higher than for the coexisting liquid.[11] The relatively slow, nearly exponential change of the dimple depth when U is varied, on the other hand, is consistent with the observation of Keshishev, Parshin, and Babkin that melting-crystallization waves are strongly damped for $T \gtrsim 1$ K.[5]

In order to extend the theory of melting-crystallization waves[1] to charged interfaces we have included a term in the usual equation of motion which represents the pressure of the ions.[12] The resulting relation between the frequency ω and the wave vector q of these excitations is

$$\omega^2 = \frac{\rho_l}{\rho_s - \rho_l} gq + \frac{\rho_l}{(\rho_s - \rho_l)^2} \bar{\alpha} q^3 - \frac{i\omega q \rho_s \rho_l}{m_4 K (\rho_s - \rho_l)^2} - \frac{\rho_l}{4\pi(\rho_s - \rho_l)^2} q^2 (\epsilon_s E_s^2 + \epsilon_l E_l^2), \tag{1}$$

where we have assumed that the ions have a mobility large enough to warrant a constant electrostatic potential along the interface.[12] Here the subscripts s and l refer to the solid and the liquid, respectively; ρ denotes the density, g the acceleration due to gravity, ϵ the dielectric constant, m_4 the atomic mass of ^{4}He, and K the kinetic growth coefficient of the interface; $\bar{\alpha}$ is related to the interfacial tension α between liquid and solid He by $\bar{\alpha} = \alpha + \partial^2\alpha/\partial\varphi^2$ [φ is the angle between the normal of the displaced interface and the (vertical) z axis].[1]

An analysis of Eq. (1) shows that above a critical field solutions exist with a *negative* damping coefficient γ, i.e., these modes will grow instead of decaying in time. If we assume, for simplicity, that the interface is maximally charged ($E_1 = 0$), an instability is predicted to develop at a critical field $E_{s,c} = [64\pi^2 g\bar{\alpha}(\rho_s - \rho_l)/\epsilon_s^2]^{1/4}$ and a critical wave vector $q_c = [g(\rho_s - \rho_l)/\bar{\alpha}]^{1/2}$. The critical values, calculated with the material parameters for the hcp-superfluid interface at 1.35 K,[3] are $E_{s,c} = 1850$ V/cm and $q_c = 10.2$ cm^{-1}. When the field is raised above $E_{s,c}$ two features are expected: (i) Since $|\gamma|$ increases as well, the instability ought to develop more and more rapidly; (ii) the range of modes with negative damping (i.e.,

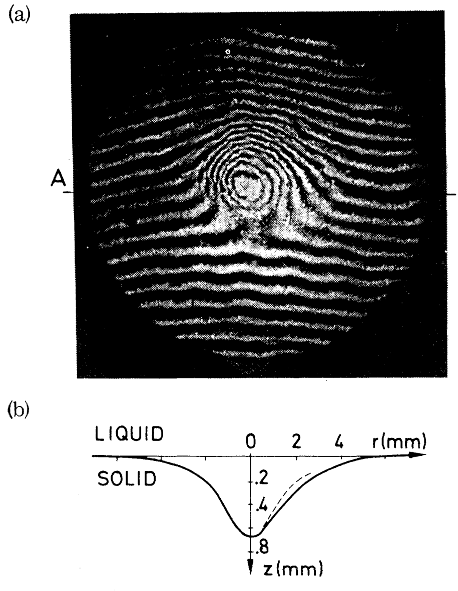

FIG. 1. (a) Interference pattern of an hcp-superfluid ^{4}He interface deformed by $\sim 10^6$ negative ions in an external field E = 700 V/cm, at a temperature T = 1.35 K. The parallel fringe pattern outside the center results from a slight inclination of the two interferometer plates. (The helium crystal in this region is flat, and hence the fringes are not distorted.) (b) Dimple profile determined from (a) along the direction A-B (full curve). A comparison with the dashed line, which is a mirror image of the left-hand profile, illustrates the asymmetry of the dimple.

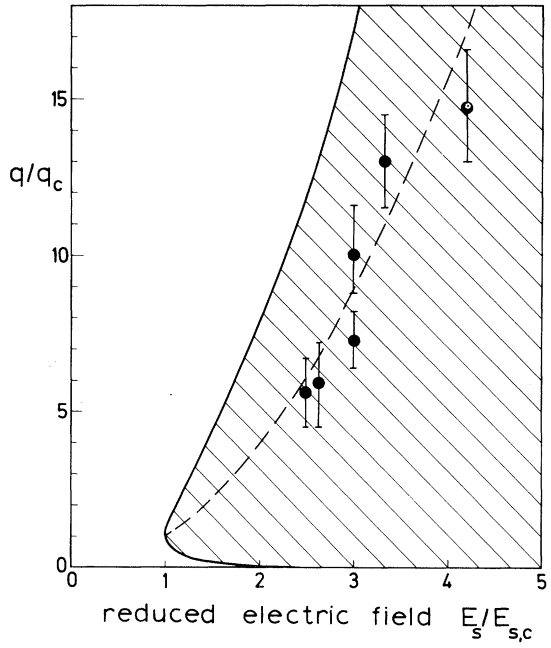

FIG. 2. Range of the charge-induced instability of the hcp-superfluid ^{4}He interface, indicated by the hatched area, as calculated from Eq. (1). The solid line is a plot of the reduced wave vectors q/q_c for which the damping coefficient vanishes; the dashed line indicates the unstable wave vectors with the largest gain. The experimental results, given by the circles, were obtained by determining the characteristic average wave vector q_m of unstable, corrugated structures like in Fig. 3.

gain) becomes larger, and hence the deformation profile of the interface will contain a wider and wider spectrum of Fourier components. This is illustrated in Fig. 2, where the range of unstable wave vectors is plotted versus the reduced electric field $E_s/E_{s,c}$.

An experimental example for an hcp-superfluid interface at supercritical conditions is shown in Fig. 3. A voltage of 1800 V was applied between the electrodes 1 sec before the picture was taken. Since the ion source was switched on continuously, the interface, because of the large mobility of the ions in superfluid ^{4}He,[13] was charged close to saturation. For a crystal height of 4 mm this implies an ion density $n = 2.7 \times 10^9$ cm^{-2} and a corresponding reduced electric field $E_s/E_{s,c} = 2.4$. As the picture demonstrates, corrugations appear consisting of a large number of small dimples with a tendency toward parallel alignment (although the influence of the cylindrical geometry is still visible). The dimples increase in depth, until finally, after a few seconds, charged liquid bubbles split off from the dimple tips and move through the solid at a velocity of 1 mm/sec. Related phenomena at the gas-liquid interface of ^{4}He have been described by Volodin, Khaikin, and Edel'man.[14]

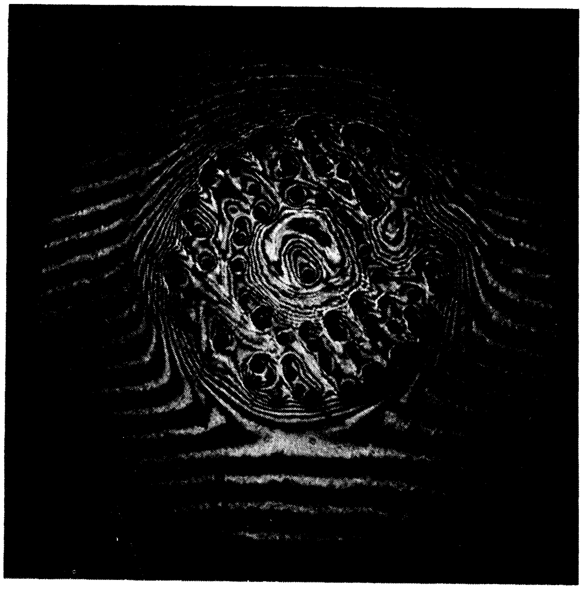

FIG. 3. Interference pattern of an hcp-superfluid ^{4}He interface undergoing a charge-induced instability, about 1 sec after a (supercritical) voltage U = 1800 V was applied between the two electrodes. An additional holding field resulting from negative charges on the cylindrical cell walls confines the ions on the crystal surface in a circular central patch about 1 cm in diameter.

The higher the applied voltage the faster this instability is found to develop (in ~ 50 sec at 1550 V, ~ 1 sec at 3000 V); simultaneously the wave vector q_m characterizing the spacing of the corrugations becomes larger and less well defined. Results for several electric fields are plotted as circles in Fig. 2. All these observations are in close agreement with the predictions derived from Eq. (1).

The direction of the stripes in Fig. 3 apparently depends on the anisotropy of the helium crystal. Although the crystal orientation has not been determined here, we have recorded the facets which appear during rapid crystal growth even above the supposed roughening transition near 1 K.[3-5] The corrugations resulting from the instability preferentially develop parallel to the plane of the largest facet (even though in these experiments the interface was only charged in thermodynamic equilibrium, long after all the visible facets had disappeared).

In summary, we have observed a charge-induced instability of the ^{4}He solid-superfluid interface analogous to the electrohydrodynamic instability of liquid surfaces. In contrast to the previously investigated liquids, however, here the excitations of the interface—melting-crystallization waves—are strongly damped. Furthermore, the present measurements extend the instability studies, which so far had concentrated on the regime $E/E_c \sim 1$ and $q/q_c \sim 1$, deep into the unstable region.

We gratefully acknowledge helpful discussions with R. Bluemel.

---

[a] Present address: Fachbereich Physik, Johannes Gutenberg-Universität, D-6500 Mainz, West Germany.

[1] A. F. Andreev and A. Ya. Parshin, Zh. Eksp. Teor. Fiz. 75, 1511 (1978) [Sov. Phys. JETP 48, 763 (1978)].

[2] S. Balibar, D. O. Edwards, and C. Laroche, Phys. Rev. Lett. 42, 782 (1979).

[3] J. Landau, S. G. Lipson, L. M. Määttänen, L. S. Balfour, and D. O. Edwards, Phys. Rev. Lett. 45, 31 (1980); J. E. Avron, L. S. Balfour, C. G. Kuper, J. Landau, S. G. Lipson, and L. S. Schulman, Phys. Rev. Lett. 45, 814 (1980).

[4] S. Balibar and B. Castaing, J. Phys. (Paris), Lett. 41, L329 (1980).

[5] K. O. Keshishev, A. Ya. Parshin, and A. V. Babkin, Pis'ma Zh. Eksp. Teor. Fiz. 30, 63 (1979) [JETP Lett. 30, 56 (1979)], and Zh. Eksp. Teor. Fiz. 80, 716

(1981) [Sov. Phys. JETP 53, 362 (1981)].

[6] T. E. Huber and H. J. Maris, Phys. Rev. Lett. 47, 1907 (1981).

[7] M. Wanner and P. Leiderer, Phys. Rev. Lett. 42, 315 (1979).

[8] P. Leiderer and M. Wanner, Phys. Lett. 73A, 189 (1979); P. Leiderer, W. Ebner, and V. B. Shikin, Surf. Sci. 113, 405 (1982).

[9] R. W. Gianetta and H. Ikezi, Phys. Rev. Lett. 47, 849 (1981).

[10] The formation of a dimple—rather than a homogeneous depression of the whole surface—is caused by a confining field due to negative charges on the walls, which leads to an accumulation of the ions near the center of the cell. At high fields this effect is reinforced by the self-trapping of the ions, as discussed for liquid surfaces in Ref. 8.

[11] The energy barrier at the solid-liquid interface was calculated as for the boundary of phase-separated ^{3}He-^{4}He mixtures. See P. Leiderer, M. Wanner, and W. Schoepe, J. Phys. (Paris), Colloq. 39, C6-1328 (1979).

[12] V. B. Shikin and P. Leiderer, Zh. Eksp. Teor. Fiz. 81, 184 (1981) [Sov. Phys. JETP 54, 92 (1981)].

[13] L. Meyer and F. Reif, Phys. Rev. 123, 727 (1961).

[14] A. P. Volodin, M. S. Khaikin, and V. S. Edel'man, Pis'ma Zh. Eksp. Teor. Fiz. 26, 707 (1977) [JETP Lett. 26, 543 (1977)].

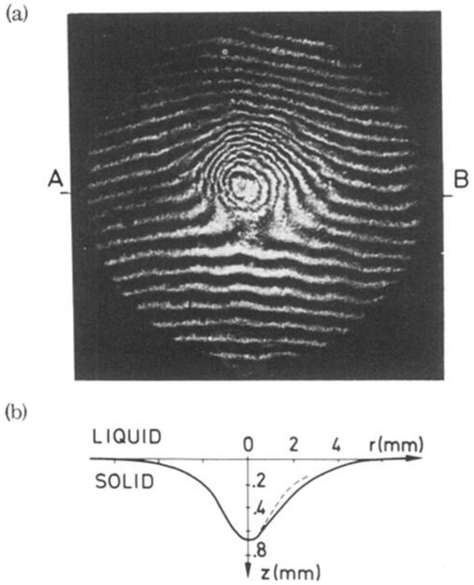

FIG. 1. (a) Interference pattern of an hcp-superfluid ^{4}He interface deformed by $\sim 10^6$ negative ions in an external field $E = 700$ V/cm, at a temperature $T = 1.35$ K. The parallel fringe pattern outside the center results from a slight inclination of the two interferometer plates. (The helium crystal in this region is flat, and hence the fringes are not distorted.) (b) Dimple profile determined from (a) along the direction A-B (full curve). A comparison with the dashed line, which is a mirror image of the left-hand profile, illustrates the asymmetry of the dimple.

FIG. 3. Interference pattern of an hcp-superfluid ^{4}He interface undergoing a charge-induced instability, about 1 sec after a (supercritical) voltage $U = 1800$ V was applied between the two electrodes. An additional holding field resulting from negative charges on the cylindrical cell walls confines the ions on the crystal surface in a circular central patch about 1 cm in diameter.