

On the Correlation between Properties of Multielectron Dimples and Bubbles

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Abstract Characteristics of multielectron dimples on the surface of liquid helium and those of multielectron bubbles arising in its bulk when the charged surface develops electrohydrodynamic instability are strongly interrelated. This circumstance allows one to explain a number of observable properties of multielectron bubbles in helium.

Keywords Electrons and ions · Superfluid ^4He

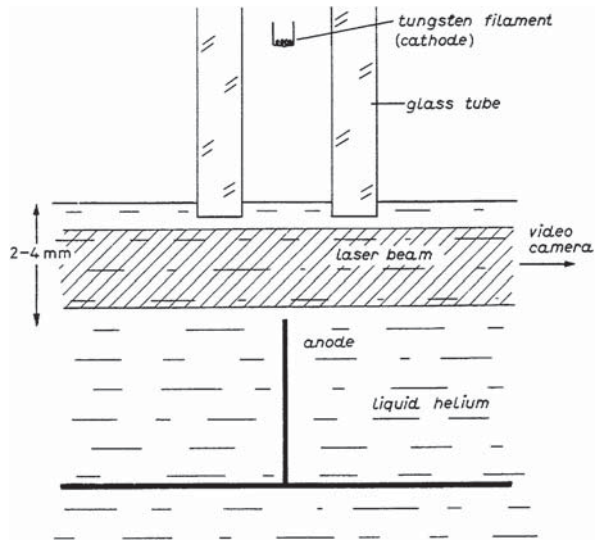
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1 Introduction

One of the problems in physics of multiply charged complexes in liquid helium is establishing their development scenario. For example, multielectron dimples on the surface of liquid helium arise when the surface instability develops in the so-called binodal mode [1]. Dimples of a similar nature, but in the dissipative kinetics mode, occur at the solid-liquid helium interface [2, 3]. An attempt to implement a quasiequilibrium scenario for multiply charged bubbles was undertaken in a series of papers [4, 5]. The purpose was to fix the number of electrons and increase the helium level until the entire cell (except for the automatically arising charged bubble) is filled with liquid. However, this appealing idea has not yet been fully realized. A different and

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Fig. 1 Experimental setup

more successful (although non-equilibrium) technique for production of bubbles was proposed by authors of [6, 7]. A glass tube with the internal radius of the order of capillary length is immersed into liquid helium. The source of electrons inside the pipette raises the electron pressure on the liquid surface up to the critical value after which a multielectron bubble (MEB) is injected into the bulk liquid which is then dragged towards the positive electrode (anode) with almost constant velocity (Fig. 1). This technique not only demonstrates the presence of charged bubbles but also allows to manipulate their sizes and measure their velocity.

It is important to note that being extremely “soft”, the multielectron bubbles in helium provide a convenient model object allowing a quantitative study of interesting (from the practical point of view) strongly fluctuating complexes such as micella, vesicles, etc. In the present Letter the obvious correlation between the properties of bubbles [6] and multielectron dimples is employed to explain some features observed in the bubble behaviour.

2 Multielectron Dimples

Let us first consider the details of the multielectron dimples behaviour. According to the measurements reported in [2, 3], there exist two critical fields. One of them, the threshold field for development of a dimple, does not depend on the total charge Q of electrons accumulated in the dimple and is only determined by the combination of constants appearing in the charged liquid surface stability criterium:

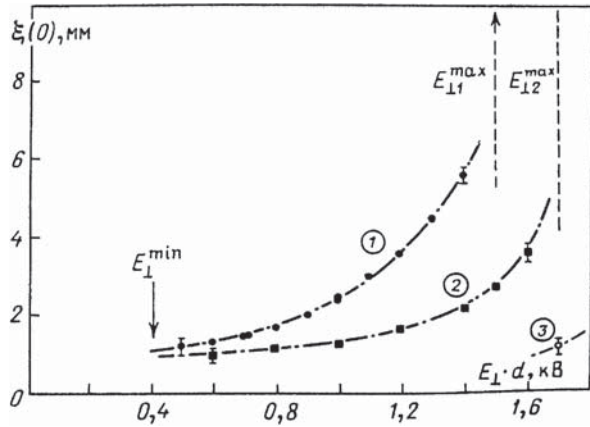
$$E_{\perp}^0 = 4\pi\sigma_0, \quad \sigma_0^4 = \rho g\alpha/(4\pi^2), \quad (1)$$

where ρ is the liquid density, g is the gravity acceleration, and α is the surface tension.

For

$$E_{\perp} > E_{\perp}^0 \quad (2)$$

Fig. 2 Dimple depth $\xi(0)$ as a function of holding field E_{\perp} at $T = 1.35$ K. Labels 1, 2 and 3 correspond to different original dimples with $Q_1 > Q_2 > Q_3$ [3]



the radius R of the charged area of a single dimple is found from the equation [8]

$$\frac{R_c^2}{R^2} = \frac{1}{2\lambda} - \sqrt{\frac{1}{4\lambda^2} - 1}, \quad \lambda = \frac{QE_{\perp}^3}{3\pi^3\alpha^2}, \quad R_c^2 = 3\pi Q/4E_{\perp}. \quad (3)$$

In the range of $\lambda \ll 1$ (corresponding to the initial stage of the reconstruction)

$$\frac{1}{R} \simeq \frac{2E_{\perp}^2}{3\pi^2\alpha}, \quad (4)$$

i.e. R does not depend on Q . However, if λ cross the border $1/2$, the definitio R (3) becomes “unstable”. In that case the critical fiel E_{\perp}^{\max} is rather sensitive to the charge Q :

$$Q_{\max} = \frac{3\pi^3\alpha^2}{2(E_{\perp}^{\max})^3}. \quad (5)$$

The experiment qualitatively confirm this relationship between Q_{\max} and E_{\perp}^{\max} (see Fig. 2 in [3]).

3 Multielectron Bubbles

Turning now to the charged bubbles from [6, 7], it is natural to assume that they arise when the inequality (5) is satisfied. This provides the grounds for comparison of (5) with available data on the number of electrons Z ($Q = Ze$) in the bubble produced by the applied holding field E_{\perp}^*

$$E_{\perp}^* = U/d, \quad (6)$$

where U is the anode voltage and d is the distance between the tube end and the anode (see Fig. 1). Although the numerical factor α here should not be taken seriously, it is natural to conclude that (5) yields a qualitatively correct relationship between Z and E_{\perp}^{\max} (see Fig. 3).

Fig. 3 MEB charge as a function of holding field E_{\perp} . Solid line corresponds to formula (5), experimental points are taken from [9]

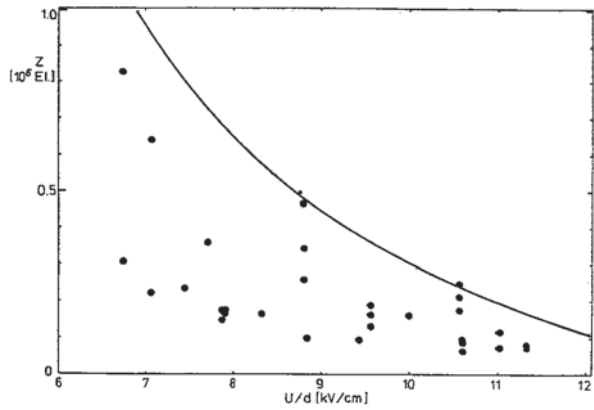
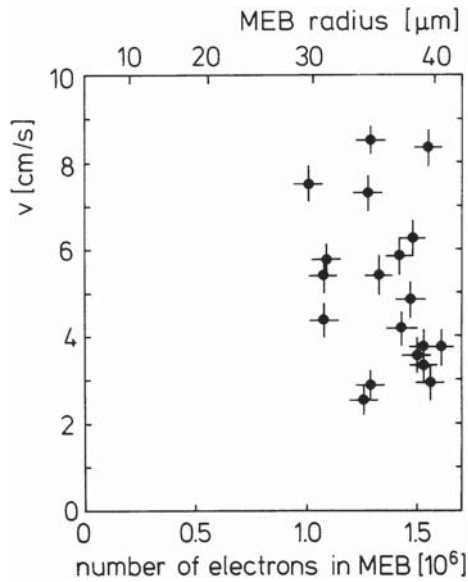


Fig. 4 Measured velocity vs. MEB charge. The calculated equilibrium radii are indicated on the upper scale



One additional result reported in [9], the bubble velocity in dependence of the driving electric field find a natural explanation within the framework of the correlation picture between the bubbles and multielectron dimples. Assuming the bubble charge to be known, one find its equilibrium radius [10]

$$R^3 = \frac{Q^2}{16\pi\alpha}. \tag{7}$$

Further, by equating the driving and drag forces applied to the charged cluster

$$Q_{\max} E_{\perp}^{\max} = F_{\text{Stokes}}, \tag{8}$$

and expressing both sides of this equation through Q_{\max}

$$F_{\text{Stokes}} = 6\pi\eta RV = 6\pi\eta V \left(\frac{1}{16\pi\alpha} \right)^{1/3} Q_{\max}^{2/3},$$

$$Q_{\max} E_{\perp}^{\max} = \left(\frac{3\pi\alpha^2}{2} \right)^{1/3} Q_{\max}^{2/3},$$

one obtains for the velocity V from the force balance given by (8) the result which is independent of Q_{\max} (in agreement with experiment [7]):

$$V = \frac{1}{3}(3\pi)^{1/3} \frac{\alpha}{\eta}. \quad (9)$$

Since the velocity V in (9) proves to be finite the viscosity η appearing in that equation can be interpreted as a turbulent viscosity [11].

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