Evaluating the Liquidity of Stocks using Transaction Data

A thesis is submitted in partial fulfilment of the requirements for the degree of Master of Arts in International Business Economics at the Department of Economics of the University of Konstanz

By: Nataliya Ivanchuk
Buecklestrasse 9/89
78467 Konstanz


1st assessor: Prof. Dr. Pohlmeier
2nd assessor Prof. Dr. Heiler

Konstanz, 30th September 2004
Contents

List of figures........................................................................................................ii
List of Tables.......................................................................................................iii
List of Abbreviations............................................................................................iv

1 Introduction........................................................................................................1

2 Theoretical Basis of the Liquidity Concept......................................................3
   2.1 Liquidity Measures.......................................................................................3
   2.2 Econometric Models for Modelling Durations Processes.........................13
   2.3 Market Microstructure Underpinnings.......................................................28

3 Empirics............................................................................................................31
   3.1 Data...........................................................................................................31
   3.2 Estimation of the liquidity dynamics.........................................................40
      3.2.1 Classical ACD Model..........................................................................40
      3.2.2 UHF-GARCH for the volatility per trade duration.............................50
      3.2.3 Excess Market Depth with VNET.......................................................54

4 Conclusion........................................................................................................60

   Appendix A: Autocorrelation of the Raw Durations......................................63
   Appendix B: Estimation Outputs for ACD (2,2)...........................................66
   Appendix C: Estimation Outputs of ACD with the Component Structure........68
   Appendix D: Autocorrelation of the Residuals from ACD (2,2) Estimation......70
   Appendix E: Estimation Outputs or UHF-GARCH.......................................72
   Appendix F: Estimation Outputs for ACD (1,1) for the Price Durations..........75
   Appendix G: VNET Estimation Outputs.......................................................77

List of References...............................................................................................79
List of Figures

Figure 1.1: Static image of the limit order book.
Figure 1.2: The slope of the quote.
Figure 3.1.1: Cubic splines for the durations of the three heavily traded stocks: C, HWP and AXP.
Figure 3.1.2: Cubic splines for the durations of the two less heavily traded stocks: JBX and FIC.
Figure 3.1.3: Cubic splines for the volumes of the three heavily traded stocks: C, HWP and AXP.
Figure 3.1.4: Cubic splines for the volumes of the two less heavily traded stocks: JBX and FIC.
Figure 3.1.5: Cubic splines for the volumes of the three heavily traded stocks: C, HWP and AXP.
Figure 3.1.6: Cubic splines for the durations of the two less heavily traded stocks: JBX and FIC.
Figure 3.2.1: Autocorrelation functions for the first 200 lags of the trade durations for C, HXP and AXP.
Figure 3.2.2: Autocorrelation functions for the first 200 lags of the trade durations for JBX and FIC.
Figure 3.2.3: Empirical hazard functions for C, HWP and AXP.
Figure 3.2.4: Empirical densities for C, HWP and AXP.
Figure 3.2.5: Empirical hazard functions for JBX and FIC.
Figure 3.2.6: Empirical densities for JBX and FIC.
List of Tables

Table 3.1: Summary statistics for five analysed stocks.
Table 3.2.1: Estimates of ACD(2,2) for the five analysed stocks.
Table 3.2.2: Estimates of ACD(2,2) with the component structure for the five analysed stocks.
Table 3.2.1: Estimated coefficients (p-values) for UHF-GARCH mean function.
Table 3.2.2: Estimated coefficients (p-values) for UHF-GARCH variance equation.
Table 3.2.3: Correlogram for volatility of JBX.
Table 3.2.4: Correlogram for volatility of FIC.
Table 3.2.5: Estimated coefficients (p-values) for ACD model.
Table 3.2.5: Estimated VNET coefficients (p-values).
List of Abbreviations

AACD – Additive Autoregressive Conditional Duration
ACD – Autoregressive Conditional Duration
AMACD – Additive and Multiplicative Autoregressive Conditional Duration
ARCH – Autoregressive Conditional Heteroscedasticity
ARMA – Autoregressive Moving Average
AXP – American Express Company
C – Citigroup Inc.
EACD – Autoregressive Conditional Duration with exponentially distributed innovations
EGARCH – Exponential Generalized Autoregressive Conditional Heteroscedasticity
EXACD – Exponential Autoregressive Conditional Duration
FIC – Fair Isaac Corporation
GARCH – Generalized Autoregressive Conditional Heteroscedasticity
HWP – Hewlett-Packard Company
JBX – Jack in the Box Inc.
NMS – Nasdaq Market System
NYSE – New York Stock Exchange
QML – Quasi Maximum Likelihood
QMLE – Quasi Maximum Likelihood Estimation
SCD – Stochastic Conditional Duration
SVD – Stochastic Volatility Duration
TAQ – Trades and Quotes database
UHF-GARCH – Ultra-High Frequency Generalized Autoregressive Conditional Heteroscedasticity
VNET – net volume of one side of the market
WACD – Weibull Autoregressive Conditional Duration
1 Introduction

In recent years a substantial amount of the relevant literature in one way or another deals with liquidity. The interest in it grows beyond the walls of the academia, as the security exchanges recognize the importance of the concept and plan to adopt unique measures of liquidity and publish them in the regular reports. But as in literature, there is still no consensus as what liquidity really means and how it should be measured, or reported, understood or predicted, as the consistent summary of what the liquidity is for a meaningful quantitative comparison is practically missing. A lot of research has been done about daily returns, daily volatilities and these measures can be now reasonably measured and forecasted; this by no means is the case with liquidity. In the present paper we try to gather and categorize the existent knowledge about liquidity and estimate the measures that can help to gain an idea of the liquidity generated on the transactions level at the stock market.

We make use of the modern development in data storage and handling capacities, such that now data can be collected and analysed at the limiting frequency: at the level of each transaction. This fact gave motivation for development of the entire new branch of econometrics: econometrics of transactions data, which has an inherent feature of irregularly spaced observations.

The interdependence of various liquidity measures was extensively studied in the literature. We consequently apply the existent knowledge on market microstructure to establish and explain the links between spreads, size of the transaction and price movements and the actual time of the trade. The market microstructure hypotheses rely on the informational context of the trading, where the essence of the revealed news as well as the proportion of market participants to whom this information is disclosed is of the crucial importance to determining the trading dynamics.

This paper carries out the analysis of the validity of the liquidity measures and transaction based econometric models for five stocks of different trading intensity in order to see how the trading intensity influences the ability to estimate the dynamics of the price formation, information unravelling and to evaluate the accuracy of the estimated models.

The rest of the paper has the following structure. Chapter two gives an overview of the modern understanding of the concept of liquidity and summarizes the developed one- and multi-dimensional measures of liquidity. The introduction into the modelling of the point processes and irregularly spaced data is presented further followed by the
comprehensive outlook on the basic tool for modelling financial durations, the ACD-type models. Next, we categorize the variety of recently proposed in the literature and estimated empirically duration models. We conclude chapter two by giving the overview of the market microstructure theory and making a smooth transition to the empirical part of the paper. Chapter three starts with the introduction of the data and the description of the preliminary data manipulation. Further we estimate the classical ACD (2,2) model and prove empirically the substantiation of the autoregressive structure in the durations. Next we estimate Ultra-High Frequency GARCH model in order to explain volatility per trade dynamics which depends on the autoregressive structure in the durations in a way to establish a link between two such important dimensions of liquidity as volatility and size of the transactions. Finally we estimate VNET, a model for the excess volume of one side of the market that can be traded without inducing a certain price change. This measure includes the estimates for the conditional price durations as well as other marks, such as spread, size and number of trades per price duration in a way to get a broader and more encompassing measure of market liquidity. The conclusions, implication for the market microstructure and suggestions for further research are presented in the final chapter.
2 Theoretical Basis of the Liquidity Concept

2.1 Liquidity Measures

There are two general broad understanding of liquidity. First of all, it is the monetary liquidity which is characterized by availability of cash or near cash in relation to the general demand of goods or financial assets. The trends of monetary liquidity are generally associated with the general state of the economy, economic cycles and consumer confidence. They are usually reflected in the short-term interest rates, as low short-term rates would work favourably for the liquidity. The other concept of liquidity is related to the way the transfer of cash and goods or financial securities is performed in the market with the reference to different dimensions of liquidity frequency of trading, price, return, volatility, market depth and the interdependencies between those. In the present paper we are only interested in the second defined concept based on the market microstructure and will completely leave out the macroeconomic dimensions of liquidity.

Recent technological developments allow for detailed data recording and processing which opens doors for the new branch of analysis in the high frequency finance and the issue of liquidity acquired there an important place.

The multidimensionality of the liquidity concept is captured by the four following dimensions:

1. The timing of transactions: the ability to execute a trade transaction at or near to the most desired time; it is measured by the transactions rate per unit of time or the inverse, the time between the subsequent trades.

2. Tightness: the ability to buy and sell without big difference in price. Tightness shows the cost of transacting at the given moment, or in other words, the cost of immediacy. It is usually measured by the size of the spread.

3. Depth: the ability to buy or sell certain volume with the smallest possible influence on price. The depth can be measured directly by the volume of shares available for the immediate trade on both sides of the market.

4. Resilience: the ability to buy and sell large volumes without incurring large change in price. The concept of resilience, in the contrast to depth regards the large volume trades that are beyond the depth of the market at the given point of time. In other words, it captures the price effect of immediacy trading incurred when the volume is beyond the depth. This measure is obtained from the market reaction curve estimated based on the data from the limit order book.
These four concepts are summarized in the following diagram based on the static image of the limit order book, where every dimension is subject to change at every point in time.

![Diagram of liquidity measures](image)

**Figure 1.1: Static image of the limit order book.**

In the following we present the overview of the liquidity measures. As liquidity is not directly measurable by itself, it is often proxied by other market variables and as the concept is rather ambiguous and lacks unanimous classification, the summary by no means claims to be complete, but nevertheless provides a rather profound insight into the matter.

The liquidity measures are generally separated into *one-dimensional measures*, that capture only one side of liquidity and *multi-dimensional measures* that try to aggregate different dimensions, find interdependencies between them and construct one aggregated measure.

The one dimensional measures can be broadly separated into four groups:
- measures that capture the size of the firm;
- measures based on the durations between the transactions;
- measures based on the traded volume;
- measures based on the spread between the bid and ask quotes.

As in the present paper liquidity is analysed in the intraday stock exchange trading concept, the measures related to the firm size do not show sufficient variation and are completely left out of the further analysis. Most of the following measures are borrowed from von Wyss (2004, 9); own interpretation and classification is given.
**Volume-related liquidity measures:** may be calculated as a certain volume traded per unit of time or the time required trading a certain amount of shares. Usually volume related liquidity measures seek to capture the depth of the market but they are also related to the time dimension, since the higher volume in the market requires less time to trade a certain number of shares. The higher is the volume related liquidity measure, the higher is the liquidity itself.

- **Volume intensity**, closely investigated by Lee and Swaminathan (2000) is calculated on the basis of the traded volume, or number of shares traded per unit of time:

\[
Vol_i = \sum_{i=1}^{N_i} vol_i
\]

where \(N_i\) denotes the number of trades between time \(t-1\) and \(t\) and \(vol_i\) is the number of shares traded in trade \(i\).

- The reciprocal of volume intensity, the **volume duration**, first proposed by Gouriéroux, Jasiak, Le Fol, (1999), measures time in which a certain threshold number of shares \(Vol^*\) is traded:

\[
Dur_i^{(Vol^*)} = t^{(Vol^*)} - t^{(Vol=0)}
\]

where \(t\) is the time of the corresponding transactions.

- **Net traded volume** is also related to the time dimension in the sense that it measures the buyer initiated volume minus seller initiated volume during a certain time period.

- The reverse of the net traded volume, **net volume duration**, measures the period of time needed to generate a certain threshold net traded volume.

- The **turnover** measure studied by Chan, Chung and Fong (2002) measures the total value of transactions over a certain period of time:

\[
TO_i = \sum_{i=1}^{N_i} p_i \times vol_i
\]

- The reciprocal measure, **turnover duration**, measures respectively the time in which a threshold turnover is achieved:

\[
Dur_i^{(TOP)} = t^{(TOP)} - t^{(TO=0)}
\]
The advantage of these two measures is that they allow for comparison between the different stocks, taking the price of the stock into consideration.

The next three measures depend on the data for quote arrivals (quote database only) and are measured at a given point in time, rather than over the period of time. These measures are especially easy to calculate, and have a simple logic behind them:

- **Volume depth** as it is referred to in Brockman and Chung (2000)

\[ D_i = vol_i^A + vol_i^B \]  \hspace{1cm} (1.5)

where \( vol_i^A \) and \( vol_i^B \) are the best quoted ask and bid volume correspondingly. If this depth measure is divided by two, we have the average bid-ask depth of the corresponding quote.

- **Log depth** is essentially same measure as volume depth, taking the logs\(^1\) of quoted bid and ask volumes in order to achieve better distributional properties of the measure as in von Wyss (2004, 11) because the distribution of the depth measure is then closer to normal and it is mathematically easier to handle:

\[ \log D_i = \log vol_i^A + \log vol_i^B \]  \hspace{1cm} (1.6)

- **Dollar depth** is calculated in the same way as the average depth, but is expressed in the monetary units and is calculated as follows:

\[ D_\$ = \frac{q_t^A \cdot p_t^A + q_t^B \cdot p_t^B}{2} \]  \hspace{1cm} (1.7)

where \( p_t^A \) and \( p_t^B \) are the best quoted bid and ask prices at time \( t \).

The described above measures take into consideration only the volume and the best bid and ask quotes. Bacidore, Battalio and Jennings (2002), quoted in von Wyss (2004, 13) estimate that approximately 16% of all NYSE orders are greater than the quoted depth\(^2\) and these orders generally cannot be executed at the best price and fractions of it are executed at the higher price than ask for an excess buy order and at the

---

\(^1\) Now and until the end of the present paper for all log valued natural logarithms are meant.

\(^2\) According to Hasbrouck, Sofianos and Sosebee (1993), an order is not a quote until it the specialist (floor broker) exposed the order to the crowd as a quote.
lower price than bid for the excess sell order. This is taken into account in the following liquidity measures.

**Time-related liquidity measures** measure how often the transactions or quotes revisions take place. Hence, the higher are the values of these measures, the higher is the liquidity.

- **Number of transactions per time unit** counts the \( N_t \), number of transactions during a time interval \( t_i - t_{i-1} \).
- The reverse of this measure it the *waiting time* between the transactions or the familiar *duration*.

Same measures can be calculated for the quote renewals, price durations, volume durations, etc.

**Spread-related liquidity measures**. The difference between the bid and ask price is approximately related to the cost incurred while trading in addition to the fees and taxes associated with executing an order. Consequently, the smaller all the spread-related liquidity measures, the greater is the liquidity itself. All spread-related liquidity measures are based on the best pair of bid and ask quotes.

- **Absolute spread and quoted spread**:

\[
Spread_i = p^A_i - p^B_i
\]

Absolute spread is the difference between the lowest ask and the highest bid quote, and this value is always positive. The quoted spread refers to the best bid-ask combination of a particular market maker who quotes the prices and is used to study the individual performance of different market makers on the same exchange, this analysis is however quite limited as most exchanges do not release this sort of data.

- **Log absolute spread**. Just like with the log depth, it is possible to calculate the log spread of the quoted bid and ask in order to improve their distributional properties:\(^3\):

\[
LogSpread_i = \log\left(Spread_i\right) = \log\left(p^A_i - p^B_i\right)
\]

\(^3\) Note, that the log depth measure is calculated as a sum of the log volumes, rather than just taking the log of the total volume.
\textbf{Proportional spread} is calculated on the basis of the midquote:

\begin{equation}
S^{(\text{prop})}_t = \frac{p^A_t + p^B_t}{p^M_t}
\end{equation}

where \( p^M_t = \frac{p^A_t + p^B_t}{2} \) is the midquote. The advantage of this measure is that it again allows for comparison of different stocks and can be computed for every bid-ask quote pair irregardless if there was a trade on this quote or not.

\textbf{Relative spread} is calculated based on a given pair of bid-ask quotes and actual price at which transaction is executed:\footnote{The prevailing quote is chosen on using Lee and Ready (1991) algorithm as described in section 3.1.}

\begin{equation}
S^{(\text{rel})}_t = \frac{p^A_t + p^B_t}{p_t}
\end{equation}

The advantage of this measure is that it takes the type of market movement into the consideration: if the trade was a “buy” the actual price would be equal or greater (if the volume is greater than the depth) than the ask quote and the market would be moving upward, whereas if the trade was a “sell” the price would be equal or lower (if the volume is greater than the depth) than the bid quote and the market would be moving downward.

\textbf{Relative spread of log quotes} is calculated in the same way as the log return is calculated, the measure is proposed by von Wyss (2004) and the meaning of it in the economic sense is not very clear:

\begin{equation}
S \log_t^{(\text{rel})} = \log(p^A_t) - \log(p^B_t) = \log\left(\frac{p^A_t}{p^B_t}\right)
\end{equation}

The measure closely related to the previous one is \textit{log relative spread of log prices}. The main purpose of this measure is also to get better distribution of the spreads, at this specification it would be close to normal, von Wyss (2004, 11):
According to the author, the purpose of this measure is to make the highly skewed distribution of the previous measure more symmetric in order to be able to approximate it with the Normal distribution.

- **Effective spread:**

\[ S_{t}^{(\text{eff})} = \left| p_t - p_t^M \right| \]  \hspace{2cm} (1.14)

where \( p_t \) is the price at which trade at time \( t \) is executed and \( p_t^M \) is the midquote calculated as before. This measure hints on a different spread concept: if the effective spread is less then half of the bid-ask spread, then the trade price occurred within the bid-ask spread. To make this measure comparable to other spread measures, one needs to multiply it by two. On the basis of this measure Battalio, Greene, and Jennings (1998) calculate the liquidity premium:

\[ LP_t = I_t \cdot (p_t - p_t^M) \]  \hspace{2cm} (1.15)

where \( I_t \) is the indicator function which is equal to 1 if \( t \) trade is a “buy” and -1 if \( t \) trade is a “sell”. The liquidity premium is positive if the buyer pays more or if the seller pays less than the midquote.

- **Relative effective spread** calculated with last trade or with the midquote; these two measures are relative and allow compatibility between the different stocks.

\[ S_{t}^{(\text{rel}eff)} = \frac{|p_t - p_t^M|}{p_t} \]  \hspace{2cm} (1.16)

\[ S_{t}^{(\text{rel}M)} = \frac{|p_t - p_t^M|}{p_t^M} \]  \hspace{2cm} (1.17)

The measures that would be described further are related to more than one dimension of liquidity at a time and therefore are **multi-dimensional** in their nature. These measures largely possess the combined properties of one-dimensional measured described above and their effect on liquidity is determined jointly by the individual effects of one-dimensional measures.
Slope of the quote:

\[
S_{\text{quote}} = \frac{\text{Spread}_t}{D \log_{r_t}} = \frac{p_t^A - p_t^B}{\log \text{vol}_{t}^A + \log \text{vol}_{t}^B}
\]

(1.18)

per construction, the greater is the value of the quote slope, the lower is liquidity. It is also possible to use log spread in the numerator to improve the distributional properties of the measure. Graphically this measure corresponds to the slope between the bid and ask quote with the log volumes on the x-axis:

![Graphical representation of the slope between bid and ask quote with log volumes on the x-axis.](image)

Figure 1.2: The slope of the quote.

Liquidity ratios combine the measures for the turnover and return or for the number of trades and return:

\[
LR_{1(\cdot)} = \frac{\sum_{i=1}^{N_i} p_i \times \text{vol}_i}{|r_i|}
\]

(1.19)

In the stated above liquidity ratio 1 (LR1), the numerator is the volume turnover measure and denominator is the absolute price change over the corresponding period. The economics behind this measure is as follows: the higher the volume, the higher the price movement that can be absorbed by the market; correspondingly, higher value of this liquidity ratio means higher liquidity. To avoid the situation “undefined” when the return over the analysed period is zero, the ratio is set to zero. The inverse of this ratio denoted return per turnover.

Liquidity Ratio (LR2) indicates the average price change per transaction; \(N_i\) denotes the number of transactions in the analysed period.
\[
LRII(t) = \frac{\sum_{i=1}^{N_t} |r_i|}{N_t}
\]  

(1.20)

Naturally, the higher is the ratio, the greater is the volatility and consequently, the lower liquidity.

- **Flow ratio** is determined by dividing the turnover by the average duration in the analysed period and makes it clear whether there were few large transactions or a number of smaller ones:

\[
FR_i = \frac{\sum_{t=1}^{N_t} p_t \cdot vol_t}{\frac{1}{N-1} \sum_{t=1}^{N_t} dur_t}
\]  

(1.21)

As liquidity is higher with the higher number of transactions and with the higher turnover, high flow ratio is the indicator of high liquidity.

- **Order Ratio** measures the size of market imbalance relative to the turnover:

\[
OR_i = \frac{vol^B_i - vol^A_i}{p_t \cdot vol_t}
\]  

(1.22)

If the market imbalance rises, the numerator becomes larger, and so does the ratio, meaning lower liquidity.

- **Market Impact** is a limit order-based measure since it requires different quotations of the bid and the ask for different size transactions. Market Impact simply calculates the spread for a given volume. The spread increases with the volume, if the increase per additional volume is greater, the liquidity of the security is lower:

\[
MI_i^{Vol^*} = p_i^{A, Vol^*} - p_i^{B, Vol^*}
\]  

(1.23)

This measure can also be calculated separately for each side of the market:

**market impact for the bid side:**  

\[
MI_i^{A, Vol^*} = p_i^{A, Vol^*} - p_i^M
\]  

(1.23a)

**market impact for the ask side:**  

\[
MI_i^{B, Vol^*} = p_i^M - p_i^{B, Vol^*}
\]  

(1.23b)
The next measure, depth for the price impact, is another limit order book measure. It measures the number of shares to be traded before the price moves beyond a certain threshold. First, the market reaction curve based on the limit order book entries is estimated and then the depth for the price impact is simply read from it. This measure can also be estimated for either side of the market separately. The greater depth for the price impact means that the market can absorb greater volume without significant movements in price, meaning more liquidity for the security.

The inverse measure of the depth for price is the price impact. It calculates the execution cost depending on the limit order book entries for the bids and the asks for the larger size transactions. A large transaction is executed at the total price:

$$\sum_{k=1}^{K} p_k \cdot vol_k$$  \hspace{1cm} (1.24)

The price impact for the buy order is:

$$PI^{(buy)} = \ln \left( \frac{\sum_{k=1}^{K} p_k \cdot vol_k}{\sum_{k=1}^{K} vol_k \cdot p^M} \right)$$  \hspace{1cm} (1.25a)

and for the sell order:

$$PI^{(buy)} = -\ln \left( \frac{\sum_{k=1}^{K} p_k \cdot vol_k}{\sum_{k=1}^{K} vol_k \cdot p^M} \right)$$  \hspace{1cm} (1.25b)

It is clear that the high price impacts imply low liquidity.

As we see, the issue of liquidity is indeed versatile and it is desirable to characterize it from different dimensions. However to be able to perform a profound and meaningful analysis of liquidity, the complete limit order book data is required. Due to the fact that this data is not currently at our disposal, we will do our best to perform adequate analysis of liquidity of the NYSE traded stocks based on the TAQ database which gives only the best pair of bid-ask quotations.
2.2 Econometric Models for Modelling Durations Processes

There have been made numerous attempts in the literature to define, measure and predict the ambiguous and elusive concept of liquidity in terms of volume or frequency of trading impact. To measure liquidity corresponding to the concept we have defined, namely as the market microstructure concept, it is obvious that it has to do with the transactions data to capture the ideas of the market microstructure. Market liquidity is considered as the capacity of the market to absorb the temporary fluctuations without a big impact on prices and disturbances of supply or demand. One most obvious measure of liquidity is the frequency of trading. More frequent trading would most surely bring more liquidity, as the asset could be turned into cash more often and therefore neither supply nor demand party can induce the premium. But with the frequency of trading alone, it is difficult to capture the issues like the price impact, volume impact and the breadth, defined as spread between the best ask and best bid. The analysis of the liquidity on the transactions level led to the development of the entire class of models named autoregressive conditional duration (ACD) which are meant to capture the trade-to-trade or quote-to-quote relationships and dependencies in the variables that from different dimensions define liquidity. The fact that the transactions data is not equally spaced makes the analysis different from the standard econometric techniques and accounts for the stochastic time component, modelled as a (marked) point process.

Recent literature on high-frequency financial data suggest that the frequency of transactions should carry an important subtle information about the state of the market. Engle, Russell (1998). The studies by Kyle (1985), Admati and Pfleiderer (1988) and Easley and O’Hara (1992) show that there are clusters of transactions followed by a period of relatively slow trading and claim that it is possible to explain this dynamics using the microstructure theory and the econometric tools.

Because of these clustering in transaction frequency, it is reasonable to assume that the liquidity might be a time varying measure rather than a fixed characteristic pertinent to a particular stock. Therefore it would make sense to analyse liquidity based on the process of transaction time arrivals, defining liquidity as ability to quickly sell or buy an asset. This definition of liquidity is however not informative enough per se as
it does not account for the volume and price impact; therefore a more precise definition of liquidity is the following proposed by Black (1971):

“...an asset is said to be liquid if it can be sold in a short time, at the price not too much below the price of what a seller would get if he took plenty of time to sell the asset ...”

In this case it is desirable to include price and volume information into the model of transaction arrival time either by modelling the volume and price durations or by including them as *marks*\(^5\) into the model of the transaction process arrival times.

Different aspects of the liquidity concept and different purposes of the modelling the time processes of quotes arrival, trades, as well as volume and price durations to analyse it, gave rise to a rich variety of duration models. Duration models, in turn became popular in the recent econometric and high-frequency finance literature due to the fact that they are easy to estimate and the logic behind them is quite intuitive and allows for straightforward application and testing of market microstructure hypotheses.

The existing models of this type can be broadly divided into two categories:

- Models for the durations given the past history of the process;
- Models for marks and the durations given the past.

*The models for the durations given the history* are modelled as a stochastic function of the previous durations. The marks such as volume, price changes, returns, bid-ask spread etc. may be included. This type of models aims to capture the most important properties of the process as clustering, overdispersion\(^6\) (except for volume durations), persistence of the process characterized by significant autocorrelations in a great number of lags. The potential uses of this type of models:

- testing market microstructure theories, for example, how informative are price durations about the volatility or volume durations about the liquidity;
- prediction of the occurrence of the next transaction, cumulative volume or price change, news arrival and quotes announcement;
- construct the implied intra-day volatility upon the price durations.

The models of this type can be further classified into models:

- with one innovation – the classical ACD, (Engle, Russell, 1998) and the extensions and modifications of it. They can but are not required to involve marks as additional regressors to help explain the process;

---

\(^5\) The covariates

\(^6\) overdispersion means that the mean of the distribution is larger than the standard deviation, in the reference to exponential distribution for which mean is equal to the standard deviation. This phenomenon can be paralleled with overcurtosis property in the Gaussian GARCH models.


The models for marks and durations given the past are modelled as stochastic process of the durations jointly with the modelling of the process in the marks.

**Basic ACD Model**

Engle and Russell (1998) first proposed dynamic model for the durations between the trades. It was observed that the financial markets have periods of high activity and more dormant periods, which reflect the time changing liquidity. One of the distinguishing characteristics of the transaction data is the clustering of the durations of the similar length: long durations tend to be followed by the long durations and short durations are followed by the short durations. This type of clustering exhibits striking resemblance with the phenomenon of the serial conditional dependence in volatilities modelled with the GARCH-type models. For this reason the model was named Autoregressive Conditional Duration and the modelling techniques are very similar to those of the models for conditional volatility of the GARCH-family.

ACD is based on the dynamic linear parameterisation of the conditional mean function:

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} \quad (2.1)$$

The model aims to explain the conditional frequency and distribution of the calendar time between the irregularly spaced events. The event of interest can be quote announcement, transaction itself or crossing the volume or the price threshold, which so far is irrelevant to the given analysis except for the fact that it is denoted as the “exit out of a given state” and is denoted as $t_i$. Then $x_i$ is the duration between the two events and a realization of a random variable $X$: $x_i = t_i - t_{i-1}$. There are different ways to describe the behaviour of $X$:

- the probability density function (pdf) $f(x)$;
- the cumulative distribution function (cdf):

$$F(x) = \int_{0}^{x} f(u)du \quad (2.2)$$

- the survivor function, which is the complement of the cdf and describes the probability of not changing the state, or “surviving” up to $t_i$. 


- the hazard rate is defined as an instantaneous rate of transition of the probability to change state within the short interval \( dx \), given that there was no state change before \( x \):

\[
\lambda(x) = \lim_{dx \to 0} \frac{P(x \leq X \leq x + dx | X \geq dx)}{dx}
\]  

(2.4)

The relationship between the hazard rate and the survivor function can be depicted as follows: using the fact that joint density can be rewritten as a product of marginal density and conditional density:

\[
P(x \leq X \leq x + dx) = P(x \leq X \leq x + dx | X \geq x)P(X \geq x) \Rightarrow \\
P(x \leq X \leq x + dx | X \geq x) = \frac{P(x \leq X \leq x + dx)}{P(X \geq x)}
\]

(2.5)

Rewrite the hazard in terms of joint density:

\[
\lambda(x) = \lim_{dx \to 0} \frac{1}{dx} \frac{P(x \leq X \leq x + dx)}{P(X \geq dx)} = \\
= \lim_{dx \to 0} \frac{1}{dx} \frac{S(x) - S(x + dx)}{S(x)} = \\
= - \frac{1}{S(x)} \frac{dS(x)}{dx}
\]

(2.6)

Substituting \( \frac{dS(x)}{dx} = -f(x) \) we have the following relationship between the survivor function, density function and the intensity (hazard) function:

\[
\lambda(x) = \frac{f(x)}{S(x)}
\]

(2.7)

which is referred to as *baseline* hazard.
Hence, the durations process can be equivalently defined in terms of probability density, cdf, survivor, or the hazard function:

\[ S(x) = \exp\left(-\int_0^x \lambda(x)dx\right) \]  
(2.8)

Duration dependence is the relationship between the time spent in the state and the probability of exit from it at a given time; it may be different and depends on the hazard function. The hazard function can be increasing, decreasing, humped or simply a constant. For example, positive duration dependence means that the longer is the time spend in a given state, the higher is the probability of leaving the state, implying increasing hazard. Duration dependence is typically modelled on some parametric families of distribution, which are defined on the positive support to ensure that the durations do not go negative. Reasonable suggestions for such distributions are exponential, Weibull, generalized Gamma, log-normal and the Burr distributions. In the following we quickly summarize the important statistical properties of these distributions.

*Exponential distribution* implies that the hazard rate is a constant:

\[ \lambda(x) = \lambda \]  
(2.9a)

The cumulative distribution function is given by:

\[ F(x) = 1 - \exp(-\lambda x) \]  
(2.9b)

The survivor function:

\[ S(x) = \exp(-\lambda x) \]  
(2.9c)

Exponential distribution implies a special relationship between first and second moments:

\[ E(X) = \frac{1}{\lambda} \quad \text{and} \quad Var(X) = \frac{1}{\lambda^2} \]  
(2.9d)

which implies that the mean is equal to the standard deviation, and digressions from this state are known as overdispersion if the standard deviation is greater than the mean and underdispersion, if the standard deviation is smaller than the mean.
Weibull distribution allows modelling a nonconstant hazard which is increasing for \( b>1 \) and is decreasing for \( b<1 \).

\[
\lambda(x) = abx^{b-1} \tag{2.10a}
\]

Probability density function of the Weibull has the following form:

\[
f(x) = abx^{b-1} \exp(-ax^b) \tag{2.10b}
\]

where both parameters \( a \) and \( b \) are in the positive parameter space.

The survivor function is given by:

\[
S(x) = \exp(-ax^b) \tag{2.10c}
\]

Log-normal distribution has a hazard function which first increases and then decreases in \( x \):

\[
f(x) = \frac{1}{\sigma x} \phi \left( \frac{\log x - m}{\sigma} \right) \tag{2.11a}
\]

with the survivor function:

\[
S(x) = 1 - \Phi(u), \text{ where } u = \frac{\log x - m}{\sigma} \tag{2.11.b}
\]

and the hazard function is:

\[
\lambda(x) = \frac{f(x)}{S(x)} = \frac{1}{x} \frac{[(1/\sigma)\phi(u)]}{1 - \Phi(u)} \tag{2.11c}
\]

Gamma, generalized Gamma and Burr distributions allow for a wide variety of humps in the hazard function.

The density of the gamma distribution family is given by:

\[
f(x) = \frac{\kappa^r}{\Gamma(r)} x^{r-1} \exp(-\kappa x), \text{ where } \Gamma(r) = \int_0^\infty \exp(-x)x^{r-1}dx \tag{2.12}
\]
The distribution depends on two parameters, $\kappa$ and $r$. Parameter $r$ might be viewed as a number of exponentially distributed durations with $\kappa=a$, summed together, in which case $\Gamma(r)=(r-1)!$

The form of the hazard depends on $r$:

a) for $r>1$ hazard is increasing and asymptotically approaches $\kappa$;
b) for $r=1$ hazard is a constant and the distribution reduces to exponential;
c) for $r<1$ hazard is decreasing and asymptotically approaches $\kappa$.

In order to model the autoregressive clustering of the durations, as in the ACD model, one has to make use of the *conditional intensity function*, where the conditioning set is the entire history of the process. The conditional intensity in the case of ACD is a linear function of past events and possibly marks. The conditional intensity function is formulated in the following way:

$$
\lambda(t|N(t),t_1,\ldots,t_{N(t)}) = \lim_{\Delta t \to 0} \frac{P(N(t + \Delta t) > N(t)|N(t),t_1,\ldots,t_{N(t)})}{\Delta t}
$$

(2.13)

where $N(t)$ is associated with the number of events that have occurred until the time $t$, where \{t_0, t_1, \ldots, t_n, \ldots\} is a *conditionally orderly* counting process *with after-effects*.

The estimation of the ACD parameters is best performed with maximum likelihood estimation making precise distributional assumptions. The log-likelihood can be expressed as a sum of conditional probability densities:

$$
L = \sum_{i=1}^{N(t)} \log p_i(t|t_0,\ldots,t_{i-1}) = \sum_{i=1}^{N(t)} \log \lambda(t|i-1,t_0,\ldots,t_{i-1}) - \int_{t_0}^{T} \lambda(u|N(u),t_0,\ldots,t_{N(t)}) du
$$

(2.13)

Denote $\psi_i$ as the expectation of the $i$-th duration as follows with the vector of parameters $\theta$:

$$
E(x_i|x_{i-1},\ldots,x_1) = \psi_i(x_{i-1},\ldots,x_1,\theta) = \psi_i
$$

(2.14)

The error term enters the model multiplicatively:

---

7 Meaning that the two events cannot occur in a small interval of time:

$$
P[|N(t + \Delta t) - N(t) = 1|t_{i-1}] = \lambda \Delta t + o(\Delta t)
$$

8 A point process is said to be „evolved with after-effects“, if for any $t>0$, the realization of points during $[t,\infty)$ is not independent of sequence $(t_0,t]$. 
\[ x_i = \psi_i \varepsilon_i \]  

(2.15)

with \( \varepsilon_i \sim i.i.d. \) following one of the described above distributions are independent of the vector of parameters. The critical assumption of ACD framework is that all the dynamic structure is captured by the conditional mean and there is no higher moment dependence\(^9\).

The baseline hazard function derived as described above based on the assumed distribution for the error term:

\[ \lambda_0 = \frac{p_0(t)}{S_0(t)} \]  

(2.16)

where \( p_0(t) \) is the probability density function and the \( S_0(t) \) is the survivor function of the error term.

The conditional intensity is then given by:

\[ \lambda(t|N(t), t_1, \ldots, t_{N(t)}) = \lambda_0 \frac{t-t_{N(t)}}{\psi_{N(t)+1}} \frac{1}{\psi_{N(t)+1}} \]  

(2.17)

It is seen that the past durations have two effects on the hazard function: the shift in baseline hazard and the shift in the conditional intensity. This concept is known as *accelerated failure time* model since the past information affects the stochastic time in the models of time deformation.

The simplest version of ACD assumes the independent identical exponential distribution of the error term \( \varepsilon \), so the baseline hazard is equal to one and the hazard function has the following simple form:

\[ \lambda(t|N(t), t_1, \ldots, t_{N(t)}) = \frac{1}{\psi_{N(t)+1}} = \psi_{N(t)+1}^{-1} \]  

(2.18)

Then the m-memory conditional intensity implies that the past \( m \) durations have effect on the expectation of the \( i \)th duration:

\(^9\) The main critique of the ACD-type model is that they assume exponential distribution of the error term and as it is known, there is a strict relationship between the expectation and higher moments of the exponential distribution.
\[ \psi_i = \omega + \sum_{j=0}^{m} \alpha_j x_{i-j} \]  

(2.19)

This leads to the general form of ACD\((p,q)\) model:

\[ \psi_i = \omega + \sum_{j=0}^{p} \alpha_j x_{i-j} + \sum_{j=0}^{q} \beta_j \psi_{i-j} \]  

(2.20)

The simplest member of the ACD family is ACD\((1,1)\):

\[ \psi_i = \omega + \alpha x_{i-1} + \beta \psi_{i-1} \]  

(2.21)

with unconditional mean:

\[ E(x_i) = \mu = \frac{\omega}{1-(\alpha + \beta)} \]  

(2.21a)

and unconditional variance:

\[ \sigma^2 = \mu^2 \left( \frac{1-\beta^2-2\alpha\beta}{1-\beta^2-2\alpha\beta-2\alpha^2} \right) \]  

(2.21b)

After some manipulations it is easy to see that unless \( \alpha = 0 \), the unconditional mean exceeds the unconditional variance, or in other words, the durations exhibit excess dispersion, the phenomenon that is parallel to excess kurtosis in the GARCH \((1,1)\) model.

By introducing the martingale difference

\[ \eta_i = x_i - \psi_i \]  

(2.22)

the ACD\((p,q)\) model can be rewritten as ARMA\((p,q)\) with non normally distributed error term \( \eta_i \):

\[ x_i = \omega + \sum_{j=0}^{\text{max}(p,q)} (\alpha_j + \beta_j) x_{i-j} + \sum_{j=0}^{q} \beta_j \eta_{i-j} + \eta_i \]  

(2.23)

As was already mentioned above, due to its popularity as well as to the potential drawbacks, the original ACD model was extended in different ways.
The first obvious extension is to assume a different type of distribution for the error term. As was already mentioned, durations can be modelled with any distribution defined on the positive support. For the sake of expositional simplicity let us analyse the extensions of the classical ACD(1,1), i.e. the number of lags is \( p=q=1 \) can be modified as WACD(1,1) with the Weibull distribution, Log normal, generalized Gamma or Burr distributions as was described earlier. To avoid the necessity of making restrictive distributional assumptions, the hazard function can be estimated semiparametrically or non-parametrically using splines or following the \( k \)-neighbours estimating procedure.

A different way to extend the ACD framework is to use different specification of the conditional mean function. Literature in this direction was rather prolific and the extensions are abundant. The lagged innovations are modelled to enter the mean function additively or multiplicatively.

Additive ACD or AACD, first considered by Hautsch (2004, 91) implies a linear news impact specification with the additive component and slope \( \alpha \). The conditional mean has the following specification:

\[
\psi_i = \omega + \alpha \epsilon_{i-1} + \beta \psi_{i-1}
\]  

(2.24)

In Additive and Multiplicative ACD or AMACD, proposed by Hautsch (2004, 91) lagged durations enter the conditional mean additively as well as multiplicatively. The model allows for more flexibility and nests the ACD as a special case when the additive component is equal to zero:

\[
\psi_i = \omega + (\alpha \psi_{i-1} + \nu) \epsilon_{i-1} + \beta \psi_{i-1}
\]  

(2.25)

Another type of extensions allows for nonlinear, kinked and regime switching news impact curves. Linear parameterization of the conditional mean quite often appears to be rather restrictive, and fails to capture the adjustment process of the duration dynamics.

The logarithmic ACD model of Bauwens and Giot (1997) targets the drawback of the original ACD that because of the stochastic component the conditional mean might end up having negative expectation. The logarithmic model is related to the EGARCH specification for the volatility process and without additional restrictions of parameters ensures positivity of the conditional mean. Bauwens and Giot (1997)
proposed two alternative parameterizations of the conditional mean to allow for concave (type I) and convex (type II) shape of conditional mean function:

\[
\psi_{i} = \omega + \alpha \ln \epsilon_{i-1} + \beta \ln \psi_{i-1} = \omega + \alpha \ln x_{i-1} + (\beta - \alpha) \ln \psi_{i-1} \tag{2.26a}
\]

\[
\psi_{i} = \omega + \alpha e_{i-1} + \beta \ln \psi_{i-1} = \omega + \alpha (x_{i-1}/\psi_{i-1}) + (\beta - \alpha) \ln \psi_{i-1} \tag{2.26b}
\]

A Box-Cox ACD proposed by Hautsch (2001) is based on power transformation of the additive ACD and allows for concave, convex or linear news impact curve, depending on the power parameters \(\delta_1\) and \(\delta_2\):

\[
\psi_{i}^{\delta_i} = \omega + \alpha e_{i-1}^{\delta_i} + \beta \psi_{i-1}^{\delta_i} \tag{2.27}
\]

EXponential ACD (EXACD) proposed by Dufour and Engle (2000) allows for piecewise linear parameterization of the conditional mean, where for the durations that are shorter than conditional mean the slope is equal to \(\alpha\) and for the durations that are longer then the conditional mean the slope is \(\alpha - c\). The kink occurs whenever durations are equal to the conditional mean: \(e_{i-1}=1\):

\[
\ln \psi_{i} = \omega + \alpha e_{i-1} + c|e_{i-1}-1| + \beta \ln \psi_{i-1} \tag{2.28}
\]

Related to EXACD is Augmented Box-Cox ACD, Hautsch (2004, 92) allows additional parameterization of the kink:

\[
\psi_{i}^{\delta_i} = \omega + \alpha (\epsilon_{i-1} - b) + c(e_{i-1} - b)^{\delta_i} + \beta \psi_{i-1}^{\delta_i} \tag{2.29}
\]

The parameter \(b\) determines the position of the kink while the parameter \(\delta_2\) determines the shape of the piece around the kink: for \(\delta_2 > 1\) the conditional mean is convex and for \(\delta_2 < 1\) it is correspondingly concave.

Fernandes and Grammig (2001) propose another type of augmented ACD model based on a multiplicative stochastic component and otherwise is closely related to the Augmented Box-Cox ACD. The idea behind this model is the basically the same as in Augmented Box-Cox ACD except that multiplicative interaction between function of \(e_{i-1}\) and \(\psi_{i-1}^{\delta_i}\):
\[ \psi^{\delta_i}_i = \omega + \alpha \psi^{\delta_i}_{i-1} \left( |e_{i-1} - b| + c(e_{i-1} - b) \right)^{\delta_i} + \beta \psi^{\delta_i}_{i-1} \] (2.30)

Hautsch (2004, 95) proposed a non-parametric specification of the news impact curve parameterized as a linear spline function with nodes at the known (or suspected) break points of \( e_{i-1} \). The range of the conditional mean is divided into \( K = K^+ + K^- \) intervals corresponding to the number of break points at \( \{ \bar{e}_{K^-}, \ldots, \bar{e}_{i-1}, \bar{e}_0, \bar{e}_1, \ldots, \bar{e}_{K^+} \} \). The model is then given by:

\[ \psi_i = \omega + \sum_{k=0}^{K^+} \alpha^+_k \mathbf{1}_{\{e_{i-1} \geq \bar{e}_k\}} \left( e_{i-1} - \bar{e}_k \right) + \sum_{k=0}^{K^-} \alpha^-_k \mathbf{1}_{\{e_{i-1} < \bar{e}_k\}} \left( \bar{e}_k - e_{i-1} \right) + \beta \psi_{i-1} \] (2.31)

where \( \alpha^+ \) and \( \alpha^- \) are the coefficients of the of the piecewise linear spline.

**Model with two innovations**

An alternative way to extend the ACD model is to allow for the conditional mean to be driven by a stochastic process (based on the marks or latent); then before the ACD model is estimated, there is a hidden process that should be inferred to justify the regime switching behaviour of the conditional mean function for the durations. Hence, there are two stochastic processes, or two innovations processes to be estimated.

Bauwens and Veredas (1999) propose a Stochastic Conditional Duration (SCD) model, which assumes that the conditional mean function \( \psi_i \), given the information up to \( i-1 \) is not deterministic but follows a latent AR(1) process, for example a hidden information flow that determines the state of the market and influences the duration process but cannot be observed directly. The specification of the SCD model is as follows:

\[ x_i = \psi_i e_i \]
\[ \ln \psi_i = \omega + \beta \ln \psi_{i-1} + u_i \] (2.32)

where \( e_i \) is the regular ACD innovation with, exponentially distributed, distribution of \( u_i \) conditional on the history of the process is normal \( u_i \bigg| \mathcal{F}_{t-1} \sim i.i.d. N(0, \sigma^2) \) and \( u_i \) is independent of \( e_i \). In this case the (marginal) distribution of \( x_i \) is determined by the mixture of log-normal and exponential distributions. The model proves to be quite flexible in spite of restrictive distributional assumptions of \( e_i \) and allows for a wide variety of different marginal hazard functions.
A related model which rests on the same idea of latent variable influencing the states of conditional distribution of durations is Markov Switching ACD model proposed by Hujer, Vuletic, and Kokot (2002) in which the conditional mean depends on the latent stochastic process which follows a Markov chain. It assumes that the regime specific conditional mean function is deterministic on the information up to $i-1$:

$$
\psi_i^{(r)} = \omega^{(r)} + \sum_{j=1}^{P} \alpha_j^{(r)} x_{i-j} + \sum_{j=1}^{Q} \beta_j^{(r)} \psi_{i-j}^{(r)}
$$

(2.33)

where $\psi_i^{(r)}$ is the regime-specific conditional mean function and the conditional mean is a sum of probability weighted regime-specific conditional means:

$$
\psi_i = \sum_{r=1}^{R} \Pr[R_i = r|S_{t-i}] \cdot \psi_i^{(r)}
$$

(2.33a)

Empiric studies suggest that many of the durations processes exhibit extremely strong persistence and close to unit root behaviour. The decay in the autocorrelation function is a lot slower than for the exponential decay. To tackle this problem, Engle (2000) applies a two-component model where first component $\psi_{1,i}$ is integrated ACD(1,1) capturing long-term dependence in duration, while the second component, $\psi_{2,i}$ is designed as a regular ACD(1,1) and takes care of the short-term dependencies. The conditional mean function for the durations is then modelled and the weighted average of the two components:

$$
\psi_{1,i} = \alpha_1 x_{i-1} + (1 - \alpha_1) \psi_{1,i-1},
\psi_{2,i} = \omega + \alpha_2 x_{i-1} + \beta_2 \psi_{2,i-1},
\psi_i = \omega \psi_{1,i} + (1 - \omega) \psi_{2,i},
$$

(2.34)

The next two innovations model proposed by Ghysels, Gouriéroux and Jasiak (2004), Stochastic Volatility Duration (SVD) Model is specifically designed to tackle the most significant drawback of the ACD-type models that they do not allow to separately parameterize higher moments dynamics due to the fact that typical duration distributions imply strict relationship between the mean and higher moments. SVD model allows separate estimation of the dynamics for the conditional volatility of the durations.
Models of the Durations and the Marks

The next type of models allows modelling the dynamics separately for the durations and for the marks\(^{10}\).

In the Threshold ACD, suggested by Zhang, Russell and Tsay (2001), the error term, \( \varepsilon_i \), depends on history of the observable components (marks) as well as the history of the process; the parameterization of the conditional mean first requires estimation of the coefficient for the marks, which can be different for the different regimes. After the parameters for the marks for the \( K \) regimes are estimated, the conditional mean function for the durations is constructed as follows:

\[
\psi_i = \omega^{(k)} + \sum_{j=1}^{p} \alpha_j^{(k)} x_{i-j} + \sum_{j=1}^{q} \beta_j^{(k)} \psi_{i-j}
\]

(2.35)

where \( \omega^{(k)}, \alpha_j^{(k)}, \beta_j^{(k)} \) are nonnegative regime switching parameters.

Closely related to Threshold ACD model is Smooth Transition ACD model suggested by Hautsch (2004, 98) allows for the smooth transition between the regimes, rather than a step function as in Threshold ACD model. The conditional mean function then has the following parameterization for the two-regime model:

\[
\psi_i = \omega^{(k)} + \sum_{j=1}^{p} \left[ \alpha_{ij} + \alpha_2 \varsigma(\varepsilon_{i-j}) \right] \varepsilon_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}
\]

(2.36)

where the transition function has the following form:

\[
\varsigma(\varepsilon_i) = 1 - \exp(-\nu \varepsilon_{i-1}), \quad \nu \geq 0
\]

(2.37)

In this case the transition between the two regimes is smooth and is determined by the function:

\[
\begin{align*}
\varsigma(\varepsilon_i) &= 1 \quad \text{if} \quad \varepsilon_i \to \infty \\
\varsigma(\varepsilon_i) &= 0 \quad \text{if} \quad \varepsilon_i = 0
\end{align*}
\]

(2.38)

\(^{10}\) A process for the marks is modelled separately from the modelling of the conditional mean for the durations, in contrast to the previously described ACD-type models, where it was possible to include the marks, but there were no special processes modelled for the marks.
Log-ACD (Bauwens and Giot, 1998) models jointly the dynamics of the durations between the two quotes and a binary variable for the direction of the mid-quote price change given the past process for the durations and information about other marks included. This model is asymmetric in its structure as it picks up the differences between the duration processes depending on whether duration end with a positive or a negative price change. This model is used to predict the next duration and the direction of the price change and to calculate the probability of price increase based on the past information.

The last of this class of models is the UHF-GARCH model (Engle, 2000). It is modelled as an ACD model for the durations and GARCH for the volatility given the history of volatility and the duration processes.

UHF-GARCH can be viewed as an alternative way to introduce regime-switching behaviour into the duration process where ACD parameters depend on the states which are determined by the marks. The possible uses of this type of models are to predict future volatility based on the predicted future durations, to test contemporaneous and lagged relationship between durations and volatility. UHF-GARCH models simultaneously the process of the trade durations and the volatility process of the trade-to-trade price changes.
2.3 Market Microstructure Underpinnings

The market microstructure proposes three explanations for the illiquidity in the markets: inventory costs, order processing costs and adverse selection or asymmetric information. A type of premium is charged for the immediacy of execution. The transaction costs in turn influence the order decisions of traders and determine the trading prices, causing the illiquidity in the market, but these do not generally vary over time, even though they vary over the size of the transactions. The inventory models of market microstructure postulate, that inventory costs together with the transactions costs, determine the bid-ask spread and therefore dictate the liquidity level. The information-based models use the insights from the theory of adverse selection and demonstrate that even if there were no transactions costs in perfectly competitive market there still would exist the bid-ask spread. Besides explaining the difference between the seller’s and the buyer’s price, information-based models allow to examine the dynamics of the trading process and by this provide insights into the adjustment process of the prices.

The information models which connect time and the price formation process are of a particular interest in the given analysis. If the time of transaction is in some way correlated with the other factors which determine liquidity, as price changes, volume and bid-ask spread, the absence of trade provides valuable information as well. It is useful to formulate the price adjustment process in terms of asymmetric information models, as in O’Hara (1995). It is assumed that there exist two types of traders: informed traders and liquidity traders. The specialist sets the bid and the ask price without knowing the number of informed and uninformed traders. The information is revealed to a certain fraction of traders and the others remain uninformed. They will buy and sell with the fixed probability. The information can involve good news or bad news. At the arrival of new information the informed traders will trade more heavily in order to use their fleeting informational advantage. The liquidity is supplied to the market by the uninformed traders presuming that there is a certain number of the informed insiders in the market, which introduces the time-varying manner to the trading relative to the prevailing market conditions. If part of the private information is revealed to the specialist, or if she observes heavy trading presumably resulting from the private information, she will widen the spread in response to large volume orders in the market.

There are some possible extensions developed along this basic framework. If as was stated above, the revealed news has effect on the trading intensity performed by the
informed traders then the intensive trading will generally imply good news. This hypothesis was first posed by Diamond and Verrecchia (1987), however the results appeared to be rather contradictory. The hypothesis can only be tested regarding the direction of trade. According to Diamond and Verrecchia (1987) framework, there is some news revealed at the beginning of the day; if the news is good, informed traders will always buy; in such a situation, intense trading means good news, this however will only be true for the buy transactions. In the presence of bad news the informed traders would be willing to use their superior information by selling the asset; then as long as the informed traders have the asset in their possession, intense trading will also follow the bad news, then the above hypothesis is contradicted. If conversely the agent does not have the asset, she would like to short it. There are different restrictions on short selling implemented by the exchanges in order to prevent a greater collapse of the price, for example the “uptick” rule of NYSE, according to which short sale is not allowed on falling prices and there should be at least one upward price move before short selling is possible. Short sales might also be subject to the “proceeds” constraints, which imply that the proceeds of the short sale will not be delivered to the trader until the short position is reversed. These two restrictions might prevent traders from taking advantage of their superior information, and they will not trade at the given prices at all. This would translate into a statement that no trades means bad news. However, market microstructure theory suggests, that there are other reasons for low trading intensity: the agent might simply decide not to transact and this would be independent of the current information and there is no informational contest in the very fact of low trading intensity. In Easley and O’Harra (1992) this is accounted as “no news”, otherwise the informed traders would willingly transact. In the presence of short sales constraints, the price adjustment to the new information is slow, and the adjustment to the bad news is particularly affected. If the prices are only subject to the proceeds constraints, then the adjustment of the prices is a lot faster, while the informed traders continue to sell. So, in the case of no restrictions on short sales, or there is a sufficient number of informed traders who are able to trade at any market conditions, they will buy in the presence of good news, sell or short sell on bad news and will not trade if they do not have superior information or no news. Hence, long durations and slow trading activity imply result in the conclusion no trades means no news.

Yet another explanation is possible. There is always the other fraction of market participants, left unaccounted for in the current analysis, but taken into consideration in Easley and O’Harra theory: the uninformed traders, who possess no superior
information and trade because of exogenous liquidity reasons. High trading period would be associated with the high proportion of uninformed traders. In this case heavy trading has nothing to do with the news arrival or asymmetric information and the durations will be short, volumes large accompanied by tight bid-ask spreads and low volatility. If however, the liquidity traders do not trade as much, there is high proportion of informed traders in the market. Due to the fact that informed traders think over their trading behaviour before entering a transaction, the activity in the market is low, spread are high and prices are volatile because each trade is information revealing. Then slow trading means informed trading.

A different interpretation is possible under which clustering of transactions occurs due to the presence of high proportion of informed traders who rush to maximize their profits from the fleeting informational advantage. This would mean that intense trading is informed trading and price discovery occurs relatively quickly.

Adamati and Pfleiderer (1988) differentiate between two types of uninformed traders: nondiscretionary liquidity traders trade due to completely exogenous reasons and will trade a certain number of shares at a particular time; the discretionary traders also have demand for liquidity, however follow more strategically substantiated trading and will transact only at the best time: when there proportion of informed traders is low, and they cannot disturb the prices by taking advantage of superior information and abstaining from trading when the proportion of informed traders is high. According to this framework it is impossible to distinguish between the above average activity which is due to a high proportion of informed traders and which is due to a high proportion of uninformed discretionary traders. It is also inconclusive how the discretionary traders distinguish between the two and decide to trade for liquidity reasons.

The newly developed econometric techniques as well as the technological advances in collecting and handling the data make the above mentioned hypothesis partially testable. These techniques go in the direction of ACD framework proposed by Engle and Russell (1998) which is based on the irregularly spaced data analysis. However, so far, no empirical work has reached unanimous and conclusive results in employing the asymmetric information framework in part due to the reason that the asymmetric information framework is in itself neither unanimous nor conclusive.
3 Empirics

3.1 Data

In the empirical part of this paper we employ data from New York Stock Exchange Trades and Quotes (TAQ) database. The NYSE TAQ database contains intraday transactions data for trades and quotes for all securities listed on NYSE, AMEX, Nasdaq Market System (NMS) and SmallCap issues. We use five stocks of (presumably) different liquidity levels in order to be able to draw comparative results. The stocks used in the analysis are: Citigroup (C), Hewlett Packard (HWP), American Express (AXP), Jack in the Box (JBX) and Fair Isaac Corp (FIC). The stocks are selected in a way to have different trading intensity. One of our tasks in the following data analysis is to check where the concept “more heavily traded stock” is equivalent to the concept of “more liquid stock” and to see where the high-frequency analysis, and it’s limiting case, transactions analysis makes equally much sense for the stocks with different trading intensity. The analysis will be tied to the market microstructure theories to find the explanations for the empirical phenomena.

To gain the initial insight into the differences in the stocks, we first compare some of the summary statistics and later will compute some stock liquidity measures for further analysis. Another task that will be addressed in this paper is whether the durations of stocks with different trading intensities have the same duration dependence pattern.

The NYSE TAQ dataset consists of two files: trades and quotes. The trades file consists of the following columns of data: SYMBOL, DATE, PRICE, SIZE, G127 and COND. The last two columns refer to the specific trades conditions which are irrelevant for the further analysis and therefore will be left out and not referred to. The quotes file consists of the following columns: SYMBOL, DATE, BID, BIDSIZE, OFFER, OFFERSIZE, MODE, MMID. The last two columns again contain no relevant information and will be left without further attention. The raw data contains every recorded entry in the corresponding database. As NYSE represents a hybrid type trading mechanism, the quotes reported in the quote database can be submitted by specialists, formed on the basis of limit orders of market participants from the limit order book or on the basis of limit orders submitted by traders on the trading floor. The peculiarity of this database, and its limitation to the current analysis is that it only contains the pair of the best bid and ask, unlike the full information limit order book contains five pairs of bids and asks.
A crucial feature of this data that determines the entire branch of modern econometric research and as well will be employed in this paper is the fact that entries are irregularly spaced: the occurrence of an event (quote announcement or trade in this case) is an outcome of a random variable.

The raw data first has to be filtered to eliminate errors, inconsecutive trades, etc. The procedure of data preparation for this paper consists of the following:

1. We analyse the three-month period starting on January 1, 2001 and ending on March 30, 2001. This gives us a sample of 61 trading days. There were no haltings in trading, partial trading days or other disturbances on the NYSE, which should have been accounted for before the analysis is carried out. Therefore the entire sample is kept on this stage.

2. The entries that occurred outside of trading hours, namely earlier than 9:30 a.m. and later than 16:00 p.m. were deleted from the sample. Inconsecutive entries were corrected to be in the sequence.

3. The two databases were merged. In merging these databases a trade is merged with the prevailing quote. We adopted the five seconds rule, which takes into account for the trade not the last recorded quote but the quote that have occurred up to five seconds before the trade. Matching trades and corresponding quotes was a long existing problems in managing the databases, which was exacerbated when the “electronic books” became widespread with the specialists. The quote that enters the electronic book at the same time as the trade, or even few (up to 5) seconds before the trade has no effect on this trade as there is not enough time to incorporate it. The quotes often are posted more quickly than transactions are recorded, meaning that a quote revision will often precede the trade from which it was instigated. Instead, Lee and Ready (1991) argue that if the “five second rule” is applied, misspecifications can be greatly reduced comparing to the situation when the last quote is used. In merging the databases, each trade is also being classified into seller initiated or buyer initiated using Lee and Ready (1991) algorithm by comparing the trade price to the bid and ask prices of the prevailing quote. If the trading price is closer to the ask than the bid quote the trade is then classified as a buy and otherwise it is a sell. If the trading price occurred exactly in the middle of the bid-ask spread of the prevailing quote, then the tick rule applies. According to this rule the trade price is compared to the trade price of the adjacent trade: if the previous trade’s price was lower, this means that the current transactions price is greater than the previous, implies that the trade was initiated by a buyer, otherwise, it must have been initiated by a seller.
4. Time is transported from the “date” measure into seconds from 9:30, which gives 23400 seconds during a trading day.

5. Next we have dropped the first trade of each day from the sample because of the opening auction. At the beginning of the day on NYSE a specialist observes electronically submitted quoted as well as the interests directly on the trading floor and sets a single opening price that clears the market. Although the opening of the exchange might convey important information of the price discovery, we choose to disregard it in the present analysis and the liquidity measures from this period will not possess the representative patterns of the rest of the day.

6. We calculate trade durations as the difference in time between two consecutive trades $t_i - t_{i-1}$, the time stamp corresponding to $i$. The quote pairs and size pairs also correspond to $i$.

7. In the current paper we define the price of the transaction as the average of the bid-ask spread rather than the actual price of the transaction. The motivation for this stems from the idea of the bid-ask bounce, which causes first-lag negative autocorrelation in asset returns. To illustrate how this actually happens, we use an example: consider a stock, for which the current bid price is equal to $b$ and the current ask price is equal to $a$. Suppose no news appears for ten minutes. But, over this period, suppose that a buy order first comes in at the ask price, $a$ followed by a sell order at the bid price $b$. This sequence of events makes it seem that the stock price has dropped by the amount of the bid-ask spread, $a-b$. This is however a totally spurious price movement.

   Even when no news is breaking, when a stock price is not changing, the “bid-ask bounce” is about prices bouncing up and down between the bid and the ask. This problem with this spurious changed is the greatest with illiquid stocks where the bid-ask spread is wide, but even with the most liquid stock it contaminates the price dynamic and leads to the fallacious conclusions. This problem is significantly mitigated when using the midquotes as the price of the stock which is a more precise way to define the fundamental price of the asset. Following, we calculate the returns per trade as the difference between the log prices.

8. It is possible that some trades demanded the volume that was greater than the current market depth on the relevant side; these transactions then were split into few of lower volume and registered at the same time stamp. This is the result of impatient trading when the impatient seller agrees to travel down the limit order book to sell at the second best price, third best price and so on, decreasing the effective price of the entire transaction; while the impatient buyer agrees to buy a part of the desired volume at the
prices higher than the best ask, increasing the effective price of the buy transaction in return to the opportunity to have the immediate order execution. This leads to the fact of having zero durations in the database. To eliminate them we apply the following procedure: the zero durations are merged to the last non-zero duration, the new volume is the sum over the volumes of the zero durations and last non-zero duration and the price is the effective price paid for the joint transaction calculated as volume-weighted average of the zero duration prices and last non-zero duration price.

After performing these steps of data preparation, we received a file of final data for each of the five analysed stocks. The stocks indeed have different trading intensity: over the same time period we received the following number of unique trade durations. For the Citigroup stock there are 163 459 trade durations in the final sample, this is the greatest number of trade durations among the analysed stocks. For the American Express stock there are 143 680; for Hewlett Packard there are 143 753 trade durations; both of these stocks can also be considered heavily traded compared to Jack in the Box and Fair Isaac Corp., which have 10 680 and 9 049 trade durations respectively. In order to carry out the optimally sensible analysis, it is reasonable to account for the outliers in the data that carry no relevant to the present analysis information and unnecessarily contaminate the corresponding processes of the durations or the marks. We eliminate the outliers defined as “too large volume” or “too large duration” or “too large spread” examining each dataset separately. The adjustments are summarized next to the output of the descriptive statistics for the corresponding data.

Table 3.1: Summary statistics for five analysed stocks.

Summary statistics for “C” (Citigroup)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Variance</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Valid</th>
<th>Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>8.8512</td>
<td>9.5753</td>
<td>91.6869</td>
<td>1.0000</td>
<td>282.0000</td>
<td>163459</td>
<td>0</td>
</tr>
<tr>
<td>Size</td>
<td>492.4371</td>
<td>14115.592</td>
<td>19924992.4415</td>
<td>100.0000</td>
<td>1786800.0000</td>
<td>163459</td>
<td>0</td>
</tr>
<tr>
<td>Spread</td>
<td>0.0824</td>
<td>0.0680</td>
<td>0.0046</td>
<td>0.0099</td>
<td>2.5000</td>
<td>163459</td>
<td>0</td>
</tr>
<tr>
<td>Quotes</td>
<td>1.4752</td>
<td>1.8398</td>
<td>3.3849</td>
<td>0.0000</td>
<td>22.0000</td>
<td>163459</td>
<td>0</td>
</tr>
</tbody>
</table>

Summary statistics for “HWP” (Hewlett Packard)

The deleted outliers: observation with volume over 3,564,000;
observation with bid-ask spread 30.1099.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Variance</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Valid</th>
<th>Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>10.0390</td>
<td>11.5998</td>
<td>134.5560</td>
<td>1.0000</td>
<td>227.0000</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Size</td>
<td>2503.8260</td>
<td>6715.1936</td>
<td>45093824.7516</td>
<td>100.0000</td>
<td>350000.0000</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Spread</td>
<td>0.0735</td>
<td>0.0629</td>
<td>0.0040</td>
<td>0.0099</td>
<td>3.1999</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Quotes</td>
<td>1.4529</td>
<td>1.7166</td>
<td>2.9467</td>
<td>0.0000</td>
<td>25.0000</td>
<td>143753</td>
<td>0</td>
</tr>
</tbody>
</table>

Summary statistics for “AXP” (American Express)

The deleted outliers: observation with volume 3,253,100;
observations with bid-ask spreads 5;
observation with duration 524.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Variance</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Valid</th>
<th>Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>10.0390</td>
<td>11.5998</td>
<td>134.5560</td>
<td>1.0000</td>
<td>227.0000</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Size</td>
<td>2503.8260</td>
<td>6715.1936</td>
<td>45093824.7516</td>
<td>100.0000</td>
<td>350000.0000</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Spread</td>
<td>0.0735</td>
<td>0.0629</td>
<td>0.0040</td>
<td>0.0099</td>
<td>3.1999</td>
<td>143753</td>
<td>0</td>
</tr>
<tr>
<td>Quotes</td>
<td>1.4529</td>
<td>1.7166</td>
<td>2.9467</td>
<td>0.0000</td>
<td>25.0000</td>
<td>143753</td>
<td>0</td>
</tr>
</tbody>
</table>
Even before any of the liquidity measures are calculated, it is already possible to make some conclusions based on the descriptive statistics provided above. The most heavily trading stock, Citigroup, also has the lowest average duration of 8.8512 seconds and highest average volume per trade (average size of 4 492.44 shares per trade), the stocks of Hewlett Packard and American Express, besides having approximately the same number of trades during the period under consideration, have very similar statistics for average duration and average size of transaction. The same fact is observed for the two relatively lightly traded stocks, Jack in the Box and Fair Isaac Corp., which also have very similar statistics for average duration and average size (average trades are 134.4 and 159.0 compared to 10.0 for Hewlett Packard and American Express average sizes of 1 249.4 and 512.7 compared to 2 503.8 and 2 535.4 respectively). The average spreads for more heavily traded stocks are also a lot tighter than for the less heavily traded stocks as well as the average number of quotes is lower for the heavier traded stocks, which confirms the logical reasoning. The dispersion measures (variances and standard deviations) for durations and spreads and for the number of quotes per duration are lower for heavier traded stocks and can in a certain base way be interpreted as greater liquidity.

8. Most of the high frequency data literature suggests the presence of pronounced diurnal patterns in the durations data, which is confirmed by the market microstructure theories, the human psychology and simple commonsensical reasoning. Intense trading is expected at the opening and closing of the exchange, whereas more lax trading is usually observed around noon indicating the existence of the “lunchtime
effect”. This pattern is expected to translate into shorter durations higher trading volume and tighter spreads at the beginning and the end of the trading with the opposite patterns around the lunchtime.

Bearing the aforementioned logics in mind, in order to take the seasonal effect out of the durations, we decomposed them into stochastic component and time of the day effect. Following Engle and Russell (1998) we define the deterministic time of the day effect as a multiplicative component:

\[ X_t = x_t \phi(t) \]  

(3.1)

where \( X_t \) represents raw durations, \( x_t \) is the stochastic component or the deseasonalized durations and \( \phi(t) \) is time of the day effect. Time of the day effect is estimated with the cubic spline function on the basis of disjointed nods constructed by averaging the 30-minute periods means over all trading days; afterwards the daily time function constructed by approximate interpolation of every second in each 30-minute period.

The cubic spline function has the following form:

\[
s(t_{i-1}) = \sum_{j=1}^{K} I_j \left[ c_j + d_{1,j} \left( t_{i-1} - k_{j-1} \right) + d_{2,j} \left( t_{i-1} - k_{j-1} \right)^2 + d_{3,j} \left( t_{i-1} - k_{j-1} \right)^3 \right] \]

(3.2)

where \( I_j \) is the indicator variable for the \( j \)-th segment of the spline, such as if \( k_{j-1}<t_{i-1}<k_j \), \( I_j=1 \), otherwise \( I_j=0 \); \( c_j \) is normalized by restricting the unconditional mean of the diurnal factor to the sample mean and \( d_{1,j}, d_{2,j}, d_{3,j} \) are the linear, quadratic and cubic coefficients of the cubic spline function.

The cubic splines for the daily trade durations of the heavily traded stocks have the following shape:

Figure 3.1.1: Cubic splines for the durations of the three heavily traded stocks: C, HWP and AXP.
Obviously, the diurnal pattern for the heavy traded stocks confirms the expectations: short durations at the beginning and at the end of the trading day and a pronounced hump around the lunch hours. The evidence even though less pronounced, but still clear is observed for the diurnal durations pattern for the less heavily traded stocks: the basic shape is still there, however what the numerous peaks and troughs in the middle of the day correspond to, remains unclear. We regard this as the evidence for the inconsistency of the diurnal pattern. But since nevertheless the theoretically confirmed basic shape is preserved, we do not disregard this pattern in the empirical analysis.

![Figure 3.1.2: Cubic splines for the durations of the two less heavily traded stocks: JBX and FIC.](image)

9. Engle (2000) suggests that size and spread also have deterministic daily pattern component. The same type of cubic spline time functions with 30 minutes nodes is applied to account for the seasonal effect. We see no motivation behind assumption of a special pattern pertaining to the day of the week therefore it was not accounted for in our data.

We check whether the diurnal pattern for the traded volumes during the trading day confirms our theoretical and logical surmises. The following illustration depicts the daily transaction size patterns for the heavier traded stocks:

![Figure 3.1.3: Cubic splines for the volumes of the three heavily traded stocks: C, HWP and AXP.](image)

The U-shape is present in all three stocks, confirming the earlier mentioned statements; the pattern is clear and pronounced. The pattern for the other two stocks with low trading intensity, as shown on the Figure 3.4, have a very unclear pattern, or to
put it bluntly, do not have a daily shape characteristic for each day of the analysed sample; the spline function is constructed by connecting the disjoi nted half-hour time period nods which are the averages of basically random trading volumes over the given period. The great dispersion of sizes of transactions (table 3.1), relative to the average size might be one explanation for such peculiar behaviour of the averaged values.

![Figure 3.1.4: Cubic splines for the volumes of the two less heavily traded stocks: JBX and FIC.](image)

10. Some relevant studies have found the presence of diurnal seasonality on the spreads, for example Chan, Chung and Johnson, H. (1995), found that spread also exhibits the U-shape daily seasonality. Based on our estimates we do not find evidence for this behaviour. The estimated cubic splines for the heavily traded stocks shows evidence of increased spreads at the beginning of the day, but less conclusive evidence of an increase at the end of the day for the C and next to no such evidence for HWP and AXP at the end of the trading day:

![Figure 3.1.5: Cubic splines for the volumes of the three heavily traded stocks: C, HWP and AXP.](image)

As in the case of seasonality with the volumes of transactions, for the not heavily traded stocks it is rather difficult to draw substantiated conclusions as to the daily pattern in spreads:
To take the diurnal effect away from the data, the deseasonalized durations, sizes and spread are computed by dividing raw data over the corresponding function of “time of the day” effect.

Instead of true transactions prices we use the midquotes of the prevailing quote as analysed prices and the changes in the midquotes respectively. This is done particularly to decrease the effect of the bid-ask bounce in the prices which causes first lag negative autocorrelation.

We calculate price durations which are defined as time elapsed before a certain absolute cumulative price change was generated. We adopted different price durations for each of the stocks based on individual durations and price changes pattern; we use the type of filtering justified by the fact that smaller changes in midquote price are transitory and might even be related to the bid-ask bounce\(^\text{11}\).

\(^{11}\) As it was mentioned, the fact that we use midquotes of the prevailing quote to examine the price change dynamics for the trade durations, is precisely done in order to decrease the bid-ask bounce effect. However Engle (2000) alleges, not all of the bid-ask bounce effect is captured by such action.
3.2 Estimation of the Liquidity Dynamics

3.2.1 Classical ACD model

In the following section we following the seminal paper of Engle and Russell (1998), estimate the Autoregressive Conditional Duration model for each of the five stocks. We account for no other marks, such as trading volume, price changes, spreads, quote revisions, etc. as we are mainly interested to show the autoregressive structure in the duration process itself.

The diurnally adjusted durations exhibit highly persistent autocorrelations which is a strong signal for long-memory dependence.

![Figure 3.2.1: Autocorrelation functions for the first 200 lags of the trade durations for C, HXP and AXP.](image)

The autocorrelation functions for three heavy traded stocks (figure 3.2.1) first declines, but then remains positive for significant number of lags, The Q-statistics at the twentieth lag has values of 14525 for C, 8705.1 for HWP and 10 796 for AXP, all of which dramatically exceed the critical level at 5% significance of 10.851 with 20 degrees of freedom\(^{12}\).

![Figure 3.2.2: Autocorrelation functions for the first 200 lags of the trade durations for JBX and FIC.](image)

The autocorrelation functions for the other two stocks (figure 3.2.2) also show a long memory dependence, with higher dispersion, where relatively high

\(^{12}\) Complete correlograms are presented in Appendix A.
autocorrelations are followed by relatively low ones, and then high ones again with the overall trend of only a gradual decline. This pattern is actually the same as for the heavily traded stocks; the values of Q statistics still by far reject the null hypothesis of no autocorrelation having values at the twentieth lag of 976.31 for JBX and of 1963.4 for FIC. On the graph of the autocorrelations function we see the persistence of the process for a large number of lags, where all autocorrelations remain positive.

These facts about the persistent autocorrelations hint on the idea of autoregressive conditional durations. We estimate the plain vanilla ACD models for each of the stocks and will argue that such a compact structure is able to capture the serial dependence in the lags.

As was already described, the ACD model has the following general parameterization of the conditional mean:

\[
\psi_i = \omega + \alpha x_{i-1} + \beta \psi_{i-1}
\] (2.21)

where \(\psi_i\) is the conditional expectation of \(x_i\):

\[
E(x_i | x_{i-1}, \ldots, x_1) = \psi_i(\{x_{i-1}, \ldots, x_1; \theta\}) \equiv \psi
\] (2.14)

The ACD framework parameterizes this conditional expectation so that the error enters a function in the multiplicative way:

\[
x_i = \psi_i \varepsilon_i
\] (2.15)

where \(\{\varepsilon\} \sim i.i.d.\) with density \(p(\varepsilon; \varphi)\) and there is no connection between \(\varphi\) and \(\theta\). The critical assumption of this ACD model is that the whole dynamics lies in the conditional expectation and there is no dependence in the higher moments. The residuals after the estimation of ACD should be therefore white noise.

The conditional intensity for the ACD model has the following form:

\[
\lambda(t | N(t), t_1, \ldots, t_{N(t)}) = \lambda_0 \left( \frac{t - t_{N(t)}}{\psi_{N(t)+1}} \right) \frac{1}{\psi_{N(t)+1}}
\] (2.17)
The simplest ACD assumes that the durations are conditionally exponential and the baseline hazard, \( \lambda_0 \) is equal to one, then the conditional intensity function has the following simple form:

\[
\lambda \left( t \mid N(t), t_1, \ldots, t_{N(t)} \right) = \frac{1}{\psi_{N(t)+1}}
\]

(2.18)

The assumption of exponential distribution might seem rather restrictive, as there is a whole class of distributions that satisfy the conditions of the model specification and allow for more general and less restrictive distributions for the error term, as was already shown\(^{13}\).

We will however show that regardless of the true density, the assumption of the exponential density for the hazard leads to consistent and asymptotically efficient results due the fact that the estimates possess the property of quasi maximum likelihood estimator of the true ACD parameters.

The log of the quasi maximum likelihood function is then specified as follows:

\[
Q(X, \theta) = -\sum_{i=1}^{N} l_i(\theta) = -\sum_{i=1}^{N} \left[ \log \psi_i - \frac{X_i}{\psi_i} \right]
\]

(3.3)

This likelihood function would be the true likelihood function if the density of \( \varepsilon \) were indeed exponential. Engle (2000) illustrates that the results of QMLE for the distribution from the exponential family with independent observations first proven by Gouriéroux, Monfort and Trognon (1984) can be applied to the ARCH-type models with data dependence and, as proven by Bolleslev and Wooldridge (1992), and therefore, is directly applicable to the EACD model.

The essential prerequisite for the Quasi Maximum Likelihood estimation is the correct specification of the conditional mean function and then, the estimator is shown to be independent of the distribution of the disturbance and is consistent Quasi Maximum Likelihood estimator.

If in the specified above likelihood function conditional mean is specified correctly, the resulting estimator will be independent of the distribution of the error term. To check if this property holds, we first find the analytic derivatives of the quasi maximum likelihood function:

\(^{13}\) As durations and expected durations are positive, the multiplicative error term should have the mean of 1 and should be defined on the positive support.
Score:
\[ \frac{\partial l_i}{\partial \theta} = \frac{(\psi_i - x_i) \frac{\partial \psi_i}{\partial \theta}}{\psi_i^2} \]  
(3.4)

Expected Hessian:
\[ H = E_{t-1} \left( H_{true} \right) = -\frac{1}{\psi_i^2} \frac{\partial \psi_i}{\partial \theta} \frac{\partial \psi_i}{\partial \theta'} \]  
(3.5)

where \( H_{true} \) is the Hessian matrix:
\[ H_{true} = \frac{\partial^2 l_i}{\partial \theta \partial \theta'} \]  
(3.6)

Under the correct specification of the mean function the following equality holds:
\[ \psi_{i,0} = \psi_i \left( \theta_0 \right) = E \left( x_i \bigg| \mathcal{I}_{k+4}; \theta_0 \right) \]  
(3.7)

where \( \psi_{i,0} \) is the true conditional mean function. It follows that \( \epsilon_i = \frac{x_i}{\psi_i} \) is stochastically independent from \( \psi_i \) and has an expectation of one and the score is a martingale difference with respect to the information set \( \mathcal{I}_{k+4} \). In this case the expectation of the Information Equality holds:
\[ E \left[ \frac{\partial^2 l_i}{\partial \theta \partial \theta'} \mathcal{I}_{k+4} \right] = \sum_{i=1}^{n} h_i, true = -\sum_{i=1}^{n} E \left[ \frac{1}{\psi_{i,0}^2} \frac{\partial \psi_{i,0}}{\partial \theta} \frac{\partial \psi_{i,0}}{\partial \theta'} \bigg| \mathcal{I}_{k+1} \right] \]  
(3.8)

Thus, under the correctly specified conditional mean function it is possible to derive the quasi maximum likelihood estimator regardless of the true density of the error term.

We will estimate the ACD model assuming the exponential distribution for the error terms. Usually the lag dependence dynamics in the ACD-type models is captured by low order lag specification, and estimation of ACD(1,1) or ACD(2,2) usually suffices. In the proceeding, we will estimate ACD(2,2) with no covariates to be able to capture the effect from the lagged durations and distinguish it from the effects of the covariates.

The model has the following specifications for the conditional mean:
\[ \psi_i = \omega + \alpha_1 x_{i-1} + \alpha_2 x_{i-2} + \beta_1 \psi_{i-1} + \beta_2 \psi_{i-2} \]  
(3.9)
The estimation is carried out by maximum likelihood maximizing the following log likelihood function:

$$
\max_{\alpha, \alpha_1, \alpha_2, \beta_1, \beta_2} Q = - \sum_{i=1}^{N} \left[ \log \psi_i - \frac{x_i}{\psi_i} \right]
$$

(3.3)

The analytical derivatives for $i$-th observation have the following form:

$$
\frac{\partial l_i}{\partial \omega} = - \frac{1}{\psi_i} + \frac{x_i}{\psi_i^2}
$$

$$
\frac{\partial l_i}{\partial \alpha_1} = - \frac{x_{i-1}}{\psi_i} + \frac{x_i}{\psi_i^2} x_{i-1}
$$

$$
\frac{\partial l_i}{\partial \alpha_2} = - \frac{x_{i-2}}{\psi_i} + \frac{x_i}{\psi_i^2} x_{i-2}
$$

$$
\frac{\partial l_i}{\partial \beta_1} = - \frac{\psi_{i-1}}{\psi_i} + \frac{x_i}{\psi_i^2} \psi_{i-1}
$$

$$
\frac{\partial l_i}{\partial \beta_2} = - \frac{\psi_{i-2}}{\psi_i} + \frac{x_i}{\psi_i^2} \psi_{i-2}
$$

(3.10)

As was described above, the estimates from this procedure possess the QML property and the estimated coefficients are consistent estimates of the true parameters.

Following Engle and Russell (1998), the model is estimated using GARCH software and taking $\sqrt{x_i}$ as dependent variable.

The simple model ACD(2,2) is estimated for all five stocks and the results are summarized in the following table:

<table>
<thead>
<tr>
<th>Stock</th>
<th>c</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“C”</td>
<td>0.002698</td>
<td>0.101602</td>
<td>-0.094581</td>
<td>1.647115</td>
<td>-0.656850</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>“HWP”</td>
<td>0.001149</td>
<td>0.027151</td>
<td>-0.023034</td>
<td>1.733239</td>
<td>-0.738561</td>
</tr>
<tr>
<td></td>
<td>(0.0123)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>“AXP”</td>
<td>0.004841</td>
<td>0.007711</td>
<td>0.024008</td>
<td>0.287474</td>
<td>0.676019</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0022)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>“JBX”</td>
<td>0.056177</td>
<td>0.035412</td>
<td>0.077884</td>
<td>0.211441</td>
<td>0.620679</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0011)</td>
<td>(0.0000)</td>
<td>(0.0091)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>“FIC”</td>
<td>0.017812</td>
<td>0.063654</td>
<td>0.076708</td>
<td>0.058374</td>
<td>0.788081</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0683)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

14 Complete outputs are presented in Appendix B.
Obviously, all the estimates for $\alpha_i$ and $\beta_i$ for all five stocks are highly significant, as well as the estimates for $\alpha_2$ and $\beta_2$ are except for FIC, where second GARCH coefficient appears to be insignificant. This might be due to the different autocorrelation structure in the series or the sufficiency of ACD(1,1) specification. On the basis of current specification, without the account for informational asymmetry, it is difficult to make valid conclusions as to the impact of the coefficients on the process, with respect to the microstructure theories. For the given data it is almost unanimous, given the positive ARCH and GARCH coefficients, that long durations are likely to be followed by long durations, and vice versa. This is consistent with the fact of liquidity clustering and autoregressive dynamics. With this framework only it is also possible to distinguish between periods when the informed traders prevail – since they are known to trade impatiently to make most of their fleeting information advantage.

A somewhat more precise specification of the model is to estimate it with the component structure proposed by Engle and Lee (1999). The component structure decomposes the model in such a way as to allow for the time varying long-run pattern and to separately identify the transitory and permanent components of the lag dependence structure. In the general ACD model, the long run unconditional mean $\omega$, as well as the rest of the coefficients is assumed to be stable over time. This specification however does not guarantee that the roots of the characteristic polynomials are real and the process might actually appear to be not stable. The ACD specification with the component structure decomposes the model in a way to separately identify the long-run permanent and the transitory components, where

$$\psi_i - q_i = \alpha (x_{i-1} - q_{i-1}) + \beta (\psi_{i-1} - q_{i-1}) \quad (3.11)$$

is the transitory component which converges to zero at the speed $(\alpha + \beta)$ and

$$q_i = \omega + \rho (q_{i-1} - \omega) + \phi (x_{i-1} - \psi_{i-1}) \quad (3.12)$$

is the permanent component which converges to $\omega$ at the speed $\rho$. Typically $\rho$ is between 0.99 and 1, so that $q_i$ approaches to $\omega$ very slowly. Combining the two equations together yields the ACD(2,2) specified in such a way as to constrain the roots of the characteristic polynomial to be real:
\[
\psi_i = \omega (1 - \rho) + (\alpha + \phi)x_{t-1} + (-\phi\alpha - \phi\beta - \alpha\rho)x_{t-2} \\
+ (\beta + \rho - \phi)\psi_{t-1} + (\phi\alpha + \phi\beta - \beta\rho)\psi_{t-2}
\] (3.13)

We estimated the ACD(2,2) with the component structure and the results are summarized in the following table:

Table 3.2.2: Estimates of ACD(2,2) with the component structure for the five analysed stocks\(^{15}\).

<table>
<thead>
<tr>
<th>Stock</th>
<th>(\omega)</th>
<th>(\rho)</th>
<th>(\phi)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“C”</td>
<td>1.014755 (0.0000)</td>
<td>0.991849 (0.0000)</td>
<td>0.007840 (0.0000)</td>
<td>0.078032 (0.0000)</td>
<td>0.810422 (0.0000)</td>
</tr>
<tr>
<td>“HWP”</td>
<td>1.012006 (0.0000)</td>
<td>0.998590 (0.0000)</td>
<td>0.009156 (0.0000)</td>
<td>0.020677 (0.0000)</td>
<td>0.931344 (0.0000)</td>
</tr>
<tr>
<td>“AXP”</td>
<td>1.011517 (0.0000)</td>
<td>0.997199 (0.0000)</td>
<td>0.018577 (0.0000)</td>
<td>-0.010943 (0.0000)</td>
<td>-0.690434 (0.0000)</td>
</tr>
<tr>
<td>“JBX”</td>
<td>1.005567 (0.0000)</td>
<td>0.971832 (0.0000)</td>
<td>0.054562 (0.0000)</td>
<td>0.000280 (0.0000)</td>
<td>0.360668 (0.0000)</td>
</tr>
<tr>
<td>“FIC”</td>
<td>1.907506 (0.1788)</td>
<td>0.998988 (0.0000)</td>
<td>0.035929 (0.0000)</td>
<td>0.061588 (0.0000)</td>
<td>0.850789 (0.0000)</td>
</tr>
</tbody>
</table>

The coefficients again are highly significant for all the series except for FIC, where the unconditional mean, is not significant. The persistence is measured by \(\rho\) and interpreted as the rate of convergence of the permanent component to the unconditional mean, is indeed high for all series: for four stocks it is between 0.99 and 1, very close to the unit root; and for JIB it is a little lower, 0.9718; however a part of this persistence might be due to the persistence in the external factors that affect the process and are left unaccounted for in the given simple specification.

If the model were specified correctly, there should be no autoregressive dynamics left in the residuals. The results for most stocks do not exactly confirm the expectations, except for FIC, where there is no significant autocorrelation in the residuals, and the values for Citigroup’s and Hewlett Packard’s Q-statistics does not exceed 20; this still exceeds dramatically the critical value of the \(\chi^2\) distribution with 5 degrees of freedom at 5% significance level 1.145; however considering the large number of observations and data imperfections (missing values were replaced with the last available values) this result is admissible. The results for JIB are slightly worse in terms of removing the autocorrelation from the residuals and for the American Express there are a few small but significant spikes at second, third and forth lags that were not removed by ACD (2,2) (for the complete correlograms of the residuals see Appendix D). One possible explanation for the remaining autocorrelation in the residuals can be

\(^{15}\) Complete outputs are presented in Appendix C.
that the autoregressive structure is not fully captured by the lags of the durations themselves, but is influenced by the covariates, such as price changes, spreads and trading volumes.

We have stressed that under the assumption of exponential distribution the estimates possess QML property; however we will estimate the hazard function and the empirical density to see their general shape for theoretical consideration.

The hazard function for this accelerated time failure model has the following form:

\[ \hat{\lambda}_i = \hat{\lambda}_0 \left( \frac{x_i}{\psi_i} \right) \frac{1}{\psi_i} \]  \hspace{1cm} (3.14)

There are different ways to empirically estimate the hazard function for the durations: it can be done by estimating the density nonparametrically, computing the survivor function and then taking the ratio of the two:

\[ \lambda(x) = \frac{f(x)}{S(x)} \]  \hspace{1cm} (2.7)

or make an estimate based on the sample hazard function. In this paper we will follow Engle (2000) and estimate the hazard by semiparametric \( k \)-neighbour estimating procedure. The hazard for a sample of events is just a failure rate per unit of time, which [the failure rate] is measured as a number of failures divided over number of events that could have failed, and this number is called “number at risk”. To find the failure rate of the \( 2k \) smallest durations one needs to divide them over the number of durations that could have failed in given time interval \((t_{i+k}-t_{i-k})\). Then the \( 2k \) nearest neighbour estimator of the hazard rate is expressed as follows:

\[ \hat{\lambda}(t_i) = \frac{2k}{n_i(t_{i+k} - t_{i-k})} \]  \hspace{1cm} (3.15)

where number \( n_i \) is the number of the durations surviving at time \( t_i \). If the true density were exponential, then he hazard rate were constant.

Since the durations data is often described by clustering of transactions, in order to account for these mass points, we chose a fixed number of nearest neighbours, creating the bandwidth of a varying size, rather than choosing a fixed bandwidth.
Because of the big sample size for the heavily traded stocks we have chosen a larger number of neighbours in order to smooth the estimator over the varying intensity of trades. We have chosen $k=7000$ for C, and $k=6000$ for both HWP and AXP, where $k$ is half the bandwidth. For the less heavily traded stocks, JBX and FIC $k$ is equal to 500 in our calculations.

The estimated empirical density and the hazard are plotted below:

Figure 3.2.3: Empirical hazard functions for C, HWP and AXP.

The results appear to be inconsistent with the assumption of exponential distribution for the durations, since the hazard function is far from being constant: the hazard for very small durations is high, then drop sharply and then only gradually declines over a longer range of durations. The probability densities for these durations hint on the idea of Weibull distribution:

$$f(x) = abx^{b-1} \exp(-ax^b) \quad (2.10b)$$

with the parameter $b<1$.

Figure 3.2.4: Empirical densities for C, HWP and AXP.

The hazards for the other two less heavily traded stocks have generally the same, but more extreme form: hazards for very short durations are very high and become
lower very fast with the increasing durations; then the decline is very slow and gradual.

Figure 3.2.5: Empirical hazard functions for JBX and FIC.

The corresponding probability density functions are as well consistent with the Weibull distribution, where the $b$ parameter is even lower than for the three stocks described above\textsuperscript{16}.

Figure 3.2.6: Empirical densities for JBX and FIC.

For more precise modelling, it is possible to find close specification from the generalized Gamma distribution or some mixture distribution to account for faster decline for particularly small and particularly large durations and considerably decline over the wide range of “middle length” durations or model the hump for the very small durations of heavily traded stocks. As was already mentioned, this does not affect the results of ACD estimation carried out based on the assumption of the exponential distribution, as the true distribution does not influence the QML property of the resulting vector of parameters.

\textsuperscript{16} If $b=1$, the Weibull distribution reduces to exponential.
3.2.2 UHF-GARCH for the volatility per trade duration

Another promising branch of literature on durations and liquidity measures is the measure of instantaneous volatility. Volatility is incorporated in the transactions prices and is a very important mark that would affect the durations in such a way that it would help to explain time varying liquidity and forecast the liquidity and volatility based on their interaction. Recent literature suggests that there is contemporaneous relationship between volume and volatility and hence, one might be able to explain the other, see for example O’Hara (1995, 160) or Lamoureux and Lastrapes (1990); however why it is so remains unclear.

Engle claims that trade durations, the reciprocal of the transactions rate, which is a proxy for volume, should be indicative of the volatility and in such a setup it is possible to carry out volume-volatility analysis. Taking the market microstructure theories into consideration, Engle (2000) first developed a model to tie together the performance of trade-to-trade volatility and the durations and called the model ultra-high-frequency GARCH of UHF-GARCH.

Defining the return per duration as a difference of the logs of the midquotes between \( t_{i-1} \) and \( t_i \), equal to \( r_i \), the conditional variance per transaction is defined as

\[
V_{i-1} \left( r_i | x_i \right) = h_i
\]

(3.16)

In order to take into account the fact that the duration, as well as return data is measured in the transaction, rather than in calendar time and in order not to neglect the stochastic time, it is useful to define the measure for expected volatility per unit of time:

\[
V_{i-1} \left( \frac{r_i}{\sqrt{x_i}} | x_i \right) = \sigma_i^2
\]

(3.17)

Naturally, these two expected volatilities are related as follows:

\[
h_i = x_i \sigma_i^2
\]

(3.18)

It is a common knowledge fact that volatility as well as durations has a specific daily pattern, so that the data needs to be deseasonalized before any reasonable analysis
is carried out. Another reasonable assumption is that the returns in themselves carry no daily pattern, otherwise it could have been exploited for profit; then the only pattern influencing the volatility is that of the durations. Then if the deseasonalized durations are used for the volatility estimation, it should suffice to remove time-of-the-day pattern from the volatilities.

Engle (2000) assumes that the volatility per trade follows a simple ARMA (1,1) process:

\[
\frac{r_i}{\sqrt{x_i}} = \rho \frac{r_{i-1}}{\sqrt{x_{i-1}}} + e_i + \phi e_{i-1}
\]  

(3.19)

where the variance of \( r_i \) per square root of time is the expected value of the square of \( e_i \). Engle (2000) also claims that the persistence in volatility depends on the persistence in durations; to account for this, durations enter the mean function:

\[
\frac{r_i}{\sqrt{x_i}} = \rho \frac{r_{i-1}}{\sqrt{x_{i-1}}} + e_i + \phi e_{i-1} + \delta \psi_i
\]  

(3.20)

In order to compute \( \sigma_i^2 \) a simple GARCH specification is used as follows:

\[
\sigma_i^2 = \omega e_{i-1}^2 + \beta \sigma_{i-1}^2
\]  

(3.21)

This model is the simplest UHF-GARCH which can be estimated as a conventional GARCH (1,1) with the ARMA (1,1) process for the return per unit of time in the mean.

If the reciprocal of the duration, which stands for intensity of trading, is included into the model, as measure of the transactions rate, we have a direct connection between the volatility and durations. This action has two economic meanings: the longer the duration, the higher is the expected return, since both are dependent on the calendar time. However according to the model of Diamond and Verrecchia (1987), long durations imply bad news and lead to declining prices. In this case expected return and respectively volatility would be lower; \( x_i \) is included into the volatility variance equation to test the impact of duration on volatility of stock return.

Engle (2000) proposes a richer extension to the model to introduce both: duration, expected duration and a new measure for a long-run volatility which is computed by Exponential Smoothed Weighted moving Average as follows:
assigning the weights of 0.005 to the last duration’s volatility and smoothing the long run volatility with the weight of 0.995.

The new UHF-GARCH has the following form:

\[
\xi_i = 0.005 \left( \frac{r_{i-1}^2}{x_{i-1}} \right) + 0.995 \xi_{i-1}
\]  

(3.22)

There are four coefficients besides GARCH coefficients that contribute to the dynamics of the volatility. The effects of durations and expected durations should be expected to be correlated, and contributions of the two coefficients are not clearly separable, but move in the same direction as described above. The effect of the ACD residual might be interpreted as follows: if duration is underforecasted, the error is larger, durations are actually longer than expected and we have again the same dubious effect of durations on volatility. The long run volatility coefficient should be indicative of the general trend in volatility: low volatility is associated with long durations and bad news and high proportion of informed traders; this trend is sustained until the pattern reverses. Hence the long run volatility coefficient is expected to be positive.

We estimate this model for all five analysed stock and see if the frequency of transactions influences the output. The results of the estimation are summarized in the following table:

Table 3.2.1: Estimated coefficients (p-values) for UHF-GARCH mean function\textsuperscript{17}.

<table>
<thead>
<tr>
<th>Stock</th>
<th>δ</th>
<th>ρ</th>
<th>ψ</th>
</tr>
</thead>
<tbody>
<tr>
<td>“C”</td>
<td>-0.004000</td>
<td>0.200000</td>
<td>-0.600000</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>“HWP”</td>
<td>0.000794</td>
<td>0.538829</td>
<td>-0.283835</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.1291)</td>
</tr>
<tr>
<td>“AXP”</td>
<td>0.017110</td>
<td>0.700771</td>
<td>0.050693</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.4652)</td>
</tr>
<tr>
<td>“JBX”</td>
<td>4.74E-06</td>
<td>0.005000</td>
<td>0.005000</td>
</tr>
<tr>
<td></td>
<td>(0.9770)</td>
<td>(0.9988)</td>
<td>(0.9988)</td>
</tr>
<tr>
<td>“FIC”</td>
<td>2.15E-05</td>
<td>0.005003</td>
<td>0.005003</td>
</tr>
<tr>
<td></td>
<td>(0.7691)</td>
<td>(0.9986)</td>
<td>(0.9986)</td>
</tr>
</tbody>
</table>

\textsuperscript{17} Complete outputs of UHF-GARCH are presented in Appendix E.
The ARMA (1,1) coefficients are all highly significant for the heavily traded stocks. This means that our assumptions about the specification of the mean function for the volatility are valid. The coefficients for both less frequently traded stocks are all insignificant, this fact, together with rather low autocorrelations in the durations (table 3.2.3 and 3.2.4) hint on the idea of no serial dependence in the first moments of volatility. The coefficient $\delta$ is also significant for the heavy traded stocks and completely insignificant for the JBX and FIC. We may draw a conclusion that volatility structure does depend on the expected durations (in fact, it might be correlated, since expected durations depend on the actual durations that enter the specification of the volatility, the dependent variable. Again, we might conclude that that there is very little dynamics in the actual durations, with no dynamics in the returns and loose connection of those with the durations; the mean function for the volatility of the two infrequently traded stocks depends neither on information nor on the processes followed by the covariates.

Table 3.2.2: Estimated coefficients (p-values) for UHF-GARCH variance equation.

<table>
<thead>
<tr>
<th>Stock</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\beta$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\gamma_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;C&quot;</td>
<td>0.149072 (0.6414)</td>
<td>0.465299 (0.0032)</td>
<td>0.662769 (0.0000)</td>
<td>0.662769 (0.0024)</td>
<td>0.671308 (0.0000)</td>
<td>0.035036 (0.0005)</td>
<td>-0.586006 (0.0009)</td>
</tr>
<tr>
<td>&quot;HWP&quot;</td>
<td>0.380600 (0.0000)</td>
<td>0.497732 (0.0000)</td>
<td>0.490171 (0.0000)</td>
<td>0.477990 (0.0000)</td>
<td>-0.069426 (0.0000)</td>
<td>0.009724 (0.0373)</td>
<td>0.083207 (0.0019)</td>
</tr>
<tr>
<td>&quot;AXP&quot;</td>
<td>0.401986 (0.2607)</td>
<td>0.423442 (0.0029)</td>
<td>0.686245 (0.0000)</td>
<td>0.176333 (0.0071)</td>
<td>0.548085 (0.0004)</td>
<td>0.030817 (0.0007)</td>
<td>-0.735032 (0.0000)</td>
</tr>
<tr>
<td>&quot;JBX&quot;</td>
<td>3.09E-05 (0.0000)</td>
<td>0.150000 (0.0000)</td>
<td>0.599998 (0.0000)</td>
<td>1.68E-08 (0.1404)</td>
<td>-4.39E-06 (0.0000)</td>
<td>-7.63E-07 (1.0000)</td>
<td>-1.43E-06 (0.0405)</td>
</tr>
<tr>
<td>&quot;FIC&quot;</td>
<td>1.28E-05 (0.0000)</td>
<td>0.150023 (0.0000)</td>
<td>0.599991 (0.0000)</td>
<td>3.64E-08 (0.0000)</td>
<td>-2.05E-06 (0.0000)</td>
<td>-1.36E-05 (0.9971)</td>
<td>1.53E-06 (0.0000)</td>
</tr>
</tbody>
</table>

The GARCH coefficients for all the stocks are highly significant, even for the two which have zero mean function; this hints on the strong dynamics in the variance of the trade returns. The rest of the estimated coefficients also reveal some significant influence of the covariates and comply with our prior expectations. The coefficient $\gamma_1$ according to Easley and O’Hara (1992) should be positive, indicating that long durations mean no news and lower volatility; in fact they are for all the analysed stocks; the coefficient $\gamma_4$ is positive for some stocks and negative for the other causing ambiguity of the interpretation. One possible explanation would be the fact that we did not account for the asymmetry of the information and for the proportion of informed traders in our model. If the proportion of informed traders is high and there was a good-news release, then regardless of the expected duration would be shorter, but with high
volatility, induced by informed trading. The coefficient for the long run volatility is significant only for heavily traded stocks, hinting on the fact that there is no such thing as “volatility trend” in the infrequently traded stocks.

Table 3.2.3: Correlogram for volatility of JBX.

<table>
<thead>
<tr>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.028</td>
<td>0.028</td>
<td>7.1418</td>
<td>0.008</td>
</tr>
<tr>
<td>0.030</td>
<td>0.029</td>
<td>15.321</td>
<td>0.000</td>
</tr>
<tr>
<td>0.032</td>
<td>0.031</td>
<td>24.713</td>
<td>0.000</td>
</tr>
<tr>
<td>0.011</td>
<td>0.009</td>
<td>25.912</td>
<td>0.000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.003</td>
<td>26.184</td>
<td>0.000</td>
</tr>
<tr>
<td>0.003</td>
<td>0.001</td>
<td>26.255</td>
<td>0.000</td>
</tr>
<tr>
<td>0.030</td>
<td>0.029</td>
<td>34.688</td>
<td>0.000</td>
</tr>
<tr>
<td>0.007</td>
<td>0.005</td>
<td>35.080</td>
<td>0.000</td>
</tr>
<tr>
<td>0.011</td>
<td>0.009</td>
<td>36.176</td>
<td>0.000</td>
</tr>
<tr>
<td>0.006</td>
<td>0.004</td>
<td>36.541</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3.2.4: Correlogram for volatility of FIC.

<table>
<thead>
<tr>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.005</td>
<td>-0.005</td>
<td>0.1877</td>
<td>0.665</td>
</tr>
<tr>
<td>0.053</td>
<td>0.053</td>
<td>25.316</td>
<td>0.000</td>
</tr>
<tr>
<td>0.031</td>
<td>0.032</td>
<td>34.214</td>
<td>0.000</td>
</tr>
<tr>
<td>0.019</td>
<td>0.016</td>
<td>37.377</td>
<td>0.000</td>
</tr>
<tr>
<td>0.027</td>
<td>0.024</td>
<td>43.888</td>
<td>0.000</td>
</tr>
<tr>
<td>0.020</td>
<td>0.017</td>
<td>47.391</td>
<td>0.000</td>
</tr>
<tr>
<td>0.009</td>
<td>0.005</td>
<td>48.099</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>-0.004</td>
<td>48.101</td>
<td>0.000</td>
</tr>
<tr>
<td>0.030</td>
<td>0.027</td>
<td>56.239</td>
<td>0.000</td>
</tr>
<tr>
<td>-0.017</td>
<td>-0.018</td>
<td>58.838</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The evidence hints conclusively that the estimation of the ARMA process for these series is totally inappropriate. With the means of 0.00109 for JBX and -5.58E-05 for FIC, the most appropriate mean function is the location parameter with the mean of zero. However the GARCH effects are still highly significant in the both stocks; as was shown earlier in the paper, the durations still follow the conditional autoregressive process; trying to tie the GARCH structure for the volatility with the expected durations was successful: the coefficients also appear significant, expect for the coefficient to the inverse of the durations has a p-value of 14.5%; however the long run volatility appears to have no effect on the volatility process in both stocks.
3.2.3 Excess Market Depth with VNET

As we have already seen, the wide range of liquidity measures and versatility of the liquidity definitions precludes us from taking a single correct approach to modelling such. The number of trades, volumes, price changes, timing of transactions, buy and sell orders are all interesting and important statistics to analyse and the interaction between them can provide additional insights into the essence of the liquidity formation. Then it is highly desirable to find a measure that would combine most of the mentioned above statistics, the existent knowledge of the modelling of the point processes of time and ability to forecast durations. One such measure was introduced by Engle and Lange (1997) and we will proceed by estimating it for the five analysed stocks. We will compare the results bearing in mind the different intensity of trading for each stock.

VNET measures the total volume that can be traded on the market without inducing a certain change in price. It is calculated as cumulative volume of one side of the market (buy side or sell side) over a given price duration.

\[ VNET = \log \left| \sum_i \left( d_i \cdot vol_i \right) \right| \]  

(3.24)

where \( d_i \) is the trade indicator equal to 1 for a buy transaction and to -1 for a sell transaction; \( vol_i \) is the volume of each transaction; VNET is then equal to the log value of the absolute value of the excess volume on either side of the market within one price duration. Intuitively this statistics conveys the depth of liquidity regarding the stability of prices.

A price durations approach is a special approach which aggregates a number of trade durations, so that the part of the information pertaining to each transaction is lost to aggregation; the advantage of such approach is however that it allows to mitigate the discreteness of the price changes and analyse price-volume relationship with the trades and quotes data and not with the more extensive and complete limit order book data.

The threshold for the price durations is set in advance and can be changed for the desired fineness of the measure. In the present analysis the price threshold is set in such a way that it captures usually more than one consecutive data points to ensure that the price durations represent real changes in prices, and not the impact of random effect and the bid-ask bounce on one hand, but allows for fine enough grid and does not lose much price formation information to aggregation. Another consideration borne in mind
when selecting the price threshold is the exact multiple of discrete price moves. This allows capturing the exact cumulative price move without the loss to approximation. Unfortunately, because of a great dispersion in transaction to transaction price changes, and due to the data imperfections (there was some data points for the ask quote missing, while processing the data we have replaced the missing points by the last available values, which led to a number of consecutive spreads with the same values), it was impossible to find the ideal threshold. In order to minimize both, the number of price durations with only one transaction and the largest number of trades per single price duration, we have set the following price thresholds: 0,09375 for Citigroup, Hewlett Packard, American Express and Jack in the Box and 0,15635 of Fair Isaac Corp.

As time is equally important measure of for evaluating liquidity, the timing of the price duration should in no case be disregarded in the given analysis. Utilizing the ability of autoregressive conditional duration (ACD) to estimate the expected duration, we will use them for explaining the VNET measure as suggested by Engle and Lange (1997). We consider the simple ACD(1,1) specification for the estimation of conditional duration, and include the spread at the end of price duration as additional covariate, assuming it carries useful information for determining the expected duration. We expect that the smaller spread (higher liquidity per se) would induce smaller price change, adding only a small value to the cumulative price change and making the price duration longer. Since we consider only the last spread of the price duration, it can be interpreted as if the new duration enters the stock in the liquid period and is expected to be longer. Therefore we expect the negative coefficient on previous duration’s spread in the ACD model.

The model for the expected price durations has the following form:

\[ \psi_t = \omega + \alpha \psi_{t-1} + \beta \psi_{t-1} + \gamma \text{spread}_{t-1} \]  

(3.25)

We assume the exponential distribution of i.i.d. innovations; as was already explained in details, even if the true density is not exponential, due to the quasi maximum likelihood property, the estimates with the assumed exponential density are consistent estimates of the true parameters.

The output coefficients are summarized in the following table\(^\text{18}\):

\(^\text{18}\) Complete output representations can be found in the appendix
Table 3.2.5: Estimated coefficients (p-values) for ACD model$^{19}$.

<table>
<thead>
<tr>
<th>Stock</th>
<th>$\omega$ (p-value)</th>
<th>$\alpha$ (p-value)</th>
<th>$\beta$ (p-value)</th>
<th>$\gamma$ (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“C”</td>
<td>3.013982 (0.0000)</td>
<td>0.070492 (0.0000)</td>
<td>0.905198 (0.0000)</td>
<td>-2.981976 (0.0000)</td>
</tr>
<tr>
<td>“HWP”</td>
<td>2.262189 (0.0001)</td>
<td>0.094070 (0.0000)</td>
<td>0.875679 (0.0000)</td>
<td>-2.222986 (0.0001)</td>
</tr>
<tr>
<td>“AXP”</td>
<td>2.295778 (0.0113)</td>
<td>0.091513 (0.0000)</td>
<td>0.890717 (0.0000)</td>
<td>-2.270082 (0.0119)</td>
</tr>
<tr>
<td>“JIB”</td>
<td>3.783865 (0.2803)</td>
<td>0.098039 (0.0005)</td>
<td>0.770313 (0.0000)</td>
<td>-3.621230 (0.2980)</td>
</tr>
<tr>
<td>“FIC”</td>
<td>0.671922 (0.1019)</td>
<td>0.123161 (0.0000)</td>
<td>0.849223 (0.0000)</td>
<td>-0.637617 (0.1153)</td>
</tr>
</tbody>
</table>

As we see, there is indeed autoregressive structure in the price durations, as both ARCH and GARCH coefficients for all five stocks are highly significant. The coefficients on the lagged spreads appear significant in three intensively trades stocks, C, HWP and AXP, and dramatically loses significance for the other two less heavily trades stocks, JIB and FIC. A possible explanation to such phenomenon is that once stocks is traded less frequently, the changes in the spread do not represent liquidity to a great enough extend and do not have explanatory power: it holds only for changes in small spreads that send a signal of a more liquid environment. In the period of a heavy trading with small durations, the specialist revises the quotes presuming a high proportion of informed traders and widens the spread; this would lead to one lad dependence only if the next duration occurs quickly enough before the trading has adjusted to the private information and to wide unattractive for trading, spreads. All the ARCH and GARCH coefficients are positive and consistent with the ACD literature of the durations clustering: longer durations trigger future longer durations.

In conveying the VNET measure, we would like to examine how other dimensions of liquidity affect the depth of one side market as measured by VNET. Since the VNET statistics is computed as the logarithmic value of the one side market depth, it makes sense to introduce the regressors in the logarithmic dimension as well$^{20}$. We examine the lagged values of the explanatory variables as we are interested in the lead lag relationship and the prediction power rather than in the contemporaneous dependence. Following Engle and Lange (2997) we introduce the following regressors that are thought to have explanatory power for the VNET liquidity statistics:

LSPREAD(-1) – log value of the previous price duration’s last spread. The logic behind this regressor is similar as before for the conditional duration model: lower

$^{19}$ Complete outputs are presented in Appendix F.

$^{20}$ All logarithmic specifications in the present analysis are represented by the natural logarithms.
spread corresponds to higher liquidity and the ability to trade greater amount of excess
volume without inducing the price change, hence we expect the positive coefficient.

LSIZE(-1) – log cumulative volume of the previous price duration. Large
cumulative volume means low price impact of the size of transactions, would lead to the
ability to generate greater excess, hence larger VNET value, the coefficient is expected
to be positive. Higher volume implies smaller imbalance and larger VNET.

LNTRADES(-1) – the log of the number of trades in the previous price duration.
The larger number of trades triggers the greater excess volume (the sum over the larger
number of transactions) and leads to a greater price impact. The most reasonable
explanation for the transactions clustering, according to the micro microstructure
reasoning, is the high proportion of informed traders taking advantage of their fleeing
information advantage. Then the coefficient would be negative, expecting that heavy
trading performed by inform traders results in the larger price impact and lower VNET.

LOG(EDUR) – log of the expected duration. The shorter is the expected
duration, the more there is presumably informed trading, the higher is the traded volume
and the greater the price impact; the faster is the threshold crossed by the cumulative
price change, and consequently the higher is the excess volume that triggers the price
impact.

LOG(ACD_RESID) – the log of the residual from the ACD estimation. This
predictions error is also expected to carry valuable for VNET information, since if the
duration is under forecasted, the error is larger, the durations is actually longer than
expected and can accumulate greater volume before the price change crosses the
threshold. The ACD error term can also be interpreted as the “trading impatience”: the
trader would trade more if there is new information and would incur the cost of the error
times the coefficient. It is logical to expect the positive coefficients.

The proposed model to estimate VNET has then the following form:

\[
VNET_i = \beta_0 + \beta_1 \text{spread}_{t-1} + \beta_2 \text{size}_{t-1} + \beta_3 \text{lntrades}_{t-1} + \beta_4 \log(\psi_{t-1}) + \beta_5 \log\left(\frac{\psi_{t-1}}{\psi_{t-1}}\right) \]

(3.26)
The estimation yields the following output coefficients\(^{21}\):

**Table 3.2.5: Estimated VNET coefficients (p-values)**\(^{22}\).

<table>
<thead>
<tr>
<th>Stock</th>
<th>(\beta_0)</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(\beta_4)</th>
<th>(\beta_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;C&quot;</td>
<td>7.737541 (0.0000)</td>
<td>16.51684 (0.0121)</td>
<td>0.300268 (0.0000)</td>
<td>-0.252841 (0.0000)</td>
<td>-0.361811 (0.0000)</td>
<td>0.985005 (0.0000)</td>
</tr>
<tr>
<td>&quot;HWP&quot;</td>
<td>7.305673 (0.0000)</td>
<td>-0.721622 (0.8979)</td>
<td>0.305936 (0.0000)</td>
<td>-0.262375 (0.0000)</td>
<td>-0.207197 (0.0000)</td>
<td>1.208840 (0.0000)</td>
</tr>
<tr>
<td>&quot;AXP&quot;</td>
<td>7.176210 (0.0000)</td>
<td>14.71970 (0.0078)</td>
<td>0.307815 (0.0000)</td>
<td>-0.226381 (0.0000)</td>
<td>-0.289364 (0.0000)</td>
<td>1.083277 (0.0000)</td>
</tr>
<tr>
<td>&quot;JBX&quot;</td>
<td>6.644517 (0.0000)</td>
<td>-17.17011 (0.0422)</td>
<td>0.207340 (0.0000)</td>
<td>-0.118242 (0.1046)</td>
<td>-0.154014 (0.4467)</td>
<td>0.857690 (0.0000)</td>
</tr>
<tr>
<td>&quot;FIC&quot;</td>
<td>5.445338 (0.0000)</td>
<td>0.435498 (0.9373)</td>
<td>0.219027 (0.0000)</td>
<td>-0.106097 (0.1257)</td>
<td>-0.515559 (0.0000)</td>
<td>0.416707 (0.0000)</td>
</tr>
</tbody>
</table>

As we see, the signs of most coefficients comply with the expectations. One coefficient that does not fully comply with the expectations is the coefficient on LSPREAD. This might be explained by the fact that spread, which immediately precedes the price duration enters via both: the LSPREAD coefficient and indirectly through ACD model in \(\log(\psi_{-1})\). The regressors are at least partially correlated and the effects are not clearly separable. The rest of the coefficients have the signs as expected. It is not directly evident in how great is the impact of each coefficient, since the model is estimated in logs, but the general influence is clear.

The important conclusion from the above estimation is that the model holds reasonably well for the heavily traded stocks and in the two less heavily trades stocks we begin to observe the insignificant coefficients: Jack in the Box: number of trades and expected durations have no more significant impact on the VNET; for Fair Isaac Corp. the spread and number of trades are also insignificant. This is due to the fact that a lot of valuable information is hidden in high spreads, low volumes and long durations, and the price impact can not be directly identified.

\(^{21}\) Complete output representations can be found in the appendix.

\(^{22}\) Complete outputs are presented in Appendix G.
4 Conclusion

This paper is a contribution to the important and growing literature on high frequency financial data. The main concern was to theoretically define and measure the elusive and ambiguous concept of liquidity in the stock market on the transactions level. The analysis rests on the developed theories of market microstructure and existing knowledge about modelling the irregularly spaced transactions processes.

We have collected and summarized the important ways of understanding the concept of liquidity, and the measures that characterize it from different perspectives including one dimensional and multidimensional measures.

Further we introduced and summarized the existing models to explain and forecast time varying liquidity: ACD-type models. Rich and growing literature in this direction allows for increasing flexibility in the modelling of the process of duration between the events of interest (quote renewals, transactions, price and volume durations) and a proxy for liquidity, as well as the covariate processes to better capture the multi-dimensionality of the concept. We have in details explained the evolution of the durations process with the underlying distributional assumptions and the description of the estimation procedure.

We performed extensive empirical analysis of the five stocks of different trading intensity in order to compare the validity of the microstructure theories for the real data and the ability of the information contained in the data to explain the time varying liquidity.

The success of the empirical analysis of the given paper to a great extend depends the availability of high quality stock exchange data. We use TAQ NYSE data which allows us to carry out only somewhat limited analysis because, unlike the full information limit order book, TAQ provides only the best pair of bid and ask quotes, leaving much of the information unveiled. We used five stocks five NYSE traded stocks: Citigroup (C), Hewlett Packard (HWP), American Express (AXP), Jack in the Box (JBX) and Fair Isaac Corp. (FIC) in our empirical analysis. The selection of the stocks was done in a way to have stocks of different trading intensities in order to compare how the dynamics, price formation, and volatility, which are in a way measures of liquidity are influenced by the frequency of trading.

We have estimated the durations processes using the basic ACD framework and applying the market microstructure theories for the interpretation of the coefficients. We also estimated the empirical densities and the hazards for the analysed stocks. The
results for all five stocks generally complied with the expectations and confirmed the existence of the autoregressive structure. We found that in the less frequently traded stocks the autoregressive dynamics is still present, but is less clear and the liquidity clustering is more difficult to distinguish. The reason for this might be that due to the nature of the trading of this particular not so popular with the investors stock, some of the information influencing the durations dynamics is being hidden between the durations: the information that should influence the trading pattern is revealed, but not to a great enough extent to induce trading, in other words, due to exogenous reasons, trading occurs less frequently than the information that should influence it, is released.

Further using the estimates for the expected durations, following Engle (2000), we estimate the volatility per unit of time, where the durations, expected durations and long run volatility enter the variance equation. We argue that informed trading and news impact have significant influence on the price discovery, price adjustment and volatility of the stocks, however the results are drastically less pronounces for the infrequently traded stocks. This is again due to the lost information between the trades.

We also estimate the VNET measure for the price durations which measures the excess volume on one side of the market that can be sustained before the price change crosses the prespecified threshold. With the explanatory variables for this one side excess volume as spread, cumulative volume traded on both sides of the market over the price duration, number of trades as well as the expectation of the previous duration, VNET conveys a multidimensional statistics for the market liquidity at the transactions level, and giving some insights to the prevailing market microstructure. The VNET statistics for the frequently traded stocks is well explained by the explanatory variables, whereas for the infrequently traded stocks the results are more dubious. Once again it confirms the hypothesis that there is information that is revealed and is important to the price formation but doesn’t induce trading and the price changes in the consecutive events; price durations in this case do not really represent total information that is able to explain the dynamics.

Summarizing all in one, the main result that follows from current analysis is that the microstructure theories can be to a certain extent successfully tested and different liquidity measures can be successfully estimated based on the information on the previous durations, price changes, volumes, spreads and other marks on the transactions level; this however is only possible if the stock is traded often enough where all the market information (or a great enough portion of it) is revealed to the econometrician via trading. If the trading is rather infrequent or completely sporadic, the important
pricing information does not induce informed traders to trade and is left unaccounted, in this case the existing pattern is very difficult to estimate.

This analysis could be extended to provide more valuable results if the additional data information could be employed. This is particularly important for the infrequently traded stocks, but would make sense for the frequently traded ones as well. By the additional data we of course mean the full limit order book data, on the basis of which it would be possible to get more reliable estimates for the price impacts of the size change, triggered by greater bid-ask spreads. Another very valuable to the current analysis data would be the data that would help to gain insights as to the proportion of informed traders in the market at each point in time when the new information is revealed. This knowledge of asymmetry of the information would make the market microstructure hypotheses directly testable, as to whether the informed trading really causes the transaction clustering; how much it hinders the liquidity and what is the role of the liquidity traders (discretionary and nondiscretionary) in the price adjustment and inhibition of market imbalance.
### Appendix A: Autocorrelation of the Raw Durations

#### Stock "C"

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.120</td>
<td>0.120</td>
<td>2336.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.108</td>
<td>0.096</td>
<td>4260.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.098</td>
<td>0.077</td>
<td>5835.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.091</td>
<td>0.084</td>
<td>7177.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.080</td>
<td>0.049</td>
<td>8216.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.069</td>
<td>0.037</td>
<td>8984.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.061</td>
<td>0.029</td>
<td>9586.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.061</td>
<td>0.031</td>
<td>10203.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.062</td>
<td>0.031</td>
<td>10827.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.054</td>
<td>0.023</td>
<td>11307.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.055</td>
<td>0.024</td>
<td>11792.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.053</td>
<td>0.023</td>
<td>12258.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>0.045</td>
<td>0.014</td>
<td>12586.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>0.045</td>
<td>0.015</td>
<td>12915.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>0.041</td>
<td>0.012</td>
<td>13185.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>0.042</td>
<td>0.014</td>
<td>13470.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>0.044</td>
<td>0.018</td>
<td>13793.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.040</td>
<td>0.013</td>
<td>14056.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>0.040</td>
<td>0.013</td>
<td>14313.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.036</td>
<td>0.009</td>
<td>14525.</td>
</tr>
</tbody>
</table>

#### Stock „HWP“

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.062</td>
<td>0.062</td>
<td>545.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.067</td>
<td>0.064</td>
<td>1199.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.067</td>
<td>0.060</td>
<td>1849.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.060</td>
<td>0.049</td>
<td>2368.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.059</td>
<td>0.046</td>
<td>2871.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.058</td>
<td>0.042</td>
<td>3349.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.058</td>
<td>0.041</td>
<td>3826.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.059</td>
<td>0.041</td>
<td>4334.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.049</td>
<td>0.029</td>
<td>4685.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.056</td>
<td>0.035</td>
<td>5136.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.052</td>
<td>0.030</td>
<td>5525.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.055</td>
<td>0.032</td>
<td>5960.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>0.052</td>
<td>0.028</td>
<td>6350.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>0.051</td>
<td>0.027</td>
<td>6730.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>0.052</td>
<td>0.027</td>
<td>7118.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>0.045</td>
<td>0.020</td>
<td>7415.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>0.052</td>
<td>0.026</td>
<td>7801.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.044</td>
<td>0.017</td>
<td>8073.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>0.046</td>
<td>0.019</td>
<td>8378.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.048</td>
<td>0.021</td>
<td>8705.1</td>
</tr>
</tbody>
</table>
### Stock „AXP“

Sample: 1 143678  
Included observations: 143678

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.049</td>
<td>0.049</td>
<td>346.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.079</td>
<td>0.076</td>
<td>1234.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.072</td>
<td>0.065</td>
<td>1971.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.083</td>
<td>0.073</td>
<td>2972.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.067</td>
<td>0.052</td>
<td>3617.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.068</td>
<td>0.050</td>
<td>4290.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.063</td>
<td>0.042</td>
<td>4867.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.065</td>
<td>0.042</td>
<td>5474.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.059</td>
<td>0.034</td>
<td>5971.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.056</td>
<td>0.030</td>
<td>6425.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.059</td>
<td>0.032</td>
<td>6920.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.057</td>
<td>0.030</td>
<td>7385.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>0.061</td>
<td>0.033</td>
<td>7920.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>0.052</td>
<td>0.024</td>
<td>8314.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>0.056</td>
<td>0.027</td>
<td>8769.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>0.052</td>
<td>0.023</td>
<td>9163.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>0.057</td>
<td>0.027</td>
<td>9636.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.054</td>
<td>0.024</td>
<td>10062.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>0.052</td>
<td>0.020</td>
<td>10447.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.049</td>
<td>0.018</td>
<td>10796.</td>
</tr>
</tbody>
</table>

### Stock „JBX“

Sample: 1 10680  
Included observations: 10680

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.069</td>
<td>0.069</td>
<td>51.136</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.106</td>
<td>0.102</td>
<td>171.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.081</td>
<td>0.068</td>
<td>241.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.106</td>
<td>0.088</td>
<td>360.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.070</td>
<td>0.046</td>
<td>413.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.085</td>
<td>0.058</td>
<td>491.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.088</td>
<td>0.059</td>
<td>574.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.061</td>
<td>0.027</td>
<td>614.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.056</td>
<td>0.022</td>
<td>648.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.054</td>
<td>0.020</td>
<td>678.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.070</td>
<td>0.038</td>
<td>730.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.066</td>
<td>0.033</td>
<td>775.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>0.061</td>
<td>0.027</td>
<td>815.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>0.054</td>
<td>0.020</td>
<td>846.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>0.060</td>
<td>0.025</td>
<td>884.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>0.033</td>
<td>0.001</td>
<td>896.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>0.059</td>
<td>0.026</td>
<td>933.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>0.038</td>
<td>0.005</td>
<td>949.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>0.034</td>
<td>0.000</td>
<td>961.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.037</td>
<td>0.008</td>
<td>976.31</td>
</tr>
</tbody>
</table>
### Stock „FIC“

Sample: 1 9048  
Included observations: 9048  

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>*</td>
<td>1</td>
<td>0.151</td>
<td>0.151</td>
<td>206.40</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>2</td>
<td>0.162</td>
<td>0.143</td>
<td>444.61</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>3</td>
<td>0.135</td>
<td>0.096</td>
<td>608.54</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>4</td>
<td>0.133</td>
<td>0.086</td>
<td>769.31</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
<td>5</td>
<td>0.099</td>
<td>0.044</td>
<td>857.67</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>6</td>
<td>0.115</td>
<td>0.062</td>
<td>977.20</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>7</td>
<td>0.095</td>
<td>0.039</td>
<td>1058.4</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>8</td>
<td>0.101</td>
<td>0.045</td>
<td>1150.5</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>9</td>
<td>0.080</td>
<td>0.023</td>
<td>1208.8</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>10</td>
<td>0.106</td>
<td>0.053</td>
<td>1311.0</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>11</td>
<td>0.079</td>
<td>0.021</td>
<td>1366.8</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>12</td>
<td>0.090</td>
<td>0.033</td>
<td>1440.0</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>13</td>
<td>0.087</td>
<td>0.031</td>
<td>1508.0</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>14</td>
<td>0.100</td>
<td>0.043</td>
<td>1599.5</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>15</td>
<td>0.080</td>
<td>0.020</td>
<td>1657.1</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>16</td>
<td>0.095</td>
<td>0.035</td>
<td>1739.8</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>17</td>
<td>0.092</td>
<td>0.032</td>
<td>1816.9</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>18</td>
<td>0.089</td>
<td>0.026</td>
<td>1889.2</td>
</tr>
<tr>
<td>*</td>
<td></td>
<td>19</td>
<td>0.066</td>
<td>0.002</td>
<td>1928.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.062</td>
<td>-0.001</td>
<td>1963.4</td>
</tr>
</tbody>
</table>
Appendix B: Estimation outputs for ACD (2,2)

Stock „C“
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04 Time: 17:52
Sample: 1 163459
Included observations: 163459
Convergence achieved after 69 iterations
Bollerslev-Wooldridge robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.002698</td>
<td>0.000333</td>
<td>8.091192</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.101602</td>
<td>0.002708</td>
<td>37.51734</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>-0.094581</td>
<td>0.002485</td>
<td>-38.05316</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>1.647115</td>
<td>0.016702</td>
<td>98.61577</td>
</tr>
<tr>
<td>GARCH(2)</td>
<td>-0.656850</td>
<td>0.015845</td>
<td>-41.45374</td>
</tr>
</tbody>
</table>

R-squared 4.417685
Mean dependent var 0.905679
Adjusted R-squared 4.417817
S.D. dependent var 0.430902
S.E. of regression 1.002976
Akaike info criterion 2.819572
Schwarz criterion 2.819878
Durbin-Watson stat 0.325029

Stock „HWP“
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04 Time: 18:14
Sample: 1 143749
Included observations: 143749
Convergence achieved after 32 iterations
Bollerslev-Wooldridge robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.001149</td>
<td>0.000459</td>
<td>2.502119</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.027151</td>
<td>0.002376</td>
<td>11.42826</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>-0.023034</td>
<td>0.001928</td>
<td>-11.94685</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>1.733239</td>
<td>0.087919</td>
<td>19.71409</td>
</tr>
<tr>
<td>GARCH(2)</td>
<td>-0.735661</td>
<td>0.089869</td>
<td>-8.01023</td>
</tr>
</tbody>
</table>

R-squared 3.778969
Mean dependent var 0.895404
Adjusted R-squared 3.779122
S.D. dependent var 0.459837
S.E. of regression 1.005260
Akaike info criterion 2.826548
Schwarz criterion 2.826892
Durbin-Watson stat 0.392579

Stock „AXP“
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04 Time: 18:36
Sample: 1 143678
Included observations: 143678
Convergence achieved after 83 iterations
Bollerslev-Wooldridge robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.004841</td>
<td>0.000503</td>
<td>9.626618</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.007711</td>
<td>0.002522</td>
<td>3.057072</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>0.024008</td>
<td>0.002166</td>
<td>11.08620</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.287474</td>
<td>0.096632</td>
<td>2.974924</td>
</tr>
<tr>
<td>GARCH(2)</td>
<td>0.676019</td>
<td>0.094675</td>
<td>7.140413</td>
</tr>
</tbody>
</table>

R-squared 3.882315
Mean dependent var 0.895220
Adjusted R-squared 3.882451
S.D. dependent var 0.459391
Akaike info criterion 2.816901
Schwarz criterion 2.817245
Durbin-Watson stat 0.389734
Stock „JBX“
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04   Time: 18:40
Sample: 1 10680
Included observations: 10680
Convergence achieved after 14 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.056177</td>
<td>0.009395</td>
<td>5.979390</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.035412</td>
<td>0.010845</td>
<td>3.265316</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>0.077884</td>
<td>0.009630</td>
<td>8.087281</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.211441</td>
<td>0.125138</td>
<td>1.689666</td>
</tr>
<tr>
<td>GARCH(2)</td>
<td>0.620679</td>
<td>0.115285</td>
<td>5.383879</td>
</tr>
</tbody>
</table>

R-squared    -1.888977     Mean dependent var 0.814856
Adjusted R-squared -1.890059     S.D. dependent var 0.592909
S.E. of regression 1.007955     Akaike info criterion 2.810270
Sum squared resid 10845.51     Schwarz criterion 2.813677
Log likelihood -15001.84     Durbin-Watson stat 0.655597

Stock „FIC“
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04   Time: 18:44
Sample: 1 9048
Included observations: 9048
Convergence achieved after 33 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.017812</td>
<td>0.003482</td>
<td>5.116138</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.063654</td>
<td>0.011177</td>
<td>5.695081</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>0.076708</td>
<td>0.009935</td>
<td>7.720739</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.058374</td>
<td>0.145537</td>
<td>0.401097</td>
</tr>
<tr>
<td>GARCH(2)</td>
<td>0.788081</td>
<td>0.133953</td>
<td>5.883247</td>
</tr>
</tbody>
</table>

R-squared    -1.420874     Mean dependent var 0.770267
Adjusted R-squared -1.421945     S.D. dependent var 0.646231
S.E. of regression 1.005703     Akaike info criterion 2.721275
Sum squared resid 9146.444     Schwarz criterion 2.725204
Log likelihood -12306.05     Durbin-Watson stat 0.693122
Appendix C: Estimation outputs for ACD with the component structure

Stock “C”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04   Time: 23:46
Sample: 1 163459
Included observations: 163459
Convergence achieved after 29 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perm: C</td>
<td>1.014755</td>
<td>0.017714</td>
<td>57.28590</td>
</tr>
<tr>
<td>Perm: [Q-C]</td>
<td>0.998419</td>
<td>0.000262</td>
<td>3817.641</td>
</tr>
<tr>
<td>Perm: [ARCH-GARCH]</td>
<td>0.007840</td>
<td>0.000628</td>
<td>12.47969</td>
</tr>
<tr>
<td>Tran: [ARCH-Q]</td>
<td>0.078032</td>
<td>0.002108</td>
<td>37.01367</td>
</tr>
<tr>
<td>Tran: [GARCH-Q]</td>
<td>0.810422</td>
<td>0.006289</td>
<td>128.8732</td>
</tr>
</tbody>
</table>

R-squared: -4.417685
Adjusted R-squared: -4.417817
S.E. of regression: 1.002976
Sum squared resid: 164428.4
Log likelihood: -230382.0
Durbin-Watson stat: 0.325029

Stock “HWP”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04   Time: 23:50
Sample: 1 143749
Included observations: 143749
Convergence achieved after 110 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perm: C</td>
<td>1.012006</td>
<td>0.023092</td>
<td>43.82527</td>
</tr>
<tr>
<td>Perm: [Q-C]</td>
<td>0.998590</td>
<td>0.000228</td>
<td>4371.676</td>
</tr>
<tr>
<td>Perm: [ARCH-GARCH]</td>
<td>0.009156</td>
<td>0.000692</td>
<td>13.22277</td>
</tr>
<tr>
<td>Tran: [ARCH-Q]</td>
<td>0.020677</td>
<td>0.001377</td>
<td>15.01385</td>
</tr>
<tr>
<td>Tran: [GARCH-Q]</td>
<td>0.931344</td>
<td>0.006456</td>
<td>144.2604</td>
</tr>
</tbody>
</table>

R-squared: -3.778989
Adjusted R-squared: -3.779122
S.E. of regression: 1.005260
Sum squared resid: 145260.2
Log likelihood: -203113.9
Durbin-Watson stat: 0.392579

Stock “AXP”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/18/04   Time: 23:53
Sample: 1 143678
Included observations: 143678
Convergence achieved after 22 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perm: C</td>
<td>1.011517</td>
<td>0.022117</td>
<td>45.73400</td>
</tr>
<tr>
<td>Perm: [Q-C]</td>
<td>0.997199</td>
<td>0.000269</td>
<td>3703.517</td>
</tr>
<tr>
<td>Perm: [ARCH-GARCH]</td>
<td>0.018577</td>
<td>0.000612</td>
<td>30.36512</td>
</tr>
<tr>
<td>Tran: [ARCH-Q]</td>
<td>-0.010943</td>
<td>0.002494</td>
<td>-4.398090</td>
</tr>
<tr>
<td>Tran: [GARCH-Q]</td>
<td>-0.690434</td>
<td>0.096889</td>
<td>-7.126054</td>
</tr>
</tbody>
</table>

R-squared: -3.882315
Adjusted R-squared: -3.882451
S.E. of regression: 1.003150
Sum squared resid: 144579.7
Log likelihood: -202357.6
Durbin-Watson stat: 0.392679
## Stock “JBX”

**Dependent Variable:** SQR(DUR)  
**Method:** ML - ARCH (Marquardt)  
**Date:** 09/18/04   **Time:** 23:56  
**Sample:** 1 10680  
**Included observations:** 10680  
**Convergence achieved after 6 iterations**  
**Bollerslev-Wooldridge robust standard errors & covariance**  
**Variance backcast:** ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perm: C</td>
<td>1.005567</td>
<td>0.039242</td>
<td>25.62455</td>
</tr>
<tr>
<td>Perm: [Q-C]</td>
<td>0.971832</td>
<td>0.005266</td>
<td>184.5559</td>
</tr>
<tr>
<td>Perm: [ARCH-GARCH]</td>
<td>0.054562</td>
<td>0.005973</td>
<td>9.135069</td>
</tr>
<tr>
<td>Tran: [ARCH-Q]</td>
<td>0.000280</td>
<td>0.012412</td>
<td>0.022524</td>
</tr>
<tr>
<td>Tran: [GARCH-Q]</td>
<td>0.360668</td>
<td>37.38685</td>
<td>0.009647</td>
</tr>
</tbody>
</table>

### Variance Equation

- **R-squared:** -1.888977  
- **Mean dependent var:** 0.814856  
- **Adjusted R-squared:** -1.890059  
- **S.D. dependent var:** 0.592909  
- **S.E. of regression:** 1.007955  
- **Akaike info criterion:** 2.811709  
- **Sum squared resid:** 10845.51  
- **Schwarz criterion:** 2.815115  
- **Log likelihood:** -15009.53  
- **Durbin-Watson stat:** 0.655597

## Stock “FIC”

**Dependent Variable:** SQR(DUR)  
**Method:** ML - ARCH (Marquardt)  
**Date:** 09/19/04   **Time:** 00:16  
**Sample:** 1 9048  
**Included observations:** 9048  
**Convergence achieved after 16 iterations**  
**Bollerslev-Wooldridge robust standard errors & covariance**  
**Variance backcast:** ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perm: C</td>
<td>1.907506</td>
<td>1.418674</td>
<td>1.344570</td>
</tr>
<tr>
<td>Perm: [Q-C]</td>
<td>0.998988</td>
<td>0.001035</td>
<td>965.0708</td>
</tr>
<tr>
<td>Perm: [ARCH-GARCH]</td>
<td>0.035929</td>
<td>0.006352</td>
<td>5.656755</td>
</tr>
<tr>
<td>Tran: [ARCH-Q]</td>
<td>0.061588</td>
<td>0.009651</td>
<td>6.381559</td>
</tr>
<tr>
<td>Tran: [GARCH-Q]</td>
<td>0.850789</td>
<td>0.028613</td>
<td>29.73418</td>
</tr>
</tbody>
</table>

### Variance Equation

- **R-squared:** -1.420874  
- **Mean dependent var:** 0.770267  
- **Adjusted R-squared:** -1.421945  
- **S.D. dependent var:** 0.646231  
- **S.E. of regression:** 1.005703  
- **Akaike info criterion:** 2.715729  
- **Sum squared resid:** 9146.444  
- **Schwarz criterion:** 2.719658  
- **Log likelihood:** -12280.96  
- **Durbin-Watson stat:** 0.693122
## Appendix D: Autocorrelation functions of the residuals from ACD(2,2) estimation

### Stock “C”
Sample: 3 163459  
Included observations: 163457

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.007</td>
<td>-0.007</td>
<td>8.2421</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.004</td>
<td>0.004</td>
<td>10.818</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.003</td>
<td>0.003</td>
<td>12.609</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.001</td>
<td>-0.001</td>
<td>12.694</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.007</td>
<td>-0.007</td>
<td>19.619</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.009</td>
<td>-0.009</td>
<td>32.530</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.010</td>
<td>-0.010</td>
<td>48.247</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.005</td>
<td>-0.005</td>
<td>51.988</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.001</td>
<td>-0.001</td>
<td>52.069</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.002</td>
<td>-0.002</td>
<td>52.725</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

### Stock “HWP”
Sample: 1 143749  
Included observations: 143749

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.006</td>
<td>-0.006</td>
<td>5.5535</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.004</td>
<td>0.004</td>
<td>7.6684</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.007</td>
<td>0.007</td>
<td>13.794</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
<td>0.001</td>
<td>13.985</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
<td>0.001</td>
<td>14.138</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.002</td>
<td>-0.002</td>
<td>14.679</td>
<td>0.023</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.001</td>
<td>0.001</td>
<td>14.812</td>
<td>0.038</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.002</td>
<td>0.002</td>
<td>15.460</td>
<td>0.051</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.004</td>
<td>-0.004</td>
<td>17.474</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>17.477</td>
<td>0.064</td>
<td></td>
</tr>
</tbody>
</table>

### Stock “AXP”
Date: 09/19/04  
Time: 19:31  
Sample: 1 143678  
Included observations: 143678

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.3012</td>
<td>0.583</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.017</td>
<td>0.017</td>
<td>39.777</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.019</td>
<td>0.019</td>
<td>89.027</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.020</td>
<td>0.020</td>
<td>147.65</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.012</td>
<td>0.012</td>
<td>169.23</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.010</td>
<td>0.009</td>
<td>183.10</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.005</td>
<td>0.004</td>
<td>186.90</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.007</td>
<td>0.006</td>
<td>193.32</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.005</td>
<td>0.004</td>
<td>196.40</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.001</td>
<td>-0.002</td>
<td>196.69</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

### Stock “JBX”
Date: 09/19/04  
Time: 19:34  
Sample: 1 10680  
Included observations: 10680

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.036</td>
<td>-0.036</td>
<td>13.554</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.057</td>
<td>0.056</td>
<td>48.655</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.004</td>
<td>0.008</td>
<td>48.820</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.029</td>
<td>0.026</td>
<td>57.631</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.007</td>
<td>-0.006</td>
<td>58.113</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.023</td>
<td>0.020</td>
<td>63.897</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.003</td>
<td>-0.001</td>
<td>63.997</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.004</td>
<td>0.001</td>
<td>64.173</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.003</td>
<td>0.004</td>
<td>64.300</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.006</td>
<td>0.005</td>
<td>64.699</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>
### Stock “FIC”

Sample: 1000  
Included observations: 1000

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.007</td>
<td>0.007</td>
<td>0.0493</td>
<td>0.824</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.008</td>
<td>0.008</td>
<td>0.1111</td>
<td>0.946</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.025</td>
<td>-0.025</td>
<td>0.7298</td>
<td>0.866</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.021</td>
<td>-0.020</td>
<td>1.1544</td>
<td>0.886</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.056</td>
<td>-0.055</td>
<td>4.2661</td>
<td>0.512</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.036</td>
<td>0.037</td>
<td>5.5962</td>
<td>0.470</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.015</td>
<td>0.014</td>
<td>5.8162</td>
<td>0.561</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.010</td>
<td>0.006</td>
<td>5.9127</td>
<td>0.657</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.015</td>
<td>-0.016</td>
<td>6.1515</td>
<td>0.725</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.017</td>
<td>-0.018</td>
<td>6.4596</td>
<td>0.775</td>
<td></td>
</tr>
</tbody>
</table>
Appendix E: Estimation outputs of UHF-GARCH

Stock “C”
Dependent Variable: R
Method: ML - ARCH (Marquardt)
Date: 09/21/04 Time: 17:56
Sample: 2 163459
Included observations: 163458
Convergence not achieved after 200 iterations
Bollerslev-Wooldrige robust standard errors & covariance
MA backcast: 1, Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUR</td>
<td>-0.004000</td>
<td>0.000158</td>
<td>-25.26262</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.200000</td>
<td>0.032212</td>
<td>6.208866</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.600000</td>
<td>0.018726</td>
<td>-32.0431</td>
</tr>
</tbody>
</table>

Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.149072</td>
<td>0.320031</td>
<td>0.465806</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.463299</td>
<td>0.157680</td>
<td>2.938214</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.862769</td>
<td>0.057088</td>
<td>14.60857</td>
</tr>
<tr>
<td>DUR</td>
<td>0.168574</td>
<td>0.055550</td>
<td>3.043633</td>
</tr>
<tr>
<td>DUR/EDUR</td>
<td>0.671308</td>
<td>0.136138</td>
<td>4.931086</td>
</tr>
<tr>
<td>LONGVOL(-1)</td>
<td>0.035036</td>
<td>0.010125</td>
<td>3.460252</td>
</tr>
<tr>
<td>1/EDUR</td>
<td>-0.586006</td>
<td>0.177071</td>
<td>-3.309438</td>
</tr>
</tbody>
</table>

R-squared 0.544893     Mean dependent var 0.474241
Adjusted R-squared 0.544868     S.D. dependent var 6.205608
S.E. of regression 4.186521     Akaike info criterion 3.350066
Sum squared resid 2864746.     Schwarz criterion 3.350678
Log likelihood -273787.6     Durbin-Watson stat 2.119098

Inverted AR Roots .20
Inverted MA Roots -.60

Stock „HWP“
Dependent Variable: R
Method: ML - ARCH (Marquardt)
Date: 09/21/04 Time: 01:05
Sample(adjusted): 2 143749
Included observations: 143748 after adjusting endpoints
Convergence achieved after 17 iterations
Bollerslev-Wooldrige robust standard errors & covariance
MA backcast: 1, Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUR</td>
<td>0.000794</td>
<td>0.000750</td>
<td>3.831640</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.538829</td>
<td>0.143884</td>
<td>3.748209</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.283835</td>
<td>0.187037</td>
<td>-1.517528</td>
</tr>
</tbody>
</table>

Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.380600</td>
<td>0.074393</td>
<td>5.116075</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.490171</td>
<td>0.031652</td>
<td>15.48940</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.497732</td>
<td>0.104082</td>
<td>4.782091</td>
</tr>
<tr>
<td>1/DUR</td>
<td>0.477990</td>
<td>0.035040</td>
<td>13.64143</td>
</tr>
<tr>
<td>DUR/EDUR</td>
<td>-0.069426</td>
<td>0.004090</td>
<td>-16.97361</td>
</tr>
<tr>
<td>LONGVOL(-1)</td>
<td>0.009724</td>
<td>0.004670</td>
<td>2.082097</td>
</tr>
<tr>
<td>1/EDUR</td>
<td>0.083207</td>
<td>0.108792</td>
<td>3.126314</td>
</tr>
</tbody>
</table>

R-squared 0.373607     Mean dependent var 0.182756
Adjusted R-squared 0.373568     S.D. dependent var 3.082105
S.E. of regression 2.439408     Akaike info criterion 3.128576
Sum squared resid 855343.4     Schwarz criterion 3.129263
Log likelihood -224853.2     Durbin-Watson stat 1.041436

Inverted AR Roots .54
Inverted MA Roots .28
Stock “AXP”

Dependent Variable: R
Method: ML - ARCH (Marquardt)
Date: 09/21/04   Time: 21:34
Sample: 2 143678
Included observations: 143677
Convergence achieved after 147 iterations
Bollerslev-Wooldrige robust standard errors & covariance
MA backcast: 1, Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUR</td>
<td>0.017110</td>
<td>0.003298</td>
<td>5.188305</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.700771</td>
<td>0.052205</td>
<td>13.42341</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.050693</td>
<td>0.069412</td>
<td>0.730318</td>
</tr>
</tbody>
</table>

Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.401986</td>
<td>0.357409</td>
<td>1.124723</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.423442</td>
<td>0.142245</td>
<td>2.976841</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.686245</td>
<td>0.054035</td>
<td>12.69996</td>
</tr>
<tr>
<td>1/DUR</td>
<td>0.176333</td>
<td>0.065484</td>
<td>2.692757</td>
</tr>
<tr>
<td>DUR/EDUR</td>
<td>0.548085</td>
<td>0.156167</td>
<td>3.505569</td>
</tr>
<tr>
<td>LONGVOL(-1)</td>
<td>0.030817</td>
<td>0.009048</td>
<td>3.406029</td>
</tr>
<tr>
<td>1/EDUR</td>
<td>-0.735032</td>
<td>0.156638</td>
<td>-4.692564</td>
</tr>
</tbody>
</table>

R-squared 0.547464  Mean dependent var 0.513105
Adjusted R-squared 0.547436  S.D. dependent var 6.501194
S.E. of regression 4.37431  Akaike info criterion 3.460787
Sum squared resid 2748042  Schwarz criterion 3.461475
Log likelihood -248607.8  Durbin-Watson stat 2.123269

Inverted AR Roots .70
Inverted MA Roots -.05
### Stock “JBX”

**Dependent Variable:** R  
**Method:** ML - ARCH (Marquardt)  
**Date:** 09/22/04  **Time:** 01:56  
**Sample(adjusted):** 3 9119  
**Included observations:** 9117 after adjusting endpoints  
**Convergence achieved after 12 iterations**  
**MA backcast:** 2, **Variance backcast:** ON

#### Coefficient Table 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUR</td>
<td>4.74E-06</td>
<td>0.000164</td>
<td>0.028862</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.005000</td>
<td>3.217929</td>
<td>0.001554</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.005000</td>
<td>3.217153</td>
<td>0.001554</td>
</tr>
</tbody>
</table>

#### Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3.09E-05</td>
<td>1.32E-06</td>
<td>23.49696</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.150000</td>
<td>0.014160</td>
<td>10.59332</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.599998</td>
<td>0.019209</td>
<td>31.23585</td>
</tr>
<tr>
<td>1/DUR</td>
<td>1.68E-08</td>
<td>1.14E-08</td>
<td>1.474404</td>
</tr>
<tr>
<td>DUR/EDUR</td>
<td>-4.39E-06</td>
<td>1.20E-07</td>
<td>-36.44582</td>
</tr>
<tr>
<td>LONGVOL(-1)</td>
<td>-7.63E-07</td>
<td>0.016281</td>
<td>-4.68E-05</td>
</tr>
<tr>
<td>1/EDUR</td>
<td>-1.43E-06</td>
<td>6.99E-07</td>
<td>-2.048102</td>
</tr>
</tbody>
</table>

**R-squared** 0.000454  
**Mean dependent var** -1.35E-05  
**Adjusted R-squared** -0.000534  
**S.D. dependent var** 0.005757  
**S.E. of regression** 0.005759  
**Akaike info criterion** -7.311969  
**Schwarz criterion** -7.304162  
**Log likelihood** 33341.61  
**Durbin-Watson stat** 1.964462

#### Inverted Roots

- Inverted AR Roots: 0.01
- Inverted MA Roots: -0.01

### FIC

**Dependent Variable:** R  
**Method:** ML - ARCH (Marquardt)  
**Date:** 09/22/04  **Time:** 01:49  
**Sample(adjusted):** 2 9048  
**Included observations:** 9047 after adjusting endpoints  
**Convergence achieved after 41 iterations**  
**MA backcast:** 1, **Variance backcast:** ON

#### Coefficient Table 2

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUR</td>
<td>2.15E-05</td>
<td>7.32E-05</td>
<td>0.293611</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.005003</td>
<td>2.881201</td>
<td>0.001736</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.005003</td>
<td>2.880028</td>
<td>0.001737</td>
</tr>
</tbody>
</table>

#### Variance Equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.28E-05</td>
<td>4.27E-07</td>
<td>29.98859</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.150023</td>
<td>0.004985</td>
<td>30.09330</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.599991</td>
<td>0.013603</td>
<td>44.10618</td>
</tr>
<tr>
<td>1/DUR</td>
<td>3.64E-08</td>
<td>2.22E-09</td>
<td>16.37449</td>
</tr>
<tr>
<td>DUR/EDUR</td>
<td>-2.05E-06</td>
<td>2.73E-08</td>
<td>-75.14054</td>
</tr>
<tr>
<td>LONGVOL(-1)</td>
<td>-1.36E-05</td>
<td>0.003789</td>
<td>-0.003591</td>
</tr>
<tr>
<td>1/EDUR</td>
<td>1.53E-06</td>
<td>7.82E-08</td>
<td>19.61654</td>
</tr>
</tbody>
</table>

**R-squared** -0.000392  
**Mean dependent var** -5.52E-05  
**Adjusted R-squared** -0.001388  
**S.D. dependent var** 0.004591  
**S.E. of regression** 0.004594  
**Akaike info criterion** -7.934376  
**Schwarz criterion** -7.926516  
**Log likelihood** 35901.15  
**Durbin-Watson stat** 2.029549

#### Inverted Roots

- Inverted AR Roots: 0.01
- Inverted MA Roots: -0.01
Appendix F: Estimation output for ACD (1,1) for the price durations

Stock “C”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/17/04   Time: 14:26
Sample(adjusted): 2 6820
Included observations: 6819 after adjusting endpoints
Convergence achieved after 49 iterations
Bollerslev-Wooldridge robust standard errors & covariance

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3.013982</td>
<td>0.598025</td>
<td>5.039895</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.070492</td>
<td>0.006437</td>
<td>10.95154</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.905198</td>
<td>0.006475</td>
<td>106.8128</td>
</tr>
<tr>
<td>SPREAD(-1)</td>
<td>-2.981976</td>
<td>0.594836</td>
<td>-5.013105</td>
</tr>
</tbody>
</table>

R-squared: -3.680161
Mean dependent var: 0.875901
S.E. of regression: 0.988051
Akaike info criterion: 2.749898
Sum squared resid: 6653.109
Schwarz criterion: 2.753903
Log likelihood: -9371.778
Durbin-Watson stat: 0.341175

Stock “HWP”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/17/04   Time: 13:16
Sample(adjusted): 2 5793
Included observations: 5792 after adjusting endpoints
Convergence achieved after 27 iterations
Bollerslev-Wooldridge robust standard errors & covariance

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.262189</td>
<td>0.589020</td>
<td>3.834857</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.094070</td>
<td>0.008318</td>
<td>11.30982</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.875679</td>
<td>0.010141</td>
<td>86.35098</td>
</tr>
<tr>
<td>SPREAD(-1)</td>
<td>-2.222986</td>
<td>0.583969</td>
<td>-3.806685</td>
</tr>
</tbody>
</table>

R-squared: -2.773254
Mean dependent var: 0.845362
S.E. of regression: 0.986407
Akaike info criterion: 2.734579
Sum squared resid: 5631.716
Schwarz criterion: 2.729247
Log likelihood: -7915.340
Durbin-Watson stat: 0.400971

Stock “AXP”
Dependent Variable: SQR(DUR)
Method: ML - ARCH (Marquardt)
Date: 09/17/04   Time: 13:15
Sample(adjusted): 2 5630
Included observations: 5629 after adjusting endpoints
Convergence achieved after 28 iterations
Bollerslev-Wooldridge robust standard errors & covariance

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.295778</td>
<td>0.906630</td>
<td>2.532211</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.091513</td>
<td>0.008121</td>
<td>11.26933</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.890717</td>
<td>0.009813</td>
<td>101.0729</td>
</tr>
<tr>
<td>SPREAD(-1)</td>
<td>-2.270082</td>
<td>0.902584</td>
<td>-2.515062</td>
</tr>
</tbody>
</table>

R-squared: -2.908969
Mean dependent var: 0.855002
S.E. of regression: 0.501265
Akaike info criterion: 2.721429
Sum squared resid: 5529.091
Schwarz criterion: 2.726145
Log likelihood: -7655.463
Durbin-Watson stat: 0.390160
### Stock “JBX”

**Dependent Variable:** SQR(DUR)

**Method:** ML - ARCH (Marquardt)

**Sample (adjusted):** 2 1137

**Included observations:** 1136 after adjusting endpoints

**Convergence achieved after 15 iterations**

**Bollerslev-Wooldridge robust standard errors & covariance**

**Variance backcast: ON**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3.783865</td>
<td>3.504712</td>
<td>1.079651</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.098039</td>
<td>0.028110</td>
<td>3.487701</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.770313</td>
<td>0.062356</td>
<td>12.35348</td>
</tr>
<tr>
<td>SPREAD(-1)</td>
<td>-3.621230</td>
<td>3.479225</td>
<td>-1.040815</td>
</tr>
</tbody>
</table>

| R-squared    | -2.318166  | Mean dependent var | 0.862115 |
| Adjusted R-squared | -2.326959 | S.D. dependent var | 0.566479 |
| S.E. of regression | 1.033256 | Akaike info criterion | 2.875380 |
| Sum squared resid | 1208.543 | Schwarz criterion | 2.893110 |
| Log likelihood | -1629.216 | Durbin-Watson stat | 0.521707 |

### Stock “FIC”

**Dependent Variable:** SQR(DUR)

**Method:** ML - ARCH (Marquardt)

**Sample (adjusted):** 2 1334

**Included observations:** 1333 after adjusting endpoints

**Convergence achieved after 16 iterations**

**Bollerslev-Wooldridge robust standard errors & covariance**

**Variance backcast: ON**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.671922</td>
<td>0.410839</td>
<td>1.635486</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.123161</td>
<td>0.021397</td>
<td>5.755858</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.849223</td>
<td>0.023902</td>
<td>35.67913</td>
</tr>
<tr>
<td>SPREAD(-1)</td>
<td>-0.637617</td>
<td>0.404900</td>
<td>-1.574752</td>
</tr>
</tbody>
</table>

| R-squared    | -1.998683  | Mean dependent var | 0.813999 |
| Adjusted R-squared | -2.005452 | S.D. dependent var | 0.575990 |
| S.E. of regression | 0.998550 | Akaike info criterion | 2.727345 |
| Sum squared resid | 1325.148 | Schwarz criterion | 2.742934 |
| Log likelihood | -1813.775 | Durbin-Watson stat | 0.496018 |
# Appendix G: VNET Estimation Outputs

## Stock “C”

Dependent Variable: VNET  
Method: Least Squares  
Date: 09/17/04  Time: 14:26  
Sample(adjusted): 2 6820  
Included observations: 6819 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7.737541</td>
<td>0.168742</td>
<td>45.85422</td>
<td>0.0000</td>
</tr>
<tr>
<td>LSPREAD(-1)</td>
<td>16.51684</td>
<td>6.578014</td>
<td>2.510916</td>
<td>0.0121</td>
</tr>
<tr>
<td>LSIZE(-1)</td>
<td>0.300268</td>
<td>0.018241</td>
<td>16.46104</td>
<td>0.0000</td>
</tr>
<tr>
<td>LNTRADES(-1)</td>
<td>-0.252841</td>
<td>0.019492</td>
<td>-12.97149</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(EDUR)</td>
<td>-0.361811</td>
<td>0.046436</td>
<td>-7.791556</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(ACD_RESID)</td>
<td>0.985005</td>
<td>0.029640</td>
<td>33.23201</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.163425  Mean dependent var 10.22552
Adjusted R-squared 0.162811  S.D. dependent var 1.292742
S.E. of regression 1.182834  Akaike info criterion 3.174582
Sum squared resid 9532.037  Schwarz criterion 3.180590
Log likelihood -10817.74  F-statistic 266.1846
Durbin-Watson stat 2.037044  Prob(F-statistic) 0.000000

## Stock “HWP”

Dependent Variable: VNET  
Method: Least Squares  
Date: 09/18/04  Time: 16:20  
Sample(adjusted): 2 5793  
Included observations: 5792 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7.305673</td>
<td>0.166715</td>
<td>43.82125</td>
<td>0.0000</td>
</tr>
<tr>
<td>LSPREAD(-1)</td>
<td>-0.721622</td>
<td>5.623182</td>
<td>-0.128330</td>
<td>0.8979</td>
</tr>
<tr>
<td>LSIZE(-1)</td>
<td>0.305936</td>
<td>0.020716</td>
<td>14.76795</td>
<td>0.0000</td>
</tr>
<tr>
<td>LNTRADES(-1)</td>
<td>-0.262375</td>
<td>0.026737</td>
<td>-9.813260</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(EDUR)</td>
<td>0.397223</td>
<td>0.046722</td>
<td>8.501880</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(ACD_RESID)</td>
<td>1.208840</td>
<td>0.031223</td>
<td>38.71621</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.243969  Mean dependent var 9.375577
Adjusted R-squared 0.243161  S.D. dependent var 1.512107
S.E. of regression 1.315345  Akaike info criterion 3.387111
Sum squared resid 10010.55  Schwarz criterion 3.394015
Log likelihood -9803.073  F-statistic 373.4249
Durbin-Watson stat 2.044826  Prob(F-statistic) 0.000000

## Stock “AXP”

Dependent Variable: VNET  
Method: Least Squares  
Date: 09/17/04  Time: 13:15  
Sample(adjusted): 2 5630  
Included observations: 5629 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7.176210</td>
<td>0.173117</td>
<td>41.45304</td>
<td>0.0000</td>
</tr>
<tr>
<td>LSPREAD(-1)</td>
<td>14.71970</td>
<td>5.530308</td>
<td>2.661642</td>
<td>0.0078</td>
</tr>
<tr>
<td>LSIZE(-1)</td>
<td>0.307815</td>
<td>0.021670</td>
<td>14.20456</td>
<td>0.0000</td>
</tr>
<tr>
<td>LNTRADES(-1)</td>
<td>-0.226381</td>
<td>0.027191</td>
<td>-8.325581</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(EDUR)</td>
<td>-0.289364</td>
<td>0.041785</td>
<td>-6.925038</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(ACD_RESID)</td>
<td>1.083277</td>
<td>0.032503</td>
<td>33.32815</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.194066  Mean dependent var 9.512221
Adjusted R-squared 0.193290  S.D. dependent var 1.393359
S.E. of regression 1.251474  Akaike info criterion 3.287587
Sum squared resid 8806.675  Schwarz criterion 3.294660
Log likelihood -9246.914  F-statistic 270.6962
Durbin-Watson stat 1.976661  Prob(F-statistic) 0.000000
### Stock “JBX”

Dependent Variable: VNET  
Method: Least Squares  
Date: 09/17/04   Time: 13:18  
Sample(adjusted): 2 1137  
Included observations: 1136 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6.644517</td>
<td>0.332865</td>
<td>19.96157</td>
<td>0.0000</td>
</tr>
<tr>
<td>LSPREAD(-1)</td>
<td>-17.17011</td>
<td>8.441844</td>
<td>-2.033928</td>
<td>0.0422</td>
</tr>
<tr>
<td>LSIZE(-1)</td>
<td>0.207340</td>
<td>0.048895</td>
<td>4.240518</td>
<td>0.0000</td>
</tr>
<tr>
<td>LNTRADES(-1)</td>
<td>-0.118242</td>
<td>0.072797</td>
<td>-1.624280</td>
<td>0.1046</td>
</tr>
<tr>
<td>LOG(EDUR)</td>
<td>-0.154014</td>
<td>0.202317</td>
<td>-0.761250</td>
<td>0.4467</td>
</tr>
<tr>
<td>LOG(ACD_RESID)</td>
<td>0.857690</td>
<td>0.065481</td>
<td>13.09838</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.160124  
Mean dependent var 7.774643  
S.D. dependent var 1.621456  
S.E. of regression 1.489263  
Akaike info criterion 3.639707  
Schwarz criterion 3.663302  
Log likelihood -2061.354  
F-statistic 43.08742  
Prob(F-statistic) 0.000000  

### Stock “FIC”

Dependent Variable: VNET  
Method: Least Squares  
Date: 09/17/04   Time: 15:21  
Sample(adjusted): 2 1334  
Included observations: 1333 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5.445338</td>
<td>0.303805</td>
<td>17.92379</td>
<td>0.0000</td>
</tr>
<tr>
<td>LSPREAD(-1)</td>
<td>0.435498</td>
<td>5.531353</td>
<td>0.078733</td>
<td>0.9373</td>
</tr>
<tr>
<td>LSIZE(-1)</td>
<td>0.219027</td>
<td>0.049934</td>
<td>4.386379</td>
<td>0.0000</td>
</tr>
<tr>
<td>LNTRADES(-1)</td>
<td>-0.106097</td>
<td>0.069248</td>
<td>-1.532129</td>
<td>0.1257</td>
</tr>
<tr>
<td>LOG(EDUR)</td>
<td>-0.515559</td>
<td>0.088962</td>
<td>-5.795256</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(ACD_RESID)</td>
<td>0.416707</td>
<td>0.059468</td>
<td>7.007197</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.060184  
Mean dependent var 6.800451  
S.D. dependent var 1.514812  
Akaike info criterion 3.672951  
Schwarz criterion 3.696335  
Log likelihood -2442.022  
Prob(F-statistic) 0.000000  

78
List of References


Hautsch (2001): “Modelling Intraday Trading Activity Using Box-Cox ACD Models”. Discussion paper 02/05, CoFE, University of Konstanz.


