Three Essays in Quantitative Macroeconomics

Dissertation

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2. Referent: Prof. Dr. Volker Hahn
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Summary

This thesis presents three essays in the field of empirical and quantitative macroeconomics. The essays focus on the macroeconomic consequences of credit-market imperfections. The first chapter is about the dynamics of the sovereign credit, while the rest two chapters are on the corporate/business credit. In terms of methodologies, panel-data approaches are applied in both Chapter 1 and Chapter 2. Structural quantitative methods, on the other hand, are used in Chapter 1 and Chapter 3.

In Chapter 1, which is joint work with Almuth Scholl, we study the dynamic interaction between sovereign default risk, taxation, and the underground economy. The study is motivated by the observation that in the wake of sovereign debt crises, many countries have adopted fiscal consolidation policies in order to reduce public debt and to restore creditworthiness. However, particularly in crisis-prone countries, substantial underground activities undermine tax enforcement. We first conduct an empirical analysis. For a large sample of countries, we find that the size of the underground economy is positively correlated with sovereign debt and interest spreads. We rationalize these empirical regularities within a quantitative model of sovereign default that explicitly accounts for underground activities. We study optimal fiscal policy in the presence of limited tax enforcement and default risk using the concept of Markov-perfect equilibria, in which the government moves first and the households form expectations about future public policies. In a quantitative exercise, we analyze the properties of optimal public policies and the private sector's responses and study the dynamics of a default event with particular focus on the underground sector. Our simulation results reveal that the theoretical framework replicates the empirical regularities very well. In particular, our model predicts that the size of the underground economy is positively correlated with sovereign debt and interest spreads. We show that during a debt crises, the dynamic interaction between sovereign default risk and the underground economy creates a vicious circle: Higher sovereign risk premia tighten the endogenous borrowing constraint and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. In turn,
falling tax revenues force the government to either implement further tax hikes or to default. Eventually, raising taxes becomes too costly and the government finds it optimal to default. Our quantitative findings suggest that the underground economy fosters sovereign default risk and deepens debt crises.

In the second chapter, we ask the following question: Has the financial sector become more efficient over time? It is natural to believe that the financial efficiency has improved, at least in an advanced economy such as the United States. However, some authors, such as Philippon (2015) and Bazot (2018), show that it might not be the case. This chapter revisits the estimation of financial efficiency. In this chapter, we propose a regression method to assess the time trend in the firm’s intermediation cost of external credit. We define the intermediation cost as the part of the unit cost of the external credit finance that is unrelated to the risk premium. We find that the intermediation cost has decreased significantly over the period of 1983Q1–2007Q4, which has substantially reduced firm’s borrowing cost. On average, the fall in the intermediation cost leads to a reduction by 0.57-0.71 percentage point in the ratio of interest expense over revenue, and a decrease by 0.78-0.79 percentage point in the ratio of interest expense over book value of debt. The estimation results are robust against different sample components and against alternative measures of the risk-free rate and inflation. The reduction in the intermediation cost implies that the financial efficiency has improved in the US corporate credit market. We also assess the cross-sectional average risk premium and unit cost over the same period of time, but find mixed results.

Chapter 3 is motivated by the empirical evidence in Chapter 2, and explores the macroeconomic implications of such decrease in the intermediation cost. The chapter is motivated by the fact that over the past decades, there has been a dramatic credit boom in the United States, coupled with decreasing asset returns and rising inequality. It analyzes whether the decreasing intermediation cost of borrowing is an explanation for these developments. We first construct a simple two-period model, followed by a fully dynamic model based on Angeletos (2007), which we apply to the United States in a quantitative exercise. We find that the macroeconomic effects of the fall in intermediation costs are amplified by two feedback loops: one is between the capital market and the credit market, and another is between the capital-credit market and the wealth distribution. We show that, due to lower intermediation costs, the credit market experiences a “simultaneous” expansion of credit demand and credit supply. As a result, the real risk-free interest rate barely changes. The capital market also expands, leading to a decrease in the returns on capital. The feedback loops exaggerate the capital-income risk and increase the average returns on investment among leveraged investors, driving up the overall wealth and income inequality. In a quantitative exercise, we find that much of the rise in the top-end wealth inequality during 1980–2007 could be explained by the reduction in the intermediation costs. In terms of welfare, we find that the
welfare decreases for the households in the bottom-90% wealth group, while it improves for households in other groups.
Zusammenfassung


zeigen, dass die dynamische Wechselwirkung zwischen staatlichen Ausfallrisiken und der Schattenwirtschaft einen Teufelskreis während einer Verschuldungskrisen schafft: Höhere staatliche Risikoprämien verschärfen die endogene Beschränkung der Kreditaufnahme und zwingen die Regierung dazu, die Steuern zu erhöhen. Steuererhöhungen induzieren jedoch die privaten Sektors, weniger zu investieren und Steuern zu hinterziehen, indem sie in der Schattenwirtschaft produzieren. Im Gegenzug zwingen sinkende Steuereinnahmen die Regierung entweder zur weiteren Steuererhöhungen oder zur Zahlungsunfähigkeit. Irgendwann wird die Steuer zu teuer sein, damit die Regierung einen Ausfall als optimal hält. Unsere quantitativen Ergebnisse legen nahe, dass die Schattenwirtschaft die Zahlungsunfähigkeit von Staaten begünstigt und Schuldenkrisen vertieft.


In einer quantitativen Untersuchung stellen wir fest, dass ein Großteil des Anstiegs der Top-End-Vermögensungleichheit in den Jahren 1980-2007 durch die Verringerung der Vermittlungskosten erklärt werden kann. In Bezug auf die Wohlfahrt stellen wir fest, dass die Wohlfahrt für die Haushalte in der unteren 90% Vermögensgruppe abnimmt, während die Wohlfahrt für Haushalte in anderen Gruppen besser geworden ist.
Chapter 1

Sovereign Default, Taxation and the Underground Economy
1.1 Introduction

In the wake of sovereign debt crises, many countries have adopted fiscal consolidation policies in order to reduce public debt and to restore creditworthiness. However, particularly in crisis-prone countries, substantial underground activities undermine tax enforcement. According to Schneider et al. (2010), the share of the underground production over official GDP averaged 25.3% in Argentina, 27.5% in Greece and 27.0% in Italy during 1999 and 2007, compared to an average of 16.0% in Germany, 8.6% in the United States and 13.4% in all OECD countries\(^1\) during the same period of time. The non-negligible size of the underground economy in such countries raises several important questions: How do tax evasion and the underground economy affect the government’s policy choices between external borrowing, taxation, and public spending? How does limited tax enforcement impact sovereign default risk? How effective are fiscal consolidation policies in the presence of a large underground economy?

Motivated by these questions, this chapter analyzes the dynamic interaction between sovereign debt, taxation, and the underground economy. We first explore the empirical relationship for a large sample of countries and find that the size of the underground economy is positively correlated with government debt and sovereign interest spreads, particularly in the sample group of emerging economies.\(^2\) We then rationalize these empirical regularities within a quantitative model of sovereign debt and default. We consider a small open economy in which a government finances a public good by taxing income and by issuing external long-term debt. The economy is divided into two sectors of production. In the official market sector, taxes are perfectly enforceable and production takes place using capital and labor. Production in the unofficial non-market (underground) sector uses labor only and is hidden from the tax authorities. Thus, households can evade taxes by running underground activities. Tax evasion, however, is illegal and bears the risk of getting detected and punished by the government. International financial markets are incomplete and debt contracts are not enforceable. In any period, the government has the option to default on its external debt. International creditors incorporate the probability of a sovereign default and charge a risk premium. In the event of a default, the government is temporarily excluded from international financial markets and finances public spending solely via taxation.

We study optimal fiscal policy in the presence of limited tax enforcement and default risk using the concept of Markov-perfect equilibria, in which the government moves first and the households form expectations about future public policies. In a quantitative exercise, we analyze the properties of optimal public policies and the private sector’s responses and study the dynamics of a default event with particular focus on the underground sector.

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\(^1\) The average in OECD countries is weighted by total GDP of each country in 2005.
\(^2\) These findings are line with Elgin and Öztunalı (2012) and Elgin and Uras (2013).
Our simulation results reveal that the theoretical framework replicates the empirical regularities very well. In particular, our model predicts that the size of the underground economy is positively correlated with sovereign debt and interest spreads. We show that during a debt crisis, the dynamic interaction between sovereign default risk and the underground economy creates a “vicious circle”: Higher sovereign risk premia tighten the endogenous borrowing constraint and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. In turn, falling tax revenues force the government to either implement further tax hikes or to default. Eventually, raising taxes becomes too costly and the government finds it optimal to default.

After the default, once the government regains access to international financial markets, the vicious circle is reversed and becomes a “virtuous circle”: Because after a default, the debt burden is low, the government is able to implement a low tax rate encouraging formal production activities. In turn, a growing formal sector raises tax revenues, which allow the government to reduce the tax rate even further. The tax cuts amplify the positive impact on formal labor and investment and foster the recovery of the economy.

This chapter builds on the quantitative literature on sovereign debt and default, initiated by Eaton and Gersovitz (1981), Arellano (2008), and Aguiar and Gopinath (2006). Recent contributions in this area of research analyze the role of fiscal policy, (Cuadra et al. (2010), Kaas et al. (2020)), capital and investment dynamics (Gordon and Guerron-Quintana (2018), Park (2017)), and the importance of long-term debt (Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Chatterjee and Eyigungor (2015)). This chapter is related to these studies as we study optimal fiscal policy in the presence of sovereign default risk allowing for endogenous production and investment dynamics. Moreover, to account for the high level of indebtedness, we include long-term debt.

Our main contribution is to explicitly account for the dynamic interaction between the underground economy and sovereign default risk. Our modeling approach of the underground economy follows Busato and Chiarini (2004), Orsi et al. (2014), and Pappa et al. (2015) who study the role of non-market production in two-sector general equilibrium models with a representative agent. These studies explore the impact of fiscal consolidations in the presence of underground economic activities, but abstract from sovereign default risk.

This chapter is related to a recent paper by Pappada and Zylberberg (2019) who study a model of sovereign default in which with a certain probability in each period entrepreneurs can choose between the adoption of a formal and an informal production technology. The authors show that fiscal consolidations may not unambiguously lower default risk, which is related to our finding of the vicious cycle presented here.
While we consider a representative agent setup, there are recent attempts to introduce the classic tax evasion mechanism of Allingham and Sandmo (1972) into dynamic macroeconomic settings with heterogeneous agents, see, e.g., Maffezzoli (2011), di Nola et al. (2018), and Kotsogiannis and Mateos-Planas (2019). Studies that focus on heterogeneous firms in the underground economy are, e.g., Ordonez (2014) and Antunes and Cavalcanti (2007). All these papers abstract from sovereign debt and default, which is the focus of this chapter.

This chapter is structured as follows. In Section 1.2, we present our empirical analysis of the relationship between the underground economy, public debt, and sovereign default risk. In Section 1.3 we describe the model environment and define the recursive equilibrium. Section 1.4 explains the calibration strategy and presents the quantitative analysis of our theoretical framework. Section 1.5 concludes.

1.2 **Empirical Facts**

In this section, we empirically analyze the interaction between the size of the underground economy, government indebtedness, and sovereign default risk. The methodology is similar to Elgin and Uras (2013), albeit with two differences: First, we adopt the Emerging Market Bond Index (EMBI) to measure the sovereign bond yield in emerging economies; and second, we differentiate between OECD countries and emerging market economies. We document two empirical facts. First, in emerging economies, the size of the informal sector is positively correlated with the government debt-to-GDP ratio. Second, the size of the informal sector is positively correlated with the government bond yield in emerging economies as well as in OECD economies. These findings are in line with Elgin and Uras (2013).

In a first step, Figure 1.1 illustrates how the size of the underground economy is related to the bond yield and to the debt-to-GDP ratio in the cross-section. For the sample of emerging economies, we measure the sovereign bond yield by the JPMorgan Emerging Market Bond Index (EMBI). For the sample of OECD countries, we employ the yield of the 10-year government bond. As shown in the left column of the figure, countries with a larger informal sector tend to have a higher government bond yield. For both samples, the Spearman’s

---

3 Specifically, we use the stripped yield to maturity of the EMBI Global index. There are four versions of the EMBI index: EMBI, EMBI+, EMBI Global and the EMBI Global Diversified. The first of the four covers only the Brady bonds. The EMBI+ extends the coverage of the original EMBI to other emerging economies and to other dollar-denominated loans and Eurobonds. The EMBI Global further extends the EMBI+ to cover even more countries and more eligible instruments. The difference between the last two indices lies only in the weighting of countries, which do not matter for our purposes. We use the EMBI Global in our research as it provides the largest coverage.
Figure 1.1: Cross-sectional correlations between sovereign debt and underground economy.

Notes: The bond yield for the OECD countries are taken as the yield of 10-year government bond; while for emerging economies is the stripped yield to maturity of the JP Morgan EMBI Global index. The red lines are best fits in the sense of least squares. “rank.corr” denotes the Spearman’s rank correlation, whose significance at the 10%, 5%, and 1% level is indicated by *, ** and ***, respectively.

We find a significant positive correlation between the size of the informal sector and the debt-to-GDP ratio in emerging economies (middle column). For OECD countries, we do not find a significant interaction between the underground economy and government indebtedness. As a supplement, we also plot the relationship between the informal sector and the GDP per capita in the right column of Figure 1.1. We find that larger underground economies are usually found in less-wealthy countries.

In a second step, to capture the dynamics between the size of the underground economy, government debt and bond yields over time, we apply the one-way error component model.

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4 We calculate the rank correlation, instead of the Pearson’s correlation, because the former is less affected by the outliers in our sample.

5 In the scatterplot of the debt-to-GDP for OECD countries, the rightmost outlier is Japan. If we exclude Japan from the sample, the rank correlation rises to 0.0920, still not significant.
with fixed individual effects. We use the following fixed-effect panel data model to explore the within-group correlations:

\[
\text{Indebtedness}_{i,t} = a + \alpha_i + \delta \text{SE}_{i,t} + \mathbf{x}_{i,t}' \boldsymbol{\beta} + \varepsilon_{i,t},
\]

(1.2.1)

\[
\text{Bond\_Yield}_{i,t} = a + \alpha_i + \delta \text{SE}_{i,t} + \mathbf{x}_{i,t}' \boldsymbol{\beta} + \varepsilon_{i,t}.
\]

(1.2.2)

Indebtedness\(_{i,t}\) refers to the gross general government debt-to-GDP ratio. Bond\_Yield\(_{i,t}\) denotes the sovereign bond yield, measured by the EMBI global in the sample of emerging economies, and by the yield on 10-year government bonds in the sample of OECD economies. SE\(_{i,t}\) stands for the size of the underground economy. \(\mathbf{x}_{i,t}\) is the vector of controls that could potentially explain the over-time variations of the dependent variables. Moreover, \(a\) is the constant term, and \(\alpha_i\) is the individual-specific intercept. Finally, \(i\) represents individual country and \(t\) stands for time. \(\varepsilon_{i,t}\) is the error term.

Equations (1.2.1) and (1.2.2) are estimated separately for the sample of emerging economies and the sample of OECD countries. The sample of emerging economies consists of 34 countries that have data on the EMBI Global index. The sample of OECD economies contains 33 countries.\(^6\) The panels are unbalanced. Data are of annual frequency\(^7\), and the time window is between 1999 and 2007. The sizes of the underground economy are from Schneider et al. (2010), who construct a comprehensive data set of the informal sector for 162 countries from 1999 to 2007.\(^8\)

The control variables represent other factors that potentially affect the dependent variables. GDP \textit{per capita}, trade openness and the GDP growth rate are taken from the Penn World Table 8.1. These variables are all adjusted by the purchasing power parity (PPP) to enable a cross-country comparison. The current-account balance and the annual rate of inflation are taken from the World Economic Outlook (WEO) data of the IMF. Moreover, we use the World Bank data for unemployment and the tax revenue. To cover the political environment of each country, we specify six political indicators as independent variables: government stability, investment profile, corruption, law and order, democratic accountability and the bureaucratic quality. These indicators are from the International Country Risk Guide (ICRG).

Table 1.1 summarizes the descriptive statistics of the dependent and independent variables for both samples. Emerging economies are characterized by higher average bond yields and larger sizes of the underground economy. They also tend to have higher inflation and unemployment, as well as lower scores on the political indicators. While the government debt-to-GDP ratios are similar in both samples, OECD countries have higher tax revenues.

\(^6\) See Table 1.7 of the Appendix 1.A for the components of each sample.
\(^7\) To the best of our knowledge, there is only annual estimates on the size of the informal sector.
\(^8\) Another wide-ranging data set of the informal sector is constructed by Elgin and Öztunalı (2012) using a model-based approach. Our empirical conclusions do not change when using alternative datasets on the underground economy.
Table 1.1: Summary statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>OECD Countries</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. Debt-to-GDP (%)</td>
<td>55.50</td>
<td>32.62</td>
</tr>
<tr>
<td>Bond Yield (%)</td>
<td>5.05</td>
<td>1.85</td>
</tr>
<tr>
<td>Underground Economy (%)</td>
<td>18.03</td>
<td>6.00</td>
</tr>
<tr>
<td>GDP per capita (Thd. USD)</td>
<td>30.27</td>
<td>11.55</td>
</tr>
<tr>
<td>Trade Openness (%)</td>
<td>85.85</td>
<td>49.46</td>
</tr>
<tr>
<td>Growth Rate (%)</td>
<td>3.86</td>
<td>3.89</td>
</tr>
<tr>
<td>Current Account (%GDP)</td>
<td>-0.10</td>
<td>6.36</td>
</tr>
<tr>
<td>Inflation (%)</td>
<td>2.66</td>
<td>1.85</td>
</tr>
<tr>
<td>Unemployment (%)</td>
<td>6.73</td>
<td>3.52</td>
</tr>
<tr>
<td>Tax Revenue (%GDP)</td>
<td>34.67</td>
<td>7.00</td>
</tr>
<tr>
<td>Gov. Stability</td>
<td>8.73</td>
<td>1.34</td>
</tr>
<tr>
<td>Investment Profile</td>
<td>10.94</td>
<td>1.28</td>
</tr>
<tr>
<td>Corruption</td>
<td>4.06</td>
<td>1.17</td>
</tr>
<tr>
<td>Law and Order</td>
<td>5.20</td>
<td>0.89</td>
</tr>
<tr>
<td>Demo. Accountability</td>
<td>5.74</td>
<td>0.46</td>
</tr>
<tr>
<td>Bureau. Quality</td>
<td>3.64</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Notes: The bond yield for the OECD countries are taken as the yield of 10-year government bond; while for emerging economies is the stripped yield to maturity of the JP Morgan EMBI Global index. Moreover, “Std.” is shorthand for the standard deviation, and “Obs.” represents the number of observations on each variable.
This implies that governments in emerging economies rely more on debt finance. One potential explanation for this phenomenon is that the larger informal sector in emerging economies limits the tax enforceability, which forces the governments to finance their expenditures through issuing debt. Importantly, the sample of the emerging economies shows a larger heterogeneity than the OECD countries, which can be seen in the high standard deviations for most variables.

Table 1.2 and 1.3 report the regression results of equations (1.2.1) and (1.2.2). For each sample, we run four regressions to study the robustness of the regression results against the specification of the covariates. The first regression uses the size of the underground economy as the only independent variable; the second adds the statistics from the national accounts. The ICRG scores are added to the third regression. The fourth regression applies all the above-mentioned independent variables plus the tax revenue and the government-debt-to-GDP ratio.\(^9\)

In emerging economies, we find a strong and robust positive correlation between the size of the informal sector and government indebtedness. Thus, government debt is higher in times of a larger underground economy. In OECD countries, the debt-to-GDP ratio is largely explained by other covariates. Among other explanatory variables, the current account balance and the unemployment rate are positive correlated with government indebtedness. These findings are in line with Kim and Roubini (2008), who, by means of VAR models, show that the historical data exhibit the “twin divergence” between fiscal and current account deficits.

In emerging economies, GDP per capita is positively correlated with government indebtedness. In contrast, in OECD countries, such correlation is weak and negative. This finding reveals the procyclical feature of fiscal policies in emerging economies.

Table 1.3 reveals that the size of the underground economy is positively and significantly correlated with the sovereign bond yield in both samples. Importantly, such correlation is stronger in the emerging economies than in the OECD countries.\(^10\)

### 1.3 The Model

We develop a small open economy in which there are three types of agents: households, international creditors, and a government. The government finances a public good by taxing income and by issuing external long-term debt. Following Busato and Chiarini (2004), Orsi

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\(^9\) The tax revenue is added only in the last regression because it has missing values for the emerging economies. The government-debt-to-GDP ratio is used only for the regression (1.2.2).

\(^10\) Note, however, that the sovereign bond yield is measured differently in the two samples. Among other things, the EMBI spread covers the government debt instruments denominated in US dollars and Euros only, and it includes bonds with all maturities.
Table 1.2: Regression results of the model (1.2.1).

<table>
<thead>
<tr>
<th>Ind. Variable</th>
<th>OECD Countries</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>Underground eco.</strong></td>
<td>3.0430***</td>
<td>-1.1967</td>
</tr>
<tr>
<td></td>
<td>(0.9595)</td>
<td>(1.7107)</td>
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<tr>
<td>GDP per capita</td>
<td>-0.4903</td>
<td>-0.6559*</td>
</tr>
<tr>
<td></td>
<td>(.3564)</td>
<td>(.3867)</td>
</tr>
<tr>
<td>Trade openness</td>
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<td>-0.0423</td>
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<tr>
<td></td>
<td>(.0744)</td>
<td>(.0783)</td>
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<td>Growth rate</td>
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</tr>
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<td></td>
<td>(.1208)</td>
<td>(.1304)</td>
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<td>Current account</td>
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<td>0.4262**</td>
</tr>
<tr>
<td></td>
<td>(.1913)</td>
<td>(.1972)</td>
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<tr>
<td>Inflation</td>
<td>0.941</td>
<td>0.0707</td>
</tr>
<tr>
<td></td>
<td>(.3855)</td>
<td>(.3978)</td>
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<tr>
<td>Unemployment</td>
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<td>1.5333***</td>
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<tr>
<td></td>
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<td>(.4267)</td>
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<td>Gov. stability</td>
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<tr>
<td></td>
<td>(.4860)</td>
<td>(.4848)</td>
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<tr>
<td>Investment Profile</td>
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<td>1.1229**</td>
</tr>
<tr>
<td></td>
<td>(.5011)</td>
<td>(.4958)</td>
</tr>
<tr>
<td>Corruption</td>
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<td>1.0211</td>
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<tr>
<td></td>
<td>(.9860)</td>
<td>(.9854)</td>
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<tr>
<td>Law and order</td>
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<tr>
<td></td>
<td>(1.3338)</td>
<td>(1.3178)</td>
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<tr>
<td></td>
<td>(1.9376)</td>
<td>(1.9030)</td>
</tr>
<tr>
<td>Bureau. quality</td>
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<td>-3.8116</td>
</tr>
<tr>
<td></td>
<td>(3.0622)</td>
<td>(3.0072)</td>
</tr>
<tr>
<td>Tax revenue</td>
<td>1.3577***</td>
<td>1.3577***</td>
</tr>
<tr>
<td></td>
<td>(.4422)</td>
<td>(.4422)</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>0.6586 (17.2990)</td>
<td>83.3044** (36.9594)</td>
</tr>
<tr>
<td></td>
<td>(1.3577***</td>
<td>1.3577***</td>
</tr>
<tr>
<td></td>
<td>(.4422)</td>
<td>(.4422)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>.9555 .9586 .9592 .9607</td>
<td>.8573 .8815 .8925 .9587</td>
</tr>
</tbody>
</table>

Notes: All regressions contain fixed individual effects for each country/economy. The numbers in parentheses are standard errors. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.
Table 1.3: Regression results of the model (1.2.2).

<table>
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<tr>
<th>Ind. Variable</th>
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<th></th>
<th>Emerging Economies</th>
<th></th>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
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<td>Underground eco.</td>
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<td>.7295***</td>
<td>.4746***</td>
<td>.5894***</td>
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<td>(.1046)</td>
<td>(.1729)</td>
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<td>(.1707)</td>
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<td>GDP per capita</td>
<td>−.0606*</td>
<td>−.0042</td>
<td>.0045</td>
<td>−.7737</td>
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<tr>
<td></td>
<td>(.0362)</td>
<td>(.0378)</td>
<td>(.0364)</td>
<td>(.6263)</td>
</tr>
<tr>
<td>Trade openness</td>
<td>−.0123</td>
<td>−.0141***</td>
<td>−.0110</td>
<td>.0723</td>
</tr>
<tr>
<td></td>
<td>(.0075)</td>
<td>(.0076)</td>
<td>(.0073)</td>
<td>(.0509)</td>
</tr>
<tr>
<td>Growth rate</td>
<td>−.0159</td>
<td>−.0291**</td>
<td>−.0248**</td>
<td>−.0403</td>
</tr>
<tr>
<td></td>
<td>(.0122)</td>
<td>(.0127)</td>
<td>(.0121)</td>
<td>(.0415)</td>
</tr>
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<td>Current account</td>
<td>−.0017</td>
<td>.0138</td>
<td>.0038</td>
<td>−.0931</td>
</tr>
<tr>
<td></td>
<td>(.0194)</td>
<td>(.0192)</td>
<td>(.0196)</td>
<td>(.0859)</td>
</tr>
<tr>
<td>Inflation</td>
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<td>.2672***</td>
<td>.1662***</td>
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<td>(.0386)</td>
<td>(.0383)</td>
<td>(.0372)</td>
<td>(.0366)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>.0706*</td>
<td>−.0434</td>
<td>−.0618</td>
<td>.4350*</td>
</tr>
<tr>
<td></td>
<td>(.0427)</td>
<td>(.0417)</td>
<td>(.0407)</td>
<td>(.2285)</td>
</tr>
<tr>
<td>Gov. stability</td>
<td>.1182**</td>
<td>.1274***</td>
<td>−1.2888***</td>
<td>−.3295</td>
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<tr>
<td></td>
<td>(.0475)</td>
<td>(.0459)</td>
<td>(.0459)</td>
<td>(.3009)</td>
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<td>Investment Profile</td>
<td>−.1637***</td>
<td>−.1756***</td>
<td>−.8076***</td>
<td>−.0168</td>
</tr>
<tr>
<td></td>
<td>(.0490)</td>
<td>(.0474)</td>
<td>(.0474)</td>
<td>(.2971)</td>
</tr>
<tr>
<td>Corruption</td>
<td>.1644*</td>
<td>.1031</td>
<td>1.2918*</td>
<td>1.1912**</td>
</tr>
<tr>
<td></td>
<td>(.0958)</td>
<td>(.0934)</td>
<td>(.0934)</td>
<td>(.6683)</td>
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<tr>
<td>Law and order</td>
<td>−.1507</td>
<td>−.1483</td>
<td>−1.5003**</td>
<td>.9061*</td>
</tr>
<tr>
<td></td>
<td>(.1307)</td>
<td>(.1247)</td>
<td>(.1247)</td>
<td>(.6510)</td>
</tr>
<tr>
<td>Demo. account.</td>
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<td>−.1941</td>
<td>.0255</td>
<td>−.8558**</td>
</tr>
<tr>
<td></td>
<td>(.1898)</td>
<td>(.1801)</td>
<td>(.1801)</td>
<td>(.5842)</td>
</tr>
<tr>
<td>Bureau. quality</td>
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<td>−1.3477</td>
<td>−.6276</td>
</tr>
<tr>
<td></td>
<td>(.3001)</td>
<td>(.2855)</td>
<td>(.2855)</td>
<td>(1.8083)</td>
</tr>
<tr>
<td>Tax revenue</td>
<td>−.0117</td>
<td>.0117</td>
<td>−.0416</td>
<td>−.0416</td>
</tr>
<tr>
<td></td>
<td>(.0427)</td>
<td>(.0427)</td>
<td>(.1872)</td>
<td>(.1872)</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>−15.4698***</td>
<td>−5.3290</td>
<td>−2.5500</td>
<td>−67.3578***</td>
</tr>
<tr>
<td></td>
<td>(1.8882)</td>
<td>(3.7389)</td>
<td>(4.3288)</td>
<td>(4.4685)</td>
</tr>
<tr>
<td>(Intercept)</td>
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<td>−2.8400</td>
<td>−6.7358***</td>
<td>−42.8808***</td>
</tr>
<tr>
<td></td>
<td>(.1882)</td>
<td>(4.3288)</td>
<td>(4.4685)</td>
<td>(8.2430)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
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<td>.8671</td>
<td>.8784</td>
<td>.5843</td>
</tr>
<tr>
<td>Observations</td>
<td>275</td>
<td>275</td>
<td>275</td>
<td>274</td>
</tr>
</tbody>
</table>

Notes: All regressions contain fixed individual effects for each country/economy. The numbers in parentheses are standard errors. Significance at the 10%, 5%, and 1% level are indicated by *, ** and *** respectively.
et al. (2014), and Pappa et al. (2015), the economy is divided into two sectors of production. In the formal (or market) sector, production takes place using capital and labor. Production in the informal (or underground) sector uses labor only and is hidden to the tax authorities. Thus, households can evade taxes by running underground activities. Tax evasion, however, bears the risk of getting detected and punished by the government. As in Arellano (2008) and Aguiar and Gopinath (2006), international financial markets are incomplete and debt contracts are not enforceable. In any period, the government has the option to default on its external debt. International creditors incorporate the probability of a sovereign default and charge a risk premium. In the event of a default, the government is temporarily excluded from international financial markets and suffers an exogenous output loss.

1.3.1 The Environment

The small open economy is inhabited by an infinitely-lived representative household who produces and consumes a homogeneous consumption good. Her preferences are given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t),$$

where $\beta \in (0, 1)$ denotes the rate of time preference. $c_t$ and $g_t$ refer to private consumption and government consumption, respectively. The utility function is differentiable, monotonically increasing and strictly concave in both arguments.

The economy is divided into two sectors of production: the formal (or market) sector and the informal (or underground) sector. In the market sector, the household employs capital and labor to produce the consumption good. Following Busato and Chiarini (2004, 2013), the production process of the underground sector uses labor only. In each period, the household is endowed with one unit of time, which she inelastically supplies as labor. Let $w_t \in (0, 1)$ denote the share of labor time that the household allocates to market production, and $(1 - w_t)$ the share of labor time used for underground production. The technologies of the market production $y_{m,t}$ and the underground production $y_{u,t}$ are given by:

$$y_{m,t} = e(z_t, d_t)k_t^\theta w_t^{1-\theta},$$
$$y_{u,t} = e(z_t, d_t)(1 - w_t)^{1-\nu},$$

where $k_t$ denotes capital in period $t$. $0 < \theta < 1$ denotes the capital share in the official production sector. With $0 < \nu < 1$ the underground economy exhibits decreasing returns to scale, capturing limited span of control (Lucas; 1978). $z_t \in \mathcal{Z}$ denotes total factor productivity, which follows a Markov process with a Markov transition function $\mu(z_{t+1}, z_t)$. We assume that productivity is common to both production sectors. $d_t$ is the sovereign's
credit status in period $t$, which takes the value of 1 if the government defaults, or has previously defaulted and has not regained access to international financial markets. $d_t = 0$ otherwise. The function $e(z_t, d_t)$ incorporates an output loss associated with a sovereign default: $e(z_t, d_t) = z_t$ if $d_t = 0$ and $e(z_t, d_t) \leq z_t$ if $d_t = 1$. We follow Busato and Chiarini (2004, 2013) and Orsi et al. (2014), and assume that the consumption good produced in the underground economy is indistinguishable from the one produced in the market sector. Therefore, in each period, we can normalize the common price of the consumption goods produced in two sectors to 1. Total production is thus given by

$$y_t = y_{m,t} + y_{u,t}.$$  

The household owns the capital stock, makes production and investment decisions, and consumes. The household pays a tax $\tau_t$ on her income in the formal sector. As in Busato and Chiarini (2004), Orsi et al. (2014), and Pappa et al. (2015), she can hide her income from the tax authorities and evade taxes by producing in the underground economy. However, tax evasion is illegal and bears the risk of getting detected by the government. We follow Allingham and Sandmo (1972) and consider an exogenous probability $p \in (0, 1)$ of being audited. Once detected, the household is forced to pay the tax on the hidden income and is punished by an additional exogenous surcharge factor $s > 1$. The expected income from underground production can be stated as:

$$\mathbb{E}_t I_{u,t} = (1 - p)y_{u,t} + p(1 - s\tau_t)y_{u,t} = (1 - ps\tau_t)y_{u,t}.$$  

The household’s budget constraint thus reads as:

$$c_t + k_{t+1} + \Theta(k_{t+1}, k_t) = (1 - \tau_t)y_{m,t} + (1 - ps\tau_t)y_{u,t} + (1 - \delta)k_t, \quad (1.3.4)$$

where $\delta$ is the capital depreciation rate and $\Theta(k_{t+1}, k_t)$ denotes convex capital adjustment costs.

The government is benevolent, which finances optimal public spending $g_t$. In addition to raising an income tax, at period $t$, the government can purchase non-contingent long-term bond discounted at price $q(b_{t+1}, k_{t+1}, z_t)$ in international financial markets. $q(\cdot)$ represents the bond-price schedule, which the sovereign takes as given, and $b_t < 0$ denotes the outstanding bond stock held by the government at period $t$. We follow the convention of the literature and model the government’s international borrowing as negative bond holdings. We adopt the approach by Chatterjee and Eyigungor (2012) and let each unit of outstanding bond $b_t$ mature with probability $\lambda$ at period $t$. If a unit of bond does not mature, the sovereign has to pay a coupon $n > 0$ to the creditor. Such mechanism implies that the new bond
purchase at period $t$ is $b_{t+1} - (1 - \lambda)b_t$. If the government fulfills the current debt obligations, its budget constraint is given by

$$g_t = \tau_t y_{r,t} + p s_t y_{u,t} + (\lambda + (1 - \lambda)n)b_t - q(b_{t+1}, k_{t+1}, z_t)(b_{t+1} - (1 - \lambda)b_t). \quad (1.3.5)$$

The term $(\lambda + (1 - \lambda)n)b_t$ captures the sum of the debt-repayments related to the fraction $\lambda$ that matures and the coupon-payments related to the fraction $(1 - \lambda)$ that remains outstanding. The term $q(b_{t+1}, k_{t+1}, z_t)(b_{t+1} - (1 - \lambda)b_t)$ is the value of the period-$(t + 1)$ outstanding bond at the current bond price.

Debt contracts are not enforceable and subject to default risk. In each period, contingent on having access to international financial markets, the government decides whether to repay or to default on its outstanding debt. Sovereign default has four consequences. First, all existing debt and coupon obligations are written off. Second, the sovereign is immediately excluded from international financial markets and enters financial autarky. During financial autarky, the government can only rely on the income tax to finance the government consumption. Third, starting from the next period after the default, the sovereign has an exogenous probability $\phi$ in each period to re-enter international financial markets. Fourth, as described above, financial autarky is associated with an asymmetric exogenous output loss, such that $e(z_t, 1) \leq z_t$. If the government defaults at period $t$ and/or the sovereign is in financial autarky, the government’s budget constraint is

$$g_t = \tau_t y_{r,t} + p s_t y_{u,t}. \quad (1.3.6)$$

International creditors provide long-term debt contracts to the government. They are risk-neutral, perfectly competitive and have perfect information about the sovereign’s past and current debt level, the capital stock, and the productivity realizations. Moreover, international creditors can borrow or lend from international financial markets at the constant risk-free interest rate $r_f$.

### 1.3.2 The Recursive Equilibrium

In the following, we consider the government-moves-first Markov-perfect equilibrium as defined by Ortigueira (2006). Specifically, in period $t$, the government takes instantaneous leadership and chooses its fiscal policies before the household decides on consumption, investment, and production. Thus, the government incorporates the private sector's response when making its optimal choices on taxation, debt issuance, and default.
Private Sector

The current state of the economy can be characterized by \((b, k, z)\). In each period, the household chooses \(c, k'\) and \(w\) to maximize her expected lifetime utility \((1.3.1)\), subject to constraints \((1.3.2), (1.3.3)\) and \((1.3.4)\). The household takes as given the government’s fiscal policies \(d, g, \tau\) and \(b'\), and expects the government to follow the policy rules \(d' = D(b', k', z'), g' = G(b', k', z'), \tau' = T(b', k', z')\) and \(b'' = B(b', k', z')\) in the future period. Let \(W(b, k, z \mid d, g, \tau, b')\) denote the household’s (conditional) value function. If the current default decision is \(d = 0\), the economy has access to international financial markets and \(W(\cdot)\) can be recursively defined as follows:

\[
W(b, k, z) = \max_{\{c, k', w\}} \left\{ u(c, g) + \beta \int_{z'} \left( D(b', k', z'), G(b', k', z'), T(b', k', z'), B(b', k', z') \right) \mu(z', z) \, dz' \right\}
\]

subject to
\[
c + k' + \Theta(k', k) = (1 - \tau)y_m + (1 - p s \tau)y_u + (1 - \delta)k,
\]
\[
y_m = e(z, 0)k^\theta w^{1-\theta},
\]
\[
y_u = e(z, 0)(1 - w)^{1-\nu}.
\]

If the current default decision is \(d = 1\), then \(b' = 0\), and the sovereign loses access to international financial markets. But starting from the next period, the sovereign has an exogenous probability \(\phi\) in each period to re-enter. The economy suffers from the output loss during financial autarky. In this case, \(W(\cdot)\) can be recursively defined as follows:

\[
W(b, k, z \mid d, g, \tau, 0) = \max_{\{c, k', w\}} \left\{ u(c, g) + \beta \int_{z'} \left[ \phi W(0, k', z') + (1 - \phi) W(0, k', z') \right] \mu(z', z) \, dz' \right\}
\]

subject to
\[
c + k' + \Theta(k', k) = (1 - \tau)y_m + (1 - p s \tau)y_u + (1 - \delta)k,
\]
\[
y_m = e(z, 1)k^\theta w^{1-\theta},
\]
\[
y_u = e(z, 1)(1 - w)^{1-\nu}.
\]
In the following, we denote the solution to the constrained maximization problem (1.3.7) or (1.3.8) as 
\[ \mathcal{V}(b, k, z) = \max_{\{d\}} \left\{ \mathcal{V}^r(b, k, z), \mathcal{V}^d(k, z) \right\}. \] (1.3.9)

where \( \mathcal{V}^r(b, k, z) \) denotes the value function associated with debt repayment, and \( \mathcal{V}^d(k, z) \) is the value function under financial autarky.

If the government fulfills its current debt obligations, \( d = 0 \), it can issue new debt. Taking the bond price schedule \( q(b', k', z) \) as given, the government solves

\[ \mathcal{V}^r(b, k, z) = \max_{\{g, \tau, b'\}} \left\{ u(c, g) + \beta \int_{z'} \mathcal{V}^r(b', k', z') \mathcal{R}(z')dz' \right\}. \] (1.3.10)

subject to

\[ g = \tau y_m + p s \tau y_u + (\lambda + (1 - \lambda)n)b - q(b', k', z)(b' - (1 - \lambda)b), \]
\[ y_m = e(z, 0)k^0w^{1-a}, \]
\[ y_u = e(z, 0)(1 - w)^{1-v}, \]
\[ k' = \mathcal{K}(b, k, z|0, g, \tau, b'), \quad c = \mathcal{C}(b, k, z|0, g, \tau, b'), \quad \text{and} \quad w = \mathcal{W}(b, k, z|0, g, \tau, b'). \]

If the government defaults, \( d = 1 \), the existing debt and coupon obligations are written off, and the government is in financial autarky, during which the productivity is reduced to \( e(z, 1) \leq z \). Taking as given the probability \( \phi \) of regaining access to international financial markets, the government solves

\[ \mathcal{V}^d(k, z) = \max_{\{g, \tau\}} \left\{ u(c, g) + \beta \int_{z'} \left[ \phi \mathcal{V}^d(0, k', z') + (1 - \phi)\mathcal{V}^d(k', z') \right] \mathcal{R}(z')dz' \right\}. \] (1.3.11)
subject to
\[ g = \tau y_m + p s \tau y_u, \]
\[ y_m = e(z, 1)k^\theta w^{1-\theta}, \]
\[ y_u = e(z, 1)(1 - w)^{1-\nu}, \]
\[ k' = K(b, k, z|1, g, \tau, 0), \quad c = C(b, k, z|1, \tau, 0), \quad \text{and} \quad w = W(b, k, z|1, g, \tau, 0). \]

In each period, \( d = 1 \) if the sovereign has defaulted in the past and has not yet regained access to international financial markets. Otherwise, the government determines \( d \) based on (1.3.9), before any other decisions in the public and private sectors are made. In the latter case, we denote the government’s default decision as \( D(b, k, z) \), and

\[
d = D(b, k, z) = \begin{cases} 1 & \text{if } V^d(b, k, z) < V^d(k, z), \\ 0 & \text{else.} \end{cases} \tag{1.3.12}
\]

By solving the constrained optimization problems (1.3.9), (1.3.10) and (1.3.11), the government obtains its optimal fiscal policies \( g = G(b, k, z) \), \( \tau = T(b, k, z) \) and \( b' = B(b, k, z) \).

**Bond Price**

Following Arellano (2008), the world risk-free interest rate \( r_f \) is taken as exogenous. Moreover, since risk-neutral international creditors are perfectly competitive, the zero-expected-profit condition holds. In this case, the bond price \( q(b', k', z) \) must solve

\[
q(b', k', z) = \int_{z'} \left[ 1 - D(b', k', z') \right] \frac{1 + (1 - \lambda)\left[n + q(b'', k'', z')\right]}{1 + r_f} \mu(z', z) \, dz'. \tag{1.3.13}
\]

The bond-price function (1.3.13) is the same as in Chatterjee and Eyigungor (2012) and Gordon and Guerron-Quintana (2018). The creditor weights the cost \( q(b', k', z) \) against the discounted future benefit (right-hand side of equation (1.3.13)) of owning a unit of sovereign bond. In the event of default in the next period \( (i.e., D(b', k', z') = 1) \), the creditor gets nothing. If the sovereign does not default, the creditor gets back the fraction \( \lambda \) of the bond that matures. For the fraction of \( (1 - \lambda) \) that does not mature, the creditor receives coupon payment \( n \) as well as the market value \( q(b'', k'', z') \) of the outstanding bond.

**Equilibrium Definition**

The recursive equilibrium for the small open economy is defined as (1) a set of value functions \( \mathcal{V}(b, k, z), \mathcal{V}^d(b, k, z), \mathcal{V}^d(k, z) \) and \( \mathcal{W}(b, k, z|d, g, \tau, b') \); (2) a set of (conditional) policy functions for household’s consumption \( C(b, k, z|d, g, \tau, b') \), next-period capital stock
$K(b, k, z \mid d, g, \tau, b')$ and official labor share $W(b, k, z \mid d, g, \tau, b')$; (3) a set of policy functions for sovereign’s default decision $D(b, k, z)$, government spending $G(b, k, z)$, taxation $T(b, k, z)$ and the next-period debt $B(b, k, z)$; and (4) a bond price function $q(b', k', z)$. Such that

1. given the government’s fiscal policies $d, g, \tau$ and $b'$, the household’s policies $C(b, k, z \mid d, g, \tau, b')$, $K(b, k, z \mid d, g, \tau, b')$ and $W(b, k, z \mid d, g, \tau, b')$ solve the constrained maximization problems (1.3.7) and (1.3.8);

2. given the bond price function $q(b', k', z)$ and the household’s optimal responses $C(b, k, z \mid d, g, \tau, b')$, $K(b, k, z \mid d, g, \tau, b')$ and $W(b, k, z \mid d, g, \tau, b')$, the government’s policy functions $G(b, k, z)$, $T(b, k, z)$, $B(b, k, z)$ and the default decision $D(b, k, z)$ solve (1.3.9), (1.3.10), (1.3.11) and (1.3.12);

3. bond prices $q(b', k', z)$ fulfill equation (1.3.13) such that risk-neutral international creditors earn zero expected profits.

### 1.4 Quantitative Analysis

#### 1.4.1 Data

In our quantitative analysis, we apply our model to the Argentine economy to study the dynamic interaction of sovereign default risk, fiscal policy, and the underground economy. Table 1.6 summarizes selected empirical facts for the Argentine economy. For the business cycle statistics, we exclude the sovereign default event of 2002 and consider quarterly data from 1994Q1 to 2001Q4.\footnote{Data sources and descriptions are summarized in Appendix 1.B.}

The estimated size of the underground economy is taken from Elgin and Öztunalı (2012).\footnote{Although the data from Schneider et al. (2010) are used in our empirical analysis, the data only date back to the year 1999. Elgin and Öztunalı (2012), on the other hand, provide data from 1950 to 2009. Recently, there has been a new dataset from Medina and Schneider (2018), which covers 158 countries from 1991 to 2015. At least for Argentina, these three datasets have similar first moments of the size of the underground economy. Figure 1.7 compares the three datasets around the Argentina’s default event in 2002.} The estimated size of the underground economy is taken from Elgin and Öztunalı (2012).\footnote{Although the data from Schneider et al. (2010) are used in our empirical analysis, the data only date back to the year 1999. Elgin and Öztunalı (2012), on the other hand, provide data from 1950 to 2009. Recently, there has been a new dataset from Medina and Schneider (2018), which covers 158 countries from 1991 to 2015. At least for Argentina, these three datasets have similar first moments of the size of the underground economy. Figure 1.7 compares the three datasets around the Argentina’s default event in 2002.}

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The empirical statistics highlight that the Argentine economy is substantially indebted, facing a mean sovereign spread of 7.7%. The size of the underground economy is non-negligible and amounts to 23% of national income. Underground activities are countercyclical. Importantly, the size of the underground economy is positively correlated with the sovereign

---

\[ \ldots \]
spread (refer to Section 1.2). Similar cyclical properties of the underground economy are reported by Busato and Chiarini (2004). In line with previous studies, e.g., Arellano (2008), Neumeyer and Perri (2005), output is negatively correlated with the sovereign spread, private consumption is more volatile than output, and net exports are countercyclical. Investment is 2.5 times more volatile than output and strongly procyclical.

1.4.2 Measuring Official Output in the Data and in the Model

Do the official GDP data, which is reported in the national accounts, already include the underground activities? This question is surprisingly hard to answer. Our data series of GDP are based on the System of National Accounts (SNA) 2008, which recommends to include the underground economy in the national accounts. However, it is not straightforward to know whether such recommendation is put into practice by the national statistics authority. There are different opinions and treatment in the literature. Let \( y_{o,t} \) denote the official measure of GDP. Busato and Chiarini (2004, 2013) and Orsi et al. (2014) take the official GDP in Italy as \( y_{o,t} := y_{m,t} + y_{u,t} \). On the other hand, when estimating the size of underground sector in Latin America, Solis-Garcia and Xie (2018) matches official GDP as \( y_{o,t} := y_{m,t} \). The latter definition is also adopted by Elgin and Öztunalı (2012).

As we calibrate our model to Argentina, we follow Solis-Garcia and Xie (2018) and assume that the official GDP measure is not all-inclusive. But different from Solis-Garcia and Xie (2018), we take the middle ground and assume that, with probability \( p \), the underground activities are detected and enter the reported official measure of GDP, namely

\[
y_{o,t} = y_{m,t} + p y_{u,t}.
\]  

(1.4.1)

Let \( i_t := k' - (1 - \delta)k + \Theta(k', k) \) represent the investment of the private sector, and \( n x_t \) denote net exports, the national account identity in our model is given by

\[
c_t + g_t + i_t + n x_t = y_{m,t} + y_{u,t}.
\]

To maintain the national account identity, we define \( c_{o,t} \) as the officially measured private consumption expenditures:

\[
\left[ c_t - (1 - p) y_{u,t} \right] + g_t + i_t + n x_t = y_{m,t} + p y_{u,t}.
\]

\[= : c_{o,t} + y_{o,t}
\]

Note that this specification relies on the assumption that underground output is consumed rather than invested. We further denote the consumption of the formal and informal good by \( c_{m,t} \) and \( c_{u,t} \), respectively, where \( c_{m,t} := c_{t} - y_{u,t} \).
Table 1.4: Model-generated properties of consumption and output.

<table>
<thead>
<tr>
<th>Variable (X)</th>
<th>$y_{u,t}$</th>
<th>$y_{m,t}$</th>
<th>$y_{o,t}$</th>
<th>$y_t$</th>
<th>$c_{m,t}$</th>
<th>$c_{o,t}$</th>
<th>$c_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}(X)$</td>
<td>0.38</td>
<td>1.64</td>
<td>1.65</td>
<td>2.02</td>
<td>1.12</td>
<td>1.13</td>
<td>1.50</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>4.01</td>
<td>4.85</td>
<td>4.82</td>
<td>4.09</td>
<td>5.37</td>
<td>5.30</td>
<td>3.85</td>
</tr>
<tr>
<td>$\rho(X, y_{m,t})$</td>
<td>0.12</td>
<td>—</td>
<td>0.99</td>
<td>0.98</td>
<td>0.87</td>
<td>0.88</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho(X, y_{o,t})$</td>
<td>0.13</td>
<td>—</td>
<td>0.98</td>
<td>0.87</td>
<td>0.87</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>$\rho(X, y_t)$</td>
<td>0.29</td>
<td>—</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
<td>0.89</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\mathbb{E}(\cdot)$, $\sigma$ and $\rho(\cdot, \cdot)$ represent mean, standard deviation and Pearson’s correlation, respectively.

To explore the sensitivity of these measures, in Table 1.4, we report the model-generated statistical properties of $c_{o,t}$ and $y_{o,t}$ together with $c_{m,t}$, $y_{m,t}$, $c_{u,t}$, and $y_{u,t}$. As can be seen from the table, comparing to $c_{m,t}$ and $c_{o,t}$, $c_t$ has a lower volatility but a higher correlation with production. The volatility and correlation of $y_t$, on the other hand, are close to that of $y_{m,t}$ and $y_{o,t}$. Most importantly, $y_{o,t}$ and $c_{o,t}$ have very similar statistical properties as $y_{m,t}$ and $c_{m,t}$. This is mainly because $p$ is very small in our calibration. The last finding indicates that our quantitative implications will not be different if we define the official GDP as $y_{m,t}$ instead.

1.4.3 Functional Forms and Calibration

To calibrate the model to the Argentine economy, we specify functional forms and choose parameter values on a quarterly basis. Table 1.5 summarizes the set of parameters and indicates whether the parameter values are chosen directly or calibrated to match empirical targets.

Following Busato and Chiarini (2004, 2013) and Cuadra et al. (2010) utility takes the conventional weighted-CRRA form:

$$u(c_t, g_t) = c_t^{1-\eta} - 1 \frac{1}{1-\eta} + \alpha g_t^{1-\eta} - 1 \frac{1}{1-\eta},$$

where $\eta > 0$, $\eta \neq 1$ denotes the parameter of relative risk aversion and $\alpha > 0$ is a preference weight. We follow the macroeconomic literature and choose $\eta = 2$. $\alpha$ is calibrated to match government consumption as a percentage share of official GDP (12%). The rate of time preference $\beta$ is chosen to replicate the debt-to-GDP ratio. According to Chatterjee and Eyigungor (2012), for the period from 1993Q1 to 2001Q4, 70% of the external Argentine
debt was defaultable. Our model generates an average debt share of 104.26%, which corresponds to 72% defaultable debt.

As in Park (2017), capital adjustment cost is given as:

\[ \Theta(k_{t+1}, k_t) = \frac{k}{2} \left( 1 - \frac{k_{t+1}}{k_t} \right)^2 k_t. \]  

(1.4.3)

\( \kappa \) is chosen to match the volatility of investment in Argentina, which is 2.56 times higher than the volatility of official GDP. Capital depreciation \( \delta \) is chosen to replicate the average ratio of investment to official GDP, which is 17% in Argentina.

Regarding the capital shares in production, it is a consensus that the \( \nu \) in the informal sector should be lower than the \( \theta \) in the formal sector, as production is widely perceived to be “markedly more capital-intensive in the formal sector than in the informal sector” (Amaral and Quintin; 2006, p.1545). However, the calibration of the capital shares differs wildly in the literature. Busato and Chiarini (2004, 2013), for example, set \( \theta = 0.30 \) and \( \nu = 0 \); Using Bayesian methods, Orsi et al. (2014) estimate \( \theta \) and \( \nu \) to be 0.36 and 0.33, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r ) the risk-free interest rate</td>
<td>0.01</td>
<td>(standard value)</td>
</tr>
<tr>
<td>( \beta ) time discount factor</td>
<td>0.951</td>
<td>defaultable debt to official GDP</td>
</tr>
<tr>
<td>( \eta ) relative risk aversion</td>
<td>2.00</td>
<td>(standard value)</td>
</tr>
<tr>
<td>( \alpha ) weight on gov’t consumption</td>
<td>0.025</td>
<td>gov. consumption to official GDP</td>
</tr>
<tr>
<td>( p_s ) expected surcharge on tax evasion</td>
<td>0.06</td>
<td>size of the underground economy</td>
</tr>
<tr>
<td>( \delta ) capital depreciation</td>
<td>0.0555</td>
<td>ratio of investment to official GDP</td>
</tr>
<tr>
<td>( \kappa ) capital adjustment cost</td>
<td>5.50</td>
<td>volatility of investment</td>
</tr>
<tr>
<td>( \theta ) capital share in market production</td>
<td>0.39</td>
<td>share of underground labor</td>
</tr>
<tr>
<td>( \nu ) parameter in underground production</td>
<td>0.36</td>
<td>(standard value)</td>
</tr>
<tr>
<td>( \sigma ) volatility of productivity shock</td>
<td>0.0194</td>
<td>volatility of official GDP</td>
</tr>
<tr>
<td>( \rho ) persistence of productivity shock</td>
<td>0.95</td>
<td>Neumeyer and Perri (2005)</td>
</tr>
<tr>
<td>( \lambda ) probability of maturity</td>
<td>0.05</td>
<td>Chatterjee and Eyigungor (2012)</td>
</tr>
<tr>
<td>( \eta ) coupon payment</td>
<td>0.03</td>
<td>Chatterjee and Eyigungor (2012)</td>
</tr>
<tr>
<td>( \phi ) probability of re-entry after default</td>
<td>0.10</td>
<td>Chatterjee and Eyigungor (2012)</td>
</tr>
<tr>
<td>( \chi_0 ) intercept of output cost</td>
<td>-0.2488</td>
<td>mean of sovereign spread</td>
</tr>
<tr>
<td>( \chi_1 ) slope of output cost</td>
<td>0.3458</td>
<td>volatility of sovereign spread</td>
</tr>
<tr>
<td>( \nu ) upper bound of default probability</td>
<td>0.75</td>
<td>Chatterjee and Eyigungor (2015)</td>
</tr>
</tbody>
</table>
Pappa et al. (2015), on the other hand, set $\theta = 0.36$ and $\nu = 0.40$. In our model, we assume that $\theta > \nu$. To calibrate the parameters, we set either $\theta$ or $\nu$ to a conventional value of 0.36, and set the other to match the empirically observed share of underground labor of 23%. This leads to our calibration of $\theta$ and $\nu$ at 0.39 and 0.36, respectively.

Following the convention in the business-cycle literature, the logarithm of $z_t$ is assumed to follow an AR(1) process:

$$\log(z_t) = \rho \zeta \log(z_{t-1}) + \epsilon_t,$$

with $\epsilon_t$ being i.i.d. $N(0, \sigma^2)$. We follow Neumeyer and Perri (2005) and set the persistence $\rho$ equal to 0.95. $\sigma$ is calibrated to replicate the volatility of official GDP in Argentina. Productivity is assumed to be given by:

$$e(z_t, d_t) = (1 - d_t)z_t + d_t(1 - \chi(z_t))z_t.$$

Thus, if the government does not default $d_t = 0$, productivity is given by $e(z_t, 0) = z_t$. If, however, the government chooses to default, $d_t = 1$, productivity takes the value $e(z_t, 1) = (1 - \chi(z_t))z_t$, where $\chi(z_t)$ denotes the asymmetric output cost as in Gordon and Guerron-Quintana (2018) and Chatterjee and Eyigungor (2012):

$$\chi(z_t) = \min \left\{ \max \left\{ \chi_0 + \chi_1 z_t, 0 \right\}, 1 \right\}.$$

$\chi_1$ and $\chi_2$ are set to match the average sovereign spread of 7.70% and the volatility of the spread of 3.51%. Following Chatterjee and Eyigungor (2015), we impose an upper bound on the default probability in order to prevent debt dilution.

In the theoretical model, two parameters affect the size of the underground economy: the probability $\theta$ that tax evasion is detected by the government, and the surcharge factor $s$ to be paid by the household on the hidden income once detected. In the calibration exercise, these two parameters are not distinguishable. Therefore, we choose the expected surcharge factor $E[s]$. We calibrate $E[s] = 0.06$, which replicates the empirically observed size of the underground economy of 23% in Argentina. Considering Italy, Orsi et al. (2014) and Busato and Chiarini (2004) use a detection probability of $\theta = 0.03$ and a surcharge factor $s = 1.3$, implying $E[s] = 0.0309$, which is slightly lower than our value.

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13 Strictly speaking, given our definition of official GDP in equation (1.4.1), $p$ matters for the accounting of the informal sector’s share in GDP. However, it is confirmed by several authors, such as Busato and Chiarini (2004, 2013), Orsi et al. (2014), Solis-Garcia and Xie (2018), etc., that the detection probability $p$ is very small, usually close to 0. In such a case, $p$ alone would have very limited quantitative effect, which is also confirmed by the Table 1.4. We thus set $p$ to be a small number ($p = 0.03$) and calibrate the $E[s]$ together.
Chapter 1. Sovereign Default, Taxation and the Underground Economy

1.4.4 Quantitative Results

Default Regions and Bond Price Schedule

The model has standard implications with respect to the default region and the bond price schedule, which we briefly discuss in this subsection. Figure 1.2 displays the optimal decision of the government whether to repay or to default on its outstanding external debt obligations. In the first row of the figure, we fix capital $k$ at three different values and plot the default decision $D(b, k, z)$ as a function of bonds $b$ and the productivity realization $z$. The second row fixes $z$ at different levels, and plots $D(b, k, z)$ against $b$ and $k$. The grey area refers to the states in which the government finds it optimal to default. Clearly, the default area is smaller for better productivity realizations, for higher capital stocks, and for lower levels of debt.

The first row of Figure 1.3 shows the sovereign bond price $q(b', k', z)$, keeping $k'$ and $z$ fixed at their respective means of the model-simulated non-default episodes. In the second
Figure 1.3: Bond price and bond policies.

Notes: The upper and lower panels are the bond prices and the sovereign’s optimal bond holdings, respectively. Mean k (k’) is close to the average capital during non-default episodes in the simulation; Higher and lower k (k’) are approximately ±25% of the mean k (k’), respectively. Mean z is close to the mean TFP; Higher and lower z are approximately ±5% of the mean TFP, respectively.

row, we plot the government’s borrowing policy $B(b, k, z)$. It is evident that, first, the bond price is increasing in b (decreasing in the sovereign’s debt burden). With the accumulation of debt, international creditors incorporate the rising sovereign default risk in their pricing decision and charge a larger premium on public debt. Second, the bond price decreases if the economy faces adverse productivity realizations. In times of low productivity, the government is less able to fulfill its external debt obligations and the higher sovereign default risk is reflected in the bond price. Third, the bond price is increasing in capital. If the economy operates with a higher capital stock, the larger production raises tax revenues and reduces the probability of a sovereign default. In summary, for high public indebtedness and in times of adverse productivity realizations, the government becomes borrowing-constrained
The Interaction between Public Debt, Taxation, and the Underground Economy

Figure 1.4 displays the government’s tax rate policy. The tax rate is increasing in the sovereign’s indebtedness, but decreasing in productivity. Except for the case where the debt level is very low and the sovereign is not credit constrained, \( T(\cdot) \) is also decreasing in capital \( k \).\(^{14}\) This pattern implies that fiscal policy is procyclical: In times of low productivity/low capital or high debt, the government becomes borrowing-constrained and has to increase the tax rate to finance government consumption. In default, however, debt is not repaid, which allows the government to reduce the tax rate. These findings correspond to those reported in previous studies, such as Cuadra et al. (2010), Kaas et al. (2020), Fink and Scholl (2016), and is in line with the broad empirical literature on the procyclical nature of fiscal policy in emerging and developing economies, see, e.g., Talvi and Vegh (2005), Ilzetzki and Vegh (2008).

Raising tax rate encourages informal activities, as the first row of Figure 1.5 suggests. When the tax rate increases, the share of formal labor force decreases (left panel) and the size of underground economy rises (right panel). When the sovereign faces a tightening

\(^{14}\) The credit constraint is much looser in the model with long-term debt. A sovereign with a very low debt burden has a large room of borrowing to replace taxes. In such a case, if capital stock is low and the marginal cost of taxation on the private sector is high, the sovereign has an incentive to reduce tax rate and increase borrowing. In the extreme case, the sovereign could even impose a negative tax rate, essentially subsidizing the private sector by means of foreign borrowing, as is suggested by the plot.
credit constraint, it has to raise taxes, but the tax hike induces informal activities. Therefore, there exists a positive relationship between the sovereign’s indebtedness and the underground activities, which is confirmed by the second row of Figure 1.5. The rise in the underground economy reduces the tax base, as households in the informal sector do not pay taxes. We can see that there exists a “vicious circle” between sovereign debt and the underground economy: in bad times, the government becomes borrowing-constrained and has to raise the tax rate. Tax hikes, however, induce the private sector to engage in underground activities. In turn, a larger underground economy reduces tax revenues, which forces the government to either implement further tax hikes or to default.

The inclusion of capital into the model further exacerbates the above-mentioned vicious circle. As Figure 1.6 suggests, in bad times, the private sector reduces investment in capital,
which drives down the capital stock. The reduction in the capital stock during bad times has two effects. First, recalling the right panel of Figure 1.4, lower capital stock leads to a higher tax rate when the sovereign is credit-constrained. Second, the size of the underground economy is larger when capital stock is lower, as illustrated in Figure 1.5. Both effects amplify the vicious circle.

**Business Cycles and the Underground Economy**

In Table 1.6, we report the model-generated business-cycle statistics in the second-last column, and compare them with the empirical counterparts. The model matches the targeted moments of the Argentine data (highlighted in bold) very well. Especially, our theoretical economy is characterized by substantial sovereign default risk, reflected by a high and volatile interest spread. At the same time, the size of the underground economy is non-negligible. Moreover, since the model allows for long-term debt contracts, it generates the empirically observed debt-to-GDP ratio. In addition to the targeted statistics, our theoretical framework is able to replicate the empirical non-targeted moments, such as the volatility of consumption relative to output. This is in line with previous studies, such as Arellano (2008), Cuadra et al. (2010). These studies have shown that the endogenous borrowing constraint implies that private consumption is more volatile than (official) output. Beside the consumption volatility, our model generates countercyclical net exports and sovereign spreads, in line with the empirical observations on typical emerging economies made by Neumeyer and Perri (2005).
Our focus is on interaction between the underground economy and sovereign debt. As reported by the last four rows of Table 1.6, the underground activities are positively correlated with sovereign debt and sovereign interest rates. Namely, the size of the underground economy and the share of informal labor are both high during times when the sovereign is facing a tight credit constraint. This finding is in line with the empirical evidences found in Section Table 1.6: Business cycle statistics.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>size of underground economy</td>
<td>$\Xi(y_u/y_o)$</td>
<td>0.23</td>
</tr>
<tr>
<td>average spread (%)</td>
<td>$\Xi(s)$</td>
<td>7.70</td>
</tr>
<tr>
<td>consumption share</td>
<td>$\Xi(c_o/y_o)$</td>
<td>0.69</td>
</tr>
<tr>
<td>government consumption share</td>
<td>$\Xi(g/y_o)$</td>
<td>0.12</td>
</tr>
<tr>
<td>investment share</td>
<td>$\Xi(i/y_o)$</td>
<td>0.17</td>
</tr>
<tr>
<td>defaultable debt to GDP</td>
<td>$\Xi(-b/y_o)$</td>
<td>0.72</td>
</tr>
<tr>
<td>default probability</td>
<td>$\Xi(d)$</td>
<td>0.9</td>
</tr>
<tr>
<td>volatility of output (%)</td>
<td>$\sigma(y_o)$</td>
<td>4.86</td>
</tr>
<tr>
<td>volatility of spread (%)</td>
<td>$\sigma(s)$</td>
<td>3.51</td>
</tr>
<tr>
<td>volatility of consumption</td>
<td>$\sigma(c_o)/\sigma(y_o)$</td>
<td>1.09</td>
</tr>
<tr>
<td>volatility of government consumption</td>
<td>$\sigma(g)/\sigma(y_o)$</td>
<td>0.41</td>
</tr>
<tr>
<td>volatility of net export</td>
<td>$\sigma(nx)/\sigma(y_o)$</td>
<td>0.30</td>
</tr>
<tr>
<td>volatility of investment</td>
<td>$\sigma(i)/\sigma(y_o)$</td>
<td>2.56</td>
</tr>
<tr>
<td>correlation of consumption</td>
<td>$\rho(c_o, y_o)$</td>
<td>0.98</td>
</tr>
<tr>
<td>correlation of government consumption</td>
<td>$\rho(g, y_o)$</td>
<td>0.67</td>
</tr>
<tr>
<td>correlation of investment</td>
<td>$\rho(i, y_o)$</td>
<td>0.98</td>
</tr>
<tr>
<td>correlation of net export</td>
<td>$\rho(nx, y_o)$</td>
<td>−0.79</td>
</tr>
<tr>
<td>correlation of spread</td>
<td>$\rho(s, y_o)$</td>
<td>−0.80</td>
</tr>
<tr>
<td>corr. underground size and spread</td>
<td>$\rho(y_u/y_o, s)$</td>
<td>0.43</td>
</tr>
<tr>
<td>corr. informal labor share and spread</td>
<td>$\rho(1 - w, s)$</td>
<td>0.56</td>
</tr>
<tr>
<td>corr. underground size and debt</td>
<td>$\rho(y_u/y_o, -b)$</td>
<td>0.26</td>
</tr>
<tr>
<td>corr. informal labor share and debt</td>
<td>$\rho(1 - w, -b)$</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Notes: The empirical statistics (“Data”) refer to quarterly data of Argentina between 1994Q1 to 2001Q4. The simulation results (“Model”) are averages over 1000 simulations. Each simulation has a sample size of 2000 quarters, but the first 200 quarters are abandoned. Model statistics are calculated for episodes in which no default occurs, where each non-default episode must be at least 30 quarters long. Both data and simulated series are linearly detrended. The targeted statistics are highlighted in bold.
1.2. In our quantitative model, as highlighted by the policy functions in Subsection 1.4.4 and 1.4.4, increasing borrowing tightens the endogenous borrowing constraint and raises the sovereign’s borrowing cost (Figure 1.3), which forces the government to raise the tax rate (Figure 1.4). However, tax hikes increase the private sector’s incentives to engage in underground activities (Figure 1.5). This mechanism is reflected in the cyclical properties of the informal labor share and the share of underground production over official GDP: the two shares are positively correlated with sovereign debt and the sovereign spreads.

The cyclical properties of the informal labor share and the size of the underground economy are further driven by the procyclicality of investment. As illustrated in Figure 1.6, households reduce investment in the capital stock when the sovereign faces a tight credit constraint. On the one hand, as discussed in Subsection 1.4.4, the reduction in investment exacerbates the vicious circle between the underground economy and the sovereign’s credit constraint during bad time. On the other hand, as the informal sector does not use capital, the reduction in the capital stock reduces the labor productivity in the formal sector. The reduction in the formal labor productivity decreases the relative size of the informal sector even if \( w \) is fixed.

**Sovereign Default and the Underground Economy**

To understand the role of the underground economy during a debt crisis, we perform an event study and show the dynamics of the simulated economy 12 quarters before and after a sovereign default. We assume that the government has access to international financial markets in \( t < 0 \) but defaults at date \( t = 0 \). Both data and the simulated series are detrended by means of HP filter with a smoothing parameter of 1600. Each variable at each date is the average of 17,628 simulated default episodes. Figure 1.7 displays the results. The solid curves are model-simulated averages, while the dashed curves are Argentine data around its 2001-2002 debt crisis. The sovereign default occurred in 2002Q1.

The bottom-right panel suggests that productivity \( e(\cdot) \) peaks about one year before default, and is followed by a sharp decline in \( t = 0 \). Lower productivity tightens the sovereign’s borrowing constraint, which is reflected by a rising sovereign spread. The vicious circle now kicks in: the tightening borrowing constraint forces the government to raise the tax rate. The tax hike induces more tax evasion and underground activities. In turn, more underground activities reduce tax revenues. Facing inadequate tax revenues, the government is forced to further raise the tax rate, which encourages even more underground production. The vicious circle keeps rolling until the period before the default. Moreover, the decreasing productivity and the tightening borrowing constraint brings down private investment in capital (Figure 1.6). As discussed earlier in Subsection 1.4.4, the falling investment during bad times further exacerbates the vicious circle. Therefore, in Figure 1.7, the sovereign spread, the tax rate,
the share of informal labor and the share of underground economy keep going up before the default occurs. GDP and official consumption fall due to lower productivity, lower investment, higher tax rate and the larger underground economy. Government consumption decreases because of debt repayment becomes more expensive.

At some point, it becomes too costly for the government to implement further tax hikes, such that the government finds it optimal to default on the external debt obligations. After the default, the government is excluded from international financial markets, and the economy suffers from the output cost. At $t = 0$, since debt is not repaid, the government budget constraint relaxes, such that the tax rate on income can be lowered. The reduction in the tax rate encourages formal production, and hence the informal sector shrinks immediately after default. Figure 1.7, shows the sudden drop of the tax rate, informal labor share and the underground economy at $t = 0$.

With the re-entering of the sovereign to international financial markets, productivity recovers. At the initial stage of the recovery, the debt burden is low and the sovereign borrows to substitute taxation. The vicious circle is now reversed and becomes a “virtuous circle”: a low tax rate encourages formal activities, and a larger formal sector increases tax revenues, which allow the government to further reduce the tax rate. The reduction in the tax rate amplifies the positive impact on formal labor. Moreover, due to the decreasing tax rate and the rising productivity, investment recovers, which fosters the virtuous circle. As a result of the increasing productivity, the larger formal sector and the rising investment, GDP, consumption and government spending recover. However, at the later stage of the recovery, the quickly built-up debt stock raises sovereign interest rates and restricts further borrowing. Consequently, the government has to gradually raise the tax rate to an average level. The reduction in the informal activities stops, and the informal labor share, as well as the size of the underground economy rebound to pre-default normality. However, because of the ongoing recovery of productivity and investment, GDP, official consumption and government spending keep increasing. The economy gradually returns to normal.

The simulated patterns of GDP, official consumption, investment, government spending, underground economy and the sovereign spread match their empirical counterparts during Argentina’s default episode in 2001-2002. Our model fails to match the pattern of net export. Especially, the model predicts a trade surplus before default but a trade deficit afterwards, which is contrary to the data. This is a common shortfall of sovereign debt models with long-term debt, and Gordon and Guerron-Quintana (2018) offers an explanation of why this is the case. As for the dynamics of the underground economy around the default event, our model predicts a spike of the underground economy prior to default, followed by a drop of the underground activities at the default, as well as during the initial stage of recovery. This is in line with the data of the underground economy by Medina and Schneider (2018) and
Figure 1.7: Default events.

Notes: Default events are plotted 12 periods (quarters) before and after default (except the spread). The solid curves are model simulations, while the dashed curves are data. In the plot of the underground economy (%GDP), “Data:MS18”, “Data:EO12” and “Data:SBM10” represent the data by Medina and Schneider (2018), Elgin and Öztunalı (2012) and Schneider et al. (2010), respectively. The default event in the data refers to Argentina in 2002Q1. Data and simulated series of GDP, consumption, government consumption, investment and productivity are logged and HP filtered with a smoothing parameter of 1600.
Schneider et al. (2010). In addition, our model predicts a gradual rise of size of the informal sector at the later stage of recovery, which is in line with the data of Elgin and Öztunali (2012).

1.5 Conclusions

This chapter studies the dynamic interaction between sovereign debt, taxation, and the underground economy. Our empirical analysis provides evidence that the size of the underground economy is positively correlated with government debt and sovereign interest spreads, particularly in emerging economies.

We rationalize these empirical regularities within a quantitative model of sovereign debt and default, which incorporates an underground production sector that is hidden from the tax authorities. In an application to the Argentine economy, we show that the dynamic interaction between sovereign default risk and the underground economy creates a vicious circle: Higher sovereign interest rates make sovereign borrowing costly and force the government to raise taxes. Tax hikes, however, induce the private sector to invest less and to evade taxes by producing in the underground sector. In turn, falling tax revenues force the government to either implement further tax hikes or to default.

The vicious circle explains the time-series correlation between the underground economy and sovereign debt: sovereign indebtedness and the sovereign bond yield tend to be higher (lower), when the size of the underground economy is rising (falling). However, our empirical analysis in Section 1.2 also points to a cross-sectional correlation. Namely, countries with larger (smaller) underground economy usually have a larger (smaller) debt-to-GDP ratio, as well as a higher (lower) yield of the sovereign bond. In future research, we could calibrate our model to a group of economies to see whether our model also explains the cross-sectional correlation.
Appendix 1.A  Samples in the Empirical Analysis

The empirical analysis is performed on two samples of countries: OECD countries and the emerging economies. The components of each sample are listed in Table 1.7. As of 2015, there are 34 members of the Organisation for Economic Co-operation and Development (OECD). However, Estonia has to be excluded from our sample, because of the missing data on the yield of 10-year government bond during 1999-2007. The sample of emerging economies consists of countries that are covered by the EMBI Global index during 1999-2007. For the within-group estimation to work, each country should have a data coverage of at least two years. As can be seen from Table 1.7, both samples lead to unbalanced panels.

<table>
<thead>
<tr>
<th>Time Window</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OECD Countries (33)</strong></td>
<td></td>
</tr>
<tr>
<td>1999 – 2007 (25)</td>
<td>Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States</td>
</tr>
<tr>
<td>2000 – 2007 (3)</td>
<td>Czech Republic, Slovakia, South Korea</td>
</tr>
<tr>
<td>2001 – 2007 (2)</td>
<td>Mexico, Poland</td>
</tr>
<tr>
<td>2002 – 2007 (1)</td>
<td>Slovenia</td>
</tr>
<tr>
<td>2004 – 2007 (1)</td>
<td>Chile</td>
</tr>
<tr>
<td>2006 – 2007 (1)</td>
<td>Turkey</td>
</tr>
<tr>
<td><strong>Emerging Economies (34)</strong></td>
<td></td>
</tr>
<tr>
<td>1999 – 2007 (20)</td>
<td>Argentina, Brazil, Bulgaria, Chile, China, Colombia, Côte d’Ivoire, Ecuador, Hungary, Lebanon, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Turkey, Venezuela</td>
</tr>
<tr>
<td>1999 – 2006 (3)</td>
<td>Nigeria, Panama, Thailand</td>
</tr>
<tr>
<td>1999 – 2004 (2)</td>
<td>Croatia, South Korea</td>
</tr>
<tr>
<td>2000 – 2007 (1)</td>
<td>Ukraine</td>
</tr>
<tr>
<td>2001 – 2007 (3)</td>
<td>Egypt, Pakistan, Uruguay</td>
</tr>
<tr>
<td>2002 – 2007 (3)</td>
<td>Dominican Republic, El Salvador, Tunisia</td>
</tr>
<tr>
<td>2004 – 2007 (1)</td>
<td>Indonesia</td>
</tr>
<tr>
<td>2006 – 2007 (1)</td>
<td>Vietnam</td>
</tr>
</tbody>
</table>
Appendix 1.B Data Sources and Calculations for the Business Cycle Statistics and the Default Event Studies

For the business-cycle statistics, we consider data of Argentina from 1994Q1 to 2001Q4. The data start from 1994Q1, due to the availability of the EMBI data. The size of the underground economy is from Elgin and Öztunalı (2012). The amount of sovereign debt is from the World Bank’s International Debt Statistics (IDS) database. The sovereign bond yield is taken as the JP Morgan Emerging Markets Bond Index (EMBI Global) for Argentina. All other time series are from the National Institute of Statistics and Censuses (INDEC) of Argentina.

We first seasonally adjust the data by means of the Census X13 method from the U.S. Census Bureau. Afterwards, nominal variables are deflated using GDP implied deflator, except for the imports and the exports. The latter two are deflated using import and export price deflator, respectively. Finally, the series are logged and linearly detrended before being used for calculating the business-cycle statistics in Table 1.6.

For the purpose of comparison in the default-event studies in Figure 1.7, we use the data of Argentina from 1994Q1 to 2013Q3. The data are processed in the same way as above. The annual size of the underground economy is transformed into quarterly frequency by means of linear interpolation. The date of default is taken as 2002Q1. The data that are used in the default-event study must extend beyond the date of default. In this case, depending on which date we extend the data to, linear filter produces different magnitudes of deviations. In order to mitigate such arbitrariness, instead of the linear filter, we apply the Hodrick-Prescott filter with a smoothing parameter of 1600.

Appendix 1.C Numerical Algorithm

The model is solved using value function iteration. The infinite-horizon optimization problems of the household and the government are approximately by the finite-horizon equilibrium. Government’s value functions, the bond-price function, as well as the public- and private-sector policy functions are updated simultaneously in each iteration. We employ cubic-spline interpolations when updating.
With the specification of the household’s utility in (1.4.2), it can be easily verified that the first-order condition of the household’s optimization problem (1.3.7) or (1.3.8) with respect to \( w \) writes

\[
\frac{w^{\theta}}{1 - w^{\gamma}} = \frac{(1 - \theta)(1 - \tau)}{(1 - \nu)(1 - ps\tau)} \cdot k^\theta. \tag{1.C.1}
\]

Equation (1.C.1) suggests that \( w \) is a function of the current state \( k \), as well as the government’s tax policy \( \tau \). With the functional form of the capital adjustment cost (1.4.3), the first-order condition regarding \( k \) writes

\[
e^{-q} \left[ 1 + \kappa \left( \frac{k'}{k} - 1 \right) \right] = \beta \int z' e^{-q} \left[ \theta(1 - \tau') \ e(z', d') k'^{(\theta - 1)} \ w^{(1 - \theta)} \right. \\
+ \ (1 - \delta) + \frac{k}{2} \left( \frac{k''}{k'} \right)^2 \left( 1 \right) \left( \mu(z', z) \ dz' \right. \tag{1.C.2}
\]

In equation (1.C.2), \( \mu(z', z) \) is implied by the TFP process (1.4.4). Note that the government default decision \( d \) affects the household’s policies through the current tax policy \( \tau \), the household’s expected future tax rate \( \tau' \), and as well as through the current and future productivity \( e(\cdot) \). Given \( \tau, \tau' \) and \( k'' \), household’s policies of \( c, w \) and \( k' \) can be obtained by simultaneously solving nonlinear equations (1.C.1), (1.C.2) and (1.3.4).

The state of the economy is characterized by \( (b, k, z) \). To solve the model numerically, we define evenly distributed grid for the sovereign bond \( b \in [\bar{b}, \bar{b}] \), private-sector capital stock \( k \in [\bar{k}, \bar{k}] \) and the productivity realizations \( z \in [\bar{z}, \bar{z}] \). Let \( \forall r(0)(b, k, z) \), \( \forall d(0)(k, z) \) and \( \forall(0)(b, k, z) \) be the initial guesses for the government’s value functions. Let \( T_r(0)(b, k, z) \) and \( T_d(0)(k, z) \) be initial guesses for the optimal tax policies if \( d = 0 \) and \( d = 1 \), respectively, and similarly denote the initial guesses for the optimal capital policies of the household by \( K_r(0)(b, k, z) \) and \( K_d(0)(k, z) \). Finally, we let \( Q(0)(b, k, z) \) be the initial guesses for the bond prices associated with government’s optimal bond holdings and the household’s optimal capital policies, if \( d = 0 \).\(^{15}\) The values, policies and the bond prices are updated by the following steps on each grid point of \( (b, k, z) \):

1. Let \( \tau^r \) and \( \tau^d \) be the candidate tax rates when \( d = 0 \) and \( d = 1 \), respectively, and \( b' \) be the candidate for the government’s bond policy when \( d = 0 \). For either case of \( d = 0 \) or \( d = 1 \), we apply cubic-spline interpolation to obtain \( \tau' = T_r(b', k', z') \) or \( T_d(k', z') \) based on \( T_r(0)(b, k, z) \) or \( T_d(0)(k, z) \), and obtain \( k'' = K_r(b', k', z') \) or \( K_d(k', z') \) based on \( K_r(0)(b, k, z) \) or \( K_d(0)(k, z) \). We then solve the nonlinear equations (1.C.1), (1.C.2) and (1.3.4) simultaneously to obtain optimal household’s policies for \( c, w \) and \( k' \), under each case of \( d = 0 \) and \( d = 1 \).

\(^{15}\) Note that if \( d = 0 \), \( Q(b, k, z) := q(B(b, k, z), K(b, k, z), z) \).
2. Taking household’s optimal \( c, w \) and \( k \) obtained in the last step, we obtain the objective functions (1.3.9), (1.3.10) and (1.3.11) for each candidate value of \( \tau^r, \tau^d \) and \( b' \). We apply cubic-spline interpolation for the future values \( \mathcal{V}^r(0)(b', k', z') \), \( \mathcal{V}^d(0)(k', z') \) and \( \mathcal{V}(0)(b', k', z') \) based on \( \mathcal{V}^r(0)(b, k, z) \), \( \mathcal{V}^d(0)(k, z) \) and \( \mathcal{V}(0)(b, k, z) \), and we approximate the \( q(b'', k'', z') \) on the right-hand side of the bond-price function (1.3.13) by a cubic-spline interpolation for \( \mathcal{Q}(b', k', z') \) based on \( \mathcal{Q}(0)(b, k, z) \). Global search procedures are implemented to find the best among the candidate values of \( \tau^r, \tau^d \) and \( b' \), such that the objective functions are maximized. After this step, we update values \( \mathcal{V}^r(1)(b, k, z) \), \( \mathcal{V}^d(1)(k, z) \) and \( \mathcal{V}(1)(b, k, z) \), as well as policies \( \mathcal{T}^r(1)(b, k, z) \), \( \mathcal{T}^d(1)(k, z) \), \( \mathcal{K}^r(1)(b, k, z) \), \( \mathcal{K}^d(1)(k, z) \).

3. Taking the household’s non-default capital policies \( \mathcal{K}^r(1)(b, k, z) \) and the government’s optimal (non-default) bond policies \( \mathcal{B}(b, k, z) \) obtained in the last step into the bond-price function (1.3.13), we update the optimal bond-price function \( \mathcal{Q}(1)(b, k, z) \). Note that we approximate \( q(b'', k'', z') \) on the right-hand side of (1.3.13) once again by a cubic-spline interpolation for \( \mathcal{Q}(b', k', z') \) based on \( \mathcal{Q}(0)(b, k, z) \), and we substitute all \( b' \) and \( k' \) in (1.3.13) by \( \mathcal{B}(b, k, z) \) and \( \mathcal{K}^r(1)(b, k, z) \), respectively.

Repeat the steps above until the value functions converges.
Chapter 2

Intermediation Cost of Credit in the US Corporate Sector


2.1 Introduction

Has the financial sector become more efficient over time? This is an important question, as an efficient financial market allocates capital to its best use, and is therefore crucial for the economic development. Given the development of the IT technology, increasing credit information sharing, financial deregulation, as well as the boom in financial innovations, it is natural to believe that financial efficiency has improved, at least in an advanced economy such as the United States. However, in his seminal paper, Philippon (2015) estimates the time series of the overall unit cost of external finance (including credit and equity finance) for the past 130 years, and claims that financial efficiency has not improved in the United States. Bazot (2018) obtains similar results for the financial sector in Europe. This chapter revisits the estimation of financial efficiency.

Instead of estimating the overall cost of external finance, we focus on the intermediation cost of credit finance in the corporate sector of the United States. We define the intermediation cost as the part of the unit cost that is unrelated to the risk premium. Figure 2.1 decomposes the cost of borrowing into three components. Taking a bank loan as an example, the costs of borrowing include all the related costs associated with an actual amount of bank borrowing. A part of the cost is the risk-free return, which is what the bank must pay to savers for the amount of lended money. In addition, the borrower has to compensate the bank for the default risk. This amount of compensation is called the (default) risk premium. The intermediation cost is defined as what is left over of the cost of borrowing from the risk premium and the risk free return. Typical examples of the intermediation cost include the cost of banks’ office and staff, screening and monitoring cost on borrowers, etc. As the risk premium is strongly associated with the firm’s characteristics, we maintain that the intermediation cost is a better measure of the financial efficiency.

Figure 2.1: Composition of the borrowing cost.

- Risk Premium
- Intermediation Cost
- Risk-free Return (Return to Saver)

Cost of Borrowing

Unit Cost

Cost of Credit Supply
In reality, there are more sources of external credit than bank loans, such as bond issuance, factoring, financial leases, etc. It is not straightforward to assess the intermediation cost of all sources of external finance. In this chapter, we propose a method of identifying and estimating the time variation in the overall intermediation cost of firm’s external credit. We show that the intermediation cost has decreased significantly over the period of 1983Q1–2007Q4. On average, the fall in the intermediation cost leads to a reduction by 0.57-0.71 percentage point in firm’s interest expense over revenue, and a decrease by 0.78-0.79 percentage point in the interest expense over book value of debt.

The intermediation cost is estimated as follows. We first apply a two-way error component model with fixed individual and time effects, and regress the cost of borrowing on firm’s characteristics that potentially affect its borrowing amount and the default-risk premium. If the risk premium is well-controlled by firm’s individual characteristics, a negative trend in the estimated time effects can be interpreted as a decrease in the overall credit-supply cost. To assess the trend in the intermediation cost, we further regress the time effects on the risk-free interest rate and the inflation rate by means of ordinary least squares (OLS). Due to the orthogonality of the OLS residuals, the trend in them, if exists, captures the movement in the intermediation cost.

Different comparison schemes are in place to ensure the robustness of our results. First, our empirical analysis is performed on both balanced and unbalanced panel of data. The original dataset is unbalanced. The balanced panel, on the other hand, contains those firms for which a data coverage over the entire time window of 1983Q1–2007Q4 is available. In addition, we define two measures of the borrowing cost: interest expense over revenue, and interest expense over book value of debt. The two panels and the two measures, although having noticeably different patterns of cross-sectional means, lead to very close estimates of the time trend in the cost of credit supply and in the intermediation cost. Furthermore, in the second regression, we use different measures of the risk-free rate and inflation, and find that using alternative measures does not change our empirical results. Last but not least, to verify the validity of our methodology, instead of the risk-free rate, we use the Moody’s Aaa-graded corporate bond yield for the second regression. The yield of the Aaa-graded corporate bond, although carrying little default-risk premium, contains the intermediation cost. We find that by using the corporate bond yield in our regression, residuals no longer exhibit a clear time trend. According to the decomposition of the borrowing cost illustrated by Figure 2.1, this is exactly what we should expect, if our regression design is correct.

The rest of the chapter proceeds as follows. Section 2.2 describes the data, variables, as well as our regression design. Section 2.3 presents the empirical results, as well as the results of the robustness check. Section 2.4 concludes.
Chapter 2. Intermediation Cost of Credit in the US Corporate Sector

**Related Literature.** There are several papers on the estimation of the historical series of financing cost. Philippon (2015) estimates the unit cost of finance for the United States from 1886 to 2012. However, his estimation includes all aspects of the financial activities, such as debt and equity finance, asset management and household finance. Similar estimation routines are applied by Bazot (2018) on European countries, who shows that during 1950 and 2007, except France, unit cost had not decreased in Europe. Of a narrow focus on the corporate finance, Eisfeldt and Muir (2016) estimate the average cost of external finance of the US firms from 1980 to 2014 by means of a structural model on the Compustat data. Their results, too, suggest that the average cost of external finance hasn’t decreased. However, Eisfeldt and Muir (2016) do not differentiate between the credit and equity finance. This chapter, on the other hand, has a more narrow focus on firm’s credit finance.

There is a large literature on the development of the financial sector in the United States and in Europe. Schularick and Taylor (2012), for example, provide historical comparison on the growth of money, credit and finance in 14 countries over the past 140 years. Philippon and Reshef (2013) and Greenwood and Scharfstein (2013) document the growth of financial sector in the United States. They find that the size of the financial sector, either measured by the income share of the financial sector over GDP, or by the quantity of intermediated assets, has grown significantly over the past decades. Graham et al. (2015) show a similar trend in the corporate finance. They find that the corporate sector in the United States has become increasingly leveraged over the past century. Our findings in this chapter shed some light on the reasons behind such credit expansion.

There are many papers on the links of the firm’s characteristics to its borrowing cost. Examples include Lin et al. (2011) on the relationship between ownership structure and the borrowing cost, and Graham et al. (2008) on the corporate mis-reporting and the bank loan contract. Especially, the design of our fixed effect regression took reference from Lin et al. (2011) and the ANOVA decomposition by Schularick and Taylor (2012).

### 2.2 Data, Variables and Methodology

#### 2.2.1 Data

We use the accounting data from the Compustat North America, which cover all listed companies in the United States and Canada. For our purpose, we obtain the data for the United States covering the period between 1980Q1 and 2007Q4. Data are of quarterly frequency, which contain 239,736 firm-quarters. We eliminate 86,864 observations that have missing values or negative entries in at least one of the following three items: interest

\footnote{There are firms that are located in the United States, but with the native currency not being US Dollar. These firms are not included in our sample.}
expense, revenue and total equity. This results in an unbalanced panel that consists of 152,872 firm-quarters. In addition, a “balanced” panel is constructed by deleting firms that do not have full data coverage by the Compustat over the time window of 1983Q1–2007Q4. The balanced panel has 68,852 firm-quarters. The same empirical analysis will be conducted on both the balanced and the unbalanced panel.

The balanced panel consists of mainly the big and/or long-lasting firms. While the unbalanced panel also contains newly-listed firms, as well as firms that have been delisted or terminated. The downside of the unbalanced panel is the existence of a large number of outliers (see Subsection 2.2.5). Our methodology is to manipulate the dataset as little as possible, hence we do not kick the “abnormal” firms out of the unbalanced panel. Rather, we compare the regression results of the unbalanced panel with that of the balanced panel, which do not have the issue of outliers. The balanced panel, on the other hand, potentially suffers from the firm-age effect. Longer-lasting firms usually enjoy better reputation, which mitigates conflicts of interest between borrowers and lenders (Diamond; 1989). Therefore, the credit availability improves with the firm-age (Petersen and Rajan; 1995), and the borrowing cost decreases with the age of firms (Sakai et al.; 2010). In our empirical analysis, the balanced and unbalanced panel serve as robustness check. In Section 2.3, we will see that the two panels yield very similar estimation results.

2.2.2 Regression

To assess the time trend in the cost of credit supply, we estimate the following two-way error component model with fixed individual and time effects:

\[
Y_{it} = \beta_0 + (x_{it}^{MA12})^T \beta_1 + (x_{it}^{MA4} - x_{it}^{MA12})^T \beta_2 + (x_{it} - x_{it}^{MA12})^T \beta_3 + \mu_i + \gamma_t + e_{it}, \tag{2.2.1}
\]

where \( Y \) is a measure of the firm’s borrowing cost (see Subsection 2.2.3). \( x \) is the (column) vector of controls for the amount of borrowing and for the default premium. \( x_{it} \) denotes the controls for firm \( i \) at time \( t \) (the spot values), \( x_{it}^{MA12} \) and \( x_{it}^{MA4} \) represent the 12-quarter and 4-quarter moving averages of controls for firm \( i \), respectively. Namely,

\[
x_{it}^{MA12} = \frac{1}{12} \sum_{s=0}^{11} x_{i(t-s)}, \quad \text{and}
\]
\[
x_{it}^{MA4} = \frac{1}{4} \sum_{s=0}^{3} x_{i(t-s)}.
\]

\(^2\) Note that the balanced panel still contains missing data on some variables at certain years. In this sense, it is different from the econometric concept of the balanced panel.
The moving averages of controls are introduced out of the consideration that firm’s leverage and cost of borrowing usually relates to its past condition and behaviors. Furthermore, we use the difference of \((x_{it}^{MA4} - x_{it}^{MA12})\) and \((x_{it} - x_{it}^{MA12})\) to capture the mid-term and short-term improvement/deterioration of the firm’s financial status, respectively. In model (2.2.1), \(\mu_i\) and \(\gamma_t\) are fixed individual and time effects, respectively. One of the advantages of using the fixed-effect model is that we do not have to control for the cross-sectional firm-characteristics, such as the firm’s industry, sector, or geography. The term \(\mu_i\) absorbs all such cross-sectional effects on the dependent variable. The OLS estimation of the model only accounts for the “within-group” effect, i.e., the variation across time for each individual firm. Finally, \(e_{it}\) is the identically and independently distributed (i.i.d.) error term.

If the variation in the borrowing amount and the risk premium are well-covered by the controls, \(\gamma_t\) measures the time effects on the borrowing cost that have nothing to do with the firm’s financial status, default risk or cross-sectional characteristics. Therefore, a negative trend in \(\gamma_t\), if exists, indicates a decrease in the cost of credit supply, and the change in \(\gamma_t\) measures the variation in the borrowing cost due to the credit supply cost. To further disentangle the time trend in the intermediation cost from that in the credit-supply cost, we regress the estimates of \(\gamma_t\) (denoted by \(\hat{\gamma}_t\)) on the risk-free interest rate \(r_t\) and the inflation rate \(\pi_t\):

\[
\hat{\gamma}_t = \alpha_0 + \alpha_1 r_t + \alpha_2 \pi_t + e_t. \tag{2.2.2}
\]

The estimates of the residuals \(e_t\) (denoted by \(\hat{e}_t\)) are orthogonal to \(r_t\) and \(\pi_t\). In other words, \(\hat{e}_t\) captures the variation in \(\hat{\gamma}_t\) that cannot be explained by the risk-free rate and inflation. According to the decomposition of the borrowing cost illustrated by Figure 2.1, a change in \(\hat{e}_t\) thus measures the variation in the cost of borrowing due to the intermediation cost.

### 2.2.3 Dependent Variables

We choose the Compustat item “Interest Expense” as a measure of the cost of borrowing. The item contains the costs related to a firm’s external credit finance, including the interest paid for short- and long-term debt, bond-issuance cost, factoring charges, as well as non-debt interest expense, etc. It is a summary of costs of interest-bearing liabilities from all sources of external credit.

The borrowing cost should be scaled by some reference items to ensure comparison.\(^3\) In this chapter, we apply two ways of scaling. First, as the interest expense is a flow variable, a

\(^3\) Beside the purpose of comparison, scaling the cost of borrowing also facilitates the use of accounting ratios as control variables (see Table 2.1). Those accounting ratios are widely accepted as tools for summarizing firm’s financial status. They are usually also directly behind the creditor’s perception of the firm’s financial soundness.
Figure 2.2: The aggregate and mean of dependent variables.

Notes: \( Y^1 \) and \( Y^2 \) are as defined in equation (2.2.3) and (2.2.4), respectively. The aggregate dependent variable for each year is the sum of the interest expense across firms divided by the sum of total revenue (or book value of debt) across firms. The mean dependent variable is the average of dependent variable across firms. Time series are seasonally adjusted by centred moving-average using an additive-component model.

natural scaling method is to scale it by another flow variable. Here we choose total revenue as the scaling factor, because, unlike other flows such as gross profit, EBIT or net profit, revenue is always positive and has less abnormal observations. The interest expense that is scaled by total revenue is denoted by \( Y^1 \), and

\[
Y^1 = \frac{\text{Interest Expense}}{\text{Total Revenue}} \times 100. \tag{2.2.3}
\]

At the same time, borrowing cost is usually described as a percentage of the amount of borrowing. Based on the definition of the Compustat item of “Interest Expense”, a reasonable
scaling factor is the book value of debt.\textsuperscript{4} The interest expense that is scaled by the book value debt is denoted by $Y^2$, namely,

$$Y^2 = \frac{\text{Interest Expense}}{\text{Book Value of Debt}} \times 100.$$ \hspace{1cm} (2.2.4)

Figure 2.2 plots the time series of the aggregate dependent variables (first row), as well as the cross-sectional simple means (second row) of the dependent variables. The aggregate $Y^1$ can be deemed as a weighted average of $Y^1$ across firms, with the total revenue as the weight. Similarly, the aggregate $Y^2$ is the weighted average of the interest expense over book value of debt, weighted by the latter.\textsuperscript{5} As the first row of Figure 2.2 shows, the aggregate interest expense represents roughly 1% of the aggregate revenue, and 4.5%-4.7% of the aggregate book value of debt. The aggregate $Y^1$ does not exhibit any time trend for both panels, while the aggregate $Y^2$ clearly shows a downward trend. The second row of the figure displays the simple means of $Y^1$ and $Y^2$. Due to the existence of outliers in the unbalanced panel, the cross-sectional simple means for the unbalanced panel are much more volatile and erratic. Especially, the spike in the number of distressed firms after the burst of the dot-com bubble in 2002 significantly dominates the calculation of the simple means of $Y^1$ in the unbalanced panel.\textsuperscript{6} However, despite all these different time patterns between the balanced and unbalanced panel, as well as between $Y^1$ and $Y^2$, the regressions performed on them lead to surprisingly similar results, as will be shown in Section 2.3.

\subsection*{2.2.4 Independent Variables}

\textbf{Controls in Model (2.2.1)}

To specify the control variables, we seek possible factors that could explain the firm’s amount of borrowing and its risk premium over time. The goal is to improve the goodness of fit of the regression model (2.2.1) as much as possible. These factors can be grouped into seven categories: size, liquidity, capital structure, solvency, profitability, market valuation and others (tangibility). Except for variables describing firm’s size, all controls are formulated

\textsuperscript{4} Lin et al. (2011) defines the book value of debt as the difference between total assets and the book value of equity, where, according to Fama and French (1993), book value of equity is the sum of total equity and the deferred tax and investment tax credit, minus the book value of preferred stock. As a result, Lin et al. (2011)’s definition of the book value of debt includes preferred stock. However, as the Compustat’s Interest Expense does not cover preferred-stock dividends, we have to subtract the book value of the preferred stock from the book value of debt. Therefore, our definition of the book value of debt is the total assets minus the book value of equity plus the book value of preferred stock. The definition of the book value of equity still follows Fama and French (1993).

\textsuperscript{5} The aggregate $Y^1$ and $Y^2$ behave in a close way as their respective medians.

\textsuperscript{6} In $Y^2$, interest expense is divided by a stock, while in $Y^1$ it is scaled by a flow. Therefore, the simple mean of $Y^1$ has more ups-and-downs than that of $Y^2$, as flows are generally much more volatile than stocks.
as accounting ratios. Table 2.1 lists the definitions of the independent variables. Note that we have multiplied all flow items of the Compustat by 4 to obtain annual-equivalent figures.

The size of a company is an important explanatory factor of the dependent variables, for two reasons. First, the size is directly or indirectly related to the denominators of $Y^1$ and $Y^2$. And second, the size is behind the creditor’s perception of firm’s credit-worthiness. Larger firms usually have less information asymmetries in the credit market (due to, e.g., more coverage by credit analysts) and enjoy a perception of lower credit risk. Three variables are used to describe the sizes of firms: total asset, total revenue and the market capitalization.

We expect liquidity to be negatively correlated with the amount of borrowing and with the risk premium. Low liquidity is associated with high burden of (the short-term) debt. At the same time, if the liquidity is low, lending to the firm is risky. In this case, creditors usually charge a high risk premium. Since the firm with lower liquidity has a higher amount of borrowing and usually faces higher risk premium, the cost of borrowing of the firm must be higher. We use three variables to measure the liquidity of a firm from different angles: current ratio, quick ratio and cash ratio.

Three variables are specified to measure the capital structure. Debt in current liabilities, which also includes long-term debt that matures within one year, matters for the near-term interest payment. The long-term debt represents firm’s interest-payment obligation for the longer future. Finally, we also control for the use of debt as a source of external finance.

For the solvency of a firm, we specify two measures: solvency ratio and the interest coverage ratio. The relationship between the two variables and the cost of borrowing is ambiguous. Increasing borrowing leads to a higher amount of liability and more interest expense on one hand, which implies an increase in the denominators of the two ratios. On the other hand, more borrowing is usually associated with an expansion of production, which generally results in a higher income. In the latter case, the numerators of the two ratios also rise. Therefore, in the end, we cannot be sure on how the two solvency measures related to the cost of borrowing.

It is also hard to predict the relationship between the profitability of a firm and its cost of borrowing. On one hand, more profitable firms have the motivation to increase borrowing; but on the other hand, they usually have lower risk premia. The impact of the firm’s market valuation on the cost of borrowing is ambiguous, because the market valuation can either signal risks or proxy for additional values (Graham et al.; 2008). Finally, following Lin et al. (2011), we include the tangibility of the firm’s assets into our regression, and we expect it to be negatively correlated with the risk premium. However, higher tangibility makes it easier for a firm to borrow, and therefore, the amount of borrowing might be higher if a firm owns more tangible assets. Hence, the impact of the tangibility on the cost of borrowing is mixed.
Table 2.1: List of independent variables.

<table>
<thead>
<tr>
<th>Variable names</th>
<th>Variable definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size</strong></td>
<td></td>
</tr>
<tr>
<td>asset</td>
<td>log of total assets.</td>
</tr>
<tr>
<td>revenue</td>
<td>log of total revenue.</td>
</tr>
<tr>
<td>market cap.</td>
<td>log of market capitalization, where market capitalization = stock price (close) × common shares outstanding.</td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
</tr>
<tr>
<td>current ratio</td>
<td>= current assets / current liabilities.</td>
</tr>
<tr>
<td>quick ratio</td>
<td>= (current assets – inventories) / current liabilities.</td>
</tr>
<tr>
<td>cash ratio</td>
<td>= cash and short-term investments / current liabilities.</td>
</tr>
<tr>
<td><strong>Capital Structure</strong></td>
<td></td>
</tr>
<tr>
<td>prop. debt in current liab.</td>
<td>= debt in current liabilities / current liabilities.</td>
</tr>
<tr>
<td>prop. long-term debt</td>
<td>= long-term debt / total liabilities.</td>
</tr>
<tr>
<td>debt ratio</td>
<td>= (debt in current liabilities + long-term debt) / total assets.</td>
</tr>
<tr>
<td><strong>Solvency</strong></td>
<td></td>
</tr>
<tr>
<td>solvency ratio</td>
<td>= (net income + depreciation and amortization) / total liabilities.</td>
</tr>
<tr>
<td>interest coverage</td>
<td>= EBIT / Interest expense, where EBIT = operating income after depreciation + non-operating income (expense).</td>
</tr>
<tr>
<td><strong>Profitability</strong></td>
<td></td>
</tr>
<tr>
<td>gross margin</td>
<td>= (total revenue – cost of goods sold) / total revenue.</td>
</tr>
<tr>
<td>return on assets (ROA)</td>
<td>= net income / total assets.</td>
</tr>
<tr>
<td><strong>Market valuation</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
</tr>
<tr>
<td>tangibility</td>
<td>property, plant, and equipment (Net) / total assets.</td>
</tr>
</tbody>
</table>
Independent Variables in Model (2.2.2)

There are many measures of the risk free rate, as well as the rate of inflation. Reasonable proxies for the risk-free rate include the yield of treasury bills, the effective federal fund rate and the US dollar-denominated London Interbank Offered Rate (LIBOR). In addition, there are three major measures of inflation — consumer price index (CPI), personal consumption expenditure (PCE) index, and the GDP implied deflator. These interest rates and inflation rates are highly correlated among themselves, and thus in principle should lead to similar regression results.

In the benchmark regression of the model (2.2.2), the risk-free interest rate \( r_f \) is measured by the 3-month treasury constant maturity rate, and the inflation rate is measured by consumer price index (CPI). But as a robustness test, in Subsection 2.3.4, we show that using alternative measures of the risk-free rate and the inflation rate do not change our empirical results.

2.2.5 Descriptive Statistics

Table 2.2 reports the descriptive statistics of the dependent and independent variables. Comparing the balanced with the unbalanced panel, the latter has larger standard deviations on all variables. Moreover, for variables involving items of the income-statement, the differences between the mean and median are much larger in the unbalanced panel than that in the balanced panel. This is due to the fact that the unbalanced panel has a large number of outliers. Many firms that dropped out of the sample in the middle of the time window reported very low earnings and revenues, as well as very high costs of borrowing. Some new firms, belonging to the “new economy” such as the internet sector, reported also near-zero or even negative earnings. The existence of such firms in the unbalanced panel affects the calculation of the means and increases the standard deviations.\(^7\) As mentioned above, our methodology is to manipulate the dataset as little as possible, hence we do not kick the “abnormal” firms out of the unbalanced panel. Rather, we compare the regression results of the unbalanced panel with that of the balanced panel, which does not have the issue of outliers. In Subsection 2.3.2 and 2.3.3 below, we will see that the existence of outliers in the unbalanced panel does not affect our empirical conclusions.

\(^7\) As an example of the existence of outliers in the unbalanced panel, one notices that the mean and standard deviation of the price-to-book ratio (PB) are over 10\(^{13}\). This is because there are many firms in the unbalanced panel having near-zero book values. Zero book-value is abnormal, which usually signals the coming bankruptcy. Those firms with almost-zero book values are not in the balanced panel, since the latter contains only firms that do not have the issue of going concern.
Table 2.2: Descriptive statistics.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Balanced Panel</th>
<th></th>
<th></th>
<th></th>
<th>Unbalanced Panel</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Std.</td>
<td>N</td>
<td>Mean</td>
<td>Median</td>
<td>Std.</td>
<td>N</td>
</tr>
<tr>
<td><strong>Dependent Variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y^1$</td>
<td>5.8972</td>
<td>2.3645</td>
<td>59.7509</td>
<td>68 852</td>
<td>20.1762</td>
<td>2.1905</td>
<td>863.0648</td>
<td>152 872</td>
</tr>
<tr>
<td>$Y^2$</td>
<td>4.7049</td>
<td>4.3707</td>
<td>10.2219</td>
<td>68 852</td>
<td>5.6420</td>
<td>4.1774</td>
<td>76.1639</td>
<td>152 864</td>
</tr>
<tr>
<td><strong>Size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>asset</td>
<td>6.8498</td>
<td>7.0199</td>
<td>2.2363</td>
<td>68 852</td>
<td>6.1369</td>
<td>6.2856</td>
<td>2.4392</td>
<td>152 872</td>
</tr>
<tr>
<td>revenue</td>
<td>6.6188</td>
<td>6.8361</td>
<td>2.1839</td>
<td>68 852</td>
<td>5.8190</td>
<td>6.1045</td>
<td>2.5107</td>
<td>152 872</td>
</tr>
<tr>
<td>market cap.</td>
<td>6.3060</td>
<td>6.4925</td>
<td>2.3515</td>
<td>59 058</td>
<td>5.7828</td>
<td>5.9099</td>
<td>2.3844</td>
<td>133 746</td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>current ratio</td>
<td>1.8097</td>
<td>1.4958</td>
<td>1.4342</td>
<td>61 987</td>
<td>2.2540</td>
<td>1.6480</td>
<td>4.4094</td>
<td>132 284</td>
</tr>
<tr>
<td>quick ratio</td>
<td>1.1523</td>
<td>.9577</td>
<td>.9212</td>
<td>58 213</td>
<td>1.4452</td>
<td>1.0418</td>
<td>1.9431</td>
<td>114 874</td>
</tr>
<tr>
<td>cash ratio</td>
<td>.3785</td>
<td>.1296</td>
<td>1.0079</td>
<td>61 838</td>
<td>.8064</td>
<td>.1911</td>
<td>4.1810</td>
<td>131 628</td>
</tr>
<tr>
<td><strong>Capital Structure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prop.debt in curr.liab.</td>
<td>.2334</td>
<td>.1922</td>
<td>.1860</td>
<td>57 970</td>
<td>.2342</td>
<td>.1810</td>
<td>.2039</td>
<td>118 692</td>
</tr>
<tr>
<td>prop.long-term debt</td>
<td>.4087</td>
<td>.4134</td>
<td>.2039</td>
<td>67 101</td>
<td>.4102</td>
<td>.4071</td>
<td>.2438</td>
<td>142 304</td>
</tr>
<tr>
<td>debt ratio</td>
<td>.3065</td>
<td>.3031</td>
<td>.1597</td>
<td>60 470</td>
<td>.3011</td>
<td>.2904</td>
<td>.1849</td>
<td>121 707</td>
</tr>
<tr>
<td><strong>Solvency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solvency ratio</td>
<td>.1751</td>
<td>.1408</td>
<td>.9217</td>
<td>68 847</td>
<td>.0805</td>
<td>.1337</td>
<td>.4166</td>
<td>152 847</td>
</tr>
<tr>
<td>interest coverage</td>
<td>18.4699</td>
<td>4.0217</td>
<td>522.3745</td>
<td>68 431</td>
<td>32.2996</td>
<td>3.7605</td>
<td>894.6041</td>
<td>152 084</td>
</tr>
<tr>
<td><strong>Profitability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gross margin</td>
<td>.2767</td>
<td>.2929</td>
<td>2.3790</td>
<td>68 567</td>
<td>-.5298</td>
<td>.3043</td>
<td>31.5183</td>
<td>152 257</td>
</tr>
<tr>
<td>ROA</td>
<td>.0440</td>
<td>.0455</td>
<td>.1268</td>
<td>68 849</td>
<td>-.0034</td>
<td>.0410</td>
<td>2.5362</td>
<td>152 857</td>
</tr>
<tr>
<td><strong>Market valuation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>2.2436</td>
<td>1.4805</td>
<td>26.4916</td>
<td>59 059</td>
<td>4.2E+13</td>
<td>1.7061</td>
<td>1.5E+16</td>
<td>133 748</td>
</tr>
<tr>
<td>Q</td>
<td>1.4415</td>
<td>1.1927</td>
<td>1.4710</td>
<td>59 059</td>
<td>1.8795</td>
<td>1.2914</td>
<td>9.5025</td>
<td>133 748</td>
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<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tangibility</td>
<td>0.4401</td>
<td>0.3956</td>
<td>0.2600</td>
<td>66 294</td>
<td>0.3707</td>
<td>0.3110</td>
<td>0.2683</td>
<td>143 417</td>
</tr>
</tbody>
</table>

Notes: “Std.” and “N” represent standard deviation and the number of observations, respectively.

2.3 Results

2.3.1 Corporate Leverage in the United States

Over the past decades, there has been a credit boom in the corporate sector. Figure 2.3 plots two measures of the corporate leverage for our sample: the book value of debt over total assets and the book value of debt over total revenue. Both measures exhibit a clear upward trend during the period between 1980Q1 and 2007Q4. This is in line with the finding by
Graham et al. (2015), who document a credit expansion of the US firms over the past century. Graham et al. (2015) argue that such credit expansion cannot be explained by the change in firm’s characteristics.

Graham et al. (2015) proposes three possible explanations behind the corporate credit boom: change in government borrowing, financial development and the change in macroeconomic uncertainty. Our empirical results, which will be presented below, indicate that the intermediation cost of borrowing has decreased dramatically. As the intermediation cost is a measure of financial efficiency, our findings point to the evidence of financial development. Namely, the financial development reduces the intermediation cost, which fosters more borrowing of the firms.

2.3.2 The Cost of Credit Supply

The model (2.2.1) is estimated by means of ordinary least squares (OLS). Table 2.7 and 2.8 in the Appendix 2.A report the regression results for the dependent variable $Y^1$ and $Y^2$, respectively. As our focus is on the goodness-of-fit and the fixed time effects, we also remove the discussion on the estimation results of the controls to the Appendix 2.A. As the tables show, the adjusted $R^2$ for the balanced panel is 0.81 for the regression of $Y^1$, and 0.76 for that of $Y^2$. The goodness-of-fit for the unbalanced panel is not very high (0.48 for $Y^1$ and 0.57 for $Y^2$). However, Figure 2.4 shows that the estimated fixed effects $\hat{\gamma}_i$ in the unbalanced panel follow closely to that in the balanced panel. In Subsection 2.3.3, we further show that the
balanced and the unbalanced panel have very similar patterns of change in the intermediation cost (see Table 2.3 and Figure 2.6). Therefore, we can conclude that most factors affecting firm’s borrowing cost are covered by the independent variables and by the two fixed effects.

The estimates of the time effects $\hat{\gamma}_t$ are plotted in Figure 2.4. As we can see, over time, the time effects on the borrowing cost are consistently decreasing. This is the case for both panels. Comparing to the cross-sectional means of the two dependent variables reported in the second row of Figure 2.2, there is no big swing in the estimated $\hat{\gamma}_t$. The large divergence of the cross-sectional means between the balanced and unbalanced panel also vanishes in the time effects. This further implies that the firm’s characters affecting the cost of borrowing are well controlled. The fixed time effects $\hat{\gamma}_t$ thus contain only the factor of the credit-supply cost. Figure 2.4 suggests that the cost of credit supply has been constantly decreasing over time.

In Figure 2.5, we fit a linear trend to the estimated time effects $\hat{\gamma}_t$, and estimate the coefficient of the time trend. The results indicate that, on average, the fall in the cost of credit supply leads to a quarterly reduction by 0.020%-0.021% in $Y^1$, or 0.025% in $Y^2$. For the entire time window (1983Q1–2007Q4, 100 periods), due to the decrease in the cost of credit supply, the interest expense over revenue ($Y^1$) has dropped by 2.00%-2.13%, and the interest expense over book-value of debt ($Y^2$) has reduced by 2.52%-2.54%. Both are significant reductions, compared with the cross-sectional means of $Y^1$ and $Y^2$ reported in Table 2.2.
Chapter 2. Intermediation Cost of Credit in the US Corporate Sector

Figure 2.5: Time trend in the cost of credit supply.

Notes: Each scatter point is the estimated time effect $\hat{\gamma}_t$ (not seasonally adjusted) of the model (2.2.1). The time trend is the estimated regression coefficient $\hat{\eta}_t$ in the simple regression model $\hat{\gamma}_t = \eta_0 + \eta_1 \times T_t + \epsilon_t^g$, where $T_t = \text{calendar quarter} – 1983Q1$, and $\epsilon_t^g$ is the residual. Numbers in the parentheses are Newey-West standard errors. *** denotes significance at the 1%-level.

2.3.3 The Intermediation Cost

By estimating the model (2.2.2), the time trend in the intermediation cost can be disentangled from the trend in the cost of credit supply. The estimation results of the model (2.2.2) are reported in Table 2.3. We see that the risk-free rate holds a strong explanatory power on the variation in the cost of credit supply. This is as expected, as the former is a component of the latter.

Our focus is on the estimates of the residuals. As has been discussed above, the residuals $\hat{\epsilon}_t$ of the regression model (2.2.2) measure the effects of the decrease in the intermediation
Table 2.3: Regression results of the model (2.2.2).

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Dependent Var. $Y^1$</th>
<th></th>
<th>Dependent Var. $Y^2$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Balanced</td>
<td>Unbalanced</td>
<td>Balanced</td>
<td>Unbalanced</td>
</tr>
<tr>
<td>treasury-bill rate</td>
<td>.2375***</td>
<td>.2350***</td>
<td>.2835***</td>
<td>.2865***</td>
</tr>
<tr>
<td></td>
<td>(.0256)</td>
<td>(.0219)</td>
<td>(.0299)</td>
<td>(.0299)</td>
</tr>
<tr>
<td>CPI-inflation rate</td>
<td>.0198</td>
<td>.0386</td>
<td>.0500</td>
<td>.0518</td>
</tr>
<tr>
<td></td>
<td>(.0591)</td>
<td>(.0558)</td>
<td>(.0663)</td>
<td>(.0678)</td>
</tr>
<tr>
<td>(intercept)</td>
<td>−1.2973***</td>
<td>−1.3426***</td>
<td>−1.6309***</td>
<td>−1.6518***</td>
</tr>
<tr>
<td></td>
<td>(.1396)</td>
<td>(.1250)</td>
<td>(.1504)</td>
<td>(.1527)</td>
</tr>
<tr>
<td>Nr. period</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>adj. R²</td>
<td>.7308</td>
<td>.6340</td>
<td>.7653</td>
<td>.7704</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.

cost on the cost of borrowing. In Figure 2.6, we fit a linear trend to the estimated residuals $\hat{e}_t$. As we can see, the intermediation cost has decreased significantly over the period between 1983Q1 and 2007Q4. On average, the drop in the intermediation cost leads to a quarterly drop by 0.0057%-0.0071% in $Y^1$, corresponding to an overall decrease by 0.57%-0.71% in the interest expense over revenue for the whole period. At the same time, the reduction in the intermediation cost also results in a quarterly drop by 0.0078%-0.0079% in $Y^2$, corresponding to an overall decrease by 0.78%-0.79% in the interest expense over book value of debt for the entire time window. The reduction in both dependent variables are significant at the 1%-level.

### 2.3.4 Robustness

As a robustness check, we use different measures of the risk-free rate and the rate of inflation as independent variables of the model (2.2.2). To be specific, we proxy the risk-free rate by the effective federal fund rate and the US dollar-denominated LIBOR rate. In addition, we measure the inflation by the PCE price index and the GDP implied deflator. Table 2.9, 2.10, 2.11 and 2.12 in the Appendix 2.B report the results of the robustness check. As the tables show, using alternative measures does not change our empirical results. The regression results and the estimated linear time trends in the intermediation cost are very close to each other under alternative measures. The only noticeable difference occurs when we use the LIBOR rate as the risk-free rate (although the difference is not large). This is because the series of LIBOR rate are only available from 1986Q1, while our estimated time effects $\hat{\gamma}_t$ start from 1983Q1. The $\hat{\gamma}_t$ before 1986Q1 has to be dropped when regressing on the
Chapter 2. Intermediation Cost of Credit in the US Corporate Sector

Figure 2.6: Time trend in the intermediated cost.

Notes: Each scatter point is the estimated residual $\hat{\varepsilon}_t$ (not seasonally adjusted) of the model (2.2.2). The time trend is the estimated regression coefficient $\hat{\delta}_1$ in the simple regression model $\hat{\varepsilon}_t = \hat{\delta}_0 + \hat{\delta}_1 \times T_t + \epsilon_t^\prime$, where $T_t$ = calendar quarter – 1983Q1, and $\epsilon_t^\prime$ is the residual. Numbers in the parentheses are Newey-West standard errors. *** denotes significance at the 1%-level.

LIBOR rate. In summary, the robustness check confirms that the effects of the decreasing intermediation cost on the cost of borrowing are robust against alternative measures of risk-free rate and inflation.

To verify the validity of our methodology, instead of the risk-free interest rate, we use the Moody’s Aaa-graded corporate bond yield and re-run the regression on model (2.2.2). The yield of the Aaa-graded corporate bond carries little default-risk premium. But unlike other interest rates that we have used so far, the corporate-bond yield contains the intermediation cost. If our methodology is correct, there should not exist a clear trend in the residuals. This
Table 2.4: Regression results with the Aaa corporate bond rate with $Y^1$.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Balanced 1</th>
<th>Balanced 2</th>
<th>Balanced 3</th>
<th>Unbalanced 1</th>
<th>Unbalanced 2</th>
<th>Unbalanced 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa rate</td>
<td>.2851***</td>
<td>.2773***</td>
<td>.2886***</td>
<td>.2771***</td>
<td>.2712***</td>
<td>.2775***</td>
</tr>
<tr>
<td></td>
<td>(.0278)</td>
<td>(.0312)</td>
<td>(.0284)</td>
<td>(.0232)</td>
<td>(.0271)</td>
<td>(.0249)</td>
</tr>
<tr>
<td>CPI</td>
<td>.0500</td>
<td>.0729</td>
<td></td>
<td>.0761</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0492)</td>
<td>(.0444)</td>
<td></td>
<td>(.0552)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE</td>
<td>.0643</td>
<td></td>
<td>.0400</td>
<td></td>
<td>.0788</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0551)</td>
<td></td>
<td>(.0626)</td>
<td></td>
<td>(.0587)</td>
<td></td>
</tr>
<tr>
<td>Def</td>
<td>−2.4321***</td>
<td>−2.3850***</td>
<td>−2.4068***</td>
<td>−2.4394***</td>
<td>−2.3683***</td>
<td>−2.4181***</td>
</tr>
<tr>
<td></td>
<td>(.2081)</td>
<td>(.1988)</td>
<td>(.2182)</td>
<td>(.1721)</td>
<td>(.1707)</td>
<td>(.1874)</td>
</tr>
<tr>
<td>(intercept)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Nr. period</td>
<td>.8573</td>
<td>.8584</td>
<td>.8538</td>
<td>.7215</td>
<td>.7194</td>
<td>.7178</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>.8573</td>
<td>.8584</td>
<td>.8538</td>
<td>.7215</td>
<td>.7194</td>
<td>.7178</td>
</tr>
</tbody>
</table>

Notes: “Aaa rate” stands for Aaa-graded corporate bond yield. “Def” represents the GDP implied deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.

is because both the variation in the risk-free rate and that in the intermediation cost are fully covered by the change in the corporate bond yield, and the cost of credit supply is the sum of the latter two components (recall Figure 2.1). The results of the regression, as reported by Table 2.4 and 2.5, confirm that this is indeed the case. The yield of the corporate bond holds almost the entire explanatory power on the cost of credit supply, leaving no significant trend in the estimated residuals.

### 2.3.5 Trend in the Average Risk Premium and Unit Cost

We have established that the intermediation cost of borrowing has decreased over time. But what about the risk-premium and the unit cost? To answer this question, we propose the following estimation method. For each panel and for each dependent variable $Y^1$ and $Y^2$, we first estimate a two-way ANOVA model

$$Y_{it} = c + \mu_i + \gamma_i + \epsilon_{it}. \quad (2.3.1)$$

The estimates $\hat{\gamma}_i$ capture the variation in the cross-sectional average cost of borrowing over time. Next, we subtract the $\hat{\gamma}_i$ by the time effects of the credit-supply cost $\hat{\gamma}_i$ obtained
Table 2.5: Regression results with the Aaa corporate bond rate with $Y^2$.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Balanced 1</th>
<th>Balanced 2</th>
<th>Balanced 3</th>
<th>Unbalanced 1</th>
<th>Unbalanced 2</th>
<th>Unbalanced 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa rate</td>
<td>0.3301***</td>
<td>0.3236***</td>
<td>0.3329***</td>
<td>0.3354***</td>
<td>0.3265***</td>
<td>0.3365***</td>
</tr>
<tr>
<td></td>
<td>(0.0361)</td>
<td>(0.0397)</td>
<td>(0.0376)</td>
<td>(0.0340)</td>
<td>(0.0377)</td>
<td>(0.0363)</td>
</tr>
<tr>
<td>CPI</td>
<td>0.0952</td>
<td>0.0958</td>
<td>0.1041</td>
<td>0.0958</td>
<td>0.0958</td>
<td>0.1041</td>
</tr>
<tr>
<td></td>
<td>(0.0625)</td>
<td>(0.0606)</td>
<td>(0.0685)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE</td>
<td>0.0959</td>
<td>0.0931</td>
<td>0.1041</td>
<td>0.0958</td>
<td>0.0958</td>
<td>0.1041</td>
</tr>
<tr>
<td></td>
<td>(0.0916)</td>
<td>(0.0857)</td>
<td>(0.0685)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Def</td>
<td>-2.9323***</td>
<td>-2.8389***</td>
<td>-2.9870***</td>
<td>-2.9763***</td>
<td>-2.8834***</td>
<td>-2.9468***</td>
</tr>
<tr>
<td></td>
<td>(0.2660)</td>
<td>(0.2658)</td>
<td>(0.2929)</td>
<td>(0.2512)</td>
<td>(0.2496)</td>
<td>(0.2744)</td>
</tr>
<tr>
<td></td>
<td>(0.3232)</td>
<td>(0.3232)</td>
<td>(0.3232)</td>
<td>(0.3232)</td>
<td>(0.3232)</td>
<td>(0.3232)</td>
</tr>
<tr>
<td>Nr. period</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>.8545</td>
<td>.8509</td>
<td>.8478</td>
<td>.8665</td>
<td>.8646</td>
<td>.8611</td>
</tr>
<tr>
<td>time trend</td>
<td>-0.0027</td>
<td>-0.0026</td>
<td>-0.0027</td>
<td>-0.0025</td>
<td>-0.0024</td>
<td>-0.0025</td>
</tr>
<tr>
<td></td>
<td>(.0022)</td>
<td>(.0023)</td>
<td>(.0024)</td>
<td>(.0021)</td>
<td>(.0022)</td>
<td>(.0023)</td>
</tr>
</tbody>
</table>

Notes: “Aaa rate” stands for Aaa-graded corporate bond yield. “Def” represents the GDP implied deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.

in Subsection 2.3.2. The resulted series thus captures the variation in the cross-sectional average borrowing cost due to the change in firms’ financial status over time. In other words, it proxies the effects of the change in firms’ risk premium on the cross-sectional average borrowing cost. The first two lines of Table 2.6 reports the linear time trend in the estimated $(\tilde{\gamma}_i^o - \tilde{\gamma}_i)$. At the same time, we apply the regression model (2.2.2), and regress the $\tilde{\gamma}_i^o$ on the risk-free rate (yield of treasury bills) and the inflation rate (CPI). By the same principle as discussed in Subsection 2.2.2, the trend in the residuals, if exists, captures the variation in the unit cost. The last two lines of Table 2.6 report the results.

Table 2.6 presents mixed results on the time trend in the average risk premium and unit cost. There is no trend in either of the two when the borrowing cost is measured by $Y^1$. When the cost of borrowing is taken as the interest expense over book value of debt ($Y^2$), only the balanced panel exhibits a clear downward trend. Therefore, it is hard to draw a conclusion on the change in the average risk premium and the average unit cost.

In Figure 2.7 we plot the “average cost of external finance” estimated by Eisfeldt and Muir (2016). The cost of external finance defined in Eisfeldt and Muir (2016) is similar to our unit cost, but the former also includes costs of firm’s equity finance. For the purpose of
Table 2.6: Time trend in the estimated \( \hat{\gamma}_t - \hat{\gamma}_t \) and unit cost.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Dependent Var. ( Y^1 )</th>
<th>Dependent Var. ( Y^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Balanced</td>
<td>Unbalanced</td>
</tr>
<tr>
<td>time trend</td>
<td>.0040</td>
<td>.0930</td>
</tr>
<tr>
<td>in ( \hat{\gamma}_t - \hat{\gamma}_t )</td>
<td>(.0099)</td>
<td>(.0604)</td>
</tr>
<tr>
<td>time trend</td>
<td>.0014</td>
<td>.0499</td>
</tr>
<tr>
<td>in unit cost</td>
<td>(.0091)</td>
<td>(.0591)</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.

comparison, we take their estimates from 1983 to 2007. Figure 2.7 shows no significant time trend. To compare, our empirical results also suggest a lack of clear trend in the unit cost. In this sense, our findings are in line with Eisfeldt and Muir (2016) as well as Philippon (2015). However, we find that a part of the unit cost in the corporate credit finance, the intermediation cost, has decreased. This part, we maintain, reflects the improvement in the financial efficiency.

2.4 Conclusions

In this chapter, we propose a regression method to assess the time trend in the firm’s intermediation cost of external credit. We define the intermediation cost as the part of the unit cost of the external credit finance that is unrelated to the risk premium. We find that the intermediation cost has decreased significantly over the period of 1983Q1–2007Q4, which has substantially reduced firm’s borrowing cost. On average, the fall in the intermediation cost leads to a reduction by 0.57-0.71 percentage point in the ratio of interest expense over revenue, and a decrease by 0.78-0.79 percentage point in the ratio of interest expense over book value of debt. The estimation results are robust against different sample components and against alternative measures of the risk-free rate and inflation. The reduction in the intermediation cost implies that the financial efficiency has improved in the US corporate credit market. We also assess the cross-sectional average risk premium and unit cost over the same period of time, but find mixed results.

Our empirical results are in line with Philippon (2015) and Eisfeldt and Muir (2016) for the lack of evidence on the reduction in the unit cost. The estimation in Philippon (2015) covers the entire financial sector, while Eisfeldt and Muir (2016) estimate the combined cost of corporate debt- and equity-finance. Our empirical work, however, has a narrow focus...
on the corporate credit finance. The main finding of this chapter is that the intermediation cost, which is a part of the unit cost, has decreased. As the unit cost contains the risk premium and the risk premium is strongly associated with the firm’s characteristics, we maintain that the intermediation cost is a better measure of the financial efficiency. In this sense, our estimation results suggest an improvement in the financial efficiency in the US corporate-credit market.

There are several interesting works yet to be done. First, we plan to push our estimation of the time trend in the intermediation cost toward more recent years. Second, as Figure 2.6 shows, there are ups-and-downs in the estimated trend in the intermediation cost, which appear to form cycles. It would be interesting to assess the cyclical properties of the intermediation cost, and discuss its macroeconomic implications. Last but not least, it would also be interesting to further focus on the intermediation cost in the loan market. But for that we need additional data on the loan-contract level.
Appendix 2.A  Regression Results of Model (2.2.1)

Table 2.7 and 2.8 report the regression results of the model (2.2.1) for the dependent variable $Y^1$ and $Y^2$, respectively. The goodness-of-fit and the estimated fixed time effects are presented and discussed in Subsection 2.3.2. Here we briefly discuss the estimation results of the controls.

Among the three variables describing the firm’s size, market capitalization has a significantly negative correlation with either measure of the borrowing cost. This is as expected. As discussed in Subsection 2.2.4, firms with larger market values usually have less information asymmetries in the credit market and thus enjoy a perception of lower credit risk. Therefore, *ceteris paribus*, their borrowing cost is usually lower. Total assets are positively correlated with $Y^1$. This is because liability is a major component of firm’s total assets. In other words, a firm with a larger size of the balance sheet usually has more interest-bearing liabilities. Note that the denominator of $Y^1$ is revenue, not a balance-sheet item as in $Y^2$. Hence total assets positively relate to $Y^1$, but the correlation between the total assets and $Y^2$ is mixed. Similar logic applies to the regression results of the revenue. As interest expense is a component of the revenue, a firm with larger revenue usually incurs more cost of borrowing. Here, revenue positively relates to $Y^2$, as the denominator of $Y^2$ is not from the income statement. The correlation between the revenue and $Y^1$, on the other hand, is mixed.

Concerning the liquidity, unexpectedly, the cash ratio has a significantly positive correlation with both measures of the cost of borrowing. A potential explanation could be that although the cash ratio measures the liquidity of a firm, firms with higher interest expenses usually hold larger quantities of cash and its equivalents, in order to be ready for the repayment of the short-term debt. The correlation between the MA12 current ratio and the cost of borrowing is consistently negative, which is what we have expected. But the cost of borrowing could go up following an improvement of the current ratio (the column of $(\text{MA12} - \text{MA4})$ and $(\text{spot} - \text{MA12})$). One of the explanations could be that firms take advantage of the improved liquidity and increase the long-term borrowing.

We see that the debt ratio has a strong positive relationship with the cost of borrowing. This can be easily understood, as debt is the main interest-bearing liability in a company’s balance sheet. The debt ratio is a measure of firm’s leverage; and the leverage is immediately related to the amount of borrowing and the risk premium. There are mixed effects of the other two capital-structure measures on the cost of borrowing. Finally, as expected in Subsection 2.3.2, the relationships between all other controls and the cost of borrowing are also mixed.

We would like to emphasize that interpreting the regression results of the controls is not our main task. The independent variables are specified in a way to achieve a goodness-of-fit as high as possible. As many controls are actually correlated with each other, it is hard to form a clean interpretation on the regression results of every control. This, however, does not
Table 2.7: Regression results of the model with $Y^1$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>asset</td>
<td>4.3919***</td>
<td>−.3594</td>
<td>3.5428***</td>
<td>4.4552***</td>
<td>−3.2937</td>
<td>7.2379***</td>
</tr>
<tr>
<td></td>
<td>(.1394)</td>
<td>(.7573)</td>
<td>(.4980)</td>
<td>(.2283)</td>
<td>(2.3885)</td>
<td>(2.2344)</td>
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<td></td>
<td>(.1323)</td>
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<td>(.3470)</td>
<td>(.3217)</td>
<td>(2.1225)</td>
<td>(2.1471)</td>
</tr>
<tr>
<td>market cap.</td>
<td>−.7764***</td>
<td>.0137</td>
<td>−.1730</td>
<td>−.4266***</td>
<td>−.7122</td>
<td>.8684**</td>
</tr>
<tr>
<td></td>
<td>(.0636)</td>
<td>(.2042)</td>
<td>(.1575)</td>
<td>(.1583)</td>
<td>(.4549)</td>
<td>(.3453)</td>
</tr>
<tr>
<td>current ratio</td>
<td>−.4824***</td>
<td>.6675***</td>
<td>−.3688**</td>
<td>−.3169***</td>
<td>1.2131**</td>
<td>−.8033*</td>
</tr>
<tr>
<td></td>
<td>(.0590)</td>
<td>(.1873)</td>
<td>(.1464)</td>
<td>(.0916)</td>
<td>(.5073)</td>
<td>(.4419)</td>
</tr>
<tr>
<td>quick ratio</td>
<td>.0661</td>
<td>−1.7644***</td>
<td>.6347**</td>
<td>−.1043</td>
<td>−3.1514**</td>
<td>2.5316*</td>
</tr>
<tr>
<td></td>
<td>(.1003)</td>
<td>(.3785)</td>
<td>(.3001)</td>
<td>(.3325)</td>
<td>(1.5647)</td>
<td>(1.4681)</td>
</tr>
<tr>
<td>cash ratio</td>
<td>.6006***</td>
<td>1.8275***</td>
<td>−.4927*</td>
<td>.8884***</td>
<td>1.3928</td>
<td>−1.6657</td>
</tr>
<tr>
<td></td>
<td>(.1570)</td>
<td>(.3901)</td>
<td>(.2991)</td>
<td>(.3413)</td>
<td>(1.1213)</td>
<td>(1.0999)</td>
</tr>
<tr>
<td>prop. debt in</td>
<td>−.7065***</td>
<td>−2.3796***</td>
<td>−.1853</td>
<td>−2.0562***</td>
<td>−1.4821</td>
<td>−2.1298**</td>
</tr>
<tr>
<td>current liab.</td>
<td>(.2085)</td>
<td>(.8220)</td>
<td>(.5276)</td>
<td>(.3843)</td>
<td>(1.1811)</td>
<td>(0.9802)</td>
</tr>
<tr>
<td>prop. long-term</td>
<td>1.4060***</td>
<td>−3.1599***</td>
<td>.4071</td>
<td>−1.3306*</td>
<td>.1605</td>
<td>−4.9745***</td>
</tr>
<tr>
<td>debt</td>
<td>(.4070)</td>
<td>(1.1291)</td>
<td>(.7638)</td>
<td>(.7911)</td>
<td>(1.9826)</td>
<td>(1.9286)</td>
</tr>
<tr>
<td>debt ratio</td>
<td>5.8718***</td>
<td>10.2188***</td>
<td>2.5190*</td>
<td>8.9729***</td>
<td>13.3313***</td>
<td>4.8621</td>
</tr>
<tr>
<td></td>
<td>(.5883)</td>
<td>(1.9783)</td>
<td>(1.2905)</td>
<td>(.9493)</td>
<td>(4.8650)</td>
<td>(3.9181)</td>
</tr>
<tr>
<td>solvency ratio</td>
<td>−1.1190***</td>
<td>−.3492</td>
<td>.6753</td>
<td>−1.7859</td>
<td>2.5569**</td>
<td>−.1094</td>
</tr>
<tr>
<td></td>
<td>(.3605)</td>
<td>(.5372)</td>
<td>(.6170)</td>
<td>(.12914)</td>
<td>(1.1122)</td>
<td>(.4090)</td>
</tr>
<tr>
<td>interest coverage</td>
<td>−1.1E−5</td>
<td>−1.5E−6</td>
<td>8.1E−6</td>
<td>9.7E−5*</td>
<td>1.6E−5</td>
<td>2.6E−5</td>
</tr>
<tr>
<td></td>
<td>(2.4E−5)</td>
<td>(1.2E−5)</td>
<td>(7.5E−6)</td>
<td>(5.2E−5)</td>
<td>(2.3E−5)</td>
<td>(3.4E−5)</td>
</tr>
<tr>
<td>gross margin</td>
<td>.8988*</td>
<td>9.4613***</td>
<td>−8.3231***</td>
<td>3.5494</td>
<td>−.7442</td>
<td>−.9307</td>
</tr>
<tr>
<td></td>
<td>(.4963)</td>
<td>(2.5779)</td>
<td>(2.5490)</td>
<td>(2.4183)</td>
<td>(1.4951)</td>
<td>(0.6852)</td>
</tr>
<tr>
<td>ROA</td>
<td>3.8118***</td>
<td>−3.5533***</td>
<td>.6666</td>
<td>1.2333</td>
<td>−6.4201*</td>
<td>1.6373</td>
</tr>
<tr>
<td></td>
<td>(.8818)</td>
<td>(1.2585)</td>
<td>(1.3696)</td>
<td>(1.5977)</td>
<td>(3.3282)</td>
<td>(1.4985)</td>
</tr>
<tr>
<td>PB</td>
<td>.0142**</td>
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Notes: Numbers in parentheses denote White robust standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.
Table 2.8: Regression results of the model with $Y^2$.

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</table>

Notes: Numbers in parentheses denote White robust standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.
affect our estimates of the fixed time effects, as long as the controls fully cover the variation in the firm characteristics over time.

Appendix 2.B  Robustness Check on Alternative Risk-free Rate and Inflation Rate

In Subsection 2.3.4 we conduct the robustness check on the regression model (2.2.2), using alternative measures of the risk-free rate and the rate of inflation. The results of the robustness check are presented in Table 2.9, 2.10, 2.11 and 2.12 below. The discussion on the robustness check is in Subsection 2.3.4.
Table 2.9: Results with different measures of risk-free rate and inflation for model, $Y^1$ with balanced panel.

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<th>6</th>
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<td>.0253 (.0660)</td>
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<tr>
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<td>.0867 (.0560)</td>
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<tr>
<td>Def</td>
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<td>.0483 (.0849)</td>
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<td>.0293 (.0851)</td>
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<tr>
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<td>-1.2837*** (.1400)</td>
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<td>-1.3092*** (.1356)</td>
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</table>

| Nr. period      | 100 | 100 | 100 | 100 | 100 | 88  | 88  | 88  |
| adj. $R^2$      | .7463 | .7315 | .7354 | .7517 | .7372 | .6493 | .6635 | .6489 |
| time trend      | -.0068*** (.0018) | -.0070*** (.0018) | -.0071*** (.0016) | -.0068*** (.0016) | -.0071*** (.0017) | -.0096*** (.0016) | -.0094*** (.0016) | -.0096*** (.0016) |

Notes: “LIBOR” stands for dollar-denominated 3-month LIBOR rate. “Def” represents the GDP-deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.
Table 2.10: Results with different measures of risk-free rate and inflation, \(Y^1\) with unbalanced panel.

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<th>5</th>
<th>6</th>
<th>7</th>
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<td>.0867 (.0550)</td>
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Notes: “LIBOR” stands for dollar-denominated 3-month LIBOR rate. “Def” represents the GDP-deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and ***, respectively.
Table 2.11: Results with different measures of risk-free rate and inflation for model, $Y^2$ with balanced panel.

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**Notes:** “LIBOR” stands for dollar-denominated 3-month LIBOR rate. “Def” represents the GDP-deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and *** respectively.
Table 2.12: Results with different measures of risk-free rate and inflation, $Y^2$ with unbalanced panel.

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Notes: “LIBOR” stands for dollar-denominated 3-month LIBOR rate. “Def” represents the GDP-deflator. Numbers in parentheses denote Newey-West standard errors of corresponding estimates. Significance at the 10%, 5%, and 1% level are indicated by *, ** and *** respectively.
Chapter 3

Intermediation Cost, Credit Expansion and Inequality
3.1 Introduction

Over the past decades, there has been a dramatic credit boom in the United States. Figure 3.1 shows that there is a clear upward trend in the non-financial corporate debt-to-GDP ratio in the United States. From the 1950s until the late 2010s, the corporate credit-to-GDP ratio expanded from around 30% to 75%. Graham et al. (2015) argue that the firm’s characteristics cannot explain the credit expansion, and instead suggest three explanations for the surge in corporate borrowing: changes in government borrowing, macroeconomic uncertainty, and the development of the financial sector. In this chapter, we focus on the latter, and analyze how the development of the financial sector affects macroeconomic and distributional outcomes.

In the previous chapter, we have verified that the financial development has reduced the intermediation cost of borrowing since the 1980s. Building on this empirical finding, this chapter explores whether the fall in the intermediation cost helps to explain the credit boom, as well as the following two stylized facts:

**A secular rise in income and wealth inequality.** The first row of Figure 3.2 plots different measures of the wealth and income distribution of the United States. Since the 1980s, the discrepancies of income and wealth among households have increased. Especially, the top-end inequality has been rising.

Figure 3.1: Non-financial corporate debt-to-GDP of the United States.
A secular fall in returns on risky assets. Over the last four decades, buying and holding risky assets has become less rewarding. The second row of the Figure 3.2 illustrates such trend. Stocks and real estates are two major assets for investors. As the figure shows, the former market is characterized by decreasing dividend and earnings yields, as well as increasing prices relative to earnings. The rent-price ratio of the housing market also falls in the long run.

As defined in the Section 2.1, the intermediation cost is the part of the unit cost of credit finance that is unrelated to the risk premium. These costs are usually charged by financial intermediaries for their services. In the previous chapter, we have shown that the intermediation cost has decreased significantly over the period of 1983Q1–2007Q4. On average, the fall in the intermediation cost leads to a reduction by 0.57-0.71 percentage point in firm’s interest expense over revenue, and a decrease by 0.78-0.79 percentage point in the interest expense over book value of debt.
In this chapter, we construct a stochastic dynamic general equilibrium model to explore the macroeconomic and distributional consequences of the decrease in the intermediation cost. In our model environment, there are two types of households: leveraged investors (H-types) and non-leveraged investors (L-types). The agents can invest in a risky asset and a safe asset. In our model, the decrease in the intermediation cost triggers a feedback-loop between the capital market and the credit market. A decrease in the intermediation cost makes it cheaper for leveraged H-type investors to borrow, such that they are able to invest more. Higher investment by the H-type investors raises aggregate capital and reduces the return on capital.\footnote{The return on capital in the model corresponds to the (buying-and-holding) return on risky asset that is discussed in the stylized facts. In this chapter, the two phrases are used interchangeably.} The decreasing returns on capital induce the non-leveraged L-type investors to reduce their investment and increase their savings. The additional savings by the L-type finance the additional borrowings by the H-type. Hence, the credit market experiences a “simultaneous” expansion of the supply- and demand-side of credit. The additional savings by the L-type agents raise credit supply and encourage the H-type investors to borrow more, which further raises aggregate capital. The falling return on capital induces L-type agents to save more. This feedback loop between the credit market and the capital market amplifies the initial impact of the decreasing intermediation costs. We show that the expansion in the capital market is less pronounced than the expansion in the credit market.

In addition to the feedback loop between the capital and credit market, there exists a second feedback loop between the capital-credit market and the wealth distribution. We show that the decrease in the intermediation cost results in a higher leverage of the H-type, which exaggerates the capital-income risk among the H-type investors and increases their average returns on investment. This drives up the wealth heterogeneity among households. Especially, the tail of the wealth distribution becomes thicker, leading to more rich L-type households who provide more credit to the H-type investors. This in turn increases the leverage of the H-type. The feedback between the capita-credit market and the wealth distribution thus further amplifies the initial effect of the decreasing intermediation cost. We find that the decrease in the intermediation costs strongly increases the welfare of the wealthy households. The bottom 90% households, however, face welfare losses.

To highlight the working of the feedback loop between the capital and credit market, we first construct a simple two-period model. We then develop a fully dynamic model based on Angeletos (2007), which we apply to the United States in a quantitative exercise. The model features incomplete markets with heterogeneous agents and idiosyncratic investment risk. Types are Markovian. Following Cao and Luo (2017), in the dynamic model, we assume that the H-type has higher expected returns on investment, which motivates the H-type to leverage. Saving and borrowing are conducted through a perfectly-competitive and risk-neutral financial intermediary. Saving is risk-free, paying out a risk-free rate. The
borrower has limited liability. The borrowing interest rate consists of two components, the intermediation cost, which is the spread between the risk-free rate and the lending interest rate to the risk-free borrowers, as well as the default-risk premium, which is determined by the borrower’s type and her leverage. The borrowing interest rate, as well as the borrowing limit is endogenously determined. We conduct counterfactual experiments by varying the intermediation cost. The model has rich implications that fulfill the stylized facts listed above. In the calibrated model, when the intermediation cost of borrowing decreases from 3.0% to 2.0%, credit-output ratio expands from 50.02% to 66.30%. This expansion corresponds roughly to the movement of the US corporate debt-to-GDP ratio from 1980 to 2007 (see Figure 3.1). Because of such a decrease in the intermediation cost, the return on capital falls, and the share of total wealth held by the top 10% (1%) wealthiest households increases from 63.01% (26.63%) to 68.70% (31.51%). In comparison to the data, we find that almost all the increase in the top 10% wealth share during 1980–2007 is explained by the decrease in \( \theta \), and the change in the intermediation cost was behind 38.85% of the surge in the top 1% wealth share during the same period. Our quantitative exercise also suggests that the reduction in the intermediation cost does not benefit the bottom-90% households, due to the distribution effect. Therefore, re-distributional policies may be implemented to balance the welfare gains/losses of different wealth groups.

The rest of the chapter proceeds as follows. In Section 3.2, we develop a simple two-period model to highlight the main theoretical feedback loops between the credit and capital market and the wealth distribution. Section 3.3 lays out the full model. Sections 3.4 and 3.5 discuss the calibration strategy and the quantitative findings. Section 3.6 concludes.

**Related Literature.** This chapter relates to three strands of literature. First, it adds to the recent discussion of the consequences of credit expansion, such as Iacoviello (2008), Kumhof et al. (2015), etc. While this literature focuses on the expansion of the consumer credit, our emphasis is on the business-credit boom. Graham et al. (2015) documents the upward trend of the corporate debt-to-GDP ratio over the past century, and offers tentative explanations. But they do not provide any discussion of the macroeconomic effects of such an expansion, which is the focus of this chapter. There is also growing literature on the macroeconomic consequences of financial deepening, such as Kiyotaki and Moore (2005), Townsend and Ueda (2006) and Baiardi and Morana (2016). However, the literature on financial deepening is not entirely in line with our topic, for the following two reasons. First, the literature is usually about the emerging economies. For example, Townsend and Ueda (2006) calibrates their model to Thailand, and Dabla-Norris et al. (2014) calibrates to a group of emerging economies. Second, the literature usually emphasizes the link of financial deepening to the economic growth, and to the concept of the “financial Kuznets curve”. Our focus, however, is on the interaction between the credit and capital market.
Chapter 3. Intermediation Cost, Credit Expansion and Inequality

Second, on the modelling side, this chapter builds on the extensive literature on heterogeneous-agent models with incomplete markets, e.g., Aiyagari (1994), Covas (2006), Angeletos (2007). In particular, we build on Angeletos (2007) who presents a heterogeneous agent model with incomplete markets and idiosyncratic investment risk. We choose this model as our starting point, because Benhabib et al. (2011, 2015) show that the capital-income risk drives the properties of the right-tail of the wealth distribution. Using Angeletos’ model, but in continuous time, Cao and Luo (2017) discuss the effects of investor’s type persistence on the wealth distribution. Finally, this chapter is related to the asset pricing model with incomplete markets by Guvenen (2009).

Finally, this chapter is related to e.g., Krusell and Smith (1997), Heaton and Lucas (2002) and Favilukis (2013), who study the properties of portfolio choice and the equity-premium puzzle. One of the main topics on the portfolio choice is the equity-premium puzzle. In our model, we follow this literature and adopt the Epstein-Zin specification of preferences to generate a reasonable return-premium on risky investment. This is based on Epstein and Zin (1989) and Guvenen (2009).

3.2 A Two-Period Model

In this section, we set up and analyze a two-period model to assess the macroeconomic implications of the decrease in the intermediation cost. In particular, we evaluate the properties and consequences of the feedback loop between the capital and the credit market. The two-period model is built up for explanatory purposes, since it can be solved analytically. As it turns out, many complicated relationships in the dynamic model are straightforward in the two-period model. The conclusions of the two-period model are the same as what we obtain in the dynamic model of the next section.

3.2.1 The Model Environment

The economy consists of an infinite number of agents. The total population of the economy is normalized to one. In the economy, there is a share of \( p \) agents that are of type H, and the rest \( (1 - p) \) are of type L. The economy lasts for two periods, \( t = 0, 1 \), and agents only consume at the end of period \( t = 1 \).

Each agent obtains endowment \( w \) with different timings: the L-type receives the endowment at the beginning of period \( t = 0 \), and the H-type receives the endowment at the end of period \( t = 1 \).

Alternatively, one can assume an infinite-horizon version of the model, in which the period \( t = 0 \) corresponds to the morning of day, and \( t = 1 \) corresponds to the afternoon of the day. If this assumption is adopted, one has to assume in addition that all endowment and production cannot be stored, so that they must be consumed at the end of the day.
beginning of period $t = 1$. The different timing across types has two consequences: first, it “artificially” creates leveraged (H-type) and non-leveraged investors (L-type). Second, the H-type will be essentially risk-neutral.\footnote{The conclusions of the model do not depend on the specific form of the type-difference, as long as the H-type has higher motivation to invest than the L-type. For example, the two-period model has the same implications as the dynamic model of the next section, although the latter assumes a different form of type-difference. Moreover, even the extreme version of the type-difference assumed in the two-period model has some support from the literature. There are empirical findings that the investors (i.e., asset market participants) have lower risk aversion and higher EIS, see, for example, Guvenen (2006), and our set-up fulfills such empirical regularity.} For simplicity, agents are assumed to have log-preferences over consumption.

Agent can invest to produce the consumption good. Investment is risky, and takes place only at the beginning of period $t = 0$. Denote by $i_L$ and $i_H$ the investment of the L-type and H-type, respectively. At the end of period $t = 0$, there is a capital formation process, which transforms investment of either type into productive capital,

$$k_L = \varepsilon i_L, \quad \text{and} \quad k_H = \varepsilon i_H,$$

(3.2.1)

where $k_L$ and $k_H$ denote capital of the L-type and the H-type, respectively. $\varepsilon$ is a Bernoulli random variable, which takes the value 1 with probability $1 - \lambda$. The realization of $\varepsilon$ is idiosyncratic to each agent. In other words, if an agent invests, there is a probability of $\lambda$ that the investment fails and all investment of the agent is erased. By the law of large numbers, at the end of period $t = 0$, the total amount of capital in the entire economy is

$$K = (1 - \lambda) \left[ pi_H + (1 - p)i_L \right].$$

(3.2.2)

It is assumed that when investing and not investing result in the same utility, the agent always invests.\footnote{Equivalently, we can also assume that investing brings a tiny utility gain of “staying-in-business”.
}

At the beginning of period $t = 1$, there is a perfectly competitive representative firm, that pools the aggregate capital produced in the last period and produces the consumption good to be consumed at the end of the period. The production function is

$$Y = K^\alpha,$$

(3.2.3)

where $\alpha \in (0, 1)$. The gross return to capital is $R^e = Y/K = K^{\alpha - 1}$. The capital return is distributed according to the capital holdings of the agents. The timing of events of the economy is summarized in Figure 3.3.
rate $r^f$ that is determined by the market-clearing condition. If an agent borrows, there is an intermediation cost proportional to the amount of borrowing. The intermediation cost is captured by the parameter $\theta \in (0, 1)$. When the agent borrows $b$, she can apply $\frac{1}{1+\theta}b$ to investment, while the rest $\frac{\theta}{1+\theta}b$ is the intermediation cost which is wasted. In the model, the agent borrows for investment, and she is prohibited from simultaneously borrowing and saving. Borrower has limited liability, namely, the agent’s borrowing is only secured by her investment. The bank sets the interest rate $\pi$ on borrowing according to the zero-profit condition. In particular, the bank makes zero profit after charging the interest rate from the borrowers and paying for the risk-free interest to the savers.

3.2.2 General Equilibrium

Since the H-type does not own resources at the beginning of $t = 0$, she has to borrow to invest. Because of the existence of the intermediation cost, to invest $i_H$, she has to borrow $b = (1 + \theta)i_H$. The bank is risk-neutral and perfectly competitive. If the bank is willing to lend, it charges the interest rate $\pi$ according to the zero-profit condition

$$ (1 + r^f)b = (1 - \lambda)(1 + \pi)b, \tag{3.2.4} $$

---

5 Since the agent has only one-time consumption at the end of $t = 1$, consumer credit is not possible in the model.

6 Allowing for simultaneous saving and borrowing does not change any conclusion, but it complicates the analysis. In the model, only the L-type agent has the chance to save and borrow simultaneously. However, it can be shown that, in the equilibrium, it is not optimal for her to do so.

7 We can imagine that each agent invests by setting up a limited-liability company. Even in the worst case, the bank can only grab the company’s asset. The agent’s endowment, if not being put into investment, cannot be claimed by the bank.
where \( r^f \) is the risk-free interest rate. Hence, the interest rate is given by

\[
1 + \pi = \frac{1 + r^f}{1 - \lambda}.
\]  

(3.2.5)

Lemma 3.2.1 states that in equilibrium, the bank is always willing to lend to the H-type. If the bank is willing to lend and the H-type decides to borrow, the latter’s optimization problem can be formulated as

\[
\max_{i_H} \left\{ (1 - \lambda) \log \left( R^c i_H - (1 + \pi)b + w \right) + \lambda \log w \right\},
\]

(3.2.6)

s.t. \( b = (1 + \theta)i_H \quad \text{and} \quad i_H \geq 0, \)

where \( R^c i_H - (1 + \pi)b + w \) is the wealth (consumption) that the H-type obtains if the capital-formation is successful. Remember that the endowment cannot be used as a commitment to borrowing. If the capital formation fails and her investment is wiped out, the H-type can still consume her endowment \( w \) in the end. The optimization problem (3.2.6) implies that the H-type is essentially risk-neutral; and she keeps investing until the marginal returns from investment equals the marginal cost,

\[
R^c = (1 + \pi)(1 + \theta).
\]

(3.2.7)

In equilibrium, the H-type makes zero profit from investment. But because the agent prefers “staying-in-business”, H-type still invests. The condition (3.2.7) builds a constant relationship between the capital return and the risk-free rate

\[
(1 - \lambda)R^c = (1 + \theta)(1 + r^f),
\]

(3.2.8)

which implies a risk premium of \( (1 - \lambda)R^c - (1 + r^f) = \theta(1 + r^f) \). The risk premium equals the opportunity cost of \( \theta \), which is the consequence of the risk-neutrality of the H-type.

The H-type is the net borrower in the credit market. If the credit market does not collapse, the L-type must be a net saver. Namely, the L-type must allocate her endowment between risky investment and risk-free saving. If the credit market exists, the L-type solves the following problem:

\[
\max_{i_L} \left\{ (1 - \lambda) \log \left( (1 + r^f)s + R^c i_L \right) + \lambda \log \left( (1 + r^f)s \right) \right\},
\]

(3.2.9)

s.t. \( s = w - i_L \quad \text{and} \quad i_L \geq 0, \)
where $s := w - i_n$ is the amount of saving by the L-type. The first-order condition of the optimization problem (3.2.9) implies that

$$i_L = \left[ 1 - \frac{\lambda R^c}{R^c - (1 + r^f)} \right] w. \quad (3.2.10)$$

**LEMMA 3.2.1.** In equilibrium, credit market exists and $0 < i_L < w$.

**Proof** see appendix. ■

We can now discuss the behavior of the H- and L-type when facing the decrease in the intermediation cost $\theta$. Suppose that the economy is initially in equilibrium and condition (3.2.7) holds. When $\theta$ decreases, $(1 + \pi)(1 + \theta)$ becomes smaller than $R^c$. Thus the first-order derivative of the H-type’s objective function (3.2.6) becomes strictly positive. The H-type increases investment $i_H$ until condition (3.2.7) binds again. Proposition 3.2.2 presents the solution to the general equilibrium of the economy. In general equilibrium, the increase in H-type’s leveraged investment drives up the risk-free rate $r^f$ and drives down $R^c$. According to equation (3.2.10), such changes in $r^f$ and $R^c$ reduce $i_L$. Thus the L-type invests less and saves more when $\theta$ decreases.

$r^f$, $R^c$, as well as the aggregate capital stock $K$ are determined in the general equilibrium. In the general equilibrium, credit market, good market, as well as the capital market clears. The general equilibrium of the economy is concluded in the following proposition.

**PROPOSITION 3.2.2.** (General Equilibrium) In the general equilibrium,

$$i_L = \left[ 1 - \frac{\lambda(1 + \theta)}{\theta + \lambda} \right] w; \quad (3.2.11)$$

$$i_H = \frac{1 - p}{p} \cdot \frac{\lambda}{\theta + \lambda} w; \quad (3.2.12)$$

$$b = (1 + \theta)i_H = \frac{1 - p}{p} \cdot \frac{\lambda(1 + \theta)}{\theta + \lambda} w; \quad (3.2.13)$$

$$s = w - i_L = \frac{\lambda(1 + \theta)}{\theta + \lambda} w. \quad (3.2.14)$$

Moreover, the aggregate capital capital of the economy is

$$K = (1 - p)(1 - \lambda) \left[ 1 - \frac{\lambda \theta}{\theta + \lambda} \right] w, \quad (3.2.15)$$
with the gross return to capital $R^c = K^{\alpha-1}$. The total intermediated asset $S$ and risk-free interest rate $r^f$ are

$$S = (1 - p) \frac{\lambda(1 + \theta)}{\theta + \lambda} w;$$  \hspace{1cm} (3.2.16)$$

$$1 + r^f = \frac{1 - \lambda}{1 + \theta} R^c.$$  \hspace{1cm} (3.2.17)

**Proof** see appendix.

When the intermediation cost $\theta$ decreases, $K$ increases in equilibrium, and $R^c$ drops accordingly. This means that when $\theta$ decreases, the additional investment by the H-type drives down $R^c$. Decreasing $R^c$ propels the L-type to reduce her investment in risky capital and increase her saving (equation (3.2.11)). Therefore, in the credit market, the L-type saves more to finance more borrowing of the H-type. Hence the total intermediated asset expands (equation (3.2.16)). We call this expansion a “simultaneous” credit expansion, because such an expansion is driven by both the supply (L-type) and demand side (H-type).

The feedback between the capital and the credit market amplifies the initial effect of the decrease in the intermediation cost. The additional saving by the L-type finances further leveraged investment of the H-type. Thus, $i_H$ increases again, while $R^c$ and $i_L$ drops further. In the end, the L-type supplies even more credit. The feedback loop knocks on until new equilibrium is reached. In the remainder of this section, we will discuss the consequences of the feedback loop on the risk-free rate, inequality and the welfare.

### 3.2.3 Risk-free Rate

In equation (3.2.17), $R^c$ and $\theta$ have opposing effect on $r^f$. In response to a decrease in $\theta$, the change of $r^f$ depends on the strength of credit-supply expansion relative to the credit-demand expansion. In the two-period model, it can be easily verified that when $\theta$ drops, the credit-demand effect dominates the credit-supply effect, and the risk-free rate $r^f$ increases.

**PROPOSITION 3.2.3.** When $\theta$ decreases, $r^f$ increases in equilibrium.

**Proof** see appendix.

The increasing risk-free rate comes from the fact that the H-type is risk-neutral and has a higher motivation to invest, compared to the L-type. Therefore, the H-type’s aggregate investment response to the decreasing intermediation cost is stronger than that of the L-type. To see this, let $I_H$ and $I_L$ denote the aggregate investment of the H-type and L-type,
respectively, so that \( I_H = p i_H \) and \( I_L = (1 - p) i_L \). Based on equations (3.2.11) and (3.2.12) we have

\[
\frac{\partial I_H}{\partial \theta} = -\frac{\lambda(1 - p)}{(\theta + \lambda)^2} w, \quad \text{and}
\]

\[
\frac{\partial I_L}{\partial \theta} = \frac{\lambda(1 - \lambda)(1 - p)}{(\theta + \lambda)^2} w.
\]

Therefore, we can see that

\[
\left| \frac{\partial I_H}{\partial \theta} \right| = \frac{1}{1 - \lambda} > 1,
\]

where the last inequality holds because \( 0 < \lambda < 1 \). This implies that when \( \theta \) drops, the increase in the aggregate investment of the H-type is faster than the decrease in that of the L-type. Therefore, the aggregate capital \( K \) increases, which results to a lower capital return \( R_c \). On the other hand, remember that the H-type’s investment is borrowed, and the reduced amount of the L-type’s investment is put into savings. Equation (3.2.18) thus also implies that when \( \theta \) decreases, the increase in the aggregate credit demand is faster than the increase in the aggregate credit supply. \( r^f \) increases as a result.

### 3.2.4 Credit Expansion versus Capital Expansion

When the intermediation cost \( \theta \) decreases, both the capital market and the credit market expand. But the strength of the credit expansion surpasses that of the capital expansion. To see this, let’s first compare aggregate savings and aggregate capital in general equilibrium. By equations (3.2.15) and (3.2.16),

\[
\frac{S}{K} = \frac{\lambda}{1 - \lambda} \cdot \frac{1 + \theta}{\lambda + (1 - \lambda) \theta}.
\]

The ratio of \( S/K \) increases when \( \theta \) decreases, because

\[
\frac{\partial (S/K)}{\partial \theta} = -\frac{\lambda}{1 - \lambda} \cdot \frac{1}{[\lambda + (1 - \lambda) \theta]^2} < 0.
\]

Equation (3.2.19) implies that when \( \theta \) drops, \( S \) increases more than \( K \). The reason behind this finding is easy to understand. When \( \theta \) decreases, the credit market experiences a “simultaneous” expansion from both demand and supply side. However, in the capital market, the increase in investment by the H-type (which equals exactly to the increase in the
credit demand) is dampened by the decrease in investment by the L-type. Hence the capital expansion is less significant than the credit expansion.

### 3.2.5 Inequality

The model has implications on distributions. Since the model has only two periods, wealth, income and consumption can be treated as the same. The H-type, no matter whether the capital formation is successful and what the level of \( \theta \) is, always consumes \( w \) in the end. Hence only the L-type bears the differences in income. Denote by \( c_L^1 \) and \( c_L^0 \) the L-type’s consumption when the capital formation is and is not successful, respectively. Namely, \( c_L^1 = (1 + r^f)s + R^c i_L \) and \( c_L^0 = (1 + r^f)s \). In equilibrium, by using equations (3.2.11), (3.2.14) and (3.2.17), we have

\[
\begin{align*}
    c_L^1 &= (1 - \lambda)R^c w; \\
    c_L^0 &= \frac{\lambda}{\theta + \lambda} c_L^1.
\end{align*}
\]

Since \( R^c \) is decreasing in \( \theta \), \( c_L^1 \) decreases when the intermediation cost drops. It can be easily verified that the opposite is true for \( c_L^0 \). Therefore, the lower intermediation cost decreases the L-type’s consumption (income) when the capital formation succeeds, and increases its consumption (income) when the capital formation fails. Figure 3.4 summarizes the income distribution among the L-types. In the two-period model, the change in L-types’ income distribution is equivalent to the change in their capital-income risk.\(^8\) To see this, we simply calculate the difference between \( c_L^1 \) and \( c_L^0 \): \( c_L^1 - c_L^0 = R^c i_L \). In the general equilibrium, both \( i_L \) and \( R^c \) decreases when \( \theta \) decreases. Hence the L-type’s income difference between when capital-formation success and when capital-formation fails becomes smaller. This can be deemed as the decrease in the capital-income risk, which is equivalent to a concentration of income among the L-types.

The two-period model cannot differentiate between the income and wealth distribution. Moreover, the model’s implication that the H-type’s income is less than the L-type’s is also counter-intuitive. These issues will be addressed in the dynamic model of the next section. But the main takeaway from the discussion in this subsection is the relationship between the capital-income risk and the (income) inequality. Among other things, the income inequality among L-types drops also in the dynamic model, due to the same reason as what we have seen in the two-period model.

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\(^8\) To be consistent with the existing literature, we define the “capital income” as the income from both risky investment and risk-free saving.
3.2.6 Welfare

Because the H-type is risk-neutral and always makes zero profit, the change of the overall \textit{ex ante} welfare depends on the L-type. It can be shown that with $\theta$ decreasing, the \textit{ex ante} welfare of the L-type is improving.

\textbf{PROPOSITION 3.2.4.} Decreasing $\theta$ improves the \textit{ex ante} welfare of the economy. 

\textbf{Proof} see appendix.

It can be easily verified that in the general equilibrium, when $\theta$ drops, $S$ increases but $\theta S$ decreases. This means that with lower $\theta$, financial sector consumes less resources, although the total quantity of the intermediated asset increases. The improved welfare thus comes from the more cost-efficient financial sector.

3.3 The Dynamic Model

This section presents the dynamic model that highlights the macroeconomic effects of the decrease in the intermediation cost. Building on Angeletos (2007), the model features an incomplete market with idiosyncratic investment risk. The model structure is similar to the two-period model, except that the time-horizon is infinite and the economy is productive. Because of this, leveraged and non-leveraged household need to be motivated in a different way. In the dynamic model, this is done by assuming that the former has a higher expected return on investment.

3.3.1 The Model Environment

Time is discrete, indexed by $t \in \{1, 2, \ldots, \infty\}$. There is a continuum of infinitely-lived households indexed by $i \in [0, 1]$. At each period, each household is endowed with one

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**Notes**: When $\theta$ drops, the L-type’s consumption changes from $c_L^0$ to $c_L^1$ when the capital-formation succeed; and from $c_L^{0'}$ to $c_L^{1'}$ when it fails.
unit of labor that is supplied inelastically to the labor market. The economy produces one homogeneous and perishable good. The household $i$’s consumption at time $t$ is denoted by $c^i_t$.

There are two types of household in the economy: H-type and L-type. The two types have different productivities in risky investment. $\tau^i_t \in \{H, L\}$ denotes the household’s type in period $t$. The type is assumed to be Markovian, with Poisson switching rates of $\lambda_{LH}$ (from L-type to H-type) and $\lambda_{HL}$ (from H-type to L-type). By the law of large numbers, in each period, there is always a fraction of $p$ H-type agents, and a fraction of $(1 - p)$ L-type agents,

$$p = \frac{\lambda_{LH}}{\lambda_{HL} + \lambda_{LH}}.$$  (3.3.1)

The household can invest in two assets: (1) risky investment in production $i^i_t \geq 0$; and (2) risk-free savings $s^i_t \geq 0$ in the financial intermediary. Households can also borrow from the financial intermediary to finance her risky investment. In this case, $s^i_t < 0$, and she has to pay a borrowing interest rate $\pi^i_t$ which depends on her type and her leverage.

### 3.3.2 Preferences

The household’s preferences take the form of the Epstein-Zin specification with a constant elasticity of intertemporal substitution (EIS) and a constant relative risk aversion. A stochastic consumption stream $\{c^i_t\}_{t=0}^\infty$ generates a stochastic utility stream $\{u^i_t\}_{t=0}^\infty$ according to the following recursive form:

$$U^i_t = \left[ (1 - \beta)(c^i_t)^{1-\mu} + \beta \left( E_t(U^i_{t+1}) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \tag{3.3.2}$$

where $\mu > 0$ measures the inverse of the EIS, and $\gamma > 0$ is coefficient of relative risk aversion. For $\mu = 1/\gamma$, equation (3.3.2) reduces to the standard sum of expected CRRA instantaneous utility. Epstein-Zin preferences allows us to generate an empirically reasonable risk premium. However, none of the conclusions of this chapter rely on the Epstein-Zin specification.

### 3.3.3 Investment and Production

Similar to the two-period model, the risky investment in production takes two stages. The first stage happens at the end of period $t$, when the household invests the amount of $i^i_t$ into

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*See Epstein and Zin (1989) and Weil (1989).*
her own capital-formation project. The project transforms investment into productive capital of the next period according to

\[ k_{t+1}^i = A_i^i(i_t^i)\nu. \]  

\[ (3.3.3) \]

\( \nu \in (0, 1) \) represents the Lucas’ span of control, and \( \nu < 1 \) is necessary to prevent the household from accumulating wealth infinitely. \( A_i^i \) is the productivity of the project, which takes the form of

\[ \log A_i^i = \eta_i^i + \varepsilon_i^i, \quad \text{with} \quad \varepsilon_i^i \sim N\left(-\sigma^2/2, \sigma^2\right), \]  

\[ (3.3.4) \]

where \( \varepsilon_i^i \) is an exogenously normally distributed random variable that represents the risk of investment. Note that the expectation of \( \varepsilon_i^i \) is set to be \(-\sigma^2/2\) to normalize \( E(A_i^i) = \exp\{\eta_i^i\} \). \( \eta_i^i \) differentiates the H- and L-type. We let \( \eta_H^i = \eta > 0 \) for the H-type (\( \tau_H^i = H \)), and \( \eta_L^i = 0 \) for the L-type (\( \tau_L^i = L \)). In other words, the H-type has higher productivity in capital formation than the L-type. The differentiation between types of household based on their productivity is commonly done in the macroeconomic literature, such as Angeletos (2007) and Cao and Luo (2017).

The second stage takes place at the beginning of the next period, when the formed capital is pooled by a perfectly-competitive representative corporation. The corporation uses aggregate capital \( K_{t+1} \) and aggregate labor \( L_{t+1} \), and produces the period-(\( t+1 \)) consumption good \( Y_{t+1} \) using a Cobb-Douglas production technology:

\[ Y_{t+1} = K_{t+1}^0 L_{t+1}^{1-a}, \]  

\[ (3.3.5) \]

where \( L_{t+1} \equiv 1 \), since we have normalized the inelastic labor supply to 1; and

\[ K_{t+1} = \int_0^1 k_{t+1}^i \, di. \]  

\[ (3.3.6) \]

The corporation solves the static optimization problem

\[
\max_{\{K_{t+1}, L_{t+1}\}} \left\{ Y_{t+1} - w_{t+1}L_{t+1} - (r_{t+1}^c + \delta)K_{t+1} \right\}, \]  

\[ (3.3.7) \]

where \( w_{t+1} \) and \( r_{t+1}^c \) are the wage and the return on capital, respectively, and \( \delta \) is the depreciation rate of capital. The price of the consumption good is normalized to 1. The

---

10 The formulation that each household runs her own capital-formation project is similar to the set-up by Angeletos (2007). The latter assumes that each household owns a single private firm.
corporation is price-taker on the labor and capital markets. Solving the optimization problem above obtains

\[ w_{t+1} = (1 - \alpha)K_{t+1}^\alpha, \]
\[ r_{t+1}^c = \alpha K_{t+1}^{\alpha-1} - \delta. \]  

(3.3.8)  
(3.3.9)

The investment-production specification resembles the formulation of the noncorporate and corporate sector by Quadrini (2000). However, in Quadrini (2000), the two sectors are parallel, while in our model, the two processes are sequential. The purpose of our set-up of investment and production is to decompose the investment returns. Specifically, the return on investment is determined by three components: the risk component, the type component (the ability or productivity of the investor), and the scarcity component. In the model, the risk component is captured by the random noise of \( \varepsilon_i^c \). The scarcity of capital is reflected by the capital return \( A_2^c \). Because of the resource constraints, especially the constraint of the total labor force, capital has diminishing marginal returns. Finally, the H-type realizes on-average higher return on investment, which is captured by the type-variable \( \eta_i^c \). Therefore, with our set-up of investment and production, different components of the investment returns can be analyzed separately.

### 3.3.4 Saving and Borrowing

Saving and borrowing are conducted through a perfectly-competitive and risk-neutral financial intermediary. Both saving and borrowing have a time duration of one period. Saving is always risk-free with an interest rate of \( r_f^c \), where \( r_f^c \) is determined by the market-clearing condition of the credit market. The interest rate of borrowing \( \pi^c \) is determined endogenously by the bank, so that the bank makes zero expected profits.

It is assumed that the bank knows the type of the household when it makes its lending decision. Similar to the two-period model, borrower has limited liability, in the sense that the agent’s borrowing is only secured by her investment. The labor income, on the other hand, cannot be used for debt repayment. Suppose that a household borrows \( s_t^c < 0 \) to finance part of her investment, and her investment forms the capital \( k_{t+1}^c \). Next period, the household’s payable to the bank is

\[ u_{t+1}^c := -(1 + \pi^c) s_t^c. \]  

(3.3.10)

---

11 We can imagine that there are many identical risk-neutral banks operating in the credit market.

12 Similar to the assumption in the two-period model, one can imagine that each agent invests by setting up a limited-liability company. Even in the worst scenario, the bank can only grab the company’s asset.
However, the maximal commitment of the household to the borrowing is

\[ v_t^i := (1 - \varphi)(1 + r_t^c)k_t^i. \] (3.3.11)

If \( u_{t+1}^i > v_{t+1}^i \), the household losses all her capital and returns; and the bank can only collect \( v_{t+1}^i \). An interpretation of this assumption is that when \( u_{t+1}^i \) is larger than \( v_{t+1}^i \), household would be forced to liquidate her capital, and the amount of \( \varphi(1 + r_{t+1}^c)k_{t+1}^i \) would be wasted on the lawsuit and the liquidation process. In this sense, \( \varphi \in (0, 1) \) is a measure of the bankruptcy deadweight loss.\(^{13}\) To summarize, let \( h_{t+1}^i \) be the household’s resource outflow due to borrowing \( s_t^i < 0 \),

\[
h_{t+1}^i = \begin{cases} 
-(1 + \pi_t^i)s_t^i & \text{if } (1 + \pi_t^i)s_t^i \leq (1 - \varphi)(1 + r_{t+1}^c)k_{t+1}^i; \\
(1 + r_{t+1}^c)k_{t+1}^i & \text{otherwise.}
\end{cases}
\] (3.3.12)

At the same time, define \( b_{t+1}^i \) as the debt repayment that the bank receives,

\[
b_{t+1}^i = \min \left\{ -(1 + \pi_t^i)s_t^i, (1 - \varphi)(1 + r_{t+1}^c)k_{t+1}^i \right\}. \] (3.3.13)

Borrowing incurs an intermediation cost, while saving is cost-free. We assume that the intermediation cost is borne by the borrower, and is proportional to the amount of borrowing. Define

\[
L_t^i := -\frac{s_t^i}{(1 - \varphi)(1 + r_{t+1}^c) \exp(\eta_t^i)(t_t^i)^\nu}
\]

as a measure of leverage of the investment, the intermediation cost \( \kappa_t^i \) writes

\[
\kappa_t^i = -\theta(L_t^i)^\zeta s_t^i, \quad s_t^i < 0, \quad L_t^i > 0.
\] (3.3.14)

The term \((L_t^i)^\zeta\) is a technical term, with \( \zeta \) being a very small positive number. The existence of \((L_t^i)^\zeta\) doesn’t affect our quantitative results, but it makes the household’s next-period state a continuously differentiable function of the current choice variables, which facilitates our

---

\(^{13}\) A more realistic assumption would be that the household liquidates when \( u_{t+1}^i > (1 - \varphi^a)(1 + r_{t+1}^c)k_{t+1}^i =: \tilde{v}_{t+1}^i \); and when liquidates, the amount of \( \varphi(1 + r_{t+1}^c)k_{t+1}^i \) is wasted, where \( 0 < \varphi^b < \varphi^a < 1 \). Under this assumption, when \( u_{t+1}^i > \tilde{v}_{t+1}^i \), it would be optimal for the bank to “force” the household to liquidate. Here, \( \varphi^b \) would be the deadweight loss of bankruptcy. However, since bankruptcy is not a focus of this chapter, we would like to make a little sacrifice on this part for the simplicity of the model by assuming that \( \varphi^a = \varphi^b =: \varphi \). Under this simplified assumption, the bank is indifferent to the liquidation of the household when \( u_{t+1}^i > v_{t+1}^i \). The liquidation is in this case only mechanical.
numerical solution to the model (see discussion below). The parameter $\theta > 0$ measures the level of the intermediation cost.

### 3.3.5 Timing of Events

The timing of events of the model is summarized in Figure 3.5. The household $i$ enters period $t$ with capital $k^i_t$ and the saving (or borrowing) from the last period $s^i_{t-1}$. The representative corporation produces at the beginning of period $t$. The wage $w_t$ as well as the capital return $r^c_t$ are realized. At this point of time, we can define “cash-on-hand” $a^i_t$, which refers to the resources that the household $i$ possesses before making current-period consumption, investment and saving/borrowing decisions. Namely,

$$a^i_t = \begin{cases} 
  w_t + (1 + r^c_t)k^i_t + (1 + r^f_t)s^i_{t-1} & \text{if } s^i_{t-1} \geq 0; \\
  w_t + (1 + r^c_t)k^i_t - h^i_t & \text{if } s^i_{t-1} < 0.
\end{cases}$$ (3.3.15)

The household makes her decisions on consumption, investment and borrowing, before she learns her current-period type $\tau^i_t$. $\varepsilon^i_t$, however, is realized after investment has been made. At the end of period $t$, the next-period productive capital $k^i_{t+1}$ is formed.

Because of the existence of the intermediation cost, households do not simultaneously borrow and save. Define $n^i_t := a^i_t - c^i_t - \hat{i}^i_t$ as the household’s residual resources after

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Figure 3.5: Timing of events.
consumption and investment. If \( n_i^t < 0 \), she has to borrow \( s_i^t = n_i^t - \kappa_i^t \). The intermediation cost is modelled as additional borrowing that has to be borne by the household, which is similar to the simplified model. For a more general representation of \( s_i^t \) that covers both saving and borrowing, we define \( I_i^t \) as an indicator, which takes the value of 1 if \( n_i^t < 0 \) and 0 otherwise. \( s_i^t \) can be represented as

\[
s_i^t = d_i^t - c_i^t - i_i^t - \kappa_i^t I_i^t. \tag{3.3.16}
\]

Notice that \( s_i^t \geq 0(< 0) \) if and only if \( n_i^t \geq 0(< 0) \). Finally, in the model, it is assumed that when the bank makes lending decisions, it observes the household’s type \( \eta_i^t \), but does not know the realization of her \( e_i^t \).

### 3.3.6 Borrowing Interest Rate

The bank pays the risk-free interest to the savers, and charges interest from the borrowers, and makes zero expected profit. For the household who borrows \( s_i^t < 0 \), the bank sets the borrowing interest rate \( \pi_i^t \) according to

\[
(1 + r_i^t) s_i^t + \int_{-\infty}^{\infty} b_{i+1} f(e_i^t) \, de_i^t = 0. \tag{3.3.17}
\]

The first term is the cost to the bank, which is the gross risk-free interest paid by the bank to the savers. The second term is the expected gross lending interest received by the bank. Equation (3.3.17) defines a non-linear equation for \( \pi_i^t \) for any given triplet \((s_i^t, i_i^t, \eta_i^t)\).

Proposition 3.3.2 states that the borrowing interest rate \( \pi_i^t \) is finite when \( L_i^t < (1 + r_i^t)^{-1} \). As it turns out, \( \pi_i^t \) defined in (3.3.17) can be represented as a nonlinear equation with respect to \( L_i^t \), as is shown in the proposition below.

**Proposition 3.3.1.** When \( i_i^t > 0, s_i^t < 0 \) and \( 0 < L_i^t < (1 + r_i^t)^{-1} \), \( \pi_i^t \) defined by the equation (3.3.17) is equivalent to the one defined by the following non-linear equation:

\[
1 + r_i^t = (1 + \pi_i^t) \left[ 1 - \Phi \left( \frac{\log \left( (1 + \pi_i^t) L_i^t \right) + \sigma^2/2}{\sigma} \right) \right] + (L_i^t)^{-1} \Phi \left( \frac{\log \left( (1 + \pi_i^t) L_i^t \right) - \sigma^2/2}{\sigma} \right), \tag{3.3.18}
\]

where \( \Phi(\cdot) \) is the cumulative density function (CDF) of the standard normal distribution.
Proof see appendix. ■

Given \( s_i^l, i_i^l \) and \( \pi_i^l \), Proposition 3.3.1 allows us to obtain \( \pi_i^l \) conveniently by solving the non-linear equation (3.3.18). This significantly simplifies our numerical solution to the model. To interpret equation (3.3.18), let’s re-arrange the equation into the following equivalent form:

\[
1 + r_i^f = (1 + \pi_i^l) \cdot \mathbb{P} \left[ e_i^l \geq \log \left( \left( 1 + \pi_i^l \right) L_i^l \right) \right] + (L_i^l)^{-1} \cdot \mathbb{P} \left[ e_i^l < \log \left( \left( 1 + \pi_i^l \right) L_i^l \right) - \sigma^2 \right]. \tag{3.3.19}
\]

The probability \( \mathbb{P} \left[ e_i^l \geq \log \left( \left( 1 + \pi_i^l \right) L_i^l \right) \right] \) is the probability of full repayment;\(^{14}\) the probability \( \mathbb{P} \left[ e_i^l < \log \left( \left( 1 + \pi_i^l \right) L_i^l \right) - \sigma^2 \right] \) can be regarded as the equivalent probability of insolvency, and the term \( (L_i^l)^{-1} \) can be deemed as the equivalent recovery rate when the borrower becomes insolvent.\(^ {15}\) The lender chooses the interest rate \( \pi_i^l \) such that the expected repayment of one dollar of lending equals the risk-free (gross) return to the dollar.

It can be shown that \( \pi_i^l \) is a well-defined monotonically increasing function of \( L_i^l \). A unique finite solution to \( \pi_i^l \) is guaranteed for any \( L_i^l \in (0, (1 + r_i^f)^{-1}) \).

**PROPOSITION 3.3.2.** When \( i_i^l > 0 \) and \( s_i^l < 0 \), \( \pi_i^l \) is a well-defined function of \( L_i^l \). We represent the function as \( \pi_i^l = \Pi(L_i^l) : \mathcal{L}_i \to \mathcal{R}_i \), where \( \mathcal{L}_i := (0, (1 + r_i^f)^{-1}) \) and \( \mathcal{R}_i = (r_i^f, \infty) \). Further, when \( L_i^l \in \mathcal{L}_i \), the first-order derivative of \( \pi_i^l \) with respect to \( L_i^l \) is

\[
\frac{d\pi_i^l}{dL_i^l} = (L_i^l)^{-2} \frac{\Phi \left( \frac{\log \left( \left( 1 + \pi_i^l \right) L_i^l \right) - \sigma^2/2}{\sigma} \right)}{1 - \Phi \left( \frac{\log \left( \left( 1 + \pi_i^l \right) L_i^l \right) + \sigma^2/2}{\sigma} \right)} > 0. \tag{3.3.20}
\]

Proof see appendix. ■

Proposition 3.3.2 provides the upper bound of leverage \( L_i^l \) such that \( \pi_i^l \) is finite. This upper bound implicitly defines the borrowing limit of the household. Take the definition of \( L_i^l \) to the inequality \( L_i^l < (1 + r_i^f)^{-1} \), we have

\[
s_i^l \geq \frac{(1 - \varphi)(1 + r_i^c) \exp\{\eta_i^l\}}{1 + r_i^f} \cdot (i_i^l)^\nu. \tag{3.3.21}
\]

Depending on values of parameters and variables, the borrowing limit can be very low; But because of the decreasing returns to scale in investment (\( \nu < 1 \)), such borrowing limit

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\(^{14}\) See the proof to the Proposition 3.3.1 in the Appendix 3.B.1.

\(^{15}\) “Equivalent” in the sense of transforming the continuous problem to the binary counterpart.
always exists. To prevent the borrowing limit to be too low, we require that in the stationary equilibrium,

\[
(1 - \varphi)(1 + r_{i+1}^c) \exp\{\bar{\eta}\} \frac{1}{1 + r_i^d} < 1. \tag{3.3.22}
\]

The assumption (3.3.22) is a restriction on the value of the parameter \( \varphi \). This restriction is meant to prevent even the H-type from investing purely by borrowing when \( i_t^H \geq 1 \). But because \( \nu < 1 \), investing purely by borrowing is always possible for either type when \( i_t^H \) is very small, as long as \( \varphi < 1 \). As it turns out, the condition (3.3.22) puts a very loose restriction on \( \varphi \). In our calibration, a \( \varphi \geq 0.1 \) guarantees that condition (3.3.22) is fulfilled.

### 3.3.7 Stationary Recursive Competitive Equilibrium

The household has choices over consumption \( c_i^t \), investment \( i_i^t \) and saving/borrowing \( s_i^t \). But by equation (3.3.16), for any \( a_i^t \), once the \( c_i^t \) and \( i_i^t \) is chosen, \( s_i^t \) is determined mechanically.\(^\text{16}\) Therefore, we can reduce the household’s choice variables to \( c_i^t \) and \( i_i^t \). Moreover, in a given period, the aggregate state of the economy is characterized by the risk-free interest rate \( r_i^d \) of the current period, as well as the aggregate capital \( K_{i+1} \), capital return \( r_{i+1}^c \) and wage \( w_{i+1} \) of the next period. We let \( G_i \) denote the period \( i \)’s aggregate state vector \((r_i^d, K_{i+1}, r_{i+1}^c, w_{i+1})\).

Given \( G_i \) and the household’s state \( a_i^t \) and \( \tau_i^t \), the household’s next-period cash-on-hand can be represented as a function of her choices \( c_i^t, i_i^t \) and of random realization of \( c_i^t, i_i^t \)\(^\text{17}\)

\[
a_{i+1} = \begin{cases} \frac{w_{i+1} + (1 + r_{i+1}^c) \exp\{\eta_i^t + \epsilon_i^t\} (i_i^t)^\nu + (1 + r_i^d) (a_i^t - c_i^t - i_i^t)}{1 + w_{i+1} r_i^d} & \text{if } s_i^t \geq 0; \\ \frac{w_{i+1} + (1 + r_{i+1}^c) \exp\{\eta_i^t + \epsilon_i^t\} (i_i^t)^\nu + (1 + r_i^d) (a_i^t - c_i^t - i_i^t - \kappa_i^t)}{1 + w_{i+1} r_i^d} & \text{if } s_i^t < 0 \text{ and solvent}; \\ \frac{w_{i+1} + (1 + r_{i+1}^c) \exp\{\eta_i^t + \epsilon_i^t\} (i_i^t)^\nu + (1 + r_i^d) (a_i^t - c_i^t - i_i^t - \kappa_i^t)}{1 + w_{i+1} r_i^d} & \text{if } s_i^t < 0 \text{ and insolvent}. \end{cases} \tag{3.3.23}
\]

\(^\text{16}\) It can be shown that given \( G_i \), \( a_i^t \) and \( \tau_i^t \), both \( L_i^c \) and \( \kappa_i^t \) are well-defined monotonic functions of \((c_i^t, i_i^t)\). See Appendix 3.C.1.

\(^\text{17}\) From quation (3.C.17), one can see how the inclusion of the technical term \((L_i^c)^\kappa\) in the formulation of the intermediation cost (3.3.14) “smooth out” the next-period state. With \((L_i^c)^\kappa\) in \( \kappa_i^t \), \( a_i^t \) is continuously differentiable in \( c_i^t \) and \( i_i^t \), especially when \((c_i^t + i_i^t)\) crosses the threshold of \( a_i^t \). This is because when \((c_i^t + i_i^t) \uparrow a_i^t, (L_i^c)^\kappa \to 0 \) and hence \( \kappa_i^t \to 0 \). The continuously differentiable state space allows us to explicitly formulate the household’s first-order conditions, which facilitates our numerical solution to the model. For a rigorous discussion of the continuous-differentiability of the state space, as well as the first-order conditions of the household’s optimization problem, please refer to Appendix 3.C.
Next, we formulate the household’s problem recursively. Following the convention of the literature, we drop the subscript \( t \), and we use the prime-notation to represent the variables in the period \( (t + 1) \). The recursive problem of the household writes

\[
V(0^\prime, g^\prime; G^\prime) = \max_{(c^\prime, H^\prime) \in D_+} \left[ (1 - \beta)(c^\prime)^{1-\mu} + \beta \left( \lambda_{\eta H} \int_{e^\prime} \mathbb{V}((a^\prime)^\gamma, H; G^\prime)^{1-\gamma} f(e^\prime) \, de^\prime + \lambda_{\eta L} \int_{e^\prime} \mathbb{V}((a^\prime)^\gamma, L; G^\prime)^{1-\gamma} f(e^\prime) \, de^\prime \right) \right]^{1-\mu},
\]

subject to (3.3.23),

where \( D_+ \) is the range of \((e^\prime, i^\prime)\) with \( c^\prime > 0 \) and \( i^\prime > 0 \) that fulfill inequality (3.3.21), ensuring a finite interest rate. The household forms rational expectation on the next period’s aggregate state \( G^\prime \) when solving the individual optimization problem (3.3.24). The individual optimal decisions give rise to the distribution of saving/borrowing and future capital over households. These distributions, along with the type distribution over households, in turn determine the aggregate state \( G^\prime \), on which the individual household’s optimal decision is based. In the recursive equilibrium, the realization of the the aggregate state \( G^\prime \) is consistent with the household’s rational expectation.

**DEFINITION 3.3.3. (Stationary Recursive Competitive Equilibrium)** A recursive competitive equilibrium is defined as: (1) The household’s individual value function \( V(a^\prime, \tau^\prime; G) \); (2) The household’s individual decision rules \( c^\prime = C(a^\prime, \tau^\prime; G) \) and \( i^\prime = I(a^\prime, \tau^\prime; G) \) for consumption and investment, respectively; (3) Law of motion of the aggregate state \( G^\prime = g(G) \), such that

1. Given the rational expectation of \( G^\prime \), the household’s individual value function and decision rules solve the optimization problem (3.3.24);

2. Given the household’s individual decision rules, \( G^\prime = g(G) \) is determined as follows:
   (1) \( K' \) is the sum of individual capital holdings, equation (3.3.6);
   (2) \( K' \) determines \( w' \) and \( (r^\prime)^\gamma \) according to equations (3.3.8) and (3.3.9);
   (3) The risk free interest rate \( r^\prime \) equates aggregate saving and borrowing,

\[
\int_0^1 s^\prime \, di = 0;
\]

3. The next period’s aggregate state \( G^\prime \) determined in 2. is consistent with the household’s rational expectation in 1.

A stationary recursive competitive equilibrium is the recursive competitive equilibrium with invariant law of motion \( g(G) = G \).
3.4 Calibration

As a benchmark, the dynamic model is calibrated annually to the US economy from 1980 to 2007. The period is characterized by relatively stable financial-market conditions. In contrast, the 1970s were characterized by the Great Inflation, and the International Financial Crisis and the subsequent Great Recession struck at the end of the year 2007. In the model, there are 13 parameters to be calibrated: $(\beta, \mu, \gamma, \sigma, \bar{\eta}, \nu, \theta, \zeta, \lambda_{HL}, \lambda_{LH}, \alpha, \delta)$. Some parameters are calibrated outside the model, and others are calibrated inside. Table 3.1 summarizes the benchmark calibration.

Table 3.1: Benchmark calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Outside the Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ Time discount factor</td>
<td>0.96</td>
<td>(conventional)</td>
</tr>
<tr>
<td>$\alpha$ Capital share of production</td>
<td>0.36</td>
<td>(conventional)</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate of capital</td>
<td>0.08</td>
<td>(conventional)</td>
</tr>
<tr>
<td>$\lambda_{HL}$ Poisson switching rate</td>
<td>0.10</td>
<td>Cao and Luo (2017)</td>
</tr>
<tr>
<td>$\lambda_{LH}$ Poisson switching rate</td>
<td>0.0111</td>
<td>10% share of H-types among household</td>
</tr>
<tr>
<td>$\sigma$ risk of investment</td>
<td>0.20</td>
<td>Angeletos (2007), Benhabib et al. (2011)</td>
</tr>
<tr>
<td>$\zeta$ technical term</td>
<td>$10^{-4}$</td>
<td>(very small positive number)</td>
</tr>
<tr>
<td>Calibrated Inside the Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$ inverse of EIS</td>
<td>0.50</td>
<td>risk-free rate</td>
</tr>
<tr>
<td>$\gamma$ relative risk aversion</td>
<td>1.50</td>
<td>risk premium</td>
</tr>
<tr>
<td>$\bar{\eta}$ return-premium of H-type</td>
<td>0.06</td>
<td>top-10% wealth share</td>
</tr>
<tr>
<td>$\nu$ Lucas’ span-of-control</td>
<td>0.9945</td>
<td>top-1% wealth share</td>
</tr>
<tr>
<td>$\theta$ intermediation cost</td>
<td>0.025</td>
<td>corporate debt-to-GDP</td>
</tr>
<tr>
<td>$\varphi$ bankruptcy dead-weight loss</td>
<td>0.10</td>
<td>income Gini-coefficient</td>
</tr>
</tbody>
</table>

3.4.1 Externally Calibrated Parameters

The “H-type” is interpreted as the best 10% investors (households). In other words, the unconditional proportion of the H-type is $p = 0.10$. Following Cao and Luo (2017), we

\[p = 0.10\]
set $\lambda_{HL} = 0.10$ such that the average duration of being H-type is 10 years. By the property of the Markov chain, $p := \frac{\lambda_{HL}}{\lambda_{HL} + \lambda_{LH}} = 0.10$, this implies that $\lambda_{LH} = \frac{p}{1-p} \lambda_{HL} = 0.0111$.

Two sources of data can be used to calibrate the risk of investment (the volatility of the capital formation) $\sigma$. The first is the volatility of returns on public equity. Between 1980 and 2007, the annual real returns of the S&P 500 have a volatility of 17.52%. However, in our model, capital investment goes beyond public equity. Regarding private equity, Moskowitz and Vissing-Jørgensen (2002) find that private equity does not offer higher returns than public equity, but the owners of the private equity (entrepreneurs) poorly diversify their portfolio. The concentrated investment on private equity implies a higher risk. However, there is no reliable measure on such risk. In his baseline calibration, Angeletos (2007) calibrates the volatility to the entrepreneurial returns to be 20%. Benhabib et al. (2011) adopt the number of 20% to be their calibration for the volatility of overall investment (namely, public and private equity combined). We follow Angeletos (2007) and Benhabib et al. (2011) and calibrate $\sigma$ to 20%.

The rest of the parameters are calibrated as follows. The subjective discount factor $\beta$, the share of capital income in production $\alpha$, and the annual capital depreciation rate $\delta$ are set to conventional value of 0.96, 0.36 and 0.08, respectively. These values are standard in the literature, and are identical to the calibration in Aiyagari (1994) and Angeletos (2007). The very small but positive technical term $\zeta$ is set to be $1 \times 10^{-4}$.

### 3.4.2 Internally Calibrated Parameters

The remaining six parameters are calibrated internally: $(\mu, \gamma, \bar{\eta}, \nu, \theta, \phi)$. $\mu$ is the inverse of the elasticity of intertemporal substitution (EIS). The empirical estimations to the EIS differ wildly, ranging from 0.1 (Hall; 1988) to 2 (Gruber; 2013). Calibrated quantitative works therefore use widely different values of the EIS. For example, Barro (2009) and Colacito and Croce (2011) calibrate the EIS to 2; Angeletos (2007) applies the value of 1 in his benchmark calibration; and in Guvenen (2009) the calibrated EIS is 0.1–0.3. In this chapter, $\mu$ is calibrated internally to match the real risk-free rate of 2.03%.

The range of estimated values of the relative risk aversion $\gamma$ is also wide. The estimates for $\gamma$ range from 0-1 (Hansen and Singleton; 1982, 1984) to 40-50 (Cochrane and Hansen; 1992). Estimates based on labor-market data usually imply a lower upper-bound of the parameter. For example, Chetty (2006) shows that the empirical evidence on the effects of wage changes to labor supply imply that $\gamma < 2$. On the other hand, empirical work using data of the financial market leads to higher estimates, such as Bliss and Panigirtzoglou (2005), who estimate $\gamma$ to be between 3.37 and 9.52, depending on the forecast horizon. Given such

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19 See Guvenen (2006) and Havranek et al. (2015) for the summary of recent published works on the estimation of the EIS. Havránek (2015) offers an explanation on the conflicting estimation results of the EIS.
Chapter 3. Intermediation Cost, Credit Expansion and Inequality

Table 3.2: Moments of the benchmark calibration.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Model (benchmark)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Targeted Statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-free rate $r^f$ (%)</td>
<td>2.03</td>
<td>2.05</td>
</tr>
<tr>
<td>Average return on investment (ROI, %)</td>
<td>8.33</td>
<td>8.65</td>
</tr>
<tr>
<td>Average risk premium (%)</td>
<td>6.30</td>
<td>6.60</td>
</tr>
<tr>
<td>(Corporate) Debt-to-GDP</td>
<td>0.59</td>
<td>0.57</td>
</tr>
<tr>
<td>Top-10% wealth share</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Top-1% wealth share</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>Income Gini-coefficient</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Untargeted Statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td>Wealth Gini-coefficient</td>
<td>0.79</td>
<td>0.69</td>
</tr>
</tbody>
</table>

large variety of empirical estimates of $\gamma$, we choose to calibrate the parameter internally. The $\gamma$ is calibrated to target the average risk premium of the risky asset of 6.30% during 1980 and 2007.

The parameters of $\overline{\eta}$ (return premium of the H-type) and the $\nu$ (the Lucas’ span-of-control) are calibrated to target the wealth distribution. To be specific, $\overline{\eta}$ is set to target the top 10% wealth share in the US, which averages 65% during 1980 and 2007; and $\nu$ is calibrated to target the top 1% wealth share of 29% during the same period. The bankruptcy deadweight loss $\varphi$ affects household’s willingness and ability to leverage. We will see in the next section that, in the model, leverage exaggerates the capital-income risk, which drives the income inequality. In this sense, we calibrate $\varphi$ to the average income Gini-coefficient of 0.48.

There is no direct indicator in the data that leads to a precise calibration of the intermediation cost of borrowing $\theta$. To obtain an idea about the range of $\theta$, we calculate the average prime-lending spread in the United States. The average of such spread between 1980 and 2007 is 2.75%. However, even the prime borrowers bear some default risk. Therefore, $\theta$ should be slightly lower than 2.75%. In our model, we choose the value of $\theta = 2.50\%$ to match the average debt-to-GDP ratio of the non-financial corporate sector of 59%.

Table 3.2 compares some key data moments to the moments generated by our model. In general, we match our calibration targets well. Moreover, the capital-output ratio of
the benchmark model is also in line with the data,\textsuperscript{20} although we don’t target this statistic. However, the wealth Gini-coefficient generated by the model is lower than what the data suggest, because our model precludes negative wealth.

3.5 Results

3.5.1 Household’s Decisions

Individual household’s policies of investment and saving as functions of cash-on-hand are reported in Figure 3.6. The baseline considers $\theta = 2.5\%$. To compare, we include the counterfactual policies of the household when $\theta$ is $2\%$ and $3\%$.\textsuperscript{21}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{policy_diagram.png}
\caption{Policies of the individual household.}
\end{figure}

\textbf{Notes:} The dotted curves are policies for $\theta = 2\%$; the solid curves are policies when $\theta = 2.5\%$; and the dashed curves represent the policies when $\theta = 3\%$.

\textsuperscript{20} To obtain the capital-output ratio in the data, we use the total tangible fixed assets as our measure of capital stock. See BEA Fixed Assets Table 1.1.

\textsuperscript{21} In Subsection 3.5.2, we show that when $\theta$ drops from $3\%$ to $2\%$, credit-output ratio in the model expands from 50.02\% to 66.30\%. Such expansion of the credit-output ratio roughly matches the movement of the US corporate debt-to-GDP ratio during the calibration period.
In any state, the H-type invests more than the L-type; and consumes less than the L-type. In the model, the rich L-type household saves; while the H-type only saves when her cash-on-hand is very large.\textsuperscript{22} The savings of the rich L-types finance the borrowings of the H-types. But interestingly, when \( a^i \) is small enough, the L-type also borrows to invest. This is because the lack of labor-income risk provides a consumption insurance to the household. In the model, household’s cash-on-hand cannot drop below the stationary wage \( w \). The lower bound on the cash-on-hand limits the marginal cost of insolvency; but because of the concavity of the preferences, the marginal benefit of leveraged investment gets higher when the household becomes poorer. Thus, the poor L-type also leverages.\textsuperscript{23} On the other hand, the H-type de-leverages when she becomes richer. As a result, her borrowing interest rate also decreases. The reason behind is straightforward. With the decreasing marginal utility from future consumption, the subjective marginal return from the leveraged investment drops when \( a^i \) increases. Hence leverage becomes less attractive to the richer H-type.

In Figure 3.6, we also plot the counterfactual policies when \( \theta = 3\% \) and 2\%. When it comes to the counterfactuals of consumption-, saving- and leverage-policies, the dynamic model has similar implications as the two-period model. The decreasing intermediation cost encourages more leveraged investment of the H-type. The interest rate of the H-type’s borrowing also rises, due to the increasing leverage. Later in Subsection 3.5.2 we will show that, in general equilibrium, a lower \( \theta \) decreases the capital return \( r^c \) and increases the risk-free rate \( r^f \) (see Table 3.3). On the individual level, the lower \( r^c \) and higher \( r^f \) discourages the (not-too-poor) L-type from investing, and encourages the rich L-type to save.\textsuperscript{24}

In Figure 3.7, we highlight the details of the value and policy functions when \( a \) is small and when \( a \) is large. First, the values of the household shown on the top-left panel indicate that the reduction in \( \theta \) is beneficial to all household, except for the rich H-type. The latter group of household suffers from the decrease in \( r^c \); while with the accumulation of wealth, the benefit of easier credit is tempered with reduced motivation to leverage. On the other hand, the rich L-type still benefits from the reduction in \( \theta \), because the saving’s interest rate \( r^f \) rises and the rich L-type allocates significant proportion of her wealth in the risk-free savings. Second, decreasing \( \theta \) encourages the poor H-type to reduce consumption; but for other households, the lower \( \theta \) is associated with higher consumption (see the bottom-left

\textsuperscript{22} With our baseline calibration (\( \theta = 2.5\% \)), H-type starts to save when \( a^i > 5000 \). In the stationary equilibrium, the probability of any household accumulating such large \( a^i \) is negligible.

\textsuperscript{23} Although the poor L-type borrows, the borrowing amount is rather small compared with the H-type, both at the individual level (as can be see from the corresponding graphs in the Figure 3.6 and Figure 3.7) and in aggregate. The high leverage of the very poor L-type shown in the bottom-left panel of the Figure 3.6 is due to the very small amount of investment. In aggregate, the borrowing of the poor L-type represents no more than 5\% of the total borrowing.

\textsuperscript{24} For the poor L-type, the reduction in \( \theta \) encourages her to leverage and to invest more. See Figure 3.7. This is because lower \( \theta \) further increases the marginal benefit of leveraged investment.
panel of Figure 3.7). This implies that the substitution effect brought by lower $\theta$ dominates its income effect only for the poor H-type. Last but not least, we look at the investment and saving/borrowing policies plotted on the right two panels of the same figure. Here, the decrease in $\theta$ encourages the H-type to leverage more to invest. However, as has been discussed above, when the L-type is poor, she also leverages. With lower $\theta$, poor L-type also leverages more, although the amount of borrowing by the poor L-type is very small. The rich L-type, on the other hand, rebalances her portfolio from the risky investment toward safe savings, when she faces lower $\theta$.

In summary, lower intermediation cost encourages the poor H-type to reduce consumption and increase leverage to invest more. When the H-type gets richer, the income effect of lower $\theta$ starts to dominate its substitution effect, making her consume more than when $\theta$ is higher. But in any state, the H-type’s leverage and investment are higher with lower $\theta$. The reduction in $\theta$ improves the subjective value of the poor H-type, but depresses the value of
the rich H-type. The rich L-type, at the same time, rebalances her portfolio toward risk-free savings. Her consumption is higher and the subjective value is improved with lower $\theta$.

### 3.5.2 General Equilibrium

In Table 3.3, we summarize the effects of $\theta$ on the general equilibrium. As a counterfactual experiment, we allow $\theta$ to vary from 3.50% to 1.50%, while keeping the values of other parameters at the benchmark. The table suggests that when $\theta$ decreases from 3.00% to 2.00%, the credit-output ratio increases from 50.02% to 66.30%. To compare, the US corporate debt-to-GDP ratio was 51.38% in the year 1980, and 67.98% in 2007. Therefore, the expansion of the credit-output ratio in the model when $\theta$ changes from 3.00% to 2.00% roughly matches the variation in the US corporate debt-to-GDP ratio during 1980–2007. Such change in $\theta$ is roughly in line with our empirical finding in the previous chapter, where we have shown that the interest expense over book value of debt in the US corporate sector has dropped by 0.78-0.79 percentage point during 1983Q1–2007Q4.

Table 3.3 shows that, when $\theta$ decreases, the credit-output ratio rises substantially, while the change in the capital-output ratio is less substantial. This can be explained as follows. The decrease in $\theta$ propels the H-type to leverage more to invest, which drives up the aggregate capital stock $K$ and lowers capital return $r^C$. The fall in $r^C$ induces the rich L-type to reduce her investment in risky capital and put more resources into savings. Therefore, the credit market sees a “simultaneous” credit expansion from both supply side (rich L-types) and demand side (H-types), which drives up the credit-output ratio substantially. In the capital market, the additional investment by the H-type is dampened by the reduction in investment by the rich L-type. This explains why the capital expansion is moderate compared to the credit expansion.

Since the additional demand for credit by the H-type is largely met by the additional supply by the rich L-type, the price of the credit $r^F$ does not change much. One sees from Table 3.3 that when $\theta$ decreases, $r^F$ increases quite moderately. Similar to the two-period model, the H-type is higher motivated to invest than the L-type. Namely, if each is given one unit of additional resource, the H-type would distribute larger proportion of the additional dollar into investment than the L-type. In this case, the response of the H-type dominates the response of the L-type. In the credit market, it implies that the demand-effect (by H-type) surpasses the supply-effect (by rich L-type). Hence we see an increase in the risk-free rate (however moderately) when $\theta$ decreases. In the capital market, the additional investment by the H-type dominates the reduction in investment by the L-type, implying larger capital-output ratio and lower $r^C$.

\footnote{The table includes the case of $\theta = 3.00\%$ and $\theta = 2.00\%$, which are cases for which we plot counterfactual policies in Figure 3.6, 3.7, 3.8 and 3.9.}
Table 3.3: Effects of *cateris paribus* change in $\theta$ on returns, aggregate capital and credit.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$r^f$</th>
<th>$r^c$</th>
<th>avg.ROI</th>
<th>capital-output ($K/Y$)</th>
<th>credit-output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50</td>
<td>2.00</td>
<td>5.42</td>
<td>8.37</td>
<td>268.18 $(-1.32%)$</td>
<td>42.89 $(-25.44%)$</td>
</tr>
<tr>
<td>3.25</td>
<td>2.01</td>
<td>5.38</td>
<td>8.44</td>
<td>269.03 $(-1.02%)$</td>
<td>46.39 $(-19.36%)$</td>
</tr>
<tr>
<td>3.00</td>
<td>2.03</td>
<td>5.34</td>
<td>8.51</td>
<td>269.84 $(-0.72%)$</td>
<td>50.02 $(-13.05%)$</td>
</tr>
<tr>
<td>2.75</td>
<td>2.04</td>
<td>5.29</td>
<td>8.58</td>
<td>270.85 $(-0.34%)$</td>
<td>53.82 $(-6.45%)$</td>
</tr>
<tr>
<td><strong>2.50</strong></td>
<td><strong>2.05</strong></td>
<td><strong>5.24</strong></td>
<td><strong>8.65</strong></td>
<td><strong>271.81</strong></td>
<td><strong>57.53</strong></td>
</tr>
<tr>
<td>2.25</td>
<td>2.06</td>
<td>5.19</td>
<td>8.71</td>
<td>272.92 $(+0.40%)$</td>
<td>61.37 $(+6.67%)$</td>
</tr>
<tr>
<td>2.00</td>
<td>2.09</td>
<td>5.14</td>
<td>8.77</td>
<td>273.92 $(+0.77%)$</td>
<td>66.30 $(+15.24%)$</td>
</tr>
<tr>
<td>1.75</td>
<td>2.12</td>
<td>5.08</td>
<td>8.82</td>
<td>275.24 $(+1.26%)$</td>
<td>72.18 $(+25.47%)$</td>
</tr>
<tr>
<td>1.50</td>
<td>2.19</td>
<td>5.03</td>
<td>8.83</td>
<td>276.28 $(+1.64%)$</td>
<td>79.87 $(+38.83%)$</td>
</tr>
</tbody>
</table>

**Notes:** "avg.ROI" is the weighted average return on investment across households. The numbers in parentheses are percentage changes from benchmark calibration ($\theta = 2.5\%$). All other numbers are in percentage.

We now look at the capital market in specific. According to the model’s set-up discussed in Subsection 3.3.3, the return on investment (ROI) is determined by three components: (1) the risk component, (2) the type component and (3) the scarcity component (measured by $r^c$). The larger capital-output ratio reduces scarcity of capital, hence $r^c$ drops. This means that investment becomes less rewarding for the L-type. The difference between the average ROI and $r^c$ measures the type component of the investment return. One can observe in Table 3.3 that the increasing leverage brought by lower $\theta$ exaggerates such type component. Regarding the risk component, while the exaggerated type component brings higher average returns to the H-type, it also introduces higher capital-income risk among H-types. This is due to two reasons. First, higher leverage translates to higher risk of insolvency; and second, leverage amplifies the profits and losses of investment. As will be discussed in the next subsection, the higher capital-income risk is the force that drives up inequality when $\theta$ decreases. For the L-types, however, the capital-income risk is subdued when $\theta$ is lower, because they allocate higher proportion of their wealth to risk-free savings.

### 3.5.3 Income and Wealth Distributions

Figure 3.8 plots the distribution of cash-on-hand in the stationary recursive competitive equilibrium. The distribution is skewed and has a heavy right-tail. Under the benchmark calibration, the top one-percent of household holds about 25% of the total cash-on-hand;

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26 Similar to the discussion in the two-period model, to be consistent with existing literature, we define the “capital income” as the total income from investment and risk-free savings.
while the top 10-percent household’s share of cash-on-hand is more than 60%. As in Benhabib et al. (2011, 2015), the skewness and the heavy tail of the wealth distribution is driven by the capital-income risk.

Figure 3.8 also plots the distributions when $\theta = 0.03$ and 0.02, respectively. On one hand, when $\theta$ decreases, the mode of the wealth distribution shifts to the left, implying a decrease in the median wealth of household. We further see in the left two panels of Figure 3.9 that the medians of the wealth distributions decrease for both of the subgroups of H- and L-types. The decrease in the median wealth is driven by the decrease in $A^2$ and the increase in risk brought by higher leverage. The reasons for the changes in medians are different for the two types. Remember that the L-type does not save when $\theta$ is low. Therefore, the poor L-type does not benefit from an increase in $r^f$, but is harmed by the falling $r^c$. Hence, when $\theta$ decreases, poor L-type’s wealth is shrinking, which drives down the median of the L-types’ wealth distribution. For the H-types, the higher leverage implies higher risk. Among other things, there are more insolvent H-types when $\theta$ decreases. This drives down the median of the wealth distribution among the H-types.

On the other hand, the right-tail of the distribution gets thicker if $\theta$ decreases (bottom-right panel of Figure 3.8). The pattern of the tail-change is the same for both of the subgroups of the H- and L-types, too, as is shown in the right two panels of the Figure 3.9. Again, the

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27 When $\theta$ drops from 0.03 to 0.02, the insolvency rate of the H-type (measured by the percentage-share of the H-types that become insolvent for the current period) increases from 2.31% to 3.65%.
reasons behind the change in tail are different for the H- and L-types. The H-type leverages more when $\theta$ is lower. As is discussed above, higher leverage of the H-type exaggerates the type component of the investment returns and amplifies the capital-income risk. The H-types are thus able to accumulate more wealth on average, albeit with more heterogeneity. The increased heterogeneity in investment returns is accountable for the increasing thickness of the right-tail. Turning to the L-types. Most of the very rich L-types accumulate wealth when they are H-types in the previous periods.\footnote{We can see this by comparing the household’s expected next-period cash-on-hand $E((a^t)')$ against the current-period state $a^t$. $E((a^t)') > a^t$ implies that the household is accumulating wealth, otherwise she is reducing wealth. Remember that $E((a^t)')$ is a monotonically increasing function of $a^t$. For $t^i = H, L$, we can solve for the equation $E((a^t)') = a^t$. We denote the solution to the equation by $\bar{a}^H$ and $\bar{a}^L$ for the H- and L-type household, respectively. For the benchmark calibration ($\theta = 2.5\%$), $\bar{a}^L = 2.87$ while $\bar{a}^H > 5000$. Therefore, only through being H-types at some time, can a household becomes very rich.} Therefore, the thicker tail of the wealth distribution among the H-types leads to the thicker tail of that among the L-types, through the Markovian transition between types. Moreover, the increasing $A^b$ brought by the decreasing intermediation cost also helps to preserve the rich L-types’ wealth better.

Table 3.4 documents different measures of wealth and income inequality for different values of $\theta$. The table shows that, in general, the decrease in the intermediation cost is
Table 3.4: Effects of *cateris paribus* change of $\theta$ on wealth and income distributions.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Gini</th>
<th>Gini L-type</th>
<th>top 10% share</th>
<th>top 1% share</th>
<th>Gini</th>
<th>Gini L-type</th>
</tr>
</thead>
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<td>64.75</td>
<td>63.39</td>
<td>60.38</td>
<td>24.34</td>
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<td>64.35</td>
<td>61.75</td>
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<td>35.75</td>
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<td>65.49</td>
<td>63.01</td>
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<td>48.37</td>
<td>35.07</td>
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<td>68.15</td>
<td>66.55</td>
<td>64.32</td>
<td>27.84</td>
<td>48.72</td>
<td>34.34</td>
</tr>
<tr>
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<td><strong>67.76</strong></td>
<td><strong>65.67</strong></td>
<td><strong>29.08</strong></td>
<td><strong>49.08</strong></td>
<td><strong>33.62</strong></td>
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<tr>
<td>2.25</td>
<td>70.70</td>
<td>69.02</td>
<td>67.15</td>
<td>30.27</td>
<td>49.44</td>
<td>32.89</td>
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<tr>
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<td>70.70</td>
<td>68.70</td>
<td>31.51</td>
<td>49.71</td>
<td>32.25</td>
</tr>
<tr>
<td>1.75</td>
<td>74.29</td>
<td>72.94</td>
<td>70.52</td>
<td>32.68</td>
<td>50.05</td>
<td>31.51</td>
</tr>
<tr>
<td>1.50</td>
<td>76.63</td>
<td>75.67</td>
<td>72.51</td>
<td>34.03</td>
<td>50.26</td>
<td>31.13</td>
</tr>
</tbody>
</table>

*Notes:* The household $i$’s wealth is defined as her cash-on-hand minus the stationary wage, namely, $a^i - w$. All numbers in the table are in percentage.

Associated with the rise of both income and wealth inequality. Such rise in inequity is associated with the change in the tail of the wealth distribution, as is evident from the change in the top 10% and 1% wealth share. Especially, when $\theta$ decreases from 0.03 to 0.02, the top 10% (1%) wealth share increases from 63.01% (26.63%) to 68.70% (31.51%). To compare, in the data, the share of total wealth held by the top 10% (1%) wealthiest households was 65.10% (22.54%) in 1980, and 69.79% (35.10%) in 2007. If we take $\theta = 0.03$ and 0.02 as the intermediation cost in 1980 and 2007, respectively (see Subsection 3.5.2), almost all the increase in the top 10% wealth share in the data during 1980–2007 could be explained by the decrease in $\theta$. At the same time, the change in the intermediation cost was behind 38.85% of the change in the top 1% wealth share in the data.

In Table 3.4, it is also interesting to notice that the income-Gini among the L-type households decreases when $\theta$ drops. This observation is similar to what we have observed in the two-period model, and the explanation remains the same. The rich L-type, facing increasing $r^f$ and decreasing $r^c$, holds more risk-free savings and invests less. This essentially reduces the capital-income risk among the rich L-types, which reduces the income dispersion among them.

As a matter of fact, capital-income risk can be viewed as the main driving force behind the change in the income and wealth distributions. As mentioned earlier, when the intermediation cost decreases, the capital-income risk is exaggerated for the H-type and is subdued for the rich L-type. Higher capital-income risk translates to higher income dispersion. For the L-type, the income-Gini shrinks due to the reduced risk of capital income; while for the
H-type, the opposite is happening. Larger income dispersion among H-types (as well as between the H- and L-type) dominates the concentration of income among L-types, resulting to a larger overall income inequality. As for the wealth, the story is the same for the H-type. But because wealth is an accumulation of previous income, the wealth inequality among L-types still increases due to L-types’ previous histories as the H-types, as well as a higher $r^f$ that better preserves the very rich L-types’ asset.

Feedback to the Capital and Credit Market. There is interaction between the wealth distribution and the general equilibrium. The decrease in $\theta$ leads to a larger share of very rich L-type households (see the bottom-right panel of Figure 3.9). This increases the credit supply, because the rich L-types are savers, and the richer the L-type gets, the more she saves (see the top-right panel of Figure 3.6). Hence the thicker tail of L-types’ wealth distribution makes credit even more available to the H-type. This contributes to the reason why the $r^f$ does not increase much when $\theta$ drops. The additional credit availability promotes more borrowing and investment of the H-type, which again leads to larger dispersion of wealth. Therefore, this feedback between the wealth distribution and the general equilibrium exaggerates the previously-discussed feedback-loop between the credit market and capital market.

3.5.4 Welfare

We apply the welfare criterion à la Aiyagari and McGrattan (1998), where the welfare is defined as the weighted-average values of the households. Let $\Omega^H$, $\Omega^L$ represents the welfare of the group of the H-types and the L-types, respectively. In the stationary recursive competitive equilibrium, we define

$$\Omega^H = \int_W^\infty \mathbb{V}(a^i, H) \, dH(a^i, H), \quad \text{and} \quad (3.5.1)$$

$$\Omega^L = \int_W^\infty \mathbb{V}(a^i, L) \, dH(a^i, L), \quad \text{and} \quad (3.5.2)$$

where $H(a^i, H)$ and $H(a^i, L)$ are distributions (probability densities) of cash-on-hand for the H- and L-types, respectively. The overall welfare of the economy, $\Omega$, is defined as $\Omega = p\Omega^H + (1 - p)\Omega^L$. The welfare for other subgroups of households can be defined similarly.

Table 3.5 reports the change in welfare for the overall and selected subgroups of household, when $\theta$ changes. Similar to the two-period model, the decrease in the intermediation cost improves the overall welfare. However, in the dynamic model, the H-type’s overall welfare is also improving, and is improving faster than that of the subgroup of the L-types. This is
Table 3.5: Welfare effects of *cateris paribus* change of $\theta$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>overall</th>
<th>H-type</th>
<th>L-type</th>
<th>bottom 90%</th>
<th>top 1%-10%</th>
<th>top 1%</th>
<th>top 0.1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50</td>
<td>-0.67</td>
<td>-2.63</td>
<td>-0.36</td>
<td>+0.14</td>
<td>-1.86</td>
<td>-12.70</td>
<td>-16.40</td>
</tr>
<tr>
<td>3.25</td>
<td>-0.52</td>
<td>-1.97</td>
<td>-0.28</td>
<td>+0.08</td>
<td>-1.21</td>
<td>-9.82</td>
<td>-13.45</td>
</tr>
<tr>
<td>3.00</td>
<td>-0.36</td>
<td>-1.42</td>
<td>-0.20</td>
<td>+0.04</td>
<td>-0.85</td>
<td>-6.57</td>
<td>-8.45</td>
</tr>
<tr>
<td>2.75</td>
<td>-0.19</td>
<td>-0.71</td>
<td>-0.10</td>
<td>+0.03</td>
<td>-0.52</td>
<td>-3.35</td>
<td>-3.73</td>
</tr>
<tr>
<td>2.50</td>
<td>+0.22</td>
<td>+0.88</td>
<td>+0.11</td>
<td>-0.00</td>
<td>+0.51</td>
<td>+3.30</td>
<td>+4.10</td>
</tr>
<tr>
<td>2.00</td>
<td>+0.42</td>
<td>+1.64</td>
<td>+0.22</td>
<td>-0.01</td>
<td>+0.97</td>
<td>+6.87</td>
<td>+7.42</td>
</tr>
<tr>
<td>1.75</td>
<td>+0.66</td>
<td>+2.68</td>
<td>+0.35</td>
<td>-0.08</td>
<td>+2.03</td>
<td>+10.51</td>
<td>+11.06</td>
</tr>
<tr>
<td>1.50</td>
<td>+0.88</td>
<td>+3.39</td>
<td>+0.48</td>
<td>-0.18</td>
<td>+2.92</td>
<td>14.73</td>
<td>+16.23</td>
</tr>
</tbody>
</table>

Notes: Welfare is defined by equation (3.5.1) and (3.5.2). All numbers in the table are percentage change from the benchmark ($\theta = 2.5\%$).

Different from the two-period model, since the H-type in the dynamic model is associated with higher ROI on average; while in the two-period model, the H-type make zero profits. Thus the H-type in the dynamic model benefits more from the decrease in $\theta$.

The dynamic model also allows us to assess the welfare effects for each wealth group. These are reported in the last four columns of the Table 3.5. As one can see, the decrease in $\theta$ reduces the welfare of the bottom-90% wealth group. For the top 10% wealth group, the reduction of the intermediation cost is welfare improving. The improvement of welfare is biased toward the rich household, as the percentage welfare gain of the top 0.1% is larger than that of the top 1%; and the top 1% also gains larger welfare than the top 1%-10%.

According to the definition of welfare in the equation (3.5.1) and (3.5.2), there are two factors that are behind the change of welfare: the first is the value $V(\cdot)$; and the second is the distribution $H(\cdot)$. We call the first factor the *value effect*; and the second *distribution effect*. In our model, the distribution effect dominates the value effect. For example, the top-left panel of the Figure 3.7 indicates that given $a^i$ not large enough, for either type, the household’s value increases when $\theta$ drops. Yet the bottom-90% wealth group see their welfare reducing. This is due to the fact that decreasing $\theta$ shifts the mode of the wealth distribution to the left.

---

29 With all calibrations that are considered in this chapter, H-type’s value is still decreasing in $\theta$ way after $a^i$ passes the threshold of the top-10% wealth group.

30 The decrease in the welfare for the bottom-90% of the household is mainly driven by the L-type households. With our calibration, when $\theta$ decreases, welfare barely changes for the H-type households.
### 3.6 Conclusions

In this chapter, we develop a dynamic general equilibrium framework to explore the macroeconomic consequences of the decreasing intermediation cost. We find that the reduction in the intermediation cost triggers two feedback loops. The first is between the capital and credit market, and the second is between the capital-credit market and the wealth distribution. The two feedback loops amplify the initial impact of the decreasing intermediation cost. We show that after the reduction in the intermediation cost, credit market experiences a “simultaneous” expansion from both credit-demand and credit-supply side. Because of the “simultaneous” nature, the real risk-free interest rate barely changes. In the meantime, the capital market also sees an expansion. But the extent of the capital-market expansion is less significant than the credit-market expansion. The feedback loop between the capital and credit market exaggerates the capital-income risk and increases the average returns to investment among the leveraged investors (H-types), and hence the wealth heterogeneity increases. We find that the overall inequality in income and wealth rises, albeit the income inequality among L-type households drops. In terms of welfare, the decrease in the intermediation cost improves the overall welfare, as well as the welfare of the wealthy households, although the bottom-90% households in terms of wealth see their welfare decreased.

Our studies suggest that the decrease in the intermediation cost explains three stylized facts in the US economy: (1) an expansion of the corporate debt-to-GDP ratio, (2) a secular rise in the income and wealth inequality, and (3) a secular fall in the (buying-and-holding) returns on risky assets. For the past decades, the real interest rate in the United States also saw a decreasing trend. Although we cannot capture such trend with our current framework, the real risk-free interest rate in our model barely changes when the intermediation cost decreases. There are many factors that could affect the real interest rate, such as the global saving glut, central bank’s policies, etc. We maintain that these factors, instead of the intermediation cost, are behind the recent secular decrease in the real interest rate.

There are several aspects on which our work can still be extended. One of the interesting extensions might be an inclusion of aggregate shocks. This enables an assessment of the impact of the intermediation cost on the dynamics of the economy, especially the dynamics of asset prices. Furthermore, the transitional dynamics associated with changing intermediation cost could also be explored.
Appendix 3.A  Proofs for the Two-Period Model

3.A.1  Proof to the Lemma 3.2.1

We first show that the demand-side of the credit market exists. The bank is willing to lend to the H-type and charge the interest rate \( \pi \) based on (3.2.5), if the H-type can pay back the principal plus interest rate when capital formation is successful (when \( \varepsilon = 1 \)). Namely, if \( R^c i_H \geq (1 + \pi)b \). By equation (3.2.5), this implies a bank’s lending condition of

\[
(1 - \lambda)R^c \geq (1 + r^f)(1 + \theta).
\]

(3.A.1)

This condition is weakly satisfied under (3.2.8). We therefore conclude that the demand-side of the credit market exists in equilibrium.

Next, we look at the supply side of the credit market. To start, it is obvious that borrowing to invest is not optimal for the L-type. Since the model doesn’t allow the agent to borrow and save simultaneously, L-type only borrows when she invests more than \( w \). But when she does this, she would receive zero-consumption when the capital formation fails. With the log utility, zero-consumption results to an utility of minus infinity. Therefore, any \( i_L > w \) is suboptimal. When the L-type do not borrow to invest, her optimal amount of investment is given by equation (3.2.10). If \( 0 < i_L < w \), it must be satisfied that

\[
0 < \frac{\lambda R^c}{R^c - (1 + r^f)} < 1,
\]

which translate to the condition of

\[
(1 - \lambda)R^c > 1 + r^f,
\]

(3.A.2)

which always holds given the \( R^c \) in (3.2.8). We can thus conclude that in equilibrium, L-type always allocates some of her endowment to saving. Hence the supply side of the credit thus also exists.

3.A.2  Proof to the Proposition 3.2.2

In general equilibrium, (3.2.8) holds; and the equation (3.2.17) of the proposition is just a re-arrangement of (3.2.8). We take the \( R^c \) implied by (3.2.8) to the L-type’s first-order condition (3.2.10) and obtain

\[
i_L = 1 - \frac{\lambda(1 + \pi)(1 + \theta)}{(1 + \pi)(1 + \theta) - (1 + r^f)} w,
\]

(3.A.3)
which, after taking in the interest-rate scheme (3.2.5) and some simple re-arrangement, leads to the equation (3.2.11) of the proposition. Because \( \theta(1 - \lambda) > 0 \) by the construction of the model,

\[
0 < \frac{\lambda(1 + \theta)}{\theta + \lambda} < 1,
\]

and hence \( 0 < i_L < w \). Therefore the amount of saving by the L-type is \( s = w - i_L \), which is given by the equation (3.2.14) of the proposition. The aggregate saving of the economy (from the L-type) is thus

\[
S = (1 - p)s = (1 - p) \frac{\lambda(1 + \theta)}{\theta + \lambda}w, \tag{3.A.4}
\]

which is exactly the equation (3.2.16) of the proposition.

The aggregate borrowing of the H-type is \( pb = p(1 + \theta)i_H \). In general equilibrium, credit market clears. The aggregate borrowing of the H-type must equal to \( S \), namely.

\[
p(1 + \theta)i_H = S = (1 - p) \frac{\lambda(1 + \theta)}{\theta + \lambda}w,
\]

from which we obtain

\[
i_H = \frac{1 - p}{p} \cdot \frac{\lambda}{\theta + \lambda}w, \tag{3.A.5}
\]

which is the equation (3.2.12) of the proposition; \( b = (1 + \theta)i_H \) is thus obtained as the equation (3.2.13) of the proposition. Moreover, from the equation (3.2.2) we obtain another representation of \( i_H \):

\[
i_H = \frac{K - (1 - p)(1 - \lambda)i_L}{p(1 - \lambda)}. \tag{3.A.6}
\]

Equating the two representations of the \( i_H \) in equations (3.A.5) and (3.A.6),

\[
\frac{1 - p}{p} \cdot \frac{\lambda}{\theta + \lambda}w = \frac{K - (1 - p)(1 - \lambda)i_L}{p(1 - \lambda)}, \tag{3.A.7}
\]

and taking the representation of \( i_L \) in equation (3.2.11) to (3.A.7) above and solving for \( K \), the equation (3.2.15) of the proposition is obtained. \( \blacksquare \)
3.A.3 Proof to the Proposition 3.2.3

Based on the equation (3.2.17) of the Proposition 3.2.2, in general equilibrium, we have

\[ 1 + r^f = \frac{1 - \lambda}{1 + \theta} K^{\alpha - 1}. \]  

(3.A.8)

The first-order partial derivative of \( r^f \) with respect to \( \theta \) is

\[
\frac{\partial r^f}{\partial \theta} = (1 - \lambda) \frac{(\alpha - 1)(1 + \theta)K^{\alpha - 2} \frac{\partial K}{\partial \theta} - K^{\alpha - 1}}{(1 + \theta)^2} \\
= \frac{(1 - \lambda)K^{\alpha - 2}}{(1 + \theta)^2} \left[ (\alpha - 1)(1 + \theta) \frac{\partial K}{\partial \theta} - K \right].
\]

Hence \( \frac{\partial r^f}{\partial \theta} < 0 \) if \( u := (\alpha - 1)(1 + \theta) \frac{\partial K}{\partial \theta} - K < 0 \). From the equation (3.2.15) we see that

\[
\frac{\partial K}{\partial \theta} = -(1 - p)(1 - \lambda)w \cdot \frac{\lambda^2}{(\theta + \lambda)^2},
\]

(3.A.9)

and hence

\[
u = (1 - \alpha)(1 + \theta)(1 - p)(1 - \lambda)w \cdot \frac{\lambda^2}{(\theta + \lambda)^2} - (1 - p)(1 - \lambda) \left[ 1 - \frac{\lambda \theta}{\theta + \lambda} \right] w \\
= (1 - p)(1 - \lambda)w \left[ (1 - \alpha)(1 + \theta) \cdot \frac{\lambda^2}{(\theta + \lambda)^2} + \frac{\lambda \theta}{\theta + \lambda} - 1 \right] \\
= (1 - p)(1 - \lambda)w \cdot \frac{-\alpha(1 + \theta)\lambda^2 - \theta(1 - \lambda)(\theta + 2\lambda)}{(\theta + \lambda)^2} < 0.
\]

Therefore, the \( r^f \) is an decreasing function of \( \theta \). The Proposition 3.2.3 follows.  

3.A.4 Proof to the Proposition 3.2.4

Denote by \( L \) the L-type’s objective function, and take the \( c^1_L \) and \( c^0_L \) (eq. (3.2.11), (3.2.14)) in the general equilibrium to \( L \):

\[
L = (1 - \lambda) \log c^1_L + \lambda \log c^0_L = (1 - \lambda) \log \left( (1 - \lambda) R^c w \right) + \lambda \log \left( \frac{\lambda}{\theta + \lambda} (1 - \lambda) R^c w \right).
\]
The first-order derivative of $L$ with respect to $\theta$ writes

$$\frac{\partial L}{\partial \theta} = (1 - \lambda) \frac{(1 - \lambda)w \cdot \frac{\partial R^c}{\partial \theta}}{(1 - \lambda)R^cw} + \lambda \frac{\frac{\lambda}{\theta + \lambda} \cdot \frac{\partial R^c}{\partial \theta} - \frac{\partial R^c}{\partial \theta}}{(1 - \lambda)R^cw}$$

$$= (1 - \lambda) \cdot \frac{1}{R^c} \cdot \frac{\partial R^c}{\partial \theta} + \lambda \cdot \frac{1}{R^c} \cdot \left[ \frac{\partial R^c}{\partial \theta} (\theta + \lambda) - R^c \right]$$

$$= \frac{1}{R^c} \cdot \frac{\partial R^c}{\partial \theta} - \frac{\lambda}{\theta + \lambda}$$

$$= \frac{1}{K^{\alpha-1}} \cdot (\alpha - 1)K^{\alpha-2} \cdot \frac{\partial K}{\partial \theta} - \frac{\lambda}{\theta + \lambda} \quad \text{(since } R^c = K^{\alpha-1} \text{ in equilibrium})$$

$$= (\alpha - 1) \frac{1}{K} \cdot \frac{\partial K}{\partial \theta} - \frac{\lambda}{\theta + \lambda}$$

$$= (1 - \alpha) \frac{1}{(1 - p)(1 - \lambda)} \left[ 1 - \frac{\lambda}{\theta + \lambda} \right] \cdot (1 - p)(1 - \lambda)w \cdot \frac{\lambda^2}{(\theta + \lambda)^2} - \frac{\lambda}{\theta + \lambda}$$

(eq. (3.2.15) and (3.A.9))

$$= \left[ \frac{(1 - \alpha)\lambda}{(1 - \theta)\lambda + \theta} - 1 \right] \cdot \frac{\lambda}{\theta + \lambda} < 0.$$

The last line follows from the fact that

$$0 > -\alpha \lambda - (1 - \lambda) \theta = (1 - \alpha)\lambda - \left[ (1 - \theta)\lambda + \theta \right];$$

$$\iff \frac{(1 - \alpha)\lambda}{(1 - \theta)\lambda + \theta} < 1.$$

Hence we show that the welfare of the L-type is decreasing in $\theta$. Decreasing $\theta$ would thus improve the welfare of the L-type.
Appendix 3.B  Proofs for the Dynamic Model

For the succinctness of the presentation, in all the proofs of this section, I drop the individual subscript $i$.

3.B.1 Proof to the Proposition 3.3.1

The borrower repays all the debt plus interest if

$$
(1 - \varphi)(1 + r_{i+1}^c) \exp\left\{ \eta_i + \varepsilon_i \right\} i_t^v \geq -(1 + \pi_i) s_t; \\
\iff \varepsilon_i \geq \log \left( -\frac{(1 + \pi_i) s_t}{(1 - \varphi)(1 + r_{i+1}^c) i_t^v \exp\{\eta_i\}} \right); \\
\text{namely} \quad \varepsilon_i \geq \log \left( (1 + \pi_i) L_t \right). 
$$

(3.B.1)

For the succinctness of the notation, in the following, we define $E(\pi_t) := \log \left( (1 + \pi_t) L_t \right)$.

Given $\pi_t > 0$, we can thus write the expected reclaim from the bank as

$$
\int_{-\infty}^{\infty} b_{t+1} f(\varepsilon_t) \, d\varepsilon_t = -(1 + \pi_t) s_t \int_{-\infty}^{E(\pi_t)} f(\varepsilon_t) \, d\varepsilon_t + \int_{E(\pi_t)}^{\infty} (1 - \varphi)(1 + r_{i+1}^c) \exp\left\{ \eta_i + \varepsilon_t \right\} i_t^v f(\varepsilon_t) \, d\varepsilon_t \\
= -(1 + \pi_t) s_t \int_{-\infty}^{E(\pi_t)} f(\varepsilon_t) \, d\varepsilon_t + \int_{E(\pi_t)}^{\infty} (1 - \varphi)(1 + r_{i+1}^c) i_t^v \exp\{\eta_i\} \exp\left\{ \varepsilon_t \right\} f(\varepsilon_t) \, d\varepsilon_t.
$$

(3.B.2)

Further,

$$
\int_{-\infty}^{E(\pi_t)} \exp\{\varepsilon_t\} f(\varepsilon_t) \, d\varepsilon_t \\
= \int_{-\infty}^{E(\pi_t)} \exp\{\varepsilon_t\} \frac{1}{\sqrt{2\pi} \sigma^2} \exp\left\{ -\frac{(\varepsilon_t + \sigma^2/2)^2}{2\sigma^2} \right\} \, d\varepsilon_t \\
= \Phi \left( \frac{E(\pi_t) - \sigma^2/2}{\sigma} \right) 
$$

(3.B.3)

Remember that $\varepsilon_t \sim \mathcal{N}( -\sigma^2/2, \sigma^2)$, hence

$$
\int_{-\infty}^{E(\pi_t)} f(\varepsilon_t) \, d\varepsilon_t = 1 - \Phi \left( \frac{E(\pi_t) + \sigma^2/2}{\sigma} \right).
$$

Further,

$$
\int_{-\infty}^{E(\pi_t)} \frac{1}{\sqrt{2\pi} \sigma^2} \exp\left\{ -\frac{(\varepsilon_t - \sigma^2/2)^2}{2\sigma^2} \right\} \, d\varepsilon_t \\
= \Phi \left( \frac{E(\pi_t) - \sigma^2/2}{\sigma} \right)
$$

(3.B.4)
Substituting the corresponding terms in the equation (3.3.18) as a function of \( \pi_t \) and \( L_t \), and denote this function as \( B(\pi_t, L_t) \). Namely,

\[
B(\pi_t, L_t) = (1 + \pi_t) \left[ 1 - \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) + \sigma^2/2}{\sigma} \right) \right] + L_t^{-1} \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) - \sigma^2/2}{\sigma} \right).
\]

The next lemma shows that for any \( L > 0 \), \( B(\cdot) \) is a monotonically increasing function in \( \pi_t \in (0, \infty) \).

**Lemma 3.B.1.** Given \( L > 0 \), the function \( B(\cdot) \) is a monotonically increasing function in \( \pi_t \in (0, \infty) \). Moreover,

\[
\frac{\partial B(\cdot)}{\partial \pi_t} = 1 - \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) + \sigma^2/2}{\sigma} \right) > 0.
\]

**Proof** For any \( L > 0 \) we have \( \pi_t > 0 \), the first-order derivative can be calculated as follows:
\[ \frac{\partial B(\cdot)}{\partial \pi_t} = 1 - \Phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t + \sigma^2/2}{\sigma} \right)}{\sigma} \right) \]  
\[ \quad - \frac{1}{\sigma} \cdot \phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t + \sigma^2/2}{\sigma} \right)}{\sigma} \right) + \frac{L_t^{-1}}{\sigma(1 + \pi_t)} \cdot \phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t - \sigma^2/2}{\sigma} \right)}{\sigma} \right), \]  
\[ = C \]  
(3.B.8)

where \( \phi(\cdot) \) is the probability density function (PDF) of the standard normal distribution.

Notice that

\[ \phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t + \sigma^2/2}{\sigma} \right)}{\sigma} \right) = \exp \left\{ - \frac{\left( \frac{\log \left( \frac{(1 + \pi_t)L_t + \sigma^2/2}{\sigma} \right)}{2\sigma^2} \right)^2}{2\sigma^2} \right\} \]

\[ \phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t - \sigma^2/2}{\sigma} \right)}{\sigma} \right) = \exp \left\{ - \frac{\left( \frac{\log \left( \frac{(1 + \pi_t)L_t - \sigma^2/2}{\sigma} \right)}{2\sigma^2} \right)^2}{2\sigma^2} \right\} \]

\[ = \frac{1}{\exp \left\{ \log \left( (1 + \pi_t)L_t \right) \right\}} \]

\[ = \frac{L_t^{-1}}{(1 + \pi_t)}. \]

Therefore, the term \( C \) in the equation (3.B.8) is 0. The equation (3.B.8) thus becomes

\[ \frac{\partial B(\cdot)}{\partial \pi_t} = 1 - \Phi \left( \frac{\log \left( \frac{(1 + \pi_t)L_t + \sigma^2/2}{\sigma} \right)}{\sigma} \right) > 0. \]  
(3.B.10)

Thus for any \( L_t > 0 \), the function \( B(\cdot) \) is monotonically increasing in \( \pi_t \in (0, \infty) \).  

The range of \( \pi_t \) that we are interested in is \( [r_i^f, \infty) \). It worths looking at the value of \( B(\pi_t, L_t) \) when \( \pi_t \) approaches to the upper and lower limit of the range. Next lemma deals with this, and the conclusion of which helps to find the condition under which the root of the equation (3.3.18) exists.

**Lemma 3.B.2.** Given \( L_t > 0 \), we have \( B(r_i^f, L_t) < 1 + r_i^f \), and

\[ \lim_{\pi_t \to \infty} B(\pi_t, L_t) = L_t^{-1}. \]  
(3.B.11)
Proof  Compare the equation (3.3.17) and (3.3.18) and we can write the $B(c, C^t)$ as

$$B(c, C^t) = -\frac{1}{s_t} \int_{-\infty}^{\infty} b_{t+1} f(\varepsilon_t) \, d\varepsilon_t \tag{3.B.12}$$

To prove the conclusion that $B(r^f_t, L_t) < 1 + r^f_t$, we use the equation (3.B.2), and

$$B(p_t, L_t) = \frac{1}{s_t} \left[ \int_{-\infty}^{\infty} E(\pi_t) (1 + \pi_t) s_t f(\varepsilon_t) \, d\varepsilon_t - \int_{-\infty}^{\infty} (1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t + \varepsilon_t\} l_t^v f(\varepsilon_t) \, d\varepsilon_t \right]. \tag{3.B.13}$$

Observe that by the condition (3.B.1), when $\varepsilon_t < E(\pi_t)$, $(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t + \varepsilon_t\} l_t^v < -(1 + \pi_t)s_t$. We thus proceed from the equation (3.B.13) that

$$B(p_t, L_t) < \frac{1}{s_t} \left[ \int_{-\infty}^{\infty} (1 + \pi_t) s_t f(\varepsilon_t) \, d\varepsilon_t + \int_{-\infty}^{E(\pi_t)} (1 + \pi_t) s_t f(\varepsilon_t) \, d\varepsilon_t \right] = (1 + \pi_t).$$

and therefore when $\pi_t = r^f_t$, we have $B(r^f_t, L_t) < 1 + r^f_t$. To prove the equation (3.B.11), we see from the equation (3.B.12) that

$$\lim_{\pi_t \to \infty} B(\pi_t, L_t) = -\frac{1}{s_t} \int_{-\infty}^{\infty} (1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t + \varepsilon_t\} l_t^v f(\varepsilon_t) \, d\varepsilon_t$$

$$= L_t^{-1} \int_{-\infty}^{\infty} \exp\{\varepsilon_t\} f(\varepsilon_t) \, d\varepsilon_t.$$

The last equality follows from the expectation of the lognormal distribution of $\exp\{\varepsilon_t\}$. ■

With the Lemma 3.B.1 and the Lemma 3.B.2, we are ready to prove the Proposition 3.3.2.

Proof to the Proposition 3.3.2  Using the notations defined above, the non-linear equation (3.3.17) can be written as

$$1 + r^f_t = B(\pi_t, L_t). \tag{3.B.14}$$
For any $L_t > 0$, $B(\pi_t, L_t)$ is increasing in $\pi_t$ (Lemma 3.B.1). Therefore, for a given $L_t > 0$, there can be no more than one root of $\pi_t$. Further, by Lemma 3.B.2, if $L_t < (1 + r^f_t)^{-1}$,

$$\lim_{\pi_t \to \infty} B(\pi_t, L_t) = L_t^{-1} > (1 + r^f_t).$$

By the same lemma, $B(r^f_t, L_t) < 1 + r^f_t$. We can therefore conclude that root exists for $\pi_t \in (r^f_t, \infty)$, when $L_t \in (0, (1 + r^f_t)^{-1})$. This proves that the function $\Pi(L_t)$ is well-defined on $L_t$.

When $L_t \in L_t$, the function $\pi_t = \Pi(L_t)$ is defined through the equation (3.B.14). The total differential on the equation (3.B.14) thus follows

$$0 = \frac{\partial B(\cdot)}{\partial \pi_t} d\pi_t + \frac{\partial B(\cdot)}{\partial L_t} dL_t. \quad (3.B.15)$$

The $\frac{\partial B(\cdot)}{\partial \pi_t}$ is given by the Lemma 3.B.1. The $\frac{\partial B(\cdot)}{\partial L_t}$ can be obtained in a similar way as the former derivative, in which we utilize again the equation (3.B.9). After calculation, we have

$$\frac{\partial B(\cdot)}{\partial L_t} = -L_t^{-2} \Phi \left( \frac{\log \left( (1 + \pi_t) L_t \right) - \sigma^2/2}{\sigma} \right).$$

Take the formulation of $\frac{\partial B(\cdot)}{\partial \pi_t}$ and $\frac{\partial B(\cdot)}{\partial L_t}$ to the equation (3.B.15) and we can solve for the equation (3.3.20). The $\frac{\partial \sigma_t}{dL_t}$ must be greater than zero since the CDF’s must be within $(0, 1)$, and $L_t > 0$ by setup.

**Appendix 3.C  Technical Appendix of the Dynamic Model**

This section has four subsections. In the first subsection, we characterize the relationship between consumption, investment and saving/borrowing. Especially, we describe the choice space of the household’s optimization problem, and relate the leverage and the borrowing interest rate to the choice variables. In Subsection 3.C.2, we show that the next-period state is a well-defined continuously-differentiable function of the current-period choice variables. This property is crucial for our method to solve for the household’s optimization problem. In Subsection 3.C.3, we derive the first-order conditions, as well as the envelope condition of the household’s dynamic optimization problem. The numerical algorithm is laid out in Subsection 3.C.4, which is built on the conditions solved in the last subsection.
3.C.1 Consumption, Investment and Saving

The purpose of the discussion in this section is to reduce the number of choice variables. We show that the household’s choice of savings (borrowings), borrowing interest rate, overall investment return, as well as the gross return from saving and borrowing can be represented as functions of investment and consumption. The properties of such functions are also discussed.

The relationship between the investment and the leverage

In this section, we discuss the relationship between the investment and the leverage. We start from the equation (3.3.16). Given \( a_t \) and \( c_t \), such that \( a_t > 0, c_t > 0 \), and \( a_t > c_t \), the equation (3.3.16) can be manipulated into

\[
a_t - c_t - i_t = (1 - \theta L_t^\zeta) s_t; \]

\[
\iff \quad \frac{i_t - (a_t - c_t)}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}i_t^\nu} = (1 - \theta L_t^\zeta) L_t; \]

\[
\iff \quad \frac{1}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}} \left[ i_t^{1-\gamma} - (a_t - c_t)i_t^{-\nu} \right] = (1 - \theta L_t^\zeta) L_t. \tag{3.C.1}
\]

The equation (3.C.1) defines a correspondence between investment \( i_t \) and the leverage \( L_t \), which we denote as \( L_t = \mathcal{F}(i_t) \). In the next proposition, we confirm that given \( a_t \) and \( c_t \), \( L_t = \mathcal{F}(i_t) \) is a well-defined monotonically increasing function of \( i_t \) on some interval of \( i_t \).

The first-order derivative of the function \( L_t = \mathcal{F}(i_t) \) can also be characterized analytically.

**PROPOSITION 3.C.1.** Given \( a_t \) and \( c_t \) such that \( a_t > 0, c_t > 0 \) and \( a_t \geq c_t \), there exist an upper bound of investment \( \bar{i}_t \in (a_t - c_t, \infty) \), such that \( L_t = \mathcal{F}(i_t) \) is a one-to-one mapping from \( \mathcal{I} = (a_t - c_t, \bar{i}_t) \) to \( \mathcal{L} = (0, (1 + r_t^f)^{-1}) \). The function \( L_t = \mathcal{F}(i_t) \) is twice continuously differentiable and strictly increasing. Further, the derivative of the function \( L_t = \mathcal{F}(i_t) \) is given by

\[
\frac{dL_t}{di_t} = \frac{1}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}} \cdot \frac{(1 - \nu) + \nu(a_t - c_t)i_t^{-1}}{1 - \theta L_t^\zeta(1 + \zeta)} > 0. \tag{3.C.2}
\]

**Proof** Denote the left-hand side of the equation (3.C.1) as \( f(i_t) \), and the right-hand side of the same equation as \( g(L_t) \). The equation (3.C.1) can thus be abbreviated denoted by \( f(i_t) = g(L_t) \). We first look at the properties of the function \( f(i_t) \). The first-order derivative of \( f(i_t) \) with respect to \( i_t \) is

\[
f'(i_t) = \frac{1}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}} \left[ (1 - \nu)i_t^{1-\gamma} + \nu(a_t - c_t)i_t^{-(\nu+1)} \right] > 0,
\]
hence \( f(i_t) \) is increasing in \( i_t \). This implies that given \( L_t \), if there is root of \( i_t \) in the equation (3.C.1), there is at most one root. We further notice that

\[
\lim_{i_t \to (a_t - c_t)} f(i_t) = 0, \quad \text{and} \quad \lim_{i_t \to +\infty} f(i_t) = +\infty.
\]

(3.C.3)

Hence \( f(i_t) \) is a monotonically increasing function over \((a_t - c_t, +\infty)\), with the range of \((0, +\infty)\). Second, we look at the properties of the function \( g(L_t) \). Again we consider the first-order derivative of the \( g(L_t) \) with respect to \( L_t \):

\[
g'(L_t) = 1 - \theta L_t^\zeta (1 + \zeta).
\]

Since \( \zeta \) is very small, \( g'(L_t) \) is always larger than zero. This implies that given \( i_t \), if there is root of \( L_t \) in the equation (3.C.1), there is at most one root of \( L_t \) in \( L \). We further notice that \( g(0) = 0 \), hence the range of \( g(L_t) \) is \( g(L_t) \in (0, g(\overline{L}_t)) \), where \( \overline{L}_t := (1 + r_t^f)^{-1} \).

Define \( \tilde{i}_t \) such that \( f(\tilde{i}_t) = g(\overline{L}_t) \), namely, \( \tilde{i}_t \) solves

\[
\frac{\tilde{i}_t - (a_t - c_t)}{(1 - \varphi)(1 + r_{t+1}) \exp(\eta_t \tilde{i}_t^\zeta)} = (1 - \theta \overline{L}_t^\zeta) \overline{L}_t.
\]

(3.C.4)

When \( i_t \in I = (a_t - c_t, \tilde{i}_t) \), there is always \( L_t \in \mathcal{L} = (0, \overline{L}_t) \) that solves \( g(L_t) = f(i_t) \), and \textit{vice versa}. Thus the function \( L_t = F(i_t) \) is a well-defined one-to-one mapping from \( I \) to \( \mathcal{L} \).

The final part of the proof concerns the first-order derivative of \( L_t \) with respect to \( i_t \). This is done by re-arranging the equation (3.C.1) into \( f(i_t) - g(L_t) = 0 \) and apply the total differential

\[
0 = f'(i_t)di_t - g'(L_t)dL_t.
\]

Taking the \( f'(i_t) \) and \( g'(L_t) \) derived above to the equation, the result of (3.C.2) follows obviously. It is obvious that the right-hand side of the equation (3.C.2) is again continuously differentiable with respect to \( i_t \), when \( i_t \in I \). Hence the function \( L_t = F(i_t) \) is twice continuously differentiable.

Since the function \( g(L_t) \) attains the maximum when \( L_t = \overline{L}_t \), it would thus not be plausible for the household to invest more than \( \tilde{i}_t \). The \( \tilde{i}_t \) is thus the higher bound of investment that a household can achieve in a given period. Because of the existence of the span-of-control parameter \( \nu \), there is always such a higher bound of investment, even when \( a_t = c_t \). However, in the latter case, the \( \tilde{i}_t \) must be very small.

The discussion above presume that the household’s choice of consumption is smaller than the cash-on-hand, \( a_t \). However, it is feasible that the household choose to consume
more than $a_t$. When $a_t < c_t$, $f(i_t)$ is not monotonically increasing any more. It is instead first decreasing than increasing. Therefore, there exists minimal value of $f(i_t)$ when $a_t < c_t$. Such a minimal value can be used to define an “absolute higher bound” of consumption, $\tilde{c}_t$, as is shown in the next proposition.

**PROPOSITION 3.C.2.** Given $a_t > 0$ and $\eta_t$, there exists an “absolute higher bound” of consumption, $\tilde{c}_t$, that depends only on the $a_t$ and $\eta_t$:

$$\tilde{c}_t = a_t + (1 - \nu) \nu^{\frac{1}{\nu}} \left[ (1 - \varphi)(1 + r_{t+1}^c) \exp(\eta_t) g(L_t) \right]^{\frac{1}{\nu}}.$$  \hspace{1cm} (3.C.5)

When $c_t \in (a_t, \tilde{c}_t)$, there exists lower and upper bound of investment $\hat{i}_t$, $\tilde{i}_t \in (0, \infty)$, as well as a lower bound of leverage $\hat{L}_t \in (0, L_t)$, such that $L_t = F(i_t)$ is a function from $\mathcal{T} = (\hat{i}_t, \tilde{i}_t)$ to $\mathcal{L} = (\hat{L}_t, \tilde{L}_t)$. The function is decreasing on the interval $(\hat{i}_t, \tilde{i}_t)$, and increasing on $(\tilde{i}_t, \hat{i}_t)$, where $\tilde{i}_t$ is given by

$$i_t^\circ = \frac{\nu}{1 - \nu}(c_t - a_t).$$  \hspace{1cm} (3.C.6)

Furthermore, $L_t \rightarrow \tilde{L}_t$ when either $i_t \rightarrow \hat{i}_t$ or $i_t \rightarrow \tilde{i}_t$. Lastly, $\hat{i}_t$, $\tilde{L}_t$, $i_t^\circ \rightarrow 0$ when $c_t \rightarrow a_t^+$.

**Proof** When $c_t > a_t$, we can arrange of formulation of the first-order derivative of $f(i_t)$ as

$$f'(i_t) = \frac{(1 - \nu) - \nu(c_t - a_t)\hat{i}_t^{-1}}{(1 - \varphi)(1 + r_{t+1}^c) \exp(\eta_t) \hat{i}_t^0}.$$  \hspace{1cm} (3.C.7)

When $c_t - a_t > 0$, $f'(i_t)$ is no longer always larger than 0 for any $i_t > 0$. We denote by $i_t^\circ$ the root to the equation $f'(i_t) = 0$, and $i_t^\circ$ can be found as

$$i_t^\circ = \frac{\nu}{1 - \nu}(c_t - a_t).$$

One can easily establish that $f(i_t)$ is decreasing when $i_t \leq i_t^\circ$ and increasing when $i_t > i_t^\circ$. Remember that $g(L_t)$ is increasing on the interval $(0, \tilde{L}_t)$. Thus consumption is only feasible if $f(i_t^\circ) < g(\hat{L}_t)$. The higher bound of consumption $\tilde{c}_t$ is thus defined as the level of consumption that equates the inequality. Namely,

$$f \left( \frac{\nu}{1 - \nu}(\tilde{c}_t - a_t) \right) = g(\hat{L}_t).$$

The root to the equation above is our formulation of $\tilde{c}_t$ in the equation (3.C.5). When $c_t < \tilde{c}_t$, the lower bound of the leverage $\hat{L}_t$ can be found by solving $f(i_t^\circ) = g(\hat{L}_t)$. Further, the lower and higher bound of investment $\hat{i}_t$ and $\tilde{i}_t$ are the two roots of $i_t$ to the equation $f(i_t) = g(\hat{L}_t)$. Since when $c_t > a_t$, $\lim_{i_t \rightarrow 0} f(i_t) = \lim_{i_t \rightarrow \infty} f(i_t) = \infty$; At the same
time, \( f(i_t) \) is decreasing on \((0, i_t^*)\) and increasing on \([i_t^*, \infty)\). Therefore, the condition that \( c_t < \bar{c}_t \) would guarantee that the two roots of \( i_t \) to the equation \( f(i_t) = g(\bar{L}_t) \) are well-established.

By the monotonic property of the function \( g(\bar{L}_t) \), we can conclude that as long as \( a_t < c_t < \bar{c}_t \), \( L_t = \mathcal{F}(i_t) \) is a well-defined function from \( I \) to \( L' \). Finally, concerning the limiting behaviour of \( \hat{\bar{L}}_t \) and \( \hat{i}_t^o \) when \( c_t \to a_t^+ \). That \( \lim_{c_t \to a_t^+} \hat{i}_t^o = 0 \) is easy to observe from the equation (3.C.6). To check for \( \hat{i}_t^o \), we notice that

\[
\lim_{c_t \to a_t^+} f(i_t^o) = \lim_{c_t \to a_t^+} f \left( \frac{\nu}{1 - \nu} (\bar{c}_t - a_t) \right)
\]

\[
= \lim_{c_t \to a_t^+} \frac{1}{(1 - \nu)^{1 - \nu}} \cdot \frac{1}{(1 + r_{t+1}^c)} \cdot \exp(\eta_t) \cdot (c_t - a_t)^{1 - \nu}
\]

\[
= 0,
\]

and remember that \( g(0) = 0 \) and \( g(\cdot) \) is continuous. Since given \( a_t \) and \( \eta_t \), for any \( c_t > a_t \), \( \bar{L}_t \) is the root to the equation \( f(i_t^o) = g(\bar{L}_t) \), we can thus conclude that \( \bar{L}_t \to 0 \) when \( c_t \to a_t^+ \). The limiting behaviour of \( \hat{i}_t \) can be established by observing that \( 0 < \hat{i}_t < i_t^o \).

If parameters are chosen appropriately, \( \bar{c}_t \) is very close to \( a_t \). In such case, it is very unlikely that the household may choose to consume more than she has. No matter what the relationship between \( c_t \) and \( a_t \) is, the function from investment to leverage is always well-defined on a meaningful range of investment. The first-order derivative of which is always given by the equation (3.C.2).

**The relationship between the investment and the saving**

We can now derive all other derivatives related to the household’s choice between investment and savings. This helps to consolidate the two choice variables \( i_t \) and \( s_t \) into one. First, we can show that for \( i_t \in I \), \( s_t \) is a well-defined function of \( i_t \), and we can characterize the first-order derivative of which analytically.

**LEMMA 3.C.3.** Given \( a_t > 0 \), \( \eta_t \) and \( c_t \in (0, \bar{c}_t) \), \( s_t \) is well-defined twice continuously differentiable function of \( i_t \) for \( i_t \in I \). Moreover, the first-order derivative of \( s_t \) with respect to \( i_t \) is

\[
\frac{\partial s_t}{\partial i_t} = -\nu(1 - \varphi)(1 + r_{t+1}^c) \exp(\eta_t) i_t^{\nu - 1} L_t - \frac{(1 - \nu) + \nu(a_t - c_t)i_t^{\nu - 1}}{1 - \theta L_t^c (1 + \zeta)}.
\] (3.C.8)
Proof. From the Proposition 3.C.1, if \( i_t \in \mathcal{I} \), then \( L_t = \mathcal{F}(i_t) \in \mathcal{L} \) (or \( \in \mathcal{L}^I \)) is well-defined. Further, based on the definition of the leverage we have

\[
s_t = -(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}i_t^\gamma L_t,
\]

hence the function from \( i_t \) to \( s_t \) is also well-defined. The first-order derivative of \( s_t \) with respect to \( i_t \) writes

\[
\frac{\partial s_t}{\partial i_t} = -\nu(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}i_t^{\gamma-1}L_t - (1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\}i_t^\gamma \cdot \frac{\partial L_t}{\partial i_t}.
\]

Take the representation of \( \frac{\partial L_t}{\partial i_t} \) (equation (3.C.2)) into the equation above and we obtain the equation (3.C.8). Further, since the equation (3.C.8) exists and is obviously continuously differentiable, thus \( s_t \) is twice continuously differentiable in \( i_t \). ■

Next, we’ll see that the interest rate \( \pi_t \) is also a well-defined function of \( i_t \) on \( \mathcal{I} \). There is nice representation of the first-order derivative of \( \pi_t \) with respect to \( i_t \).

**Lemma 3.C.4.** Given \( a_t > 0 \), \( \eta_t \) and \( c_t \in (0, \bar{c}_t) \), \( \pi_t \) is a well-defined twice continuously differentiable function of \( i_t \) for \( i_t \in \mathcal{I} \). Moreover, the first-order derivative of \( \pi_t \) with respect to \( i_t \) is

\[
\frac{\partial \pi_t}{\partial i_t} = -\frac{1}{s_t L_t} \cdot \Phi \left( \frac{\log \left( (1+\pi_t)L_t \right) - \sigma^2/2}{\sigma} \right) \cdot (1 - \nu) + \nu (a_t - c_t) i_t^{-1} \left[ 1 - \theta L_t^c (1 + \zeta) \right].
\]

(3.C.9)

If \( c_t \in (0, a_t] \), \( \pi_t \) is strictly increasing in \( \mathcal{I} \); while if \( c_t \in (a_t, \bar{c}_t) \), \( \pi_t \) is first decreasing on \( (\hat{i}_t^c, \hat{i}_t^\circ) \) then increasing on \( (\hat{i}_t^\circ, \hat{i}_t) \).

Proof. From the Proposition 3.C.1, if \( i_t \in \mathcal{I} \), then \( L_t = \mathcal{F}(i_t) \in \mathcal{L} \) (or \( \in \mathcal{L}^I \)) is well-defined. It is strictly increasing on \( \mathcal{I} \) when \( c_t \in (0, a_t] \); By the Proposition 3.C.2, when \( c_t \in (a_t, \bar{c}_t) \), the function is first decreasing on \( (\hat{i}_t^c, \hat{i}_t^\circ) \) and then increasing on \( (\hat{i}_t^\circ, \hat{i}_t) \).

From the Proposition 3.3.2, when \( L_t \in \mathcal{L}^I \subset \mathcal{L} \), \( \pi_t = \Pi(L_t) \in \mathcal{R} \) is also well-defined and strictly increasing. Therefore, a function from \( i_t \in \mathcal{I} \) to \( \pi_t \in \mathcal{R} \) is always well-defined, with the monotonic property follows that of the function \( \mathcal{F}(\cdot) \). The first-order derivative of \( \pi_t \) with respect to \( i_t \) can be calculated by the chain rule

\[
\frac{\partial \pi_t}{\partial i_t} = \frac{\partial \pi_t}{\partial L_t} \cdot \frac{\partial L_t}{\partial i_t},
\]
where the two derivatives on the right-hand side of the equation can be found in the equation (3.3.20) and (3.C.2), respectively. Moreover, since the equation (3.C.9) exists and is obviously continuously differentiable, thus \( \pi_t \) is twice continuously differentiable in \( i_t \). ■

The relationship between the consumption and the saving

In this section, similar calculations are performed for the relationship between the consumption and the saving. First, given \( a_t \) and \( i_t \), the equation (3.C.1) defines a correspondence between the consumption and the leverage, \( L_t = \mathcal{J}(c_t) \). Similar to the Proposition 3.C.1 and 3.C.2, the following proposition confirms that given \( a_t \) and \( c_t \), \( L_t = \mathcal{J}(c_t) \) is a well-defined monotonically increasing function of \( c_t \) on some interval of \( c_t \), with a well-defined first-order derivative.

**Proposition 3.C.5.** Given \( a_t > 0 \) and \( \eta_t \), there exists an “absolute higher bound” of investment, \( \tilde{i}_t \), that depends only on the \( a_t \) and \( \eta_t \), such that investment must be less than \( \tilde{i}_t \). Given \( a_t > 0 \) and \( i_t \in (0, \tilde{i}_t) \), there exists a lower bound of consumption \( \hat{c}_t = \max\{0, a_t - i_t\} \) and an upper bound of consumption \( \bar{c} \in (\hat{c}_t, \tilde{c}_t) \), as well as a lower bound of the leverage \( \tilde{L} \in [0, \tilde{L}_t) \), such that \( L_t = \mathcal{J}(c_t) \) is a one-to-one mapping from \( C = (\tilde{c}_t, \bar{c}_t) \) to \( L^c = (\tilde{L}_t, \bar{L}_t) \). The function \( L_t = \mathcal{J}(c_t) \) is twice continuously differentiable and strictly increasing. Further, the derivative of the function \( L_t = \mathcal{J}(c_t) \) is given by

\[
\frac{dL_t}{dc_t} = \frac{1}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\} i_t^c} \cdot \frac{1}{1 - \theta L_t^c (1 + \zeta)} > 0. \tag{3.C.10}
\]

**Proof** The proof is similar to that of the Proposition 3.C.1. We denote the left-hand side of the equation (3.C.1) as \( l(c_t) \), so that the equation (3.C.1) becomes \( l(c_t) = g(L_t) \). The function \( l(c_t) \) is linear in \( c_t \), with the first-order derivative

\[
l'(c_t) = \frac{1}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\} i_t^c} > 0,
\]

hence if \( i_t \neq 0 \), \( l'(c_t) \) is monotonically increasing in \( c_t \). Further,

\[
\lim_{c_t \to 0} l(c_t) = \frac{i_t - a_t}{(1 - \varphi)(1 + r_{t+1}^c) \exp\{\eta_t\} i_t^c}, \quad \text{and} \quad \lim_{c_t \to +\infty} l(c_t) = +\infty. \tag{3.C.11}
\]

The properties of \( g(L_t) \) is the same as before. We discuss the solution of \( L_t \) in the following case:
We let \( \tilde{L}_t := 0 \). Similar to the logic in the proof of the Proposition 3.C.1, the function \( L_t = \mathcal{J}(c_t) \) is a well-defined strictly increasing bijective function from \( C = (\tilde{c}_t, \bar{c}_t) \) to \( \mathcal{L}_c = (\tilde{L}_t, \bar{L}_t) \). The function is also twice continuously differentiable.

**Case of** \( a_t < i_t \). Define \( \tilde{c}_t := 0 \). Further, define \( \tilde{L}_t \) such that \( \lim_{c_t \to 0} l(c_t) = g(\tilde{L}_t) \), namely, \( \tilde{L}_t \) solves the following non-linear equation

\[
\frac{i_t - a_t}{(1 - \varphi)(1 + r_{i,t+1}^c)} \exp\{\eta_t\}i_t^\gamma = \left[1 - \theta\tilde{L}_t^\zeta\right] \tilde{L}_t. \tag{3.C.13}
\]

At the same time, the definition of the \( \bar{c}_t \) is the same as the equation (3.C.12). Since the upper bound of consumption cannot drop below or equal to 0, investment must satisfy \( g(\tilde{L}_t) < g(\bar{L}_t) \). Therefore, the “absolute higher bound” of investment \( \tilde{c}_t \) can be defined as the level of investment that equates \( g(\tilde{L}_t) \) to \( g(\bar{L}_t) \), which depends only on \( a_t \) and \( \eta_t \). More specifically, \( \tilde{c}_t \) can be found by solving the following nonlinear equation

\[
\frac{\tilde{c}_t - a_t}{(1 - \varphi)(1 + r_{i,t+1}^c)} \exp\{\eta_t\}\tilde{c}_t^\gamma = \left[1 - \theta\tilde{L}_t^\zeta\right] \tilde{L}_t. \tag{3.C.14}
\]

To prevent consumption dropping below (or equal to) zero, investment must satisfy \( i_t < \tilde{c}_t \). If \( i_t < \tilde{c}_t \), the function \( L_t = \mathcal{J}(c_t) \) is a well-defined strictly increasing bijective function from \( C = (\tilde{c}_t, \bar{c}_t) \) to \( \mathcal{L}_c = (\tilde{L}_t, \bar{L}_t) \). The function is also twice continuously differentiable.

When \( c_t \in C \), the first-order derivative \( (dL_t/dc_t) \) can be found by applying the total differential on the equation (3.C.1), in a similar way as what in the proof of the Proposition 3.C.1:

\[ 0 = l'(c_t) dc_t - g'(L_t) dL_t. \]

Next, similar to the Lemma 3.C.3 and 3.C.4, the first-order derivative of \( s_t \) and \( \pi_t \) with respect to \( c_t \) can also be analytically characterized.

**Lemma 3.C.6.** Given \( a_t > 0, \eta_t \) and \( i_t \in (0, \tilde{i}_t) \), \( s_t \) is a well-defined twice continuously differentiable and strictly decreasing in \( c_t \) for \( c_t \in C \). Moreover, the first-order derivative of \( s_t \) with respect to \( c_t \) is

\[
\frac{\partial s_t}{\partial c_t} = -\frac{1}{1 - \theta L_t^\zeta (1 + \zeta)} < 0. \tag{3.C.15}
\]
Proof From the Proposition 3.C.5, if \( c_t \in \mathcal{C} \), then \( L_t = \mathcal{F}(c_t) \in \mathcal{L}^c \) is well-defined. Further, based on the definition of the leverage we have

\[
s_t = -(1 - \varphi)(1 + r^c_{i+1}) \exp \{ \eta_t \} i_t^\nu L_t,
\]

hence the function from \( c_t \) to \( s_t \) is also well-defined. The first-order derivative of \( s_t \) with respect to \( c_t \) writes

\[
\frac{\partial s_t}{\partial c_t} = -(1 - \varphi)(1 + r^c_{i+1}) \exp \{ \eta_t \} i_t^\nu \cdot \frac{\partial L_t}{\partial c_t}.
\]

Take the representation of the \( \frac{\partial L_t}{\partial c_t} \) (equation (3.C.10)) into the equation above and we obtain the equation (3.C.15). Because when \( c_t \in \mathcal{C} \), we have \( L_t \in \mathcal{L}^c \) and \( g'(L_t) = 1 - \theta L_t^{\xi}(1 + \zeta) > 0 \). Therefore, the first-order derivative in (3.C.15) is always larger than zero. Furthermore, since the equation (3.C.15) exists and is obviously continuously differentiable, \( s_t \) is twice continuously differentiable in \( c_t \).

**Lemma 3.C.7.** Given \( a_t > 0 \), \( \eta_i \) and \( i_t \in (0, \tilde{i}_t) \), \( \pi_t \) is a well-defined twice continuously differentiable and strictly increasing in \( c_t \) for \( c_t \in \mathcal{C} \). Moreover, the first-order derivative of \( \pi_t \) with respect to \( c_t \) is

\[
\frac{\partial \pi_t}{\partial c_t} = -\frac{1}{s_t L_t} \cdot \frac{\Phi \left( \log \left( \frac{(1+\pi_t)L_t}{\sigma} \right) - \sigma^2/2 \right)}{1 - \Phi \left( \log \left( \frac{(1+\pi_t)L_t}{\sigma} \right) + \sigma^2/2 \right)} \cdot \frac{1}{1 - \theta L_t^{\xi}(1 + \zeta)} > 0.
\] (3.C.16)

Proof From the Proposition 3.C.5, if \( c_t \in \mathcal{C} \), \( L_t = \mathcal{F}(c_t) \in \mathcal{L}^c \) is well-defined and strictly increasing. From the Proposition 3.3.2, when \( L_t \in \mathcal{L}^c \subset \mathcal{L} \), \( \pi_t = \Pi(L_t) \in \mathcal{R} \) is also well-defined and strictly increasing. Therefore, a function from \( c_t \in \mathcal{C} \) to \( \pi_t \in \mathcal{R} \) is always well-defined and strictly increasing. The first-order derivative of \( \pi_t \) with respect to \( c_t \) can be calculated by the chain rule

\[
\frac{\partial \pi_t}{\partial c_t} = \frac{\partial \pi_t}{\partial L_t} \cdot \frac{\partial L_t}{\partial c_t},
\]

where the two derivatives on the right-hand side of the equation can be found in the equation (3.3.20) and (3.C.10), respectively. Moreover, since the equation (3.C.16) exists and obviously continuously differentiable, \( \pi_t \) is twice continuously differentiable in \( c_t \).
3.C.2 Next-period State

Household’s next-period cash-on-hand can be represented as a function \( c_t, i_t \) and \( \epsilon_t \), as well as \( a_t \) and \( \eta_t \):

\[
a_{t+1} = \mathcal{A}(c_t, i_t, \epsilon_t; a_t, \eta_t) \tag{3.C.17}
\]

\[
= \begin{cases} 
  w_{t+1} + (1 + r_{t+1}^C) \exp\{\eta_t + \epsilon_t\} i_t^\gamma + (1 + r_{t}^f)(a_t - c_t - i_t) & \text{if } s_t \geq 0; \\
  w_{t+1} + (1 + r_{t+1}^C) \exp\{\eta_t + \epsilon_t\} i_t^\gamma + (1 + \pi_t)s_t & \text{if } s_t < 0 \text{ and solvent}; \\
  w_{t+1} & \text{if } s_t < 0 \text{ and insolvent}.
\end{cases}
\]

In this section, we first derive the first-order derivative of \( \mathcal{A}(\cdot) \) with respect to \( c_t \) and \( i_t \), given \( a_t > 0 \) and \( \eta_t \). It is crucial to divide our discussion into two cases, one is when \( c_t + i_t \leq a_t \), and another \( c_t + i_t > a_t \). In the former case, there is no borrowing, and hence the transaction cost \( k_t \) is zero. The household only faces the risk-free interest rate if she doesn’t borrow. In the latter case, the borrowing interest rate depends on the amount of borrowing. The two cases have two different formulations of the first-order partial derivatives w.r.t. \( c_t \) and \( i_t \). But thanks to the technical term \( L_t^C \) in our formulation of the transaction cost, the partial derivatives are smooth when \( (c_t + i_t) \) passes through \( a_t \). In the end we are able to show that \( \mathcal{A}(\cdot) \) is continuously differentiable in both argument of \( c_t \) and \( i_t \).

We will also derive the first-order derivative of \( \mathcal{A}(\cdot) \) with respect to \( a_t \), given \( c_t, i_t \) and \( \eta_t \). This partial derivative enable us to apply envelope condition to back out the formulation of \( \frac{\partial \mathcal{V}}{\partial a} \), which shows up in the first-order conditions of the consumption and investment.

Next-period Cash-on-hand with respect to choices

**Case of** \( c_t + i_t \leq a_t \). In this case, the equation (3.C.17) becomes

\[
\mathcal{A}(c_t, i_t, \epsilon_t; a_t, \eta_t) = (1 + r_{t+1}^C) \exp\{\eta_t + \epsilon_t\} i_t^\gamma + (1 + r_{t}^f)(a_t - c_t - i_t) + w_{t+1}. \tag{3.C.18}
\]

The first-order partial derivative of \( \mathcal{A}(i_t, \epsilon_t) \) with respect to \( c_t \) and \( i_t \) are

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial c_t} = -(1 + r_{t}^f); \tag{3.C.19}
\]

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial i_t} = v(1 + r_{t+1}^C) \exp\{\eta_t + \epsilon_t\} i_t^{\gamma-1} - (1 + r_{t}^f). \tag{3.C.20}
\]

**Case of** \( c_t + i_t > a_t \). Household is insolvent, namely \( a_{t+1} = w_{t+1} \). when

\[
(1 - \varphi)(1 + r_{t+1}^C) \exp\{\eta_t + \epsilon_t\} i_t^\gamma + (1 + \pi_t)s_t \leq 0 \iff \epsilon_t \leq \log \left( (1 + \pi_t)L_t \right). \tag{3.C.21}
\]
Based on the discussion in the last section, when \( c_t \in \mathcal{C} \) and \( i_t \in \mathcal{I} \), both \( \pi_t \) and \( L_t \) can be represented as well-defined functions of \( c_t \) and \( i_t \). We thus adopt the notation \( \mathcal{E}(c_t, i_t) := \log \left( (1 + \pi_t)L_t \right) \).

When \( \varepsilon_t \leq \mathcal{E}(c_t, i_t) \), \( \mathcal{A}(\cdot) \equiv w_{t+1} \). In this case, \( \frac{\partial \mathcal{A}(\cdot)}{\partial c_t} = \frac{\partial \mathcal{A}(\cdot)}{\partial i_t} = 0 \). When \( \varepsilon_t > \mathcal{E}(c_t, i_t) \), using the Lemma 3.C.3, Lemma 3.C.4, Lemma 3.C.6 and Lemma 3.C.7, it can be shown that

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial c_t} = (1 + \pi_t) \frac{\partial s_t}{\partial c_t} + s_t \frac{\partial \pi_t}{\partial c_t}
= -\frac{1}{1 - \theta L_t^\zeta (1 + \zeta)} \left[ (1 + \pi_t) + \frac{1}{L_t} \cdot \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) - \sigma^2/2}{\sigma} \right) \right]
\]

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial i_t} = \nu(1 + r_{t+1}^c) \exp \{ \eta_t + \varepsilon_t \} i_t^{\nu - 1} + (1 + \pi_t) \frac{\partial s_t}{\partial i_t} + s_t \frac{\partial \pi_t}{\partial i_t}
= \nu(1 + r_{t+1}^c) \exp \{ \eta_t + \varepsilon_t \} i_t^{\nu - 1} - \nu(1 - \varphi)(1 + r_{t+1}^c) \exp \{ \eta_t \} i_t^{\nu - 1}(1 + \pi_t)L_t
\]

\[
- \frac{(1 - \nu) + \nu(a_t - c_t)i_t^{\nu - 1}}{1 - \theta L_t^\zeta (1 + \zeta)} \left[ (1 + \pi_t) + \frac{1}{L_t} \cdot \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) - \sigma^2/2}{\sigma} \right) \right]
\]

Applying the equation (3.3.18) and the definition of the leverage \( L_t \), we are able to show that

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial c_t} = -\frac{1}{1 - \theta L_t^\zeta (1 + \zeta)} \cdot \frac{1 + r_{t}^f}{1 - \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) + \sigma^2/2}{\sigma} \right)}
\]

\[
\frac{\partial \mathcal{A}(\cdot)}{\partial i_t} = \nu(1 + r_{t+1}^c) \exp \{ \eta_t + \varepsilon_t \} i_t^{\nu - 1} + \nu(1 + \pi_t) \frac{s_t}{i_t}
- \frac{(1 - \nu) + \nu(a_t - c_t)i_t^{\nu - 1}}{1 - \theta L_t^\zeta (1 + \zeta)} \cdot \frac{1 + r_{t}^f}{1 - \Phi \left( \frac{\log \left( (1 + \pi_t)L_t \right) + \sigma^2/2}{\sigma} \right)}
\]

**Continuous Differentiability of the function \( \mathcal{A}(\cdot) \).** We now show that given \( a_t > 0 \) and \( \eta_t \), the next-period state \( \mathcal{A}(c_t, i_t, \varepsilon_t; a_t, \eta_t) \) is a continuously differentiable function in \( c_t \) and \( i_t \), for \( c_t \in \mathcal{C}^+ := (0, \bar{c}_t) \) and \( i_t \in \mathcal{I}^+ := (\bar{i}_t, \bar{i}_t) \), where \( \bar{i}_t = 0 \) if \( c_t \leq a_t \). The range \( \mathcal{C}^+ \) and
\(\mathcal{I}^+\) are ranges of \(c_t\) and \(i_t\) on which the optimization is performed. Note that \(\bar{c}_t\) depends on the household’s choice of \(i_t\), while \(\hat{i}_t\) depends on her choice of \(c_t\). If \(c_t > a_t, \hat{i}_t\) also depends on \(c_t\). The crucial thing is that, when household passes through the point where she has to borrow, her next-period period state should be continuously differentiable in the two choice variables. This is obvious, since if \(c_t + i_t \to a_t^+, s_t \to 0, L_t \to 0\) and \(\mathcal{E}(c_t, i_t) \to -\infty\). The \(\Phi(\cdot)\)-term in the equation (3.C.24) and (3.C.25) approaches to zero as a result. Thanks to the technical term \(\Theta\), when \(\Theta \to 0\), \((1 - \Theta L_t^\xi (1 + \zeta))^{-1} \to 1\). Therefore, the equation (3.C.24) and (3.C.25) converges to (3.C.19) and (3.C.20) when \(c_t + i_t \to a_t^+\). This is concluded in the following proposition.

**Proposition 3.C.8.** Given \(a_t > 0\) and \(\eta_t\) and if \(c_t \in \mathcal{C}^+\) and \(i_t \in \mathcal{I}^+\), \(A(c_t, i_t, \varepsilon_t; a_t, \eta_t)\) is a continuously differentiable function in \(c_t\) and \(i_t\).

**Next-period Cash-on-hand with respect to current cash-on-hand**

Given \(\eta_t\) and the choice of \(c_t \in \mathcal{C}^+\) and \(i_t \in \mathcal{I}^+\), if \(c_t + i_t > a_t\), the equation (3.C.1) defines a function from \(a_t\) to \(L_t\). The first-order derivative of \(L_t\) with respect to \(a_t\) writes

\[
\frac{dL_t}{da_t} = -\frac{1}{(1 - \varphi(1 + r_{t+1}^e) \exp(\eta_t) i_t^\nu)} \cdot \frac{1}{1 - \theta L_t^\xi (1 + \zeta)} < 0. \tag{3.C.26}
\]

Under the case of \(a_t < c_t + i_t\), following the same methods in the proof of Lemma 3.C.3, Lemma 3.C.4, Lemma 3.C.6 and Lemma 3.C.7, the first-order derivative of \(s_t\) with respect to \(a_t\) writes

\[
\frac{\partial s_t}{\partial a_t} = \frac{1}{1 - \theta L_t^\xi (1 + \zeta)} > 0; \tag{3.C.27}
\]

and the first-order derivative of \(\pi_t\) with respect to \(a_t\) writes

\[
\frac{\partial \pi_t}{\partial a_t} = \frac{1}{s_t L_t} \cdot \frac{\Phi \left( \frac{\log \left( \frac{(1 + \pi_t) L_t}{\sigma} + \sigma^2 / 2 \right)}{\sigma} \right)}{1 - \Phi \left( \frac{\log \left( \frac{(1 + \pi_t) L_t}{\sigma} + \sigma^2 / 2 \right)}{\sigma} \right)} \cdot \frac{1}{1 - \theta L_t^\xi (1 + \zeta)} < 0. \tag{3.C.28}
\]

We are now ready to derive the first-order derivative of \(A(\cdot)\) with respect to \(a_t\). The derivation is again divided into two cases. If \(c_t + i_t \leq a_t\),

\[
\frac{\partial A(\cdot)}{\partial a_t} = (1 + r_{t}^f); \tag{3.C.29}
\]
while if \( c_t + i_t > a_t \), \( \frac{\partial A(\cdot)}{\partial a_t} \equiv 0 \) when \( \varepsilon_t \leq E(c_t, i_t) \); When \( \varepsilon_t > E(c_t, i_t) \),

\[
\frac{\partial A(\cdot)}{\partial a_t} = (1 + \pi_t) \frac{\partial s_t}{\partial a_t} + s_t \frac{\partial \pi_t}{\partial a_t}
\]

\[
= \frac{1}{1 - \theta L_t^2 (1 + \zeta)} \left[ (1 + \pi_t) + \frac{1}{L_t} \cdot \Phi \left( \frac{\log \left( \frac{(1+\pi_t) L_t}{\sigma} \right) - \sigma^2/2}{\sigma} \right) \right]
\]

\[
= \frac{1}{1 - \theta L_t^2 (1 + \zeta)} \cdot \frac{1 + r_t^f}{1 - \Phi \left( \frac{\log \left( \frac{(1+\pi_t) L_t}{\sigma} \right) + \sigma^2/2}{\sigma} \right)}
\]

(3.C.30)

One notices that \( \frac{\partial A(\cdot)}{\partial a_t} = -\frac{\partial A(\cdot)}{\partial c_t} \). Following the same logic as the Proposition 3.C.8, \( A(\cdot) \) is continuously differentiable in \( a_t \).

### 3.C.3 Optimal Investment and Consumption

We are now ready to obtain the first-order conditions of the household’s optimization problem. The purpose is to obtain a well-defined non-linear equation systems of \( c_t \) and \( i_t \), to be solved numerically for the optimal decisions of household. Following the convention of the literature, we drop the subscript \( t \), and we use the prime-notation to represent the variables in the period \( (t+1) \).

**Household’s problem revisited**

Recall that the recursive representation of the household’s problem as follows:

\[
\mathcal{V}(a, \eta; G) = \max_{\{c, i\}} \left[ (1 - \beta) e^{1-\mu} + \beta \left( \lambda_{\eta H} \int_E \mathcal{V}(a', \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_E \mathcal{V}(a', 0; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon \right)^{\frac{1-\mu}{1-\gamma}} \right]^{\frac{1}{1-\mu}}
\]

subject to \( a' = A(c, i; \varepsilon; a, \eta), \quad c \in C^+, \quad i \in I^+ \).
Remember from the Proposition 3.C.8, the function $\mathcal{A}(c, i, \varepsilon; a, \eta)$ is a continuously differentiable function in $c \in \mathcal{C}^+$ and $i \in \mathcal{I}^+$. Therefore, we can derive the first-order conditions for $c$ and $i$.

We further define

$$\tilde{\alpha}(c, i; \eta) := \varphi(1 + r') \exp\{\mathcal{E}(i, c) + \eta\} \tau' + w$$

as the bankruptcy deadweight loss.

**First-order conditions of investment and consumption**

If $W > 1$ and $\gamma > 1$, after the consumption is chosen, the household only needs to choose the investment $i^*$, such that

$$i^* = \arg \min_{i \in \mathcal{I}^+} \left\{ \lambda_H \int_{\mathcal{E}} \mathcal{V}(a', \bar{\eta}; \mathcal{G}')^{1-\gamma} f(\varepsilon) \, d\varepsilon + \lambda_L \int_{\mathcal{E}} \mathcal{V}(a', 0; \mathcal{G}')^{1-\gamma} f(\varepsilon) \, d\varepsilon \right\}.$$  

The next proposition provides the first-order condition of investment, if the optimal investment exists.

**PROPOSITION 3.C.9. (First-order Necessary Condition for Investment)** Suppose that the value function $\mathcal{V}(a, \eta; \mathcal{G})$ is continuously differentiable in $a$. Given $a > 0$ and $\eta$, and for given $c \in (0, \bar{c})$, if $i^* \in \mathcal{I}^+$ exists, $i^*$ must solve

$$0 = (1 - \gamma) \int_{\mathcal{E}(c,i^*)}^{\infty} \mathbb{D}(c, i^*, \varepsilon) \frac{\partial \mathcal{A}(c, i^*, \varepsilon)}{\partial i^*} \phi\left(\frac{\varepsilon + \sigma^2/2}{\sigma}\right) \, d\varepsilon$$

$$+ \phi\left(\frac{\mathcal{E}(c, i^*) + \sigma^2/2}{\sigma}\right) \cdot \frac{\partial \mathcal{E}(c, i^*)}{\partial i^*} \cdot \mathcal{C}(c, i^*),$$

where $\mathcal{E}(c, i^*) = -\infty$ if $c + i^* \leq a$. $\phi(\cdot)$ is the probability density function of the standard normal distribution. Moreover,

$$\mathbb{D}(c, i^*, \varepsilon) = \lambda_H \mathcal{V}(\mathcal{A}(c, i^*, \varepsilon), \bar{\eta}; \mathcal{G}')^{-\gamma} \cdot \frac{\partial \mathcal{V}(\mathcal{A}(c, i^*, \varepsilon), \bar{\eta}; \mathcal{G}')}{\partial \mathcal{A}(c, i^*, \varepsilon)} +$$

$$\lambda_L \mathcal{V}(\mathcal{A}(c, i^*, \varepsilon), 0; \mathcal{G}')^{-\gamma} \cdot \frac{\partial \mathcal{V}(\mathcal{A}(c, i^*, \varepsilon), 0; \mathcal{G}')}{\partial \mathcal{A}(c, i^*, \varepsilon)}.$$  

---

31 In the following, to keep the formulation succinct, we denote the $\mathcal{A}(c, i, \varepsilon; a, \eta)$ as $\mathcal{A}(c, i, \varepsilon)$. 
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and \( C(\cdot) \) can be deemed as the “marginal utility cost” of bankruptcy, which is

\[
C(c, i^*) = \lambda_{\eta H} \left\{ \mathbb{V}(w, \eta; G')^{1-\gamma} - \mathbb{V}(\tilde{a}, \eta; G')^{1-\gamma} \right\} + \\
\lambda_{\eta L} \left\{ \mathbb{V}(w, 0; G')^{1-\gamma} - \mathbb{V}(\tilde{a}, 0; G')^{1-\gamma} \right\}. \tag{3.C.33}
\]

**Proof** For given \( a > 0, \eta \) and \( c \in (0, \bar{c}) \), define \( \mathcal{H}(i; \eta) \) to be

\[
\mathcal{H}(i; \eta) = \mathbb{E}_{\varepsilon} \left[ \mathbb{V}(A(c, i, \varepsilon), \eta; G')^{1-\gamma} \right] = \int_{\varepsilon} \mathbb{V}(A(c, i, \varepsilon), \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon.
\]

We can thus equivalently represent the \( i^* \) as

\[
i^* = \arg\min_{i \in \mathcal{I}^+} \left\{ \lambda_{\eta H} \mathcal{H}(i; \eta) + \lambda_{\eta L} \mathcal{H}(i; 0) \right\}. \tag{3.C.34}
\]

When \( i \in \mathcal{I}^+ \), the first-order derivative of \( \mathcal{H}(\cdot) \) with respect to \( i \) is

\[
\frac{\partial \mathcal{H}(\cdot)}{\partial i} = \frac{\partial}{\partial i} \left[ \int_{-\infty}^{\varepsilon(c,i)} \mathbb{V}(w, \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon + \int_{\varepsilon(c,i)}^{\infty} \mathbb{V}(A(c, i, \varepsilon), \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon \right]. \tag{3.C.35}
\]

By the Leibniz rule, we have

\[
\frac{\partial}{\partial i} \int_{-\infty}^{\varepsilon(c,i)} \mathbb{V}(w, \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon = \mathbb{V}(w, \eta; G')^{1-\gamma} f(\varepsilon(c,i)) \cdot \frac{\partial \varepsilon(c,i)}{\partial i}; \tag{3.C.36}
\]

and since \( \mathcal{A}(c, i, \varepsilon(c,i)) = \tilde{a}(c, i) \),

\[
\frac{\partial}{\partial i} \int_{\varepsilon(c,i)}^{\infty} \mathbb{V}(A(c, i, \varepsilon), \eta; G')^{1-\gamma} f(\varepsilon) \, d\varepsilon = -\mathbb{V}(\tilde{a}, \eta; G')^{1-\gamma} f(\varepsilon(c,i)) \cdot \frac{\partial \varepsilon(c,i)}{\partial i} \tag{3.C.37}
\]

\[+(1-\gamma) \int_{\varepsilon(c,i)}^{\infty} \mathbb{V}(A(c, i, \varepsilon), \eta; G')^{-\gamma} \frac{\partial \mathbb{V}(A(c, i, \varepsilon), \eta; G')}{\partial A(c, i, \varepsilon)} \cdot \frac{\partial A(c, i, \varepsilon)}{\partial i} f(\varepsilon) \, d\varepsilon.
\]

Take the equation (3.C.36) and (3.C.37) to the equation (3.C.35) and we have
\[
\frac{\partial \mathcal{H}(\cdot)}{\partial i} = (1-\gamma) \int_{\mathcal{E}(\cdot,i)}^{\infty} \left[ \nabla(A(c,i,\varepsilon),\eta; \mathcal{G})^{-\gamma} \cdot \frac{\partial \nabla(A(c,i,\varepsilon),\eta; \mathcal{G})}{\partial A(c,i,\varepsilon)} \cdot \frac{\partial A(c,i,\varepsilon)}{\partial i} \right] f(\varepsilon) \, d\varepsilon + f(\mathcal{E}(c,i)) \cdot \frac{\partial \mathcal{E}(c,i)}{\partial i} \left[ \nabla(w,\eta; \mathcal{G})^{1-\gamma} - \nabla(\tilde{a},\eta; \mathcal{G})^{1-\gamma} \right].
\]

(3.C.38)

By the Proposition 3.C.8, the function \( A(\cdot) \) is continuously differentiable in \( \eta \in I^+ \). Therefore, if the value function \( V(\cdot) \) is continuously differentiable in \( \eta \), the first-order derivative (3.C.38) is well defined.

By the equation (3.C.34), if \( i^* \in I^+ \) exists, it must solve

\[
\lambda_{\eta\mathcal{H}} \frac{\partial \mathcal{H}(i^*; \eta)}{\partial i^*} + \lambda_{\eta\mathcal{L}} \frac{\partial \mathcal{H}(i^*; 0)}{\partial i^*} = 0.
\]

Taking the representation of (3.C.38) to the equation above and we obtain

\[
0 = (1 - \gamma) \int_{\mathcal{E}(c,i^*)}^{\infty} \nabla(c,i^*,\varepsilon) \frac{\partial A(c,i^*,\varepsilon)}{\partial i^*} f(\varepsilon) \, d\varepsilon + f(\mathcal{E}(c,i^*)) \cdot \frac{\partial \mathcal{E}(c,i^*)}{\partial i^*} \cdot \Theta(c,i^*).
\]

(3.C.39)

Since \( \varepsilon \sim \mathcal{N}(-\sigma^2/2, \sigma^2) \), hence \( f(\varepsilon) = \frac{1}{\sigma} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right) \). Take this to the equation (3.C.39), we obtain the equation (3.C.31). \( \blacksquare \)

To put the first-order condition into numerical calculation, one needs to obtain the explicit form of the second term of the equation (3.C.31). To obtain the partial derivative of \( \mathcal{E}(\cdot) \) when \( c + i > a \), we perform the following calculation and apply the Proposition 3.C.1 and the Lemma 3.C.4:

\[
\frac{\partial \mathcal{E}(c,i)}{\partial i} = \frac{\partial}{\partial i} \log \left( (1 + \pi)L \right) = \frac{1}{(1 + \pi)L} \left[ (1 + \pi) \frac{\partial L}{\partial i} + L \frac{\partial \pi}{\partial i} \right]
\]

\[
= \frac{1}{L} \cdot \frac{\partial L}{\partial i} + \frac{1 + \pi}{1 + \pi} \cdot \frac{\partial \pi}{\partial i}
\]

\[
= \frac{1}{s} \cdot \frac{(1 - \nu) + \nu(a - c)i^{-1}}{g'(L)}
\]

\[
= \frac{1}{s} \cdot \frac{(1 - \nu) + \nu(a - c)i^{-1}}{g'(L)} \cdot \frac{1}{(1 + \pi)L} \cdot \Phi \left( \frac{\log \left( (1 + \pi)L - \sigma^2/2 \right)}{\sigma} \right) - 1 - \Phi \left( \frac{\log \left( (1 + \pi)L + \sigma^2/2 \right)}{\sigma} \right)
\]
where $g'(L) = 1 - \theta L\xi (1 + \zeta)$. Applying the non-linear equation for interest rate (Proposition 3.3.1) we obtain that

$$
\frac{\partial \mathcal{E}(c, i)}{\partial i} = -\frac{(1 - \nu) + \nu(a - c)i^{-1}}{(1 + \pi)s g'(L)} \cdot \frac{1 + r^f}{1 - \Phi\left(\frac{\log \left(\frac{(1+\pi)\xi + \sigma^2/2}{\sigma}\right)}{\sigma}\right)}.
$$

(3.C.40)

Taking the equation (3.C.40) to the equation (3.C.31) and we obtain

$$
0 = \int_{\mathcal{E}(c,i^*)}^\infty \mathbb{D}(c, i^*, \epsilon) \frac{\partial \mathcal{A}(c, i^*, \epsilon)}{\partial i^*} \phi\left(\frac{\epsilon + \sigma^2/2}{\sigma}\right) \text{d}\epsilon
- \frac{1 + r^f}{1 - \gamma} \cdot \frac{(1 - \nu) + \nu(a - c)i^{-1}}{s(1 + \pi)g'(L)} \cdot \phi\left(\frac{\mathcal{E}(c,i^*) + \sigma^2/2}{\sigma}\right) \cdot \mathbb{C}(c, i^*);
$$

(3.C.41)

or we can write

$$
0 = \int_{\mathcal{E}(c,i^*)}^\infty \mathbb{D}(c, i^*, \epsilon) \frac{\partial \mathcal{A}(c, i^*, \epsilon)}{\partial i^*} \phi\left(\frac{\epsilon + \sigma^2/2}{\sigma}\right) \text{d}\epsilon
- \frac{1 + r^f}{1 - \gamma} \cdot \frac{(1 - \nu) + \nu(a - c)i^{-1}}{s(1 + \pi)g'(L)} \cdot m\left(-\frac{\mathcal{E}(c,i^*) + \sigma^2/2}{\sigma}\right) \cdot \mathbb{C}(c, i^*),
$$

(3.C.42)

where $m(\cdot) = \phi(\cdot)/\Phi(\cdot)$ is the inverse Mills ratio. When $c + i^* \leq a$, the second term of the equation (3.C.41) and (3.C.42) vanishes, and the first-order condition becomes

$$
0 = \mathbb{D}(c, i^*, \epsilon) \frac{\partial \mathcal{A}(c, i^*, \epsilon)}{\partial i^*} \phi\left(\frac{\epsilon + \sigma^2/2}{\sigma}\right).
$$

We now derive the first-order condition of consumption. If $\gamma > 1$ and $\mu > 1$, after the investment is chosen, the household chooses the consumption $c^*$, such that

$$
c^* = \arg\min_{c \in C^*} \left\{(1 - \beta)c^{1 - \mu} + \beta \left(\lambda_{\eta H} \int_{\mathbb{V}} \mathbb{V}(a', \eta; \mathcal{G}')^{1 - \gamma} f(\epsilon) \text{d}\epsilon + \lambda_{\eta L} \int_{\mathbb{V}} \mathbb{V}(a', 0; \mathcal{G}')^{1 - \gamma} f(\epsilon) \text{d}\epsilon\right)^{\frac{1 - \mu}{1 - \gamma}}\right\}.
$$
In a similar manner as is in the proof of the Proposition 3.C.10, we can derive the first-order necessary condition for consumption. The next proposition provides the formulation of the first-order condition, if the optimal consumption exists in $C^+$.

**PROPOSITION 3.C.10.** (First-order Necessary Condition for Consumption) Suppose that the value function $V(a, \eta; G)$ is continuously differentiable in $a$. Given $a > 0$ and $\eta$, and for given $i \in (0, \tilde{i})$, if $c^* \in C^+$ exists, $c^*$ must solve

$$0 = \sigma (1 - \beta) (c^*)^{-\mu} + \beta \frac{1}{1 - \gamma} \cdot \mathbb{CE} \left( V(a', \eta'; G') \right)^{\gamma - \mu} \cdot \left[ \phi \left( \frac{\mathcal{E}(c^*, i) + \sigma^2/2}{\sigma} \right) \cdot \frac{\partial \mathcal{E}(c^*, i)}{\partial c^*} \cdot \mathcal{C}(c^*, i) + (1 - \gamma) \int_{\mathcal{D}(c^*, i, \varepsilon)}^{\infty} \mathcal{D}(c^*, i, \varepsilon) \frac{\partial \mathcal{A}(c^*, i, \varepsilon)}{\partial c^*} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right) d\varepsilon \right],$$

where $\mathbb{CE}(\cdot)$ is the certainty equivalence of the future value, and

$$\mathbb{CE} \left( V(a', \eta'; G') \right) = \left[ \lambda_{\eta H} \int_{a}^{a'} V(a', \eta'; G')^{-1 - \gamma} f(\varepsilon) d\varepsilon + \lambda_{\eta L} \int_{a}^{0} V(a', 0; G')^{-1 - \gamma} f(\varepsilon) d\varepsilon \right]^{1/\gamma},$$

and $\mathcal{D}(c^*, i, \varepsilon)$ is similar as (3.C.32) and is

$$\mathcal{D}(c^*, i, \varepsilon) = \lambda_{\eta H} V(A(c^*, i, \varepsilon), \eta; G')^{-\gamma} \cdot \frac{\partial V(A(c^*, i, \varepsilon), \eta; G')}{\partial A(c^*, i, \varepsilon)} + \lambda_{\eta L} V(A(c^*, i, \varepsilon), 0; G')^{-\gamma} \cdot \frac{\partial V(A(c^*, i, \varepsilon), 0; G')}{\partial A(c^*, i, \varepsilon)}.$$  

Note that we set $\mathcal{E}(c^*, i) = -\infty$ if $c^* + i \leq a$. Note that the partial term $\frac{\partial \mathcal{A}(\cdot)}{\partial c^*}$ does not depend on the random term $\varepsilon$.

**Proof** The first-order condition writes

$$0 = (1 - \mu)(1 - \beta)c^{-\mu} + \beta \frac{1}{1 - \gamma} \cdot \mathbb{CE} \left( V(a', \eta'; G') \right)^{\gamma - \mu} \cdot \left[ \phi \left( \frac{\mathcal{E}(c^*, i) + \sigma^2/2}{\sigma} \right) \cdot \frac{\partial \mathcal{E}(c^*, i)}{\partial c^*} \cdot \mathcal{C}(c^*, i) + (1 - \gamma) \int_{\mathcal{D}(c^*, i, \varepsilon)}^{\infty} \mathcal{D}(c^*, i, \varepsilon) \frac{\partial \mathcal{A}(c^*, i, \varepsilon)}{\partial c^*} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right) d\varepsilon \right],$$

re-arrange and we obtain
Chapter 3. Intermediation Cost, Credit Expansion and Inequality

\[ 0 = (1 - \beta)c^{-\mu} + \frac{\beta}{1 - \gamma} \cdot \mathbb{E} \left( \mathcal{V}(a', \eta'; \mathcal{G}') \right)^{\gamma - \mu}. \]  

(3.C.45)

\[
\frac{\partial}{\partial c} \left( \lambda_{\eta H} \int_{\mathcal{E}} \mathcal{V}(a', \eta'; \mathcal{G}')^{1 - \gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_{\mathcal{E}} \mathcal{V}(a', 0; \mathcal{G}')^{1 - \gamma} f(\varepsilon) \, d\varepsilon \right).
\]

Using the same method as the proof of the Proposition 3.C.9., we have

\[
\frac{\partial}{\partial c} \left( \lambda_{\eta H} \int_{\mathcal{E}} \mathcal{V}(a', \eta'; \mathcal{G}')^{1 - \gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_{\mathcal{E}} \mathcal{V}(a', 0; \mathcal{G}')^{1 - \gamma} f(\varepsilon) \, d\varepsilon \right) = (1 - \gamma) \int_{\mathcal{E}(c,i)} \mathbb{D}(c, i, \varepsilon) \frac{\partial A(c, i, \varepsilon)}{\partial c} f(\varepsilon) \, d\varepsilon + f(\mathcal{E}(c, i)) \cdot \frac{\partial \mathcal{E}(c, i)}{\partial c} \cdot \mathbb{C}(c, i),
\]

which is taken to the equation (3.C.43), and we obtain the equation (3.C.43).

We need again to determine an explicit form of the first-order condition above. By Proposition 3.C.1 and Lemma 3.C.4, when \( c + i > a \), we see that

\[
\frac{\partial \mathcal{E}(c, i)}{\partial c} = \frac{\partial}{\partial c} \log \left( (1 + \pi)L \right) = \frac{1}{(1 + \pi)L} \left( (1 + \pi) \frac{\partial L}{\partial c} + L \frac{\partial \pi}{\partial c} \right)
\]

\[= \frac{1}{L} \cdot \frac{\partial L}{\partial c} + \frac{1}{1 + \pi} \cdot \frac{\partial \pi}{\partial c}
\]

\[= -\frac{1}{s \, g'(L)} - \frac{1}{s \, g'(L)} \cdot \frac{1}{(1 + \pi)L} \cdot \frac{\Phi \left( \log \left( \frac{1 + \pi L}{\sigma} \right) - \sigma^2/2 \right)}{1 - \Phi \left( \log \left( \frac{1 + \pi L + \sigma^2/2}{\sigma} \right) \right)}
\]

(3.C.46)

\[= -\frac{1}{(1 + \pi)s \, g'(L)} \cdot \left( (1 + \pi) + \frac{1}{L} \cdot \frac{\Phi \left( \log \left( \frac{1 + \pi L}{\sigma} \right) - \sigma^2/2 \right)}{1 - \Phi \left( \log \left( \frac{1 + \pi L + \sigma^2/2}{\sigma} \right) \right)} \right)
\]

\[= -\frac{1}{(1 + \pi)s \, g'(L)} \cdot \frac{1 + r_f}{1 - \Phi \left( \log \left( \frac{1 + \pi L + \sigma^2/2}{\sigma} \right) \right)}.
\]

Take the equation (3.C.46) into the FOC (3.C.43) and we obtain
\[
0 = \sigma(1 - \beta)(e^*)^{-\mu} + \beta \mathbb{E}\left( \bar{V}(a', \eta'; \mathcal{G}') \right)^{\gamma - \mu} \cdot \int_{\mathcal{E}(e^*, i)}^{\infty} \mathbb{D}(c^*, i, \varepsilon) \frac{\partial A(e^*, i, \varepsilon)}{\partial c^*} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right) \, d\varepsilon,
\]

\[
- \left( \frac{1 + r^f}{s(1 - \gamma)(1 + \pi)} g'(L) \right) \cdot \frac{\phi \left( \frac{\mathcal{E}(c^*, i) + \sigma^2/2}{\sigma} \right)}{1 - \Phi \left( \frac{\mathcal{E}(c^*, i) + \sigma^2/2}{\sigma} \right)} \cdot \mathbb{C}(c^*, i),
\]

(3.C.47)

or can we write

\[
0 = \sigma(1 - \beta)(e^*)^{-\mu} + \beta \mathbb{E}\left( \bar{V}(a', \eta'; \mathcal{G}') \right)^{\gamma - \mu} \cdot \int_{\mathcal{E}(e^*, i)}^{\infty} \mathbb{D}(c^*, i, \varepsilon) \frac{\partial A(e^*, i, \varepsilon)}{\partial c^*} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right) \, d\varepsilon
\]

\[
- \left( \frac{1 + r^f}{s(1 - \gamma)(1 + \pi)} g'(L) \right) \cdot m \left( \frac{\mathcal{E}(c^*, i) + \sigma^2/2}{\sigma} \right) \cdot \mathbb{C}(c^*, i),
\]

(3.C.48)

where \( m(\cdot) = \phi(\cdot)/\Phi(\cdot) \) is the inverse Mills ratio. When \( c^* + i \leq a \), the first-order condition (3.C.47) and (3.C.48) becomes

\[
0 = \sigma(1 - \beta)(e^*)^{-\mu} + \beta \mathbb{E}\left( \bar{V}(a', \eta'; \mathcal{G}') \right)^{\gamma - \mu} \cdot \mathbb{D}(c^*, i, \varepsilon) \frac{\partial A(e^*, i, \varepsilon)}{\partial c^*} \phi \left( \frac{\varepsilon + \sigma^2/2}{\sigma} \right).
\]

The envelope condition for the state

The equation (3.C.31) and (3.C.43) defines a nonlinear equation system for \( e^* \in \mathcal{C}^+ \) and \( i^* \in \mathcal{I}^+ \). In the numerical exercise, the non-linear equation system will be solved. To solve this, one needs to know \( \bar{V}(a', \eta'; \mathcal{G}') \) as well as its derivative with respect to \( a' \). The value function \( \bar{V}(\cdot) \) can be updated through value function iterations. But we still need a way to update the \( \frac{\partial \bar{V}(\cdot)}{\partial a} \). In the next proposition, we use the envelope condition to formulate the \( \frac{\partial \bar{V}(\cdot)}{\partial a} \), which can be used to update the partial derivative of the value.

**PROPOSITION 3.C.11.** Suppose that the value function \( \bar{V}(a, \eta; \mathcal{G}) \) is continuously differentiable in \( a \). Given \( a > 0 \) and \( \eta \), if \( c \in \mathcal{C}^+ \) and \( i^* \in \mathcal{I}^+ \) both exists,

\[
\frac{\partial \bar{V}(a, \eta; \mathcal{G})}{\partial a} = (1 - \beta)(e^*)^{-\mu} \cdot \bar{V}(a, \eta; \mathcal{G})^{\mu / \rho}.
\]

(3.C.49)

**Proof** It is not hard to notice that

\[
\frac{\partial \mathcal{E}(c, i)}{\partial a} = - \frac{\partial \mathcal{E}(c, i)}{\partial c}.
\]

(3.C.50)
Moreover, we have \( \frac{\partial A(\cdot)}{\partial a} = -\frac{\partial A(\cdot)}{\partial c} \). Therefore, it is easy to see that

\[
\frac{\partial}{\partial a} \left( \lambda_{\eta H} \int_{E} \mathcal{V}(a', \eta; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_{E} \mathcal{V}(a', 0; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon \right)
= -\frac{\partial}{\partial c} \left( \lambda_{\eta H} \int_{E} \mathcal{V}(a', \eta; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_{E} \mathcal{V}(a', 0; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon \right). \tag{3.C.51}
\]

In the optimum, the first-order derivative of the value with respect to the state is

\[
\frac{\partial \mathcal{V}(a, \eta; G)}{\partial a} = \frac{\beta}{1 - \gamma} \cdot \mathcal{V}(a, \eta; G)^{\frac{\mu}{1-\gamma}} \mathcal{EB} \left( \mathcal{V}(a', \eta; G') \right)^{\gamma - \mu} \cdot \frac{\partial}{\partial a} \left( \lambda_{\eta H} \int_{E} \mathcal{V}(a', \eta; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon + \lambda_{\eta L} \int_{E} \mathcal{V}(a', 0; G')^{-1-\gamma} f(\varepsilon) \, d\varepsilon \right).
\]

By the condition (3.C.51), we can apply the equation (3.C.45) to write the equation above into the envelope condition (3.C.49).

\[\blacksquare\]

### 3.C.4 Numerical Algorithm

On the individual-level, we define two evenly distributed grid-vectors of cash-on-hand \( A^H \in [w, a^{sup}] \) and \( A^L \in [w, a^{sup}] \) for the H-type and L-type, respectively. On each grid point, we define a search grid of consumption \( C \in [\chi^a, \chi^a] \) to perform the global search for the optimal level of consumption, where \( \chi^a \) is close to 0 and \( \chi^a \) is close to 1. Let \( G_0 \) be the initial guess of the vector of the aggregate state, and let \( \mathcal{V}_{0}^H(a, G_0) \) and \( \mathcal{V}_{0}^L(a, G_0) \) denote the initial guesses for the value functions of the H- and L-type, respectively. Moreover, we let \( C_{0}^H(a, G_0) \) and \( C_{0}^L(a, G_0) \) denote the initial guesses of consumption for the H- and L-type, respectively. On each grid point on \( A^H \) or \( A^L \) and on each search point on \( C \), the optimal investment (given consumption) can be obtained by solving the non-linear equation (3.C.31). The term of \( \frac{\partial \mathcal{V}(\cdot)}{\partial a} \) in the non-linear equation (3.C.31) can be obtained through the envelop condition (3.C.49). On each grid point of the cash-on-hand, we find the consumption \( C_{1}^H(a, G_0) (C_{1}^L(a, G_0)) \), such that \( C_{1}^H(a, G_0) (C_{1}^L(a, G_0)) \) and the implies optimal investment correspond to the maximal level of the value \( \mathcal{V}_{1}^H(a, G_0) (\mathcal{V}_{1}^L(a, G_0)) \). We repeat this procedure until both values and consumptions converge. Values and consumptions are interpolated using Akima cubic-splines. Expected continuation values are computed using Gaussâ€”Hermite quadrature points and weights.

After obtaining household’s optimal policies, we iterate over the distribution of cash-on-hand in the economy. For this purpose, we define a much larger grid \( A^H \in [w, a^{sup}] \) and \( A^L \in [w, a^{sup}] \) for the H- and L-type, respectively. As initial guesses, we assign equal probability for each grid point on \( A^H (A^L) \). Using household’s policy functions and the probability distribution, we calculate the probability that households end up on each grid.
point of $A_H$ ($A_L$) in the next period. We iterate until the probability distribution converges. The aggregate state $G_0$ can thus be updated to $G_1$, based on the probability distribution and household’s policy functions. Repeat the steps above, until aggregate state $G$ converges.
Gesamtzusammenfassung

In dieser Arbeit präsentieren wir drei Aufsätze zur empirischen und quantitativen Makroökonomie.


Der Teufelskreis erklärt die Zeitreihen-Korrelation zwischen der Schattenwirtschaft und der Staatsverschuldung: die Staatsverschuldung und die Renditen von Staatsanleihen sind tendenziell höher (niedriger), wenn der Umfang der Schattenwirtschaft steigt (fällt). Unsere empirische Analyse im Abschnitt 1.2 weist ebenfalls auf eine Querschnittskorrelation hin. Nämlich Länder mit weitere (geringere) Schattenwirtschaften besitzen in der Regel größeren (kleineren) Schuldenstand im Verhältnis zum BIP, sowie höhere (niedrigere) Rendite der Staatsanleihe. Um zu sehen, ob unser Modell die Querschnittskorrelation auch erklärt, können wir unser Modell mit unterschiedlichen Größenordnungen der Schattenwirtschaft kalibrieren und schauen, was unser Modell vorhersagt.

Im Kapitel 2 schlagen wir eine Regressionsmethode vor, um die zeitliche Entwicklung der Vermittlungskosten für externe Firmenkredite zu analysieren. Wir definieren die


Gesammtzusammenfassung

die allgemeine Wohlfahrt sowie die Wohlfahrt der wohlhabenden Haushalte, während die Wohlfahrt der untersten 90% Haushalten, gemessen am Vermögen, abnimmt.


Eigenabgrenzung

Das erste Kapitel habe ich zusammen mit Almuth Scholl verfasst (Universität Konstanz). Meine individuelle Leistung bei der Erstellung dieses Kapitel beträgt 50%.

Ich versichere hiermit, dass ich Kapitel 2 und Kapitel 3 ohne Hilfe Dritter verfasst habe.
References


References


