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Critical Currents in Superfluid Helium Near T_λ

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The temperature dependence of the critical heat current Q_c in He II has been measured in the temperature region $3 \cdot 10^{-5}$ (°K) $< T_\lambda - T < 1.2 \cdot 10^{-2}$ (°K). The result $Q_c \propto (T_\lambda - T)^{1.07 \pm 0.01}$ is consistent with a divergent mutual friction near T_λ proposed recently by AHLERS.

I. Introduction

The resistanceless flow of superfluid helium is limited by critical relative velocities $(v_n - v_s)_c$ between its normal and superfluid components. The breakdown of superfluidity of flowing He II at temperatures $T < T_\lambda$ has been demonstrated by different methods[1-7]. The corresponding "intrinsic critical velocities" show a characteristic temperature dependence in the critical region. In contrast the critical velocity introduced by FEYNMAN[8] to explain the breakdown of irrotational flow of He II at the first occurence of vortex lines due to the kinetic energy of the liquid is temperature independent. As different as the methods of the mentioned experiments have been their interpretations.

In this paper we present measurements of the temperature dependence of the critical heat current in He II in the range $3 \cdot 10^{-5}$ °K $< T_\lambda - T <$ $1.2 \cdot 10^{-2}$ °K where T_λ is the λ-temperature of the quiescent liquid. The critical heat current is found to be proportional to $(T_\lambda - T)^{1.07 \pm 0.01}$ and will be discussed in connection with the interpretations of the above mentioned experiments and related theories.

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II. Experimental

In the present experiment the thermomechanical effect is used to obtain a relative velocity between the two components of He II. The set-up shown in Fig. 1 is similar to that described earlier[3]. The inner

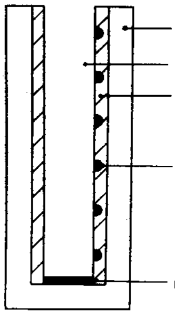

Fig. 1. Schematic representation of the experimental set-up: lucite cylinder (1.0 cm inner diam., 10 or 30 cm length); thermometers (1/8 W Allen-Bradley carbon resistors with $R(T_\lambda)\approx 6\ \mathrm{k\Omega}$ or $R(T_\lambda)\approx 50\ \mathrm{k\Omega}$)

helium filled lucite tube is surrounded by a vacuum jacket. It is open at the top so that the liquid there is in direct contact with the outer helium bath. Lucite rods of different diameter could be placed into the cylinder changing the arrangement into helium filled annuli of different widths (1.0; 0.51; 0.23 mm). Heating the bottom of the cylinder by the heater resulted in a heat flow Q and a mean counterflow velocity $(v_n - v_s)$ of the normal and superfluid components of the liquid in the tube according to the relation

$$(v_n - v_s) = Q/\rho_s S T \tag{1}$$

(ρ_s = density of the superfluid component; S = entropy).

Starting below the λ-point the liquid inside the cylinder was slowly heated in one run at a constant heat input and in the next run with zero input into the heater to a temperature above T_λ by reducing pumping. Six carbon resistors were glued into the walls of the lucite cylinder at different heights so that they could not disturb the flowing helium and had only contact to it at their filed side. The ac-voltage drop across one of these thermometers as a function of time was measured with a PAR lock-in amplifier and registered on a recorder. The λ-temperature of the quiescent liquid or the critical temperature when a counterflow was present was identified by a characteristic discontinuity in the helium warming rate due to the breakdown of the unique thermal conductivity of He II at these temperatures[9]. At the mostly used power level in the thermo-

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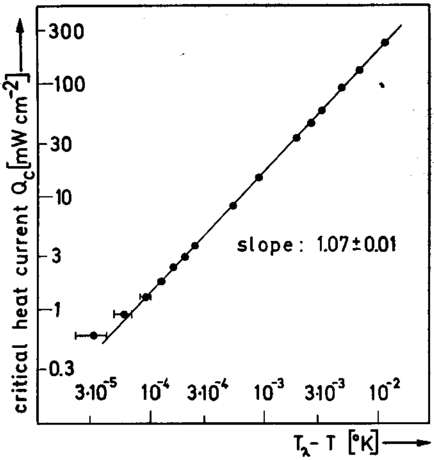

Fig. 2. Critical heat current Q_c as a function of the temperature difference $T_\lambda - T$ (T_λ: λ-point of quiescent helium). The temperatures were measured with a thermometer 1.0 cm above the heater in the free lucite tube (10 cm length, 1.0 cm diam.). The corresponding critical velocity for example at $Q_c = 17$ mW/cm² and $T_\lambda - T = 10^{-3}$ °K is $(v_n - v_s)_c = 2.5$ cm/sec

meters ($4 \cdot 10^{-9}$ W) the resistance resolution corresponded to 5 µdeg. Measured temperature differences $(T_\lambda - T)$ were not influenced by changing the warming rate, changing the starting temperature, or changing the thermometer power between $2 \cdot 10^{-6}$ W and $2 \cdot 10^{-10}$ W[9].

With a heat current present in the liquid a discontinuity in the thermogram identical in shape to that at the usual λ-temperature occured at a temperature $T < T_\lambda$. This means that He^4 had lost its superfluid properties at a temperature $T < T_\lambda$ due to the counterflow of the two components of He II at a critical heat current Q_c.

In Fig. 2 the critical heat current Q_c is shown as a function of the difference between the λ-temperature T_λ of the quiescent liquid and the temperature T at which the discontinuity in the thermogram of flowing He II occurs. Within the given accuracy (10^{-5} °K) the data can be represented by

$$Q_c = A \cdot (T_\lambda - T)^{1.07 \pm 0.01} \quad \{\text{W/cm}^2\} \tag{2}$$

for

$$3 \cdot 10^{-5} \, \{°\text{K}\} < T_\lambda - T < 1.2 \cdot 10^{-2} \, \{°\text{K}\}$$

in agreement with Ahlers' recent results for $T_\lambda - T < 10^{-3}$ °K[6]. The temperature dependence of the corresponding critical velocity according to Eqs. (1) and (2) is given by

$$(v_n - v_s)_c \propto (T_\lambda - T)^{0.41 \pm 0.01} \tag{3}$$

The coefficient A was found to depend on *geometry* which on the other hand *did not affect the exponent* in our temperature region. We did not observe a transition to a region with a smaller (geometry depending) temperature exponent of Q_c which was observed by[1,2,4,6,7]. This transition should occur in our measurements at $T_\lambda - T \simeq 10^{-3} \div 10^{-4}$ (°K) following Ahlers' discussion. The coefficient A measured with the thermometers at different heights in the tube is shown in Fig. 3 as a function of the position of the thermometers. Furthermore A was measured at a fixed position in the free tube (diam. 1.0 cm) and in three annuli of different widths (1.0; 0.51, and 0.23 mm). It was found to increase continuously by about 20% in decreasing the free width for the helium flow from 1 to 0.023 cm.

III. Discussion

A simple case of the breakdown of the superfluid properties of liquid helium at $T<T_\lambda$ is the homogeneously flowing bulk liquid where the density ρ_s of the superfluid component is decreased by a relative velocity $(v_n - v_s)$ according to $\rho_s \approx \rho_{s,0}\left(1-\frac{(v_n-v_s)^2}{(v_n-v_s)_m^2}\right)$ and superfluidity should vanish at a temperature $T<T_\lambda$ as predicted by the phenomenological theory[10,11] and by thermodynamic arguments[12,13]. In the experiments mentioned above[1-3,5-7] where the critical velocity was estimated by isothermal flow, by persistent current or by heat current methods as well as in the present measurements the superfluid properties of He II have been found to break down at velocities which are up to a factor 15 smaller than the above $(v_n-v_s)_m$[14,15]. In addition, in the experiments[1,3-6] the critical temperature exponents of Q_c or $(v_n-v_s)_c$ were

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15. Ahlers[6] has shown that also the high value of $(v_n-v_s)_c$ measured in [4] by observing the formation of bubbles at a heated wire can be explained by a divergent mutual friction (see below).

smaller than 4/3 or 2/3, respectively which are expected by phenomenological theories of superfluidity[10,11]. Including logarithmic corrections[13,16] on the basis of the singular specific heat of liquid helium can not account for this deviation of the exponents*.

The heat conductivity as well as persistent currents of He II can be limited not only by the transition to the normal state but also by production of a high degree of vorticity in He II which normally takes place at appreciably smaller velocities. The critical velocities of the persistent current experiment of CLOW and REPPY[2] have been discussed by LANGER and FISHER[17]. In their theory thermally activated fluctuations of the order parameter in flowing He II result in an exponentially increasing vortex production in approaching a critical temperature $T<T_\lambda$. LANGER and FISHER predict an exponent 2/3 for the temperature dependence of the critical velocity which has been confirmed by CLOW and REPPY[2]. The more recent persistent current experiments[5] performed at $T_\lambda - T > 5 \cdot 10^{-3}$ °K show a dependence of the critical velocity on the pore size. In addition exponents between 0.5 and 0.6 instead of 2/3 were found in these experiments. Hence the decrease of the exponent also at temperatures rather near to T_λ is not a characteristic feature only of the heat current experiments.

The interpretations of the experiments concerning the critical heat current seem to be still more confusing. AHLERS[6] has explained the critical heat current by a mutual friction force[18] between the normal fluid and vortices in the superfluid proportional to $(v_n - v_s)^m$ and diverging near T_λ (m between 3 and 4). His results on bulk helium are in good agreement with ours both in their magnitude and their temperature dependence. In addition the limitation of the heat current by mutual friction offers a possibility to understand the dependence of the critical current Q_c on the length L of the tube which should be $Q_c \propto L^{-1/m}$ and is shown in Fig. 3.

But which mechanism is responsible for the creation of vorticity necessary for mutual friction? In bulk He II the kinetic energy at the critical velocity in the present measurements and in the experiments[3,6] is more than sufficient to overcome the energy barrier

* A correction like $Q_c \propto (T_\lambda - T)^{4/3} \cdot \sqrt{\log (T_\lambda - T)}$ [13] gives an effective exponent 1.26 at $T_\lambda - T = 10^{-3}$ °K which moreover is temperature dependent; hence the exponent 1.14 given in [4] is not in agreement with Mikeska's[13] more recent calculation.

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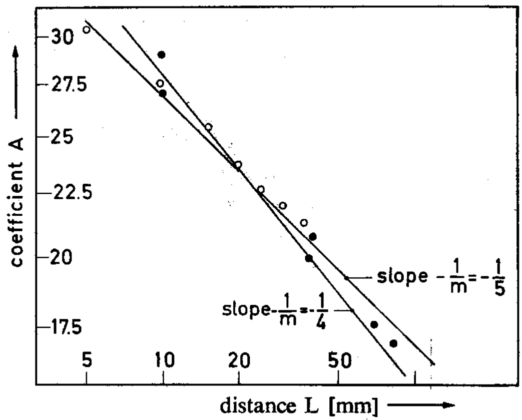

Fig. 3. Coefficient A (see Eq. (2)) as a function of the distance L of the thermometer from the starting point of the heat current Q_c. The full points are measured in the cylinder (1.0 cm diam.), the open points are measured in an annulus (0.1 cm width). The two lines are the dependence expected for mutual friction with Gorter-Mellink exponents[18] $m=4$, and $m=5$, respectively (the values given in the literature are between $m=3$ and $m=4$, see AHLERS[6])

for vortex production. Therefore in bulk He II the production of vortices should be mainly estimated by the temperature independent Feynman critical velocity which depends on the tube diameter[8]. This is supported by our observation on the increase of the critical current by decreasing the free diameter for the helium flow. In slits, pores, and films[1,2,5,7] the Feynman critical velocity is higher, and temperature dependent fluctuations of the order parameter as proposed by LANGER and FISHER[17] should be dominating the vortex production, – but both mechanisms can be present. The relative importance of these two mechanisms depends on temperature and geometry. Therefore one expects different critical temperature exponents of $(v_n - v_s)_c$ or Q_c in experiments with different geometries, e.g. with bulk liquid, in pores or in films of different dimensions[19]. In addition these exponents can be temperature dependent[1,2,4,6,7]. The resulting mutual friction between the normal component of He II and the vorticity in its counterflowing superfluid component limits the current. A divergent mutual friction has been discussed up till now only as a limit for the current in bulk helium[6] but it also may be of influence in the measurements with

19. A further reason for the different critical exponents of $(v_n - v_s)_c$ and Q_c in bulk helium and in films or pores, respectively, may be a different temperature dependence of the superfluid density ρ_s in bulk helium and in restricted geometries (see GUYON, E., and J. RUDNICK: J. Physique **29**, 1081 (1968), and BURKHARDT, STAUFFER[11]).

slits, pores and films. The above discussed different mechanisms for vortex production may be the reason for the different exponents x which AHLERS found in his nearly identical experiments on the critical current in helium films ($x = 1.399$) which he discussed only in terms of the Langer and Fisher theory [7], and in bulk helium ($x = 1.077$) which he discussed only with a divergent mutual friction constant [6,19].

It remains to prove whether the critical velocity mechanisms discussed above are able to explain that in our measurements the discontinuity in the thermogram at the critical temperature of flowing He II has the same shape as the discontinuity of quiescent helium at T_λ where $\rho_s = 0$. The discontinuity in the thermogram of flowing He II if explained by mutual friction should then occur due to a sudden breakdown of superfluidity near the heater or when T_λ is surpassed there, limiting the transport of heat from the heater. Oppositely, in the quiescent liquid $T = T_\lambda$ is observed when $\rho_s = 0$ near the thermometer. There was no difference between the two discontinuities in the thermogram even if the thermometer power or the heating rate was changed. – Measurements of the temperature gradient along the tube at different heat currents near the critical value should give more information.

On the other hand measurements of the singularity of the specific heat and of the velocity of second sound in flowing He II [13] ought to show at which temperature the He II/He I transition ($\rho_s = 0$) occurs in the flowing liquid and should inform about the order of this transition [11].

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