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A formal version of the Guided Search (GS2) model

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Guided Search 2 (GS2) is currently one of the most detailed models of visual search and has been used to predict search times for different stimulus conditions by means of detailed computer simulations. The present article goes a step further and presents formulas that allow for the calculation of the search times and their variances. Moreover, these formulas can be applied to fit GS2 to data. An example is provided in which GS2 is fitted to search functions representing search asymmetries.

Visual search is a widely used experimental paradigm for investigating properties of the visual system. In respective studies, participants have to decide as quickly as possible whether a target item is present among a variable number of distractor items or not. The results are usually presented as so-called search functions (i.e., functions that relate the response times to the number of distractors). The slopes and offsets of these functions provide valuable information about the efficiency with which the items or item features are processed. For instance, when the target differs from the distractors by a unique feature, the search functions are flat (i.e., the decision time or search time depends little on the number of distractors). On the other hand, when the target is defined by a conjunction of certain features, the resulting search function are usually steep.

A first prominent model that tried to account for these results was the *feature integration theory* (FIT, e.g., Treisman, 1988; Treisman & Gelade, 1980; Treisman & Gormican, 1988). FIT assumes that features produce activation on specific retinotopic feature maps. When the target possesses a unique feature, the monitoring of the activation on the respective map is sufficient for deciding whether a target is present. Since the coding of a simple feature occurs in parallel across the visual field, this explains the flat search functions. In the case where the target is defined by a conjunction of features, the items have to be checked by combining the features on a so-called map of locations. Since this process requires attention, it proceeds serially until the target is found, which accounts for the observed steep linear search functions.

Meanwhile, however, the distinction between parallel and serial search has been questioned. There have been many results showing that search efficiency varies continuously, which suggests that such a dichotomy does not hold (Duncan & Humphreys, 1989, 1992). A recent account that can explain a gradually varying search efficiency is the Guided Search 2 (GS2) model, developed by Wolfe and his colleagues (Chun & Wolfe, 1996; Wolfe, 1994; Wolfe, Cave, & Franzel, 1989). They assume, as do Treisman and her colleagues, that item features produce activation on respective retinotopic maps. But, different from FIT, the activations that each item produces on the different feature maps are summed and the result is represented on a so-called activation map. Search is based on the activation on this map and proceeds serially. However, the order in which the items are checked is not random but guided by the relative strength of the activations on the activation map. Thus, when the target produces the highest activation on the map, it is checked first. If this occurs frequently across trials, a fast average search time results.

A great advantage of GS2 is that it is implemented as a computer program, so that search behavior can be simulated. For instance, Chun and Wolfe (1996) presented a detailed flow chart and simulated search behavior under different conditions. Although this is undoubtedly great progress, as compared with verbally stated models, in some cases one would prefer to have formulas that allow one to compute the model's behavior exactly. For instance, one might want to specify some characteristics of the model precisely. Also, comparing the performance of the model for different parameter values might be faster and easier with formulas. However, the greatest advantage of formulas would probably be that they allow the model to be fit to empirical data by means of minimization programs. Such a formal version for the search and decision part of GS2 is presented in this article. Furthermore, two examples are provided. In the first example, the exact values for search functions are provided, which were simulated in Chun and Wolfe. In the second example, GS2 is fit to empirical data.

As already mentioned, in GS2 it is assumed that the items produce activations on the activation map. These

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activations can be described by random variables. Let X_s and X_n be random variables representing target and distractor activations, respectively. The corresponding densities are denoted by $f_s(x)$ and $f_n(x)$ and the distribution functions by $F_s(x)$ and $F_n(x)$. Given a set size of m items, there are $m - 1$ or m distractors in target-present or target-absent trials, respectively. We will first consider the case in which a target is present.

Target-Present Trials

The crucial assumption of GS2 is that the items are checked one by one in descending order of their activation strength. Thus, the higher the rank of target activation, the fewer steps that are required to find it. Let m be the number of items in the display. We will first examine the simple situation in which search always proceeds until the target is found. In this case, search time T depends only on the number of distractors whose activation exceeds that of the target. Let R denote a discrete random variable representing this number. To compute the expected search time across trials, we must know the probability mass function $P\{R = r\}$ of R.

Let a represent the target activation and assume that the activation of an individual distractor exceeds this value with probability q. We have to determine the probability that $0, 1, \ldots, m - 1$ distractor activations are larger than a. The event that r activations are above a (successes) and $m - 1 - r$ activations are not (failures) can occur in as many ways as r letters S can be distributed across $m - 1$ positions. The possible number of distributions is

$$\binom{m-1}{r},$$

where each has a probability of $q^r(1 - q)^{m-1-r}$.

This shows that the number R of distractor activations exceeding the target activation has a binomial distribution (cf., e.g., Ross, 1981) with parameters $(m - 1, q)$

$$P_m\{R=r\} = \binom{m-1}{r} q^r (1-q)^{m-1-r}, \quad r = 0,1,\ldots,m-1$$

where q is given by

$$q = 1 - F_n(a).$$

However, in GS2, the situation is more complicated, since the target activation is itself a random variable X_s. To take this into account, we have to compute the respective probabilities by conditioning on the values of X_s. This yields

$$P_m\{R=r\} = \binom{m-1}{r} \int_{-\infty}^{\infty} [1-F_n(x)]^r [F_n(x)]^{m-1-r} f_s(x)dx,$$
$$r = 0,1,\ldots,m-1. \tag{1}$$

By using Equation 1, we can compute the expected search time $E[T]$ for the target, given a set size of m. Let $p_m(r) = P_m\{R = r\}$, then we have

$$E_m[T] = c + t \cdot \sum_{r=0}^{m-1} r \cdot p_m(r) \tag{2}$$

and

$$E_m[T^2] = c + t^2 \cdot \sum_{r=0}^{m-1} r^2 \cdot p_m(r). \tag{3}$$

Here, t is the search and processing time per item, and c is a constant representing the time required for different processes such as stimulus coding and making the response. As can be seen, when no distractor activation exceeds the target, nothing is added to the constant. Since it is assumed that at least the location of one item is checked in any case, the time consumed by this process is also part of c.

When we combine Equations 2 and 3, we can compute the variance of the search times by

$$\text{Var}_m(T) = E_m[T^2] - E_m[T]^2. \tag{4}$$

The formulas so far can be used for calculating the expected time required for finding the target item. Since the target is found in any case, there are no misses.

However, according to GS2, search does not always proceed until the target is found. Rather, there is an activation threshold, and search is terminated when this threshold is reached. Consequently, misses are possible when the target activation is below that threshold, which occurs with probability

$$P(\text{miss}) = F_s(th).$$

As can be seen, the miss rate is independent of set size. It only depends on the threshold and on the distribution of the target activation.

To take this search strategy into account, we consider the probability mass function of R under the condition that the target activation X_s is above the activation threshold th. Thus, we consider only those cases in which the target is found. This is in line with most studies in which only correct responses are included in the calculation of the mean search times (i.e., error trials are discarded).

The probability that the target activation is above threshold is

$$P\{X_s \geq th\} = 1 - F_s(th).$$

Therefore, the probability mass function of R for this case $(r = 0, 1, \ldots, m - 1)$ is

$$P_m\{R=r \mid X_s \geq th\} =$$
$$\frac{1}{1-F_s(th)} \binom{m-1}{r} \int_{th}^{\infty} [1-F_n(x)]^r [F_n(x)]^{m-1-r} f_s(x)dx. \tag{5}$$

Equation 5 is similar to Equation 1, except that we consider only target activations above threshold (i.e., integration starts from th). To guarantee that the probabilities of the mass function sum up to 1, the first fraction was included as a normalization factor.

When we substitute the corresponding probability mass function, we can use Equations 2 and 4 for computing the mean and variances for the present case.

Target-Absent Trials

When no activation threshold is given, we have exhaustive search on target-absent trials. Therefore, the search time is proportional to set size

$$E_m[T_{\text{absent}}] = c + t \cdot (m - 1).$$

(Note that $m - 1$ is used instead of m, because the time for going to one item is already contained in c.)

On the other hand, when an activation threshold is assumed, the search time on absent trials is determined by the number of distractors whose activations are above that threshold. Thus, we have to calculate the probability that $k = 0, 1, \ldots, m - 1$ of $m - 1$ distractor activations are above threshold. (Again, note that one location is checked in any case, therefore $m - 1$.) Let N denote a random variable representing the number of activations above activation threshold. This random variable has a binomial distribution with parameters $(m - 1, q)$

$$P_m\{N = k\} = \binom{m-1}{k} q^k (1-q)^{m-1-k}, k = 0, 1, \ldots, m-1, \quad (6)$$

where q is the probability that a single distractor activation is above the activation threshold;

$$q = 1 - F_n(th).$$

Since the mean and variance of the binomial distribution given by Equation 6 is $(m - 1)q$ and $(m - 1)q(1 - q)$, the expected search time is

$$E_m[T] = c + t \cdot (m - 1)q, \quad (7)$$

and the variance

$$\text{Var}_m(T) = c + t^2 \cdot (m - 1)q(1 - q). \quad (8)$$

Example 1. In the first example, the derived formulas are applied in order to compute the exact search times for parameter values used in a simulation by Chun and Wolfe (1996). The main goal of their simulation was to show that an activation threshold can be used to terminate search when no target is present. For the computations presented here, the same parameter values as for their simulation were used. As constant c, they chose 450 msec and, as processing time per item t, a value of 50 msec. They further assumed that target and distractor activations were normally distributed, with mean 400 and 300 msec, respectively. The standard deviation σ_n of the distractor activation was set to 100, whereas the standard deviation σ_s of the target distribution depended on that of the distractor distribution and on the target mean μ_s. Chun and Wolfe related these parameters in such a way as to obtain a slope ratio of approximately 2. The functional relationship they used was

$$\sigma_s = \sigma_n \cdot \frac{1}{1 + 2d'}, \quad (9)$$

where d' is defined by (cf. Green & Swets, 1966)

$$d' = \frac{\mu_s - \mu_n}{\sigma_n}.$$

Given the parameters above, a standard deviation σ_s of 33.3 results.

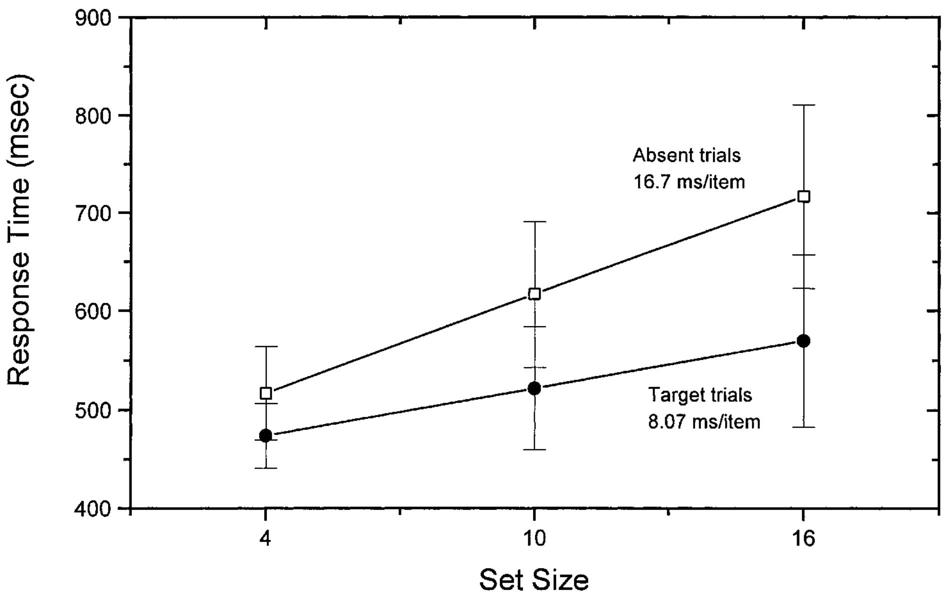

Figure 1. This figure shows the exact search times and variances obtained by applying the derived formulas and using the same parameter values as in Chun and Wolfe (1996), who simulated their data.

Figure 1 shows the resulting search functions that correspond to the simulated data presented in Figure 4 of Chun and Wolfe (1996). They obtained a slope of 20.5 msec/item and of 9.5 msec/item for target-absent and target-present responses, respectively. This corresponds to a ratio of 2.16. Here, the respective slopes are 16.6 msec/item and 8.07 msec/item, which gives a ratio of 2.07.

Chun and Wolfe (1996) included in their simulation an adaptive mechanism that modifies the activation threshold in such a way that, on average, the desired error rate is obtained. Here, a fixed threshold *th* was used and set to 343, which corresponds to a miss rate of 4.36% (Chun & Wolfe had a miss error rate of 4.3%). These parameters are sufficient to compute the mean search times and their variances. In Figure 2, the corresponding probability mass functions are shown for the three employed set sizes of 4, 10, and 16. Each function gives the probability that a certain number of distractor activations is above that of the target. As can be seen, the maximum probability of being above the target activation shifts to a larger number of distractors with increasing set size.

Example 2. The second example serves to show how the formulas can be used to fit GS2 to empirical data. To make the example interesting and to demonstrate how useful and flexible the present approach is, data were selected that reflect a search asymmetry. This phenomenon describes situations where the slopes of the search functions change considerably when the role of target and distractors are exchanged. Search asymmetries are still a great challenge to visual-search theories. Although, for instance, Treisman and Gormican (1988) discuss several possible accounts for search asymmetries and relate them to FIT, it remains unclear which one is valid or whether other accounts might be more appropriate. What makes the issue rather complex, is that search asymmetries occur for different types of items (e.g. Malinowski & Hübner, 2001; Treisman & Gormican, 1988; Wang, Cavanagh, & Green, 1994; Wolfe, 1994, 1998), and it is open to question whether a single theory can account for all of them.

Here we will consider the case in which the target differs from the distractors in degree on a quantitative dimension. The specific data that will be modeled are from Treisman and Gormican (1988) and describe search behavior for line length. These authors found that search is more efficient when a longer line has to be found among shorter lines of the same orientation than vice versa. They explain this type of search asymmetry by means of a pooled-response account in combination with Weber's law. Treisman and Gormican assume that the activity elicited by the items on the corresponding feature map is pooled and the result is used to decide whether a target is present or not. When target and distractors differ in their activity levels, it follows from Weber's law that it is easier to find a high activity target among low activity distractors than vice versa. Thus, they proposed that an asymmetric discriminability was responsible for the search asymmetry. To support their hypothesis, they additionally varied discriminability by using an easy condition and a difficult condition. In the former case, the difference in line length between target and distractors was larger than in the latter case. Finally, they matched for discriminability and, indeed, the search asymmetry vanished. Thus, altogether, their account is in line with the data. However, as Treisman and Gormican mentioned, the pooling account is not without problems. For instance,

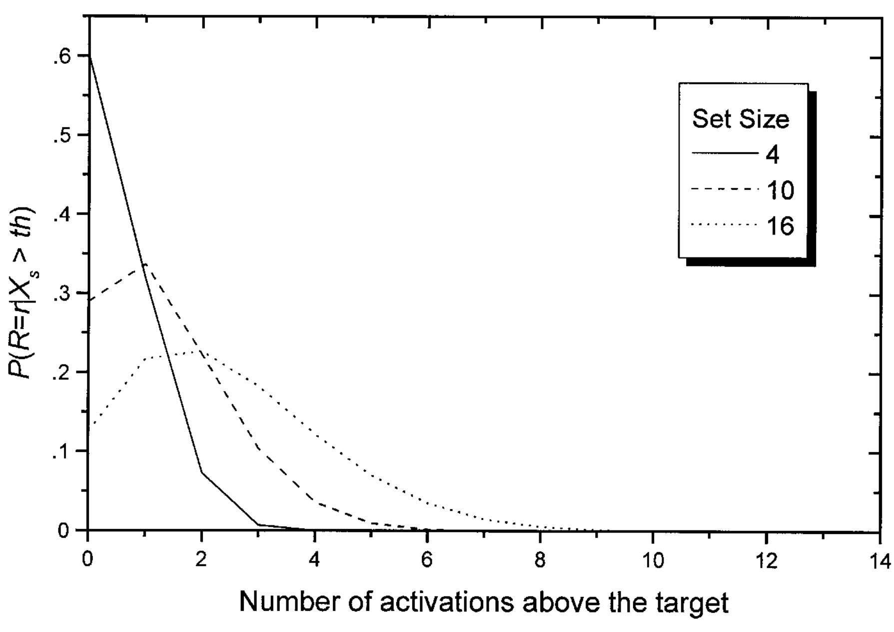

Figure 2. The probability mass functions corresponding to the parameter values in the example and the three set sizes of 4, 10, and 16. Each function gives the probability that a certain number of distractor activations is above that of the target.

with a pooled activation one cannot distinguish between some high activations and many low ones. Therefore, they suggested that the average activity in the pooled detectors be considered instead.

Here, I demonstrate that, alternatively to the pooling account, GS2 also can explain the data. This is done by fitting the GS2 model to the search data for line length in the easy and difficult conditions of Treisman and Gormican (1988). The corresponding data points are shown in Figure 3.

Four conditions were considered. A long line served as target and short lines as distractors, or vice versa. Moreover, these two conditions were realized with line lengths that were easy or difficult to discriminate. The four conditions were labeled as *easy long*, *easy short*, *difficult long*, and *difficult short*, where *short* and *long* denote the target. Since Treisman and Gormican (1988) used three different set sizes, there were six search times for each condition: three for target-present responses, and three for target-absent responses. Also, error rates were reported. However, they were not only quite small but, unfortunately, misses and false alarms were summed. Therefore, I decided to ignore the empirical error rates and fix the activation threshold to a value that produces 5% misses. As a consequence, there were six data points for each of the four conditions.

To keep the number of free parameters small, most of the parameters were fixed to reasonable values. I chose a value of 400 for the mean and 80 for the standard deviation of the distractor distributions. For calculating the standard deviation of the target distribution I used Equation 9. Finally, I chose a reasonable value for the processing time per item (50 msec). Thus, only the constant c and the mean of the target distribution μ_s remained as free parameters. Indeed, when search asymmetries are mainly due to discriminability, the varying of the mean activation of the target (i.e., d') should be sufficient for producing a search asymmetry also in connection with GS2.

The values of the free parameters were estimated by fitting GS2 to the respective search times by means of a minimization algorithm (the function "fminsearch" from MATLAB) that tries to minimize the squared deviation of the predicted search times from the empirical ones. As expected, given these values of the fixed parameters, a good fit could be obtained for both easy conditions. It also turned out that the quality of the fit remained largely unchanged even with moderately different values of the fixed parameters. This led merely to different estimates of the free parameters. In any case, the decrease in performance from the condition with the long target (efficient) to that with a short target (less efficient) could sufficiently be modeled by a reduction of d'.

However, it was not possible to model the decrease in performance from the easy to the difficult conditions simply by reducing d'. To obtain the steep search functions for the difficult conditions, I also had to increase the processing time per item. Actually, this is a reasonable assumption. The time for deciding whether the currently inspected item is the target or not should depend on the discriminability between the target and distractors. To

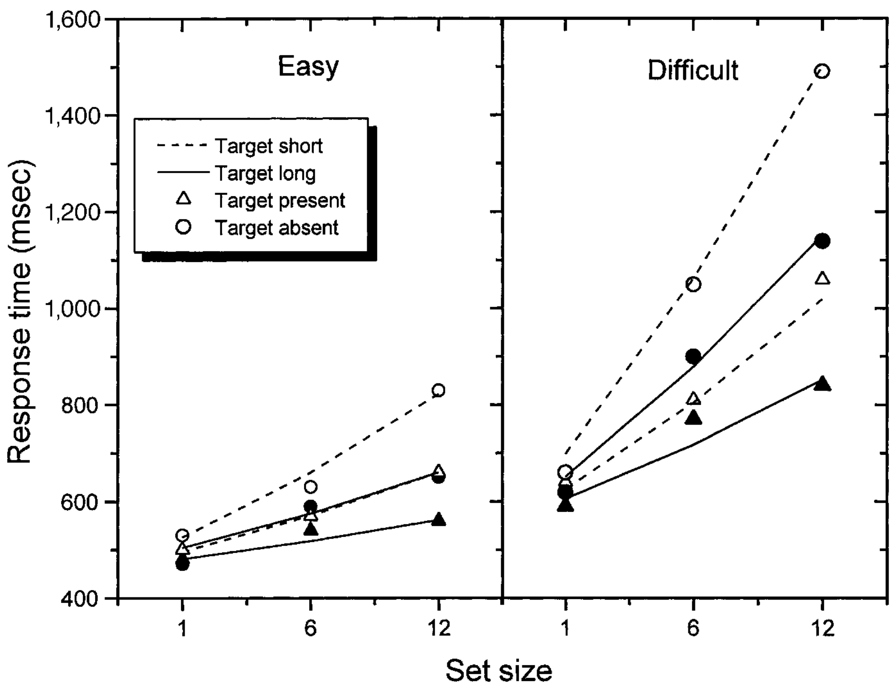

Figure 3. The result of fitting GS2 to the data of Treisman and Gormican (1988). The points represent the data, and the lines, the fit of the model.

Table 1
Parameter Values for the Search Behavior of GS2 in the Different Conditions Considered in Figure 3

Condition	Constant c	Target Mean μ_s	d'	Time per Item t (in Milliseconds)	Activation Threshold th
Easy long	481	473	0.91	41	426
Easy short	500	452	0.65	51	394
Difficult long	606	433	0.42	67	362
Difficult short	625	418	0.23	89	328

Note—Only the constant c and the mean μ_s of the target activation were free parameters and were estimated from the data. The other values depended on the target mean and were calculated.

avoid including the processing time as a second free parameter, I related it to d' in a similar manner as the standard deviation of the target distribution

$$t = t_{\max} \cdot \frac{1}{1 + 3d'}, \qquad d' \geq 0. \qquad (10)$$

The value of parameter $t_{\max}$ in Equation 10 was set to 150 msec. This value and the constant 3 were chosen in order to obtain processing times that lie in a reasonable range for the expected values of d'. With this additional relation, all eight search functions could be fit by one set of fixed parameter values and the constant c and the mean of the target activation as free parameters. The result is given in Figure 3, in which the lines represent the prediction of GS2 with the obtained parameters. In Table 1, the corresponding parameter values are given. Note that only the values in the first two columns were estimated. The other values were simply calculated from those in the second column.

This example shows that even fitting a small number of data points is not trivial with only two free parameters. However, by assuming reasonable relations between several parameters, GS2 can explain the data quite well. As a result, the model fit suggests that, when visual search proceeds serially, as assumed for GS2, discriminability not only affects the mean activations but also the processing or decision time per item. Thus, GS2 provides a serious alternative to the pooling account of Treisman and Gormican (1988).

Discussion

In the present article, formulas were derived for computing visual search times according to the GS2 model (Chun & Wolfe, 1996; Wolfe, 1994). They can be used for deriving and investigating general properties of the model. Moreover, with these formulas it is possible to fit GS2 to data by means of standard minimization procedures. The usefulness of this has been demonstrated by an example in which GS2 was fit successfully to search functions representing a search asymmetry. It should be mentioned that Wolfe (1994) already simulated a search asymmetry with respect to orientation. However, in his study, the simulation relied on the prototype account of Treisman and Gormican (1988). Moreover, he simulated only the qualitative aspects of this phenomenon and did not fit GS2 to real data.

A further application of the present approach is provided in Hübner and Malinowski (2001). They even extend GS2 and provide an account of the phenomenon that, under certain conditions, an absent advantage occurs (i.e., absent responses are faster than present responses; cf. Humphreys, Quinlan, & Riddoch, 1989; Müller, Humphreys, & Donnelly, 1994).

The present derivations focus mainly on search times and deal only partly with errors. One reason is that errors are not well elaborated in GS2. To account for errors, particularly for false alarms, educated guesses are proposed. For instance, Chun and Wolfe (1996) assumed that the likelihood of a guess increases with elapsed search time. When a guess occurs, the probability of an absent response was set to .80 and that of a present response to .20. However, there are alternative and more plausible accounts for errors. One way would be to abandon the assumption of perfect matches (cf. Zenger & Fahle, 1997).

Nevertheless, since the most important information is usually contained in the search times, the provided formulas should be useful for examining and modeling visual search behavior. Moreover, the formulas also allow variances to be predicted. Therefore, future studies should take them into account.

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