Pricing of cap-interest rates based on renewal processes

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Abstract

Pricing of cap insurance contracts is considered for political mortgage rates. A simple stochastic process for mortgage rates is proposed. The process is based on renewal processes for modelling the length of periods with downward and upward trend respectively. Prices are calculated by simulation of conditional future sample paths. Future conditional quantiles can be obtained to assess the risk of a contract. The method is illustrated by applying it to observed quarterly mortgage rates of the Swiss Union of Raiffeisenbanks for the years 1970 to 2001.

Key words: cap, cap rate, cap insurance, interest rate, mortgage, premium, renewal process, Poisson process, prediction.

1 Introduction

In the last few years, several new types of mortgages appeared on the Swiss market. Typical examples are interest rate cap insurance (cap rate), roll-over and portfolio of market rates. In particular, the interest rate cap insurance became quite popular in the retail market. It provides an insurance against the event that the floating interest rate rises above a certain fixed level, the so-called cap. Caps are offered for market rates and political mortgage rates. Here, we consider caps on political mortgage rates.

Pricing of cap rates is based on forecasts of future interest rates. In this paper, a stochastic model is proposed that mimics the main features of observed interest rates. This allows for maximum likelihood estimation of the parameters, prediction of future interest rates, Monte-Carlo simulation of a fair cap rate price and risk assessment. In the literature, models used for predicting interest rates include ARIMA and fractional ARIMA models, heteroskedastic models such as GARCH, regression models, structural systems of equations, multivariate models (e.g. multivariate ARIMA), neural networks, regime switching models, consensus forecasting, state space models and exponential smoothing. References can be found, for instance, in a review paper by Fauvel et al. (1999).

The model proposed here falls into the category of univariate regime switching models. The purpose is to obtain a simple model that does not require any knowledge of explanatory variables, but still provides reliable short- and medium term forecasts.

The outline of the article is as follows. The general pricing equation is given in section 2. In section 3, statistical properties of mortgage rates are discussed and a stochastic model with these properties is proposed. Maximum likelihood estimation is considered in section 4. In section 5, prediction of future interest rates and estimation of a fair cap price are discussed. The performance of the proposed pricing strategy is illustrated by applying it to the quarterly mortgage rates of the Swiss Union of Raiffeisenbanks.

2 Pricing of a cap-mortgage

When a cap on a loan and the loan itself are both provided by the same financial institution, then the cost of the option is usually incorporated into the underlying, such as the interest rate. Denote by K the nominal value of the mortgage and by Z_t the floating mortgage rate at time t ($t = t_o + 1, ..., t_o + T$, where T denotes maturity). Given a cap $C > 0$, the rate paid by the customer at time t is

$$R_t = \min(Z_t, C).$$

The financial institution's cash-flow D_t at time t is then

$$D_t = (R_t - Z_t) \cdot K \qquad (t_o + 1 \leq t \leq t_o + T).$$

We assume that, in order to give a credit, the financial institution itself has to take a loan from another creditor. Also, it is assumed that the last n interest rates $Z_{t_o-n+1}, ..., Z_{t_o}$ are known, the first installment of the interest rate is to be paid at time $t = t_o + 1$, and repayment of the nominal is made at maturity only. If no additional premium were charged to the customer, then the total profit (or loss) Y over the term of the contract would be

$$Y = K \sum_{t=1}^{T} \left[\min\left(Z_t, C\right) - Z_t\right]. \tag{1}$$

For $u \leq t$, let $\mathcal{F}_{u,t}$ be the $\sigma-$algebra generated by $Z_s(u \leq s \leq t)$. A fair total price for the contract, given the observations Z_s $(t_o - n + 1 \leq s \leq t_o)$, is

$$\mu_\alpha = K\{\alpha T + \sum_{t=1}^{T} [\min(Z_{t_o+t}, C) - Z_{t_o+t}]\} \tag{2}$$

where

$$\alpha = -\frac{1}{KT} E\left(Y \mid \mathcal{F}_{t_o-n+1,t_o}\right). \tag{3}$$

Thus, we define a fair cap-interest-rate

$$Z_{t,\alpha} = R_t + \alpha \tag{4}$$

where α is the fair premium defined by

$$\alpha(\mathcal{F}_{t_o-n+1,t_o}, C, T) = -\frac{1}{T}\sum_{t=1}^{T}\{E[\min(Z_{t_o+t}, C)|\mathcal{F}_{t_o-n+1,t_o}] - E[Z_{t_o+t}|\mathcal{F}_{t_o-n+1,t_o}]\}, \tag{5}$$

The premium α depends on the past behaviour of the interest rates Z_t, on the cap-level C, and the maturity T. Note, in particular, that $\mathcal{F}_{t_o-n+1,t_o}$ may contain more information than just the last observed value Z_{t_o}.

3 A model for mortgage rates based on renewal processes

3.1 Qualitative features of observed mortgage rates

The definition of the model proposed below is motivated by essential qualitative properties of observed data, as illustrated by the following example: Figure 1a shows the quarterly middle rates Z_t for new mortgages of the Swiss Union of Raiffeisenbanks for the period 1970 to 2001 ($n = 125$). Quarterly data are of particular importance, since most financial institutions charge the cost of a cap insurance for interest rates every three months. Moreover, most floating-rate contracts include the possibility of quarterly adjustments. The following qualitative features can be observed for this type of data: 1. Z_t is a positive step function, with $\Delta Z_t = Z_t - Z_{t-1}$ assuming only a finite number of possible values (multiples of 0.25%); 2. ΔZ_t is mostly zero; 3. there are long stretches where Z_t remains constant; 4. time is divided in long periods where Z_t is monotonically nondecreasing and in periods where it is monotonically nonincreasing; 5. the conditional distribution of $|\Delta Z_t|$ given $\{\Delta Z_t \geq 0\}$ may differ from the conditional distribution of $|\Delta Z_t|$ given $\{\Delta Z_t \leq 0\}$.

3.2 Definition of the process

The following assumptions will be used: U is a random variable with $P(U = -1) = P(U = 1) = \frac{1}{2}$, $W_j > 0$ are independent random variables with distribution F_W on $\{w : w = j,\ j \in N\}$ and such that $P(0 < W_j < \infty) = 1$. Furthermore $S_o = 0$ and

$$S_i = \sum_{j=1}^{i} W_j \ (i = 1, 2, ...). \tag{6}$$

Also, it is assumed that $W_j (j \in N)$ are independent of U. Note that S_i $(i = 1, 2, ...)$ is a recurrent periodic renewal process with period 1 and positive waiting times. Furthermore, define $M_t = \max\{j : S_j \leq t\}$ and let F_1 and F_2 be distribution functions on $\{0, 1, ..., k\}$ for some fixed $k \in N$ and $p_1(i) = F_1(i) - F_1(i-1)$, $p_2(i) = F_2(i) - F_2(i-1)$ the corresponding probabilities. A simple switching random walk type process can now be defined as follows:

Definition: *Let* $Z_o = z_o$, $I_t = (-1)^{M_t - 1} U$, *define*

$$Z_t - Z_{t-1} = (-1)^{j-1} U A_t \tag{7}$$

where $A_1, A_2, ...$ *is a sequence of random variables, independent of* $W_j (j \in N)$ and U, *such that*

$$P(A_t = i | \mathcal{B}_{t-1} \cap \{S_{M_{t-1}} = t-1\}) = 1\{I_t = 1\} p_1(i|A > 0) + 1\{I_t = -1\} p_2(i|A > 0) \tag{8}$$

and

$$P(A_t = i | \mathcal{B}_{t-1} \cap \{S_{M_{t-1}} < t-1\}) = 1\{I_t = 1\} p_1(i) + 1\{I_t = -1\} p_2(i) \tag{9}$$

where by $\mathcal{B}_t$ the $\sigma-$algebra generated by U, A_r $(r \leq t)$ and M_t, and

Remarks:

1. The process S_t divides the time axis in periods where Z_t is monotonically nondecreasing or nonincreasing respectively. At a given time point t, Z_t is in the period number $M_t = \min\{j : t \leq S_j\}$. The value of $I_t = (-1)^{M_t - 1} U$ determines which of the two types of periods we are in and $W_{M_t} = S_{M_t} - S_{M_{t-1}}$ determines how long this period is. Note that, since S_t is a recurrent periodic renewal process, the asymptotically expected number of changes

between upward and downward periods in a time interval $[s,t]$ is equal to $(t-s)\mu_W^{-1}$ where $\mu_W = E(W)$ (see e.g. Cinlar 1975).

2. The definition implies that, for $S_{j-1}+1 \leq t \leq S_j$,

$$Z_t = z_o + U[\sum_{i=1}^{j-1}(-1)^{i-1}\sum_{r=1}^{W_i} A_{S_{i-1}+r} + (-1)^{j-1}\sum_{s=1}^{t-S_{j-1}} A_{S_j+s}]$$

3. For $t = S_{j-1}+1$, we have $A_t > 0$ with probability one. This condition is needed in order that S_j ($j \in N$) can be reconstructed uniquely from an observed series Z_t ($t \in N$).

4. The process Z_t may become negative. A simple modification that avoids this problem can be made as follows:

$$P(A_t = i|\mathcal{B}_{t-1} \cap \{S_{M_{t-1}} = t-1\}) =$$
$$1\{I_t = 1\}p_1(i|A > 0) + 1\{I_t = -1, Z_{t-1} > 0\}p_2(i|0 < A \leq Z_{t-1})$$
$$+1\{A_t = 0, I_t = -1, Z_{t-1} = 0\}$$

and

$$P(A_t = i|\mathcal{B}_{t-1} \cap \{S_{M_{t-1}} < t-1\})$$
$$= 1\{I_t = 1\}p_1(i) + 1\{I_t = -1\}p_2(i|A \leq Z_{t-1}).$$

5. If the observed interest rate changes in steps that are multiples of a fixed step size d, then $\widetilde{Z}_t = d \cdot Z_t$ is used.

6. Observation of the process Z_t ($t \in N$) may start an arbitrary time point which does not necessarilty coincide with the beginning of a period. Similarily, the last observation may not be at the end of a period. This means that W_1 and W_{M_n} cannot be reconstructed exactly from the observed values w_1, w_{M_n}. Instead, the observed information consists of the events $\{W_1 \geq w_1\}$ and $\{W_{M_n} \geq w_{M_n}\}$.

4 Maximum likelihood estimation

4.1 General maximum likelihood equation

Consider Z_t in definition 1. Suppose that $F_W(x) = F_w(x;\eta)$, $F_1(x) = F_1(x;\tau)$ and $F_2(x) = F_2(x;\zeta)$ are characterized by finite dimensional parameter vectors

$\eta = (\eta_1, ..., \eta_p)$, $\tau = (\tau_1, ..., \tau_q)$, and $\zeta = (\zeta_1, ..., \zeta_r)$. For an observed series $Z_1, ..., Z_n$, the unknown parameter vector $\theta = (\eta, \tau, \zeta)^t$ can be estimated by maximimizing the likelihood function. For simplicity, we assume $Z_2 - Z_1 \neq 0$, and write $U = sign(A_2)$. The quantities U, $w_1, W_2, ..., W_{M_n-1}, w_{M_n}$ and $A_2, ..., A_n$ can be obtained from $Z_1, ..., Z_n$ by

$$A_t = |\Delta Z_t| = |Z_t - Z_{t-1}|,\ U = sign(A_2),$$

$$w_1 = \min\{t : sign(\Delta Z_t) \neq U\} - 1,$$

$$S_1^* = w_1, S_{M_n}^* = n,$$

$$S_j = \min\{t : S_{j-1} + 1 \leq t, sign(\Delta Z_t) \neq (-1)^{j-1} U\}\ (2 \leq j \leq M_n - 1),$$

$$W_j = S_j - \ S_{j-1}\ (2 \leq j \leq M_n - 1),$$

$$w_{M_n} = n - S_{M_n - 1}^*.$$

The conditional likelihood function, given $Z_1 = z_1$, then follows directly from definition 1. For instance, if $U = 1$, the loglikelihood function is equal to

$$L(\theta) = \log\{1 - F_W(w_1; \eta)\} + \log\{1 - F_W(w_{M_n}; \eta)\} \tag{10}$$

$$+ \sum_{j=2}^{M_n - 1} \log p_W(w_j; \eta) + \sum_{t:I_t=1} \log p_1(a_j; \tau) + \sum_{t:I_t=-1} \log p_2(a_j; \zeta)$$

where a_t are the observed values of A_t.

Remarks:

1. For the modified model with $X_t \geq 0$, $p_1(a_t; \tau)$ and $p_1(a_t; \zeta)$ have to be replaced by the corresponding conditional probabilities, unless $\min\{Z_t : I_t = -1\} \geq k$.

2. For $n \to \infty$, the contributions of w_1 and w_{M_n} are negligible. Note that, in contrast to censored data as they occur in survival analysis, omitting the contribution of W_1 and W_{M_n} does induce a bias in the estimation of θ.

4.2 Maximum likelihood estimation in the case of a Poisson renewal process

Suppose that W_j are iid Poisson distributed with intensity η, $p_1(i) = \tau_i$ $(i = 1, ..., k)$, $p_1(0) = 1 - \sum_{i=1}^k \tau_i$ and $p_2(i) = \zeta_i$ $(i = 1, ..., k)$, $p_2(0) = 1 - \sum_{i=1}^k \zeta_i$.

Omitting w_1 and w_{M_n} yields the following approximate explicit formulae for $\hat{\theta}$:

$$\hat{\eta} = \frac{\sum_{i=2}^{M_n-1} w_i}{M_n - 2} \tag{11}$$

$$\widehat{\tau}_i = \frac{1}{n_1} \sum_{I_t=1}^{n} 1\{|\Delta Z_t| = i\},\ \hat{\zeta}_i = \frac{1}{n_2} \sum_{I_t=-1}^{n} 1\{|\Delta Z_t| = i\}, \tag{12}$$

where $n_1 = \sum_{t=1}^{n} 1\{I_t = 1\}$ and $n_2 = n - n_1$.

Applying these estimates to the observed series in figure 1a, we obtain $\hat{\eta} = 19.6$, $\hat{\tau} = (0.060, 0.140, 0.040, 0.040)$ and $\hat{\zeta} = (0.149, 0.108, 0.0135, 0.0135)$.

5 Pricing by simulated predictions

5.1 Prediction of future interest rates and conditional pricing

Given observations $Z_1, ..., Z_n$, the premium α is obtained by estimating the conditional expected values $E[\min(Z_{n+t}, C)|\mathcal{F}_{1,n}]$ and $E[Z_{n+t}|\mathcal{F}_{1,n}]$. Moreover, in order to assess the risk of the contract, the distribution or at least certain extreme quantiles of Z_t $(t = n+1, ..., n+T)$ and of the loss $Y = K\sum_{t=1}^{T}[\min(Z_{n+t}, C) - Z_{n+t}]$ need to be estimated. This is done in two steps: 1. maximum likelihood estimation of θ; and 2. simulation of future sample paths $Z_{n+1}, ..., Z_{n+T}$, *conditionally* on $\mathcal{F}_{1,n}$. Since only a lower bound for the length of the last period W_{M_n} is known, the conditional distribution of Z_{n+k} is given by

$$P(Z_{n+k} = z|\mathcal{F}_{1,n}) = \sum_{i=w_{M_n}}^{\infty} P(W = i|W \geq w_{M_n})P(Z_{n+k} = z|\mathcal{F}_{1,n} \cap \{W_{M_n} = i\}). \tag{13}$$

Note that conditioning on all information is essential in oder to obtain a realistic assesment of the future distribution of Z_t (and Y). For instance, if $Z_n - Z_{n-1} > 0$ and W_{M_n} is relatively small, then it is quite likely that Z_t will increase in the near future. The reason is that the conditional probability $P(W > w_{M_n}|W \geq w_{M_n})$ is large. This results in a relatively high premium α. In contrast, if $Z_n - Z_{n-1} < 0$ and W_{M_n} is small, then a lower premium can be charged, since Z_t is unlikely to increase (much) in the near future.

Figures 1b and c illustrate the performance of the model for the Raiffeisenbank mortgage rates. The parameters were estimated from the first 70 observations. Figure 1b shows observations 71 to 102 (corresponding to a period of 8 years) and 10 sample paths simulated conditionally on the first 70 observations - after θ has been estimated from the first 70 observations. Note that the predicted sample paths exhibit a nonlinear behaviour that strongly ressembles the shape of the actual future observations. Figure 1c shows the predicted medians and the 2.5%- and 97.5%-quantiles obtained from 1000 simulated sample paths. The actually observed values are all in the 95% prediction interval, with the exception of a few unexpectedly low values at the beginning. Overall, the median and quantile curves provide a realistic prediction of the future S-shaped up and down movement.

5.2 Simulated prices for contracts with fixed cap

To illustrate the proposed pricing method, consider a cap-contract signed at time t_o, with $T = 12$ and a conditional cap defined by $C = Z_{t_o} + x$ where x is fixed. For the observed interest rate series (figure 1a), the following calculations were made:

1. For $t_o = 70, 71, ..., 109$, θ is estimated from the last 70 observations Z_{t_o-69}, $Z_{t_o-68}, ..., Z_{t_o}$. Thus, we obtain 40 estimates $\widehat{\theta}(t_o)$.
2. For each $70 \leq t_o \leq 109$, four hundred series $[\widetilde{Z}_{t_o+1}\{i; \widehat{\theta}(t_o)\}, ..., \widetilde{Z}_{t_o+T}\{i; \widehat{\theta}(t_o)\}]$ $(i = 1, ..., 400)$ and the corresponding values of Y are simulated, conditionally on $Z_{t_o-69}, ..., Z_{t_o}$, using $\widehat{\theta} = \widehat{\theta}(t_o)$. The simulated value of $\alpha(t_o, x)$ for a contract starting at time $t_o + 1$,
$$\alpha(t_o, x) = -\frac{1}{400KT} \sum_{i=1}^{400} \widetilde{Y}_i,$$
is calculated, where $\widetilde{Y}_i = K \sum_{t=1}^{T} [\min\left(\widetilde{Z}_{t_o+t}, C_{t_o,x}\right) - \widetilde{Z}_{t_o+t}]$, $K = 1$ and $C_{t_o,x} = Z_{t_o} + x$. Moreover, the actually observed loss, if no additional premium is charged, $Y(t_o, x) = \sum_{t=1}^{T} [\min(Z_{t_o+t}, C_{t_o,x}) - Z_{t_o+t}]$ is calculated. Figures 2a and 2b display $Y(t_o, x)$ and $\alpha(t_o, x)$ respectively, plotted agains $t_o = 70, ..., 109$, for $x = 0.25, 0.50, ..., 4.0$. In both pictures, the upper most line corresponds to $x = 0.25$ and the lowest to $x = 4.0$, since

the observed and expected loss increase with decreasing cap. A direct comparison of $Y(t_o, x)$ with $\alpha(t_o, x)$ is given in figures 2c and d. Figure 2c displays the loss $Y(t_o, x) - \alpha(t_o, x)$ plotted agains time for $0.25 \leq x \leq 4$. The overall observed average loss (Figure 2d) due to contracts (with the pricing policy described here) that started in the period $70 \leq t_o \leq 109$ is negative for all values of x, i.e. the bank would have made a slight profit with any choice of x.

5.3 Simulated prices for contracts with variable cap

In the first quarter of the period considered in figures 2a to d, the observed loss rises far above its expected value (figure 2c) but remains zero for the rest of the time. This is typical for cap contracts with a fixed cap: occasional exreme losses are compensated by long periods where the interest rate remains below the cap rate. The reason is that variability and thus uncertainty increases for longer term forecasts. A financial institution offering longer term contracts with a fixed cap must have enough reserves in order to survive the possibly extreme temporary losses. An alternative that avoids extreme losses is to offer contracts where the cap is adjusted on a regular (e.g. quarterly) basis. In the extreme case, we have adjustments of α and C at every time point t. The premium $\alpha(t)$ is then simply a one-step-ahead forecast of $\min(Z_t, C_t) - Z_t$. The overall expected loss (per time unit) is then equal to

$$L(t_o) = \frac{1}{T} \sum_{t=t_o+1}^{t_o+T} \alpha(\mathcal{F}_{t_o-n,t-1}, C_t) \tag{14}$$

where n =number of observations and

$$\alpha(\mathcal{F}_{t_o-n,t-1}, C_t) = E[\min(Z_t, C_t) - Z_t | \mathcal{F}_{t_o-n,t-1}]. \tag{15}$$

An application of this contract is shown in figures 3a and b, with $C_t = C_{t,x} = Z_t + x$ where x is fixed at the beginning of the contract. Figure 3a displays estimated values of $\alpha(\mathcal{F}_{t_o-n,t-1}, C_{t,x})$ for $t = 71, ..., 122$. Note that, since the new cap rate is adapted at each time point, the largest possible loss is equal to the maximal possible increase of 1%. Therefore, only the values $x = 0.25, 0.5$ and 0.75 are meaningful. The observed losses for contracts starting at time points $71 \leq t \leq 109$, and $x = 0.25, 0.50$ and 0.75 are displayed in figure 3b. In this period, no losses would have occured for any of the values of x, except for one slight loss for $x = 0.75$ at $t = 79$. This is so, in spite of the small values of α.

In conclusion, cap pricing can involve considerable risk (occasional large losses) if the cap is fixed throughout the term of the contract. The more flexible the adjustment scheme, the less risk is involved and the premium can be kept relatively low - compared to the current interest rate. This is also illustrated in figure 4 with boxplots of α for fixed rate contracts (with $x = 1, 2, 3$) on one hand and the values of $\alpha(\mathcal{F}_{t_o-n,t-1}, Z_{t-1} + x)$ for $x = 0.25, 0.5, 0.75$.

6 Final Remarks

In this paper, a simple model was introduced to model political mortgage rates without any additional explanatory information. The model can be used to calculate fair cap interest rates and to assess the risk of cap insurance contracts. In spite of the simple structure of the model, predictions appear to yield realistic results when applied to observed interest rates. In particular, future nonlinear behaviour can be predicted. This is in contrast to models that combine deterministic trends with linear stochastic components in an additive way (see e.g. Beran and Ocker 1999, Beran and Feng 2002a,b). The model can also provide an alternative explanation to the long-memory phenomenon in interest rate series reported in the literature (see e.g. Tkacz 2001; for references on fractional ARIMA and other long-memory models see also Granger and Joyeux 1980, Hosking 1981, Beran 1994, Beran et al. 1997 and references therein). The apparent long memory (or fractional integration) may be due to switching regimes rather than fractional integration. For similar comments on long memory versus switching regimes see e.g. Mikosch and Starica (2000).

Finally, note that the accuracy of forecasts may be enhanced further by including additional explanatory variables and by suitable modelling of term structure. How and which type of information should be included in the model will need to be looked at in future research.

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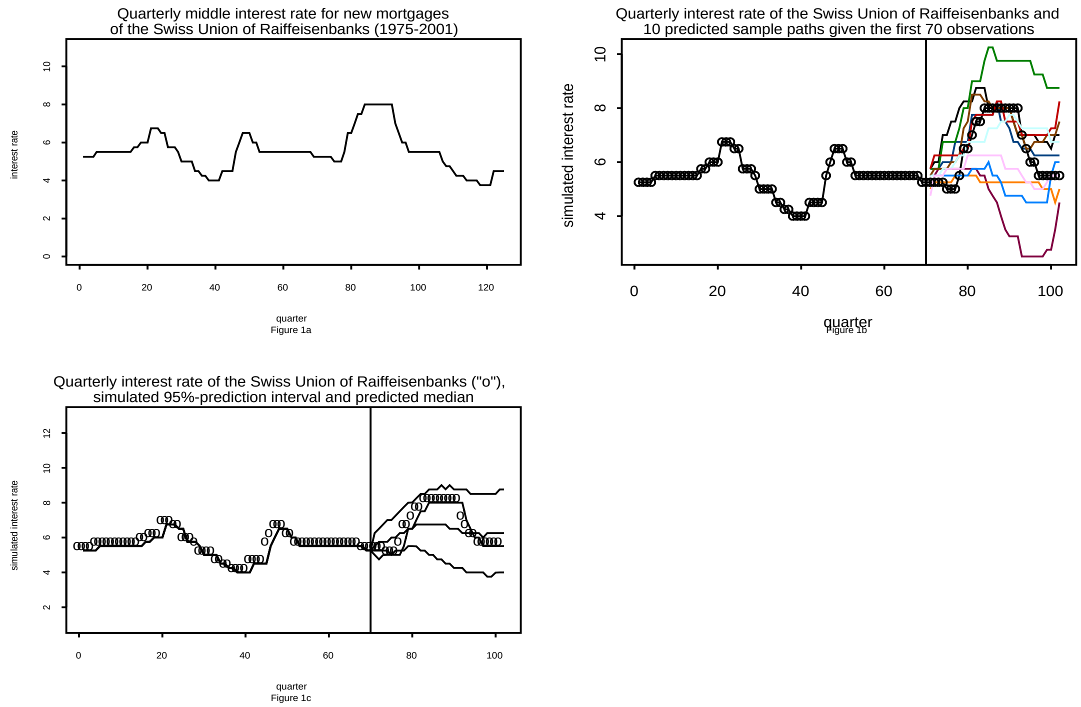

Figure 1: Quarterly interest rates for new mortgages by the Swiss Union of Raiffeisenbanks (1975-2001), predicted sample paths (figure 1b) and predicted 2.5%-, 50%- and 97.5%-quantiles (figure 1c).

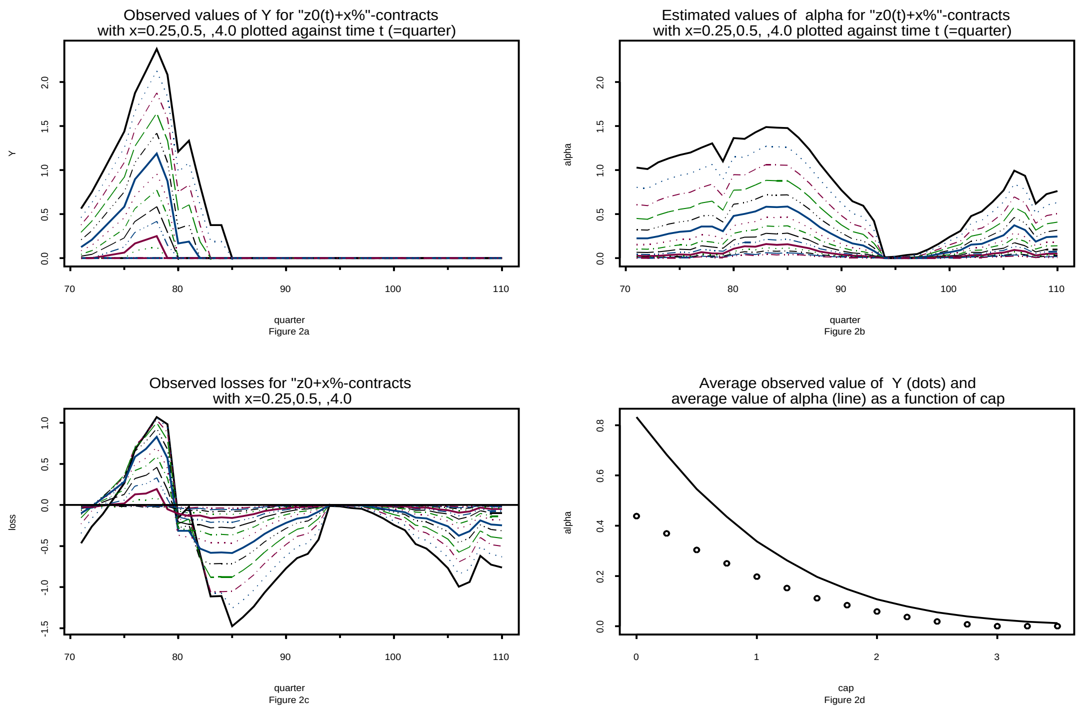

Figure 2: Simulated values of $\overline{Y}(t_o, x)$ and $\widehat{\alpha}(t_o, x)$ respectively (figures 2a,b), plotted agains $t_o = 70, ..., 109$, for $x = 0.25, 0.50, ..., 4.0$, loss $\overline{Y}(t_o, x) - \widehat{\alpha}(t_o, x)$ plotted agains time for $0.25 \leq x \leq 4$ (figure 2c) and overall observed average loss (figure 2d) due to contracts that started in the period $70 \leq t_o \leq 109$.

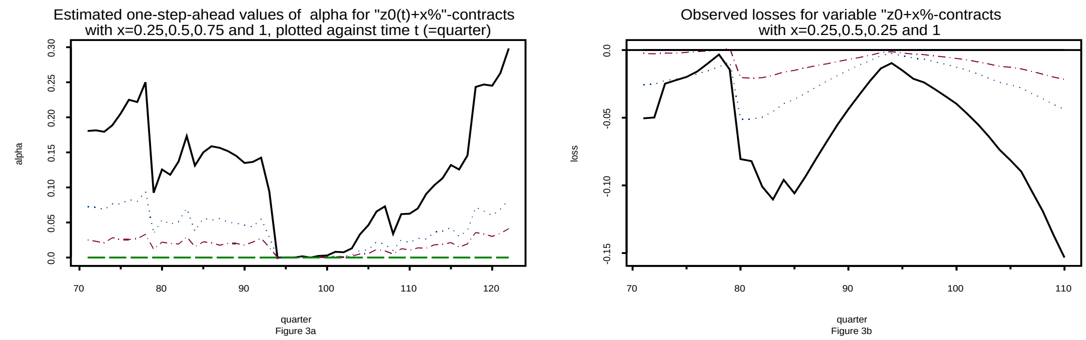

Figure 3: Estimated values of $\alpha(\mathcal{F}_{t_o-n,t-1}, C)$ for $t = 71, ..., 122$ (figure 3a), and observed losses for contracts starting at time points $71 \leq t \leq 109$, and $x = 0.25, 0.5, 0.75$ and 1 (figure 3b).

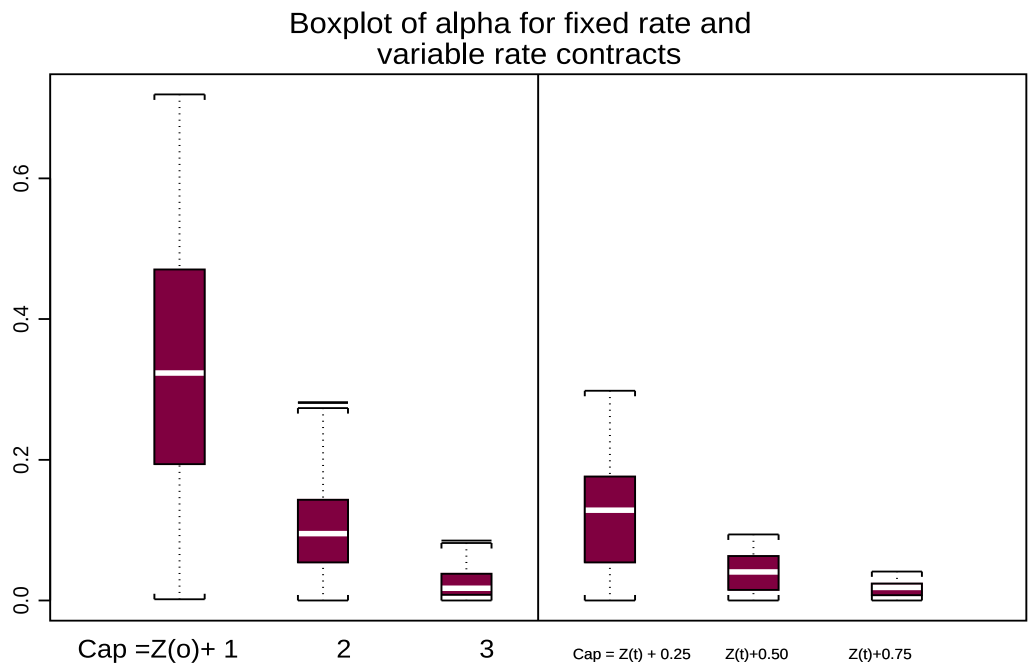

Figure 4: Boxplots of α for fixed rate contracts (with $x = 1, 2, 3$) (left) and the values of $\alpha(\mathcal{F}_{t_o-n,t-1}, Z_{t-1} + x)$ (right) for $x = 0.25, 0.5, 0.75$.