Phase-controlled spin and charge currents in a superconductor-ferromagnet hybrid

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We investigate spin-dependent quasiparticle and Cooper-pair transport through a central node interfaced with two superconductors and two ferromagnets. We demonstrate that voltage biasing of the ferromagnetic contacts induces superconducting triplet correlations on the node and reverses the supercurrent flowing between the two superconducting contacts. We further predict that such triplet correlations can mediate a tunable spin current flow into the ferromagnetic contacts. Our key finding is that noncollinearity in combination with spin-mixing results in equal-spin-triplet correlations on the node and leads to a net charge current between the unbiased two magnets. Our proposed device thus enables the generation, control, and detection of the typically elusive equal-spin-triplet Cooper pairs.

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I. INTRODUCTION

Cooper pairs from a superconductor (S) placed in the vicinity of a ferromagnet (F) may diffuse into the latter, thus modifying their electronic properties [1–5]. The engineering of this proximity effect in hybrid structures generated over the past few years considerable interest in thermoelectricity [6–16], spin caloric transport [17,18], and topological superconductivity [19–24]. Somewhat unexpected is that not only can ferromagnets [25–28] and antiferromagnetic insulators [29] cause spin imbalances into superconductors, but also normal metals [30] exploiting a superconductor itself may serve as a spin filter [31].

By sandwiching a normal metal or ferromagnet between two superconductors, one can realize a Josephson junction featuring a current of Cooper pairs between them, which is characterized by the junctions’ free energy [32–34]. Its global minimum determines the ground state of the system. While a ground state at zero phase difference indicates a Josephson current mediated by singlet Cooper pairs, a shifted ground state about \( \pi \), occurring in magnetic Josephson junctions [35–44], signals triplet superconductivity [45–48] manifesting in a reversed current phase relation (CPR) [49–51]. Such magnetic Josephson junctions are interesting for quantum computation [52–54] and cryogenic memories [55,56].

Recently, the generation of equal-spin-triplet pairs has been demonstrated in S/F systems consisting of two s-wave superconducting \((n = S_1, S_2)\) and two ferromagnetic \((n = F_1, F_2)\) terminals with a common contact region [Fig. 1(a)]. Our key finding is that a voltage applied to the two ferromagnetic leads in combination with noncollinear magnetizations can induce triplet superconducting correlations in the system and therewith transitions in the CPR. We show that the mechanism sketched in Fig. 1(b) allows the relative magnetization angle \( \theta \) to control the spin currents in the ferromagnetic and superconducting contacts for voltages above the superconducting gap. As shown in the Fig. 1(b), left image, the interaction between the quasiparticle spin chemical potential and the condensate induces triplet correlations which are being spin split and distributed within the superconducting leads, resulting in the net spin supercurrent. Above the superconducting gap, the left and right magnetic leads inject up- and down-spin electrons into the node, respectively [see the right image of Fig. 1(b)]. This implies that there is (an up-)spin flow from the left to the right magnet. Note that \( I^L_F \) is a purely-quasiparticle-mediated spin transport and does not depend on the superconducting phase difference. Finally, we predict that asymmetric spin-mixing conductances (e.g., \( G^L_F = 0 \) and \( G^R_F > 0 \)) in a noncollinear magnetization configuration leads a finite net charge current between the ferromagnetic contacts. As illustrated in Fig. 1(c), the origin is the generation of equal-spin-triplet correlations in the central node. As depicted in the right image, introducing the finite \( G^R_F \) results in an imbalance between the injected Andreev currents to the left and right magnets, which in return gives rise to the current. Altogether, our proposal allows for the experimental detection and exploitation of spin-polarized

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c account for losses of superconducting correlations. The ducting and two ferromagnetic terminals via corresponding of a central node which is interfaced with two superconductors. Here we map our system to a layout consisting structures are discretized as a network of nodes, terminals, and superconducting spintronics technologies [64,65].

various ways, making it a potential building block for future multiterminal topology allows the currents to be channeled in spin-triplet correlations and supercurrents. Furthermore, its our proposed device allows an easy voltage control of the (anti)parallel magnetization of the ferromagnetic leads for (0). (b) The left (right) sketch of the central node illustrates a net magnetization angle [67–71] circuit theory [72,73]. In this framework, hybrid spin-triplet conductances are governed by the ferromagnetic contacts are governed by the BCS bulk Green’s functions (2)

where

is the normalization condition

The conservation of the transport properties. The normalization condition

of dimension 8 matrixes and the haˇcek indicates the Keldysh characterizing the transport properties. The conservation of the total matrix current at this node, \(0_n = \sum I_n\), together with the normalization condition \(\tilde{G}_n^R = \frac{1}{8}\) determines \(\tilde{G}_n\) and therefore with the individual matrix currents \(I_n\) \(\tilde{G}_n\) between terminal \(n\) \(\in\{S1, S2, F1, F2, \text{leak}\}\) and the central node.\(^\dagger\)

The superconducting contacts are characterized by the BCS bulk Green’s functions \((2/G_0)\mathcal{M}_{S0} = \tilde{G}_S\), with \(G_0\) the conductance of the corresponding connector to the central node and \(\alpha = 1, 2\). Its retarded/advanced component reads, in the spinor basis \{\(\Psi_1^+, \Psi_1^-\), \(\Psi_2^+, -\Psi_2^+\}\} [6,7],

\[
\tilde{G}_{S\alpha}^R = \frac{\pm \text{sgn}(\epsilon)}{} (\epsilon \pm i\Gamma)^2 - |\Delta_u|^2 \mp i\Gamma - \epsilon \otimes I_2, \tag{1}
\]

where \(\Delta_{1,2} \equiv \Delta \exp(\pm i\theta/2)\) denotes the superconducting gap with phase difference \(\phi\) across the junction. A small imaginary component in the denominator of Eq. (1) (here \(\Gamma = 10^{-3}\Delta\)) accounts for a finite lifetime \(\Gamma/\Delta\) of the quasiparticles, with energy \(\epsilon\), smearing out the superconducting gap [74]. The ferromagnetic contacts are governed by \(\mathcal{M}_{F\alpha} = (1/2)[G_N(\hat{a}_\uparrow + PK_a)\hat{G}_F - iG_0^\sigma\hat{a}_\downarrow\] with \(G_N\) \((G_0^\sigma)\) the normal (spin-mixing) conductances of the corresponding connectors. In addition, \(P\) denotes the contact spin polarization and \(\hat{a}_\downarrow = \hat{a}_\downarrow \otimes \sigma \otimes (m_\alpha \cdot \sigma)\) is the spin matrix which is diagonal in Keldysh space. In the last expression, \(\sigma = \{\sigma_x, \sigma_y, \sigma_z\}\) labels the vector of Pauli matrices and \(m_\alpha\) is the magnetization vector corresponding to the ferromagnet \(F\alpha\). Here we set \(m_1 = (0, 0, 1)\) in the \(z\) direction and consider \(m_2 = (\sin \theta, 0, \cos \theta)\) tilted by an arbitrary angle \(\theta\). We further consider fully polarized ferromagnetic contacts, i.e., \(P = 1\). The retarded/advanced component of the ferromagnetic Green’s function is given by \(\tilde{G}_F^{R,A} = \pm \sigma_z \otimes I_2\). Finally, the leakage terminal is described by \(\mathcal{M}_{I\text{leak}} = -iG_F(\epsilon/\epsilon_{\text{Th}})I_2 \otimes \sigma \otimes I_2\), with \(\epsilon_{\text{Th}}\) the Thouless energy. The Keldysh component of the S and F Green’s functions follows from \(\tilde{G}_n^R = \tilde{G}_n^R\hat{h}_n - \hat{h}_n\tilde{G}_n^A\), with the distribution function \(\hat{h}_n = \text{diag}([\text{tanh}(\epsilon - \epsilon V_{S1})/2k_BT], \text{tanh}(\epsilon + \epsilon V_{S2})/2k_BT]) \otimes I_2\). Hereafter, we assume all contacts at zero temperature \(k_BT = 0\), the superconductors at zero (reference) voltage \(V_{S1} = V_{S2} = 0\), equally biased ferromagnetic contacts \(V = V_{F1} = V_{F2}\), and equal conductances \(G_S = G_N\). The cases of asymmetric device configurations and partially polarized ferromagnets are discussed in Appendixes A and B, respectively.

The Keldysh component of the matrix currents \(\tilde{I}_n\) leads to the charge currents

\[
I_n = \frac{1}{8e} \int_{-\infty}^{\infty} d\epsilon \text{tr} \left[ (\sigma_z \otimes I_2) \tilde{I}_n^R(\epsilon) \right] \tag{2}
\]

and the \(p\)-z polarized spin currents

\[
I_n^p = \frac{\hbar}{16e^2} \int_{-\infty}^{\infty} d\epsilon \text{tr} \left[ (I_2 \otimes \sigma_z) \tilde{I}_n^p(\epsilon) \right] \tag{3}
\]

\(^\dagger\)The symbol \(0_n (I_n)\) labels the zero (identity) matrix in \(n \times n\) dimensions and the haˇcek indicates the Keldysh\@Nambu\@spin space of dimension \(8 \times 8\).

II. METHOD

We study diffusive transport, within a semiclassical [67–71] circuit theory [72,73]. In this framework, hybrid structures are discretized as a network of nodes, terminals, and connectors. Here we map our system to a layout consisting of a central node which is interfaced with two superconducting and two ferromagnetic terminals via corresponding connectors [see Fig. 1(a)]. An additional leakage terminal can account for losses of superconducting correlations. The

FIG. 1. (a) Four-terminal circuit. Two superconductors with phases \(\pm \varphi/2\) and gap \(\Delta\) (green) and two ferromagnets with relative magnetization angle \(\theta\) (blue) are connected to a central node (yellow). The left (right) sketch of the central node illustrates a net spin current injection into the superconductors (ferromagnets) due to (antiparallel magnetization of the ferromagnetic leads for \(\epsilon_{\text{Th}} \ll \Delta\) and \(V \gtrsim \Delta/\epsilon\) in the absence of spin-mixing conductances. Singlet (mixed-triplet) correlations are indicated by encircled arrows containing a minus (plus) sign. (c) Finite spin mixing \(G_S^r > 0\) allows the generation and detection of equal-spin-triplet correlations (encircled arrows of the same color) by measuring a net charge current between the ferromagnetic leads.
[73,75–77]. In particular, we will analyze the charge $I_c = I_{S1} - I_{S2}$ and spin net current $I_S = \varepsilon f_{\uparrow \downarrow} - \varepsilon f_{\downarrow \uparrow}$ between the superconductors/ferromagnets ($\chi = S, F$). The matrix elements $f_{ss'} = \langle \Psi_s | \hat{G}_k | \Psi_{s'} \rangle$, with $s, s' = \uparrow, \downarrow$, contain the spectral information about spin-pair correlations in nonequilibrium. Here we consider the integrated quantities over positive energies $\epsilon > 0$ (triplet correlations are odd in $\epsilon$) to quantify singlet $F_S = \int d\epsilon (f_{\uparrow \downarrow} - f_{\downarrow \uparrow})/\sqrt{2}$, mixed-spin triplet $F_{T\uparrow \downarrow} = \int d\epsilon (f_{\uparrow \downarrow} + f_{\downarrow \uparrow})/\sqrt{2}$, and equal-spin-triplet correlations $F_{T\uparrow \uparrow} = \int d\epsilon f_{\uparrow \uparrow}$.

III. SCALING

Before analyzing the interplay between Cooper-pair and quasiparticle transport, let us recall that in equilibrium only a Josephson current $I^0_J = I_c \sin \varphi$ may flow between both $s$-wave superconductors, which has a purely sinusoidal CPR; all other currents require quasiparticle excitations. Depending on the effective size $L$ of the central node, the Josephson current scales in the diffusive regime for $\epsilon_{th} \ll \Delta$ (large island) with the Thouless energy $\epsilon_{th} \propto hD/L^2$, where $D$ is the diffusion constant (see the black solid line in Fig. 2(a)]. For $\epsilon_{th} \gg \Delta$ (small island), however, it is characterized by the superconducting gap $\Delta$ [34]. While the superconducting condensate is in equilibrium entirely formed by spin-singlet Cooper pairs, finite voltages may cause triplet correlations in the system. They can lead, for voltages below the gap [dashed line in Fig. 2(a)], to a reduction of the Josephson current; here we consider parallel collinear magnetization. For voltages above the gap and intermediate values of the Thouless energy $\epsilon_{th}$, such triplet correlations can even induce current reversals in the Josephson current $I_S$. In Fig. 2(a), showing the modulus of $I_S$ on a double logarithmic scale, these zero crossings (at which the logarithm diverges) result in the two sharp dips of the dash-dotted curve. Also the corresponding critical current $I_c = \max_{\epsilon} I_S$ [purple line in Fig. 2(a)] indicates with the kinks the presence of triplet correlations.

For Thouless energies much smaller (larger) than the superconducting gap, the corresponding net supercurrent stays always positive and is dominated by spin-singlet correlations. We show in the following that applied voltages in combination with nonparallel magnetization $\vartheta \neq 0$ can induce triplet correlations and transitions in the CPR also in the regimes $\epsilon_{th} \ll \Delta$ and $\epsilon_{th} \gg \Delta$.

IV. PHASE TRANSITIONS AND SPIN CURRENT CONTROL

First, let us consider the large-island regime $\epsilon_{th} \ll \Delta$, where the Cooper-pair transport is characterized by the Thouless energy, i.e., $I_S, I^0_J \propto \epsilon_{th}$. While the net current between both superconductors follows in equilibrium the usual sinusoidal CPR, $I_S \propto \sin \varphi$, nonequilibrium in combination with noncollinear magnetization, $0 < \vartheta < \pi$, can induce $0-\pi$ transitions for voltages $V > 0$ (and $P \gtrsim 0.6$; cf. Appendix B) below the gap $\Delta$, due to mixed-spin-triplet correlations [black solid line in Fig. 2(b)]. Above the gap, quasiparticle transport sets in and $I_S$ saturates. In this regime, a finite net spin current $I^0_J \propto V$ (red dash-dotted line) emerges between the ferromagnetic contacts for a nonzero magnetization angle, irrespective of $\varphi$. For the chosen symmetric configuration, however, no corresponding net charge current $I_F$ flows.

A special feature of our setup is the occurrence of a finite net spin current $I^0_J$ between both superconductors (blue dashed line) for voltages $V \gtrsim \Delta/e$. This effect is maximal for parallel magnetization $\vartheta = 0$, as can be seen in the inset, for which only mixed-triplet and singlet correlations are present. It vanishes for antiparallel orientation $\vartheta = \pi$, where no triplet correlations arise, and when the Josephson phase $\varphi$ is a multiple of $\pi$. Notice that $I^0_J$ is antisymmetric in $\varphi$, and $I^0_J$ and $\hat{I}^0_J$ are symmetric in $\vartheta$. While voltages $V$ above the gap can trigger net spin currents $I^0_J$ and $\hat{I}^0_J$, the magnetization angle $\vartheta$ can control their ratio (see the inset), making the proposed setup thus attractive for future applications.

V. SPIN-MIXING-INDUCED CHARGE CURRENT

Let us now turn to the small-island regime $\epsilon_{th} \gg \Delta$, where losses of superconducting coherence become
Remarkably, the noncollinear magnetization in this system gives rise to equal-spin-triplet correlations \( |F_{T1}| \) and \( |F_{T1}| \) [see Fig. 3(b)]. A distinctive feature, however, is that these equal-spin-triplet correlations can induce a finite net charge current \( I_F \) into the ferromagnets [green dotted line in Fig. 3(a)] for asymmetric spin mixing \( G_{S} \neq G_{T} \). This feature is attributed to the creation of an imbalance in the ferromagnetic spin channels [see Fig. 1(c)]. This effect also persists for vanishing Josephson phase \( \varphi \). Under a mutual exchange of the spin-mixing conductances \( (G_{S} \leftrightarrow G_{T}) \), the net charge current \( I_F \) just inverts. However, \( I_F \) additionally contains an offset for asymmetric device configurations, detailed in Appendix A. An experimentally measurable charge current \( I_F \) serves also in the large-island regime as a signature of equal-spin-triplet correlations. It features in this regime a similar curve progression, but scales instead with \( \epsilon_{Th} \).

VI. CONCLUSION

Spin-dependent quasiparticle and Cooper-pair transport have been analyzed in a proximity-coupled multiterminal \( S/F \) heterostructure in nonequilibrium. We have shown that 0-\( \pi \) transitions can be induced in the CPR by biasing the ferromagnetic contacts and bearing noncollinear magnetic moments, as long as the loss of superconducting coherence is large, \( \epsilon_{Th} \ll \Delta \). In this limit, voltages exceeding the superconducting gap, \( V \gg \Delta/e \), trigger net spin currents into the ferromagnets/superconductors, which can be controlled by the relative magnetization angle \( \theta \). The small-island regime, however, requires additionally finite spin mixing to induce a 0-\( \pi \) transition in the CPR. The proposed heterostructure constitutes an ideal platform for the generation of triplet correlations of different spin projection and as a voltage- and phase-controlled switch for spin and electron currents. In particular, it constitutes a minimal setup for generating, controlling, and detecting equal-spin-triplet correlations by current measurements.

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APPENDIX A: ASYMMETRIC DEVICE SETUP

In this Appendix, we address the effect of an asymmetric setup and discuss in particular asymmetric boundary conductances of the connectors between the central node and the ferromagnetic/superconducting terminals. Unlike the voltages, they cannot be tuned externally and differ from sample to sample, although a low asymmetry can generally be achieved within a single device and fabrication process. We distinguish here the conductances to the ferromagnetic terminals \( n = Fa \) with \( G_{Sa} \) (\( \alpha = 1, 2 \)) and define the conductances to the superconducting terminals \( n = S \alpha \) with \( G_{Sa} \). Note that
Now let us turn to the case of asymmetric conductances of the connectors to the superconductors. Here the net charge current $I_F$ [colored lines in Fig. 4(e)] and the equal-spin-triplet correlations $F_{T↑}$ [colored lines in Fig. 4(f)] show only small deviations from the symmetric configuration (black lines). However, the net supercurrent is roughly shifted due to the asymmetry $G_{S1} \neq G_{S2}$ with an offset of $I_S^{\text{offset}} \approx (G_{S1} - G_{S2})V$ [see Fig. 4(d)]. Here we have to subtract this offset contribution from $I_S$ to obtain the supercurrent from which one can estimate the 0-π transitions.

**APPENDIX B: PARTIALLY-SPIN-POLARIZED FERROMAGNETS**

While we focused in the main text on the ideal case of fully polarized ferromagnets permitting us to more easily separate occurring transport processes, we discuss in this Appendix what changes when rather partially-spin-polarized ones are used. In particular, we consider here typical ferromagnets such as Ni, Co, and Fe, which have a spin polarization of about $3–0.5$; higher values may be achieved by using specific compound semiconductors or alloys [38,78,79].

To get an overview of the chief differences with respect to the fully polarized case, we depict in Fig. 5(a) how the charge and spin currents, given in Fig. 3(a), change for a polarization of $P = 0.3–0.5$; higher values may be achieved by using specific compound semiconductors or alloys [38,78,79].
a function of dependence of \( P \) for which the supercurrent changes sign, thereby manifesting a 0-\( P \) and IF scaling factor. While the net charge current in the fully polarized case. However, there is no global rescaling factor. While the net charge current \( I_F \) scales roughly quadratically with \( P \) [green dotted line in Fig. 5(b)], the net spin current \( I_F^S \) changes rather linearly [red dash-dotted line in Fig. 5(b)].

Now let us turn to the net supercurrent \( I_S \), which for a spin polarization of \( P = 0.4 \) no longer features a current reversal [see Fig. 5(a)]. This disappearance is just related to the fact that a weaker polarization also implies that the triplet correlations are less pronounced. Figure 5(b) reflects this interplay between the singlet and the triplet correlations in the monotonic rise of \( I_S \) for a decreasing \( P \). It can be seen that the system stays for a polarizations below \( P \approx 0.6 \) (purple circle) always in a 0-like state. Finally, let us return to the choice of the phase difference used in Fig. 5. To this aim, we depict in Fig. 6 the current-phase relation of the net supercurrent \( I_F \) for different spin polarizations. We see that the \( \sin \varphi \) behavior for strong polarization (\( P = 1 \)) toward a \( \sin \varphi \) behavior for weak polarization (\( P = 0.4 \)) goes along with a shift of the minimum occurring at smaller values of \( \varphi \). Moreover, one sees that for an intermediate spin polarization of \( P = 0.9 \) the net supercurrent \( I_S \) picks up a large \( -\sin 2\varphi \) component.

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[10] S. Kolenda, M. J. Wolf, and D. Beckmann, Observation of Thermoelectric Currents in High-Field Superconductor-


