

Portfolio Choice and Transactions Taxes

Markus Haberer *

August 2003

Abstract

In a simple portfolio choice model of two assets a foreign exchange transactions tax is implemented. We show that the graph in the μ - σ^2 -range is still a parabola and delineate its characteristics for altering tax rates. We presumed a risk averse investor seeking to minimize investment risks by international diversification of two uncorrelated assets. The main finding is that setting up a portfolio under the new tax condition leads to a higher transaction volume on international financial markets. In contrast, the transactions tax has got a stabilizing character when adjusting the portfolio to increased foreign investment risks.

Keywords: International Financial Markets; Portfolio Choice; Risk Diversification; Tobin Tax; Transactions Tax.

JEL classifications: G11, G15, G18, H29, H39, O16, O23.

*Thanks to Bernd Genser, Erik Lueders and Dirk Schindler for helpful comments, and to Gundula Hadjiani and Wolfgang Eggert for technical support. All errors are mine. Markus Haberer, Department of Economics, University of Konstanz, PBox D133, 78457 Konstanz, Germany. Email: markus.2.haberer@uni-konstanz.de

Contents

1	Introduction	1
2	The Model	2
3	Analysis	4
3.1	The Possibilities Curve	4
3.2	The Minimum Variance Portfolio	7
3.3	A Comparative Static Analysis	8
3.3.1	Uncorrelated Assets	9
3.3.2	Adaptation Process After the Tax Levy	10
3.3.3	Changing Foreign Investment Risks	14
4	Summary and Conclusions	15
	Appendix	17
	References	21

1 Introduction

The outbreak of financial crises and monetary turmoil in some developing countries in the 1990s has given reason to the so-called globalization debate. Above all, critics hold the tremendous transaction volume on international financial markets responsible for the erroneous trend of these economies. Asymmetric information and herd behavior of investors, pulling in and out huge amounts of money within seconds, misguide international capital flows and thus having negative impact on concerned economies. Transactions taxes are said to be one way out and are put forward as a political measure to diminish globalization risks.

A tax on foreign exchange transactions should make foreign investments more expensive dependent on the time of holding the foreign asset. Thus, the Tobin Tax - named after its first proposer James Tobin in 1978¹ - discriminates short term investments against investments of longer holding periods. There exists a broad literature about the Tobin Tax discussing the pros and cons in respect of its desirability, effectiveness, and feasibility².

In his model Frankel (1996) shows mathematically that the tax burden goes contrary to the holding period of the foreign asset. He concludes that the Tobin Tax is an incentive not to trade foreign exchange that often, and therefore the transaction volume on the foreign exchange market will decline. Assets are assumed to yield a fixed return or at least an expected interest rate neglecting any risk. In addition to Frankel (1996), and also Stiglitz (1989), Summers/Summers (1989) and Eichengreen/Tobin/Wyplosz (1995), who revive Tobin's arguments for putting sand in the wheels of financial markets,

¹See Tobin (1978).

²An overview offers Haberler (2003).

most contributions to the Tobin Tax discussion do not focus on portfolio decisions³. In this paper we will examine how a forex transactions tax affects the portfolio choice. Our framework is based on Markowitz's pioneering findings of the 1950s, in which a (representative) investor's decision is based on the expected return and the risk of the portfolio⁴.

This article is structured as follows:

Chapter 2 explains the model with its assumptions and definitions. Our analysis takes place in chapter 3. In 3.1 the portfolio possibilities curve and the efficient frontier are defined. Presuming a very risk averse investor we concentrate on the minimum variance portfolio in chapter 3.2 and will do some comparative static analysis in 3.3 for the case of uncorrelated assets. Chapter 4 summarizes and concludes.

2 The Model

Our framework is a two-country-model. In the home country as well as in the foreign country there is only one risky asset available. Extending the model to n assets available in many countries would be unessentially more complex, since it does not offer any additional insights, and we have the possibility to show the effects of taxation graphically in the μ - σ -range. Since always residual risks of default and inflation remain, and moreover foreign

³Most cited papers are that of Arestis/Saywer (1997), Bird/Rajan (2001), Davidson (1997), Davidson (1998), De Grauwe (2000), Dooley (1996), Goodhart (1996), Lyons (1997), Menkhoff/Michaelis (1993) and Palley (1999).

⁴About the portfolio choice theory see Elton/Gruber (1995).

investments face exchange rate risk we except riskless lending and borrowing⁵. Let r_1 be the return of the domestic asset with the variance σ_1^2 , and r_2 the return of the foreign asset with the variance σ_2^2 , then $\bar{r}_1 > 0$ is the expected return of the domestic asset, $\bar{r}_2 > 0$ of the foreign asset respectively. A_1 is the fraction invested in the home asset, A_2 the fraction invested in the foreign country's asset (net, without tax payment).

Then the return of the portfolio r can be written as

$$r = A_1 r_1 + A_2 r_2 - (1 - A_1)T. \quad (2.1)$$

T denotes the foreign exchange transactions tax modelled as a withholding tax, which is due only at the time of buying foreign currency⁶.

From equation 2.1 we get the expected return of the portfolio μ_r as

$$\mu_r = A_1 \bar{r}_1 + A_2 \bar{r}_2 - (1 - A_1)T. \quad (2.2)$$

The side condition is $A_1 + A_2 + (1 - A_1)T = 1$ or rather

$$A_2 = (1 - T)(1 - A_1) \quad (2.3)$$

what means that the fractions invested in the two assets and the tax payment must sum up to 1.

Finally, the variance of the portfolio as the measure for the risk is

$$\sigma_r^2 = A_1^2 \sigma_1^2 + A_2^2 \sigma_2^2 + 2\sigma_{12} A_1 A_2 \quad (2.4)$$

⁵The investment behavior of a manager of a fond of one industrial sector gives us another rational for the exclusion of riskless lending and borrowing. Such a portfolio only consists of risky assets of pharmaceutics e.g. and does not involve "riskless" financial assets like government bonds.

⁶The Tobin Tax proposal is a transactions tax due at the point of buying and selling the foreign currency. In the sense of Haberer (2003) - in contrast to Frankel (1996) - equation 2.1 should be $r = A_1 r_1 + A_2 r_2 - (1 - A_1)T - A_2(1 + r_2)T$. But to avoid complexity without losing any insight we model the transactions tax to be due only once.

with σ_{12} as the covariance between the returns of the two assets.

It is assumed that the representative investor is risk averse and makes up his mind only on the basis of the expected portfolio return μ_r and the variance σ_r^2 . According to the home bias that can be justified by asymmetric information amongst the domestic and the foreign country, we assume the risk of the foreign investment to be higher than the domestic, and therefore

$$\sigma_2^2 > \sigma_1^2. \quad (2.5)$$

Since the investor is risk averse, he will only take more risk if he expects a higher return, and thus

$$\bar{r}_2 > \bar{r}_1. \quad (2.6)$$

3 Analysis

In this chapter we want to analyze the effects of the transactions tax on investor's portfolio choice. At first, we develop the possibilities curve in the μ_r - σ_r^2 -range before determining the minimum variance portfolio. By doing some comparative static analysis we find out in chapter 3.3 that a transactions tax might increase the transaction volume on international financial markets after imposing the tax.

3.1 The Possibilities Curve

Having implemented a transactions tax into a standard portfolio choice model in the previous chapter we now want to illustrate the set of all μ_r - σ_r^2 -combinations of the portfolio return that are possible. Moreover we will show graphically

in this chapter how this possibilities curve or opportunity set will behave against the tax rate. For risk averse investors we will detect dominated portfolios so that we can expose an efficient frontier.

Equation 2.3 applied to equation 2.2 and solved for A_1 and A_2 yields

$$A_1 = \frac{\mu_r + T - \bar{r}_2(1 - T)}{\bar{r}_1 - \bar{r}_2(1 - T) + T} \quad (3.7)$$

and

$$A_2 = \frac{(\bar{r}_1 - \mu_r)(1 - T)}{\bar{r}_1 - \bar{r}_2(1 - T) + T}. \quad (3.8)$$

Equations 3.7 and 3.8 plugged into 2.4 gives us an expression for the variance σ_r^2 dependent on the expected return μ_r and the exogenous variables T , σ_1^2 , σ_2^2 , σ_{12} and the asset returns r_1 and r_2 . Substituting A, B and D for expressions of the exogenous variables (see appendix A1), we can rewrite the variance σ_r^2 against the return μ_r as follows:

$$\sigma_r^2 = A(\mu_r - B)^2 + D. \quad (3.9)$$

Equation 3.9 is that of a parabola in the μ_r - σ_r^2 -range.

Figure 3.1 shows the possibilities curve for the case of uncorrelated assets ($\sigma_{12} = 0$) with a transactions tax of 1 per cent⁷. P_1 is the portfolio if only asset 1 is bought, P_2 is the portfolio if only asset 2 is bought. M is the minimum variance portfolio. All combinations of assets on the ascending part of the parabola dominate the portfolios below, since higher returns with the

⁷In figure 3.1 and 3.2 all the other parameters are constant in their values: $\bar{r}_1 = 0.05$, $\bar{r}_2 = 0.10$, $\sigma_1^2 = 0.25$ and $\sigma_2^2 = 0.5$.

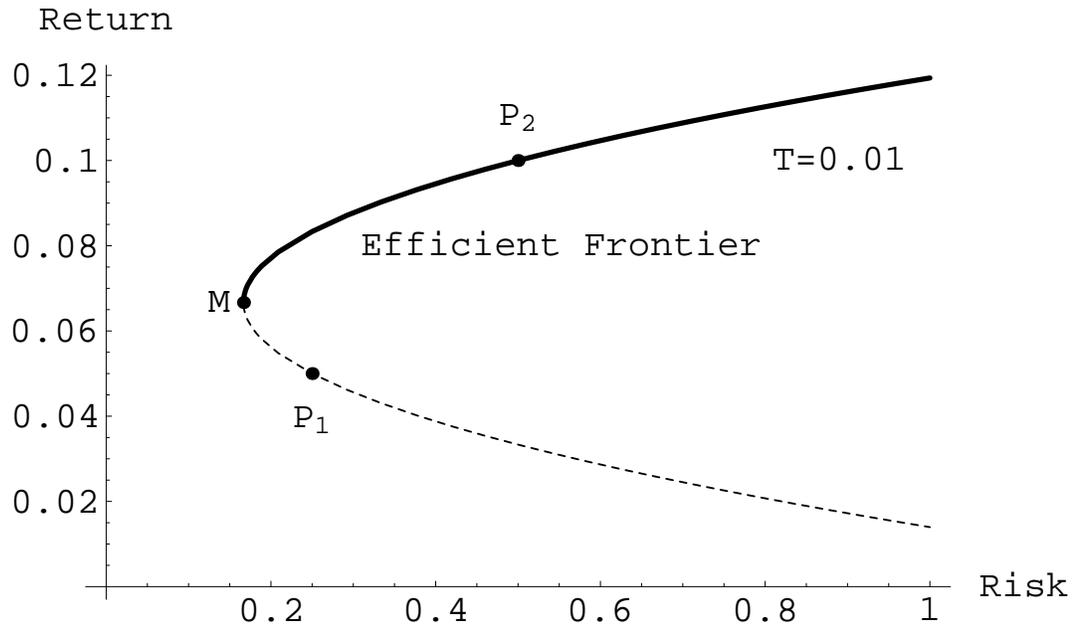


Figure 3.1: The Possibilities Curve and its Efficient Frontier.

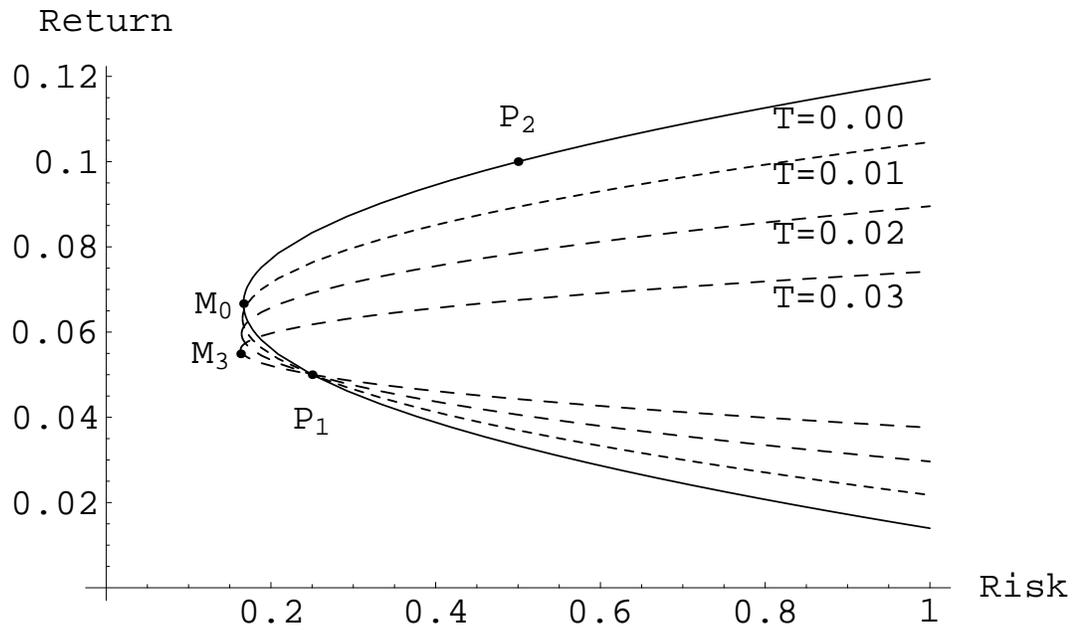


Figure 3.2: The Possibilities Curve with Different Tax Rates.

same risk can be realized. Thus, the efficient frontier is situated between M and P_2 if short-selling is not allowed or it is the total ascending part of the curve, if short-selling is allowed (default in asset 1).

Figure 3.2 shows the possibilities curve with altering tax rates from 0 per cent to 3 per cent. As we can see, the parabola gets narrower with increasing tax rates. No matter of the tax rate, P_1 can be reached in every case in contrast to P_2 , which can be realized only in the case of $T = 0$. Another finding from the graphic is, that the minimum variances become smaller with increasing tax rates.

3.2 The Minimum Variance Portfolio

The attractiveness of international financial markets is that of diversifying risk internationally. As we can see from the graphics in the previous chapter there exist always efficient portfolios of lower risk than that of one single asset even for uncorrelated assets. The minimum variance portfolio is that combination of assets, in which risk can no longer be reduced by diversification. The objective function is equation 2.4 with the side condition 2.3. Solving this minimizing problem in Lagrangian mode we get

$$L = A_1^2\sigma_1^2 + A_2^2\sigma_2^2 + 2\sigma_{12}A_1A_2 + \lambda[1 - A_1 - T(1 - A_1) - A_2]. \quad (3.10)$$

The three conditions

$$\frac{\partial L}{\partial A_1} = 0 \quad (3.11)$$

$$\frac{\partial L}{\partial A_2} = 0 \quad (3.12)$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad (3.13)$$

must be satisfied in the minimum variance portfolio. From condition 3.11 and 3.12 we get the ratio between the two fractions in the minimum variance portfolio:

$$\frac{A_{1min}}{A_{2min}} = \frac{(1-T)\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - (1-T)\sigma_{12}}, \quad T \neq 1. \quad (3.14)$$

Together with equation 3.13 we get the expressions for the fractions invested in the two assets to minimize the portfolio risk:

$$A_{1min} = \frac{(1-T)[\sigma_2^2(1-T) - \sigma_{12}]}{(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2} \quad (3.15)$$

$$A_{2min} = \frac{(1-T)[\sigma_1^2 - (1-T)\sigma_{12}]}{(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2}. \quad (3.16)$$

The tax payment $T(1 - A_{1min})$ is given by

$$T(1 - A_{1min}) = \frac{T[\sigma_1^2 - \sigma_{12}(1-T)]}{(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2}. \quad (3.17)$$

We now have delineated the possibilities curve, the efficient frontier and the minimum variance portfolio as the optimal choice for an very risk avers investor, who wants to minimize his portfolio risk by international diversification. In the following section we want to examine the impact of changes in the tax rate on the investor's portfolio choice.

3.3 A Comparative Static Analysis

We now turn to the ceteris paribus analysis of the investment decision. We first examine the optimal adjustment when the transactions tax is introduced or the tax rate changes. Note that these findings hold for portfolios set up

after imposing the tax, since only capital flows and not capital stocks are taxed. In the last part we study how a transactions tax governs the portfolio choice of the investor when foreign investment risks change.

Most of the findings refer to a situation of uncorrelated assets. We argue for that restriction as follows:

3.3.1 Uncorrelated Assets

The ratio of the fractions invested in the two assets in the risk minimum is given by equation 3.14. The first derivative of 3.14 is

$$\begin{aligned} \frac{\partial \frac{A_{1min}}{A_{2min}}}{\partial T} &= \frac{\sigma_{12}^2 - \sigma_1^2 \sigma_2^2}{[\sigma_1^2 - (1 - T)\sigma_{12}]^2} \\ &= \frac{\sigma_1^2 \sigma_2^2 ((\rho_{12}^2 - 1))}{[\sigma_1^2 - (1 - T)\sigma_{12}]^2} \leq 0. \end{aligned} \quad (3.18)$$

ρ_{12} is the correlation coefficient between the returns of the two available assets and is given by $\rho_{12} = \sigma_{12}/\sigma_1\sigma_2$ and therefore in the range between -1 and +1. The ratio of the fractions does not change for perfectly correlated assets, thus $\rho_{12} = \pm 1$. The smaller the correlation the higher the impact on the ratio. In the case of uncorrelated assets, the change of the tax rate influences the investor's decision at most.

Moreover, the clue of the portfolio theory is the reduction of the portfolio risk by diversification even in the case of uncorrelated assets. It is clear-cut that risk can be reduced by buying negatively correlated assets or by buying and short-selling positively correlated assets, but the more interesting case is that of $\rho_{12} = 0$.

Another reason for examining a portfolio of uncorrelated assets is based on

the fact that most of the investors do not sell short. Partially short-selling is forbidden by law. This means that diversification must be done under the constraint

$$A_{1min} \geq 0 \quad \text{and} \quad A_{2min} \geq 0. \quad (3.19)$$

Therefore the condition

$$\rho_{12} \leq \min\left\{(1-T)\frac{\sigma_2}{\sigma_1}; \frac{1}{(1-T)}\frac{\sigma_1}{\sigma_2}\right\} \quad (3.20)$$

must be fulfilled (see appendix A2). This holds always for $\rho_{12} = 0$ whatever the variances of the two assets are.

3.3.2 Adaptation Process After the Tax Levy

With $\rho_{12} = 0$ the expressions for the two fractions can be reduced to

$$A_{1min} = \frac{(1-T)^2\sigma_2^2}{(1-T)^2\sigma_2^2 + \sigma_1^2} \quad (3.21)$$

and

$$A_{2min} = \frac{(1-T)^2\sigma_1^2}{(1-T)^2\sigma_2^2 + \sigma_1^2}. \quad (3.22)$$

To examine how the fraction invested in the domestic assets must be rearranged due to changes of the transactions tax rate we take the first derivative of equation 3.21 that yields

$$\frac{\partial A_{1min}}{\partial T} = \frac{-2\sigma_1^2\sigma_2^2(1-T)}{[(1-T)^2\sigma_2^2 + \sigma_1^2]^2} < 0. \quad (3.23)$$

This expression is negative since the denominator is positive and all terms of the numerator are positive as well ($T < 0$). This means that the fraction,

which the representative investor invests in the domestic assets, decreases with rising tax rates.

The tax payment given by $T(1 - A_{1min})$ behaves as follows:

$$\frac{\partial[T(1 - A_{1min})]}{\partial T} = 1 - A_{1min} - \frac{\partial A_{1min}}{\partial T} > 0. \quad (3.24)$$

This derivative with respect to the tax rate is positive because the last term as seen above is negative and A_{1min} is smaller than 1 since we exclude short-selling. Thus, the rise of the tax rate increases the tax payment.

The much more interesting issue is the optimal adjustment of the foreign investment. Taking the first derivative of A_{2min} with respect to the tax rate gives us

$$\frac{\partial A_{2min}}{\partial T} = \frac{\sigma_1^2[\sigma_2^2(1 - T)^2 - \sigma_1^2]}{[(1 - T)^2\sigma_2^2 + \sigma_1^2]^2}. \quad (3.25)$$

Whether this expression is positive or negative depends on the term in square brackets. As we can see, each parameter is positive and squared so that we can take the square root. For all combinations of the variances solving

$$(1 - T)\sigma_2 > \sigma_1 \quad (3.26)$$

the equation 3.25 is positive. Condition 3.26 is fulfilled for $\sigma_2 > \sigma_1$ as we have already presumed (see condition 2.5) and small values of T what is recommended by actual literature about the foreign exchange transactions tax. Therefore, against all findings and persuasions of the proponents of the Tobin tax, in this simple framework of portfolio choice a transactions tax on the foreign exchange market would raise the fraction invested in the foreign

asset by the representative investor. The transaction volume on the foreign exchange market would increase quite after imposing the tax or changing the tax rate.

The rational for the raise of the foreign fraction runs as follows: The tax payment on foreign exchange transactions distorts the investor's optimal portfolio by reducing the fraction of foreign assets. To reach the optimal ratio after changing the tax rate, the investor has to remargin into the foreign asset in order to minimize the portfolio risk.

The variance of the minimum variance portfolio is given by

$$\sigma_{r,min}^2 = A_{1min}^2 \sigma_1^2 + A_{2min}^2 \sigma_2^2. \quad (3.27)$$

Equations 3.21 and 3.22 plugged into 3.27 gives

$$\sigma_{r,min}^2 = \frac{(1-T)^2 \sigma_1^2 \sigma_2^2}{(1-T)^2 \sigma_2^2 + \sigma_1^2} < \sigma_2^2 < \sigma_1^2. \quad (3.28)$$

The first derivative of the portfolio variance with respect to the tax rate can be written as

$$\frac{\partial \sigma_{r,min}^2}{\partial T} = -2\sigma_1^4 \sigma_2^2 (1-T) [\sigma_2^2 (1-T)^2 + \sigma_1^2] < 0. \quad (3.29)$$

Since all terms on the right hand side are positive, the derivative is negative. Thus, a raise of the tax rate results in lower portfolio risk after adjusting the portfolio. The transactions tax can be regarded as an riskless asset with negative return. Since the investor's objective is to minimize the portfolio risk and the portfolio adjustment results from exalting the foreign fraction, the tax levy lowers the portfolio risk.

One might rashly think that this risk reduction goes inevitably with lower return. The portfolio return in the variance minimum is given by

$$\begin{aligned}\mu_{r,min} &= A_{1min}\bar{r}_1 + A_{2min}\bar{r}_2 - (1 - A_{1min})T \\ &= A_{1min}(\bar{r}_1 + T) + A_{2min}\bar{r}_2 - T.\end{aligned}\tag{3.30}$$

The first derivative with respect to the tax rate is

$$\begin{aligned}\frac{\partial\mu_{r,min}}{\partial T} &= -1 + \frac{\partial A_{2min}}{\partial T}\bar{r}_2 + \frac{\partial A_{1min}}{\partial T}(\bar{r}_1 + T) + A_{1min} \\ &= \sigma_1^2 \frac{\bar{r}_2[(1-T)^2\sigma_2^2 - \sigma_1^2] - \sigma_1^2 - \sigma_2^2(1-T)^2[1 + \frac{2(\bar{r}_1+T)}{1-T}]}{[(1-T)^2\sigma_2^2 + \sigma_1^2]^2}.\end{aligned}\tag{3.31}$$

The condition for $\frac{\partial\mu_{r,min}}{\partial T} > 0$ is

$$\bar{r}_2\left[1 - \frac{\sigma_1^2}{(1-T)^2\sigma_2^2}\right] > 1 + \frac{\sigma_1^2}{(1-T)^2\sigma_2^2} + \frac{2(\bar{r}_1 + T)}{1-T}.\tag{3.32}$$

That means that for extrem values of the foreign return⁸ and a much more higher risk of the foreign investment, not only can the portfolio variance be reduced by adjusting the optimal portfolio, but also may the expected return be increased.

The question that rises immediately is why the risk averse investor does not hold back a certain amount of money and does invest it in a riskless asset like the tax payment in our case but with a positive return. The answer is clear-cut: In our model we excluded riskless lending and borrowing from the set of available assets and thus the tax payment is the only riskless tool but with negative return.

⁸In the case of imminent financial crises very high returns are expected.

3.3.3 Changing Foreign Investment Risks

In the previous chapter we examined the adjustment of a portfolio set up under the new tax condition. In this chapter we study how the investor behaves when the investment risk in the foreign country rises e.g. due to political or economic turmoil.

The fraction invested into the foreign asset is given by (see 3.22)

$$A_{2min} = \frac{(1 - T)^2 \sigma_1^2}{(1 - T)^2 \sigma_2^2 + \sigma_1^2}. \quad (3.33)$$

A change of the return risk of asset 2 in the foreign country influences the optimal fraction according to the first derivative of equation 3.33 with respect to the variance σ_2^2

$$\frac{\partial A_{2min}}{\partial \sigma_2^2} = -\frac{\sigma_1^2 (1 - T)^4}{[\sigma_1^2 + \sigma_2^2 (1 - T)^2]^2} < 0. \quad (3.34)$$

This derivative is negative what means, that a higher foreign risk leads to smaller investments in the foreign country. What we want to know is how a transactions tax influences this restructuring of the portfolio. Therefore we take the first derivative of equation 3.34 with respect to the tax rate T that is

$$\frac{\frac{\partial A_{2min}}{\partial \sigma_2^2}}{\partial T} = -\frac{4\sigma_1^4 (T - 1)^3}{[\sigma_1^2 + \sigma_2^2 (T - 1)^2]^3} > 0. \quad (3.35)$$

This derivative is positive, since T is smaller than 1. The conclusion is the following:

A higher tax rate raises the first derivative of the foreign fraction with respect to the assumed foreign risk. Since this derivative (equation 3.34) is negative,

its value approximates 0 what means that the restructuring of the portfolio under transactions taxes is lower, since less money will be pulled out of the foreign country, if foreign investment risk goes up.

4 Summary and Conclusions

In this article we examined the effect of transactions taxes on the investor's portfolio choice. We concentrated on the case of two uncorrelated assets, one available in the home country and the other in a foreign country. Short-selling was not allowed. We found out, that the opportunity set in the μ_r - σ_r^2 -range is still a parabola with its efficient frontier at the ascending part. We presumed a very risk averse investor reaching to minimize risks. Only in the case of totally correlated assets the ratio between the fractions are independent from the tax rate. Otherwise decreases the investment in the domestic asset with increasing tax rate. Uncorrelated assets assure that diversification without short-selling one asset takes place.

We distinguished two examinations: The portfolio adjustment due to the tax levy or changing tax rates, and the adaptation due to changes in assumed investment risks. For low tax rates the fraction invested in the foreign asset would increase by adjusting the portfolio due to the tax levy. Hence, as a temporary effect of adjustment the transaction volume on the foreign exchange market would increase and is the opposite effect of what the proponents of the Tobin tax intend. The transactions tax lowers the portfolio risk without necessarily lowering the return. In contrast, a transactions tax has a stabilizing effect when the investment risk abroad increases, since the fraction of the foreign asset would be shifted less to adjust the portfolio.

Further research would be generalizing the approach to correlated assets, introducing a specific utility function of the representative investor to characterize his risk aversion and allow riskless lending and borrowing.

Appendix

A1

To delineate the possibilities curve we take the expressions for the two fractions (equations 3.7 and 3.8)

$$A_1 = \frac{\mu_r + T - \bar{r}_2(1 - T)}{\bar{r}_1 - \bar{r}_2(1 - T) + T}$$

and

$$A_2 = \frac{(\bar{r}_1 - \mu_r)(1 - T)}{\bar{r}_1 - \bar{r}_2(1 - T) + T}$$

and plug them into the equation for the portfolio variance (2.4)

$$\sigma_r^2 = A_1^2 \sigma_1^2 + A_2^2 \sigma_2^2 + 2\sigma_{12} A_1 A_2.$$

This yields

$$\begin{aligned} \sigma_r^2 &= \frac{[\mu_r + T - \bar{r}_2(1 - T)]^2 \sigma_1^2}{[\bar{r}_1 - \bar{r}_2(1 - T) + T]^2} \\ &+ \frac{[(\bar{r}_1 - \mu_r)(1 - T)]^2 \sigma_2^2}{[\bar{r}_1 - \bar{r}_2(1 - T) + T]^2} \\ &+ 2\sigma_{12} \frac{[\mu_r + T - \bar{r}_2(1 - T)][(\bar{r}_1 - \mu_r)(1 - T)]}{[\bar{r}_1 - \bar{r}_2(1 - T) + T]^2} \end{aligned}$$

or

$$\begin{aligned}
\sigma_r^2 &= \frac{1}{[\bar{r}_1 - \bar{r}_2(1 - T) + T]^2} \{ \sigma_2^2(1 - T)^2(\bar{r}_1^2 + \mu_r^2 - 2\bar{r}_1\mu_r) \\
&+ \sigma_1^2[\mu_r^2 + T^2 + \bar{r}_2^2(1 - T)^2 + 2\mu_r T - 2\mu_r\bar{r}_2(1 - T) - 2T\bar{r}_2(1 - T)] \\
&+ 2\sigma_{12}(1 - T)[- \mu_r^2 + \mu_r(\bar{r}_1 - T + (1 - T)\bar{r}_2) + T\bar{r}_1 - (1 - T)\bar{r}_1\bar{r}_2] \}.
\end{aligned}$$

Since we want to display σ_r^2 against μ_r we can rewrite

$$\begin{aligned}
\sigma_r^2 &= \frac{1}{[\bar{r}_1 - \bar{r}_2(1 - T) + T]^2} \{ \mu_r^2[\sigma_1^2 + \sigma_2^2(1 - T)^2 - 2\sigma_{12}(1 - T)] \\
&- 2\mu_r[-\sigma_1^2 T + \sigma_1^2 \bar{r}_2(1 - T) + \sigma_2^2(1 - T)^2 \bar{r}_1 - \sigma_{12}(1 - T)(\bar{r}_1 - T + (1 - T)\bar{r}_2)] \\
&+ \sigma_1^2[T^2 + \bar{r}_2^2(1 - T)^2 - 2T\bar{r}_2(1 - T) + \sigma_2^2(1 - T)^2 \bar{r}_1^2] \\
&+ 2\sigma_{12}(1 - T)\bar{r}_1[T - (1 - T)\bar{r}_2] \}. \tag{A.1}
\end{aligned}$$

We define the term following μ_r^2 in equation A.1

$$a = \sigma_1^2 + \sigma_2^2(1 - T)^2 - 2\sigma_{12}(1 - T)$$

and the term following $-2\mu_r$

$$b = -\sigma_1^2 T + \sigma_1^2 \bar{r}_2(1 - T) + \sigma_2^2(1 - T)^2 \bar{r}_1 - \sigma_{12}(1 - T)(\bar{r}_1 - T + (1 - T)\bar{r}_2)$$

and the last two lines of equation A.1

$$c = \sigma_1^2[T^2 + \bar{r}_2^2(1-T)^2 - 2T\bar{r}_2(1-T)] + \sigma_2^2(1-T)^2\bar{r}_1^2 + 2\sigma_{12}(1-T)\bar{r}_1[T - (1-T)\bar{r}_2].$$

By defining

$$A = \frac{a}{[\bar{r}_1 - \bar{r}_2(1-T) + T]^2}$$

$$B = \frac{b}{a}$$

$$C = \frac{c}{a}$$

we can rewrite

$$\begin{aligned} \sigma_r^2 &= A[\mu_r^2 - 2\mu_r B + C] \\ &= A[\mu_r - B]^2 + A(C - B^2) \\ &= A[\mu_r - B]^2 + D \end{aligned} \tag{A.2}$$

with

$$D = A(C - B^2).$$

Equation A.2 is a parabola in the μ_r - σ_r^2 -range.

A2

The condition for real diversification without short-selling is equation 3.19:

$$A_{1min} \geq 0 \quad \text{and} \quad A_{2min} \geq 0.$$

Together with equations 3.15 and 3.16 it yields

$$\frac{(1-T)[\sigma_2^2(1-T) - \sigma_{12}]}{(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2} \geq 0$$

$$\frac{(1-T)[\sigma_1^2 - (1-T)\sigma_{12}]}{(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2} \geq 0.$$

We first ignore the denominators and concentrate on the numerators, which are positive for

$$\sigma_2^2(1-T) \geq \sigma_{12} \quad \text{and} \quad \sigma_1^2 \geq (1-T)\sigma_{12}.$$

The last conditions divided by $\sigma_1\sigma_2$ and with $\rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2}$ can be rewritten as

$$\rho_{12} \leq \min\left\{(1-T)\frac{\sigma_2}{\sigma_1}; \frac{1}{(1-T)}\frac{\sigma_1}{\sigma_2}\right\}$$

and is always fulfilled for $\sigma_{12} = 0$.

This condition includes that the collective denominator $(1-T)[\sigma_2^2(1-T) - 2\sigma_{12}] + \sigma_1^2$ is positive, because solving for $2\sigma_{12}$ and dividing by $\sigma_1\sigma_2$ gives

$$2\rho_{12} \leq (1-T)\frac{\sigma_2}{\sigma_1} + \frac{1}{(1-T)}\frac{\sigma_1}{\sigma_2}.$$

References

- Arestis, Philip/ Sawyer, Malcolm (1997): How Many Cheers for the Tobin Transactions Tax? *Cambridge Journal of Economics*, No. 21, pp. 753-768.
- Bird, Graham/ Rajan, Ramkishen S. (2001): International Currency Taxation and Currency Stabilisation in Developing Countries. *The Journal of Developing Studies*, Vol. 37, No. 3, pp. 21-38.
- Davidson, Paul (1997): Are Grains of Sand in the Wheels of International Finance Sufficient to Do the Job When Boulders Are Often Required? *The Economic Journal*, Vol. 107, No. 442, pp. 671-686.
- Davidson, Paul (1998): Efficiency and Fragile Speculative Financial Markets: Against the Tobin Tax and For a Credible Market Maker. *American Journal of Economics and Sociology*, Vol. 57, No. 4, pp. 639-662.
- De Grauwe, Paul (2000): Controls on Capital Flows. *Journal of Policy Modeling*, Vol. 22, No. 3, pp. 391-405.
- Dooley, Michael P. (1996): The Tobin Tax: Good Theory, Weak Evidence, Questionable Policy. in: ul Haq/Kaul/Grunberg (Ed.): *The Tobin Tax: Coping with Financial Volatility*: Oxford University Press, New York, pp. 83 - 106.
- Eichengreen, Barry/ Tobin, James/ Wyplosz, Charles (1995): Two Cases for Sand in the Wheels of International Finance. *The Economic Journal*, Vol. 105, No. 428, pp. 162-172.

- Elton, E./Gruber, M. (1995): *Modern Portfolio Theory and Investment Analysis*. John Wiley & Sons, New York, fifth edition.
- Frankel, Jeffrey (1996): How Well do Foreign Exchange Markets Work: Might a Tobin Tax Help?. in: ul Haq/Kaul/Grunberg (Ed.): *The Tobin Tax: Coping with Financial Volatility*: Oxford University Press, New York, pp. 41 - 81.
- Goodhart, Charles A. (1978): A Currency Transactions Tax. Why and How: Discussant to Professor J. Tobin. *Economic Systems merged with Journal of International and Comparative Economics*, Vol. 20, No. 2-3, pp. 91-95.
- Haberer, Markus (2003): Some Criticism of the Tobin Tax. *Center of Finance and Econometrics (CoFE)*, University of Konstanz, Discussion Paper No. 03/01, January 2003.
- Lyons, Richard K. (1997): A Simultaneous Trade Model of the Foreign Exchange Hot Potato. *Journal of International Economics*, Vol. 42, pp. 275-298.
- Menkhoff, Lukas/ Michaelis, Jochen (1993): Exzessive Markteffizienz und Tobin-Steuer. Diskussionsbeitraege 24/93, Institut fr Finanzwissenschaft der Albert-Ludwigs-Universitaet Freiburg im Breisgau.
- Palley, Thomas (1999): Speculation and Tobin Taxes: Why Sand in the Wheels Can Increase Economic Efficiency. *Journal of Economics*, Vol. 69, No. 2, pp. 113-126.
- Stiglitz, Joseph E. (1989): Using Tax Policy to Curb Speculative Short-Term Trading. *Journal of Financial Services Research*, Vol. 3, pp. 101-115.

Summers, Lawrence H./ Summers, Victoria P. (1989): When Financial Markets Work Too Well: A Cautious Case For a Securities Transactions Tax. *Journal of Financial Services Research*, Vol. 3, pp. 261-286.

Tobin, James (1978): A Proposal for International Monetary Reform. *The Eastern Economic Journal*, Vol. 4, No. 3-4, pp. 153-159.