Electron Conductivity on Helium Films

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Abstract Electrons on liquid helium film form a two-dimensional (2D) array with a wide range of electron density. This system is also very interesting for applications in restricted geometry. The conductivity σ of the electron arrays, however, strongly depends on the thickness d of the helium film adsorbed above solid substrates. This behaviour of σ is discussed in detail for a randomly rough substrate. It turns out that for the dependence of the conductivity $\sigma(d)$ there exist three regions of helium thicknesses: $d > d_{min}$, $d \sim d_{min}$, and $d < d_{min}$. Here d_{min} is the helium film thickness which corresponds to a relatively deep minimum of the 2D conductivity. In the first interval, $d > d_{min}$, a two-fraction scenario determines the behaviour of $\sigma(d)$. In the vicinity of d_{min} percolation phenomena develop and the conductivity exhibits different types of the so-called dip effect. For even thinner helium films i.e., when $d < d_{min}$, an activation type of mobility is stimulated. The presented model fits quite well to existing data of *ac* and *dc* electron mobility.

Keywords 2D electron systems · Thin helium film · Electronic transport · Surface roughness

PACS 67.70.+n · 72.10.-d · 72.60.+g · 73.50.-h

1 Introduction

Two-dimensional (2D) electron arrays on thin helium film (2DES) are challenging systems since they are sensitive to many different parameters. Such are the dielectric

constant both of the helium film and the substrate, the characteristics of their van-der-Waals interaction ΔC_3 , the temperature T , the 2D electron density n_s , the pressing electric field $E_\perp$, the level of substrate roughness, and so on. Most of these parameters can be regarded as known ones. However, in the case of the substrate roughness, this characteristic is difficult to determine. As a rule it has to be extracted from experimental data. Some qualitative possibilities of such diagnostics, using the dynamics of the 2DES, are presented below.

2 Corrugation Features

We start from the periodically corrugated surfaces. Corrugation is a special type of substrate roughness $\delta(x)$, i.e., the amplitude of perturbation has the limitation from the top: $|\delta(x)| < \delta_0$, see Fig. 1. An investigation of this type of surface perturbation reveals two typical phenomena—the ‘screening’ effect and the electron localization—applied to the system ‘2DES + surface roughness of substrate’.

Figures 1 and 2 show that the thickness $\zeta(x)$ of the helium film does not follow the profile $\delta(x)$ (screening effect). The screening radius R (i.e., the Laplace radius, caused by the finite surface tension) can be large ($R \gg b$; strong screening) or small ($R < b$; weak screening). R is defined as $R = \alpha/(\rho gh)$, where α and ρ are the surface tension and density of the liquid helium film, g is the acceleration due to gravity, and h is the distance between the bulk and average plane of the distribution $\delta(x)$. In Fig. 2 the measured shape of the helium meniscus and a fit of the experimental data are shown for the structure from Fig. 1, with $b = 300 \mu\text{m}$ and $h = 0.42 \text{ mm}$, see [1].

The second effect is the localization of some electron fraction along the thin part of the helium film (e.g. along distance a in Fig. 1). So the total electron density n_s can be presented as the sum

$$n_s = n_l + n_e \quad (1)$$

where n_l is the trapped electron fraction with low mobility and n_e is the free electron fraction along the thick part of the helium film. Figure 2b indirectly shows the existence of the two electron fractions: without a trapped electron fraction on the tops of

Fig. 1 Schematic sketch of a corrugated surface where, due to capillary condensation, a suspended thick liquid helium film exists. $\delta(x)$ is the corrugation profile ($a + b$) is the corrugated structure period, and $\zeta(x)$ is the profile of the helium film. R is the radius of curvature of the capillary condensed thick helium film and h ($> 2\delta_0$) is the distance between the average structure of the substrate and the bulk liquid helium surface

Fig. 2 (a) Video image of interference fringes resulting from a suspended helium film $h = 0.42$ mm and the pressing field $E_{\perp} = 172$ V. *Top*: uncharged film *Bottom*: charged with electrons. (b) Film thickness versus position within channel, determined from the video images. Due to the electron pressure the helium film is thin in the center of the channel. For large electron densities the width of the thin film region does not increase any more. For details see [1]

Fig. 3 (a) Helium film profile for different $R(h)$: $R_A > R_B > R_C$. (b) Balance of forces holding the electron on the helium surface: $eE_{\perp}$ is the electric force due to non-zero pressing field $E_{\perp}$ and F_{im} is the image force

the channel structure an increase of n_s should result in an increase of the width of the electron area at the bottom within the channel. This, however, is not seen in the measurements, see [1].

Qualitatively interesting is the extreme case of relatively low density of 2D electrons (opposite to that presented in Fig. 2) where

$$n_s < a^2, b^2$$

i.e., there are less than one electron per corrugated lattice cell. Let us consider a fragment of such a lattice shown in Fig. 3a. It is clear that different values of $R(h)$ correspond to different helium filling of the well changing from completely filled (meniscus A) to practically empty one (meniscus C). The problem is to find out where the electron resides which happened to occur in the indicated lattice fragment. In the absence of electric field $E_{\perp}$ pressing the electrons to the liquid helium surface the most energetically favorable places are located at the peaks of the $\delta(x)$ profile (here the helium film thickness is minimal). On the contrary, if $E_{\perp} \neq 0$, the electron position is governed by the balance of forces indicated in Fig. 3b. Electron is localized in the transition domain of the helium surface profile $\zeta(x)$ so that the net force is normal to the film surface. Without going into details one can state, bearing in mind the experiments discussed below, that electrons are indeed localized following the mechanism presented in Fig. 3b descending into the $\delta(x)$ profile as $R(h)$ is reduced.

Fig. 4 (a) Trajectories of 1D electrons on the periodical surfaces of $\delta(x, y)$ with different curvatures and the corresponding separatrix. It is clearly seen that the separatrix is the only trajectory along which electrons can travel from cell to cell. (b) Electron conductivity for the system of 1D electrons as a function of helium film thickness d

The arising picture results in interesting geometry of localized states on the two-dimensional lattice $\delta(x, y)$. Its fragment along one of the directions is shown in Fig. 4a. Obviously, in the vicinity of maxima and minima of $\delta(x, y)$ the trajectories of 1D electrons are closed and span the peaks of $\delta(x, y)$ (clockwise arrows) or prove to be “inserted” into the minima of $\delta(x, y)$ (anticlockwise arrows). The only exception is a single plane containing the separatrix separating the states with different curvatures. Moving along the separatrix, the 1D electrons can travel between the cells thus producing a finite conductivity along the corrugated helium film. Qualitatively, the resulting conductivity σ as a function of the average thickness of the helium film in the well demonstrates the behaviour depicted in Fig. 4b. We believe that this property obviously taking place for the periodic corrugation should also be valid for the substrates with arbitrary roughness.

3 Random Substrate Roughness

As a rule the roughness cannot be presented via simple corrugation. The suitable alternative is the introduction of Gaussian roughness. Under these conditions ac and dc conductivities inevitably start to be sensitive to the average helium film thickness $\langle d \rangle$ due to the substrate roughness. To explain the situation we need the basic Gaussian roughness definitions. Mainly the one-dimensional presentation is sufficient. However, some qualitative statements are valid just in the 2D roughness picture.

We now consider a random one-dimensional roughness $\delta(x)$ with a Gaussian distribution of amplitudes $D(\delta)$ and autocorrelation function for $\delta(x)$, see [2]:

$$D(\delta) = \frac{\exp(-\frac{\delta^2}{2\Delta^2})}{(2\pi\Delta^2)^{1/2}} \quad \text{and} \quad \langle \delta(x)\delta(x-x') \rangle = \Delta^2 \exp\left(-\frac{x'^2}{2\eta^2}\right) \quad (2)$$

where $\Delta^2 = \langle \delta^2 \rangle$ is the mean-square deviation of the roughness amplitude and η is the correlation length.

Using (2) one obtains

$$\langle \delta''\delta'' \rangle = \beta^2 \equiv \frac{3\Delta^2}{\eta^4} \quad \text{and} \quad D(\gamma) = \frac{\exp(-\frac{\gamma^2}{2\beta^2})}{(2\pi\beta^2)^{1/2}}, \quad \gamma = \delta''. \quad (3)$$

The definition in (3) are used below for introducing the distribution of the continuum electron oscillator frequencies ω_r , caused by interaction between electrons and a rough solid substrate: $\omega_r^2 \propto \delta''$.

Besides we need the correlation function [2]

$$P(\delta, \delta'') = \langle \delta \delta'' \rangle = \frac{\eta^2}{2^{3/2} \pi \Delta^2} \exp\left(-\frac{\delta^2}{2\Delta^2}\right) \exp\left[\frac{(-\delta + \delta'' \eta^2)^2}{4\eta^2}\right] \quad (4)$$

which is necessary in the discussion of the activation type mobility.

In the presence of roughness, (2), there are three essential intervals of the helium film thickness:

- (a) $\kappa^{-1} \geq R \geq \eta$; this interval is suitable for the two-fraction separation in the 2D electron system, $\kappa^2 = \rho g / \alpha$;
- (b) $R \simeq \eta$; in this situation the development of an infinite conducting cluster becomes possible (so-called dip effect);
- (c) $R \ll \eta$; the area for the activation type conductivity.

Here R is the Laplace radius, with

$$\frac{2\alpha}{R} \simeq \rho g^* h, \quad (5)$$

where the height h (distance between substrate and bulk liquid level) is defined in Fig. 1, and g^* is the effective gravity acceleration with $g^* = g + \Delta C_3 / \rho d^4$.

All these limiting cases are interesting and have to be discussed separately.

3.1 Two-Fraction Situation

If the helium film is thick then just a few roughness tops will be at the level (or above) of the free helium surface. The 2D density of these so-called active tops n_a is [2]

$$n_a^{-1} \simeq b_a^2, \quad R \simeq \frac{b_a}{2\sqrt{2}} \left(\frac{b_a}{\Delta}\right) \sqrt{\ln\left(\frac{b_a}{s\eta}\right)} \quad (6)$$

with $s^2 = 2\pi$, $b_a^2 > \Delta^2$, and $R \gg \Delta$ is defined in (5), η is the correlation length from (2).

Definition (6) helps to introduce the two-fraction scenario for the electron behaviour. As in (1) we have $n_s = n_l + n_e$, where

$$n_l = \frac{n_a}{\exp\left(\frac{V_a - \mu_0}{T}\right) + 1}, \quad V_a \geq -\frac{\Lambda}{d_a}, \quad \Lambda = \frac{e^2(\epsilon_d - 1)}{4(\epsilon_d + 1)}, \quad d_a^3 \simeq \frac{\Delta C_3}{\rho g h} \quad (7)$$

and

$$n_e = \frac{n_e^0 \exp\left(\frac{T_e}{T}\right)}{\exp\left(-\frac{\mu_0}{T}\right) + 1}, \quad n_e^0 = \frac{mT}{2\pi \hbar^2}, \quad T_e = \frac{eE_\perp \Delta}{2\sqrt{2}[\ln\left(\frac{(R\Delta)^{1/2}}{\eta}\right)]^{1/2}}, \quad (8)$$

Fig. 5 (a) The electro-chemical potential, μ_0 , as function of coupling energy, V_a , and density ratio of the number of localized electrons due to the active tops, n_a , to the total number of electrons, n_s . When $n_a = n_s$ then there is a sharp drop in μ_0 , which increases as the coupling energy gets bigger, see [3]. (b) Electron conductivity against the helium film thickness for two pressing electric fields. Reproduced from [4]. In contrast to Figs. 8 and 9, these data do not show the dip effect

where ΔC_3 is the constant of van-der-Waals interaction. The fraction n_e corresponds to the free electron motion along the helium surface. In Fig. 1 this fraction is distributed along the minimum of the meniscus. The second fraction, n_l , represents the density of electrons localized to potential wells of the roughness of the solid substrate. These electrons are localized in the vicinity of the tops with density n_a above the helium film. The total density n_s is usually fixed. The ratio between n_e and n_l is controlled by the potential V_a , temperature T , and the electro-chemical potential μ_0 . The dependence of μ_0 is shown in Fig. 5a, see [3].

In terms of n_e and electron mobility μ the *dc* conductivity σ can be written as

$$\sigma = n_e \mu. \quad (9)$$

Definition (8) and (9) show, at least qualitatively, that σ decreases monotonically when the thickness of the helium film gets thinner. Indeed, the mobility μ goes down with decreasing thickness of the helium film and the fraction n_e gets smaller. Experiments demonstrate the expected tendency in $\sigma(d)$, see Fig. 5b and [4]. However, from the monotonic behaviour of $\sigma(d)$ it is difficult to deduce the contribution due to $\mu(d)$ and $n_e(d)$.

A more informative example of the two-fraction scenario is given by cyclotron resonance (CR) measurements on thin helium films, see Figs. 6 and 7. In the two-fraction area the trapped electrons cannot participate in the *dc* conductivity (see (9)). However, these electrons contribute to the *ac* dynamics seen in the eigenfrequencies $\omega_r^2 \sim \delta''$. Therefore the effective absorption CR line has two contributions: one from the free electron motion, and one from the trapped electron states. For a Gaussian distribution of the random potential the trapped fraction continuously grows as d gets thinner. Consequently, the fraction n_e shows the opposite behaviour. The experimental confirmation of this prediction is demonstrated in Figs. 6 and 7 [5].

The data in Fig. 6 show the two contributions to the electron density: the free electron part which decreases with decreasing helium film thickness and the trapped

Fig. 6 The absorption Q^{-1} as function of magnetic field for up (Δ) and down (∇) sweeps. The dashed and dotted lines represent the free and localized electron fraction, the full line is the sum of both fitted to the data. From (a) to (b) the helium film thickness decreases. In (a) $n_e \approx 67\%$ and in (b) $n_e \approx 49\%$. For details see [5]

electron part with opposite tendency. To prove this interpretation it is necessary to express all components of the absorption kinetics in terms which are sensitive to Gaussian roughness. Using the motion equation in a Drude form with the parameter τ , the statistical information (2), (3), and (5) to (8) with the parameters Δ and η , and the excitation scenario neglecting the resonator features, one can fit the data using $\Delta = 8$ nm, $\eta = 6$ nm, $n_s = 10^9$ cm $^{-2}$, and $\omega\tau \approx 4$, see [5].

3.2 Percolation Area, Dip Effect in dc Mobility

Now let us turn to the reproducibly observable N-shaped feature in the $\sigma(d)$ plot for electrons above the helium film on rough substrates (the so-called dip effect). This remarkable effect was first observed by Andrey [6] for electrons above the helium

Fig. 7 Dependence of free electron fraction n_e as function of h . (●) and (○) are from fitting to all measured data. Both in (a) and (b) the *solid lines* present the best fit with the same parameters of a and T_0 . The *dashed* and *dotted lines* in (a) show fit with different T_0 but keeping a fixed, and in (b) with fixed T_0 but varying a . This shows the good agreement with one set of parameters to describe the measured data. The (○), labeled (1) to (3), correspond to the same data-points as shown in Fig. 6. For details see [5]

fil on the sapphire substrate is clearly seen on Fig. 8b. Further independent results presented in Fig. 8a [7] demonstrate the importance of roughness (the dip effect is reduced as the substrate made of solid hydrogen employed in [7] is annealed, i.e. the amplitude of $\delta(x, y)$ is decreased). One should also note a series of measurements of the *dc* conductivity in the cell first providing this possibility for electrons above helium (see [3, 8] and Fig. 9). Most important here are the data presented in Fig. 9a revealing a gap between the threshold of the 2D conductivity from the side of $d > d_{min}$ and its peak in the region of $d < d_{min}$ indicating the composite nature of the dip effect (here d_{min} is the threshold thickness for the appearance of 2D conductivity).

Our CR data on the two-component behaviour of 2D electrons on the fil above a rough substrate plotted in Figs. 6 and 7 allow to explain a sharp fall of the 2D conductivity in the domain of

$$n_a > n_s \quad \text{and simultaneously} \quad d < d_{min} \quad (10)$$

Fig. 8 (a) Conductivity versus temperature measured for a growing helium film on a freshly prepared quench-condensed hydrogen film (lower curve) and after thermal annealing of the hydrogen film (upper curve). Reproduced from [7].
(b) Mobility as function of helium film thickness on sapphire as substrate at two different electron densities. Reproduced from [6]

Fig. 9 Measurement of dc currents of electrons travelling along a helium film adsorbed on different substrates. The currents are proportional to the dc conductivity. (a) The data show the existence of a quasi-zero conductivity for a finite interval of helium thickness [8]. (b) Dip in conductivity measured on helium film adsorbed on a Au, Cu, and, as shown in the inset, on a Si substrate. For details see [3]

(for the chemical potential shown in Fig. 5a the fraction n_e of free electrons in this domain becomes exponentially small).

However, what is the origin of the sharp rise in $\sigma(d)$ in the range of $d < d_{min}$? We believe that just as for the periodic corrugation shown in Fig. 4a, rough substrates also possess the δ -percolation effect (similar to that shown in Fig. 4b) for 1D localized electrons in the domain of $d < d_{min}$. We do not attempt to prove the existence of δ -percolation effect in the conductivity of 1D electrons above the helium film on an arbitrary rough substrate. Even the ordinary percolation involving the proof of the existence of the threshold is a tough mathematical problem which was only rigorously analyzed with numerical methods [9]. However, by adopting this hypothesis one obtains a natural explanation of the dip effect through a simple combination of two percolation phenomena. One of them is the 2D percolation with the threshold (10), and the second one the 1D percolation with the peak in $\sigma(d)$ corresponding to the helium level crossing the averaged saddle point of the $\delta(x, y)$ profile. Schematic picture of this combination of the two contributions to the conductivity is presented in Fig. 10. The thresholds d_{min} and d_{peak} are generally different. For $d_{min} > d_{peak}$ we have the situation of Fig. 9a. If $d_{min} \geq d_{peak}$ then Figs. 8 and 9b are realized. Finally, if $d_{min} < d_{peak}$, the dip effect may be absent (see Fig. 5b).

Fig. 10 Schematic structure of the dip effect in the $\sigma(d)$ behaviour. In the range of $d > d_{min}$, the two-fraction dc conductivity is developed. For $d < d_{min}$, the 1D percolation around $d = d_{peak}$ may occur

Fig. 11 Logarithm of the 2D electron mobility on thin liquid helium film with different substrates as a function of d^{-2} . Reproduced from [11] (and see references therein)

3.3 Activation Type of Mobility

For the limit $R \ll \eta$ all electrons are practically trapped below the 1D percolation level (see Fig. 3a, situation “C”) in the minimum of $\delta(x)$. In this case the optimal minimum δ_* follows from (4) and the correct description of the curvature of this effective minimum. We believe that δ_*'' coincides with the inverse screening radius $R_*^2 \simeq \alpha/\rho g^*$. If so, the value δ_* follows from

$$\frac{\partial P(\delta, \delta_*'')}{\partial \delta} = 0 \quad (11)$$

where $P(x, y)$ is from (4). After calculations we have from (11) the condition $\delta_* = -2\eta^2/(3R_*)$.

The activation energy is $E_a \simeq eE_\perp | \delta_* | \propto d^{-2}$ [10]. This result explains the experimental data in Fig. 11 and supports the general picture of the contribution of surface roughness in the conductivity $\sigma(d)$.

4 Conclusions

We have shown that surface roughness of the substrate supporting a helium film can strongly influence the conductivity of a 2DES on top of the film. The model which

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