}{\hat{p}'(e)(\Delta y - \Delta w)e'_{w^L} - (1 - \hat{p}(e))}$$

Insert expressions $e'_{w^H} = dv'(w^H)/\pi'(e)$ and $e'_{w^L} = -dv'(w^L)/\pi'(e)$ to obtain equation (2.4.8). Differentiate equation (2.4.4) with respect to w^H to obtain:

$$J^H(w^H, w^L) = \left[-\frac{\partial J^H/\partial w^H}{\partial R^H/\partial w^H} \right] \frac{1 - \eta_q}{\eta_q} R^H(w^H, w^L, U)$$

Insert expressions for $\partial J^H/\partial w^L$, $\partial R^H/\partial w^L$ to obtain the risk-adjusted Hosios condition (2.4.9).

Appendix 2.10.3: Proof of propositions 2.3-2.4

In the search equilibrium with ex-post wage setting the firm is maximizing its surplus with respect to the wage value w^H given that the wage w^L is adjusting according to the worker participation constraint. This gives rise to the following optimization problem:

$$\max_{w^H, w^L} J^H(w^H, w^L) + \lambda_u R^H(w^H, w^L, U)$$

where λ_u stands for the Lagrange multiplier. The first order conditions for this optimization problem are given by:

$$\frac{\partial J^H}{\partial w^H} + \lambda_u \frac{\partial R^H}{\partial w^H} = 0 \quad (2.10.6)$$

$$\frac{\partial J^H}{\partial w^L} + \lambda_u \frac{\partial R^H}{\partial w^L} = 0 \quad (2.10.7)$$

$$\lambda_u R(w^H, w^L, U) = 0 \quad (2.10.8)$$

where the last equation represents a complementary slackness condition. In the unrestricted firm optimization problem with the ex-post wage setting the worker participation constraint is binding, which means that $\lambda_u \neq 0$, while $R(w^H, w^L, U) = 0$. Then equations (2.10.6) - (2.10.7) can be rearranged to produce the risk sharing curve given by equation (2.4.10).

Now consider the restricted firm optimization problem with a binding limited liability constraint of the type $w \geq \bar{w}$. The firm optimization problem can be written as follows:

$$\max_{w^H} J^H(w^H, \bar{w}) + \lambda_R R^H(w^H, \bar{w}, U)$$

where λ_R stands for the Lagrange multiplier. The first order condition for this optimization problem is given by:

$$\frac{\partial J^H}{\partial w^H} + \lambda_R \frac{\partial R^H}{\partial w^H} = 0 \quad (2.10.9)$$

$$\lambda_R R(w^H, w^L, U) = 0 \quad (2.10.10)$$

where the last equation stands for the complementary slackness condition. If the worker participation constraint is not binding then $\lambda_R = 0$ and the optimal bonus payment is given by $\partial J^H / \partial w^H = 0$ implying that:

$$(\Delta y - b) \hat{p}'_e e'_b = \hat{p}(e), \quad \text{where } e = e(b, \bar{w})$$

which is the restricted risk sharing condition (2.5.3). Solution to this equation is denoted by b^* . If the worker participation constraint is binding meaning that b^* is too low, then $\lambda_R \neq 0$, so that the optimal bonus payment b^{**} is given by equation $R(b^{**}, \bar{w}, U) = 0$. Overall, the optimal bonus payment is given by: $b_{PR} = \max(b^*, b^{**})$.

Appendix 2.10.4: Proof of propositions 2.5-2.6

The current value Hamiltonian for the unconstrained planner problem (first best) is:

$$\begin{aligned} H &= uv(z) + (1-u)\hat{v}(w^L, w^H, e) - \gamma_u[(1-u)\gamma - u\lambda(\theta)] \\ &+ \alpha[cu\theta - (1-u)(\hat{p}(e)(y^H - w^H) + (1-p(e))(y^L - w^L))] \end{aligned}$$

where α is a Lagrange multiplier and γ_u is a costate variable corresponding to u . The optimal social planner solution must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma_u \Rightarrow \alpha J^H + R^H = \gamma_u \quad (2.10.11)$$

Maximizing H with respect to e , w^H , w^L and θ yields:

$$\frac{\partial H}{\partial e} = 0 \Rightarrow d[(\Delta y - \Delta w)\alpha + \Delta v] = \pi'(e) \quad (2.10.12)$$

$$\frac{\partial H}{\partial w^H} = 0 \Rightarrow \alpha = v'(w^H) \quad (2.10.13)$$

$$\frac{\partial H}{\partial w^L} = 0 \Rightarrow \alpha = v'(w^L) \quad (2.10.14)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma_u(1 - \eta_q)q(\theta) = c\alpha \quad (2.10.15)$$

where $\Delta w = w^H - w^L$ and $\Delta v = v(w^H) - v(w^L)$. Equations (2.10.13)-(2.10.14) imply $w^H = w^L$ and $\alpha = v'(w)$ - shadow price of an output unit. These results transform equations (2.10.12), (2.10.15) and (2.10.11) into:

$$d\Delta y v'(w) = \pi'(e) \quad \text{and} \quad J^H = \frac{1 - \eta_q}{\eta_q} \frac{R^H}{v'(w)}$$

Consider the current value Hamiltonian function for the constrained social planner problem (second best). Denote μ - Lagrange multiplier corresponding to the budget constraint of the social planner, and γ_c - a costate variable corresponding to u . The optimal social planner solution in this case must satisfy:

$$\frac{\partial H}{\partial u} = -r\gamma_c \Rightarrow \mu J^H + R^H = \gamma_c \quad (2.10.16)$$

$$\frac{\partial H}{\partial w^H} = 0 \Rightarrow \hat{p}(e) - dp'(e)e'_{w^H}(\Delta y - \Delta w) = \hat{p}(e) \frac{v'(w^H)}{\mu} \quad (2.10.17)$$

$$\frac{\partial H}{\partial w^L} = 0 \Rightarrow (1 - \hat{p}(e)) \frac{v'(w^L)}{\mu} = 1 - p(e) - dp'(e)e'_{w^L}(\Delta y - \Delta w) \quad (2.10.18)$$

$$\frac{\partial H}{\partial \theta} = 0 \Rightarrow \gamma_c(1 - \eta_q)q(\theta) = c\mu \quad (2.10.19)$$

The ratio of equations (2.10.17),(2.10.18) can be rearranged to produce equation (2.4.8).

In addition equations (2.10.16) and (2.10.19) imply the following optimal rent split:

$$J^H = \frac{1 - \eta_q}{\eta_q} \frac{R^H}{\mu}$$

Chapter 3

Temporary Layoffs with Incomplete Worker Attachment in Search Equilibrium

3.1 Introduction

The process of job destruction is well understood and incorporated into the models of job search. The seminal work in this field is accomplished by Mortensen and Pissarides (1994) with the following studies by Pissarides (2000), Bontemps, Robin and Van den Berg (2000) and Postel-Vinay and Robin (2002a). The general framework for the analysis of job destruction builds up on the mechanism of permanent, independent and idiosyncratic productivity shocks inducing agents to separate. As a result of the negative productivity shock jobs are destroyed while workers are unemployed and search for a new employment. Nevertheless the common assumption of permanent separations and memoryless behavior of workers and firms contradicts the existing empirical literature. Mavromaras and Rudolph (1998) show, that 26.5% of the individuals finding employment in Germany are recalled to their former employers (table 3.1). Similar frequencies of recalls are registered in Austria and Spain being respectively 32.4% and 35.7%. Even higher recall ratios are estimated in Scandinavian countries ranging from 32.2% in Norway to about 50% in Denmark. In addition, empirical relevance of temporary layoffs is supported by the fractions of attached unemployed (expecting a recall) in the pool of unemployed workers. These ratios range from approximately 10% in Sweden to 22.2% in Austria.

Study	Results	Sample (spells)	Country
K. Mavromaras, H. Rudolph (1998)	Recalls: 26.5%	N=22601 (L)	Germany 1980-1990
G. Fischer, K. Pichelmann (1991)	Recalls: 32.4% AU: 22.2%	N=2499 (T)	Austria 1985
A. Alba-Ramirez, J. Arranz, F. Munoz-Bullon (2007)	Recalls: 35.7%	N=23035 (L)	Spain 1999-2002
P. Jensen, M. Svarer (2003)	Recalls: 50% AU: 20%	N=35000 (T)	Denmark 1981-1990
F. Jansson (2002)	Recalls: 40-47% AU: 10%	N=3668 (T)	Sweden 1995-1996
K. Roed, M. Nordberg (2003)	Recalls: 32.2% AU: 13.3%	N=815373 (T)	Norway 1989-1998

AU – attached unemployment; L – layoff unemployment; T – total unemployment;

Table 3.1: Empirical research on temporary layoffs (Europe)

Temporary layoffs are also a wide-spread phenomenon in the U.S. According to the data of the U.S. Bureau of Labor Statistics for the period 2000-2007, approximately 1 million of registered unemployed in the U.S. expect to be recalled to their former employers¹. This corresponds to the ratios of 13.6% of total unemployment and 26.4% of layoff unemployment in the U.S.

Following the empirical evidence this study considers the problem of temporary layoffs in a model of job search. The starting point of this paper is to introduce temporary productivity shocks and worker-firm attachment into the search and matching framework of Mortensen and Pissarides (1994), where search is random and undirected, and wages are set via the Nash bargaining. Bargaining as a wage determination mechanism is supported on the empirical level, e.g. using the data from Princeton Data Improvement Initiative for the year 2008 Hall and Krüger (2008) find, that about a third of all workers in the sample bargained with their current employers rather than treated their job offer as take-it-or-leave-it. Further, this study considers wage contracts with limited commitment and allows for wage renegotiations if either of the participation constraints is binding.

¹Individuals on a temporary layoff are defined as those "who have been given a date to return to work or who expect to return within 6 months", U.S. Bureau of Labor Statistics, Handbook of Methods, Chapter 1, available at www.bls.gov/opub/hom

Conditionally on productivity shocks being sufficiently severe for the threat of layoff to be credible, there are two different equilibria. The first equilibrium obtains at low variation in productivity, the layoff threat is then eliminated by wage renegotiation implying a wage reduction after the first production spell. The second equilibrium with temporary layoffs obtains at high productivity variation and is in the focus of the present study. First of all, search costs incurred by firms as well as a temporary nature of productivity fluctuations mutually motivate the worker-firm attachment upon a separation. Nevertheless, worker's attachment is incomplete, since workers search for new job alternatives during the low productivity spells. Both workers and firms gain from their attachment. Firms obtain a valuable option to recall the worker, while workers gain from an additional possibility to be recalled. There is also a second gain for the workers: attached unemployed have a higher reservation wage than the unattached, which means they can bargain a higher wage, when contacted by a new employer. The ex-post differentiation of reservation wages among attached and unattached unemployed produces a binary wage distribution in the equilibrium. The model can thus contribute to the debate on endogenous wage dispersion following the seminal study by Burdett and Mortensen (1998).

Furthermore, this study confirms theoretical predictions of the model using the data from the German Social-Economic Panel for the years 2003-2007. The probit regression model shows that workers recalled to their previous employer face approximately 8% lower probability of wage improvement compared to those finding a job with a new employer. This means that the worker-firm attachment and recalls have significant predictive power for wage changes and therefore provide an additional explanation of wage heterogeneity in Germany. Other significant explanatory variables include age of the individual, the reason for separation as well as comparison of job characteristics. This study shows that voluntary separations are associated with 6.5% higher probability of wage improvement upon a job change, at the same time the probability is 8.2% lower in the case of involuntary separation. Moreover, additional benefits, better promotion possibilities and improved job security are positively associated with wage gains.

Finally, this study considers welfare properties of an economy with search frictions and temporary layoffs. I find that the decentralized equilibrium with temporary layoffs is constrained inefficient, even if search externalities are internalized. Hosios (1990) shows, that search externalities are an inherent feature of models with stochastic matching and wage bargaining, since matching takes place before the bargaining, so that wages do not perform any allocative or signaling function. This study shows that mutual attachment of workers and firms upon a negative productivity shock introduces a new source of the equilibrium inefficiency. The novel attachment externality results from the fact, that workers on a temporary layoff, accepting new jobs, do not internalize the losses imposed on their previous employer. The previous firm is losing an option to recall the former employee, which is immediately translated into a value loss, since hiring is costly and time-consuming in the model.

To separate search and attachment externalities I set the bargaining power parameter equal to the elasticity of the matching function. According to Hosios (1990) this condition guarantees, that search externalities are internalized. Then the decentralized equilibrium with temporary layoffs is characterized by excessive job creation. Profits of firms hiring workers from attached unemployment are inefficiently high, so that too many jobs are created in the equilibrium. This paper also shows, that efficiency of the decentralized equilibrium may be restored by imposing an income tax on attached unemployment starting job with a new employer. The present value of tax payments from a match should then be equal to the value loss of the previous employer of the worker.

The plan of the paper is as follows. Section 3.2 contains an overview of the literature, while section 3.3 explains notation and the general economic environment of the model. Section 3.4 presents a model with temporary layoffs and section 3.5 explains the model with wage renegotiation. Section 3.6 contains welfare analysis of the decentralized equilibrium with temporary layoffs, while section 3.7 contains an empirical test of model predictions using data from the German labour market. Section 3.8 concludes.

3.2 Overview of the related literature

There are a number of features relating this study to the existing literature. Originally the theory of temporary layoffs has been developed in the implicit contract framework represented by the studies of Baily (1977), Feldstein (1976, 1978) and Burdett and Wright (1989). Feldstein (1976) considers the option of firms to reduce employment versus the option to reduce working hours in response to random demand fluctuations. Workers are assumed to be permanently attached to the firm and receive unemployment benefits if not employed. Unemployment benefits are financed by a tax on firms, that is related to the previous benefits collected by the firm's employees (imperfect experience rating). Feldstein (1976) shows, that imperfect experience rating magnifies the effect on employment of changes in demand and increases the change in employment relative to the change in average hours. Burdett and Wright (1989) allow firms to choose both the number of workers under the contract (firm size) and the number of workers producing output in a given period of time, so the model is characterized by attached and unattached unemployment, this properties however are achieved at the expense of assuming indivisible labour supply. Given this properties Burdett and Wright (1989) show, that the major result of Feldstein (1976) is reversed, so that an increase in experience rating increases unemployment under reasonable conditions.

One-sided labour demand analysis of the implicit contract literature is extended to consider the labour supply side of the market in the literature of job search. This is represented by the studies of Burdett and Mortensen (1980), Pissarides (1982) and Mortensen (1990). Burdett and Mortensen (1980) is the first study to synthesize the search and the implicit contract approaches. This study considers a labour market with an exogenous wage offer distribution, where each job is additionally characterized by a particular layoff probability. Moreover, Burdett and Mortensen (1980) allow workers to search on a temporary layoff, however search is costly implying a positive reservation wage for attached workers. Burdett and Mortensen (1980) characterize a retention equilibrium, where expected wage obtained by the worker is sufficiently high to prevent search in attachment. This is different from the current study, where workers search on a temporary layoff and change the attachment as soon as a new wage offer is obtained. This property is achieved by the use of Nash bargaining in wage setting, leaving positive rent to the worker.

Pissarides (1982) considers search behavior of workers in attached unemployment facing an exogenous wage offer distribution. The new feature of the model is that recall probability is endogenous and is optimally chosen by firms. Pissarides (1982) shows, that workers search for an alternative job, only if the probability of recall falls below a critical level, and that firms may recall workers before the recovery of demand, depending on the costs of laying off and hiring. Another study considering the problem of temporary layoffs in a partial equilibrium framework is Mortensen (1990), who considers a situation, where workers search and receive wage offers both, when employed and unemployed, and the worker's productivity on any specific job is subject to continual stochastic disturbance over time. This setup provides explanations for job to job transitions of workers as well as the phenomena of temporary layoffs and recalls. The focus of Mortensen (1990) is on the effect of unemployment benefits on worker's optimal search behavior, in particular he shows, that both the incidence and duration of unemployment increase with the UI benefit ratio, but the effect on the incidence of attached unemployment is larger than that on the incidence of unattached unemployment. This study differs from Pissarides (1982) and Mortensen (1990) in that it considers endogenous wage setting obtained by bargaining between workers and firms in the absence of on-the-job search and given a constant recall probability. The new focus of the current study on wage setting in search equilibrium with temporary layoffs permits analysis of an endogenous wage dispersion arising from the differences in outside options of attached and unattached unemployed. In addition, this study allows for agency problems in wage setting such as the limited commitment of workers and firms as well as the two-sided resistance to unfavorable changes of wages.

Wage dispersion is a well studied phenomenon arising in models with on-the-job search. Originally wage dispersion has been documented in the studies of random search with wage posting such as Burdett and Mortensen (1998), Postel-Vinay and Robin (2002b), Burdett and Coles (2003) and Stevens (2004). Burdett and Mortensen (1998) consider wage-posting in a labour market, where firms offering higher wages gain from a reduced quit rate of the worker. In the equilibrium firms are indifferent between offering a low wage and experiencing a high worker turnover versus a high wage and a low worker turnover. This mechanism gives rise to a continuous wage distribution among identical workers and firms. Burdett and Coles (2003) as well as Stevens (2004) extend this approach by al-

lowing firms to post wage-tenure contracts, and show, that there exists a nondegenerate equilibrium distribution of initial wage offers. A similar result is obtained by Postel-Vinay and Robin (2002b), who construct an equilibrium search model with on-the-job search and allow employers to counter the wage offers received by their employees.

The first attempt to analyze features of a model with on-the-job search and Nash bargaining has been done in Pissarides (1994). However, the simplifying assumption, that workers quit their previous job once a match with a new employer is formed, does not give rise to the endogenous wage dispersion. Shimer (2006) argues, that in a model with on-the-job search and strategic bargaining, the set of feasible payoffs is typically nonconvex because an increase in the wage raises the duration of an employment relationship. He further finds, that the subgame perfect equilibrium of such a bargaining model is no longer unique, nevertheless there exist market equilibria with a continuous wage distribution in which identical firms bargain to different wages.

Finally, Cahuc, Postel-Vinay and Robin (2006) propose an equilibrium search model with strategic wage bargaining, on-the-job search and counteroffers by competing firms. The cross-sectional distribution of wages is then composed of three components: a worker fixed effect, an employer fixed effect and a random effect, characterizing the most recent wage mobility of the worker. This study differs from the existing literature on wage dispersion in that it does not consider on-the-job search by employed individuals. Instead the focus of the present study is on search-in-attachment by unemployed individuals on a temporary layoff. Thus the endogenous differentiation of unemployed workers into attached and unattached gives rise to a binary equilibrium wage distribution.

3.3 Labour market modeling framework

The labour market is characterized by the following properties. There is a unit mass of infinitely lived workers and an endogenous number of firms. Workers and firms are ex-ante identical, risk neutral and do not have access to credit markets. Both types of agents are assumed to have short memory meaning that they only can keep records of their latest attachment. Firms and workers share a common constant discount factor r .

There are two types of idiosyncratic productivity shocks in the model. Persistent productivity shocks arrive at the Poisson rate γ and imply a permanent separation between a job and a worker. As a result of the persistent productivity shock the job is permanently destroyed and the worker becomes an unattached unemployed. Temporary productivity shocks arrive with a Poisson arrival rate δ . Upon the temporary productivity shock, the productivity variable \tilde{y} can take one of the two possible realizations $\{y, y^0\}$ so that the following productivity switching rule applies:

$$\tilde{y} = \begin{cases} y & \text{with probability } p \\ y^0 & \text{with probability } 1 - p \end{cases}$$

where $y > y^0$ and $0 \leq p \leq 1$. The initial productivity of a hired worker is assumed to be high $\tilde{y} = y$. When the productivity realization is low $\tilde{y} = y^0$, firms have an option to use a temporary layoff, so that each job position can be either filled with a worker and producing output, filled with a worker but neither producing nor searching (temporary layoff) or vacant and searching for a worker. Workers on a temporary layoff are referred to as attached unemployed, while jobs attached to a worker on a temporary layoff are referred to as inactive. Workers on a temporary layoff do not receive wages, but are attached to the firm and may be recalled to continue producing.

Independent of the type of unemployment (attached or unattached) workers participate in job search and receive an exogenous unemployment insurance denoted by z . I assume that search is costless for both types of the unemployed, but the cost is prohibitively high for the employed workers so, that workers do not search on the job. Searching unemployed workers find a new job with the flow probability $\lambda(\theta)$, which is an increasing function of its argument, where $\theta = v/u$ denotes the market tightness variable, u – the unemployment rate and v – the vacancy rate. In contrast to workers, search is costly for firms, who pay a constant flow cost of maintaining a vacancy – denoted by c – and find a worker at the corresponding Poisson arrival rate $q(\theta) = \lambda(\theta)/\theta$, decreasing in θ . This follows from the standard assumptions concerning the properties of a matching function: homogeneous of degree one, increasing and concave in both arguments u and v .

Wages are determined via the concept of Nash bargaining, and there is a single wage to be defined in the contract. Furthermore, it is assumed, that a contract can commit the two parties to future payments to be made while the match continues, however either

counterparty may terminate the contract at any time, hence contracts are characterized by two-sided limited commitment. In addition, either counterparty has an option but not an obligation to offer, reject or accept the terms for contract renegotiation. This means that contract renegotiations may only take place as a result of a binding participation constraint for either of the contracting parties.

3.4 Search equilibrium with temporary layoffs

3.4.1 Decentralized equilibrium: workers

Suppose first that the necessary condition for the existence of the equilibrium with temporary layoffs is fulfilled, this condition is further derived in section 3.4.4. Denote U – surplus of an unattached unemployed worker and W^1 – surplus of a worker employed at wage w^1 (hired from unattached unemployment or recalled). Bellman equations for these two groups of workers are given by:

$$rU = z + \lambda(\theta)(W^1 - U) \quad (3.4.1)$$

$$rW^1 = w^1 - \delta(1 - p)(W^1 - L) - \gamma(W^1 - U) \quad (3.4.2)$$

where L denotes surplus of a worker on a temporary layoff, not producing and searching for a job. Additionally, let W^2 denote surplus of a worker hired from attached unemployment and receiving wage w^2 . Bellman equations for workers in attached unemployment and those who changed the employer can be written as follows:

$$rL = z + \lambda(\theta)(W^2 - L) + \delta p(W^1 - L) - \gamma(L - U) \quad (3.4.3)$$

$$rW^2 = w^2 - \delta(1 - p)(W^2 - L) - \gamma(W^2 - U) \quad (3.4.4)$$

The labour market dynamics corresponding to the equilibrium with temporary layoffs is presented in figure 3.1.

As follows from equation (3.4.3) surplus value L can be additionally written as:

$$L = \frac{z + \delta p W^1 + \lambda(\theta) W^2 + \gamma U}{r + \gamma + \delta p + \lambda(\theta)}$$

so that the surplus of an attached unemployed worker is increasing in the probability to find a new job $\lambda(\theta)$ and in the probability to be recalled back to the previous job δp . Denote $d_2(\theta) = \lambda(\theta)/(r + \gamma + \delta p + \lambda(\theta))$ – conditional probability to exit the temporary

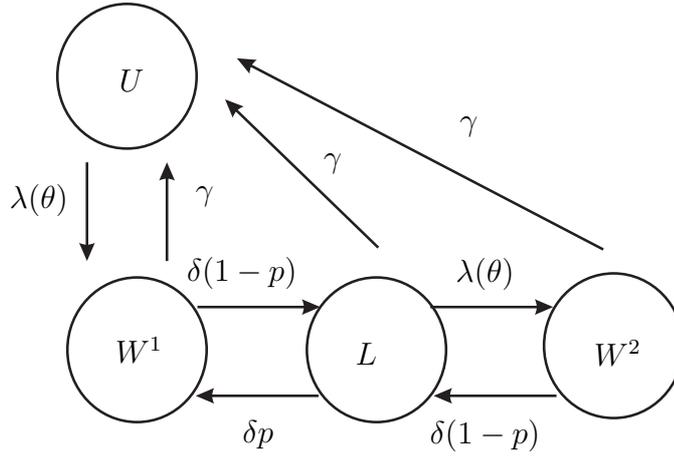


Figure 3.1: Labour market dynamics

layoff state into a new job, similarly denote $d_1(\theta) = \delta p / (r + \gamma + \delta p + \lambda(\theta))$ – conditional probability to be recalled to the previous employer and $d_0(\theta) = r / (r + \gamma + \delta p \lambda(\theta))$. Then the surplus value of a worker on a temporary layoff becomes:

$$L - U = d_0(\theta)(Z - U) + d_1(\theta)(W^1 - U) + d_2(\theta)(W^2 - U)$$

where $Z = z/r$, which can also be written as:

$$L - U = d_1(\theta)(W^1 - U) + d_2(\theta)\Delta W \tag{3.4.5}$$

where $\Delta W = W^2 - W^1$, since it is true that $d_0(\theta)(Z - U) + d_2(\theta)(W^1 - U) = 0$. Note here that $d_1(\theta)$ is a decreasing function of θ and $d_2(\theta)$ is an increasing function of θ . Workers employed from unattached unemployment enter wage negotiations with their employer and obtain wage w^1 with a corresponding surplus value W^1 . Similarly, workers recalled to their previous employer sign a new labour contract but continue receiving wage w^1 since their outside option (unattached unemployment) remains unchanged. Workers employed from attached unemployment enter wage negotiations with their employer and obtain wage w^2 with a corresponding surplus value W^2 . It is assumed that attachment to a previous employer is destroyed, as soon as a labour contract with a new employer is signed, so that every worker can have at most one attachment. This assumption implies, that workers, who were employed at high wage w^2 but experienced a spell of layoff unemployment, obtain a lower wage w^1 after they are recalled.

3.4.2 Decentralized equilibrium: firms

Denote J^1 – surplus of a job paying wage w^1 (filled with a worker from unattached unemployment or recalled). Additionally let T denote surplus of a job filled with a worker on a temporary layoff. Bellman equations for J^1 and T can be written as:

$$rJ^1 = y - w^1 - \delta(1 - p)(J^1 - T) - \gamma J^1 \quad (3.4.6)$$

$$rT = \delta p(J^1 - T) - \lambda(\theta)T - \gamma T \quad (3.4.7)$$

Surplus value of an inactive firm T can be expressed in a simplified way:

$$T = d_1(\theta)J^1$$

Finally let J^2 denote surplus of a job filled with a worker from attached unemployment and paying wage w^2 . The Bellman equation for J^2 is given by:

$$rJ^2 = y - w^2 - \delta(1 - p)(J^2 - T) - \gamma J^2 \quad (3.4.8)$$

which means that firms obtain net flow profits $y - w^2$ and become inactive at the Poisson arrival rate $\delta(1 - p)$.

3.4.3 Wage determination

Both wages w^2 and w^1 are determined via the concept of Nash bargaining. Consider a worker on a temporary layoff negotiating with a new employer. Outside option of such a worker is to remain in attached unemployment and search for another job or to continue producing upon a recall from the previous employer, so that the rent of such a worker is given by $W^2 - L$. The rent of a firm negotiating with an attached worker is given by $J^2 - V$, where V denotes surplus of an open vacancy. Wage w^2 is then determined in the following way:

$$\max_{w^2} (W^2 - L)^\beta (J^2 - V)^{1-\beta} \quad (3.4.9)$$

$$\text{where } W^2 - L = \frac{w^2 - (r + \gamma)L + \gamma U}{r + \gamma + \delta(1 - p)} \quad \text{and} \quad J^2 = \frac{y - w^2 + \delta(1 - p)T}{r + \gamma + \delta(1 - p)}$$

Here β denotes the worker's bargaining power. Firms and workers treat values L and T exogenously and in the equilibrium the free entry condition implies $V = 0$, this gives rise to the following wage expression:

$$w^2 = \beta[y + \delta(1 - p)T] + (1 - \beta)[(r + \gamma)L - \gamma U] \quad (3.4.10)$$

Now consider a worker in unattached unemployment negotiating with some employer. Outside option of such a worker is to remain in unattached unemployment, so that the rent of this worker is given by $W^1 - U$. The firm rent is given by $J^1 - V$. Note that this bargaining problem is the same for a worker on a temporary layoff recalled by his previous employer. The optimization problem is given by:

$$\max_{w^1} (W^1 - U)^\beta (J^1 - V)^{1-\beta} \quad (3.4.11)$$

$$\text{where } W^1 - U = \frac{w^1 - rU + \delta(1-p)(L-U)}{r + \gamma + \delta(1-p)} \quad \text{and} \quad J^1 = \frac{y - w^1 + \delta(1-p)T}{r + \gamma + \delta(1-p)}$$

Wage expression resulting from this optimization problem is then:

$$w^1 = \beta[y + \delta(1-p)T] + (1-\beta)[rU - \delta(1-p)(L-U)] \quad (3.4.12)$$

so that $w^2 - w^1 = (1-\beta)(L-U)(r + \gamma + \delta(1-p))$. This means that attached unemployed negotiate a higher wage $w^2 > w^1$ than the unattached due to the fact, that $L > U$ which also means, that attached unemployed have a higher reservation wage, since they can be recalled to their previous employer. Overall, attached unemployed negotiate a higher wage w^2 with a new employer as opposed to attached unemployed negotiating with their previous employer.

Given the equilibrium wage equations (3.4.10) and (3.4.12) the tuple of surplus values $\{U, T, W^1, W^2, J^1, J^2\}$ can be expressed in terms of the total surplus $S^1 \equiv J^1 + W^1 - U$ and the total surplus $S^2 \equiv J^2 + W^2 - L$:

$$W^1 = U + \beta S^1 \quad J^1 = (1-\beta)S^1 \quad (3.4.13)$$

$$rU = z + \lambda(\theta)\beta S^1 \quad T = d_1(\theta)(1-\beta)S^1 \quad (3.4.14)$$

$$W^2 = L + \beta S^2 \quad J^2 = (1-\beta)S^2 \quad (3.4.15)$$

In addition the surplus value $L - U$ can be obtained from the following expression:

$$(L - U) = d_1(\theta)\beta S^1 + d_2(\theta)\Delta W$$

This means that a reduced tuple of variables $\{\theta, S^1, S^2, \Delta W\}$ is now sufficient to characterize surplus values $\{U, L, T, W^1, W^2, J^1, J^2\}$.

3.4.4 The free-entry condition

Necessary condition for the existence of the equilibrium with temporary layoffs requires rents from a potential wage renegotiation to be negative, meaning that the productivity value y^0 should be sufficiently low. Otherwise workers and firms would benefit from sharing positive rents from renegotiation and continuing the production process. To sum up, workers and firms separate upon a negative productivity shock, if the continuation surplus is lower than the total surplus of a temporary layoff:

$$\frac{\bar{y}^0 - rU}{r + \gamma} \leq T + L - U \quad \text{where} \quad \bar{y}^0 = y^0 + \frac{\delta p \Delta y}{r + \gamma + \delta} \quad (3.4.16)$$

\Leftrightarrow

$$y^0 \leq y - \frac{y - [rU + (r + \gamma)(T + L - U)]}{r + \gamma + \delta(1 - p)}(r + \gamma + \delta) \equiv y_*^0 \quad (3.4.17)$$

where the left-hand side of inequality (3.4.16) stands for the surplus from continued production, while the right-hand side is the surplus from temporary separation. Equation (3.4.17) implies that the productivity value y^0 should be low enough for the rent from renegotiation to be negative. The equilibrium with an expectation of wage renegotiation is described in the next section.

Now assume that condition (3.4.17) is fulfilled, this case gives rise to the equilibrium with temporary layoffs and between-job wage dispersion. Denote α – probability for a vacant job to be contacted by an unattached unemployed, so that $1 - \alpha$ is the probability for a vacant job to be contacted by an attached unemployed. These probabilities can be found as:

$$\alpha = \frac{u_1}{u_1 + u_2} \quad \text{and} \quad 1 - \alpha = \frac{u_2}{u_1 + u_2}$$

where u_1 denotes a share of unattached unemployed workers in the economy and u_2 denotes a share of the attached unemployed. Then the surplus of a vacant job paying the flow cost c can be written as follows:

$$rV = -c + q(\theta)(\alpha J^1 + (1 - \alpha)J^2) \quad (3.4.18)$$

In the equilibrium it should hold that $V = 0$, then equation (3.4.18) becomes:

$$\frac{c}{q(\theta)} = \alpha J^1 + (1 - \alpha)J^2 \quad (3.4.19)$$

This means that the expected cost from an open vacancy should be equal to the expected firm surplus from a filled job. Denote e_1 – share of workers employed at wage w^1 and e_2

– share of workers employed at wage w^2 . Given that the total labour force is normalized to 1 it holds that $u_1 + u_2 + e_1 + e_2 = 1$. Flow transition rates between the four groups of workers are presented in table 3.2.

Table 3.2: Flow transition rates between states

State	u_1	u_2	e_1	e_2
u_1	–	–	$\lambda(\theta)$	–
u_2	γ	–	δp	$\lambda(\theta)$
e_1	γ	$\delta(1-p)$	–	–
e_2	γ	$\delta(1-p)$	–	–

These transition rates correspond to the following system of differential equations in variables u_1, u_2, e_1 and e_2 :

$$\begin{cases} \dot{u}_2 = \delta(1-p)(e_1 + e_2) - \delta p u_2 - \lambda(\theta)u_2 - \gamma u_2 \\ \dot{e}_1 = \lambda(\theta)u_1 + \delta p u_2 - \delta(1-p)e_1 - \gamma e_1 \\ \dot{e}_2 = \lambda(\theta)u_2 - \delta(1-p)e_2 - \gamma e_2 \\ u_1 = 1 - u_2 - e_1 - e_2 \end{cases} \quad (3.4.20)$$

Each of the equations above implies, that change in a given state variable is equal to the inflow of workers into the state minus the outflow of workers. The unique stable stationary solution with $\dot{u}_2 = 0$, $\dot{e}_1 = 0$ and $\dot{e}_2 = 0$ is then:

$$u_1 = \frac{\gamma}{\gamma + \lambda(\theta)} \quad u_2 = \frac{\delta(1-p)}{\gamma + \delta + \lambda(\theta)} \cdot \frac{\lambda(\theta)}{\gamma + \lambda(\theta)} \quad (3.4.21)$$

$$e \equiv e_1 + e_2 = \frac{\gamma + \delta p + \lambda(\theta)}{\gamma + \delta + \lambda(\theta)} \cdot \frac{\lambda(\theta)}{\gamma + \lambda(\theta)} \quad (3.4.22)$$

This means that the probability for a firm to contact an unattached unemployed is:

$$\frac{1 - \alpha}{\alpha} = \frac{u_2}{u_1} = \frac{\lambda(\theta)\delta(1-p)}{\gamma(\gamma + \delta + \lambda(\theta))} \quad (3.4.23)$$

Probability α is a decreasing function of the market tightness θ . This means that a higher job-finding rate $\lambda(\theta)$ reduces the number of unattached unemployed and therefore also the probability for a firm to contact an unattached worker.

To simplify the following representation of the model, denote $s(\theta) = \gamma + \delta(1-p)(1 - d_1(\theta))$ – endogenous job separation rate in the model. Job separations are due to a permanent productivity shock arriving at rate γ or due to a temporary productivity shock arriving at rate $\delta(1-p)$. In the state of a temporary layoff workers are not available

for a recall with a probability $(1 - d_1(\theta))$, so that the total separation rate becomes: $s(\theta) = \gamma + \delta(1 - p)(1 - d_1(\theta))$. The job separation rate is an increasing function of θ , since a higher probability of finding an external job for a worker on a temporary layoff reduces the probability, that the worker is still available for a recall $d_1(\theta)$. Using the definition of $s(\theta)$, surplus value $S^1 = J^1 + W^1 - U$ can be written as:

$$S^1 = J^1 + W^1 - U = \frac{y - w^1}{r + s(\theta)} + \frac{w^1 - rU + d_2(\theta)\Delta W}{r + s(\theta)} = \frac{y - rU + d_2(\theta)\Delta W}{r + s(\theta)}$$

The resulting search equilibrium with temporary layoffs is characterized in proposition 3.1.

Proposition 3.1: *In the presence of negative rents from renegotiation, the layoff risk is realized, the equilibrium is characterized by between-job wage dispersion and is represented by a reduced tuple of variables $\{\alpha, \theta, S^1, S^2, \Delta W\}$, satisfying equation (3.4.23), equations (a)-(c) below as well as the free-entry condition (d). The necessary condition for the equilibrium existence is $y^0 \leq y_*^0$.*

(a.) *The total surplus value S^2 is given by:*

$$S^2 = S^1(1 - d_1(\theta)\beta) - d_2(\theta)\Delta W$$

(b.) *The total surplus S^1 is given by:*

$$S^1 = \frac{y - z + c\theta + \delta(1 - p)d_2(\theta)\Delta W}{r + \lambda(\theta) + s(\theta)}$$

(c.) *The surplus difference ΔW is given by:*

$$\Delta W = \frac{(1 - \beta)d_1(\theta)\beta S^1}{1 - (1 - \beta)d_2(\theta)} \quad (3.4.24)$$

(d.) *The free-entry condition defines θ :*

$$\frac{c}{q(\theta)} = (1 - \beta)S^1 \left[1 - \frac{(1 - \alpha)d_1(\theta)\beta}{1 - (1 - \beta)d_2(\theta)} \right] \quad (3.4.25)$$

Proof: Appendix 3.10.1.

The free entry condition (3.4.25) equates expected costs from creating a vacancy on the left-hand side to the expected surplus of a filled job on the right-hand side. Note that in the absence of worker-firm attachment the probability for the firm to contact an

unattached worker is $\alpha = 1$, so that the right-hand side of equation (3.4.25) is simplified to $(1 - \beta)S^1$, which means, that firms obtain a surplus share $(1 - \beta)$ of the total job surplus S^1 . For $0 < \alpha < 1$ expression in square brackets in (3.4.25) is strictly smaller than 1, which means that the expected firm surplus is less than $(1 - \beta)S^1$. This is due to the fact that firms hiring attached unemployed have to pay a higher wage w^2 .

Also note that wage dispersion in the model is a consequence of the interior value of the bargaining power $0 < \beta < 1$. As follows from (3.4.24) $\beta = 0$ implies $w^1 = w^2 = rU$ due to the fact that $U = L = W^1$, so that neither employment, nor an attachment to the previous employer is valuable for the worker. For $\beta = 1$ the situation is similar in that $w^1 = w^2 = y + \delta(1 - p)T$, so that workers obtain the full maximum rent of the job and do not profit from an additional attachment.

The final step to characterize the model with temporary layoffs is to describe the properties of the Beveridge curve. The market tightness variable is defined as $\theta = v/u$, where $u = u_1 + u_2$ – total unemployment rate in the economy. This means that equations (3.4.21) define an implicit functional relationship between the number of open vacancies v and the equilibrium unemployment u – the Beveridge curve:

$$u = u_1 + u_2 = \frac{\gamma}{\gamma + \lambda(\theta)} \left[1 + \frac{\delta(1 - p)\lambda(\theta)}{\gamma(\gamma + \delta + \lambda(\theta))} \right], \quad \text{where } \theta = v/u$$

Proposition 3.2: *In the equilibrium with temporary layoffs and incomplete worker attachment, the Beveridge curve is downward-sloping, in particular $\partial u/\partial v < 0$ under the assumption that $\eta_q < 1$, where*

$$\eta_q = -\frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)} \quad - \text{elasticity of the job filling rate } q(\theta)$$

Proof: Denote μ_θ – elasticity of the unemployment rate with respect to θ :

$$\mu_\theta \equiv -\frac{\partial u}{\partial \theta} \frac{\theta}{u}$$

Appendix 3.10.2 shows that $0 < \mu_\theta < 1$ if $\eta_q < 1$. Additionally, the elasticity of the Beveridge curve can be expressed as:

$$\frac{\partial u}{\partial v} \theta = -\frac{\mu_\theta}{1 - \mu_\theta} < 0$$

This means that a higher market tightness θ is associated with a higher number of open vacancies v and a lower unemployment rate u .

3.5 Wage renegotiation in the presence of layoff risk

If condition (3.4.16) is violated, labour contracts are renegotiated upon a negative productivity shock. This section characterizes an equilibrium with wage renegotiations. Denote w^L – new wage negotiated between the worker and the firm in the low productivity state. Similarly denote w^H – initial wage negotiated between a firm and a worker upon hiring in the expectation of wage renegotiation. The corresponding firm and worker surplus values are denoted J^H and W^H respectively. After the first production spell workers and firms bargain over a new wage w^L with the corresponding surplus values J^L and W^L :

$$\begin{aligned}(r + \gamma)J^L &= \bar{y}^0 - w^L \\ (r + \gamma)(W^L - U) &= w^L - rU\end{aligned}$$

Note that wage w^L applies till the end of the employment relationship (including periods of high and low productivity) since worker's threat to quit a productive firm into unemployment is not credible. Outside options of a worker-firm pair are given by T and L , so that the Nash-bargaining problem over w^L becomes:

$$\begin{aligned}\max_{w^L} (W^L - L)^\beta (J^L - T)^{1-\beta} \\ \text{and } w^L = \beta[\bar{y}^0 - rU - (T + L - U)(r + \gamma)] + (r + \gamma)(L - U) + rU\end{aligned}\quad (3.5.1)$$

where the outside option values T and L can be obtained as:

$$T = d_1(\theta)J^H \quad (L - U) = \frac{d_1(\theta)(W^H - U)}{1 - (1 - \beta)d_2(\theta)}\quad (3.5.2)$$

Above expression for w^L is an optimal solution to the Nash bargaining problem between the firm and the worker as long as equation (3.4.16) is violated, meaning that the total surplus from a layoff $T + L - U$ is sufficiently low: $(T + L - U)(r + \gamma) \leq \bar{y}^0 - rU$. Bellman equations for W^H and J^H are then:

$$\begin{aligned}rW^H &= w^H - \delta(1 - p)(W^H - W^L) - \gamma(W^H - U) \\ rJ^H &= y - w^H - \delta(1 - p)(J^H - J^L) - \gamma J^H\end{aligned}$$

so that the surplus values $W^H - U$ and J^H can be expressed as:

$$\begin{aligned}(r + \gamma + \delta(1 - p))(W^H - U) &= w^H - rU + \delta(1 - p)(W^L - U) \\ (r + \gamma + \delta(1 - p))J^H &= y - w^H + \delta(1 - p)J^L\end{aligned}$$

The Nash bargaining problem over w^H can be summarized as follows:

$$\max_{w^H} (W^H - U)^\beta (J^H - V)^{1-\beta}$$

Given that in the equilibrium $V = 0$ expression for w^H takes the following form:

$$\begin{aligned} w^H &= \beta[y + \delta(1-p)J^L] + (1-\beta)[rU - \delta(1-p)(W^L - U)] \\ &= \beta[y + \delta(1-p)T] + (1-\beta)[rU - \delta(1-p)(L - U)] \end{aligned} \quad (3.5.3)$$

The functional form of w^H exactly coincides with the functional form of w^L , this is due to the fact that the net surplus in the low output state $W^L - L + J^L - T$ is split in the proportion β :

$$\beta(J^L - T) = (1-\beta)(W^L - L) \quad (3.5.4)$$

so that the initial labour contract is exactly the same *ceteris paribus* regardless of whether wage negotiations will take place in the future or not. Nevertheless surplus values W^H and J^H are such that $W^H \geq W^L$ and $J^H \geq J^L$ since workers and firms expect to share the rents in the future.

Let $S^H \equiv J^H + W^H - U$ - total surplus of a new job and $S^L \equiv J^L + W^L - U$ - total surplus in a low productivity state. Then Nash bargaining implies that:

$$W^H = U + \beta S^H \quad W^L - L = \beta[S^L - (T + L - U)] \quad (3.5.5)$$

$$J^H = (1-\beta)S^H \quad J^L - T = (1-\beta)[S^L - (T + L - U)] \quad (3.5.6)$$

$$rU = z + \lambda(\theta)\beta S^H \quad (3.5.7)$$

Equations (3.5.2) in a combination with (3.5.5)-(3.5.7) imply that search equilibrium with wage renegotiation can be summarized as a reduced vector of variables $\{S^H, S^L, \theta\}$ which is sufficient to characterize surplus values $\{U, T, L, W^L, J^L, W^H, J^H\}$. Properties of the equilibrium with wage renegotiation are summarized in proposition 3.3.

Proposition 3.3: *In the presence of positive rents from renegotiation the equilibrium is characterized by within-job wage dispersion and is represented by a tuple of variables $\{S^H, S^L, \theta\}$ satisfying conditions (a)-(c). The necessary condition for the equilibrium existence is: $y^0 \geq y_*^0$.*

(a.) The total surplus value S^L is given by:

$$S^L = \frac{\bar{y}^0 - rU}{r + \gamma}$$

(b.) The total surplus value S^H is given by:

$$S^H = \frac{\bar{y} - z + c\theta - \delta(1-p)rU/(r + \gamma)}{(r + \gamma + \delta(1-p) + \lambda(\theta))}$$

where $\bar{y} = y + \delta(1-p)\bar{y}^0/(r + \gamma)$.

(c.) The free-entry condition defines θ :

$$\frac{c}{q(\theta)} = (1 - \beta)S^H$$

The above equilibrium is characterized by within-job wage dispersion meaning that workers with the same actual productivity y may be obtaining different wages w^H or w^L depending on the history of their relationship with the employer. Denote e_H – equilibrium share of workers employed at wage w^H and e_L – equilibrium share of workers employed at wage w^L . In the equilibrium it should be true that $\dot{e}_H = 0$ and $\dot{e}_L = 0$, so

$$\begin{aligned} 0 &= \lambda(\theta)u - (\gamma + \delta(1-p))e_H \\ 0 &= \delta(1-p)e_H - \gamma e_L \end{aligned}$$

Proposition 3.4: *In the presence of layoff risk and positive rents from renegotiation the equilibrium shares of workers employed at wages w^H and w^L respectively and the equilibrium unemployment rate are given by:*

$$\begin{aligned} e_H &= \frac{\lambda(\theta)u}{\gamma + \delta(1-p)} & u &= \frac{\gamma}{\gamma + \lambda(\theta)} \\ e_L &= \frac{\delta(1-p)\lambda(\theta)u}{\gamma(\gamma + \delta(1-p))} \end{aligned}$$

Denote α_H – equilibrium fraction of workers employed at wages w^H . In the presence of layoff risk and positive rents from renegotiation equilibrium value of α_H is given by:

$$\alpha_H \equiv \frac{e_H}{e_H + e_L} = \frac{\gamma}{\gamma + \delta(1-p)}$$

The equilibrium fraction α_H is independent of the market tightness, and only depends on the exact characteristics of the production process γ and δ . In the absence of temporary productivity shocks $\delta = 0$, all workers obtain the initial wage $w^H = \beta y + (1 - \beta)rU$.

3.6 Social welfare and optimal policy

Hosios (1990) and further Pissarides (2000) show, that the Nash wage equation is not likely to internalize search externalities resulting from the dependence of transition probabilities $\lambda(\theta)$ and $q(\theta)$ on the tightness of the market. Nevertheless Hosios (1990) proves that search externalities may be internalized, if the following condition is satisfied: $\beta = \eta_q$, where η_q – elasticity of the job-filling rate $q(\theta)$. This section investigates efficiency properties of the equilibrium with Nash bargaining and temporary layoffs and shows, that the classical Hosios condition is not sufficient for the constrained efficiency of the decentralized equilibrium. To obtain this result, consider the problem of a social planner, whose objective is to maximize the expected net output per worker:

$$\max_{\theta} \int_0^{\infty} e^{-rt} \left[y(e_1 + e_2) + z(1 - e_1 - e_2) - c\theta(1 - e_1 - e_2) \right] dt \quad (3.6.1)$$

The social planner is subject to the same matching constraints as firms and workers, therefore the dynamics of employment and unemployment is the same as in the decentralized equilibrium:

$$\begin{aligned} \dot{u}_2 &= \delta(1 - p)(e_1 + e_2) - \delta p u_2 - \lambda(\theta)u_2 - \gamma u_2 \\ \dot{e}_1 &= \lambda(\theta)(1 - e_1 - e_2 - u_2) + \delta p u_2 - \delta(1 - p)e_1 - \gamma e_1 \\ \dot{e}_2 &= \lambda(\theta)u_2 - \delta(1 - p)e_2 - \gamma e_2 \end{aligned}$$

The social optimum satisfies the following first-order condition:

$$\frac{c}{q(\theta)} = (1 - \eta_q) \frac{y - z + c\theta}{r + \lambda(\theta) + s(\theta)} (1 - (1 - \alpha)d_1(\theta)) \quad (3.6.2)$$

The derivation of this condition is presented in Appendix 3.10.3. Comparing now the social condition (3.6.2) and the decentralized free entry condition (3.4.25) I find that the equilibrium is constrained inefficient. To see this recall that the free-entry condition is obtained from expression:

$$\frac{c}{q(\theta)} = \alpha(1 - \beta)S^1 + (1 - \alpha)(1 - \beta)S^2 \quad (3.6.3)$$

where $S^2 = J^2 + W^2 - L$ denotes the total surplus of a match with an attached unemployed:

$$S^2 = S^1 - (L - U) = S^1(1 - d_1(\theta)\beta) - d_2(\theta)\Delta W$$

The equilibrium inefficiency comes from the fact that the firm and an attached unemployed do not internalize the losses imposed on the previous employer of the worker. In

particular, the previous employer is losing an option to recall the worker, with a corresponding surplus value T . The social planner is taking this loss into account, so that $S^2 = S^1 - (T + L - U) = (1 - d_1(\theta))S^1$ in the optimal planner's solution. This externality imposed on the previous employer of the worker is not accounted for in the bargaining process between the worker and a new employer, so that firms hiring attached unemployed create too many jobs compared to the socially optimal level. These results are summarized in the following proposition.

Proposition 3.5: *Let $\beta = \eta_q < 1$, then:*

- (a). *Search equilibrium with temporary layoffs and wage dispersion described in proposition 3.1 is constrained inefficient;*
- (b). *The market tightness in the decentralized equilibrium is above the socially optimal level, implying excessive job creation;*

Proof: The proof of part (b) of the proposition follows from the fact that:

$$\frac{\beta}{1 - (1 - \beta)d_2(\theta)} = \frac{\beta}{\beta + (1 - \beta)(1 - d_2(\theta))} < 1$$

and $\Delta W > 0$, so that $S^1 > (y - z + c\theta)/(r + \lambda(\theta) + s(\theta))$.

The next question addressed in this section is: which tax policy of the planner can decentralize the efficient labour allocation in the equilibrium? As shown above the main source of the inefficiency of the decentralized equilibrium is surplus loss of the previous employer of the worker resulting from worker's decision to start a new job. Throughout the paper it is assumed, that firms can observe worker's attachment status and so does also the social planner. This means the tax imposed on attached unemployed taking on new employment should restore efficiency of the decentralized equilibrium. Let τ denote an income tax imposed on attached unemployed starting job with a new employer and s - income subsidy for every worker. Bellman equations for W^1 , W^2 , L and U are then modified in the following way:

$$\begin{aligned} rU &= z + s + \lambda(\theta)(W^1 - U) \\ rW^1 &= w^1 + s - \delta(1 - p)(W^1 - L) - \gamma(W^1 - U) \\ rL &= z + s + \lambda(\theta)(W^2 - L) + \delta p(W^1 - L) - \gamma(L - U) \\ rW^2 &= w^2 - \tau + s - \delta(1 - p)(W^2 - L) - \gamma(W^2 - U) \end{aligned}$$

Then the surplus difference ΔW is given by:

$$\Delta W = \frac{w^2 - \tau - w^1}{r + \gamma + \delta(1 - p)} = \frac{\beta[(1 - \beta)d_1(\theta)S^1 - F]}{1 - (1 - \beta)d_2(\theta)} \quad (3.6.4)$$

where F denotes the present value of tax payments: $F = \tau/(r + \gamma + \delta(1 - p))$. Note that from equation (3.4.14) it follows that $T = (1 - \beta)d_1(\theta)S^1$, where T is surplus of an inactive firm attached to the worker. This means imposing a tax such that $T = F$ will eliminate the real wage inequality: $w^2 - \tau = w^1$. The surplus value S^2 becomes:

$$S^2 = S^1 - (L - U) - F = S^1 - (L - U) - T = S^1(1 - d_1(\theta)) \quad (3.6.5)$$

This equation in a combination with $\beta = \eta_q$ (to internalize the search externality) guarantees, that the market tightness in the decentralized equilibrium is set optimally, and that job creation coincides with the solution of the social planner. The amount of subsidies s is then obtained from the balanced budget constraint of the planner:

$$(\tau - s)e_2 = s(e_1 + u_1 + u_2)$$

This means that the total net income flow $\tau - s$ paid by attached unemployed starting job with a new employer is distributed to the other three groups of workers $e_1 + u_1 + u_2$. This result is stated in proposition 3.6:

Proposition 3.6: *Let $\beta = \eta_q < 1$. Welfare in the decentralized equilibrium with temporary layoffs can be raised by imposing a tax τ on attached unemployed starting job with a new employer, such that $F = T = d_1(\theta)(1 - \eta_q)S^1$. This tax policy eliminates real wage inequality $w^2 - \tau = w^1$ and is equivalently written as:*

$$F \equiv \frac{\tau}{r + \gamma + \delta(1 - p)} = d_1(\theta)(1 - \eta_q) \frac{y - z + c\theta}{r + s(\theta) + \lambda(\theta)} \quad (3.6.6)$$

The balanced budget constraint of the planner implies that taxes are paid out as subsidies s obtained from: $s = \tau e_2$.

3.7 Empirical estimation

In this section a testable hypothesis based on the theoretical model from section 3.4 is formulated and confronted with the statistical data. The model predicts, that workers on a temporary layoff recalled to the previous employer obtain low wage $w_{t+1} = w_1$; this result endogenously obtains in the model due to the bargaining process between workers

and firms, since the outside option of a worker, bargaining with a previous employer, is to become an unattached unemployed. Wage w_1 prevails in this case and is independent of the previous wage of the worker w_t . In addition, the model allows to formulate an expression for the expected wage of a worker taking job with a new employer. With probability α the worker is an unattached unemployed and will bargain a wage w_1 , but with probability $1 - \alpha$ the worker is attached to the previous employer and has a higher reservation wage, so the contract wage with a new employer will be w_2 . This means that the expected value of wage for a worker taking employment with a new firm is: $\alpha w_1 + (1 - \alpha)w_2$. This allows to formulate the following hypothesis:

Hypothesis: *For any value of the previous wage w_t expected wage change Δw of an employee recalled to work for the previous employer is lower than the expected wage change of an employee taking job with a new employer:*

$$\begin{aligned} E_t[\Delta w | Recall_{t+1} = 1] &= E[w_{t+1} | Recall_{t+1} = 1] - w_t = w_1 - w_t \\ E_t[\Delta w | New job_{t+1} = 1] &= \alpha w_1 + (1 - \alpha)w_2 - w_t \geq w_1 - w_t \end{aligned}$$

To estimate the effect of recalls on wage changes I use the data from the German Socio-Economic Panel (GSOEP), a large micro-dataset administered by the Deutsches Institut für Wirtschaftsordnung. The sample covers the period of 5 years from 2003 to 2007 and includes the total of 7328 observations on job movers. The net of missing data sample contains 2595 observations. The wage change Δw is coded in the questionnaire as a dummy variable:

$$y_i = \begin{cases} 1 & \text{if } \Delta w_i = w_{it+1} - w_{it} > 0 \\ 0 & \text{if } \Delta w_i = w_{it+1} - w_{it} \leq 0 \end{cases} \quad (3.7.1)$$

so that the probit regression model is used to forecast the direction of wage changes. Index $i = 1, \dots, 2595$ here denotes the observation of wage change, while indices t and $t + 1$ are used to mark the previous and the new wage of the employee. The probability of a positive change $y_i = 1$ is then given by

$$P\{y_i = 1 | X_i\} = P\{\Delta w_i > 0 | X_i\} = \Phi(X_i' \beta) \quad (3.7.2)$$

where $\Phi(\cdot)$ is the cumulative density function of the normal distribution, β is the parameter vector and X_i is the vector of explanatory variables of individual i .

About 44% of the respondents in the final sample have reported a wage improvement compared to the previous job. Table 3.3 presents an overview of the explanatory variables. The list of individual characteristics consists of the following variables *Age*, *Education*, *German* and *Gender*. Table 3.3 shows that the representative employee in the sample is 36 years old and has completed approximately 13 years of schooling, 93.8% of the employees have German nationality and 52.4% of the employees are males. These variables create an overview of the representative individual in the sample, at the same time variables *Education*, *German*, and *Gender* are deterministic for the same individual so that their effect on the probability of wage improvement is predicted to be insignificant. A number of empirical studies show that variable *Age* enters quadratically into the wage equation, meaning that wage is increasing with age up to some maximum level and is decreasing thereafter. Variable *Age* for this reason is then predicted to have a negative effect on the probability of wage improvement.

Variable	Mean	Description
<i>Dependent variable</i>		
Pay improved	0.443	1=Earnings have improved in the new job
<i>Individual characteristics</i>		
Age	36.06	Age of the individual in years [18, ..., 68].
Education	12.81	Amount of education or training in years [7, ..., 18]
German	0.938	1=German nationality
Gender	0.524	1=Male
<i>Previous job characteristics</i>		
Tenure	4.625	Number of years with a previous employer [0, ..., 43]
Recall	0.048	1=Returned to the previous employer
<i>Reason for separation</i>		
Quit	0.404	1=Previous employment ended in a quit
Layoff	0.185	1=Previous employment ended in a layoff
Job closure	0.121	1=Previous employment ended due to job closure
Temp. contract	0.164	1=Temporary contract expired
<i>Job comparison</i>		
Promotion	0.330	1=Promotion possibilities have improved in the new job
Benefits	0.228	1=Social benefits provision has improved in the new job
Security	0.262	1=Work security has improved in the new job

Table 3.3: Explanatory variables

The major variable of interest in this study is *Recall*, this variable takes value 1 if the worker returns to the previous employer, and zero otherwise. In the original sample of 7328 observations recall rate is estimated to be 8.3%, but is reduced to only 4.8% in the

final sample. The sign on the regression coefficient of *Recall* should then be negative and significant in order to support the above hypothesis. Variable *Tenure* measures workers' experience with a previous employer. This variable traditionally has positive effect on wages, but job changes are associated with a loss of the accumulated tenure, so this variable is predicted to have a negative impact on the probability of wage improvement.

The group of variables *Quit*, *Layoff*, *Job closure* and *Temp. contract* are included in order to capture the "gains" from mobility. Note, that these variables are self-reported, specifically the respondents were asked "How did your previous job end?". Based on this data, quits comprise the largest category and amount to about 40% of the final sample; about 30% of job changes are due to layoffs and job closures and only 16.4% – are due to the end of a fixed-term contract. The omitted variable *Mutual separations* amounts to 12.6% of the sample and serves as a reference category. Variables *Layoff* and *Job closure* capture involuntary separations with a possible spell of involuntary unemployment and are therefore expected to have negative effects. In contrast, variable *Quit* captures voluntary mobility decisions and gains from possible job-to-job transitions; this variable is therefore expected to have a positive effect.

The final group of variables *Promotion*, *Benefits* and *Security* are included into the model to capture qualitative differences between the jobs. 33% of the respondents have obtained a promotion in the new job, while only 23% have obtained additional benefits and 26% have claimed an improved job security. A negative sign of the regression coefficient on each of these variables would imply substitution between wages and the respective job characteristic, while a positive sign implies complementarity.

Probit estimation results are presented in table 3.4. The second column of this table contains coefficients from the original estimation, while the reduced form regression including only significant variables is presented in the third column of table 3.4. A lower number of variables allows to increase the number of observations (to 3241) and therefore the precision of the estimated coefficients. The last column of table 3.4 contains marginal effects of the explanatory variables, which can be interpreted as a change in the probability of wage improvement corresponding to a unit change in the respective explanatory variable. All of the explanatory variables in the sample, except *Age* and *Tenure*, are bi-

nary variables, so the change in the probability of wage improvement given a unit change in the explanatory variable X_{ij} is given by:

$$\Delta P\{y_i = 1|X_0\} = P\{y_i = 1|X_0, X_{ij} = 1\} - P\{y_i = 1|X_0, X_{ij} = 0\} \quad (3.7.3)$$

where X_0 denotes characteristics of the representative individual:

$$X_0 = \{Age = 36, Recall = 0, Promotion = 0, Benefits = 0, \\ Security = 0, Layoff = 0, Quit = 0, Bancruptcy = 0\}$$

Table 3.4: Probit estimation results

Dependent variable $y_i = 1$ if wage improvement in the new job						
Variable	Coefficient	Standard deviation	Reduced form	Standard deviation	Probability change	Standard deviation
Constant	-.059	(.240)	-.030	(.155)		
Age	-.014**	(.006)	-.013**	(.005)	-.005**	(.002)
	<i>Previous job characteristics</i>					
Tenure	-.005	(.005)				
Recall	-.288**	(.132)	-.244**	(.110)	-.079**	(.034)
	<i>Job comparison</i>					
Promotion	.627**	(.059)	.638**	(.052)	.246**	(.020)
Benefits	.620**	(.067)	.612**	(.059)	.235**	(.024)
Security	.217**	(.064)	.186**	(.057)	.068**	(.021)
	<i>Reason for separation</i>					
Quit	.264**	(.084)	.180**	(.057)	.065**	(.021)
Layoff	-.165*	(.098)	-.254**	(.069)	-.082**	(.022)
Job closure	-.266**	(.111)	-.340**	(.090)	-.107**	(.027)
Temp. contract	.091	(.100)				
Observations	2595		3241		3241	
Pseudo R^2	0.1482		0.1415			
Log likelihood	-1518.3		-1911.2			

Standard deviations are given in parentheses; Two-tailed significance: * 10%, ** 5%;

Variables *Education*, *German*, *Gender* and year dummy variables are included at the initial stage but not significant at 10% significance level.

The Likelihood ratio test indicates an overall significance of the probit regression at 1% significance level: $LR = 528.14$. Variable *Recall* is significantly negative, meaning that recall to the job with a previous employer is associated with 7.9% lower probability of wage improvement compared to a job with a new employer. These result supports the hypothesis of worker-firm attachment and its implications for wages, suggested in the theoretical part of this paper. To some extent, this result is also anticipated in Burda

and Mertens (2001), who have used a merged German data sample from GSOEP and IAB (the social insurance data) to test for sample homogeneity including and excluding recalled individuals. Their findings show that the Chow test consistently rejects homogeneity of the two samples.

In line with the prediction, variable *Age* has negative effect on the probability of wage improvement for job movers. Age of the individual is often seen as a proxy for the potential experience, and so this finding is in accordance with the existing studies, i.e. for Germany Dustmann and Pereira (2005) have found that "wage gains at job changes... become negative towards the end of individuals' careers." (p.18). A similar finding is reported in Topel and Ward (1992), who find that between-job wage gains decline with experience in the US.

The coefficient on *Tenure* is negative but not significant, meaning that the loss of job-specific experience does not have effect on the probability of wage improvement. This finding is not unique for Germany, for example, Dustmann and Pereira (2005) find insignificant tenure effect in wage growth regressions; this is however different in the US, where Topel and Ward (1992) report that between-job wage gains decline with prior job tenure. One of the explanations of this difference is presented in Dustmann and Pereira (2005), who attribute the difference to a heavy use of apprenticeship training in Germany as opposed to the US. Apprenticeship training provides job-specific knowledge to workers prior to their first employment and therefore has a flattening effect on the ex-post wage growth of German workers.

Voluntary separations (quits) are associated with about 6.5% higher probability of wage improvement, while involuntary separations reduce this probability by 8.2% in the case of layoff and 10.7% in the case of job closure. At the same separations due to the end of a temporary contract are not significantly different from mutual separations, which are used as a reference category. These results are fully supported in the empirical literature: Mincer (1993) finds that voluntary transitions in the US lead to wage gains of between 10% and 20%, while Bartel and Borjas (1981) find that layoffs reduce wage growth over the two-year period by about 19 cents per hour. For Germany Burda and Mertens (2001) find that full-time men displaced in 1986 and subsequently reemployed in 1987 suffer a

reduction of wage growth of about 3.6% when compared with a reference group of continuously employed workers. Garcia-Perez and Rebollo-Sanz (2005) find that German workers tend to experience larger wage losses compared to the rest of countries, around 22%, followed by French, Spanish and Portuguese workers, who suffer wage losses of 14%, 10% and 9% respectively. Moreover, Garcia-Perez and Rebollo-Sanz (2005) report that in France, Germany and Portugal voluntary movers experience a small but positive return when changing jobs of around, 1% in France, 2% in Germany and 4% in Portugal.

Finally, variables *Promotion*, *Benefits* and *Security* have strong positive effects on the probability of wage improvement. In particular, job promotion is associated with 24.6% higher probability of wage improvement, followed by 23.5% increase for additional benefits and 6.8% increase for the improved job security. Table 3.7 shows empirical correlations of wages with the additional benefits paid in Germany in 2003:

13th Month Pay	0.44
14th Month Pay	0.19
Christmas Bonus	0.19
Vacation Bonus	0.23
Profit-sharing Bonus	0.32

Table 3.5: Correlations between benefit payments and wages

All of the benefit variables are positively associated with wages, in particular, strongest correlations are attained for the 13th month pay (0.44) and for the profit-sharing bonus payment (0.32). For a more detailed theoretical treatment of the correlation between wages and bonus payments see Chizhova (2008). These findings indicate strong complementarity between wages and other benefits in Germany, rather than substitution, and mean that firms paying higher wages also tend to provide higher benefits, better promotion possibilities and improved job security to the workers. For the theoretical explanation of the complementarity effect between wages and job security see Chizhova (2007).

3.8 Conclusions

This paper develops a search model with stochastic idiosyncratic productivity shocks and worker-firm attachments. The possibility to recall the previous attachment as well as the temporary nature of productivity fluctuations mutually motivate existence of temporary layoffs in the equilibrium. This equilibrium obtains for large productivity fluctuations,

sufficient to induce a temporary separation, otherwise mutual agreement on wage reduction between workers and firms eliminates the necessity for a layoff. In the equilibrium with temporary layoffs attachment is incomplete implying, that workers search for better job alternatives during the low productivity spells. Ex-post differentiation of unemployed workers into attached and unattached combined with Nash bargaining produces a binary equilibrium wage dispersion. Here attached unemployed bargain higher wages upon a match with a new employer as opposed to the unattached unemployed. So the paper contributes to the debate on endogenous wage dispersion.

Furthermore, this paper investigates welfare properties of the decentralized equilibrium with temporary layoffs by comparing it to the solution of the utilitarian social planner. As a result, the Hosios value of the bargaining power parameter does not any longer provide the constrained efficiency. The new type of the inefficiency in the model is explained by the fact, that workers bargaining with a new firm impose a negative externality on their previous employer, who is losing a valuable option to recall the employee upon a good productivity realization. This attachment externality is complementary to the classical search externality described in Hosios (1990). In order to separate the two effects I set the bargaining power parameter equal to the elasticity of the job filling rate and show, that job creation is excessive in the decentralized equilibrium with temporary layoffs. Efficiency may be restored by imposing a tax on firms hiring workers from attached unemployment.

Finally, theoretical implications of the model are tested against the empirical data using the German Social Economic Panel for the years 2003-2007. The probit regression for wage gains shows that recalls have significant impact on future wage changes of workers. In particular, being recalled to the previous employer is associated with approximately 8% lower probability of wage improvement. This means that worker-firm attachment and recalls provide an additional explanation of the observed wage heterogeneity in Germany. Other significant variables employed in the estimation include the reason for separation and job comparison variables. This paper shows that being laid off from the previous job imposes 8.2% lower probability of wage improvement, while voluntary separations (quits) increase this probability by 6.5%. Moreover, additional benefits, better promotion possibilities and improvements in the job security act as complements to wage gains.

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3.10 Appendix

Appendix 3.10.1: Proof of proposition 3.1.

The worker surplus $W^1 - U$ can be written in the following way:

$$W^1 - U = \frac{w^1 - z + \delta(1-p)(L-U)}{r + \gamma + \delta(1-p) + \lambda(\theta)}$$

since $rU = z + \lambda(\theta)(W^1 - U)$. Additionally, the firm surplus is:

$$J^1 = \frac{y - w^1 + c\theta + \delta(1-p)T}{r + \gamma + \delta(1-p) + \lambda(\theta)}$$

this allows to obtain the value of S^1 since $S^1 = J^1 + W^1 - U$. Additionally it is true that $T + L - U = d_1(\theta)S^1 + d_2(\theta)\Delta W$, then the total surplus S^1 becomes:

$$S^1 = \frac{y - z + c\theta + \delta(1-p)d_2(\theta)\Delta W}{r + s(\theta) + \lambda(\theta)}$$

Now rewrite the free-entry condition (3.4.19) in the following way:

$$\frac{c}{q(\theta)} = \alpha(1-\beta)S^1 + (1-\alpha)(1-\beta)S^2$$

where

$$S^2 = S^1 - (L - U) = S^1(1 - d_1(\theta)\beta) - d_2(\theta)\Delta W$$

From the wage setting equations (3.4.12), (3.4.10) it follows that:

$$\Delta W = \frac{w^2 - w^1}{r + \gamma + \delta(1-p)} = (1-\beta)(L-U)$$

which allows to rewrite the surplus difference ΔW in the following way:

$$\Delta W = \frac{(1-\beta)d_1(\theta)\beta S^1}{1 - (1-\beta)d_2(\theta)}$$

so that the free-entry condition becomes:

$$\begin{aligned}
\frac{c}{q(\theta)} &= (1 - \beta)[\alpha S^1 + (1 - \alpha)S^2] \\
&= (1 - \beta)S^1 \left[\alpha + (1 - \alpha) \left[1 - d_1(\theta)\beta - d_2(\theta) \frac{(1 - \beta)d_1(\theta)\beta}{1 - (1 - \beta)d_2(\theta)} \right] \right] \\
&= (1 - \beta)S^1 \left[1 - \frac{(1 - \alpha)d_1(\theta)\beta}{1 - (1 - \beta)d_2(\theta)} \right]
\end{aligned}$$

Appendix 3.10.2: Proof of proposition 3.2.

The elasticity variable μ_θ can be expressed as follows:

$$\mu_\theta = [1 - \eta_q] \frac{\lambda(\theta)}{\gamma + \lambda(\theta)} k(\theta) < 1, \quad \text{where}$$

$$k(\theta) = \frac{\gamma(\gamma + \delta p + \lambda(\theta)) + \delta(1 - p)(\gamma + \lambda(\theta)) \frac{\lambda(\theta)}{\gamma + \delta + \lambda(\theta)}}{\gamma(\gamma + \delta p + \lambda(\theta)) + \delta(1 - p)(\gamma + \lambda(\theta))} < 1$$

Appendix 3.10.3: Social Planner

The current value Hamiltonian for the social planner problem is:

$$\begin{aligned}
H &= y(e_1 + e_2) + z(1 - e_1 - e_2) - c\theta(1 - e_1 - e_2) \\
&+ \mu_1 \left[\delta(1 - p)(e_1 + e_2) - \delta p u_1 - \lambda(\theta)u_1 - \gamma u_1 \right] \\
&+ \mu_2 \left[\lambda(\theta)(1 - e_1 - e_2 - u_1) + \delta p u_1 - \delta(1 - p)e_1 - \gamma e_1 \right] \\
&+ \mu_3 \left[\lambda(\theta)u_1 - \delta(1 - p)e_2 - \gamma e_2 \right]
\end{aligned}$$

where μ_1 , μ_2 , and μ_3 are costate variables corresponding to u_1 , e_1 , and e_2 respectively.

The optimal social planner solution must satisfy:

$$\begin{aligned}
\frac{\partial H}{\partial \theta} &= 0 \Rightarrow -(1 - e_1 - e_2)c = \\
&= \left[\mu_1 u_1 - \mu_2(1 - e_1 - e_2 - u_1) - \mu_3 u_1 \right] \lambda'(\theta) \tag{3.10.1}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H}{\partial u_1} &= r\mu_1 \Rightarrow -\mu_1(\delta p + \lambda(\theta) + \gamma) + \mu_2(\lambda(\theta) + \delta p) = \\
&= \mu_3 \lambda(\theta) + r\mu_1 \tag{3.10.2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H}{\partial e_1} &= r\mu_2 \Rightarrow y - z + c\theta + \mu_1 \delta(1 - p) - \mu_2 \lambda(\theta) = \\
&= \mu_2(\delta(1 - p) + \gamma) + r\mu_2 \tag{3.10.3}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H}{\partial e_2} &= r\mu_3 \Rightarrow y - z + c\theta + \mu_1 \delta(1 - p) - \mu_2 \lambda(\theta) = \\
&= \mu_3(\delta(1 - p) + \gamma) + r\mu_3 \tag{3.10.4}
\end{aligned}$$

From equations (3.10.3)-(3.10.4) it follows that $\mu_2 = \mu_3$, then from equations (3.10.1), (3.10.3) it is true that:

$$\begin{aligned}\frac{c}{q(\theta)} &= (1 - \eta_q) \left[(1 - \alpha)(\mu_2 - \mu_1) + \alpha\mu_2 \right] \\ y - z + c\theta &= \mu_2(r + \gamma + \lambda(\theta)) - (\mu_1 - \mu_2)\delta(1 - p)\end{aligned}$$

where $\alpha = u/(u + u_1)$. From equation (3.10.2) it follows that $\mu_1 = d_1(\theta)\mu_2$, so

$$\mu_2 = \frac{y - z + c\theta}{r + \lambda(\theta) + s(\theta)}$$

Finally, the optimal market tightness is obtained from:

$$\frac{c}{q(\theta)} = (1 - \eta_q) \frac{y - z + c\theta}{r + \lambda(\theta) + s(\theta)} (1 - (1 - \alpha)d_1(\theta))$$

Chapter

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Erklärung

Ich erkläre hiermit, dass ich die vorliegende Arbeit mit dem Thema

**Three Essays on Worker Turnover and
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ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus den anderen Quellen direkt oder indirekt übernommenen Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Weitere Personen, insbesondere Promotionsberater, waren an der inhaltlich materiellen Erstellung dieser Arbeit nicht beteiligt. Die Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.

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