

Is energy conserved when nobody looks?

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Conservation principles are essential to describe and quantify classical and quantum mechanical processes. Classically, the conservation holds objectively because the description of reality can be considered independent of observation. In quantum mechanics, however, invasive observations change quantities drastically, even if they are expected to be conserved classically. One may hope to overcome this problem by considering weak, non-invasive quantum measurements. Interestingly, we find that the non-conservation is manifest even in weakly measured correlations if some of the observables don't commute with the conserved quantity. Our observation casts some doubt on the fundamental compatibility of conservation laws and quantum objectivity.

Introduction Conserved quantities play an important role in both classical and quantum mechanics. According to the classical Noether theorem the invariance of the dynamics of a system under specific transformations [1] implies the conservation of certain quantities: translation symmetry in time and space result in energy and momentum conservation, respectively rotational symmetry in angular momentum conservation and gauge invariance leads to a conserved charge. In quantum mechanics the observables (in the Heisenberg picture) are time-independent when they commute with the Hamiltonian. Furthermore, some conserved quantities, like total charge, commute also with all observables. We shall call them superconserved. Classically all conserved quantities are also superconserved. In high energy nomenclature the former are known as *on-shell* conserved whereas the latter are called *off-shell* conserved [2]. The concept of superconservation is closely related to the superselection rule which constitutes an additional postulate that the set of observables is restricted to those commuting with the superconserved operators [3].

Conservation principles become less obvious when one tries to verify them experimentally. While an ideal classical measurement will keep the relevant quantities unchanged, neither a nonideal classical nor any quantum measurement will necessarily reflect the conservation exactly. Even the smallest interaction between the system and the measuring device (detector) may involve a transfer of the conserved quantity. The system might become a coherent superposition of states with different values of a conserved quantity (e.g. energy), or in the case of a superconserved quantity an incoherent mixture (e.g. charge). The problem of proper modeling of the measurement of quantities incompatible with conserved ones has been noticed long ago by Wigner, Araki and Yanase [4], later been discussed in the context of consistent histories [5] and modular values [6], till recent research [7]. The generation, measurement and control of quantum conserved quantities, in particular angular momentum, has become interesting recently, both experimentally and theoretically [8–10].

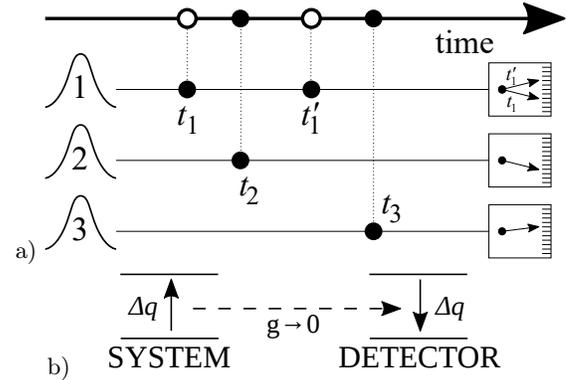


FIG. 1: a) Three weak measurements. The detectors are initially independent and couple instantly to the system at t_1 (or t'_1), t_2 and t_3 . The conserved quantity is measured (empty circles) either at time $t_1 < t_2$ or $t'_1 > t_2$. The outcomes 1 and 1' inferred from the three-point correlator might differ, even for a conserved quantity in the quantum case. b) Failure of conservation in the weak measurement. The transfer Δq of the conserved quantity q between the system and detector does not scale with the coupling g .

The objectivity of conservation is one aspect of the general concern of Einstein [11] and Mermin [12] if the (quantum) Moon exists when nobody looks. Objective observations should be represented by weak system-detector interaction. However, the objectivity based on weak measurements can lead to unexpected results such as weak values [13] or the violation of the Leggett-Garg inequality [14–16]. Since weak quantum measurements are the closest counterpart of classical measurements, they should define objective reality and the conservation principles are expected to hold in systems with an appropriate invariance.

In this Letter, we will show that for quantum measurements in the weak limit superconservation holds but quantities such as energy, momentum and angular momentum apparently violate conservation in this limit even if an appropriate symmetry results in a classical conservation law. The violation of conservation appears in third-

order time correlations as we illustrate in simple model systems (Fig. 1). We also propose a feasible experimental probing position and magnetic field of a charge in a circular trap, related to angular momentum. The violation is caused by another observable not commuting with the conserved one. We write down an operational criterion to witness the violation of a conservation principle and discuss when it is satisfied. In addition, a Leggett-Garg-type test of objective realism can be performed, even if the conservation or measurement of the quantity is only approximate.

Superconservation Operationally, the quantity \hat{Q} , defined within the system is conserved when $[\hat{H}, \hat{Q}] = 0$ for the system's Hamiltonian \hat{H} and superconserved if additionally $[\hat{A}, \hat{Q}] = 0$ for every measured \hat{A} . Let us assume the decomposition $\hat{Q} = \sum_q q \hat{P}_q$ where \hat{P}_q are (mutually commuting) projections onto the eigenspace of the value q (i.e. $\hat{Q}|\psi_q\rangle = q|\psi_q\rangle$). Now, the superselection postulate says that the state of the system $\hat{\rho}$ is always an incoherent mixture $\sum_q \hat{P}_q \hat{\rho} \hat{P}_q$, if \hat{Q} is superconserved. Then the projective measurement of \hat{A} will not alter the q -eigenspace as there exists a decomposition $\hat{A} = \sum_{q,a} a \hat{P}_{qa}$ with \hat{P}_{qa} being the projection onto the joint eigenspace of \hat{Q} and \hat{A} with respective eigenvalues q and a . For instance, if the initial state is already a q -eigenstate then it will remain such an eigenstate after the projection. For general measurements, positive operator-valued measures (POVM), represented by Kraus operators \hat{K}_c (the index c can represent an eigenvalue of \hat{A} , \hat{Q} or both but in general it can be arbitrary) such that $\sum_c \hat{K}_c^\dagger \hat{K}_c = \hat{1}$, the state $\hat{\rho}$ will collapse to $\hat{\rho}_c = \hat{K}_c \hat{\rho} \hat{K}_c^\dagger$, normalized by the probability $\text{Tr} \hat{\rho}_c$. In principle \hat{K}_c can act within q -eigenspaces, i.e. $c = qa$ and $\hat{K}_{qa} = \hat{P}_q \hat{K}_a \hat{P}_q$. In the most general case the superconserving Kraus operator reads $\hat{K}_{q'aq} = \hat{P}_{q'} \hat{K}_a \hat{P}_q$. It means that the superconserved value can change but the system remains an incoherent mixture of q -eigenstates. This applies e.g. to a charge measurement in a quantum dot (which is superconserved), where the charge can leak out into an incoherent bath. The (normally) conserved quantities do not impose any additional postulates so the state can be a coherent superposition of the states of different values of energy, angular momentum, etc. A projective measurement of \hat{A} which does not commute with \hat{Q} is enough to turn a q -eigenstate into a superposition. Now, if we try to postulate a POVM with superconserving Kraus operators then the actual measured average involves a linear combination of $\sum_{qq'} \hat{P}_q \hat{K}_a^\dagger \hat{P}_{q'} \hat{K}_a \hat{P}_q$ so it must commute with \hat{Q} which would become superconserved. The unavoidability of coherent superpositions of only conserved values is the key problem considered here.

Nonconservation in weak measurement We shall work with the established concept of quantum weak measurement [13] where

$$\hat{K}_a = (2g/\pi)^{1/4} \exp(-g(\hat{A} - a)^2), \quad (1)$$

with $g \rightarrow 0$. Note that this construction is still correct in the superconserved case because \hat{A} , \hat{K}_a and the state $\hat{\rho}$ is commuting with \hat{Q} so \hat{K}_a splits into a simple sum of \hat{K}_{qa} . The actual form of \hat{K}_a can be different but the outcome is almost independent in the limit $g \rightarrow 0$. In the lowest order we can also neglect all $\hat{K}_{q'aq}$. In the $g \rightarrow 0$ limit n -correlation of the sequence of measurements \hat{A} , \hat{B} , \hat{C} , reads [17, 18]

$$\begin{aligned} \langle a(t) \rangle &= \langle \hat{A}(t) \rangle, \quad \langle a(t_1)b(t_2) \rangle = \langle \{\hat{A}(t_1), \hat{B}(t_2)\} \rangle / 2 \\ \langle a(t_1)b(t_2)c(t_3) \rangle &= \langle \{\hat{A}(t_1), \{\hat{B}(t_2), \hat{C}(t_3)\} \} \rangle / 4 \end{aligned} \quad (2)$$

for $t_1 < t_2 < t_3$ with the anticommutator $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$.

The conservation in the weak measurement limit means that the measurable correlations involving the conserved quantity $q(t)$ corresponding to $\hat{Q}(t) = \hat{Q}$ will not depend on t . It is true at the single average, where $\langle q(t) \rangle = \langle \hat{Q} \rangle$. Interestingly, also for second order correlations the order of measurements has no influence on the result, since $\langle q(t_1)a(t_2) \rangle = \langle \{Q, A(t_2)\} \rangle$ is independent of t_1 . However, the situation changes for the three consecutive measurements (see Fig. 1), since in the last line of (2) the time order of operators matters, which has been demonstrated also experimentally [19]. Considering the difference of two measurement sequences $Q \rightarrow A \rightarrow B$ and $A \rightarrow Q \rightarrow B$, we obtain

$$\begin{aligned} &\langle \{\hat{Q}, \{\hat{A}(t_2), \hat{B}(t_3)\} \} \rangle / 4 - \langle \{\hat{A}(t_2), \{\hat{Q}, \hat{B}(t_3)\} \} \rangle / 4 \\ &= \langle [[\hat{Q}, \hat{A}(t_2)], \hat{B}(t_3)] \rangle / 4 \equiv \langle \Delta qa(t_2)b(t_3) \rangle. \end{aligned} \quad (3)$$

This quantity will show up as jump at $t_1 = t_2$, when measuring $\langle q(t_1)a(t_2)b(t_3) \rangle$. The jump will be non-zero for Q not commuting with A and B . Obviously, for superconserved quantities Q (commuting with every measurable observable) the jump is absent. The violation of the conservation principle is caused by the measurement of \hat{A} – not commuting with \hat{Q} – which allows transitions between spaces of different q with the jump size Δq not scaled by the coupling g , see Fig. 1. This difference is transferred to the detector, assuming that the total quantity (of the system and detector) is conserved regardless of the system-detector interaction.

As an example we can take the basic two-level system ($|\pm\rangle$ basis) with the Hamiltonian $\hat{H} = \hat{Q} = \hbar\omega|+\rangle\langle+|$ and $\hat{A} = \hat{B} = \hat{X} = |+\rangle\langle-| + |-\rangle\langle+|$. Then, with $\omega > 0$ the ground state is $|-\rangle$ and the third order correlation $\langle h(t_1)x(t_2)x(t_3) \rangle$ at zero temperature for $t_3 > t_{1,2}$ reads $\hbar\omega(1 - \theta(t_2 - t_1)) \cos(\omega(t_2 - t_3))/2$. The jump is $\langle \Delta hx(0)x(\tau) \rangle = \hbar\omega \cos(\omega\tau)/2$. The result can be generalized to finite temperature

$$\langle \Delta hx(0)x(\tau) \rangle = \hbar\omega \cos(\omega\tau) \tanh(\hbar\omega/2kT)/2. \quad (4)$$

For increasing temperature the jump diminishes as illustrated in Fig. 2.

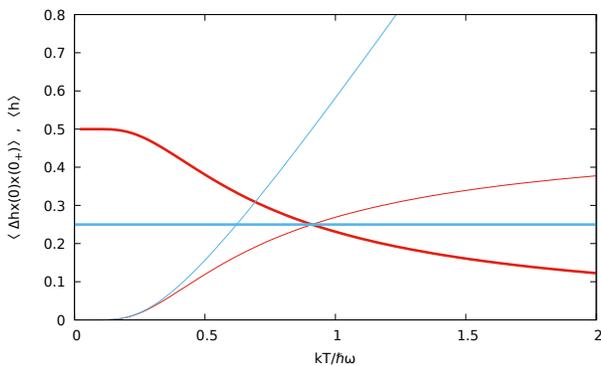


FIG. 2: The non-conserving jump for $\tau = 0^+$ (thick lines) compared to the average energy (thin lines) for the two-level system (red) and the harmonic oscillator (blue). At high temperatures the jump becomes unobservable and the classical conservation is restored. All quantities are normalized to $\hbar\omega$.

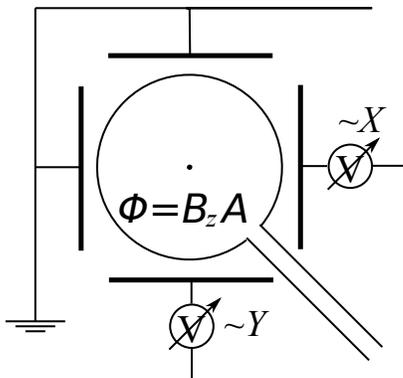


FIG. 3: A trap for a charge invariant under rotation about z axis with a detector weakly coupled to angular momentum by magnetic field B in z direction and position in xy plane detected by capacitors. Magnetic flux $\Phi = B_z A$ is measured by SQUID surrounding a loop of area A

Another basic example is the harmonic oscillator with $\hat{H} = \hat{Q} = \hbar\omega\hat{a}^\dagger\hat{a}$ with $[\hat{a}, \hat{a}^\dagger] = 1$. Taking the dimensionless position $\sqrt{2}\hat{X} = \hat{a}^\dagger + \hat{a} = \hat{A} = \hat{B}$, we find for the jump $\langle \Delta h x(0)x(t) \rangle = -\hbar\omega \cos(\omega t)/4$ independent of the state of the system. As illustrated in Fig. 2, the jump becomes unobservable at high temperatures since the average energy $\langle h \rangle = \hbar\omega/[\exp(\hbar\omega/kT) - 1]$ increases with temperature.

The previously discussed very simple examples illustrate the fundamental finding of our manuscript. If one tries to verify the conservation of energy while measuring other observable not commuting with the Hamiltonian, it is possible to find a violation of the energy conservation. It constitutes a pure quantum effect since it vanishes at high temperature, where the classically expected conservation holds. One could object that performing a series of measurements already breaks time-translational symmetry and therefore energy is not conserved. We will

illustrate in the following that this is not the case by considering other conserved observables like angular momentum in a rotationally invariant system.

Angular momentum conservation We propose an experiment to demonstrate the failure of a conservation principle for angular momentum in third order correlations in weak measurements. Instead of energy we consider one component of angular momentum, say $\hat{L}_z = \hat{X}\hat{P}_y - \hat{Y}\hat{P}_x$ which can be measured for a charged spinless particle (e.g. α) of magnetic moment μ_B by a Zeeman coupling to the magnetic field $\mu_B\hat{L}_z\hat{B}_z$. In other words \hat{L}_z will be proportional to a magnetic field generated by the particle and can in principle be detected by a sensitive magnetometer (e.g. a superconducting quantum interferometer device). The other two observables will be the particle's positions \hat{X} and \hat{Y} which can be measured e.g. by the voltage of a capacitor depending linearly on x and y for small changes in position, see the setup sketch in Fig. 3. The two positions x and z will be measured at times t_2 and t_3 , respectively.

The quantity of matter is $\langle l_z(t_1)x(t_2)y(t_3) \rangle$. Suppose the particle is in a harmonic trap rotationally invariant about z axis. The xy part of the trap Hamiltonian reads $\hat{H}_\perp = \hbar\omega(\hat{a}_x^\dagger\hat{a}_x + \hat{a}_y^\dagger\hat{a}_y)$, with $[\hat{a}_{x,y}, \hat{a}_{x,y}^\dagger] = 1$, $[\hat{a}_x, \hat{a}_y] = [\hat{a}_x^\dagger, \hat{a}_y^\dagger] = 0$. Then $\sqrt{2}\hat{X} = \hat{a}_x^\dagger + \hat{a}_x$ and $\sqrt{2}\hat{P}_x/i\hbar = \hat{a}_x^\dagger - \hat{a}_x$ (rescaled by a length unit), similarly for y , and $\hat{L}_z = i\hbar(\hat{a}_x\hat{a}_y^\dagger - \hat{a}_y\hat{a}_x^\dagger)$. In the ground state $|0\rangle$ we have $\hat{L}_z|0\rangle = 0$ so only $\langle \hat{Y}(t_3)\hat{L}_z(t_1)\hat{X}(t_2) \rangle$ and $\langle \hat{X}(t_2)\hat{L}_z(t_1)\hat{Y}(t_3) \rangle$ contribute in (2). These terms can appear only when $t_2 < t_1$ or $t_3 < t_1$. We find

$$\begin{aligned} \langle l_z(t_1)x(t_2)y(t_3) \rangle &= (1 - \theta(t_2 - t_1)\theta(t_3 - t_1)) \times (5) \\ &\langle \hat{X}(t_2)\hat{L}_z(t_1)\hat{Y}(t_3) + \hat{Y}(t_3)\hat{L}_z(t_1)\hat{X}(t_2) \rangle / 4 = \\ &(1 - \theta(t_2 - t_1)\theta(t_3 - t_1))\hbar \sin \omega(t_2 - t_3) / 4 \end{aligned}$$

The jump is therefore given

$$\langle \Delta l_z x(t_2)y(t_3) \rangle_0 = \hbar \sin[\omega(t_2 - t_3)] / 4 \quad (6)$$

and is again state-independent. It illustrates that the angular momentum conservation is violated by this experiment. At finite temperature T for $t_1 < t_{2,3}$, the correlator $\langle l_z x(t_2)y(t_3) \rangle = \hbar \sin[\omega(t_2 - t_3)] / 4 \sinh^2(\hbar\omega/2k_B T)$ increases with temperature and makes the (temperature-independent) jump unobservable.

Since in this setup the detectors are coupled permanently, a frequency-domain measurement might be more appropriate. In frequency domain the observables are $A(\alpha) = \int dt e^{i\omega t} A(t)$. Taking all our previous arguments to frequency domain, the conservation of a quantity $\hat{Q}(\alpha)$ means that correlators vanish for $\alpha \neq 0$. Interestingly, transforming to frequency domain we find at zero temperature and for $\gamma, \alpha, \beta \neq 0$ that

$$\langle l_z(\gamma)x(\alpha)y(\beta) \rangle = \frac{i\pi\hbar\omega(\alpha - \beta)\delta(\gamma + \alpha + \beta)}{2(\alpha^2 - \omega^2)(\beta^2 - \omega^2)} \quad (7)$$

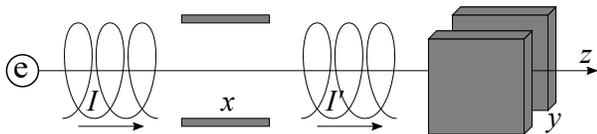


FIG. 4: A charged particle (e.g. electron) goes along the tube with angular momentum measured by current in either of two coils, I or I' while the position in x and y direction is measured by two perpendicular capacitors.

Clearly, the conservation principle for angular momentum is violated by (7). Either by time or frequency resolved measurements one should see experimentally non-conservation of angular momentum.

To realize a time-resolved measurement, we suggest to test conservation the angular momentum conservation with a charge moving inside a round tube along z direction, similar to the recent test of the order of measurements [19]. In the simplest model take $\hat{H} = \hat{H}_z + \hat{H}_\perp$ and we keep the same harmonic potential in the xy plane as above and add some $\hat{H}_z = -i\hbar v \partial_z$ with velocity v . Preparing a wavepacket as a product of the ground state of \hat{H}_\perp and $\psi(z)$ of sufficiently short width, we can measure essentially the same quantity (5) by putting a sequence of weak detectors along the tube, see Fig. 4. The angular momentum can be measured by the current signature in the coil, like in the recent experiment [8]. We simplify the coil-electron beam interaction to $\lambda(z)\hat{I}\hat{L}_z$ where λ only non-zero inside the coil. Similarly, the measurement of x and y can be modeled by local capacitive couplings. In this way, the measurement times are translated into position according to $t = z/v$. The jump (6) can then be detected by placing the coil at two different positions, see Fig. (4).

The above proposals face some practical challenges. The velocity v should be sufficiently high to ignore decoherence effects when Lindblad-type terms must be added to Hamiltonian dynamics. The test of conservation makes sense only for times/frequencies within coherence timescale. The measurement of x and y does not need to be precise. In fact, any observable roughly tracking the charge in two perpendicular directions will suffice. The tube may be not perfectly harmonic or inhomogeneous in z -direction, and \hat{L}_z can be only approximately conserved. This can be tested by measuring the difference $\Delta L_z = \hat{L}_z(t_1) - \hat{L}_z(t'_1)$ (for the two coils) assuring that the variance $\langle \Delta L_z^2 \rangle$ is small enough.

Leggett-Garg In the setup with four measurements, we can also construct a Leggett-Garg-type test of the conservation law. Let us consider the measurement of four observables: $q = q(t_1)$, $q' = q(t'_1)$, $x = x(t_2)$ and $y = y(t_3)$ with q being the conserved quantity and $t_1 < t_2 < t'_1 < t_3$. According to the macrorealism assumption, the values of (q, q', x, y) exist independent of the measurement. Hence, there is a corresponding

joint positive probability $\rho(q, q', x, y)$. Then correlations with respect to ρ must satisfy the following two Cauchy-Bunyakovsky-Schwarz inequalities

$$\begin{aligned} \langle (q - q')^2 \rangle_\rho \langle x^2 y^2 \rangle_\rho &\geq \langle (q - q') x y \rangle_\rho^2, \\ \langle (q - q')^2 y^2 \rangle_\rho \langle x^2 \rangle_\rho &\geq \langle (q - q') x y \rangle_\rho^2. \end{aligned} \quad (8)$$

In writing that we have implicitly assumed that the (classical) measurement is noninvasive, so it allows measuring each value independent of the other measurements. However, for quantum weak measurements the probabilities are found by applying the POVM (1). The measured probabilities are convoluted with a large detection noise $D(a) = (2g/\pi)^{1/2} e^{-2ga^2}$ in the limit $g \rightarrow 0$ for each observable q, q', x, y . Hence, the probability is given by $\rho(a) = \int da D(\bar{a} - a) \varrho(a)$ generalized to all observables. Here, $\varrho(q, q', x, y)$ has a well defined limit for $g \rightarrow 0$ but can be negative and constitutes a generalized quasiprobability. Hence, if in Eqs. (8) the correlators are taken after the detector noise is averaged out, the inequalities could be violated. Note that the correlators are then given by the quantum expectation values Eqs. (2). Now, the left hand sides of Eqs. (8) vanish. First, $\langle (q - q')^2 \rangle_e = 0$ because $\hat{Q}(t) = \hat{Q}$ is independent of time. Second, $\langle (q - q')^2 y^2 \rangle_e = 0$ because in addition y is measured after both q and q' . Classically, the measurements of the conserved quantity at two different time should not depend on whether another observable is measured in between and the right hand sides of Eqs. (8) vanish. On the other hand, the right hand side of (8) exactly corresponds to the quantum mechanical jump in the third-order correlator as defined in (6). Therefore, both inequalities will be violated in general. These violations can be readily tested in the setup suggested in Fig. 4. Note that the inequalities must involve fourth moments because of the so-called weak positivity [21] stating that lower moments are insufficient to violate macrorealism.

Conclusion We have shown that conservation laws in quantum mechanics need to be considered with care since their experimental verification might depend on the measurement context even in the limit of weak measurements. We can distinguish superconserved observables, which will be conserved whatever measurement will be performed, which as consequence are subject to the superselection rule – forbidding superpositions of different eigenvalues. In contrast, usual conserved quantities like energy or angular momentum might be found to be non-conserved if measured in the context of other non-commuting variables. We have defined an operational criterion to detect the non-conservation by third-order correlation functions. The non-conservation can also be formulated as Leggett-Garg-type test showing the connection to the absence of macrorealism in quantum mechanics. In future, it might be interesting on one hand to study more realistic scenarios for quantum measurements taking into account decoherence or more general

detectors [22]. Furthermore, one might generalize these findings to more fundamental relativistic field theories [23].

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