Diskussionspapiere der DFG-Forschergruppe (Nr.: 3468269275):

Heterogene Arbeit: Positive und Normative Aspekte der Qualifikationsstruktur der Arbeit

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Competitive Screening in Insurance Markets with Endogenous Labor Supply

April 2006
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Abstract:
We examine equilibria in the sense of Rothschild and Stiglitz (1976) in competitive insurance markets when individuals take unobservable labor supply decisions. Precautionary labor motives introduce countervailing incentives in the insurance market, and imperfect type separation can occur in the standard case in which individuals differ only in risk. We then extend the model to allow for both unobservable risks and labor productivities. Under these circumstances, both imperfect risk separation and genuine pooling of different risk-productivity types can arise. We show that such equilibria, with endogenous income heterogeneity, generally differ from those under exogenous income heterogeneity analyzed by Smart (2000) and Wambach (2000). We provide necessary and sufficient equilibrium existence conditions.

JEL Classification: D82, G22, J22
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April 24, 2006

Abstract

We examine equilibria in the sense of Rothschild and Stiglitz (1976) in competitive insurance markets when individuals take unobservable labor supply decisions. Precautionary labor motives introduce countervailing incentives in the insurance market, and imperfect type separation can occur in the standard case in which individuals differ only in risk. We then extend the model to allow for both unobservable risks and labor productivities. Under these circumstances, both imperfect risk separation and genuine pooling of different risk-productivity types can arise. We show that such equilibria, with endogenous income heterogeneity, generally differ from those under exogenous income heterogeneity analyzed by Smart (2000) and Wambach (2000). We provide necessary and sufficient equilibrium existence conditions.

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1 INTRODUCTION

In the standard screening model going back to Rothschild and Stiglitz (1976), individuals differ only in a single dimension, namely their risk of incurring a loss, and the choice of an insurance contract is their only action explained endogenously. In this simple framework, insurance companies can typically induce customers to fully reveal their private information by offering contracts that separate the risk types. In reality, however, individuals are heterogeneous in other dimensions than risk and make choices that cannot be observed by insurance companies. We focus on the case that individuals differ in their productivities and choose their labor supply optimally, which is particularly relevant for normative questions concerning, for instance, optimal taxation or social insurance. When this is accounted for, various interesting economic effects emerge that are ignored in the canonical model. First, optimal labor supply reacts to the level of uncertainty and thus depends on the insurance market outcome. On the other hand, the endogeneity of labor supply introduces countervailing incentives in the insurance market as the individuals’ marginal willingness to pay for insurance is influenced not only by their risk, but also by their labor income. In this paper, we demonstrate how the resulting interactions between labor and insurance markets affect insurers’ ability to screen their customers.

Models of adverse selection in competitive insurance markets in which individuals differ in more than one private characteristic have been examined by Smart (2000) and Wambach (2000). They assume that insurance customers differ in wealth in addition to risk, which introduces a second dimension of exogenous heterogeneity. In that case, individual preferences satisfy standard monotonicity properties in each of the two dimensions. Countervailing incentives and thus deviations from perfect risk separation emerge in these models only if individuals differ in both characteristics such that the resulting effects work in opposite directions. We show that it makes a difference whether heterogeneity is given exogenously in all dimensions or whether some dimensions are determined by the individuals’ endogenous choices. Even when individuals differ only in risk but not in productivity, labor supply decisions in reaction to risk introduce a second dimension of heterogeneity. Hence countervailing incentives and imperfect separation can occur already in this case. In addition, the nature of equilibria under both risk and productivity heterogeneity is generally different from the case of varying exogenous wealth levels. This demonstrates that the models with exogenous differences in wealth are not good representations for a world in which such endowments arise endogenously from choosing labor supply
under uncertainty.

The structure of this paper is as follows. In section 2, fundamental results on labor supply under uncertainty are derived. We demonstrate that, under broad and meaningful assumptions, there is a motive for precautionary labor, i.e. individuals work more in response to increases in risk. After having introduced the model of the insurance market in section 3, we first examine the resulting equilibria if there is only one-dimensional heterogeneity and individuals differ only in risk, not in productivity. As will be shown in section 4, the endogeneity of labor supply may make perfect screening impossible even in this simple framework. We then proceed to the two-dimensional case in section 5. On the one hand, this aims at demonstrating how the results of Smart (2000) and Wambach (2000) are modified when the second dimension of heterogeneity is not imposed exogenously but arises from endogenous labor supply. On the other hand, this provides an important tool for models of taxation and social insurance, in which both individual risks and productivities are typically assumed to be unobservable. In addition to imperfect separation as identified in the one-dimensional problem, genuine pooling equilibria are possible under these circumstances. In such equilibria, several different risk-productivity types will purchase the same contract. Finally, section 6 concludes and provides a research outlook. Some of the proofs and propositions are relegated to the appendix.

2 Labor Supply under Uncertainty

In order to introduce endogenous labor supply in the standard adverse selection selection model by Rothschild and Stiglitz (1976), we first need to derive some results on labor supply under uncertainty. Notably, we focus on situations in which uncertainty results from an income independent risk to consumption and labor supply is chosen before this risk is realized.\footnote{Eaton and Rosen (1980) considere the case of endogenous labor with wage uncertainty, which gives rise to different effects.} This problem was first examined by Netzer and Scheuer (2005) who used the insights of Kimball (1990) to establish a theory of precautionary labor. We briefly discuss their results in this section.

Consider a Bernoulli random variable $\tilde{\theta}(\beta)$ that results from a possible damage $D$, which occurs with probability $p$ and where the parameter $\beta \in [0, 1]$ stands for the share of the damage that is insured. It can be used to vary both expected value $E[\tilde{\theta}(\beta)] = p (1 - \beta)D$ and variance $\text{Var}[\tilde{\theta}(\beta)] = p (1 - p) [(1 - \beta)D]^2$ of the risk.
Preferences are characterized by an additively separable utility function \( U(c, L) = u(c) + v(L) \) where \( c \) denotes consumption and \( L \) denotes labor supply.\(^2\) The standard conditions \( u'(c) > 0, \quad u''(c) < 0, \quad v'(L) < 0 \) and \( v''(L) < 0 \) are assumed. The productivity of an individual is denoted by \( w \). Firms can observe \( w \) and pay wages according to marginal productivity such that earned income is \( wL \). Individuals have an additional, exogenous and state independent income \( T \).

Note that the separability of preferences implies that leisure is a normal good. In addition, let us assume the following:

**Assumption 1.** Individual utility functions \( u(c) \) display non-increasing absolute risk-aversion, and consumption is a normal good.

The first part of Assumption 1 is not particularly restrictive since increasing risk-aversion is not generally viewed as a realistic property of preferences. The second part puts an upper bound on income effects on labor supply. Both assumptions are needed to obtain clear-cut results on the direction of precautionary labor effects in the following.

The first-order condition for labor supply \( L^* \) that maximizes expected utility in the presence of a given consumption risk \( \tilde{\theta}(\beta_0) \) is

\[
wp E\left[u'(wL^* + T - \tilde{\theta}(\beta_0))\right] = -v'(L^*),
\]

where \( E \) is the expectations operator. \(^3\) To answer the question how risk affects labor supply, we examine the move from \( \tilde{\theta}(\beta_0) \) to the risk \( \tilde{\theta}(\beta) + (\beta - \beta_0)pD \), which constitutes a change in variance, leaving the expected value unaffected. Define the corresponding *equivalent precautionary premium* \( \Psi(\beta_0, \beta) \) for such a move implicitly as follows:\(^4\)

\[
E\left[u'(wL^* + T - \tilde{\theta}(\beta_0) - \Psi(\beta_0, \beta))\right] = E\left[u'(wL^* + T - \tilde{\theta}(\beta) - (\beta - \beta_0)pD)\right].
\]

Its interpretation is as follows. The expectation-neutral change in risk will have the same effect on the LHS of (1) and therefore on labor supply as a lump-sum reduction.

\(^2\)We need the assumption of separability only to keep the exposition of our labor supply theory concise. As shown by Kimball (1990), the results can be transferred to the case of nonseparable utility. We assume the function \( v \) to be at least twice, \( u \) at least three times differentiable.

\(^3\)The sufficient second order condition for a maximum is satisfied.

\(^4\)As shown by Kimball (1990), the discussed premium is simply the equivalent risk premium developed by Pratt (1964), applied to the first derivative of \( u \).
of income by $\Psi(\beta_0, \beta)$. Both changes affect the optimality condition in the same way. Therefore, statements about the adjustment of labor supply induced by a change of risk can be restated as income effects triggered by a decrease of income by $\Psi$.

Using the moments of the Bernoulli distribution, we can obtain an explicit expression for $\partial \Psi(\beta_0, \beta) / \partial \beta$ by differentiating (2). Notably, we are interested in the value of this derivative at $\beta = \beta_0$, which gives the income change that would have the same effect on labor supply as a small change in insurance, starting from a situation with insurance $\beta_0$. We obtain after a few rearrangements

$$\frac{\partial \Psi(\beta_0, \beta)}{\partial \beta} \bigg|_{\beta = \beta_0} = \left( -\frac{\Delta u''(\cdot)}{(1 - \beta_0)D} \right) \left( \frac{1}{2} \frac{\partial \text{Var}}{\partial \beta} \right),$$

where $\Delta u''(\cdot)$ stands for the difference of $u''(\cdot)$ between consumption levels in case of no damage and damage, and $E[u''(\cdot)]$ is the expected value of $u''(\cdot)$.

The first bracketed term on the RHS of (3) is the generalized coefficient of absolute prudence $\eta^G$. As $\beta_0$ converges to 1, i.e. the examined situation converges to a situation without risk, the coefficient $\eta^G$ converges to the prudence $\eta$ as defined by Kimball (1990), which is simply the coefficient of absolute risk aversion for the function $u'(\cdot)$, i.e. $\eta(c) = -u'''(c)/u''(c)$. From (3) follow first implications for labor supply under uncertainty. First, note that a sufficient condition for $\eta^G$ to be positive is that $u'''(\cdot) > 0$. This, in turn, is implied by non-increasing risk aversion and hence by Assumption 1. An increase in insurance coverage $\beta$ (compensated for its effect on expected damage) will therefore have the same effect on labor supply as an increase in income. Given that leisure is a normal good, this increases the demand for leisure and decreases labor supply. Conversely, larger labor supply will be the reaction to higher risk. The individual hence has a motive for precautionary labor, which could also be referred to as self-insurance. The size of $\eta^G$ indicates how strong this motive is.

So far, changes in risk were considered that left the expected damage unaffected. If this is not the case, changes in risk entail additional income effects. It is still useful to distinguish between pure risk effects via the variance and income effects via the expected value. As was shown by Netzer and Scheuer (2005), the effect of marginal increases in $\beta$ on labor supply can be decomposed as

$$\frac{\partial L^*}{\partial \beta} = \frac{\partial L^*}{\partial T} \left[ pD - \frac{\partial \Psi}{\partial \beta} \right],$$

The derivation makes use of the fact that $\Psi(\beta_0, \beta_0) = 0$.\footnote{The derivation makes use of the fact that $\Psi(\beta_0, \beta_0) = 0$.}
where $\partial L^*/\partial T$ denotes the negative income effect and $\partial \Psi / \partial \beta$ stands short for the expression (3). First, higher coverage increases expected income by $pD$. This effect would vanish if an insurance premium was adjusted actuarially fairly. Second, the change in the variance has the same effect as a decrease of income by the premium $\Psi$ that is raised by an increased insurance coverage $\beta$. By Assumption 1, this premium will be negative so that a lower variance through larger insurance coverage reduces labor supply.

3 The Model

Consider a society of individuals characterized by their productivity $w_i$, $i = L, H$, and probability $p_j$, $j = L, H$, of incurring a damage $D$ with the conventions $w_L < w_H$ and $p_L < p_H$. Let $n_{ij}$ denote the share of individuals with productivity $w_i$ and risk $p_j$ in the population. Individuals are offered insurance contracts specifying the share $\beta$ of the damage that is covered and the respective premium $d$. With this, consumption in case of loss is $c^0_{ij} = w_iL^*_ij + T - (1 - \beta)D - d$ and $c^1_{ij} = w_iL^*_ij + T - d$ otherwise. Labor supply $L^*_ij$ is chosen optimally according to the condition

$$p_ju'(c^0_{ij}) + (1 - p_j)u'(c^1_{ij}) = -v'(L^*_ij)/w_i.$$  

(5)

Note that, by (5), $L^*_ij$ is a function of $T$, $\beta$, $d$ and the other parameters and will therefore depend on the insurance contract. Substitution of $L^*_ij(T, \beta, d)$ into the expected utility function yields the indirect expected utility function $V_{ij}(T, \beta, d)$.

When considering an individual’s preferences in the $(\beta, d)$-space, we need to account for changes in labor supply and thus consumption levels as we move along an indifference curve. This introduces effects that may alter the shape and crossing properties of indifference curves compared to the canonical model by Rothschild and Stiglitz (1976). First, consider the slope of an indifference curve of an individual with productivity $w_i$ and risk $p_j$:

$$MRS_{ij} = \left. \frac{dd}{d\beta} \right|_{V_{ij}=\bar{V}} = \frac{Dp_ju'(c^0_{ij})}{p_ju'(c^0_{ij}) + (1 - p_j)u'(c^1_{ij})} > 0,$$

(6)

\footnote{Throughout the rest of the paper, we stick to this convention. $\partial \Psi / \partial \beta$ always stands for the partial derivative of $\Psi(\beta_0, \beta)$ with respect to $\beta$, evaluated at $\beta_0 = \beta$, and therefore captures the pure variance effect of additional insurance on labor supply.}

\footnote{Note that indifference curves are still continuously differentiable because optimal labor supply is a continuously differentiable function of $\beta$ and $d$.}
which is positive as in the standard model. However, while the curvature of indifference curves in the \((\beta, d)\)-space is always concave in the model with exogenous income, this does not necessarily hold when labor supply is endogenous. If an increase in insurance along an indifference curve leads to a strong reduction in labor supply, consumption may decrease so much that the individual actually has a higher marginal willingness to pay for insurance given decreasing risk aversion. This would imply indifference curves that are not globally concave and complicate our equilibrium analysis substantially. In the following lemma, we derive a sufficient condition to exclude this problem.

**Lemma 1.** Indifference curves are concave in the \((\beta, d)\)-space if the labor supply reaction is never so strong that an increase in insurance along the indifference curve leads to lower consumption in the case of damage.

**Proof.** See Appendix A

Obviously, Lemma 1 puts an upper bound on the precautionary labor effect that will be assumed to be satisfied for the rest of this paper.

Apart from the shape of a given individual’s indifference curves, the crossing properties of different individuals’ indifference curves in a given insurance contract are also crucial for the equilibrium outcomes. Let us first ignore productivity differences and consider individuals that only differ in their risk. In the standard adverse selection model where income is exogenous, at any given contract, high risks have a steeper indifference curve than low risks. Put formally, the marginal rate of substitution between coverage and premium given in (6) is increasing in \(p_j\). Clearly, the property immediately follows from (6) if \(L_{ij}^*\) is held fixed. By the following definition, we will refer to this as “regular-crossing” indifference curves.

**Definition 1.** The indifference curves of two individuals that differ in risk exhibit “regular-crossing” at a given contract if the high risk’s indifference curve is not flatter. Otherwise, they exhibit “irregular-crossing”.

Note that Definition 1 introduces a local concept at a given contract. If regular crossing holds in the whole contract space, as it does in the Rothschild-Stiglitz model, it implies the global property of single-crossing for indifference curves of two individuals that differ only in risk.

As was shown by Netzer and Scheuer (2005), the single-crossing property is not necessarily satisfied, however, if there are precautionary labor effects. In this case, high risk individuals supply more labor than low risks at any given contract with
less than full insurance. If this effect is strong, the resulting higher consumption may reduce the high risks’ marginal willingness to pay for insurance below that of the low risks, again due to decreasing risk aversion. The following lemma provides conditions that ensure regular-crossing even if labor supply is endogenous.

**Lemma 2.** The indifference curves of individuals that differ only in risk exhibit regular-crossing at a contract \((\beta, d)\) if either:

(i) the ratio \(p_H/p_L\) is sufficiently large,
(ii) preferences exhibit constant absolute risk-aversion or a sufficiently small degree of decreasing risk-aversion,
(iii) the coefficient of prudence is sufficiently small,
(iv) the contract \((\beta, d)\) provides full coverage.

**Proof.** See Netzer and Scheuer (2005), Appendix D.

If none of the conditions (i) to (iv) is satisfied at a given contract, the indifference curve of a low-risk individual cuts the high-risk individual’s indifference curve from below in the \((\beta, d)\)-space. This possibility is the crucial difference between our model and both the standard screening model by Rothschild and Stiglitz (1976) and the extensions by Smart (2000) and Wambach (2000). Even without two-dimensional heterogeneity, the existence of precautionary labor effects introduces counterveiling incentives in the sense of Lewis and Sappington (1989) in the insurance market that may prevent a simple ordering of the risks with respect to their marginal rate of substitution between coverage and premium.\(^8\)

We next turn to individuals that have the same risk and only differ in their productivities. Lemma 3, proved in the appendix, shows that high-productivity individuals have a flatter indifference curve at any contract with less than full coverage.

**Lemma 3.** The slope of an indifference curve at any contract \((\beta, d)\) with \(\beta < 1\) decreases in productivity if risk-aversion is decreasing. It is independent of productivity if the contract provides full coverage or if preferences exhibit constant absolute risk-aversion.

\(^8\)One could interpret the model as a combination of an adverse selection and a moral hazard model. While the adverse selection aspect is obvious, moral hazard arises from the fact that the insurers cannot observe labor supply (if this were the case, they would be able to infer back to risk). Of course, the insurers’ (principals) objective is quite special in the moral hazard context as it does not directly depend on the individuals’ labor supply. However, the moral hazard component affects the selection problem as it introduces a second dimension of heterogeneity, which may affect incentives in the opposite direction than risk. This is what we refer to as *countervailing incentives*. 
Proof. See Appendix A.

Clearly, this *local* property again implies the *global* property of single-crossing for indifference curves of individuals that differ only in productivity.

While Lemma 3 establishes a single-crossing property in the productivity dimension, indifference curves of two individuals that differ in risk may cut more than once if the conditions of Lemma 2 are not satisfied everywhere. This is of course also possible if individuals differ in both risk and productivity.\(^9\) With precautionary labor effects, we cannot generally exclude the possibility that they cut more than twice. This, however, would require that utility functions exhibit highly irregular patterns, which we shall exclude with the following assumption:

**Assumption 2.** Any two indifference curves of individuals that have different damage probabilities cut at most twice.

This double-crossing assumption reduces the number of possible equilibria as it allows us to establish a relationship with the *local* concepts from Definition 1. Namely, whenever two indifference curves of individuals that differ in risk exhibit regular-crossing at a given contract, they do not cross again anywhere to the northeast in the \((\beta, d)\)-space. Conversely, irregular-crossing at a contract implies that indifference curves do not again cross in the south-west.

As in Rothschild and Stiglitz (1976), there is a large number of risk-neutral insurance companies. Each of them offers a single insurance contract \((\beta, d)\). The expected profit of this contract if it is purchased by a low-risk and \(b\) high-risk individuals is given by

\[
\pi(\beta, d, a, b) = a[d - p_L \beta D] + b[d - p_H \beta D].
\]

The timing of the game is as follows. In stage one, insurers offer one contract each. At stage two, customers select their preferred contract and choose their labor supply. At stage three, risk and payoffs are realized. A set of contracts is an equilibrium if (i) all contracts that are offered earn zero profits in expectation, and (ii) there is no contract outside that set which would earn positive expected profits if offered in addition. A difference between this definition and the concept introduced by Rothschild and Stiglitz (1976) concerns requirement (i). In the canonical model, non-negative profits are required and zero profits arise in equilibrium due to competition between insurance companies. As it will turn out, even perfect competition

\(^9\)However, at points of full insurance, regular-crossing holds independently from productivity.
might not eliminate positive profits in our framework, which would lead to excessive entry of firms. Smart (2000) deals with this problem by assuming the existence of fixed entry costs. This, however, contradicts the assumption of truly perfect competition. A different approach, which is taken here, is to incorporate zero profits into the equilibrium concept. Situations with perfect competition and positive profits are not considered as equilibria in the following. We demonstrate that this assumption does not substantially restrict the set of equilibria but allows us to determine equilibrium mixing probabilities that would be indeterminate otherwise.

Before turning to the explicit equilibrium analysis for this game in sections 4 and 5, we define a number of terms and insurance contracts that will be repeatedly used in the following. First, a “type $ij$” is a particular combination of productivity $w_i$ and risk $p_j$. When we refer to a certain type, we usually mean all individuals in the population with the respective risk and productivity. Using this, Definition 2 clarifies what we understand by “separating” and “pooling” equilibria.

**Definition 2.** An equilibrium set of contracts is “weakly pooling” if there exists a high-risk type such that all contracts purchased by this type are also purchased by low-risk individuals. It is “strictly pooling” if it contains exactly one contract, purchased by all individuals. An equilibrium set of contracts is “weakly separating” if it is not weakly pooling. It is “strictly separating” if it contains no contract that is purchased by different risks.

Note first that this definition categorizes equilibria only with respect to which damage risks purchase which contract. This is because the major interest in terms of the insurance market is how different risks select themselves, or are “screened”. Note second that the focus on high risks for the definition of pooling will prove useful later. Pooling requires all individuals of at least one type $iH$ to be bunched in contracts with low risks. As an opposite example, observe that when some but not all individuals of each high-risk type purchase the same contract as low-risks, we have a situation with weak separation according to Definition 2. Such equilibria, which are weakly but not strictly separating, will sometimes be referred to as “imperfect separation”.

Finally, Definition 3 introduces particular insurance contracts that will be used to characterize equilibria discussed in sections 4 and 5 of the paper.

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10For this reason Smart (2000) examines the case of small fixed costs.
Definition 3. The contracts $A$, $B_m^m$, $m, n = L, H$ and $C(\beta^P, d^P)$ are defined as follows:

\[ A = (1, p_H D), \]
\[ B_m^m = (\beta_m^m, d_m^m) = \arg\max V_{mL}^*(\beta, d) \quad \text{s.t.} \quad \]
\[ (i) \; V_{mH}^*(1, p_H D) = V_{mH}^*(\beta, d), \]
\[ (ii) \; d \geq p_L \beta D, \]
\[ (iii) \; \beta \leq \beta^P. \]

\[ C = (\beta^C, d^C) = \arg\max V_{HL}^*(\beta, d) \quad \text{s.t.} \quad \]
\[ (i) \; V_{HH}^*(\beta^P, d^P) = V_{HH}^*(\beta, d), \]
\[ (ii) \; d \geq p_L \beta D, \]
\[ (iii) \; \beta \leq \beta^P. \]

Contract $A$ provides full coverage at a fair premium for the high risks. Contract $B_m^m$ maximizes the utility of the low risks with productivity $w_n$, subject to a non-negative-profit condition and the constraint that the high risks with productivity $w_m$ obtain the same utility from $A$ and from $B_m^m$.$^{11}$ Contract $C$ maximizes the utility of the $HL$-type subject to a non-negative profit condition and the constraint that the $HH$-type is indifferent between $C$ and some given contract $P = (\beta^P, d^P)$. Furthermore, the contract $C$ may not be larger than $P$. In case constraint $(iii)$ is binding, the contracts $P$ and $C = C(\beta^P, d^P)$ are identical and will not be treated as two different contracts.

4 One-Dimensional Heterogeneity

We assume first that individual productivities are publicly observable. In that case, insurance companies offer contracts conditional on productivity, such that an insurance market for each productivity group $w_i$ emerges.$^{12}$ We concentrate on one such market. As seen in the previous section, indifference curves of the two types in the $w_i$-market can cross regularly (RC) or irregularly (IRC) in a given contract $(\beta, d)$:

\[
\begin{array}{|c|c|c|}
\hline
1 & RC & MRS_{iH} \geq MRS_{iL} \\
\hline
2 & IRC & MRS_{iL} > MRS_{iH} \\
\hline
\end{array}
\]

Table 1: Possible Crossing Properties of Indifference Curves

$^{11}$Using Lemma 3, one can show that $B_m^m \geq B_m^m$, where $\geq$ refers to both elements of the contract vector. This convention will be used throughout the paper. If a contract $A$ is “larger” (“smaller”) than $B$, both coverage and premium are larger (smaller).

$^{12}$The same results obtain if $w_L = w_H$ such there is no heterogeneity with respect to productivity in the society, or if risk-aversion is constant. In that case, productivity has no influence on indifference curves by Lemma 3.
We will proceed as follows. First, we prove general properties of possible equilibria, including restrictions on where in the contract space the respective contracts can be found. More specific properties will then depend on the exact constellations of marginal rates of substitution in the respective area.

4.1 Separating Equilibria

**Proposition 1.** If productivities are observable, equilibrium in the $w_i$-market, if it exists, consists of two contracts: contract $A$ and contract $B_i$. Low risks purchase contract $B_i$. High risks mix between the two contracts, where the share (probability) of high risks purchasing $B_i$, $\gamma_i$, is implicitly defined by

$$\pi(\beta_i, d_i, n_iL, \gamma_i n_iH) = 0.$$  

Equilibrium exists if the average zero profit line of the market does not cut the low risks’ indifference curve through $B_i$.

**Proof.** We first prove that no pooling equilibrium exists. For the strictly pooling equilibrium, this follows from an argument similar to the one used by Rothschild and Stiglitz (1976). Any such contract $P$ would have to lie on the zero profit line of the pool. If $\text{MRS}_{iH} \neq \text{MRS}_{iL}$ in $P$, a contract $P'$ close to $P$ exists that attracts only low risks and earns profits. If $\text{MRS}_{iH} = \text{MRS}_{iL}$ in $P$, some contract $P' > P$ above the pool’s zero profit line attracts both types and earns profits. Weak pooling allows for additional contracts purchased by low (and possibly high) risks. However the above argument continues to hold for the contract(s) $P$ purchased by a mix of different risks.

Hence only weakly separating equilibria are possible, in which some $iH$-individuals purchase a contract that is not also purchased by low risks and hence lies on the high-risks’ zero profit line. This must be contract $A$, since otherwise a contract $A'$ above that line attracts them (and possibly others) away and earns profits. The fact that low risks obtain $B_i$ now follows from competition (constraint (i) in Definition 3) and the firms’ exit decision (constraint (ii)). The mixing probability $\gamma_i$ follows from the zero profit requirement on contract $B_i$. The existence condition is as in Rothschild and Stiglitz (1976) and requires the non-existence of a contract above the market’s average zero profit line that attracts all individuals away from the equilibrium candidate.

We now know that equilibrium will always be weakly separating. However, more
specific results obtain. A crucial distinction arises depending on whether or not the individuals’ indifference curves exhibit regular-crossing at the point in the \((\beta, d)\)-space where the high risks’ indifference curve through \(A\) intersects the low risks’ zero profit line.

**Corollary 1.** If \(RC\) (case (1)) holds at the contract where the high risks’ indifference curve through \(A\) intersects the low risks’ zero profit line, equilibrium is strictly separating.

*Proof.* Definition of regular-crossing implies that the constraint \((ii)\) in the definition of \(B_i^j\) is binding, such that the contract \(B_i^j\) corresponds to the standard Rothschild-Stiglitz contract for low risks. It earns zero profits if only low risks purchase it. The mixing probability \(\gamma_i\) from Proposition 1 is therefore zero. \(\square\)

The higher the productivity \(w_i\) is in the considered market, the flatter are the high risks’ indifference curves, if risk aversion is indeed decreasing (Lemma 3). Therefore, the coverage of the fair contract \(B_i^j\) (weakly) decreases in \(w_i\). The equilibrium is illustrated in the left graph of Figure 1.

**Corollary 2.** With \(IRC\) (case (2)) at the contract where the high risks’ indifference curve through \(A\) intersects the low risks’ zero profit line, \(\gamma_i > 0\) holds. Hence equilibrium is not strictly separating.

*Proof.* From the definition of irregular-crossing and from continuous differentiability of indifference curves it follows that the constraint \((ii)\) in definition 3 will be slack. Therefore, \(B_i^j\) earns positive profits if only low risks purchase it, and \(\gamma_i > 0\) follows from the zero-profit requirement. \(\square\)

The equilibrium is illustrated in the right graph of Figure 1. The outcome described in corollary 2 differs from previously known results. In all of the above mentioned models strict separation emerges if heterogeneity is one-dimensional. In our model, endogeneity of labor supply can lead to irregularly crossing indifference curves and hence to imperfect separation of risk types. Intuitively, since individuals can take an unobservable action that affects their marginal willingness to pay for insurance, perfect separation may not be possible in equilibrium.
Figure 1: Observable Productivities

5 TWO-DIMENSIONAL HETEROGENEITY

We now assume that both individual characteristics cannot be observed by the insurance companies.\footnote{This would, for example, also be a natural information assumption in a model of optimal taxation in the presence of risk, where the government cannot observe productivities and risk but has to rely on the observation of realized income. Private insurance markets in such models might work as described here.} This implies that all four types of individuals act on the same market, such that in a given contract \((\beta, d)\) there are in principle several possible constellations of marginal rates of substitution. We categorize them according to whether regular-crossing holds within productivity group \(w_i\), i.e. for individuals that differ in risk but have the productivity \(w_i\) in common. We further assume that risk-aversion is indeed decreasing, since otherwise productivity has no influence on indifference curves and the analysis of the previous section applies. The six constellations of marginal rates of substitution that are displayed in Table 2 can occur in some given contract \((\beta, d)\). They follow immediately by noting that \(\text{MRS}_{L_j} > \text{MRS}_{H_j}\) holds due to Lemma 3, and by using the additional (weak) inequalities implied by the assumed crossing properties in each of the four cases.

We proceed as follows. We again prove general properties of possible separating and pooling equilibria, including restrictions on where in the contract space the respective contracts will be located. More specific equilibrium properties again depend on the exact constellations of marginal rates of substitution in the respective area.
| (1a) | RC for both $w_L$ and $w_H$ | $\text{MRS}_{LH} > \text{MRS}_{HH} \geq \text{MRS}_{LL} > \text{MRS}_{HL}$ |
| (1b) | | $\text{MRS}_{LH} \geq \text{MRS}_{LL} > \text{MRS}_{HH} \geq \text{MRS}_{HL}$ |
| (2)  | RC for $w_H$, IRC for $w_L$ | $\text{MRS}_{LL} \geq \text{MRS}_{LH} > \text{MRS}_{HH} > \text{MRS}_{HL}$ |
| (3)  | RC for $w_L$, IRC for $w_H$ | $\text{MRS}_{LH} \geq \text{MRS}_{LL} > \text{MRS}_{HL} > \text{MRS}_{HH}$ |
| (4a) | IRC for both $w_L$ and $w_H$ | $\text{MRS}_{LL} \geq \text{MRS}_{LH} \geq \text{MRS}_{HL} > \text{MRS}_{HH}$ |
| (4b) | IRC for both $w_L$ and $w_H$ | $\text{MRS}_{LL} \geq \text{MRS}_{HL} > \text{MRS}_{LH} > \text{MRS}_{HH}$ |

Table 2: Possible Crossing Properties of Indifference Curves

They are summarized in the subsequent corollaries.

5.1 Separating Equilibria

**Proposition 2.** If productivities and risk are unobservable, in any weakly separating equilibrium low risks with productivity $w_i$ purchase the contract $B^H_i$. High risks with productivity $w_L$ purchase $A$. High risks with productivity $w_H$ mix between $A$, $B^L_i$ and $B^H_i$, where the probability of choosing $B^H_i$, $\mu_i$, is implicitly defined by

$$\pi(\beta^H_i, d^H_i, n_{iL}, \mu_i n_{HH}) = 0.$$

**Proof.** In any weakly separating equilibrium, some individuals of both the $HH$- and the $LH$-type purchase a contract which is not purchased by low risks and hence lies on the high risks’ zero profit line. With the same argument as in the proof of Proposition 1, this must be contract $A$. By Lemma 3, the indifference curve of the $HH$-type through $A$ will then be relevant for incentive compatibility. Therefore, the contracts $B^H_i$, $i = L, H$, will be offered in equilibrium, again with the same argument as for Proposition 1. The mixing probabilities $\mu_i$, $i = L, H$, follow from the zero profit requirement on both contracts $B^H_i$. □

Proposition 2 does not mention existence conditions for the weakly separating equilibrium in the spirit of the condition given in Proposition 1. There, existence required that the average zero profit line of the market does not cut the low risks’ indifference curve through their equilibrium contract. Such conditions exist in the present case as well, but are more complicated. It has to be checked which of the four types a new contract would attract away from the equilibrium candidate. Profitability of such a contract is then calculated by comparing its position relative to
the relevant zero profit line. Opposed to the standard case, where there is only one zero profit line for the pool, here we can have several different pools and corresponding zero profit lines. Hence there will be more than one existence condition. The entire set of existence conditions for the weakly separating equilibrium is derived and discussed in Appendix B. Instead of one, we obtain four necessary conditions for equilibrium existence.

As before, the specific characteristics of separating equilibria depend on the slopes of the low risks’ indifference curves at the contract where the $HH$-type’s indifference curve through $A$ intersect the low-risks’ zero profit line. The first possibility is analogous to the case considered by Smart (2000) and Wambach (2000).

**Corollary 3.** If case (1a) prevails at the contract where the $HH$-type’s indifference curve through $A$ intersects the low-risks’ zero profit line, any separating equilibrium will be strict.

*Proof.* If case (1a) holds at the respective contract, the constraint $(ii)$ in the definition of $B^H_i$ is binding for both low risk types, such that $B^H_L = B^H_H$ lies on the low risks’ zero profit line. The mixing probabilities $\mu_i$, $i = L, H$, as defined in Proposition 2, are therefore zero and risks are strictly separated. \qed

The equilibrium is illustrated in the left graph of Figure 2. Despite the existence of four different types in the market, only two different contracts will be offered. These two contracts suffice to strictly separate the two different risks.

**Corollary 4.** If case (1b) or case (2) prevails at the contract where the $HH$-type’s indifference curve through $A$ intersects the low-risks’ zero profit line, the equilibrium mixing probabilities $\mu_i$ are given by $\mu_H = 0$ and $\mu_L > 0$, such that the separating equilibrium will not be strict.

*Proof.* In both cases (1b) and (2) the constraint $(ii)$ in the definition of $B^H_L$ will be slack while it will be binding for $B^H_H$. \qed

The equilibrium is illustrated in the right graph of Figure 2. As in the case of one-dimensional heterogeneity, we find that imperfect separation of risks is possible when labor supply is endogenous. In the case captured in Corollary 4, individuals that choose the contract $B^H_L$ can either be high or low risks. The $HL$-type, however, is still unequivocally identified by its equilibrium choice. This, however, is not a feature that all separating equilibria have in common, as is shown by the next Corollary.
Corollary 5. Whenever case (3), case (4a), or case (4b) prevails at the contract where the $HH$-type’s indifference curve through $A$ intersects the low-risks’ zero profit line, it holds that $\mu_i > 0$ for both $i = L, H$. Therefore, equilibrium will not be strictly separating.

Proof. The constraint (ii) in Definition 2 will be slack for both $B_L^H$ and $B_H^H$, as the indifference curves of both low risk types are steeper than the $HH$-type’s indifference curve at the relevant contract in cases (3), (4a) and (4b).

The equilibrium is illustrated in Figure 3. Individuals with high risk and high productivity will mix between three contracts, two of which are also purchased by low risks. Hence separation is even “less perfect” as in the case discussed in Corollary 4, because it is now no longer possible to unambiguously identify the $HL$-type by its choice of contract.
5.2 Pooling Equilibria

Proposition 3. If productivities and risk are unobservable, any weakly pooling equilibrium consists of the contracts \( A, P = (\beta^P, d^P) \), and \( C = C(\beta^P, d^P) \). It holds that:

(i) The LH-type purchases \( A \).
(ii) \( P \) is purchased by the LL-type and a share \( 1 - \delta > 0 \) of the HH-type. \( P \) lies on the indifference curve of the LH-type through \( A \), and \( MRS_{LH} \geq MRS_{LL} > MRS_{HH} \) is satisfied in \( P \).
(iii) \( C \) is purchased by the HL-type and the share \( \delta \) of the HH-type. If \( C \neq P \), then \( \delta \) is defined by the condition \( \pi(\beta^C, d^C, n_{HL}, \delta n_{HH}) = 0 \).

Proof. For statement (i), assume a contract \( Q \) purchased by LH-individuals were also purchased by some low risks. Then the LH indifference curve through \( Q \) cannot be the steepest of all those who purchase \( Q \). Otherwise, a contract \( Q' < Q \) exists that attracts all but the LH-individuals away from \( Q \) and earns profits. This excludes cases (1a), (1b) and (3) except if \( MRS_{LH} = MRS_{LL} \) in \( Q \) and both LH- and LL-individuals purchase \( Q \). But then a contract \( Q' > Q \) above the initial pool’s zero profit line could attract all LH- and LL-individuals away from \( Q \). Using analogous arguments, a profitable contract \( Q' \) can also be constructed if cases (2), (4a) or (4b) prevail in \( Q \). Therefore, the LH-type’s contract is \( A \) with the same argument used in the previous propositions. For the definition of weak pooling to be satisfied, all HH-individuals must therefore purchase contracts that are also purchased by low risks. For incentive compatibility to be satisfied, none of those contracts can lie to the right of the LH-type’s indifference curve through \( A \) and thus they all have less
than full coverage.

Assume first that the HL- and LL-types purchase the same contract \( P \), and hence this contract is also chosen by the HH type. The HH indifference curve must be the flattest of all through \( P \) since otherwise a contract \( P' < P \) could attract away only low risks. Now, to prevent a firm doing the same with a contract \( P' > P \), \( P \) has to lie on the indifference curve of the LH-type through \( A \) and \( \text{MRS}_{LH} \geq \text{MRS}_{LL} \) has to hold there. In that case, the discussed \( P' \) would also attract the LH-individuals.\(^{14}\) Going through the possible cases (1a) - (4b) we find that \( \text{MRS}_{LL} > \text{MRS}_{HL} > \text{MRS}_{HH} \) must then hold in \( P \). This in turn yields \( C(\beta^P, d^P) = P \) because constraint (iii) in the definition of \( C \) will be binding. Therefore, all statements in the proposition will be satisfied.\(^{15}\)

Now assume that not all low risk individuals purchase the same contract. Due to the single-crossing property of LL- and HL-indifference curves (Lemma 3), there must be a contract \( D_{LL} \) purchased by some LL-individuals and a contract \( D_{HL} \) purchased by some HL-individuals such that \( D_{LL} > D_{HL} \) holds and furthermore the move from \( D_{HL} \) to \( D_{LL} \) comes at a larger than fair premium adjustment for low risks.\(^{16}\) Both contracts must earn zero profits, which implies that some HH-individuals must purchase \( D_{LL} \), which would otherwise earn positive profits given that \( D_{HL} \) earns nonnegative profits. With the same argument as above, \( D_{LL} \) must again lie on the LH-type’s indifference curve through \( A \) with \( \text{MRS}_{LH} \geq \text{MRS}_{LL} > \text{MRS}_{HH} \) holding in \( D_{LL} \). Competition will then result in \( D_{HL} \) being the preferred contract for the HL-type on the HH-type’s indifference curve through \( D_{LL} \). The contracts \( D_{LL} \) and \( D_{HL} \) constructed like this are simply the contracts \( P \) and \( C \) given in the proposition. The mixing probability \( \delta \) follows from the zero profit requirement on \( C \).

\[ \square \]

Corollary 6. A strictly pooling equilibrium does not exist.

As Proposition 3 shows, pooling can indeed occur when endogenous labor supply induces countervailing incentives in a situation with two-dimensional heterogeneity.

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\(^{14}\) Wambach (2000) makes a mistake in his argument at a similar point. He claims that in any such contract \( P \), the indifference curves of the pooled types must be tangent, since otherwise a profitable contract \( P' \) exists close to \( P \). This standard Rothschild-Stiglitz argument is not applicable here, since the LH-indifference curve through \( A \) goes through \( P \) as well. Hence he erroneously concludes that pooling equilibria do generically not exist.

\(^{15}\) Since \( C \) and \( P \) are the same contract, the mixing probability \( \delta \) is irrelevant.

\(^{16}\) Fixing \( D_{HL} \) and \( \beta_{LL} \), the smallest premium \( d_{LL} \) such that some HL-types still purchase \( D_{HL} \) results from moving along the HL-type’s indifference curve through \( D_{HL} \). This curve is steeper than \( p_{L}D \) in all contracts with less than full coverage.
It differs from imperfect separation by the fact that all $HH$-individuals are bunched in contracts with low risks.

There will be a contract $P$ in which two types, for whom risk and productivity have exactly opposite effects on the marginal willingness to pay for insurance, $LL$ and $HH$, are united. However, Proposition 3 still leaves open the possibility that (a) there are only two contracts ($P = C$) and all types but $LH$ are pooled in one of them, and (b) the two low risk types obtain different contracts. The latter case resembles our imperfect separation results: if both the $LH$- and the $LL$-type are ignored for the moment, the situation concerning the remaining types looks similar to Figure 1. A share of the high risks ($HH$) purchases $P$, while the low risks ($HL$) obtain their preferred contract on the high risks’ indifference curve. The mixing probability $\delta$ is determined so as to induce zero profits for $C$.

As before, an extensive discussion of existence conditions for the pooling equilibria is provided in Appendix B. We proceed to show how more specific properties of the equilibrium depend on local characteristics of indifference curves. By (ii) in the above proposition, either case (1b) or case (3) is satisfied in $P$ in any pooling equilibrium.

**Corollary 7.** If case (1b) prevails in $P$, it holds that $C \neq P$ in the weakly pooling equilibrium. Then, $\delta = 0$ if $MRS_{HH} \geq MRS_{HL}$ in the contract where the $HH$-type’s indifference curve through $P$ intersects the low risks’ zero profit line, and $\delta > 0$ otherwise.

*Proof.* If case (1b) holds in $P$, the constraint (iii) in the definition of contract $C$ is slack. Hence $C \neq P$. Furthermore, if $MRS_{HH} \geq MRS_{HL}$ in the contract where the $HH$-type’s indifference curve through $P$ intersects the low risks’ zero profit line, the constraint (ii) in the definition of $C$ will be binding. Hence $C$ lies on the low risks’ zero profit line and $\delta = 0$ follows from the zero profit condition on $C$. The reverse statement follows analogously.

Corollary 7 first gives a condition under which there will indeed be three distinct contracts in the equilibrium contract set. Then a further distinction arises according to whether or not the $HL$-type will uniquely be identified by its equilibrium contract choice ($\delta = 0$ or $\delta > 0$). A graphical representation of this pooling equilibrium is displayed in the left graph of Figure 4. Only the case in which $C$ lies on the low risks’ zero profit line and hence $\delta = 0$ is illustrated there. As the indifference curve of the $HL$-type in $C$ becomes steeper, the contract $C$ will eventually move up on
the $HH$-type’s indifference curve, requiring $\delta > 0$ for the zero profit condition to be satisfied.

Finally, in contrast to the results found by the existing literature, three different types might be pooled in one contract in equilibrium, as is shown by the last corollary.

**Corollary 8.** If case (3) prevails in $P$, it holds that $C = P$ in the weakly pooling equilibrium. Hence the $HH$, $LL$- and $HL$-types are pooled in one contract.

*Proof.* If case (3) prevails in $P$, the constraint (iii) in the definition of $C$ will be binding, implying $C = P$.

A graphical representation of this last case can be found in the right graph of figure 4. In comparing the two graphs of figure 4, note that the contract $P$ is lying on different zero-profit lines. This is simply because a different mix of individuals is purchasing the contract $P$ in the two cases.

![Figure 4: Unobservable Productivities / Pooling](image)

As a final remark, note that the necessary and sufficient conditions for the existence of separating and pooling equilibria in the two-dimensional case do not contradict each other. The various case considerations for the existence and shape of the equilibria take place at different regions of the contract space. While the contract where the $HH$-type’s indifference curve through $A$ cuts the low risks’ zero profit line is of crucial importance for the separating equilibrium (Corollaries 3-5), this role is
taken by the contract \( P \) for the pooling equilibrium (Corollaries 7 and 8). Hence the possibility of co-existence of the two kinds of equilibria cannot be excluded.\(^\text{17}\)

6 Conclusion

It has been realized by the literature that screening in real-world situations will have to deal with more than one dimension of heterogeneity, and that the resulting countervailing incentives do significantly shape the nature of equilibrium. What happens, however, if heterogeneity in some dimensions is not given exogenously but arises from the individuals’ choices? The existing literature has little to say about this. At a first glance it is indeed not obvious why the endogeneity should play a crucial role. As it turns out, however, the individuals’ choices depend on the resulting equilibrium and the equilibrium depends on the individuals’ choices. Such interdependencies do have a substantial impact on the outcome.

In the context of competitive insurance markets, we show that the endogeneity of labor supply will in general make perfect screening impossible. Individuals’ reaction to an increase in risk, i.e. an insurance contract with less coverage, will be an increase in labor supply. This constitutes an opposite effect on their marginal willingness to pay for insurance, such that countervailing incentives can arise.

Even if customers do only differ in risk, the equilibrium of the Rothschild-Stiglitz type will not in general be fully separating. Some high risk individuals might purchase a contract that is also purchased by low risks. The same holds if society is additionally characterized by different productivity levels, which are also unobservable to insurance companies. Besides the imperfectly separating equilibria, pooling equilibria in which some high-risk types are completely bunched in contracts with low risks do emerge. They can differ from the equilibria in models with exogenous wealth heterogeneity. For example, pooling equilibria can contain a contract that is purchased by three out of four types of individuals. Furthermore, no group of low risks with common productivity might be completely separated in equilibrium.

Besides the insight of how endogenous heterogeneity can affect selection problems, our model provides a helpful tool for the analysis of problems concerning public policy, taxation and social insurance. Meaningful models in such areas will have to combine multidimensional heterogeneity with the endogenous choice of private insurance and labor supply. We show how this set of assumptions might affect

\(^{17}\)Closer inspection reveals that the global double-crossing property is indeed not strong enough to exclude one of the equilibria given the local properties necessary for the existence of the other.
the working of insurance markets. Natural questions to ask are about the effects of social insurance in the current framework, and, more general, about the efficiency properties of the arising equilibria.

7 APPENDIX

7.1 Appendix A

Proof of Lemma 1
Consider an indifference curve $d(\beta)$ of a type $ij$ in $(\beta, d)$-space. The slope of this curve at a given contract is

$$\text{MRS}_{ij}(\beta, d) = -\frac{\partial V_{ij}/\partial \beta}{\partial V_{ij}/\partial d} = \frac{\partial V_{ij}/\partial \beta}{\partial V_{ij}/\partial T} > 0,$$

which can also be written as in (6) by noting that $\partial V_{ij}/\partial \beta = p_j u'(c_{ij}^0) D$ and $\partial V_{ij}/\partial T = p_j u'(c_{ij}^0) + (1 - p_j) u'(c_{ij}^1)$. In order to examine how this slope changes as we move up on the indifference curve, we need to evaluate the sign of

$$\frac{\partial \text{MRS}_{ij}(\beta, d(\beta))}{\partial \beta} = p_j(1 - p_j) D \frac{u''(c_{ij}^0)u'(c_{ij}^1) \frac{\partial c_{ij}^0}{\partial \beta} - u''(c_{ij}^1)u'(c_{ij}^0) \frac{\partial c_{ij}^1}{\partial \beta}}{(\partial V_{ij}/\partial T)^2},$$

where the expression on the RHS follows from differentiating (8), substituting $d = d(\beta)$ everywhere, and some simplifications. Note that $u''(c_{ij}^0)u'(c_{ij}^1) \leq u''(c_{ij}^1)u'(c_{ij}^0) < 0$ under Assumption 1 since $c_{ij}^0 \leq c_{ij}^1$ if $\beta \leq 1$. It is also clear that $\partial c_{ij}^1/\partial \beta < \partial c_{ij}^0/\partial \beta$ since the higher premium has to be paid in both states of the nature while the larger benefits are only received in case of damage. Hence $\partial c_{ij}^0/\partial \beta \geq 0$ along the indifference curve is a sufficient condition for (9) to be negative and thus for the indifference curve to be concave.

Note that this requirement puts an upper bound on the precautionary labor effect. If labor supply were fixed and did not react to the insurance contract, $\partial c_{ij}^0/\partial \beta > 0$ along an indifference curve would always hold as it is needed to hold the individual’s utility constant.

Proof of Lemma 3
In order to examine how productivity affects the marginal rate of substitution be-
between coverage and premium given in (6), we need to evaluate the sign of
\[
\frac{d}{dw_i} \frac{dd}{d\beta} = \left( L_{ij}^* + w_i \frac{\partial L_{ij}^*}{\partial w_i} \right) p_j (1 - p_j) D \frac{u''(c_{ij}^0)u'(c_{ij}^1) - u''(c_{ij}^1)u'(c_{ij}^0)}{(p_j u'(c_{ij}^0) - (1 - p_j) u'(c_{ij}^1))^2}. \tag{10}
\]

It is immediate to show that
\[
u''(c_{ij}^0)u'(c_{ij}^1) - u''(c_{ij}^1)u'(c_{ij}^0) = 0 \]
if absolute risk-aversion is constant or if the insurance contract provides full coverage so that \( c_{ij}^0 = c_{ij}^1 \). This proves the second part of Lemma 3. For the first part, note that \( u''(c_{ij}^0)u'(c_{ij}^1) - u''(c_{ij}^1)u'(c_{ij}^0) < 0 \) in the case of decreasing absolute risk-aversion and \( \beta < 1 \). Then, (10) is negative if and only if \( L_{ij}^* + w_i \frac{\partial L_{ij}^*}{\partial w_i} > 0 \). By the Slutsky-decomposition, this is equivalent to
\[
L_{ij}^* + w_i \frac{\partial L_{ij}^*}{\partial T} + w_i \frac{\partial L_{ij}^c}{\partial w_i} > 0, \tag{11}
\]
where \( \frac{\partial L_{ij}^c}{\partial w_i} > 0 \) denotes the pure substitution effect based on the Hicksian labor supply function \( L_{ij}^c \). A sufficient condition for (11) to hold is therefore that \( 1 + w_i \frac{\partial L_{ij}^*}{\partial T} > 0 \), which is just saying that consumption is a normal good and hence implied by Assumption 1.

### 7.2 Appendix B

In section 5, equilibria were characterized but the question was not addressed whether such equilibria in fact exist. In this appendix, we provide necessary and sufficient conditions for the existence of the equilibria. As in the model by Rothschild and Stiglitz (1976), the fundamental condition for existence is that there is no contract outside the equilibrium set of contracts that attracts a profitable pool of individuals. In looking for such potentially profitable deviations, we can confine ourselves to the area between the zero profit lines of the high and low risks. Clearly, a contract below the low risks’ zero profit line could never be profitable. Contracts above the high risks’ zero profit line, in turn, would not attract any individual given the equilibria from section 5.

Figure 5 illustrates this area. The thick black lines represent the high and low risks’ zero profit lines. In addition, the indifference curves of the four types through the contracts \( A \) and \( B = B_H^H = B_H^H \) of the strictly separating equilibrium defined in Corollary 3 are depicted. Based on this graphical representation, the necessary and sufficient conditions for the existence of this equilibrium can be stated as follows:

**Corollary 9.** The separating equilibrium defined in Corollary 3 exists if and only if case (1a) prevails in the contract where the HH-type’s indifference curve through A
intersects the low risks’ zero profit line and the following conditions are satisfied:

(i) the zero profit line of the pool of HH- and LL-types does not intersect area I in figure 5,
(ii) the zero profit line of the pool of HH-, HL- and LL-types does not intersect area II,
(iii) the zero profit line of the pool of HH-, LH- and LL-types does not intersect area III and
(iv) the zero profit line of the pool of all individuals in the society does not intersect area IV.

Proof. Necessity follows from Corollary 3 and the fact that, if one of the conditions (i) to (iv) is not satisfied, a profitable pooling contract exists that destroys the equilibrium. For sufficiency, note first that a contract in any other area between the zero profit lines of the low and high risks either attracts no type or only high risks. It therefore cannot be a profitable deviation. Moreover, the crossing properties of the indifference curves implied by Lemma 3 and Assumption 2 rule out the emergence of other relevant areas.

Hence, in contrast to the standard case considered by Rothschild and Stiglitz
(1976), four conditions instead of just one need to be satisfied in order to guarantee existence. In the proof of the following Corollary, we show that the existence conditions from Corollary 9 analogously apply to the other (weakly) separating equilibria defined in section 5.1.

**Corollary 10.** Conditions (i) to (iv) from Corollary 9, together with the relevant relations between the marginal rates of substitution, are also sufficient for the existence of the separating equilibria defined in Corollaries 4 and 5.

**Proof.** For the equilibrium from Corollary 4, note that there exists an additional area to the left of area II and below the HL-types’ indifference curve through $B_H^H$ representing contracts that would attract HH- and HL-types (see the right graph in figure 2). However, if condition (ii) from Corollary 10 is satisfied, there cannot be a profitable deviation in this new area. First, the zero profit line of the pool of HH- and HL-types lies above the zero profit line from condition (ii). Second, the HL-types’ indifference curve through $B_H^H$ is concave by Lemma 1. Together, this ensures that, if condition (ii) is satisfied, the zero profit line of the pool of HH- and HL-types lies above the new area where only these types are attracted. It hence cannot intersect it.

For the equilibrium from Corollary 5, the area with contracts attracting HH- and HL-types only, which was described above, also exists but cannot contain profitable contracts if condition (ii) is satisfied by the same argument as above. In addition, in this case, our double-crossing assumption does not rule out that the indifference curves of the LH- and of the LL-types and those of the LH- and the HL-types cross again above the low risks’ zero profit line (see figure 3). Then, new areas compared to figure 5 can emerge. However, contracts in these areas would either attract only high risks or a pool of HL-, HH- and LH-types, which cannot be profitable if condition (iv) is satisfied.

We now turn to the existence of pooling equilibria. First, the weakly pooling equilibrium from Corollary 7 is considered. Based on the illustration in figure 6, we can derive sufficient and necessary conditions for the existence of this pooling equilibrium in the following corollary.

**Corollary 11.** The pooling equilibrium defined in Corollary 7 exists if and only if case (1b) prevails in $P$ and the following conditions are satisfied:

(i) the zero profit line of the pool of HH-, HL- and LL-types does not intersect area $I$ in figure 6,
(ii) the zero profit line of the pool of LH- and LL-types does not intersect area II and
(iii) the zero profit line of the pool of all types does not intersect area III.

Proof. Necessity is implied by Corollary 7 and the fact that, if one of the conditions (i) to (iii) is not satisfied, there is a contract outside the equilibrium set that attracts a profitable pool.

Sufficiency is established by showing that if conditions (i) to (iii) hold, no other area in figure 6 can contain profitable deviations. This is obvious for the areas in which contracts would attract no individual or only high risks. In addition, contracts in the area to the left of area I cannot be profitable if condition (i) is satisfied. This follows from the fact that, in this area, only HH- and HL-types are attracted and hence the corresponding zero profit line must lie above the one from condition (i). Together with the concavity of the HL-types’ indifference curve through C, this ensures that there cannot be a profitable deviation in this area. By the same argument, contracts in the area between areas II and III in figure 6 are not profitable if condition (ii) is satisfied. Moreover, the contracts represented by the area above area I cannot be profitable as they would attract the LH- and LL-types
only but lie below this pool’s zero profit line. Finally, the crossing properties of the indifference curves implied by Lemma 3 and Assumption 2 rule out other relevant areas. The only exception occurs if contract $C$ is not on the low risks’ zero profit line but at a point of tangency of the HH- and HL-types indifference curves. Then, another area compared to figure 6 appears. However, it represents contracts that only attract high risks and are therefore not profitable.

Hence, in contrast to the separating equilibria, only three existence conditions are needed for the pooling equilibrium. Finally, figure 7 graphically represents the pooling equilibrium defined in Corollary 8. The necessary and sufficient conditions for its existence are as follows:

**Corollary 12.** The weakly pooling equilibrium defined in Corollary 8 exists if and only if case (3) prevails in contract $P$ and the following conditions are satisfied:

(i) the zero profit line of the pool of HH- and HL-types does not intersect area I in figure 7,

(ii) the zero profit line of the pool of LH- and LL-types does not intersect area II and

(iii) the zero profit line of the pool of LH-, HL- and LL-types does not intersect area III.

**Proof.** Both necessity and sufficiency are established as in the proof of Corollary 11. For sufficiency, note that the area to the right of area I in figure 7 cannot contain profitable contracts as they would attract all the individuals pooled in contract $P$, which lies above this area and just makes zero profits. Moreover, contracts in the area below area III cannot be profitable if condition (iii) is satisfied. Finally, it can be easily shown that even if the indifference curves of the HH- and HL-types or of the HH- and LL-types cut again (be it below or above contract $P$), the resulting new areas cannot contain profitable contracts given conditions (i) to (iii).
Figure 7: Existence of Pooling II

References


