Mutual Friction and the Thermal Conductivity of Superfluid Helium Near T_λ

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Temperature gradients in superfluid helium carrying a heat current in a 13.8-mm-wide tube or in a 0.1-mm-wide slit have been measured at $0.09 \text{ m}^\circ\text{K} < T_\lambda - T < 16 \text{ m}^\circ\text{K}$. The results are interpreted in terms of a mutual friction force between the normal component of He II and vortices in its counterflowing superfluid component. This force is found to diverge near T_λ with a mutual friction constant proportional to $(T_\lambda - T)^{-0.35 \pm 0.06}$.

1. INTRODUCTION

The two-fluid properties of superfluid helium offer the possibility of maintaining two velocity fields, v_n and v_s (for normal and superfluid components, respectively). The flow of entropy S is associated with the flow of the normal component of density ρ_n only. If a heat current Q is transported by He II through a tube its normal component has to flow from the hot to the cold end of the tube at a velocity

$$v_n = Q/\rho_s T \quad (1)$$

The condition of zero net mass flow j in a closed system will be satisfied by a counterflow of the superfluid component of density ρ_s :

$$j = \rho_n v_n + \rho_s v_s = 0 \quad (2)$$

The unusually high thermal conductivity of superfluid helium is brought about by this counterflow of the two components with a relative velocity

$$w = v_n - v_s = Q/\rho_s T \quad (3)$$

the so-called "thermomechanical effect."

For small relative velocities or small heat currents the thermal conductivity of superfluid helium reaches very high values because the only dissipating process arises from the viscosity of the normal component. As a result very small temperature gradients can produce large fluxes of heat in He II. In this region the heat

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current is proportional to the temperature gradient. But above a certain relative velocity w_c (which in most cases is of the order of cm/sec), quantized vortices are produced in the superfluid and collide with the thermal excitations comprising the counterflowing normal component. Due to their resulting interaction the two components of superfluid helium do not flow independently. The interaction between thermal excitations and counterflowing vortices reduces the thermal conductivity of superfluid helium. Experiments have shown that in this case the temperature gradient is approximately proportional to Q^3 .

Gorter and Mellink¹ have added a mutual friction force term to the hydrodynamic equations for He II to take this interaction into account. According to them the temperature gradient resulting from mutual friction is given by

$$\nabla T = A' \cdot w^{m'} \cdot \rho_n / S \quad (4)$$

where m' was supposed to have the value 3 and A' is a constant. Most experiments have been performed below 2.1°K and are in qualitative agreement with this equation. But in explaining quantitatively experimental results with an equation like (4), the exponent m' was found to be temperature-dependent with values ranging from 3 up to 4.²⁻⁸ In addition, the mutual friction constant A' showed an increase by about a factor of eight between 1.3 and 2.1°K.^{2,4,8} Furthermore, there is some discrepancy in the literature in which way, and if at all, an onset velocity for the mutual friction should be taken into account in Eq. (4)²⁻⁵—some of these results may be influenced by classical turbulence in the normal component.^{5,9}

In heat-current experiments at temperatures near T_λ one can reach high velocities v_s and w and therefore large temperature gradients at rather modest heat-current densities and velocities v_n of the normal component due to the decrease of the superfluid density near T_λ . Recently, Ahlers⁶ has shown that a divergent mutual friction constant A' can explain the measurement of heat flow in He II near T_λ reported in Refs. 4, 6, 7, 10, and 11. Gaining more quantitative information about the vortex-thermal excitation interaction near T_λ is of interest.

In this paper we report measurements of temperature gradients in He II as a function of temperature for the range $0.09 \text{ m}^\circ\text{K} < T_\lambda - T < 16 \text{ m}^\circ\text{K}$ performed at different heat currents ($1.3 \text{ mW/cm}^2 < Q < 158 \text{ mW/cm}^2$). Our results are interpreted in terms of the mutual friction theory.^{1,2} Also, at temperatures as near as $0.09 \text{ m}^\circ\text{K}$ to T_λ they can be fitted to an equation similar to (4) taking into account an onset velocity for the mutual friction and a divergence of the mutual friction constant with $(T_\lambda - T)^{-0.35 \pm 0.06}$. With these modifications no temperature dependence and no change of the exponent of the heat current in Eq. (4) from its originally¹ postulated value of 3 are necessary to explain the data.

2. EXPERIMENTAL APPARATUS AND RESULTS

The experimental setup was similar to the one used in our earlier experiments.^{7,10} A helium-filled stainless-steel cylinder (13.8-mm i.d., 0.1-mm wall, 100-mm length) was surrounded by a vacuum jacket, except at its top where it

Fig. 1. Temperature difference $T_1 - T_2$ measured by two thermometers at a distance of 16 mm in He II in a 13.8-mm-wide tube as a function of $T_\lambda - T_1$. T_1 is the temperature at the lower thermometer which is nearer to the heater. ∇ $Q = 1.3$ mW/cm²; $\blacksquare$ $Q = 2.2$ mW/cm²; $\bullet$ $Q = 8.84$ mW/cm².

was open to the main helium bath. The bath served as a thermal reservoir and determined the temperature at the upper end of the cylinder. A constant heat current was maintained in the liquid inside the cylinder by switching on a heater near its bottom. The temperature of the superfluid was measured by carbon resistors [1/8-W Allen Bradley; dissipated power 5×10^{-9} W; $R(T_\lambda) \simeq 50$ k Ω] at different heights inside the cylinder. The heater and the thermometers had no direct contact to the cylinder or its bottom. The thermal conductivity of the walls was negligible for our measurements.

In order to achieve higher heat-current densities in He II without overheating the heater a constriction was used in the cylinder for some measurements. A cylindrical piece of nylon of 1-mm length was put into the cylinder leaving an annular gap of only 0.1 mm. The temperature difference across this constriction was measured by thermometers above and below it.

Starting below the λ temperature the liquid in the cylinder was slowly warmed by raising the temperature of the main bath and dissipating a constant heat input in the heater. The temperatures measured by two thermometers at different heights in the tube were registered simultaneously on a recorder.* Temperature differences could be measured to within $5 \mu^\circ\text{K}$. All runs were performed only up to temperatures for which the temperature at the heater was below T_λ to avoid the uncertainties which arise as soon as the heater is surrounded by He I or by helium gas.

*No relaxation times longer than the time constant (3 sec) of our equipment for measuring temperatures were observed for establishing or vanishing of the temperature gradients after switching on or off the heater.

Fig. 2. Temperature difference $T_l - T_2$ measured by two thermometers at a distance of 1 mm in He II in a 0.1-mm-wide slit as a function of $T_\lambda - T_l$. T_l is the temperature at the lower thermometer nearer to the heater $\nabla Q = 15.5 \text{ mW/cm}^2$; $\bullet Q = 41 \text{ mW/cm}^2$ (both left and lower scales); $\blacksquare Q = 158 \text{ mW/cm}^2$ (right and upper scales).

As examples, in Figs. 1 and 2 the differences in temperature measured by two thermometers at different heat currents in the liquid are shown as a function of the temperatures of the lower thermometer. The rapid increase of the temperature gradient in approaching T_λ demonstrates the decrease of the thermal conductivity of He II in the nonlinear region. In addition, comparison of runs performed at different heat-current densities Q verifies roughly $\nabla T \propto Q^3$.

3. INTERPRETATION OF THE DATA AND DISCUSSION

3.1

For all our measurements the contribution of the normal fluid viscosity to the temperature gradient can be neglected. Because we have measured rather large temperature differences ΔT instead of temperature gradients ∇T , the equation for ∇T had to be integrated rather than to replace ∇T by $\Delta T/L$ to fit the results. For a first interpretation of the measured temperature differences the following equation was integrated and fitted to the experimental results*:

$$\nabla T = A' w^{m'} \rho_n / S = A' (Q / \rho_s S T)^{m'} \rho_n / S \quad (5)$$

*In our temperature range S , T , and ρ_n are taken as constants, which introduces errors of less than 1 % in the temperature differences calculated with Eq. (5) or (6).

with

$$A' = a'(T_\lambda - T)^{\alpha'}$$

A temperature-dependent, arbitrary A' as well as an arbitrary parameter m' take into account deviations from Eq. (4).

In the earlier experiments of Ahlers⁶ and us,^{7,10} the temperature at one thermometer was constant at T_λ .^{*} The result $Q \propto (T_\lambda - T)^{1.07}$ from our measurements⁷ together with Eq. (5) and $\rho_s \propto (T_\lambda - T)^{2/3}$ ¹² gave the following relation for the exponents:

$$\frac{2}{3} = \frac{1 - \alpha'}{m'} = 1.07 \pm 0.01 \quad \text{for } 0.03 \text{ m}^\circ\text{K} \leq T_\lambda - T \leq 12 \text{ m}^\circ\text{K}$$

which was found to be independent of geometry. With this relation only two unknown constants remain in Eq. (5). Fitting the data of our present runs performed at $1.3 \text{ mW/cm}^2 \leq Q \leq 158 \text{ mW/cm}^2$ and with different distances between the thermometers and the heater we found as mean values

$$a' = 110 \pm 20; \quad \alpha' = -0.41 \pm 0.03; \quad m' = 3.49 \pm 0.05$$

$$\text{for } 0.09 \text{ m}^\circ\text{K} \leq T_\lambda - T \leq 16 \text{ m}^\circ\text{K}$$

No dependence of these values on the position of the thermometers in the tube was found showing that the temperature gradient and velocities in the liquid were uniform in the flow direction along the cylinder. The above value of m' is in agreement with earlier measurements which indicated that m' increases from about 3 at $T < 1.7^\circ\text{K}$ to about 3.5 near T_λ .^{2,5,6} Plotting m' as a function of temperature we still found a temperature dependence of this exponent, showing an increase from about 3.4 at $T_\lambda - T = 10 \text{ m}^\circ\text{K}$ to about 3.6 at $T_\lambda - T = 0.1 \text{ m}^\circ\text{K}$.

3.2

For the following reasons we think that the above interpretation—which has mostly been used in the literature—may not represent an appropriate description of temperature gradients in He II due to mutual friction:

(1) If m' is temperature-dependent then its temperature dependence—as well as the temperature dependence of A' —should be taken into account in integrating Eq. (5). This integration is rather difficult and has never been done for the discussion of experimental results.

(2) It is difficult to see a physically meaningful interpretation of the temperature gradients in terms of the mutual friction theory in its present state^{1,2} if $m' \neq 3$ and is temperature-dependent (at least at temperatures where the number of vortices at a constant velocity is temperature-independent). The

^{*}In our previous experiments^{7,10} the second “thermometer” was the heater which then was in contact with the bottom of the tube. When the heater and the bottom of the tube reached T_λ a break in the thermogram of a thermometer inside the tube was observed. The temperature T at the thermometer at that instant was called “critical temperature” and the corresponding heat current was called “critical heat current.”

temperature may have an influence on the produced vorticity only at temperatures very near T_λ where thermally activated fluctuations of the order parameter become of importance.¹³

(3) The critical velocity for onset of superfluid friction in the flowing liquid should be about 4 mm/sec in our tube.⁹ This velocity is reached at a heat current of 3 mW/cm² at $T_\lambda - T = 1$ m°K. Only above this critical heat current can mutual friction occur and therefore a finite onset value should be taken into account.

The above arguments that suggested that the experimental results should be fit to an equation which contains a finite onset velocity for the mutual friction. Different formulas of this kind have been used.²⁻⁴ The appropriate one seemed to us a generalized Gorter–Mellink equation for a temperature gradient caused by mutual friction similar to the one suggested and discussed in detail by Vinen²:

$$\nabla T = Aw^{m-2}[w - w_0]^2 \rho_n / S = A(Q/\rho_s ST)^m [1 - w_0 \rho_s ST / Q]^2 \rho_n / S \quad \text{for } w > w_0$$

$$\nabla T = 0 \quad \text{for } w < w_0$$

where

$$A = a(T_\lambda - T)^\alpha \quad (6)$$

The exponent m has mostly been set equal to 3.^{2,4} We kept it—as well as the onset velocity w_0 and the temperature-dependent mutual friction coefficient A —arbitrary. Our aim was to test this equation with our data and see whether $m = 3$ may result by introducing an onset velocity $w_0 \neq 0$ and also to find the behavior of A near T_λ .

If $m = 3$ then Eq. (6) means that the mutual friction force between vortices and the normal component is proportional to their relative velocity w , proportional to the densities ρ_n and ρ_s , and proportional to the “intensity of turbulence” in the superfluid component. Vinen found this intensity to be proportional to $(w - w_0)^2$, where w_0 is an onset velocity.^{2,4} The correction term in brackets in Eq. (6) takes into account that mutual friction can only occur for $w > w_0$. The coefficient of proportionality A in Eq. (6) describes the interaction between vortex lines moving with the superfluid, and the thermal excitations comprising the counterflowing normal fluid.

Equation (6) with the four unknown parameters a , α , m , and w_0 can be written as

$$I_k = \int_{\varepsilon_{1,k}}^{\varepsilon_{2,k}} d\varepsilon / \varepsilon^{\alpha-2/3m} (1 - \varepsilon^{2/3} w_0 b ST / Q)^2 = aL \left(\frac{Q}{bST} \right)^m \rho_n / S \quad (7)$$

where L is the distance between thermometers, $\rho_s = b\varepsilon^{2/3}$,¹² $\varepsilon = T_\lambda - T$, $\varepsilon_{i,k} = T_\lambda - T_{i,k}$, $T_{i,k}$ is the temperature at thermometer i , and k is the number of the data point.

The right-hand side of Eq. (7) shows that the integrals I_k must have the same value for all data points $(\varepsilon_{1,k}; \varepsilon_{2,k})$ taken in a run at constant L and Q —if m is a constant. Our analysis showed that this is actually the case: $I_k = \text{const}$ independent

Fig. 3. Sum S of the root-mean-square deviations of the integrals I_k [see Eq. (7) in text] from their mean value I divided by I as a function of $\alpha - \frac{2}{3}m$. Each steep parabola was calculated for the indicated constant onset velocities w_0 . Their minima build the flatter parabola, the minimum of which gives the optimal values for $\alpha - \frac{2}{3}m$ and w_0 for this run ($Q = 26.6 \text{ mW/cm}^2$ in the 13.8-mm-wide tube).

of $(\varepsilon_{1,k}; \varepsilon_{2,k})$ for given values of $\alpha - \frac{2}{3}m$, w_0 , and Q ; hence $m = \text{const}$, as a first result.

Because we were unable to integrate (7) analytically we performed numerical integrations on a computer. For given values of the parameters $\alpha - \frac{2}{3}m$ and w_0 , the integrals I_k for all data points $(\varepsilon_{1,k}; \varepsilon_{2,k})$ of a run at constant Q were calculated. The parameters $\alpha - \frac{2}{3}m$ and w_0 were then changed to find their values at which the root-mean-square deviation $S = \sqrt{[\sum_k (I_k - I)^2]}$ of the integrals I_k from their mean value I has its minimum.* In Fig. 3 these root-mean-square deviations S divided by I are plotted as a function of $-(\alpha - \frac{2}{3}m)$ for different values of w_0 . For a given onset velocity w_0 the values of S/I lie on a parabola with a sharp minimum. The minima of all these parabolas build another, flatter parabolic curve. The minimum of this latter curve represents the best fit of Eq. (7) to our data. The corresponding optimum value of $\alpha - \frac{2}{3}m$ calculated in this way for $0.09 \text{ m}^\circ\text{K} \leq T_\lambda - T \leq 16 \text{ m}^\circ\text{K}$ and $1.3 \text{ mW/cm}^2 \leq Q \leq 158 \text{ mW/cm}^2$ for all runs is

$$\alpha - \frac{2}{3}m = -2.35 \pm 0.05$$

The broad minimum shows that the results are not very sensitive to small changes of the onset velocity. But it is obvious from Fig. 3 that the data are fitted

*For the calculation of the root-mean-square deviations the value of each integral I_k was weighted with $\varepsilon_{2,k} - \varepsilon_{1,k}$ which is a measure of the relative accuracy of the data and increases with $T \rightarrow T_\lambda$.

better with $w_0 \neq 0$ than with $w_0 = 0$, which would correspond to Eq. (5). The optimal values for the onset velocities are*

$$w_0 = 0.3 \pm 0.15 \text{ (cm/sec)} \quad \text{for the 13.8-mm-wide tube}$$

$$w_0 = 1.8 \pm 0.3 \text{ (cm/sec)} \quad \text{for the 0.1-mm-wide slit}$$

Both values are in agreement with values given in the literature⁹ for the onset of dissipation in heat-current experiments performed in similar geometries.

The values of $\alpha - \frac{2}{3}m$ and of w_0 might be calculated from the results of one run performed at one heat current Q . To separate the values of α and m one has to consider the results of runs performed at different Q . The value of m can be calculated from $I \propto Q^m$. In Fig. 4 we have plotted $I^{1/3}$ as a function of the heat current Q on logarithmic scales. The best fit to the data points is given by a straight line with a slope 1.00 ± 0.01 . Therefore, we get for the temperature-independent value of the exponent,

$$m = 3.01 \pm 0.04$$

Combining this result with the above given value of $\alpha - \frac{2}{3}m$, we find

$$\alpha = -0.35 \pm 0.06$$

Finally, from the values of the integrals I one can calculate with the right-hand side of Eq. (7) the prefactor in $A = a(T_\lambda - T)^\alpha$:

$$a = 560 \pm 180$$

All these results are found to be independent from the used geometry in the temperature range of this experiment ($0.09 \text{ m}^\circ\text{K} \leq T_\lambda - T \leq 16 \text{ m}^\circ\text{K}$).

The given errors are standard deviations calculated from the scatter of the data and do not include systematic errors. The relatively large errors of α and a are a result of the sensitivity of these quantities to changes in m : 1% change in m results in about 10% change in α and a .

3.3

It has been shown theoretically¹⁴ and experimentally¹⁵ that the thermal conductivity of He I *increases* with $(T - T_\lambda)^{-1/3}$ in approaching T_λ . We could show here that the thermal conductivity of He II *decreases* in approaching T_λ with a mutual friction constant $A \propto (T_\lambda - T)^{-0.35 \pm 0.06}$ —of course the origins

The obtained data for the onset velocity w_0 indicated a weak temperature dependence. According to the results of Clow and Reppy¹³ the critical velocity for our 13.8-mm-wide tube should be temperature-independent but geometry-dependent whereas for our 0.1-mm-wide slit the transition from the temperature-independent to the temperature-dependent but geometry-independent intrinsic critical velocity should occur at about $T_\lambda - T \approx 0.1 \div 1 \text{ m}^\circ\text{K}$. Our temperature region sometimes overlapped with both regions of the critical velocity, which might explain the observed temperature dependence and scatter in the results for w_0 . In fact, fitting with an ansatz $w_0^ \propto \varepsilon^{2/3}$ gave an improvement for the root-mean-square deviations for the runs performed in the slit as soon as $T_\lambda - T < 1 \div 2 \text{ m}^\circ\text{K}$, and reduced for both geometries the scatter in the results for the onset velocity. The geometry dependence of w_0 is verified by the difference of the given values of w_0 in the tube and in the slit.

Fig. 4. Cube root of the mean values I of the integrals I_k [see Eq. (7) in text] of different runs as a function of the heat-current density Q at which these runs have been performed. The slope of this curve is equal to $m/3$. ● Measurements in the 13.8-mm-wide tube; ■ measurements in the 0.1-mm-wide annular slit.

for these temperature dependences are quite different. In addition, the absolute magnitude of the thermal conductivity of He II at $T_\lambda - T > 10^{-5}$ °K is still orders of magnitude higher than the thermal conductivity of He I at $T - T_\lambda > 10^{-6}$ °K for example.

Our basic result is that temperature gradients due to mutual friction in superfluid helium can well be described by Eq. (6) with the above temperature- and geometry-independent values of the parameters a , α , and m even at temperatures only 0.09 m°K below T_λ . The data are fitted better by Eq. (6)—which is similar to the equation originally proposed by Gorter and Mellink¹ and modified later by Vinen² and Ahlers⁶—than by Eq. (5) where $w_0 = 0$ and $m' = f(T)$. The temperature-independent exponent of the heat current in this equation is $m = 3.01 \pm 0.04$. The mutual friction constant diverges with $A = (560 \pm 180)(T_\lambda - T)^{-0.35 \pm 0.06}$. The value of A is appreciably larger than values measured earlier at lower temperatures^{2,4,8} and extrapolated into our temperature region, due to the introduction of w_0 in Eqs. (6) and (7).

At present there is unfortunately no theoretical explanation for the temperature dependence of the mutual friction constant. This temperature dependence is also observed well outside of the temperature region where the number of vortices produced at a given velocity w will become temperature-dependent.¹³ The temperature dependence of A should then have its origin in a temperature-

dependent scattering between the thermal excitations of the normal component of He II and the counterflowing vortices. The thermal excitations are scattered at the vortex cores. A measure of the cores is the coherence length ξ of He II. Theoretical^{14,16} and experimental¹⁷ results show that ξ diverges with $(T_\lambda - T)^{-2/3}$ near T_λ . The critical exponent $-2/3$ of the coherence length is about a factor of two larger than the exponent of the temperature dependence of the mutual friction constant. Therefore, we are tempted to suggest that the changes in the properties of vortex lines as well as of thermal excitations (as shown, for example, by neutron scattering experiments¹⁸) at temperatures near T_λ are responsible for the temperature dependence of the mutual friction constant A . This is supported by the fact that also the temperature dependence of A at temperatures below 2.1°K ^{2,4,8} does not agree with the temperature dependence of the coherence length at these lower temperatures.¹⁷

4. ON THE DENSITY ρ_s OF THE SUPERFLUID COMPONENT OF He II IN THE PRESENCE OF A HEAT CURRENT

In Refs. 16 and 19 a depression of the transition temperature of liquid helium in the presence of a heat current Q or a relative velocity w in the liquid was predicted. This depression of T_λ is due to a depletion of the superfluid density ρ_s , or, in other terms, due to a reduction of the roton gap²⁰ when a relative velocity

Fig. 5. Temperatures T_1 and T_2 (solid lines) below and above a 0.1-mm-wide and 1-mm-long annular slit as function of time. The He II carried a heat current of 15.5 mW/cm^2 through the constriction. The temperature T_2 above the constriction was increased at about 10^{-5} K/min . At $T_\lambda - T = 0.18 \text{ m}^\circ\text{K}$ ¹⁶ or $0.10 \text{ m}^\circ\text{K}$,¹⁹ respectively, the transition to the normal state was expected to occur for He II carrying a heat current of 15.5 mW/cm^2 . This transition should result in the dashed temperature increase at one of these temperatures below the constriction, which was not observed in the experiment.

between the two components of He II is present. The depletion of ρ_s and the transition to the normal state of liquid helium at a temperature $T = T_\lambda(Q) < T_\lambda(0)$ is expected to be observable in measurements of the thermal conductivity of liquid helium near the transition temperature. In our experiments we could not find a sudden change in the behavior of the thermal conductivity of liquid helium even up to temperatures for which the difference $T_\lambda(0) - T$ was appreciably smaller than the theoretically predicted^{16,19} depression $T_\lambda(0) - T_\lambda(Q)$ for He II carrying a heat current.

The result of one measurement performed with the setup containing a 0.1-mm-wide and 1-mm-long constriction in the tube is shown in Fig. 5. The heat current transported by the liquid through the constriction was 15.5 mW/cm^2 and the temperature above it was slowly raised by warming the bath at about $10^{-5} \text{ }^\circ\text{K/min}$. The temperature below the constriction was observed to raise smoothly up to $T = T_\lambda(0) - 25 \mu^\circ\text{K}$.^{*} The mentioned theories predict the transition of liquid helium from the superfluid to the normal state to occur at $T = T_\lambda(0) - 180 \mu^\circ\text{K}$ ¹⁶ or $T = T_\lambda(0) - 100 \mu^\circ\text{K}$,¹⁹ respectively, if a heat current of 15.5 mW/cm^2 is present in the liquid. These two temperatures are indicated by the two dots in the figure; the uncertainty in the temperature measurement is smaller than the diameter of these dots.

If the liquid in the constriction had performed the transition to the normal state at one of the two indicated temperatures then we expected to see the sudden temperature increase below the constriction indicated by the dashed line at the first dot due to the worse thermal conductivity of He I.[†] Instead, up to temperatures which are about $0.1 \text{ m}^\circ\text{K}$ higher than the theoretically predicted transition temperature $T_\lambda(Q)$ we observed a smooth increase of T_1 and a thermal conductivity of liquid helium in the 0.1-mm-wide constriction which is at least in qualitative agreement with Eq. (6) and is about four orders of magnitude higher than the thermal conductivity of He I at only $1 \mu^\circ\text{K}$ above the unshifted $T_\lambda(0)$ when no heat current is present. Experiments with liquid helium in the free, 13.8-mm-wide tube without the constriction gave similar results. Hence, either the theoretically predicted shift of the transition temperature did not occur in our experiments and $T_\lambda(0) - T_\lambda(Q)$ is appreciably smaller than predicted, or the used method is not appropriate to detect this shift.

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^{*}At this temperature the heater reached T_λ .

[†]This temperature increase has been calculated by assuming that the thermal conductivity of He I is the same above a shifted λ -temperature $T_\lambda(Q)$ as it is above the unshifted λ -temperature $T_\lambda(0)$,¹⁵ taking also into account the specific heat of liquid helium and the time constant (3 sec) of our setup for measuring temperatures. At T_λ a jump of $1 \text{ m}^\circ\text{K}$ is then expected to occur within less than 10 sec.

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