Horizontal mixing in lakes

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Abstract. Horizontal mixing in the upper hypolimnion of lakes far from the boundaries was studied in lake basins with surface areas between 5 and 220 km² by observing the growth of the concentration distribution of the fluorescent dye sodium fluorescein (uranin). In each of the eight experiments, between 0.2 and 2 kg of uranin was instantaneously released into the lake in the appropriate depth (between 15 and 25 m) in such a way to keep the initial cloud size as small as possible. The horizontal extension of the cloud was repeatedly determined by integration of numerous vertical profiles. These surveys served to test theoretical models for horizontal mixing. The temporal development of the size and of the variances along the principal axes of the tracer concentration distribution was the main property considered here. The experiments cover a range of cloud sizes between 3×10^2 and 3×10^5 m². None of them support the hypothesis that cloud size grows with elapsed time raised to the power of 3 as predicted by the inertial subrange turbulence model first applied to dispersion by *Batchelor* [1950]. The shear-diffusion model of *Carter and Okubo* [1965] was found to provide a good description of the development of cloud size with time. This model also accounts for the fact that the tracer distributions were not radially symmetric. Effective horizontal diffusivities lie between 0.02 and 0.3 m² s⁻¹. Reevaluation of published data from experiments in Lake Ontario [e.g., *Murthy*, 1976] and in the ocean [e.g., *Okubo*, 1971] supports both the applicability of the shear-diffusion model and the doubts raised against the appropriateness of the inertial subrange model for scales up to 1000 m.

Introduction

The distribution of dissolved and suspended compounds in lakes and oceans is strongly influenced by the movement of the water. Water currents are turbulent in most situations. Therefore currents cannot be solely described by a set of deterministic equations. There always remains a stochastic component of motion superimposed on the average large-scale current pattern. Its adequate mathematical description has been, and still is, the aim of numerous theoretical and experimental investigations. Quantification of mixing processes is based on the understanding of the stochastic component of current patterns.

Progress has been different for the case of vertical and horizontal mixing, respectively. (Note that the terms vertical and horizontal are used instead of diapycnal and isopycnal.) Since in most situations vertical density gradients and/or the limited vertical extension of the water body keep vertical turbulent structures small, the local description of vertical turbulent fluxes by the first Fickian law is common practice for many, though not for all situations [Powell and Jassby, 1974; Wüest *et al.*, 1996]. Various techniques have been developed to determine the coefficient of vertical turbulent diffusivity, e.g., the temperature and the tracer method [Powell and Jassby, 1974; Ledwell *et al.*, 1993] as well as the method based on microstructure measurements of temperature or current velocity [Osborn and Cox, 1972; Oakey, 1982; Gregg, 1987]. In addition, there exist theoretical concepts to link vertical mixing with other hydrodynamic properties of the water column such as

vertical stability and turbulent kinetic energy dissipation [Gargett, 1984] or internal wave shear [Gregg, 1989].

In contrast to the numerous field investigations on vertical turbulent mixing, research on horizontal turbulent mixing has been scarce. An explanation often invoked is that horizontal mixing is fast compared to vertical mixing so that the major concentration gradients develop along the vertical axis. While this is commonly true in small- to medium-sized lakes, large lakes and the ocean pose a different problem. In these water bodies, horizontal transport is dominated by the large-scale currents. Thus horizontal mixing is a consequence of both the fluctuations of the velocity field and the shear of the mean advective currents. This difficulty does rarely arise for vertical mixing, since vertical currents are usually absent and transport can be attributed to turbulence alone.

Recently, several researchers have used drifter experiments to study horizontal mixing [Kawai, 1985; Sanderson and Booth, 1991; Pal and Sanderson, 1992; Sanderson *et al.*, 1995]. The statistics of drifter trajectories and its consequence on patch dispersion have found much attention [e.g., Sanderson and Booth, 1991, and references therein]. However, commonly, the number of drifters deployed is small (e.g., 5 to 10 as given by Sanderson *et al.* [1995]), and the calculation of average quantities by combining measurements from an ensemble of drifter experiments seems problematic.

Most experimental investigations on horizontal mixing were made with artificial tracers. For example, *Okubo* [1971], *Kullenberg* [1972], *Ewart and Bendiner* [1981], *Yanagi et al.* [1982], and *Ward* [1985] studied horizontal mixing in the ocean, and *Murthy* [1976] and *Palmer et al.* [1987] studied horizontal mixing in Lake Ontario. *Quay et al.* [1979] and *Colman and Armstrong* [1983] investigated horizontal mixing in very small lakes.

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dimensional distribution. If mixing is horizontally isotropic and the distribution radially symmetric, then $2K_{app} = K_{ma} = K_{mi}$.

According to (4a) the apparent horizontal diffusivity K_{app} can be determined from the actual rate of change of the cloud size σ^2 . Since in several field investigations the cloud size was determined only once after the release of the tracer, K_{app} is often approximated by the effective diffusivity K_{eff} that is defined by replacing in (4a) the temporal derivative by a finite difference and by assuming $\sigma^2(t = 0) = 0$. Thus

$$K_{eff} = \sigma^2 t \quad (4b)$$

The effective diffusivity is the temporal mean of the apparent diffusivity: $K_{eff} = t^{-1} \int_0^t K_{app}(t') dt'$. In order to compare different published experiments, in the above equations, σ^2 is replaced by σ_r^2 whenever the specific data were evaluated in terms of σ_r^2 instead of σ^2 .

Mixing by turbulence is a stochastic process. Theoretical models for horizontal mixing are based on the statistical properties of the small-scale turbulent velocity field and on its interaction with the large-scale velocity field. Thus in the strict sense of the statistical nature of mixing the models can only predict the mean temporal development of the concentration distribution for many identical experiments. The comparison of data with theoretical mixing models requires that the above quantities σ_{ma}^2 , σ_{mi}^2 , and σ^2 are evaluated for the ensemble mean concentration distribution determined from many identical experiments at identical diffusion times. Yet, the transient characteristics of most mixing situations do not allow us to conduct several mixing experiments that can be considered to be identical. Therefore as an approximation we interpret the temporal evolution of a concentration distribution of a single experiment within the framework of a specific mixing model.

In the next section, some published mixing models are described and their inherent assumptions analyzed. Next the experiments conducted during this research are presented. The final two sections are devoted to the interpretation of the results and to conclusions.

2. Models for Horizontal Diffusion

Theoretical diffusion models describe the temporal development of the statistical properties of a concentration distribution based on the properties of the velocity field that are assumed to be relevant for the mixing process. A turbulence model that has been used to describe tracer experiments in the ocean is the inertial subrange model [Kolmogorov, 1941] (application to dispersion by Batchelor [1950]; application to experiments by, e.g., Okubo [1971] and Ward [1985]). In this model the velocity field is assumed to be fully turbulent, isotropic, and stationary at scales of the cloud size. Eddies larger than the cloud size are supposed to cause advection, while smaller eddies are held responsible for diffusion relative to the center of mass of the concentration distribution. If the cloud size ranges within the scales of the inertial subrange of the turbulence spectrum, dimensional arguments can be used to predict the cloud size to increase with time t as

$$\sigma^2 = \text{const } \varepsilon t^3 \quad (5)$$

where ε is the dissipation rate of turbulent kinetic energy. In this model the velocity field is assumed to be three dimensional. Thus the statistical properties of the tracer distribution must be spherical symmetric; that is, the variance must have

the same size in all directions. However, vertical density stratification and the limited vertical size of a well-mixed water body limit turbulence in the vertical. As shown by Monin and Ozmidov [1985], (5) remains valid for two-dimensional (horizontal) isotropic turbulence if the cloud size is within the inertial subrange of the turbulence spectrum. The horizontal diffusivity K_{app} is calculated from (4a) and (5):

$$K_{app} = \text{const } \varepsilon^{1/3} \sigma^{-4/3} \quad (6)$$

The mathematical form of the inertial subrange model (equations (5) and (6)) suggests using, as a more general approach, a power law with unknown exponent m to describe the growth of σ^2 with time:

$$\sigma^2 = \text{const } t^m \quad (7)$$

The corresponding horizontal diffusion coefficient is calculated from (4) and (7) and expressed as a function of the actual cloud size σ :

$$K_{app}(\sigma) = \text{const } \sigma^{2l} \quad (8)$$

Note that in all these models (except (4b)) the initial time $t = 0$ does not necessarily correspond to the time when a tracer was introduced to the water body as an infinitesimally small cloud ($\sigma^2 = 0$). If the initial tracer cloud has a finite size, $t = 0$ represents the hypothetical time when the tracer cloud had zero size. In fact, a more general form of (7) would be $\sigma^2 = \text{const } (t + t_0)^m$ where t_0 is defined by the initial cloud size at $t = 0$: $\sigma_0^2 = \text{const } t_0^m$.

Okubo [1971], on the basis of many different dye experiments in the surface layer of the ocean, compiled the most reliable data set. He combined the results from several experiments into an ensemble of measurements and subsequently compared it to model predictions. Deviations of the data of such an ensemble from a model curve (e.g., (7)) were then regarded as randomly distributed. The different environmental conditions prevailing during the different experiments were neglected. The application of the empirical model (7) to the entire data set suggested an exponent of 2.34, i.e., $\sigma_r^2 \sim t^{2.34}$. Okubo suggested that for scales up to 1 km, between 1 km and 10 km, and above 10 km the data could be interpreted to follow $\sigma_r^2 \sim t^3$ and thus support the inertial subrange model. He, however, stated that his interpretation based on the inertial subrange model "is by no means decisive" [Okubo, 1971]. As it turns out, for scales up to 1 km (for this range Okubo [1971] cited mainly data of Foxworthy et al. [1966]), individual clouds from a single experiment grow more slowly with time than predicted for the ensemble of experiments (see Figure 9b). This is also true for the experiments conducted by Murthy [1976] in the hypolimnion of Lake Ontario. All these experiments combined predict the distribution to increase according to $\sigma_r^2 \sim t^{3.3}$, but individual experiments give exponents between 0.5 and 1.5.

Joseph and Sendner [1958] developed a model for horizontal diffusion by postulating that the distance between an individual water parcel and the center of mass of the tracer distribution increases on average at a constant velocity. This leads to an exponent in (7) of $m = 2$ and to an effective diffusivity K_{eff} which increases linearly with time. Finally, we note that the Fickian law with constant diffusivity corresponds to $m = 1$. Thus all these models combined cover a range for m lying between 1 and 3.

Yet, the most severe restriction of these models is their

A detailed overview on horizontal mixing studies using tracer experiments is given by Peeters [1994].

In this work we describe experiments with uranin (an artificial fluorescent dye) that were conducted in several Swiss lakes. The largest of these lakes (Lake Neuchâtel) has a surface area of 218 km², and the smallest (Lake Alpnach, a basin connected to Lake Lucerne) has a surface area of 4.8 km². The experiments were carried out far from the lake boundaries where the isopycnals are horizontal, except for temporal distortions due to internal waves. In the following discussion, horizontal mixing is thus considered to be equivalent to isopycnal mixing.

When comparing experiments on horizontal mixing, problems arise not only due to the different physical and orographic conditions in these lakes but also because different investigators evaluate their results with different concepts. In this work, mixing is defined by those effects of the horizontal velocity field which cause the concentration distribution of a passive tracer cloud to spread relative to its center of mass (relative diffusion). The movement of the center of mass is considered to be due to (large scale) advection. To avoid confusion, the above definition of mixing should be carefully distinguished from other concepts which also can be found in the literature. Consider, e.g., the fate of a chemical which is introduced into the water body by some local source. Advection is responsible for the (fast) transport of the chemical away from the source to other locations in the lake. The time to reach the far end of the lake is often called "mixing time." Note that this time depends on the large-scale current pattern which contributes only little to the dilution of the tracer cloud by surrounding water. It is by dilution that the concentration within the moving cloud decreases. Mixing as employed in this study includes all effects causing the tracer cloud to spread, i.e., the small-scale turbulent motions which cause dilution of the tracer cloud and the interaction between this small-scale motion with the large-scale current pattern.

To quantify the term "mixing" as defined above, a measure for the spread of a concentration distribution is required. The distribution can be characterized by quantities such as the total mass M_0 , the coordinates of the centre of mass x_i , and variance and covariance σ_{xx} .

$$M_0 = \int c(x, y, z) dx dy dz$$

$$x_i = \frac{1}{M_0} \int \int \int x_i c(x, y, z) dx dy dz \quad 1)$$

$$\sigma_{ij} = \frac{1}{M_0} \int \int \int (x_i - x_{i0})(x_j - x_{j0}) c(x, y, z) dx dy dz$$

where $i, j = 1, 2, 3$, x_i are the Cartesian coordinates ($x_1 = x$, $x_2 = y$, and $x_3 = z$) and $c(x, y, z)$ is the spatial concentration field at a given time. Since mixing neither influences the total mass nor the movement of the centre of mass, mixing is solely described by the temporal change of the variances and covariances $\sigma_{x_i x_j}$. Moments of higher order than 2 are not considered here. Thus the complex information of a general three-dimensional concentration distribution is reduced to that contained in a three-dimensional normal distribution.

To analyze horizontal mixing, the concentration distribution is integrated vertically. This leaves us with a two-dimensional

distribution that generally is not isotropic. The eigenvalues of the matrix

$$\Sigma_2 = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix}$$

yield the variances in the direction of the major and minor principal axes (σ_{ma}^2 and σ_{mi}^2) of the vertically integrated distribution. The directions of the principal axes are determined by the eigenvectors of Σ_2 . The instantaneous rate of mixing can be characterized by turbulent diffusion coefficients K_{ma} and K_{mi} that are related to the temporal evolution of σ_{ma}^2 and σ_{mi}^2 :

$$d\sigma_{ma}^2/dt = 2K_{ma} \quad d\sigma_{mi}^2/dt = 2K_{mi} \quad (2)$$

If K_{ma} and K_{mi} are constant, (2) corresponds to the results of the first Fickian law for the spreading of a distribution far from the boundaries. However, as shown below the variances rarely grow linearly with time. Therefore the diffusion coefficients defined by (2) are time (or scale) dependent.

The horizontal extension of the vertically integrated concentration distribution (thereafter referred to as cloud size) can be defined as [Okubo, 1971]

$$\sigma^2 = 2\sigma_{ma}\sigma_{mi} \quad (3)$$

Note that σ^2 , in contrast to, e.g., the total variance, $\sigma_{ma}^2 + \sigma_{mi}^2$, does not change under the influence of laminar flow, even if velocity gradients distort the shape of a tracer patch. Consider, as an example, a circular tracer cloud influenced by laminar currents along the x axis with velocity gradient in the y direction. The currents transform the circle into an ellipse. When time passes, the major axis of the ellipse will grow and turn into the x direction, while the minor axis will shrink. Thus σ_{ma} increases and σ_{mi} decreases with time, while σ^2 remains constant.

In the case of a two-dimensional normal distribution the lines of equal concentration are ellipses. The ellipse that corresponds to e^{-1} (37%) of the maximum concentration encloses 63% of the total mass of the distribution and has an area of $F = \pi\sigma^2$. The ellipse with major and minor axis of length $3\sigma_{ma}$ and $3\sigma_{mi}$, respectively, encloses 99% of the total mass of the distribution.

In many publications on horizontal mixing, symmetric distributions were constructed from measured concentrations [e.g., Okubo, 1971; Murthy, 1976]. For different concentration values c the areas enclosed by iso-concentration lines were determined. The equivalent radius $R(c)$ is then defined as the circle with an area equal to the area enclosed by the iso-concentration line. Instead of using σ^2 the size of the cloud is then defined by the variance σ_r^2 of the inverse function $c(R)$. In the case of a two-dimensional normal distribution the iso-concentration line which represents e^{-1} of the maximum concentration encloses an area of $F = \pi\sigma_r^2$ and thus σ_r^2 and σ^2 are equal.

In addition to cloud size σ^2 and variance σ_r^2 , which both are measures of the extension of a tracer cloud, a scale of diffusion defined as $l_r = 3\sigma_r$ is commonly in use [e.g., Okubo, 1971].

In analogy to (2) the temporal evolution of σ^2 serves to define an apparent horizontal diffusion coefficient K_{app} by the relation

$$d\sigma^2/dt = 4K_{app} \quad (4a)$$

The factor 4 on the right-hand side reflects the fact that in contrast to (2), (4a) describes the evolution of a two-

radial symmetry. This is in contradiction with most experimental results. Non-radially symmetric distributions can be described by shear-diffusion models [Carter and Okubo, 1965; Okubo, 1967, 1968; Kullenberg, 1972; Young *et al.*, 1982]. For the development of these models one assumes that the velocity field can be separated into two parts, one including the turbulent fluctuations and the other corresponding to the (nonturbulent) large-scale advective flow. The diffusion time (elapsed time since tracer release) is supposed to be sufficiently large so that the autocorrelation of the velocity fluctuations becomes negligible. Consequently, the spread of a tracer distribution caused by the turbulent part of the velocity field can be modeled using the Fickian law [Taylor, 1921]. Assuming the turbulent diffusion coefficients K_x , K_y , and K_z to be spatially constant, the advection-diffusion equation can be written as

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z} = K_x \frac{\partial^2 c}{\partial x^2} + K_y \frac{\partial^2 c}{\partial y^2} + K_z \frac{\partial^2 c}{\partial z^2} \quad (9)$$

where u , v , and w are the components of the nonturbulent part of the velocity field in the x , y , and z direction, respectively. If the mean velocity field is assumed to be purely horizontal and to lie along the x axis with $u = -ya - zb$, where the horizontal and vertical shear, a and b , respectively, are constant, one obtains

$$\frac{\partial c}{\partial t} - (ay + bz) \frac{\partial c}{\partial x} = K_x \frac{\partial^2 c}{\partial x^2} + K_y \frac{\partial^2 c}{\partial y^2} + K_z \frac{\partial^2 c}{\partial z^2}. \quad (10)$$

This corresponds to the model by Carter and Okubo [1965], except for a constant advection term in the x direction.

The variances and covariances of the concentration distribution obeying the Carter and Okubo [1965] model are given by

$$\begin{aligned} \sigma_{xx} &= 2K_x t + \frac{2}{3}(a^2 K_y + b^2 K_z)t^3; \\ \sigma_{yy} &= 2K_y t; \\ \sigma_{zz} &= 2K_z t; \\ \sigma_{xy} &= -K_y a t^2; \\ \sigma_{xz} &= -K_z b t^2; \\ \sigma_{yz} &= 0. \end{aligned} \quad (11)$$

These results were obtained by introducing a momentum generating function (this method is demonstrated by Okubo [1967]) and assuming the initial tracer distribution to be point-like. The solution for $c(x, y, z, t)$, found by Carter and Okubo [1965] (see also Okubo [1968]) is in agreement with the variances and covariances given in (11). However, our (11) disagrees with the expressions given by Okubo [1968] for σ_{xx} . Okubo's expressions must be incorrect; for example, his equation for σ_{xx} is not symmetric in the z and y coordinates although the velocity field is.

From (11) the variance in the direction of the minor and major principal axes and the cloud size can be calculated. Assuming $K_x = K_y = K_h$, one obtains

$$\begin{aligned} \sigma_{ma}^2 &= 2K_h t + \frac{1}{3}(a^2 K_h + b^2 K_z)t^3 \\ &\quad + \sqrt{K_h^2 a^2 t^4 + [\frac{1}{3}(a^2 K_h + b^2 K_z)t^3]^2}; \\ \sigma_{mi}^2 &= 2K_h t + \frac{1}{3}(a^2 K_h + b^2 K_z)t^3 \\ &\quad - \sqrt{K_h^2 a^2 t^4 + [\frac{1}{3}(a^2 K_h + b^2 K_z)t^3]^2}; \\ \sigma^2 &= 2\sigma_{ma}\sigma_{mi} = 2\sqrt{4K_h^2 t^2 + \frac{1}{3}K_h(a^2 K_h + 4b^2 K_z)t^4}. \end{aligned} \quad (12)$$

If the velocity shear is zero ($a = b = 0$), one obtains $\sigma^2 = 4K_h t$. This is the solution to (4a) for a constant diffusivity K_h (Fickian diffusion) and zero initial variance. For large times the cloud size σ^2 increases as t^2 like in the Joseph and Sendner [1958] model. However, the latter describes radially symmetric distributions, in contrast to the Carter and Okubo [1965] model for which σ_{ma}^2 and σ_{mi}^2 are usually different. In fact, for long diffusion times the Carter and Okubo model predicts that $\sigma_{ma}^2 \sim t^3$ and $\sigma_{mi}^2 \sim t$ (see (12)). The major principal axis of the tracer distribution then turns into the direction of the current ($\sigma_{xy}/\sigma_{xx} \xrightarrow{t \rightarrow \infty} 0$).

If the tracer distribution at $t = 0$ is not point-like but a three-dimensional normal distribution with initial variance σ_{xx0} , σ_{yy0} and σ_{zz0} , one obtains:

$$\begin{aligned} \sigma_{xx} &= 2K_x t + \frac{2}{3}(a^2 K_y + b^2 K_z)t^3 + \sigma_{xx0} + (a^2 \sigma_{yy0} + b^2 \sigma_{zz0})t^2; \\ \sigma_{yy} &= 2K_y t + \sigma_{yy0}; \\ \sigma_{zz} &= 2K_z t + \sigma_{zz0}; \\ \sigma_{xy} &= -K_y a t^2 - a \sigma_{yy0} t; \\ \sigma_{xz} &= -K_z b t^2 - b \sigma_{zz0} t; \\ \sigma_{yz} &= 0. \end{aligned} \quad (13)$$

From these results, σ_{ma}^2 , σ_{mi}^2 , and σ^2 can be calculated as above.

Other shear-diffusion models differ from the model by Carter and Okubo [1965] with respect to their nonturbulent part of the velocity field. Okubo [1967] extended the Carter and Okubo model by including oscillating currents. Kullenberg [1972] considered horizontally rotating currents with current speed decreasing with depth. In these models the cloud size still increases with time but superimposed there is an oscillatory variation. For instance, for large times the Kullenberg model causes the cloud size to increase as $\sigma^2 \sim t^2 [\frac{10}{3} + \frac{2}{3} \cos(2\omega t)]^{1/2}$, where ω is the angular frequency of the rotating current. Expressions of $\sigma_{x_i x_j}$ for the Kullenberg model are given by Peeters [1994].

The above models were generalized by Young *et al.* [1982], who investigated the interaction between turbulent diffusion and the velocity field produced by internal waves. They found that for large diffusion times the spread of a concentration distribution is dominated by the velocity shear of a steady current compared to the oscillating components and that $\sigma_{xx} \sim t^3$ as in the model by Carter and Okubo [1965].

Sanderson and Okubo [1988] analyzed the influence of a random field of internal waves by perturbation theory on the Navier-Stokes equation. They assumed the spectrum of the horizontal velocity to be proportional to the square of the frequency and found, as for the inertial subrange model, that $\sigma^2 \sim t^3$ for cloud sizes between 10^2 and 10^4 m². Yet for cloud sizes larger than 10^4 m² the exponent of the time dependence drops below 3.

In connection with the interpretation of drifter experiments, recent research relates the fractal properties of drifter trajectories to particle dispersion [e.g., Sanderson and Booth, 1991]. Sanderson and Booth find that particle dispersion can be modeled as accelerated fractional Brownian motion providing Okubo's [1971] $\sigma^2 \sim t^{2.34}$ power law. Note that, especially for small scales of diffusion, drifter and tracer experiments are not

Table 1a. General Information on the Eight Tracer Release Experiments

Experiment ^a	Date	Surface Area of Basin, km ²	Maximum Depth of Basin, m	Duration of Experiment, hours	Number of Surveys ^b	Tracer Mass Released, g	Depth of Tracer Release, m	Stability at Depth of Tracer Release, s ⁻²
	Oct. 8, 1990	4.8	35	73	7	1000		2×10^{-4}
	March 25, 1991	22.0	151	52	6 (7)	2000		7×10^{-6}
	June 24, 1991	217.9	154	48	4	1500		4×10^{-4}
	Sept. 19, 1991	22.0	200	49	4 (5)	1000		8×10^{-4}
	Sept. 30, 1991	22.0	200	12	3	250		3×10^{-4}
	Oct. 10, 1991	22.0	200	9	2 (3)	200		4×10^{-4}
	June 10, 1992	22.0	151	50	7	305		1×10^{-4}
	Oct. 6, 1992	22.0	151	52	8	265		3×10^{-4}

^aThe names of the experiments stand for Alpnachersee (Alp), Vitznauersee (Vitz), Urnersee (Urn), and Lac de Neuchâtel (Neu). The first three basins are part of Lake Lucerne.

^bNumbers in parentheses are total number of surveys including those with insufficient tracer mass recovery. The latter were not considered in the following chapters.

directly comparable since they are sensitive to different ranges of the velocity spectrum.

The following discussion will mainly focus on two models, the inertial subrange model [Batchelor, 1950] and the shear-diffusion model by Carter and Okubo [1965]. The former received wide attention through the work of Okubo [1971], while the latter provides better agreement with most measurements due to its anisotropy. The shear-diffusion models with oscillating currents are not considered any further, since most field data do not allow us to properly determine the various parameters of these models.

3. Experiment

A total of eight tracer experiments were conducted at different times of the year in the upper hypolimnion of several Swiss lakes. Table 1a summarizes information on these lakes as well as on the experiments. Alpnachersee (Alp), Vitznauersee (Vitz), and Urnersee (Urn) are all basins of Lake Lucerne located in the center of Switzerland (for a general description of the basins, see Aeschbach-Hertig [1994]). Lac Neuchâtel (Neu), located in the western part of Switzerland, is one of the largest Swiss lakes.

The fluorescent dye sodium fluorescein (uranin, $C_{20}H_{10}Na_2O_5$) was used as artificial tracer in all experiments. This dye does not significantly adsorb on suspended particles and is nontoxic. However, it is decomposed by photolysis and strongly affected by hydrolysis at a pH < 7 [Smart and Laidlaw, 1977]. All experiments took place in carbonate lakes (pH well above 7) at depths of about 20 m where the light intensity is less than 2% of the intensity at the lake surface. Thus, during the experiments of a few days, decomposition of uranin was negligible.

The tracer was released through a tube of 1.5-m length containing 40 holes of 2.5-mm diameter. Groups of five holes around the tube were located about 20 cm apart. The tube was connected to a hose and placed vertically into the water at the target depth. Between 200 and 2000 g of uranin (see Table 1a) was dissolved in 70 L of warm surface water on board the ship and pumped through the holes into the lake within about 60 s. By this procedure the tracer was diluted with at least 100 m³ of water immediately after release. The specific expansion coefficient of water due to uranin, $\rho^{-1} \partial \rho / \partial C$, was measured to be approximately $4.4 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}$. Except for Vitz 1, the vertical stability frequency of the water column at the depth of tracer release, $N^2 = -g\rho^{-1} d\rho/dz$, was 10^{-4} s^{-2} or greater

(Table 1a). Thus, even under the most conservative assumption (dilution of 2 kg of uranin into 100 m³ of water), the uranin/water mixture sank by less than 1 m after release (except for Vitz 1).

The dye concentration was measured in situ at a rate of 10 Hz using a fluorometer (either "Variosens II," Impulsphysik GmbH, or "Backscat," Geological and Marine Instrumentation). The output signal was passed through an isolation amplifier and subsequently made available to the computer by analogue-digital conversion using a data acquisition board (National Instruments NB-MIO-16L25). A conductivity-temperature-depth (CTD) probe (Multisonde MS 040, Meerestechnik Elektronik GmbH) attached to the fluorometer provided information on pressure (resolution 0.01 dbar), temperature (resolution 0.001°C), and electrical conductivity (resolution $0.1 \mu\text{S cm}^{-1}$) at a rate of 8 Hz. Depth and water density were calculated from these data. The exact position of the ship was obtained from a microwave positioning system (Trisponder Systems, Del Norte Technology Inc.) at a rate of 1 Hz (resolution $\pm 1 \text{ m}$). The data flow from the different instruments was coordinated by a data acquisition program developed for a Macintosh computer. On-line display of pressure and tracer concentration enabled an efficient survey of the tracer distributions.

During each experiment the tracer cloud was repeatedly mapped, each mapping requiring a few hours. Each mapping will be called a survey. The number of surveys for each experiment is listed in Table 1a. Examples of tracer clouds constructed from individual surveys from different experiments are shown in Figure 1. During the surveys the fluorometer and the CTD probe were moved up and down in a yo-yo-like manner. Vertical direction was changed whenever the probes were above or below the tracer cloud. Usually the ship was steered along straight lines through the tracer cloud at a constant speed of about 1 m s^{-1} and turned outside the cloud. The position of the probes was calculated from the position of the ship by using a velocity-dependent horizontal drag distance between ship and probe.

Every up or down cast is called a vertical profile. Each survey typically consists of more than 100 vertical profiles. The distance between vertical profiles along the ship track is typically less than 15 m, i.e., less than the length of the dashes shown in Figure 1. Vertical integration of the concentration in each profile provides a two-dimensional concentration distribution $c(x, y)$. In addition, horizontal integration allowed calculation

Figure 1. Examples of concentration distributions constructed from surveys of different experiments: (a) Vitznauersee (Vitz 1), 30 hours after release; (b) Urnersee (Urn 1), 25 hours; (c) Vitz 3, 13 hours; (d) Lac de Neuchâtel (Neu), 11 hours; (e) Alpnachersee (Alp), 28 hours. The dashed lines show the ship track during the surveys. The open circles mark the center of mass of the dye distribution. Note that the total amount of released tracer varies between experiments (see Table 1b).

for each survey of the total observed tracer mass. In most experiments the tracer mass recovered was on average more than 50% of the released mass (Table 1b).

During most of the experiments, drifters were released at the same depth as the tracer. From the position of the drifters before and after a survey an average linear drift velocity was determined. The coordinates of the individual profiles were corrected for the average drift in proportion to the time difference between the start of the survey and the time when the particular profile was taken. As it turned out, this correction is

small and does not significantly change the tracer distribution. Two-dimensional integration of the linearly interpolated distribution $c(x, y)$ provides the total tracer mass, the coordinates of the center of mass, the variances in the directions of the principal axes, and the cloud size of the tracer distributions.

The initial patch size of the tracer immediately after tracer release was estimated by assuming the concentration to be at a constant value inside and to be zero outside a region having a radius of about 10 m in the horizontal and a total height of approximately $h \approx 1.5$ m in the vertical (only in the experi-

Table 1b. Characteristic Data of the Eight Tracer Release Experiments

Experiment	Estimated Cloud Size at $t = 0$, m^2	Tracer Mass Recovered ^a (Min. – Max.), %	Tracer Mass Recovered, (Average), %	σ_{ma}/σ_{mi} (Min. – Max.)	σ_{ma}/σ_{mi} (Average)	Average Velocity of Center of Mass, $cm\ s^{-1}$
Alp	300	14 – 51	33 $\pm$ 5	1.4 – 9.9	2.8 $\pm$ 1.1	0.7 $\pm$ 0.1
Vitz 1	160	(19) 44 – 200	110 $\pm$ 20	1.8 – 6.7	3.3 $\pm$ 0.7	1.8 $\pm$ 0.3
Neu	200	31 – 51	40 $\pm$ 5	1.8 – 2.4	2.2 $\pm$ 0.2	1.8 $\pm$ 0.3
Urn 1	50	(9) 40 – 76	60 $\pm$ 10	1.8 – 3.1	2.5 $\pm$ 0.3	0.9 $\pm$ 0.1
Urn 2	50	54 – 122	90 $\pm$ 20	1.5 – 4.8	2.8 $\pm$ 1.0	2.6 $\pm$ 0.7
Urn 3	90	(13) 78 – 99	90 $\pm$ 10	2.8 – 5.6	4.2 $\pm$ 1.4	2.9 $\pm$ 0.5
Vitz 2	50	73 – 188	130 $\pm$ 20	1.1 – 7.9	3.6 $\pm$ 1.0	1.1 $\pm$ 0.2
Vitz 3	50	39 – 153	110 $\pm$ 20	1.1 – 4.5	2.5 $\pm$ 0.5	0.9 $\pm$ 0.2

^aNumbers in parenthesis are tracer mass recovered in the dismissed surveys.

ment Vitz 1 $h \approx 7$ m). A drift of the ship during tracer release causes an elongation of the tracer distribution in the drift direction. The resulting increase in the initial size of the tracer distribution can be estimated from the monitored ship movement during tracer release. Initial cloud sizes were calculated according to (3) to range between 50 and 300 m^2 (Table 1b). The estimated initial height h was compared to the concentration distribution of several profiles taken immediately after tracer release. The height h estimated from the profiles was $h \leq 1.5$ m in all experiments except Vitz 1 where $h \approx 7$ m. The initial vertical variance of the tracer cloud is given by $\sigma_{z0}^2 = \frac{1}{12}h^2$, which is approximately $\sigma_{z0}^2 \approx 0.2$ m^2 for all experiments except for Vitz 1 where $\sigma_{z0}^2 \approx 4.4$ m^2 .

4. Results

As an example, Figure 2a shows the temporal development of the tracer cloud in the experiment Vitz 1, represented by the ellipses with principle axes of length $3\sigma_{ma}$ and $3\sigma_{mi}$, respectively. Orientation and size of the principle axes were calculated from the matrix Σ_2 (see (1)). For a two-dimensional normal distribution the area of the ellipses is equal to 4.5π times the cloud size (see (3)). The ellipses would then represent the iso-concentration line corresponding to about 1% of the maximal concentration and enclose 99% of the total tracer mass. Cloud 6 represents the tracer distribution shown in Figure 1a. Cloud 7 was measured about 2 days after the tracer release and marks the end of this particular experiment.

The linear velocity of the center of mass was calculated for each time interval between two consecutive surveys. In the experiment Vitz 1 these velocities reach a maximum value of 2.7 $cm\ s^{-1}$ (2.3 $km\ day^{-1}$). For each experiment the average velocity was calculated as the mean of the interval velocities (Table 1b). They range between 0.8 and 2.9 $cm\ s^{-1}$ (0.6 and 2.5 $km\ day^{-1}$). Direction and magnitude of these velocities can strongly vary within short periods of time. For example, in the tracer experiments Urn 2 and Urn 3 which were conducted on 2 consecutive days, the direction of the advective current changed from southwest to northeast.

Most tracer distributions are elongated rather than radially symmetric (Figures 1 and 2a), in spite of the initial circular shape of the cloud in some of the experiments (Vitz 2, Vitz 3, Urn 1, Urn 2). The variance along the major principal axis σ_{ma}^2 mostly grows faster than the variance along the minor principal axis σ_{mi}^2 (Figures 2b and 2c), although it should be noted that the spatial orientation of these axes is not fixed. For each experiment the temporal evolution of σ_{ma}^2 and σ_{mi}^2 is de-

scribed by power laws of the form $\sigma_{ma}^2 \sim t^p$ and $\sigma_{mi}^2 \sim t^q$. Averaging the exponents obtained for all experiments yields the following mean values: $p = 1.5 \pm 0.2$ for σ_{ma}^2 and $q = 1.0 \pm 0.1$ for σ_{mi}^2 . Thus the ratio $\sigma_{ma}^2/\sigma_{mi}^2$ increases with time. Cloud size and variance in the direction of the principal axes observed during the experiments Vitz 1 and Vitz 2 are shown in Figures 2b and 2c.

The temporal increase of the cloud size σ^2 , calculated according to (3), is shown in Figure 3 for all experiments. Except for a constant factor, the functional dependence of the cloud size on time is rather equal. The same functional dependence is found for σ_r^2 , the variance of a cloud assumed to be radially symmetric. However, the σ_r^2 values are 2–3 times smaller than σ^2 . Remember that for two-dimensional normal distributions, the two values, σ^2 and σ_r^2 , should be equal. Obviously, the clouds have more complex shapes. For instance, the center of mass of the tracer distributions does not usually coincide with the location of maximum concentration as required for a normal distribution (see Figure 1). In addition, the development of “islands” with enhanced concentration was often observed. These islands and the shift of the center of mass relative to the location of maximum concentration result in variances of the tracer distribution that are larger than the variances of the constructed radially symmetric distribution. Since the variances σ_r^2 range between 100 and 10^5 m^2 , our experiments cover scales of diffusion between 30 and 1000 m.

In order to test different models for horizontal mixing, the errors of the derived cloud characteristics (cloud size σ^2 , variance in the direction of the principle axes σ_{ma} and σ_{mi}) have to be known. They were estimated by a procedure that is similar to the bootstrap estimator. For each survey a hypothetical concentration distribution was generated by constructing a hypothetical ship track. Concentrations along the hypothetical track were determined by interpolation from the measured tracer concentrations. The distance between the hypothetical profiles was chosen to give the same spatial resolution as the real profiles of the survey. The variances $\sigma_{ma,H}^2$ and $\sigma_{mi,H}^2$ and the cloud size σ_H^2 of the hypothetical concentration distribution were calculated and compared to the values obtained for the measured distribution.

The difference between the two sets of variances gives a semiquantitative measure for the uncertainty originating from the reconstruction of a continuous tracer distribution from discrete data points with limited spatial resolution. However, this is only true provided that the subgrid concentration variance is smaller than the variance measured for the grid points.

Figure 2. Temporal development of tracer distributions in Vitznau basin. (a) Schematic representation of the changing tracer cloud during the experiment Vitz 1. Each cloud is represented by an ellipse with principle axes of length $3\sigma_{ma}$ and $3\sigma_{mi}$ calculated from the matrix Σ_2 (see (1)). The solid circles are the center of mass. Ellipse 6 corresponds to the tracer cloud shown in Figure 1a. (b) Temporal development of cloud size $\sigma^2 = 2\sigma_{ma}\sigma_{mi}$ (solid circles) and variance in the direction of the major (open squares) and minor (crosses) principal axes for experiment Vitz 1. (c) As in Figure 2b, for Vitz 2. Fitting the cloud size with a power function $\sigma_m^2 = A_1 t^{A_2}$ (solid line) yields the exponent $A_2 = 1.4$.

This assumption seems to be reasonable given the mostly smooth shape of the tracer clouds and the fact that the errors turn out to be rather similar for the various surveys. Typically, the error (defined as 1 standard deviation) of the cloud size σ^2 is 20%, and the errors of the variances σ_{ma}^2 and σ_{mi}^2 are 25 and 35%, respectively (see, e.g., Figure 2c). The errors can be slightly larger for those surveys which were carried out very shortly after the release of the tracer.

From the cloud size as function of time the horizontal diffusion coefficient can be determined either from (4a) or (4b). Our experiments, conducted in the upper hypolimnion of lakes of medium size, yield K_{eff} values between 0.02 and 0.3 m² s⁻¹ in agreement with measurements in the hypolimnion of Lake Ontario [Murthy, 1976]. Typical K_{eff} values in the ocean are larger by a factor of 3–5 [e.g., Okubo, 1971], while in very small ice-covered lakes, K_{eff} can be much smaller than in our experiments [e.g., Colman and Armstrong, 1983].

During the observational period of each experiment (3–4 days) the vertical variance of the tracer distributions, calculated by averaging tracer concentrations at constant density for all vertical profiles of a survey, increased only very slowly. The average concentration profile as a function of density was then transferred to a concentration profile as a function of depth using the mean density profile determined for the particular survey. Except for the experiments Vitz 1 and Urn 3 the precision of the measurements was sufficient to rule out that the vertical variance increases by more than 0.2 m² day⁻¹. Thus the vertical diffusion coefficient, $K_z = (1/2)(d\sigma^2/dt)$, must be of the order of 1×10^{-6} m² s⁻¹ or less. Diapycnal diffusion coefficients in lakes that are determined from water temperature or dissolved chemical substances are usually larger by a factor of 3 or more [e.g., Li, 1973; Imboden and Wüest, 1995]. However, these tracers are usually spread across the whole lake area. Recent investigations in the ocean [Ledwell and

Watson, 1991] and in lakes [Wüest *et al.*, 1996] suggest that vertical mixing occurs mainly at the bottom boundary of the water basin followed by horizontal mixing of the vertically mixed components. Since during the above experiments the tracer did usually not hit the boundaries, the small K_z value can be interpreted as reflecting the vertical diffusivity in the “interior” of the water body.

5. Discussion

In this section the experimental data are compared with the different diffusion models described in section 3. A χ^2 test is employed to check whether the cloud size predicted by a specific model, $\sigma_{\text{mod}}^2(t; A_1, \dots, A_p)$, is compatible with the experimental data. The parameters A_1 to A_p are adjusted by a weighted least squares fit. The quantity

$$\chi_{\text{mod}}^2 \equiv \sum_{i=1}^n \left(\frac{\sigma_i^2 - \sigma_{\text{mod}}^2(t_i; A_1, \dots, A_p)}{w_i} \right)^2 \quad (14)$$

is calculated, where w_i is the absolute error of σ_i^2 and i denotes the i th survey of an individual experiment. The w_i were calculated by assuming that the relative error of the cloud size is 20%. The variable χ^2 is supposed to be χ^2 distributed with k degrees of freedom, where k is the difference between the number of data points (surveys) and the number of fitting parameters. If the probability of the variable χ^2 to be larger than or equal to χ_{mod}^2 , $P(\chi^2 \geq \chi_{\text{mod}}^2)$, is less than 1%, then the model $\sigma_{\text{mod}}^2(t; A_1, \dots, A_p)$ is rejected.

Figure 3. Temporal increase of cloud size σ^2 of all experiments calculated according to (3).

Figure 4. Fitting of individual experiments to the empirical model $\sigma_m^2 = A_1 t^{A_2}$. The relative error was assumed to be equal for all measurements. A growth of the cloud size as t^3 is indicated by the dashed line.

5.1. Inertial Subrange Model

According to (5) the inertial subrange model implies that the cloud size grows with time as t^3 . Fitting the cloud sizes to the power law $\sigma_{\text{mod}}^2 = A_1 t^{A_2}$ yields exponents A_2 between 1.1 and 1.7 for the different experiments. As an example, three fits are shown in Figure 4. They clearly deviate from the t^3 curve. The fitting parameters for all experiments are listed in Table 2. The mean exponent calculated for those experiments consisting of at least four surveys is 1.4 ± 0.1 . According to the χ^2 test the probability $P(\chi^2 \geq \chi_{\text{mod}}^2)$ for the model function $\sigma_{\text{mod}}^2 = A_1 t^3$ is less than 1%, even if the error of the measured cloud sizes is raised to 50%. Thus the model $\sigma_{\text{mod}}^2 = \text{const } t^3$ must be rejected.

As pointed out by Csanady [1973], for short diffusion times a finite size of the initial cloud σ_0^2 may simulate an almost linear increase of the cloud size with time. In order to test the potential influence of the initial cloud size on the above result, the model is extended by a third fitting parameter: $\sigma_{\text{mod}}^2 = A_1(t + |A_3|)^{A_2}$. The time $t = -|A_3|$ can be interpreted as the hypothetical release time when the cloud size was zero. The modified model function was fitted to all experiments for which at least three surveys exist (Figure 5). This includes all experiments except Urn 3. The estimated initial cloud sizes (Table 1b) were included in the data sets with a relative error of 100%. Now the exponents A_2 range between 1.3 and 2.3 with a mean value of 1.6 ± 0.1 (see Table 2). This is not compatible with an increase of σ^2 with elapsed time raised to the power of 3. The inertial subrange model also can be extended to account for the finite size of the initial cloud: $\sigma_{\text{mod}}^2 = A_1(t + |A_2|)^3$. The χ^2 test of this model yields the probability $P(\chi^2 \geq \chi_{\text{mod}}^2)$ less

Figure 5. Cloud size as function of elapsed time since tracer release modeled by $\sigma_m^2 = A_1(t + |A_3|)^{A_2}$. The initial cloud size at $t = 0$, σ_0^2 , is included into the fit with a relative error of 100%. The errors for the other cloud sizes are 20%.

than 2% for Alp and Vitz 3 and less than 1% for the other experiments. Thus, the modified inertial subrange model also has to be rejected as a meaningful interpretation of the data set.

5.3. The Shear Model of Carter and Okubo [1965]

Shear diffusion models allow the description of non-radially symmetric tracer distributions. The model by Carter and Okubo [1965] yields the following function for the cloud size (see (12)):

$$\sigma_{\text{mod}}^2$$

15)

Table 2. Parameters and χ^2 for Models That Include the Initial Cloud Size σ_0^2

Experiment	Vitz 1 (7)	Neu (5)	Urn 1 (5)	Urn 2 (4)	Vitz 2 (8)	Vitz 3 (9)
$A_1, \text{m}^2 \text{s}^{-A_2}$						
A_2						
$P(\chi^2 \approx \chi_{\text{mod}}^2), \%$						
$A_1, \text{m}^2 \text{s}^{-A_2}$						
A_2						
A_3, s						
$P(\chi^2 \approx \chi_{\text{mod}}^2), \%$						
$A_1, 10^{-12} \text{m}^2 \text{s}^{-3}$						
A_2, s						
$P(\chi^2 \approx \chi_{\text{mod}}^2), \%$						
$P(\chi^2 \approx \chi_{\text{mod}}^2), \%$						
σ_{mod}^2						

Assumed errors were 100% for σ_0^2 and 20% for $\sigma^2 (t > 0)$. The number of surveys (including σ_0^2) are given in parenthesis in the top line.

Figure 6. Fit of measured cloud sizes of individual experiments to the shear-diffusion model by *Carter and Okubo* [1965] using the function $\sigma_m^2 = 2(4A_1^2t^2 + \frac{1}{3}A_1A_2t^4)^{1/2}$. All data have the same relative error of 20% as shown by the error bar on the right side of the figure.

where $A_1 = K_h$ and $A_2 = K_h(\partial u/\partial y)^2 + 4K_z(\partial u/\partial z)^2$. The dependence of the cloud size on time should therefore gradually change from $\sigma^2 \sim t$ to $\sigma^2 \sim t^2$. From the tests performed for the power law models (7) it is to be expected that this model is in fair agreement with the field data. Indeed, fitting of the individual experiments (i.e., all experiments with at least three surveys) provides an excellent description of the development of the cloud size with time (Figure 6). The probability $P(\chi^2 \geq \chi_{\text{mod}}^2)$ is about 5% for Vitz 1 and ranges between 25 and 80% for all other experiments (Table 2). Thus the model (15) is in accordance with all tested experiments. This is true whether the initial cloud size σ_0^2 is zero or assumes the value listed in Table 1b. For Vitz 1, the only experiment with a somewhat larger initial vertical variance ($\sigma_{zz0} \approx 4.4 \text{ m}^2$), the influence of σ_{zz0} on the spread of the tracer cloud is discussed at the end of this section.

The parameters A_1 and A_2 and the values of χ_{mod}^2 are listed in Table 2. Since according to the shear diffusion model the fitted parameters are linked to observable quantities (velocity shear), it has to be checked whether the results are plausible and consistent with the velocity shear that was observed during the tracer experiments. For the experiments conducted in Lake Lucerne (all except Neu), the average value for A_1 , which in the model corresponds to the horizontal diffusivity K_h , is $0.04 \text{ m}^2 \text{ s}^{-1}$ with variation between 0.016 and $0.09 \text{ m}^2 \text{ s}^{-1}$. Note that these values are smaller than the effective diffusivities calculated above since the former represent real turbulent exchange while the latter additionally describe the interaction of the turbulence with the velocity shear. Horizontal turbulent

diffusivity in Lac Neuchâtel (Neu) is significantly larger than in Lake Lucerne ($0.18 \text{ m}^2 \text{ s}^{-1}$). This lake is larger and exposed to stronger winds.

The fitting values of the second parameter, A_2 , vary between 1×10^{-11} and $400 \times 10^{-11} \text{ m}^2 \text{ s}^{-3}$. According to the shear diffusion model, A_2 depends on the square of the velocity shear in both horizontal and vertical direction. This may explain its great variability even between experiments conducted in the same lake basin (compare Urn 1 and Urn 2 in Table 2). Note that the shear diffusion model of *Carter and Okubo* [1965] assumes the shear to be constant in space in time. In near-surface flows of the sea, however, velocity shear appears to be scale dependent [Kawai, 1985]. In addition, the spectrum of horizontal velocity has no spectral gap, and thus a clear distinction between a purely turbulent and a nonturbulent part of the velocity field is impossible. However, although detailed (space and time dependent) measurements of the velocity shear are not available for our measurements, we want at least to check whether the order of magnitude of A_2 is reasonable. We choose $K_z = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$, as suggested by the observed vertical spread of the tracer during the experiments, and $K_h = 4 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$, as determined from A_1 . Drifters and moored current meters employed during the tracer experiments provided values for the vertical shear, $(\partial u/\partial z)$, in the range of 1×10^{-3} to $5 \times 10^{-3} \text{ s}^{-1}$. For the estimation of the horizontal shear it is assumed, guided by Figure 2a, that the typical horizontal current velocity of 2 cm s^{-1} decays over a distance of 1 km. This yields $\partial u/\partial y = 2 \times 10^{-5} \text{ s}^{-1}$. With these values $A_2 = K_h(\partial u/\partial y)^2 + 4K_z(\partial u/\partial z)^2$ should lie between 10^{-11} and $10^{-10} \text{ m}^2 \text{ s}^{-3}$, in accordance with the range of values determined from the experiments.

The *Carter and Okubo* [1965] model describes not only the cloud size but also the variance in the direction of the major and minor principal axes as function of time (see (12)). The corresponding model functions contain three fitting parameters A_i :

$$\begin{aligned} \sigma_{ma,\text{mod}}^2 &= 2A_1t + 1/3(A_1A_2^2 + A_3)t^3 \\ &+ \sqrt{A_1^2A_2^4 + [1/3(A_1A_2^2 + A_3)t^3]^2} \\ \sigma_{mi,\text{mod}}^2 &= 2A_1t + 1/3(A_1A_2^2 + A_3)t^3 \\ &- \sqrt{A_1^2A_2^4 + [1/3(A_1A_2^2 + A_3)t^3]^2}, \end{aligned}$$

where $A_1 = K_h$, $A_2 = \partial u/\partial x$, and $A_3 = K_z(\partial u/\partial z)^2$. Note that these equations imply a point-like initial concentration distribution. The parameters are determined to minimize the sum

$$\chi_{\text{mod}}^2 = \sum_{i=1}^n \left(\frac{\sigma_{ma,i}^2}{w_{ma,i}} \frac{\sigma_{ma,\text{mod}}^2(t_i, A_1, A_2, A_3)}{\sigma_{ma,i}^2} \right) + \sum_{i=1}^n \left(\frac{\sigma_{mi,i}^2}{w_{mi,i}} \frac{\sigma_{mi,\text{mod}}^2(t_i, A_1, A_2, A_3)}{\sigma_{mi,i}^2} \right) \quad (17)$$

where $w_{ma,i}$ and $w_{mi,i}$ are the absolute errors (1 standard deviation) of $\sigma_{ma,i}^2$ and $\sigma_{mi,i}^2$, respectively. Only experiments with at least four surveys (yielding eight independent data points, i.e., four values of σ_{ma}^2 and σ_{mi}^2 each) were considered for the fitting procedure (Table 3). The agreement between model and data is good (Figure 7, solid lines and solid circles and triangles). From the fit parameters the cloud size was

Table 3. Fit Parameters for the Shear-Diffusion Model by *Carter and Okubo* [1965]

	Alp	Vitz 1	Neu	Urn	Vitz 2	Vitz 3
No. of surveys	14	12	8	8	14	16
$A_1, \text{m}^2 \text{s}^{-1}$	0.012 ± 0.006	0.018 ± 0.005	0.13 ± 0.05	0.07 ± 0.02	0.027 ± 0.002	0.014 ± 0.004
$A_2, 10^{-5} \text{s}^{-1}$	2.3 ± 0.7	4.5 ± 1.5	1.7 ± 1.2	2.8 ± 0.6	4.3 ± 0.7	0.9 ± 0.7
$A_3, 10^{-11} \text{m}^2 \text{s}^{-3}$	-0.006 ± 0.6	3 ± 3	4.8 ± 8.5	3 ± 4	1 ± 1	4 ± 3
$P(\chi^2 \geq \chi^2_{\text{mod}}), \%$	1×10^{-4}	0.4	7.5	31	11	5.5

	Lake Ontario ED1	Lake Ontario ED5	Lake Ontario ED3	Mikawa Bay	Banana River	off Southern California (e)	Pacific (1000-m depth)
No. of surveys	4	4	3	11	5	3	3
$A_1, \text{m}^2 \text{s}^{-1}$	0.05 ± 0.01	0.16 ± 0.05	0.02 ± 0.02	0.066 ± 0.003	0.09 ± 0.05	0.11 ± 0.05	0.04 ± 0.02
$A_2, 10^{-11} \text{m}^2 \text{s}^{-3}$	4 ± 3	10 ± 10	0.06 ± 3	2900 ± 500	2 ± 10	7000 ± 4000	-0.01 ± 1
$P(\chi^2 \geq \chi^2_{\text{mod}}), \%$	22	3	0.3	99	0.007	8	2.5

The top part of this table shows application of the model to the variances in the direction of the principal axes (see (16).) The values for σ_{ma}^2 and σ_{mi}^2 were assumed to have an error of 35%. The bottom part of this table shows application of the model (15) to published data for the cloud size σ^2 or σ_r^2 : Lake Ontario [Murthy, 1976], Mikawa Bay [Yanagi et al., 1982], Banana River and "off Southern California (e)" [Okubo, 1971], and in the Pacific [Ewart and Bendiner, 1981]. The value of χ^2 was calculated assuming an error of 20% for all data.

calculated and compared to the measured size (Figure 7, dashed line and circles with dot inside). Note that minimizing (17) does not exactly yield the same parameter values as (15). However, the parameter values obtained from the fits are similar in both cases.

Figure 7 demonstrates that the model of *Carter and Okubo* [1965] is suitable for the description of the cloud size as well as of the variances in the direction of the principal axes. According to the χ^2 test the Carter and Okubo model is consistent with the data from the experiments Vitz 2 and 3, Neu, and Urn 1 (Table 3). Using an error of about 35% for the variance in the direction of both principal axis, the probability $P(\chi^2 \geq \chi^2_{\text{mod}})$ is small for the experiments Alp and Vitz 1 (Table 3). One possible explanation may be the rather large and elongated initial concentration in these particular experiments.

In order to explore the influence of the initial cloud size, the data from Vitz 1 were tested by the modified *Carter and Okubo* [1965] model (13). The initial cloud size is assumed to be a three-dimensional normal distribution with initial variance of 210 m^2 , 30 m^2 , and 4.4 m^2 for the major principle, the minor principal, and the vertical axis, respectively. The agreement between measurement and model is good (Figure 8). However, statistics do not significantly improve since the errors of the initial values are large and a fourth fitting parameter is required.

5.4. Application of the *Carter and Okubo* [1965] Model to Published Data

The above evaluation clearly demonstrates that for scales up to 1000 m, horizontal transport can be better described by a combination of velocity shear and (small scale) turbulent diffusion than by the inertial subrange turbulence model. One might argue that this conclusion is biased due to the small size of the lakes in which the experiments were conducted. Since the size of the lake limits the scale of turbulence, the cloud might outgrow the inertial subrange scale and thus invalidate the conditions for the model.

In order to test this possible objection, several published experiments which were conducted in large lakes (e.g., in Lake Ontario [Murthy, 1976] or in the ocean [Okubo, 1971]) and which cover diffusion scales up to 1000 m are compared with our own results (Figure 9). Some authors present their data in terms of the radially symmetric cloud size σ_r^2 (Figure 9a) and

others in terms of σ^2 as defined in (3) (Figure 9b). The published data presented in Figure 9b exclusively stem from ocean experiments. In Figure 9a the agreement between the different experiments is excellent. At identical diffusion times the vari-

Figure 7. Application of the shear-diffusion model to the measured variance along the major (solid circles) and minor (solid triangles) principal axes from four experiments. The fit (solid lines) was carried out simultaneously for σ_{ma}^2 and σ_{mi}^2 by assuming a relative error of 35% to all measurements (see error bar in top left panel for Vitz 2). Cloud size was calculated from the fit parameters (dashed lines) and compared to the measurements (circles with dot inside).

ance σ_r^2 measured in the hypolimnion of Lake Ontario are approximately the same as in our experiments. In the ocean the clouds are generally larger than in our lakes. This was to be expected since most of the results for the ocean stem from experiments in the surface mixed layer [Okubo, 1971; Yanagi *et al.*, 1982]. However, for diffusion scales up to 1000 m the exponents of the power law model in individual experiments are similar for all the published and for our experiments, independent of whether they were conducted in the surface layer of the ocean or in the epilimnion or hypolimnion of a lake. In fact, the power law model yields exponents of $m = 0.5$ to 1.5 for the hypolimnion and of $m = 1.6$ to 2.2 for the epilimnion of Lake Ontario (data of Murthy [1976] and of Palmer *et al.* [1987]). For the ocean experiments by Yanagi *et al.* [1982] and those compiled by Okubo [1971], exponents range between 1 and 1.9 for the diffusion scales up to 1000 m discussed here (for this range Okubo's data mostly stem from Foxworthy *et al.* [1966] cited by Okubo [1971]).

Many authors state that the tracer distributions observed were usually elongated [e.g., Okubo, 1971; Murthy, 1976]. Therefore we have reevaluated some of the published data for scales up to 1000 m with the model by Carter and Okubo [1965]. Since errors of σ^2 or σ_r^2 are usually not available, it is not possible to formally calculate the probability for rejection of the various models. However, the results shown in Figure 10 demonstrate that the shear-diffusion model would provide a better description of these experiments than the inertial sub-range model. The χ^2_{mod} given in Table 3 were calculated by assuming that the relative errors of σ^2 and σ_r^2 are 20%.

Figure 8. Comparison of fits to Vitz 1 applying the shear-diffusion model with (bold lines) and without (bold dashed lines) inclusion of the initial variances. The solid circles represent the variance along the major principle axis, σ_{ma}^2 ; the solid triangles represent the variance along the minor principle axis, σ_{mi}^2 ; and the circles with the dot inside represent the cloud size σ^2 . The influence of the initial variance on the calculated cloud size (thin solid line) is negligible.

Figure 9. Comparison of our results with experiments conducted in Lake Ontario and in the ocean. (a) Radially symmetric variance σ_r^2 for the experiments Neu (solid squares) and Vitz 3 (solid triangles) and for different experiments in Lake Ontario (open squares, triangles, and circles, Murthy [1976]) and in the ocean (squares with cross inside, Okubo [1971]; crosses with circle in center, Yanagi *et al.* [1982]). (b) Cloud size σ^2 for the experiments Neu (solid circles) and Vitz 3 (solid diamonds) and for individual experiments in the ocean (crosses and circles with plus inside, Kullenberg [1972]; squares with plus inside, squares with dot inside, squares with slash inside, plusses, circles with dot inside, and open diamonds, Okubo [1971]). Lines connect measurements of individual experiments.

6. Conclusions

The main results of our investigation are that in none of the experiments did the cloud size grow as t^3 . Furthermore, the growing clouds always became elongated and not radially symmetric. According to the χ^2 test the inertial subrange model has to be rejected for almost all of our experiments independent of whether the initial cloud size of the tracer distribution is assumed to be finite or zero. The inertial subrange model reached probabilities $P(\chi^2 \geq \chi^2_{\text{mod}})$ larger than 1% only for the experiments Alp and Vitz 3, but even in these cases, $P(\chi^2 \geq \chi^2_{\text{mod}})$ remained below 2%. In contrast, the shear diffusion model of

Figure 10. Application of the shear-diffusion model by Carter and Okubo [1965] to data from experiments in Lake Ontario (solid triangles, circles, and squares, Murthy [1976]) and in the ocean (squares with cross inside and squares with plus inside, Banana River and "off Southern California," Okubo [1971]; crosses with circle in center, Yanagi et al. [1982]; open circles, Ewart and Bendiner [1981]). Only experiments with at least three surveys were considered. Since some of the authors evaluated their data in terms of the variance σ_r^2 , it was assumed that the latter is equal to the cloud size σ^2 . For comparison the dashed line shows the model for the experiment Vitz 3.

Carter and Okubo [1965] provides a good description of the temporal development of cloud size as well as of the elongated cloud shape. According to the χ^2 test the model description of the cloud size is consistent with all experiments. Although this does not prove the shear diffusion model to be the only model to describe the horizontal spread of tracer distributions, it clearly shows that the shear diffusion model is compatible with the data and cannot be rejected.

Effective horizontal diffusion coefficients lie between 0.02 and $0.3 \text{ m}^2 \text{ s}^{-1}$. If the effect of the velocity shear is removed, as is the case in the shear diffusion model, the remaining horizontal (turbulent) diffusivity is between 0.02 and $0.18 \text{ m}^2 \text{ s}^{-1}$. Assuming vertical diffusion to follow Fick's law, the vertical spread of the tracer distributions can be described with a diffusivity of about $1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ or less. This value tends to be smaller than those determined from water temperature or natural tracers. The difference between these diffusivities can be understood since the former value is only affected by the interior mixing, whereas the latter includes boundary mixing which commonly is considered to cause a large vertical transport.

A new analysis of several published diffusion experiments shows that the development of individual cloud sizes for scales up to 1000 m can be described by the shear diffusion model.

The relation $\sigma^2 \sim t^3$ suggested by, e.g., Okubo [1971] and Murthy [1976], appears, at least for scales up to 1000 m , to be a consequence of combining measurements from different experiments into one data set and assuming it to represent a single experiment.

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