Three Essays in Macroeconomics

Doctoral thesis for obtaining the academic degree of
Doctor of Economics
(Dr. rer. pol.)

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Universität Konstanz

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Konstanz, 2018
Date of Oral Examination: 08.10.2018
First Reviewer: Prof. Dr. Volker Hahn
Second Reviewer: Prof. Dr. Almuth Scholl
Acknowledgments

First of all, I would like to thank my supervisor Prof. Dr. Volker Hahn. His comments and suggestions have made this dissertation substantially better than it would otherwise have been. Apart from Chapter 1, to which he has contributed tremendously as a co-author, Chapter 2 was largely inspired by discussions with him. I am deeply indebted to him.

I am also indebted, both personally and professionally, to the macro group at the University of Konstanz. In particular, I thank Leo Kaas, Almuth Scholl and Nawid Siassi for their valuable comments on my first paper (Chapter 1), and Volker Hahn, Leo Kaas, Georgi Kocharkov, Alessandro Di Nola and Nawid Siassi, for their insightful suggestions on my third paper (Chapter 3). In addition, I would like to thank my former office mate Finn Martensen and current office mate Michal Marenčák for their tremendous help.

I thank Dirk Krueger and my former colleague Dominik Sachs for their helpful comments on my third paper (Chapter 3). I want to thank particularly my friend and co-author Oliko Vardishvili for all the joy she has brought me. It has been fun to work with her.

I am grateful to my second supervisor Prof. Dr. Almuth Scholl for evaluating my thesis and to Prof. Dr. Heinrich Ursprung for chairing the oral defence committee.

Finally, I owe my deepest debt of gratitude to my parents and girlfriend for their love and support, without which completing this thesis would not have been possible.
Summary

This thesis consists of three chapters. Each chapter represents a self-contained research paper. Except for Chapter 2, the other two chapters are co-authored. Chapter 1 is a joint work with Prof. Dr. Volker Hahn at the University of Konstanz. Chapter 3 is a joint work with Oliko Vardishvili at the European University Institute.

Chapter 1 studies, in the context of a fully non-linear new Keynesian economy, the multiplicity of discretionary Markov equilibria in the presence of the zero lower bound. We confirm previous findings in the literature that multiple equilibria occur in such economies. However, while previous papers, which have used log-linearized equilibrium relationships and comparably simple shock structures, have found that the equilibria may entail very different outcomes, our non-linear model with a more complex shock structure implies that all equilibria are very similar to one another. We conclude that the practical relevance of equilibrium multiplicity in new Keynesian models with a zero lower bound constraint may be limited.

Chapter 2 examines, in the context of a fully non-linear new Keynesian economy, the mechanism and implications of pursuing a higher inflation target temporarily at the zero lower bound. We find that raising the inflation target temporarily has two opposite effects on the nominal interest rate. On the one hand, it raises inflation expectations and therefore makes the zero lower bound less likely to bind. On the other hand, a higher target also requires more vigorous actions. Which of these two effects dominates depends crucially on the average duration of pursuing the high target. In addition, we find that raising the inflation target permanently is more effective in reducing the frequency of zero lower bound events, albeit with higher welfare costs.
Chapter 3 evaluates the role of education affordability in shaping earnings inequality in the context of an overlapping generations model where agents, heterogeneous in terms of learning ability, initial wealth, and productivity, decide whether to attend college, subject to borrowing constraints. After calibrating the model to the US economy, we perform a number of counterfactual experiments. We find that the Gini coefficient for before-tax wage income would decrease by as much as 16.2 percent if the current education policy, the fraction of higher education costs borne by the government, were replaced with its German counterpart. Poor individuals with medium and medium-high ability would benefit the most from it. Apart from distributional gains, the hypothetical policy reform would also boost macroeconomic activity by increasing labor productivity. In contrast with the existing literature, we find that labor tax progressivity plays a less significant role in explaining earnings inequality.
Zusammenfassung


Das zweite Kapitel beschäftigt sich mit den Mechanismen und Implikationen einer temporären Befolgung eines höheren Inflationsziels an der Null-Prozent-Untergrenze für nominelle Zinssätze. Der Kontext ist wiederum ein vollständig nicht-lineares neukeynesianisches Modell. Wir beobachten, dass die vorhergehende Anhebung des Inflationsziels zwei gegenläufige Effekte auf den nominellen Zinssatz zu Folge hat. Auf

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Chapter 1

Discretionary Monetary Policy, Multiple Equilibria and the Zero Lower Bound

1.1 Introduction

The global financial crisis of 2007/2008 has pushed nominal interest rates in major developed economies to historical lows. This has spurred an active literature about monetary-policy making that takes into account that nominal interest rates cannot drop below zero to a large extent because the possibility to hold cash effectively acts as a lower bound on nominal interest rates.

In a recent paper, Armenter [2017] examines inflation targeting in a log-linearized new Keynesian model with a zero-lower-bound constraint. He shows that a Markov equilibrium, if it exists, is never unique. He concludes that inflation targets, and other nominal targets like price-level or nominal-GDP targets, are perilous.

This paper revisits the issue of equilibrium uniqueness in a fully non-linear model with a zero-lower-bound constraint and a slightly more complex shock structure. Our main conclusions are twofold. First, we confirm numerically that the new Keynesian model with the zero lower bound typically has multiple Markov equilibria. Second, we demonstrate that all equilibria are very close to one another. As a consequence,
while equilibrium multiplicity is theoretically interesting, we argue that it may not be of practical importance.

To understand what drives the difference between our findings and those in Armenter 2017, we log-linearize the model around its zero inflation non-stochastic steady state and compare the resulting equilibria with those that arise in the fully non-linear economy. We find that log-linearization leads to equilibria that are very different from one another. Hence log-linearizing equilibrium relationships may be problematic.

Our paper is also related to other works. First, our paper builds on the vast literature on discretionary monetary policy, including inflation targeting, in the presence of the zero lower bound: see, for example, Adam and Billi 2007, Nakov 2006 and more recently, Billi 2011, Nakata 2017, Ngo 2014 and Hahn 2017. Except for Ngo 2014 and Hahn 2017, these authors consider models that are log-linearized around the zero inflation steady state. As Miao and Ngo 2014, Fernández-Villaverde et al. 2015, Boneva, Braun, and Waki 2016, and Eggertsson and Singh 2016 show, log-linearization can produce large approximation errors. In contrast with these papers, we consider equilibrium multiplicity and identify the entire set of a class of plausible equilibria.

Second, this paper is also related to the strand of literature on equilibrium multiplicity: see, for example, Albanesi, Chari, and L. J. Christiano 2003, Armenter 2008, Armenter and Bodenstein 2008 and more recently, Armenter 2017. Most of this literature has ignored the zero lower bound, with the notable exceptions of Mertens and Ravn 2014, which assumes that the central bank follows a Taylor rule, and Armenter 2017. We study the zero lower bound explicitly, but in contrast to Armenter 2017, whose elegant solution method is only applicable to a log-linearized model, we use a simple numerical procedure that can be used in fully nonlinear economies to determine all equilibria that satisfy a plausible property about the set of state variables for which the zero lower bound binds.

The rest of the paper is organized as follows. Section 1.2 describes the model environment and presents the definition of equilibrium. Section 1.3 discusses our calibration strategy. We report the main findings of this paper in section 1.4. In Section 1.5 we elaborate on why our results are different from the existing literature.
Finally, we present our conclusions in Section 1.6.

1.2 The Model Economy

We consider a simple new Keynesian model. The model economy contains a representative household, a perfectly competitive final good firm, a continuum of monopolistically competitive intermediate goods firms and a central bank. Prices of the intermediate goods are sticky à la Rotemberg [1982]. The model features discount factor shocks and mark-up shocks. In addition, we take into account the zero lower bound on nominal interest rates explicitly.

1.2.1 Households

The representative household chooses a state-contingent consumption plan and a labor supply schedule so as to maximize its expected present value of lifetime utility

$$
E_0 \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t-1} \beta_\tau \right) \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \lambda \frac{L_t^{1+\eta}}{1 + \eta} \right),
$$

(1.1)

where $C_t$ is consumption of a single final good, $\sigma > 0$ is the coefficient of relative risk aversion, $L_t$ is the labor supply, $\eta$ is the inverse of the Frisch elasticity of labor supply, and $\beta_\tau$ denotes the stochastic discount factor, which takes two values, $\beta_l$ and $\beta_h$, where $\beta_l < \beta_h$. One can interpret state $l$ with discount factor $\beta_l$ as the normal state and state $h$ with $\beta_h$ as a negative real interest rate shock.\footnote{A stochastic discount factor is commonly used in the literature as a way of generating real rate shocks, see for example Braun and Körber [2011] and Nakata [2017]}

We assume that the evolution of $\beta_\tau$ is governed by a two-state Markov chain with transition matrix

$$
\begin{pmatrix}
    p_{ll} & p_{lh} \\
    p_{hl} & p_{hh}
\end{pmatrix}
$$

(1.2)
where \( p_{s_1s_2} \) denotes the transition probability from \( s_1 \in \{h,l\} \) to \( s_2 \in \{h,l\} \). The representative household’s budget constraint is given by

\[
C_t + \frac{B_t}{R_tP_t} = \frac{B_{t-1}}{P_{t-1}} + \frac{W_t}{P_t}L_t + T_t
\]

where \( B_t \) are risk-free bonds, \( R_t \) is the gross rate of return on those bonds from period \( t \) to period \( t + 1 \), \( W_t \) is the economy-wide nominal wage, \( P_t \) is the price level, and \( T_t \) are dividend payments from the firms.

### 1.2.2 Final Good Firm

A perfectly competitive firm produces a single final good using a continuum of intermediate goods \( i \in [0, 1] \) according to

\[
Y_t = \left[ \int_0^1 y_t(i) \frac{\theta_t - 1}{\theta_t} \, di \right]^{\frac{\theta_t}{\theta_t - 1}},
\]

where \( \theta_t \) denotes the elasticity of substitution between intermediate goods. The greater the value of \( \theta_t \), the easier to substitute one good for another, and thus the less monopoly power each intermediate goods firm enjoys. Hence one can interpret variations in \( \theta_t \) as mark-up shocks. \( \theta_t \) evolves according to

\[
\ln(\theta_t - 1) = (1 - \rho_\theta) \ln(\bar{\theta} - 1) + \rho_\theta \ln(\theta_{t-1} - 1) + \epsilon_{\theta,t},
\]

with \( \rho_\theta \in [0, 1) \), \( \epsilon_{\theta,t} \overset{iid}{\sim} N(0, \sigma_{\theta}^2) \).

The profit maximizing demand for intermediate goods is

\[
y^d_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta_t} Y_t,
\]

where \( p_t(i) \) denotes the price of intermediate good \( i \).

\(^2\text{This specification is also used by other authors, see for example Paciello and Wiederholt } 2013\)
1.2.3 Intermediate Goods Firms

There is a continuum of monopolistically competitive firms producing intermediate goods. Each firm \( i \in [0, 1] \) produces a differentiated intermediate good according to the following technology

\[
y_t(i) = AL_t(i),
\]

(1.7)

where \( L_t(i) \) is the labor employed by firm \( i \) and \( A \) is a positive constant. Prices of intermediate goods are sticky in the sense that adjusting prices entails real costs, as in Rotemberg [1982]. The decision of each intermediate goods firm can be formulated in two steps: a cost minimization problem and a price-setting problem. First, each firm \( i \) chooses the optimal level of labor input \( L_t(i) \), taking the nominal wage \( W_t \), the general price level \( P_t \) and output \( y_t(i) \) as given. Cost minimization implies

\[
MC_t(i) = \frac{W_t}{AP_t},
\]

(1.8)

where \( MC_t(i) \) is the real marginal cost. Second, each firm \( i \) chooses a price schedule so as to maximize its expected present discounted sum of profits

\[
E_0 \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t-1} \beta_\tau \right) Q_t,0 \left\{ \left( \frac{p_t(i)}{P_t} - MC_t(i) \right) y_t(i) - \frac{\psi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 Y_t \right\},
\]

(1.9)

where \( Q_t,0 \) denotes the stochastic discount factor and the demand for good \( i, y_t(i) \), is given by (1.6). The expression in curly brackets in (1.9) stands for per-period profits, which can be broken down into three components: total revenue \( \frac{p_t(i)}{P_t} y_t(i) \), total production cost \( MC_t(i) y_t \), and price adjustment cost \( \frac{\psi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 Y_t \), where \( \psi \) is a positive parameter. The price index \( P_t \) is given by

\[
P_t := \left[ \int_0^1 p_t(i)^{1 - \theta_t} \, di \right]^{\frac{1}{1 - \theta_t}}.
\]

(1.10)
1.2.4 Private Sector Equilibrium Conditions

Before turning to monetary policy, we first collect the conditions that characterize the equilibrium behavior of the private sector. The representative household’s optimization problem leads to the so-called the new Keynesian IS curve

\[ 1 = \beta_t R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\}, \tag{1.11} \]

where \( \Pi_{t+1} = \frac{P_{t+1}}{P_t} \), and the labor supply schedule

\[ \frac{W_t}{P_t} = \chi L_t^\eta C_t^\sigma. \tag{1.12} \]

The profit-maximization problems of firms lead to (1.6) and (1.8) as well as the new Keynesian Phillips curve.

\[ \psi(\Pi_t - 1) \Pi_t = \beta_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \psi (\Pi_{t+1} - 1) \Pi_{t+1} \right\} + \theta_t MC_t - \theta_t + 1. \tag{1.13} \]

In addition, the goods-market clearing condition is given by

\[ C_t = \left( 1 - \frac{\psi}{2} (\Pi_t - 1)^2 \right) Y_t. \tag{1.14} \]

Note that only a fraction of final output is used for consumption because a proportion of final output is spent on adjusting prices.

Using the intermediate goods production function (1.7) and (1.14) we rewrite the IS curve and the Phillips curve as follows:

\[ \beta_t E_t \left[ \frac{(\Psi_{t+1} Y_{t+1})^{-\sigma}}{\Pi_{t+1}} \right] - \frac{(\Psi_t Y_t)^{-\sigma}}{R_t} = 0 \tag{1.15} \]

\[ \beta_t E_t \left[ \Psi_{t+1}^{-\sigma} Y_{t+1}^{1-\sigma} \psi (\Pi_{t+1} - 1) \Pi_{t+1} \right] - \Psi_t^{-\sigma} Y_t^{1-\sigma} \left[ \psi (\Pi_t - 1) \Pi_t + \theta_t - 1 - \chi \theta_t \Psi_t Y_t^{\sigma+\eta} \right] = 0, \tag{1.16} \]
where $\Psi_t = 1 - \frac{\psi}{2} (\Pi_t - 1)^2$. Thus the private-sector equilibrium is described by equations (1.15) and (1.16).

1.2.5 Monetary Policy

Monetary policy is delegated to an independent central bank that operates under discretion. We consider two variants of discretionary monetary policy: First, we consider inflation targeting, under which the central bank is assigned an objective function that may differ from the social welfare function. Second, we consider the case where the central bank’s objective function coincides with the social welfare function.

Inflation Targeting

The first scenario we consider in this paper is inflation targeting, which requires the central bank to keep the rate of inflation at or near a prescribed target over some time horizon. As is common in the literature, we consider a quadratic loss function for the central bank. Specifically, the central bank chooses nominal interest rate $R_t$ so as to minimize

$$E_t \sum_{s=t}^{\infty} \left( \prod_{\tau=t}^{s-1} \beta_{\tau} \right) \left\{ (\Pi_s - \Pi^*)^2 + \alpha (Y_s - Y^*)^2 \right\}, \quad (1.17)$$

taking private sector expectations and future monetary policies as given, subject to the IS curve (1.15), the Phillips curve (1.16) as well as the zero-lower-bound constraint

$$R_t - 1 \geq 0. \quad (1.18)$$

$\Pi^*$ and $Y^*$ are the inflation target and output target, respectively, and $\alpha$ is the weight on output stabilization. A larger value of $\alpha$ means that the central bank is more tolerant with regard to inflation fluctuations.
Maximizing Social Welfare

In the second scenario, the central bank’s objective function coincides with the representative household’s utility function. In each period, the central bank sets the current nominal interest rate, taking as given the exogenous state variables as well as private agents’ expectations about future policies. More precisely, the central bank’s problem is

$$
\max_{R_t, Y_t, \Pi_t} \mathbb{E}_t \sum_{s=t}^{\infty} \left( \prod_{\tau=t}^{s-1} \beta_{\tau} \right) \left\{ \frac{(\Psi_s Y_s)^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{Y_s^{1+\eta}}{1 + \eta} \right\},
$$

(1.19)

for given private-sector expectations and future monetary policies, subject to the IS curve (1.15), the Phillips curve (1.16) as well as the zero-lower-bound constraint (1.18).

1.2.6 Equilibrium

As is common in the literature, we focus on Markov-perfect rational expectations equilibrium, which can be defined in the following way:

**Definition 1** A Markov-perfect rational expectations equilibrium consists of a monetary policy $R(\beta_t, \theta_t)$ and private sector response functions, $\Pi(\beta_t, \theta_t)$ and $Y(\beta_t, \theta_t)$, that satisfy the private sector equilibrium conditions as well as the optimality conditions for the central bank, which are stated in Appendix A.1.

We restrict our attention to a particular set of Markov equilibria. To motivate our choice, we note that high realizations of $\theta_t$ lead to a reduction in markups and thereby lower inflation. One would therefore expect that sufficiently high realizations of $\theta_t$ make the zero lower bound binding. In line with this observation, we postulate that, for each of the two possible realizations of $\beta_t$, there is a level of $\theta$ such that the zero lower bound binds for all realizations of $\theta$ larger than this threshold but does not bind for all smaller realizations. We call such Markov equilibria “coherent,” as all values of $\theta$ for which the zero lower bound binds are connected. In the following, we state this definition formally:
Definition 2 A Markov-perfect rational expectations equilibrium of our economy is coherent if there are $\theta_l^*$ and $\theta_h^*$ such that the following properties hold:

1. For $\beta_l$ and all $\theta < \theta_l^*$, the interest rate satisfies $I_t > 1$ in equilibrium. For $\beta_l$ and all $\theta \geq \theta_l^*$, the interest rate satisfies $I_t = 1$.

2. For $\beta_h$ and all $\theta < \theta_h^*$, the interest rate satisfies $I_t > 1$. For $\beta_h$ and all $\theta \geq \theta_h^*$, the interest rate satisfies $I_t = 1$.

Note that we would typically expect $\theta_h^* < \theta_l^*$, as the zero lower bound is more likely to bind for the higher discount factor $\beta_h$ than for $\beta_l$. Our numerical simulations will confirm this conjecture.

1.3 Calibration

We calibrate the model to the U.S. economy. There are fourteen parameters to which we need to assign values. Some of those are standard technology and preference parameters; for those parameters we assign conventional values. The coefficient of relative risk aversion $\sigma$ is set to 1 (see, e.g., Miao and Ngo 2014). Following L. Christiano, Eichenbaum, and Rebelo 2011, we set the level of productivity $A$ and the inverse of the Frisch elasticity of labor supply $\eta$ to 1. The weight on the disutility of labor $\chi$ is set to 1, as in Fernández-Villaverde et al. 2015. For $\psi$, the parameter that governs the cost of price adjustment, we select 129, the value chosen by Leith and Liu 2016.

The calibration of the remaining parameters, which are associated with the discount factor shock process and the markup shock process, is worth explaining in more detail. We start with the observation that during the Great Moderation nominal interest rates stayed comfortably above the zero lower bound. This indicates that there were no major real rate shocks during that period, the reason being that, in the presence of a negative real rate shock, markup shocks can easily push nominal interest rates to the zero lower bound. Therefore, it is plausible to assume that the discount factor was in the normal state during the Great Moderation. The normal state discount factor $\beta_l$ is set to 0.9927, targeting an average annual real rate of interest of
2.9\%, as computed for the period from 1983\:1 to 2007\:3.

Taking the low discount factor as given, we then try to recover the historical markup shocks for the period from 1983\:1 to 2007\:3. We apply the method employed by Adam and Billi [2006] and Rotemberg and Woodford [1997]. First, we fit a VAR model in terms of output, inflation and the nominal interest rate. Second, we use the VAR one-period ahead predictions to estimate the conditional expectations in the Phillips curve (1.16). Third, substituting the actual values of output, inflation and the estimated expectations obtained from the previous step into the Phillips curve, taking the other parameters as given, we recover the historical markup shocks. Finally, we use the time series obtained in the previous step to estimate the parameters associated with the markup shock process (1.5). The estimated process does not exhibit a strong degree of autocorrelation, with the persistence parameter, $\rho_\theta$, equal to 0.1423. The other parameters are $\bar{\theta} = 10.1166$ and $\epsilon_\theta = 0.3026$. We observe that $\bar{\theta} = 10.1166$ implies a markup of approximately 11\% over marginal costs in the absence of shocks, which appears to be a plausible value.

\footnote{For a more detailed description of the identification procedure, see Appendix A.2}
The next step is to assign values to the remaining three parameters associated with the evolution of the discount factor, including the high discount factor $\beta_h$, and two transition probabilities $p_{lh}$ and $p_{hl}$. We choose these parameters in an attempt to reproduce the frequency and duration of zero lower bound episodes observed in the data as well as the level of real interest rates during such periods.\footnote{For convenience, we call an episode during which the zero lower bound is binding a zero lower bound episode.} Over the past century, there were five zero lower bound episodes, with an average duration of 7 quarters and an average annual real rate of $-0.30$ percent. In order to come up with a meaningful estimation of the frequency and duration of zero lower bound episodes, we use data on the effective federal funds rate or 3-month treasury rate, depending on data availability, for the past 100 years. In addition, we consider 15 basis points as the effective zero lower bound.

The calibration of these three parameters is policy-regime dependent. Since the Federal Reserve has been conducting monetary policy since the mid-1980s, first implicitly and then officially, within the framework of flexible inflation targeting, we calibrate these three parameters under flexible inflation targeting. We first set the inflation target $\Pi^*$ equal to 2%. Then we choose the weight on output stabilization $\alpha$, and the output target $Y^*$, the high discount factor $\beta_h$, as well as the transition probabilities $p_{lh}$ and $p_{hl}$ to minimize the sum of squared differences between simulated standard deviations of inflation and output and those observed in the data and to match the frequency, duration and real rate of interest of zero lower bound episodes.\footnote{We assign equal weight to the squared differences between simulated standard deviations of inflation and output and those observed in the data.} This procedure results in $\alpha = 0.0715$, an output target of 0.9511, a high discount factor of $\beta_h = 1.002$, and transition probabilities $p_{lh} = 0.0079$ and $p_{hl} = 0.0357$. Our calibration implies that high realizations of the discount factor occur only infrequently, but once they occur, they can last as long as seven years.\footnote{We would like to note that a high discount factor does not automatically lead to zero nominal interest rates. Zero lower bound episodes are the result of the interplay of a high discount factor and large realizations of $\theta_t$.}

Table 1.2 reports some summary statistics for the data and benchmark calibration. The first three columns report the frequency and duration of zero lower bound episodes,
episodes observed in the data and those simulated from the benchmark specification as well as the corresponding real interest rates. We consider two definitions of a zero lower bound episode in our simulations. According to the first definition (see the third row), zero lower bound episodes are periods during which nominal interest rates are exactly zero. The second definition (see the second row), involves that the economy is at the zero lower bound in all periods where the nominal interest rate falls below 15 basis points. This corresponds to the definition we employ when we identify zero lower bound episodes in the data (see the first row).

While we can match the frequency of zero lower bound episodes and the real interest rate during these episodes, the simulated duration falls short of that observed in the data. The durations of zero lower bound episodes in our model are short because even in periods with the high discount factor $\beta_h$ small realizations of $\theta$ lead to positive interest rates. Hence for the high discount factor, interest rates are typically low but slightly positive. The last two columns report the standard deviations of output and inflation. The benchmark specification matches the observed standard deviation of output and inflation reasonably well.

Since there are multiple equilibria, our calibration has to take a stand on which equilibrium prevailed over the period from 1983 : 1 to 2007 : 3. We make the assumption that it was the equilibrium with the highest level of social welfare. As will be seen below, this assumption is innocuous in that the entire set of equilibria are rather similar to one another.
1.4 Results

1.4.1 Multiple Equilibria

It is well-known that multiple equilibria may arise under discretionary monetary policy: see, for example, Albanesi, Chari, and L. J. Christiano 2003, King and Wolman 2004, and Armenter 2017. In a model with the zero-lower-bound, Armenter 2017 designs an algorithm building on the problem of vertex enumeration in convex geometry to compute the entire set of Markov-perfect equilibria, but that algorithm is only applicable to log-linearized models. To identify the set of coherent Markov-perfect equilibria in our model, we employ a simple numerical procedure that takes advantage of the coherence property.

We first discretize the state space on a finite support. For each pair of critical values \( \theta^*_l \) and \( \theta^*_h \), we proceed as follows. The pair defines a set of states where the zero lower bound is binding, which we label \( S_b \), and a set where it is not binding, \( S_{nb} \). For all states in \( S_b \), the equilibrium has to satisfy \( I_t = 1 \), the IS curve (1.15), and the Phillips curve (1.16). In all states in \( S_{nb} \), IS curve (1.15), the Phillips curve (1.16) as well as the central bank’s first-order conditions (A.1)-(A.3) or (A.6)-(A.8), respectively, have to hold. We iterate on these conditions. Provided that convergence is reached, we verify whether the candidate equilibrium is indeed an equilibrium. For this purpose we check whether, for all states in \( S_b \), the central bank could not profitably deviate to a positive interest rate and whether, for all states in \( S_{nb} \), the interest rates are weakly positive. All combinations of \( \theta^*_l \) and \( \theta^*_h \) that pass this test correspond to coherent discretionary Markov equilibria.

In line with Armenter 2017, we find that multiple equilibria exist. For the baseline calibration and using a grid of 269 nodes for \( \theta_t \), our numerical procedure identifies several equilibria both under welfare-maximizing policy and under inflation targeting. Figure 1.1 shows the areas (in yellow) where equilibria under each policy scenario exist. The horizontal axis and the vertical axis indicate the pair of critical values

---

7 The number of equilibria obviously depends on the fineness of the grid. When discretizing the state space, we assign more nodes around the region where previous experiments have shown the zero bound to be binding.

8 The space for the pair of critical values is much larger than displayed in Figure 1.1; the entire
that identify a Markov-perfect equilibrium. We have argued before that $\theta^*_t$ should plausibly be larger than $\theta^*_h$ because the zero lower bound can be expected to be binding already for comparably low values of $\theta_t$ if the discount factor is low. Figure 1.1 confirms this conjecture. In addition, it shows that markup shocks alone rarely make the zero lower bound binding under either policy scenario—the chance of a $\theta$ shock at least as large as 18.6 is 1.58%. This result is in line with Adam and Billi 2007, who also find that markup shocks alone typically do not make the zero lower bound binding. The figure also shows that, in the presence of a discount factor shock, the zero lower bound is reached more frequently under inflation targeting.

set of equilibria is concentrated in a small area of it.
More importantly, our results show that all equilibria that we have identified appear to be rather similar, as the critical values of $\theta$, $\theta_l^*$ and $\theta_h^*$, do not differ substantially across equilibria for a given policy regime.

To illustrate the similarity of the equilibria further, Table 1.3 reports some statistics about the entire set of equilibria under each policy scenario. The first row shows the range of values for the frequency of zero lower bound episodes. Under inflation targeting, the zero lower bound binds more often. But for a given policy regime, the difference in the frequency of zero lower bound episodes among different equilibria is quite small. The second row presents the welfare differences—in consumption equivalent terms—between the worst equilibrium and the best one. It indicates that the welfare differences are negligible. In addition, we also report statistics on the unconditional mean and standard deviation of inflation and output. The picture is clear. The set of equilibria are rather similar to each other.

<table>
<thead>
<tr>
<th>Welfare Difference</th>
<th>Welfare maximizing policy</th>
<th>Inflation Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_c$ (%)</td>
<td>$-0.1899 \times 10^{-4}$</td>
<td>$-0.9357 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

### Inflation

<table>
<thead>
<tr>
<th></th>
<th>Welfare maximizing policy</th>
<th>Inflation Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\Pi)$ (%)</td>
<td>2.2454–2.2456</td>
<td>2.0148–2.0148</td>
</tr>
<tr>
<td>$\sigma(\Pi)$ (%)</td>
<td>0.7781–0.7786</td>
<td>0.7429–0.7429</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th></th>
<th>Welfare maximizing policy</th>
<th>Inflation Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(Y)$</td>
<td>0.9519–0.9519</td>
<td>0.9515–0.9515</td>
</tr>
<tr>
<td>$\sigma(Y)$ (%)</td>
<td>1.0156–1.0163</td>
<td>1.2375–1.2375</td>
</tr>
</tbody>
</table>

### Zero Lower Bound

<table>
<thead>
<tr>
<th>$Pr(R = 1)$ (%)</th>
<th>Welfare maximizing policy</th>
<th>Inflation Targeting</th>
</tr>
</thead>
</table>

## 1.5 Differences from the Literature

We are not the first to study multiple equilibria in new Keynesian models. For example, Armenter 2017 studies equilibrium multiplicity in a similar, yet log-linearized, model, and he finds large differences among equilibria. Since the major difference
between Armenter [2017] and our model is that we study a fully nonlinear model, it is useful to examine the role of log-linearization in shaping his results. For this reason, we log-linearize our model and compare the results from the log-linearized model with those from the fully non-linear model. Eggertsson and Singh [2016], on the other hand, consider a fully non-linear model, yet with less sophisticated stochastic structure; they find that there are large differences among equilibria. To understand the differences between our results and theirs, we shut down markup shocks in our model, so that the stochastic structure is comparable to Eggertsson and Singh [2016] and compare the results with those obtained in the benchmark model. In the following, we conduct these exercises in turn.

1.5.1 Log-linear Approximation Versus Exact Solution

To understand what drives the difference between our findings and the findings of Armenter [2017] we now log-linearize the Phillips curve and the IS curve and compare the resulting model, where the only non-linearity stems from the zero lower bound on interest rates, with the fully non-linear model.

Following the literature on the consequences of monetary policy for welfare in log-linearized models, we introduce subsidies to firms, which are proportional to total revenues, such that the zero-inflation non-stochastic steady state is efficient (see, e.g., Woodford [2003]). This change affects the Phillips curve in the following way:

\[
\beta_t E_t \left[ \Psi_t^{-\sigma} Y_t^{-1-\sigma} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \right] = \Psi_t^{-\sigma} Y_t^{-1-\sigma} \left[ \psi(\Pi_t - 1)\Pi_t + (\theta_t - 1)Z - \chi \theta_t \Psi_t^{\sigma} Y_t^{\sigma+\eta} \right],
\]

(1.20)

where \( Z := \frac{\sigma}{\sigma - 1} \) determines the size of the subsidies. To be more precise, each intermediate firm receives subsidies equal to \( Z - 1 \) times its revenues.

It is straightforward to show that the zero inflation non-stochastic steady state is efficient and that the steady state levels of output and marginal costs, \( Y \) and \( MC \), satisfy \( MC = Y = 1 \). Let \( \tilde{\Pi}_t, \tilde{Y}_t, \) and \( \tilde{R}_t \) denote the log deviations of inflation \( \Pi_t \), output \( Y_t \) and nominal interest rate \( R_t \) from their respective steady state levels.

---

9 These subsidies are financed through lump-sum taxes on households.
Similarly, let $\tilde{\beta}_t$ and $\tilde{\theta}_t$ denote the log deviations of $\beta_t$ and $\theta_t$ from their respective steady state levels. The IS curve (1.15) and the modified Phillips curve (1.20) are log-linearized as follows:

\begin{align*}
\tilde{R} &= \sigma \left[ E_t \tilde{Y}_{t+1} - \tilde{Y}_t \right] + E_t \tilde{\Pi}_{t+1} - \tilde{\beta}_t, \quad (1.21) \\
\tilde{\Pi}_t &= \kappa \tilde{Y}_t + \beta E_t \tilde{\Pi}_{t+1} + X \tilde{\theta}_t, \quad (1.22)
\end{align*}

where

\[ \kappa := \frac{\chi \bar{\theta}(\sigma + \eta) - (1 - \sigma)(1 - \chi)\bar{\theta}}{\psi} \]

and

\[ X := \frac{Z - \chi \bar{\theta}}{\psi}. \]

Let $\mu_t := X \tilde{\theta}_t$ and $\sigma_\mu$ denote the standard deviation of $\mu_t$. We observe that $X < 0$ holds. As a consequence, large realizations of $\theta_t$ correspond to small realizations of $\mu_t$.

Social welfare is described by a social loss function, which can be derived from a linear-quadratic approximation to the utility function of the representative household. In line with Woodford (2003), social losses are then given by

\[ L = -\tilde{\Pi}_t^2 - \lambda \tilde{Y}_t^2, \quad (1.23) \]

where $\lambda = \frac{\kappa}{2\bar{\theta}}$ is the relative weight on output stabilization.

For our numerical analysis, we consider the case where the central bank’s loss function equals the social loss function (1.23) and choose parameter values that coincide with those in Armenter (2017) whenever possible. Specifically, we set the steady state discount factor to 0.994, the coefficient of relative risk aversion $\sigma$ to 1, the parameter governing price adjustment cost $\psi$ to 500, the steady state of $\theta$ to 6, and $\rho_\theta = 0.8$. These parameters result in a value of 0.024 for $\kappa$ and 0.002 for $\lambda$. In addition, we set $\sigma_\mu = 0.03$, in percentage terms. The AR(1) process for the markup shocks $\mu_t$ is discretized using the Rouwenhorst (1995) method. We assign 49 nodes to the support of $\mu$. The real rate shock $\tilde{\beta}_t$ follows a two state Markov chain with $\tilde{\beta}_t \in \{-0.003, 0.003\}$
Figure 1.2: Policy functions in the two equilibria of the log-linearized model. A red (blue) curve indicates a positive (negative) real rate shock.
and transition matrix matrix

\[
\begin{pmatrix}
0.95 & 0.05 \\
0.05 & 0.95
\end{pmatrix}.
\] (1.24)

We find that there are only two equilibria in the log-linearized model, while there are multiple equilibria in the fully nonlinear model. Figure 1.2 displays the policy functions for both equilibria of the log-linearized model. In Figure 1.3, we plot the policy functions in the best equilibrium and the worst equilibrium of the fully nonlinear model. Comparing the policy functions in the left columns with the respective policy functions in the right columns reveals that the equilibria are very similar in the fully-nonlinear model (Figure 1.3) but are rather different in the model where the IS curve and the Phillips curve have been log-linearized (Figure 1.2).

We explore this observation more thoroughly in Table 1.4, which reports various moments of our simulated data. In the case of the log-linearized model, we present statistics for the two equilibria that we found. For the fully nonlinear model, we present, for each statistic, the range of values across all equilibria\(^\text{10}\). The table shows that log-linearization has a dramatic impact on the results: while the differences across equilibria in the fully non-linear version are minimal, the two equilibria in the log-linearized version are vastly different. In addition, as a byproduct, Table 1.4 shows that the fully non-linear economy exhibits a slightly positive average rate of inflation even when the classic inflation bias is assumed away.

1.5.2 Difference from Eggertsson and Singh 2016

Though not the focus of their paper, Eggertsson and Singh 2016 also consider multiple equilibria due to self-fulfilling expectations in a similar new Keynesian model. For a model with Calvo 1983 pricing, they report two equilibria that are vastly different from each other. So why do our findings above differ from theirs? The difference, it turns out, lies in the fact that we consider a richer stochastic structure: our model features both real rate shocks and markup shocks, while they consider only a shock to the discount factor, where the discount factor is described by a two-state Markov

\(^\text{10}\)We note that the quantitative results are different from the ones presented in Table 1.3 because of the introduction of the subsidy to intermediate firms’ sales, for example.
Figure 1.3: Policy functions in the socially best equilibrium (left-hand side) and the socially worst equilibrium (right-hand side) of the fully nonlinear model. A red (blue) curve indicates a positive (negative) real rate shock.
Table 1.4: Log-linear approximation vs exact solution

<table>
<thead>
<tr>
<th></th>
<th>Log-linear Approximation</th>
<th>Fully Non-linear Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_\pi$</td>
<td>0.00</td>
<td>-0.89</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.25</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>Output gap (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_y$</td>
<td>0.00</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.72</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>Zero Lower bound (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Pr(R = 1)$</td>
<td>4.84</td>
<td>49.81</td>
</tr>
</tbody>
</table>

chain with an absorbing state.

To understand the importance of having a richer stochastic structure, consider for the moment Markov equilibria of the fully non-linear model economy in the absence of shocks. We consider strict inflation targeting with a two percent inflation target and assume $\beta = \beta_1, \theta = \tilde{\theta}$. The other parameters remain unchanged. Since there are no shocks, there are only two possible scenarios: the zero lower bound is always binding or it is never binding. If the zero lower bound is always binding, the equilibrium is characterized by the IS curve and Phillips curve; conversely, if the zero bound is not binding, the equilibrium can be solved using the first order condition and the Phillips curve. In the first scenario, the zero lower bound is not binding, and the equilibrium is characterized by a triplet $\{\Pi = 2\%, Y = 0.95, R = 1.05\}$ (here we report the annual rates). In the second scenario, the zero lower bound is binding—due to purely self-fulfilling expectations, and the equilibrium is characterized by $\{\Pi = -2.9\%, Y = 0.951, R = 1\}$. Therefore, when it comes to studying multiple equilibria, the behavior of a deterministic economy and an economy with a reasonably rich stochastic structure can be very different.

In fact, we can generate results similar to Eggertsson and Singh [2016] once we shut down markup shocks. As a numerical example, we consider a two-state Markov chain
for $\beta$ with $\beta = \{0.9867, 0.9987\}$ and a symmetric transition matrix

$$
\begin{pmatrix}
0.75 & 0.25 \\
0.25 & 0.75
\end{pmatrix}. \tag{1.25}
$$

For this simple economy, we find three equilibria, reported in Table 1.5. This example confirms again the importance of having a reasonably rich stochastic structure.

Table 1.5: Multiple equilibria in the fully non-linear economy with a two-state Markov chain for discount factor. The inflation rate and the interest rate are reported in percentage terms (annual rates).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Equilibrium 1</th>
<th>Equilibrium 2</th>
<th>Equilibrium 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.9867$</td>
<td>$2.05$</td>
<td>$0.61$</td>
<td>$-1.21$</td>
</tr>
<tr>
<td>$\beta = 0.9987$</td>
<td>$0.95$</td>
<td>$0.95$</td>
<td>$0.96$</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>$7.61$</td>
<td>$5.13$</td>
<td>$0.00$</td>
</tr>
<tr>
<td>$R$</td>
<td>$2.63$</td>
<td>$-1.23$</td>
<td>$-2.24$</td>
</tr>
<tr>
<td>$%$</td>
<td>$0.95$</td>
<td>$0.94$</td>
<td>$0.94$</td>
</tr>
</tbody>
</table>

1.6 Conclusion

This paper examines quantitatively the welfare effects of the zero lower bound on nominal interest rates and the welfare implications of multiple equilibria under two policy regimes: inflation targeting and a regime where the central bank attempts to maximize social welfare. We consider a fully non-linear new-Keynesian model that features discount factor shocks and markup shocks.

In contrast to Armenter 2017, we find that equilibrium multiplicity does not pose a serious threat under discretionary monetary policy. Though equilibrium multiplicity is pervasive, the equilibria are rather similar to one another, indicating that private-sector expectations cannot stray far away from economic fundamentals and cause significant welfare losses. Hence our analysis provides support for the view that liquidity traps are more likely to be due to fundamental shocks rather than self-fulfilling expectations.
Bibliography


Chapter 2

Inflation Targeting with a Temporarily Higher Inflation Target at the Zero Lower Bound

2.1 Introduction

In the wake of the 2008 global financial crisis, major central banks around the world cut nominal interest rates to historical lows. Nearly a decades on, policy rates have remained stubbornly low. The big concern is that when the next recession arrives, policy makers may not have much room to lower interest rates to prop up employment and keep inflation near target.

That dire scenario has led to a rethinking of the appropriate level of the inflation target. In particular, Olivier Blanchard and other prominent economists (see, e.g., Blanchard, Dell’Ariccia, and Mauro 2010 and Ball 2014) propose that the Fed and other inflation-targeting central banks consider raising the inflation target to 4%. Advocates of a higher inflation target point out that, with a higher target, the zero lower bound events are less likely to occur. Skeptics, however, voice concerns that a permanently higher inflation target may prove to be costly. They advocate for a temporary burst of inflation instead to stimulate the economy.
CHAPTER 2. A TEMPORARILY HIGHER INFLATION TARGET

This paper aims to shed light on the topic by studying the mechanism and implications of raising the inflation target in the context of a fully non-linear new Keynesian economy. First, we compare four percent inflation targeting with two percent inflation targeting. Second, since a permanently higher inflation target may be costly, we also examine whether raising the inflation target temporarily can mitigate the zero lower bound problem.

Three findings emerge from this study. First, raising the inflation target temporarily at the zero lower bound has two opposite effects on the nominal interest rate: an expectations effect and a direct policy effect. On the one hand, pursuing a higher inflation target raises inflation expectations; on the other hand, a higher target also requires more vigorous actions. Which of these two effects dominates depends crucially on the average duration of pursuing the higher target. For example, when the average duration of pursuing the higher target is less than one year, the frequency of zero lower bound events can actually increase. The reason is that, when pursued for only a short period of time, raising the inflation target temporarily does little to raise inflation expectations, and yet a higher target does require more vigorous actions. Second, raising the inflation target temporarily can help lower real interest rates, and thus boost output, during a liquidity trap. Third, raising the inflation target permanently to four percent can greatly reduce the frequency of zero lower bound events, albeit with higher welfare costs.

Before turning to the model, we would like to briefly review the existing literature. First, our paper is related to the vast literature on discretionary monetary policy, including inflation targeting, in the presence of the zero lower bound: see, for example, Adam and Billi 2007, Nakov 2006, Billi 2011, and more recently Nakata 2017, Ngo 2014, Hahn 2017, and Hahn and Wang 2018. Most of this literature, with the exception of Ngo 2014, Hahn 2017, and Hahn and Wang 2018, uses models that are log-linearized around the zero inflation steady state. But as Miao and Ngo 2014, Fernández-Villaverde et al. 2015, Boneva, Braun, and Waki 2016, and Eggertsson and Singh 2016 recently point out, log-linearization can produce large approximation errors. For this reason, this paper considers a fully non-linear model. More importantly, none of these papers studies the implications of raising the inflation target.
temporarily at the zero lower bound.

Second, this paper is loosely related to the strand of literature on equilibrium multiplicity: see, for example, Albanesi, Chari, and L. J. Christiano 2003, Armenter 2008, Armenter and Bodenstein 2008 and more recently, Armenter 2017, and Hahn and Wang 2018. In particular, Hahn and Wang 2018 show, in a similar model, that the set of Markov equilibria are rather similar to one another. Since the focus of this paper is not on equilibrium multiplicity, we consider only the best equilibrium in terms of social welfare.

Finally, and most importantly, this paper is related to the recent debate on whether inflation-targeting central banks should raise their inflation targets, see, for example, Blanchard, Dell’Ariccia, and Mauro 2010 and Ball 2014 and the strand of literature on the optimal rate of inflation (see Diercks 2017 for a nice review). One major reason for calling for a higher inflation target points to a downward trend in the natural rate of interest (see, e.g., Williams et al. 2016 and Holston, Laubach, and Williams 2017), which implies that zero lower bound events may happen more frequently than before. Other motivations for calling for a higher target include new evidence on the elusive costs of inflation (see, e.g., Nakamura et al. 2016) and potential measurement bias (see, e.g., Redding and Weinstein 2016). This paper does not address the downward trend in equilibrium interest rate or measurement bias, rather it focuses on the mechanism and implications of raising the inflation target.

The rest of the paper is organized as follows. Section 2.2 describes the model environment and presents the definition of equilibrium. Section 2.3 illustrates the two forces at work when raising the inflation target. Section 2.4 discusses our calibration strategy. We report the main findings of this paper in section 2.5 and present a robustness analysis in section 2.6. Finally, we conclude in section 2.7.

2.2 The Model Economy

We consider a fully non-linear New Keynesian economy. The model economy consists of a representative household, perfectly competitive final good firms, a continuum of monopolistically competitive intermediate goods firms and a central bank. Prices
of the intermediate goods are sticky à la Rotemberg 1982. In addition, the model features discount factor shocks and mark-up shocks. In the following, we will briefly describe the equilibrium conditions and the law of motion for exogenous shocks. We refer the reader to Hahn and Wang 2018 for a more detailed description of the model economy.

On the consumer side, the representative household’s optimal behavior is characterized by a consumption Euler equation, or the so-called the New Keynesian IS curve

\[
\beta_t E_t \left[ \frac{(\Psi_{t+1}Y_t)^{1-\sigma}}{\Pi_{t}} \right] - \frac{(\Psi_tY_t)^{-\sigma}}{R_t} = 0 \tag{2.1}
\]

where \(\beta_t\) is a stochastic discount factor, \(\sigma > 0\) is the coefficient of relative risk aversion, \(R_t\) is the gross rate of return on risk-free bonds from period \(t\) to period \(t+1\), \(C_t\) denotes consumption, \(\Pi_{t+1}\) is inflation between periods \(t\) and \(t+1\), \(Y_t\) denotes output, \(\Psi_t\) satisfies \(\Psi_t = 1 - \frac{\psi}{2}(\Pi_t - 1)^2\), \(\eta\) is the inverse of the Frisch elasticity of labor supply, \(\chi > 0\), and \(\psi > 0\) is a parameter that governs the magnitude of price adjustment cost.

We assume that \(\beta_t\) takes two values: \(\beta_l\) and \(\beta_h\), with \(\beta_l < \beta_h\). One can interpret the low discount factor \(\beta_l\) as the normal state, \(\beta_h\) as a negative real interest rate shock\(^1\). We assume that the evolution of \(\beta_t\) is governed by a two-state Markov chain with transition matrix

\[
\begin{pmatrix}
1 - p_{lh} & p_{lh} \\
p_{hl} & 1 - p_{hl}
\end{pmatrix} ,
\tag{2.2}
\]

where \(p_{lh}\) (\(p_{hl}\)) denotes the transition probability from the low (high) discount factor to the high (low) discount factor.

On the supply side, optimal behavior is characterized by the New Keynesian

\(^1\)A stochastic discount factor is commonly used in the literature as a way of generating real rate shocks, see for example Braun and Körber 2011 and Nakata 2017.
CHAPTER 2. A TEMPORARILY HIGHER INFLATION TARGET

Phillips curve
\[ \beta_t E_t [\Psi_{t+1}^{-\sigma} Y_{t+1}^{1-\sigma} \psi(\Pi_{t+1} - 1) \Pi_{t+1}] - \Psi_t^{-\sigma} Y_t^{1-\sigma} [\psi(\Pi_t - 1) \Pi_t + \theta_t - 1 - \chi \theta_t \Psi_t^{\sigma} Y_t^{\sigma+\eta}] = 0, \]  
(2.3)

where \( \theta_t \) denotes the elasticity of substitution between intermediate goods. The greater the value of \( \theta_t \), the easier to substitute one good for another, and thus the less monopoly power each intermediate goods firm enjoys. Hence one can interpret variations in \( \theta_t \) as mark-up shocks. \( \theta_t \) evolves according to

\[ \ln(\theta_t - 1) = (1 - \rho_\theta) \ln(\bar{\theta} - 1) + \rho_\theta \ln(\theta_{t-1} - 1) + \epsilon_{\theta,t}, \]  
(2.4)

with \( \rho_\theta \in [0, 1) \), \( \epsilon_{\theta,t} \) \( \overset{iid}{\sim} N(0, \sigma_\theta^2) \). When setting monetary policy, the central bank takes private sector behavior (2.1) and (2.3) as constraints.

Finally, social welfare is evaluated using the unconditional expected present value of the representative household’s life-time utility

\[ E \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t-1} \beta_\tau \right) \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{L_t^{1+\eta}}{1 + \eta} \right). \]  
(2.5)

2.2.1 Monetary Policy

Monetary policy is delegated to an independent central bank that operates under discretion. We consider two variants of inflation targeting: First, we consider standard inflation targeting, under which the central bank strives to keep inflation near or at a prespecified inflation target. Second, we consider a variation on the standard inflation targeting framework by allowing the central bank to raise the inflation target temporarily when short-term nominal interest rate hits the zero lower bound.

Inflation Targeting

The benchmark policy we consider in this paper is inflation targeting, which requires the central bank to keep the rate of inflation at or near a prescribed target over some

---

2This specification is also used by other authors, see for example Paciello and Wiederholt [2013]
time horizon. Following the literature, we consider a quadratic loss function for the
central bank. Specifically, under inflation targeting the central bank, taking private
sector expectations and future monetary policies as given, chooses nominal interest
rate $R_t$ so as to minimize

$$
E_t \sum_{s=t}^{\infty} \left( \prod_{r=t}^{s-1} \beta_r \right) \{ (\Pi_s - \Pi^\ast)^2 + \alpha (Y_s - Y^\ast)^2 \}
$$

subject to the IS curve (2.1), the Phillips curve (2.3) as well as the zero lower bound

$$
R_t - 1 \geq 0,
$$

for period $t$, where $\Pi^\ast$ and $Y^\ast$ are the inflation target and output target, respectively,
and $\alpha$ is the weight on output stabilization. A larger $\alpha$ means that the central bank
is more tolerant with inflation fluctuation.

**Inflation Targeting with a Temporarily Higher Inflation Target**

The other policy we consider is a variation on the standard inflation targeting frame-
work: instead of pursuing a single inflation target, inflation targeting with a tem-
porarily higher inflation target (hybrid IT henceforth) allows the central bank to
raise the inflation target temporarily when short-term nominal interest rate hits the
zero lower bound. The idea is that a higher inflation target, at least when pursued
for a sufficiently long period of time, can potentially help raise inflation expectations,
which in turn mitigates a liquidity trap.

More precisely, we consider two inflation targets for the central bank: a normal
(low) target ($\Pi^\ast$) and a high target ($\Pi^{\ast\ast}$). The high target is triggered once the
zero lower bound is reached. One can think of the zero lower bound being triggered
immediately when the realization of $\theta_t$, conditional on the realization of $\beta_t$, is larger
than a certain threshold. Once the high target has been triggered, the central bank
may switch back to the normal target with an exogenous probability $p > 0$. Hence
$p$ determines the expected duration of pursuing the temporarily higher target. Let
$\delta_t$ be an indicator variable. It is equal to zero if the low target is being pursued,
and one when the central bank is pursuing the high target. $\delta_t$ therefore serves as an endogenous state variable. The law of motion of $\delta_t$ can be described in terms of the probability that $\delta_t$ equals one:

$$
Pr(\delta_t = 1) = \begin{cases} 
1, & \text{if } R_t = 1 \\
0, & \text{if } R_t > 1 \text{ and } \delta_{t-1} = 0 \\
1 - p, & \text{otherwise.}
\end{cases} \tag{2.8}
$$

In each period, the central bank sets the nominal interest rate for that period, taking as given the exogenous state variables as well as private agents’ expectations about future policies. More precisely, the policy planning problem under this policy regime is

$$
\min_{R_t, Y_t, \Pi_t} \mathbb{E}_t \sum_{s=t}^{\infty} \left( \prod_{\tau=t}^{s-1} \beta_\tau \right) \left\{ (1 - \delta_s) (\Pi_s - \Pi^*)^2 + \delta_s (\Pi_s - \Pi^{**})^2 + \alpha (Y_s - Y^*)^2 \right\} \tag{2.9}
$$

subject to the IS curve \(2.1\), the Phillips curve \(2.3\), the zero lower bound \(2.7\) as well as the law of motion of $\delta_s \tag{2.8}$, taking expectations as given.

### 2.2.2 Equilibrium

The resulting equilibrium under each policy regime is a Markov perfect rational expectations equilibrium, which we define as follows.

**Definition 3** A Markov perfect rational expectations equilibrium consists of a payoff-relevant policy strategy $R(s_t)$ and private sector response functions, $\Pi(s_t)$ and $Y(s_t)$, that satisfy the private sector equilibrium conditions as well as the optimality conditions for the central bank (see appendix A of this paper), where $s_t = (\beta_t, \theta_t, \delta_t)$ denotes the state vector.

We are aware that, under discretionary monetary policy, there may exist multiple equilibria (see, e.g., Armenter \[2017\] and Hahn and Wang \[2018\]), but equilibrium multiplicity is not the focus of this paper. The results in this paper are based on the best
equilibrium (in terms of social welfare) under each policy variant. To identify the best equilibrium, we employ the method used in Hahn and Wang 2018.

### 2.3 Mechanism: Two Opposing Forces at Work

Before calibrating the model and performing quantitative analyses, it is useful to set the scene by identifying the forces at work. The primary motivation for a higher inflation target is that it will raise inflation expectations and therefore take some of the burden off the shoulders of the monetary policy maker; raising inflation expectations becomes particularly desirable in the face of the zero lower bound. An equally important force is that a higher inflation target also requires more vigorous policy actions. In the following, we examine the two opposing forces in the context of the log-linearized model, which allows us to derive analytic results.

Let $x_t$ and $z_t$, both in logs, be the actual output and the natural level of output, respectively. The latter is the level of output that would prevail if prices and wages were fully flexible. The difference between the two is often referred to as the output gap. For convenience, we define:

$$y_t \equiv x_t - z_t.$$

In addition, let $\pi_t$ and $i_t$ denote the log-deviations of inflation and the gross nominal interest rate from their respective steady state level.

After log-linearizing the model, the private sector is characterized by two linear equations. The first equation is the New Keynesian IS curve

$$i_t - E_t \pi_{t+1} = \sigma \left( E_t y_{t+1} - y_t \right),$$

where $E_t \pi_{t+1}$ and $E_t y_{t+1}$ are expectations of one-period ahead inflation and output gap, both formed at period $t$. The second equation is the New Keynesian Phillips

---

$^3$Hahn and Wang 2018 shows that, although equilibrium multiplicity is pervasive, the set of Markov equilibria are rather similar to one another.

$^4$The linear economy is derived by log-linearizing the underlying nonlinear economy around its efficient steady state.
curve
\[ \pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}, \]  
(2.11)
where \( \kappa > 0 \). For simplicity, we ignore the zero lower bound and shut down all exogenous shocks.\(^5\)

Now consider strict inflation targeting under which the central bank is concerned solely with inflation stabilization. The loss function for the central bank is of the following form
\[ l_t = (1 - \delta_t)(\pi_t - \pi^*)^2 + \delta_t(\pi_t - \pi^{**})^2, \]
where \( \pi^* \) and \( \pi^{**} \) (\( \pi^* < \pi^{**} \)) denote the low target and the high target, respectively.
As in the case of hybrid inflation targeting discussed above in the fully non-linear model, we assume that, once the high target has been triggered, there is a positive probability \( p \) of switching back to the low target.\(^6\) In the absence of the zero lower bound, the central bank can always adjust the nominal interest rate to keep inflation on target. We are interested in the impact of raising the inflation target temporarily on the nominal interest rate \( i_t \). Combining (2.10) and (2.11), we obtain the nominal interest rate under the high target
\[ i_t^{**} = \sigma \mathbb{E}_t y_{t+1} + \left(1 + \frac{\sigma \beta}{\kappa}\right) \mathbb{E}_t \pi_{t+1} - \sigma \frac{\pi^{**}}{\kappa} - \sigma \frac{\kappa \pi^{**}}{\kappa}, \]
(2.12)

\(^5\)By ignoring the zero lower bound, we can focus on a single equilibrium. Otherwise, there would exist multiple equilibria, even in the case of a fully deterministic economy.

\(^6\)It is important to note that once the central bank has returned to the low target, the high target will never be triggered again because we have shut down exogenous shocks.
CHAPTER 2. A TEMPORARILY HIGHER INFLATION TARGET

The impact of raising the inflation target temporarily on the nominal interest rate can be seen more clearly by taking the partial derivative of $i_t^{**}$ with respect to $\pi^{**}$:

$$
\frac{\partial i_t^{**}}{\partial \pi^{**}} = \sigma \frac{\partial E_t y_{t+1}}{\partial \pi^{**}} + \left(1 + \frac{\sigma \beta}{\kappa}\right) \frac{\partial E_t \pi_{t+1}}{\partial \pi^{**}} - \frac{\sigma}{\kappa} \\
= \sigma (1 - p) \frac{1 - \beta (1 - p)}{\kappa} + \left(1 + \frac{\sigma \beta}{\kappa}\right) (1 - p) - \frac{\sigma}{\kappa}.
$$

Equation (2.13) shows that raising the inflation target has two effects: an expectations effect and a direct policy effect. On the one hand, pursuing a higher inflation target temporarily raises inflation and output expectations. On the other hand, pursuing a higher target also requires more vigorous actions. Which of these two forces dominates depends crucially on $p$, which governs the expected duration of pursuing the high target.

In this simple case considered here, the direct policy effect tends to dominate for plausible parameter values. Figure 2.1 plots the effect of raising the inflation target temporarily on the nominal interest rate, $\frac{\partial i_t^{**}}{\partial \pi^{**}}(\pi^{**} - \pi^*)$, for different values of $p$. It shows that only when pursuing the high target sufficiently long, that is, $p$ is sufficiently small, the positive effect dominates.

In addition, it is interesting to examine the two extreme cases. First, when $p = 1$, raising the inflation target has no influence on inflation and output expectations. And yet the central bank has to engineer a lower interest rate to achieve the higher target. So the net effect on the nominal interest rate is negative. Second, when $p$ goes to zero, the high target becomes permanent. $\frac{\partial i_t^{**}}{\partial \pi^{**}}(\pi^{**} - \pi^*)$ is always positive. In other words, raising the inflation target permanently always has a positive impact on the nominal interest rate.

2.4 Calibration

For the numerical results below, we rely on the fully non-linear model. We consider standard inflation targeting as a benchmark and calibrate the model to the U.S.
Figure 2.1: The effect of raising the inflation target temporarily on the nominal interest rate, $\frac{\partial i_t^{**}}{\partial \pi^{**}}(\pi^{**} - \pi^*)$, for different values of $p$. For this exercise, we $\pi^* = 0.5\%$, $\pi^{**} = 1\%$, $\beta = 0.994$, $\sigma = 1$ and $\kappa = 0.024$. Note that $\pi^*$ and $\pi^{**}$ are quarterly rates.

economy. In the benchmark model, we set the inflation target $\Pi^*$ to 2%, in line with the official target of the Federal Reserve. There are fourteen parameters for which we need to assign values. Some of those are standard technology and preference parameters; for those parameters we assign reasonably conventional values. The coefficient of relative risk aversion $\sigma$ is set to 1, corresponding to a log utility function in consumption. This value is commonly used in the literature (e.g., Miao and Ngo 2014). Following L. Christiano, Eichenbaum, and Rebelo 2011, we set the level of productivity $A$ and the inverse of the Frisch elasticity of labor supply $\eta$ to 1. The weight on the disutility of labor $\chi$ is set to 1, as in Fernández-Villaverde et al. 2015.
Our choice of the value for $\psi$, the parameter that governs the cost of price adjustment, is also fairly conventional, see, for example, Leith and Liu [2016].

For the calibration of the remaining parameters, I follow the same procedure as in Hahn and Wang [2018]. The parameter values of the benchmark calibration is reported in Table 2.1.

### Table 2.1: Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>productivity</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>coeff. of relative risk aversion</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi$</td>
<td>weight to disutility of labor</td>
<td>1.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>inverse of the Frisch elasticity</td>
<td>1.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>parameter associated with price adj. cost</td>
<td>129.0</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>low discount factor</td>
<td>0.9927</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>high discount factor</td>
<td>1.002</td>
</tr>
<tr>
<td>$p_{th}$</td>
<td>$Pr(\beta_{t+1} = \beta_h \mid \beta_t = \beta_l)$</td>
<td>0.0079</td>
</tr>
<tr>
<td>$p_{hl}$</td>
<td>$Pr(\beta_{t+1} = \beta_l \mid \beta_t = \beta_h)$</td>
<td>0.0357</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>persistence parameter</td>
<td>0.1423</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>mean</td>
<td>10.1166</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>standard deviation of $\epsilon_\theta$</td>
<td>0.3026</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>inflation target</td>
<td>1.005</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>output target</td>
<td>0.9511</td>
</tr>
</tbody>
</table>

2.5 Results

In this section, we conduct a number of numerical experiments to examine the implications of pursuing a higher inflation target. First, we compare two percent inflation targeting with four percent inflation targeting. Second, we explore hybrid inflation targeting by varying the average duration of pursuing the high target, which is governed by the parameter $p$. For simplicity, we consider the case where the high target is four percent (annually).\(^7\) We discretize the markup shock process \(^{[2.4]}\) using the

---

\(^7\) For example, Olivier Blanchard and other prominent economists (see, e.g., Blanchard, Dell’Ariccia, and Mauro [2010] and Ball [2014]) propose that the Fed and other inflation-targeting central banks consider raising the inflation target to 4%.
Rouwenhorst method (see, Kopecky and Suen 2010 for a description of the method). We assign 69 nodes along the dimension of $\theta$.

### 2.5.1 Two Percent Versus Four Percent

![Figure 2.2: Standard inflation targeting: $\Pi^* = 2\%$ vs $\Pi^* = 4\%$. The upper (lower) left panel shows the policy function for two percent (four percent) inflation targeting. The right panels are the associated conditional inflation expectations.](image)

In this subsection, we show that raising the inflation target from two percent to
four percent permanently would make the zero lower bound less likely to bind. In Figure 2.2 we plot the policy function of standard inflation targeting with different inflation targets: $\Pi^* = 2\%$ vs $\Pi^* = 4\%$. $\theta_l^*$ and $\theta_h^*$ denote the threshold at which the zero lower bound starts to bind for low discount factor and high discount factor, respectively.

Clearly, under four percent inflation targeting, the zero lower bound binds less frequently: the thresholds $\theta_l^*$ and $\theta_h^*$ lie strictly to the right of the thresholds under two percent inflation targeting. This is the case where the expectations effect dominates the direct policy effect.

In the next section, we will show that the same opposing forces are also at work under hybrid inflation targeting, and that which of these two effects dominates depends crucially on the average duration of pursuing the high target.

### 2.5.2 Three Scenarios

Figure 2.3 shows the policy function and inflation expectations under hybrid IT with three different values of $p$. Policy functions are shown on the left panels while inflation expectations are shown on the right panels. Apart from inflation expectations under hybrid IT, we also show (dotted lines on the right panels of Figure 2.3) inflation expectations formed under standard two percent IT and four percent IT. Those expectations, especially expectations formed under the benchmark two percent IT, are helpful for understanding how raising the inflation target temporarily affects inflation expectations. A few observations can be made from Figure 2.3.

First, raising the inflation target temporarily has different effects on inflation expectations depending on whether the central bank is pursuing the high target or the low target. When the economy is operating on the high target, anticipating that the high target may still be pursued for a few more periods, inflation expectations are uplifted. On the other hand, inflation expectations are affected to a lesser extent when the economy is on the low target.

---

8Plots for inflation and output are relegated to the appendix.
Figure 2.3: Hybrid IT with $p = 0.5$ (upper panels), $p = 0.25$ (middle panels), and $p = 0.05$ (lower panels). The panels on the left side show the policy function for each scenario. Solid lines are policy rate under the low target while dash-dot lines are policy rate under the high target. Conditional inflation expectations are shown on the right hand side. Solid lines are inflation expectations for the corresponding hybrid IT, while dotted lines are inflation expectations under standard two percent IT (lower curves) and standard four percent IT (upper curves).
CHAPTER 2. A TEMPORARILY HIGHER INFLATION TARGET

Second, the degree to which inflation expectations are affected by pursuing a temporarily higher target depends crucially on the average duration of such efforts. This is not surprising: the longer the average duration, the closer hybrid IT is to permanent four percent inflation targeting. For example, when $p = 0.05$, corresponding to the case where the average duration of pursuing the high target is five years, inflation expectations under hybrid IT is close to those formed under permanent four percent inflation targeting.

Third, related to the second point, which of the two forces mentioned in the previous section dominates depends on the average duration of pursuing the high target. When the average duration is two quarters ($p = 0.5$), the effects of higher inflation expectations are not strong enough to offset the opposing force associated with a higher target. In this case, pursuing a higher target temporarily makes exiting a liquidity trap harder. In the second scenario ($p = 0.25$), the two opposing forces roughly offset each other. The third scenario ($p = 0.05$) shows the case where the effects of higher inflation expectations dominate.

2.5.3 Quantitative Results

In this subsection, we quantify the effects of pursuing a higher target. Table 2.2 reports some summary statistics, including welfare—the unconditional expected present value of utility of the representative household, the first and second moments of inflation and output, the frequency of zero lower bound events, and average real interest rate during zero lower bound episodes. Inflation, interest rate and standard deviations are reported in terms of annual rates. For this exercise, we simulate ten paths of the economy for 110,000 quarters. We assume that the central bank is pursuing the low target in the first period. After dropping the first 10,000 quarters, we use the last 100,000 quarters of the first path to estimate the statistics and all 10 paths to estimate the standard error of each statistic. Note that we report standard errors only when they are greater than $1 \times 10^{-4}$.

Two observations are in order. First, pursuing a higher target does help reduce the frequency of zero lower bound events. Pursuing a four percent target permanently
can reduce the frequency of zero lower bound events by as much as 6.89 percent. On the other hand, pursuing a higher target temporarily has a modest effect on the frequency of zero lower bound events. It reduces the frequency by about three percent.

Second, raising the inflation target, whether temporarily or permanently, reduces social welfare. The reason is that adjusting prices is costly in this model—a common feature in the new Keynesian literature, and pursuing a higher target leads to higher inflation on average. Yet there is growing evidence that the costs of inflation may be overestimated, see, for example, Nakamura et al. 2016. In the next section, we conduct robustness analysis by varying the cost of price adjustment, governed by the parameter ψ.

### 2.6 Robustness Analysis

In this section we examine the robustness of our findings by varying the cost of price adjustment, which is governed by the parameter ψ. A larger value of ψ means that inflation is more costly. Specifically, we consider two scenarios. In the first scenario, we set ψ at half of the value of the same parameter in the benchmark. In the second scenario, we consider the case where ψ is twice as high as that in the benchmark.
configuration.\footnote{Granted, it would be better if we recalibrated the model after changing the price adjustment cost parameter. But this exercise is nonetheless interesting in that it helps us get a sense of the importance of price adjustment cost for the main results.} The same set of statistics are reported in table \ref{table:2.3}.

The results show that, if the true cost of inflation is lower than the benchmark by one half, inflation will be more volatile and the zero lower bound is far more likely to bind. In this scenario, pursuing a higher target, whether temporarily or permanently, can greatly reduce the frequency of zero lower bound events. Moreover, the results suggest that pursuing a permanently higher target can be more appealing than pursuing a higher target temporarily. The reason is that, in this scenario, inflation is less costly than variations in consumption. For example, pursuing a four percent target for an average duration of five years during zero lower bound episodes would deliver much lower social welfare than pursuing a four percent target permanently.

On the other hand, if the true cost of inflation is even higher than in the benchmark, inflation and output will be less volatile, and the zero lower bound is less likely to bind as opposed to the benchmark. As a result, raising the inflation target is less effective in reducing the frequency of zero lower bound events. It can even increase the frequency of zero lower bound events when the average duration of pursuing the high target is too short. For instance, when pursuing a four percent target for an average duration of two quarters during zero lower bound episodes, hybrid IT can increase the frequency of zero lower bound events by more than five percent. Moreover, since the cost of inflation is higher than the benchmark, pursuing a higher target, whether temporarily or permanently, can be very costly.

\section{Conclusion}

This paper studies the mechanism and implications of pursuing a higher inflation target temporarily at the zero lower bound in a conventional new Keynesian economy. We find that raising the inflation target temporarily has two opposite effects on the nominal interest rate. On the one hand, it raises inflation expectations and therefore makes the zero lower bound less likely to bind. On the other hand, a higher target
Table 2.3: Summary statistics

<table>
<thead>
<tr>
<th>Welfare</th>
<th>$TT$ (2%)</th>
<th>HIT (p = 0.5)</th>
<th>HIT (p = 0.25)</th>
<th>HIT (p = 0.05)</th>
<th>IT (4%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(V)$</td>
<td>−70.74</td>
<td>−76.77</td>
<td>−76.88</td>
<td>−82.53</td>
<td>−77.11</td>
</tr>
<tr>
<td>Standard error</td>
<td>$1.8 \times 10^{-3}$</td>
<td>$2.3 \times 10^{-3}$</td>
<td>$2.3 \times 10^{-3}$</td>
<td>$2.6 \times 10^{-3}$</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>Inflation ($%$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Pi)$</td>
<td>1.71</td>
<td>1.98</td>
<td>2.11</td>
<td>2.17</td>
<td>4.07</td>
</tr>
<tr>
<td>$\sigma(\Pi)$</td>
<td>1.48</td>
<td>1.27</td>
<td>1.19</td>
<td>1.14</td>
<td>0.99</td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(Y)$</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma(Y)$ ($%$)</td>
<td>2.36</td>
<td>2.49</td>
<td>2.58</td>
<td>2.71</td>
<td>2.74</td>
</tr>
<tr>
<td>ZLB ($%$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Pr(R = 1)$</td>
<td>27.38</td>
<td>23.12</td>
<td>18.91</td>
<td>15.13</td>
<td>7.16</td>
</tr>
<tr>
<td>$E(r</td>
<td>R = 1)$</td>
<td>−0.53</td>
<td>−0.59</td>
<td>−0.60</td>
<td>−0.63</td>
</tr>
<tr>
<td>$E(\Pi</td>
<td>R = 1)$</td>
<td>1.03</td>
<td>1.61</td>
<td>1.87</td>
<td>2.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Welfare</th>
<th>$TT$ (2%)</th>
<th>HIT (p = 0.5)</th>
<th>HIT (p = 0.25)</th>
<th>HIT (p = 0.05)</th>
<th>IT (4%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(V)$</td>
<td>−81.02</td>
<td>−87.42</td>
<td>−87.46</td>
<td>−88.46</td>
<td>−97.52</td>
</tr>
<tr>
<td>Standard error</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Inflation ($%$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\Pi)$</td>
<td>1.66</td>
<td>1.73</td>
<td>1.73</td>
<td>1.78</td>
<td>2.84</td>
</tr>
<tr>
<td>$\sigma(\Pi)$</td>
<td>0.43</td>
<td>0.43</td>
<td>0.43</td>
<td>0.44</td>
<td>0.39</td>
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<tr>
<td>Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(Y)$</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma(Y)$ ($%$)</td>
<td>0.60</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
<td>0.77</td>
</tr>
<tr>
<td>ZLB ($%$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Pr(R = 1)$</td>
<td>4.23</td>
<td>9.47</td>
<td>4.47</td>
<td>2.80</td>
<td>1.66</td>
</tr>
<tr>
<td>$E(r</td>
<td>R = 1)$</td>
<td>$-1.4 \times 10^{-3}$</td>
<td>$-0.34 \times 10^{-3}$</td>
<td>$-1.6 \times 10^{-3}$</td>
<td>$-1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$E(\Pi</td>
<td>R = 1)$</td>
<td>1.67</td>
<td>1.84</td>
<td>1.81</td>
<td>1.90</td>
</tr>
</tbody>
</table>

also requires more vigorous actions. Which of these two effects dominates depends crucially on the average duration of pursuing the high target. We also find that raising the inflation target permanently is more effective in reducing the frequency of zero lower bound events, albeit with higher welfare costs. Finally, pursuing a higher target temporarily may be more desirable in practice because it can potentially offset the rise in real debt burdens that is typically associated with a liquidity trap.
Bibliography


Chapter 3

Education Affordability and Earnings Inequality

3.1 Introduction

This paper studies the role of education affordability in shaping earnings inequality. We begin by documenting an empirical fact about the correlation between education affordability and earnings inequality across countries. Table 3.1 reports the Gini index of before-tax gross earnings for full-time male workers, the private and public direct cost of a person attaining tertiary education as well as the fraction of tertiary education costs borne by the government for the United States, the United Kingdom and six continental European countries. It shows a strong correlation between education affordability and earnings inequality: the Pearson correlation coefficient between the Gini index and the private cost of a person attaining tertiary education is 0.939.

This empirical fact motivates us to consider education affordability as a potentially important determinant of earnings inequality. To evaluate the role of education affordability in driving earnings inequality, we build an overlapping generations model where agents, heterogeneous with respect to learning ability and initial wealth

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1 The table is compiled using data from OECD.stat online database.
2 The correlation coefficient between the Gini index and the private cost of education as a fraction of per capita GDP is 0.932.
endowment, decide whether to attend college, subject to borrowing constraints and idiosyncratic income risks. To capture the various aspects of government policy, our model features progressive labor income taxation, consumption tax, capital income tax as well as subsidies for college. After calibrating the model to the U.S. economy, we conduct a number of counterfactual experiments. In particular, we replace the status quo US education policy, the fraction of higher education costs borne by the government, with its German counterpart and study the change in earnings inequality. Besides, we also examine the roles of consumption taxes, capital income taxes as well as progressive labor income taxation. Though this paper is motivated by observations from cross-country comparisons, it focuses squarely on the US.

We find that earnings inequality, as measured by the Gini coefficient for before-tax gross earnings, would decrease by as much as 16.2 percent if the current education policy, the share of higher education costs borne by the government, were replaced with its German counterpart. Two groups of individuals would benefit from the

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1. The Gini index is compiled using before-tax gross earnings for full-time male workers for the period from 2005 to 2014.
2. Private and public direct costs of a person attaining tertiary education are in 2011 PPP international dollars.

---

### Table 3.1: Education affordability and earnings inequality

<table>
<thead>
<tr>
<th></th>
<th>Gini index</th>
<th>Private costs</th>
<th>Public costs</th>
<th>Subsidy rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_1$</td>
<td>$c_2$</td>
<td>$z = \frac{c_2}{c_1 + c_2}$</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>0.285</td>
<td>4300</td>
<td>98 400</td>
<td>0.958</td>
</tr>
<tr>
<td>Finland</td>
<td>0.268</td>
<td>3400</td>
<td>91 300</td>
<td>0.964</td>
</tr>
<tr>
<td>France</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>0.314</td>
<td>5200</td>
<td>87 500</td>
<td>0.944</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.286</td>
<td>16 900</td>
<td>73 000</td>
<td>0.812</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.272</td>
<td>200</td>
<td>97 200</td>
<td>0.998</td>
</tr>
<tr>
<td>U.K.</td>
<td>0.341</td>
<td>25 900</td>
<td>27 700</td>
<td>0.517</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.410</td>
<td>55 000</td>
<td>55 900</td>
<td>0.503</td>
</tr>
</tbody>
</table>

---

3We choose Germany for the counterfactual experiments for the simple reason that Germany is a relatively large country with higher subsidies for higher education and more progressive labor income taxes.
hypothetical policy reform. First, poor individuals with medium and medium-high ability would benefit the most from increased government subsidies for college: this group would enjoy an increase in life-time utility equivalent to an increase in consumption by as much as 80%. Those are the individuals that cannot afford to go to college under the status quo policy, although they would benefit greatly from a college education. Second, low ability individuals would also benefit from the hypothetical education policy reform, albeit for different reasons. They do not go to college anyway. But when the college wage premium decreases they enjoy a higher wage rate. On the other hand, our results suggest that a small fraction of rich and high-ability individuals would lose out from the hypothetical education policy reform due to compressed college wage premium. For example, an individual with an initial wealth at the 90th percentile of the initial wealth distribution and a learning ability at the 99th percentile of the ability distribution would suffer a utility loss equivalent to a 17.5% decrease in consumption. Finally, in contrast with the existing literature (see, for example, Guvenen, Kuruscu, and Ozkan 2014), we find that labor tax progressivity plays a less significant role in explaining earnings inequality.

The rest of the paper is organized as follows. Section 3.2 relates this paper to the literature. Section 3.3 sets up the model and defines the equilibrium. Section 3.4 describes our calibration strategy. We present the results in section 3.6. We briefly conclude the paper in section 3.7.

3.2 Relation to The Literature

A major goal of this paper is to evaluate the role of education affordability in shaping earnings inequality. In terms of modelling choices, our paper is built on the public economics literature on tax and education policy, for example, Abbott et al. 2013, Benabou 2002, Bovenberg and Jacobs 2005, Findeisen and Sachs 2015, 2016a,b, Heathcote, Storesletten, and G. L. Violante 2017, and Krueger and Ludwig 2016, 2013. This tradition, pioneered by Benabou 2002, emphasizes the distortionary effects of progressive taxation on human capital accumulation; it also recognizes that education subsidies can partially mitigate the tax distortions and therefore serve as
a complement to progressive labor income taxation. Most of this literature allows for general equilibrium effects of government policies on relative factor prices. In line with this literature, this paper considers the distortionary effects of taxation, the complementary role of education subsidies as well as the general equilibrium effects of government policy on relative factor prices in general and the college wage premium in particular.

In particular, our paper is closely related to Abbott et al. 2013, which studies, in a similar but more complex framework, the general equilibrium effects of various financial aid policies intended to foster college participation and Krueger and Ludwig 2016, which characterizes the optimal mix of progressive taxation and education subsidies. Our paper overlaps with Abbott et al. 2013 in that college participation, the main variable of interest in their paper, is an intermediate variable of interest for us. Our paper, however, is not solely concerned with college attainment, but also evaluates the importance of education policy, progressive labor income taxes, consumption taxes and capital income taxes in driving earnings inequality. Moreover, our paper also differs from Abbott et al. 2013, who use the data for the early 2000s and find that increased subsidies for college would only have modest effects on college attainment, in that we calibrate the model to the latest data (2011) which exhibits higher college wage premium and earnings inequality than those used in their paper. This may partially explain why we come to different conclusions about the impact of increased government spending on higher education on college attainment. Our paper differs from Krueger and Ludwig 2016, 2013 in terms of research questions: while we study the roles of various government policies in shaping earnings inequality, they quantitatively characterize the optimal tax and education policy and are largely silent on earnings inequality.

Our paper is also related to the strand of literature that explores the determinants of cross-countries differences in wage inequality (see, for example, Guvenen, Kuruscu, and Ozkan 2014) and intergenerational earnings persistence (see, for example, Holter 2015). Guvenen, Kuruscu, and Ozkan 2014 attribute the wage inequality gap to differences in labor income tax progressivity: More progressive labor income taxes, as
CHAPTER 3. EDUCATION AFFORDABILITY AND WAGE INEQUALITY

seen in continental European countries, distort the incentives for individuals to accumulate human capital, which compresses the wage income distribution. However, Guvenen, Kuruscu, and Ozkan 2014 abstract from the general equilibrium effects of government policy on relative factor prices and it assumes that markets are complete so that individuals that wish to invest in human capital accumulation are always able to do so by borrowing. We instead consider an environment where markets are incomplete and those who wish to attend college may find themselves financially constrained. In addition, we allow for endogenous evolution of college premium. Importantly, while taking inspiration from cross-country comparisons, our paper focuses squarely on the US. The reason is that factors affecting cross-countries differences in earnings inequality may go well beyond government policies.

Finally, our paper is related to the strand of empirical literature on the returns to skills, for example, David 2014, Goldin and Katz 2009, Hanushek et al. 2015 and Lemieux 2006. A central message of this literature is that the rising wage premium associated with higher education is a key driver in the evolution of earnings inequality. Although we take a static view and do not look at transition dynamics, our paper captures the pivotal role of the interplay of supply and demand for skills.

3.3 The Model Economy

We consider an overlapping generations model. The model economy consists of individuals that are heterogeneous with respect to age, wealth, learning ability, education and labor productivity, firms that produce a final good by hiring labor and capital on competitive spot markets, and a government that operates the tax system and the pension system. The decisions of individuals differ depending on the phase in which they find themselves. First, in the first period of its life, having drawn its learning ability and education-contingent initial wealth endowment, each individual decides whether to attend college. Second, after making their college decisions, individuals make their consumption and labor supply decisions in each period of their working life. Finally, individuals retire and live on capital income and pension benefits. We will describe the model environment and individual life cycle decisions in detail in the
followed.

3.3.1 Demographics

Time is discrete, indexed by $t$, and it goes forever. At each point in time, the economy is populated by $J$ overlapping generations indexed by $j = 1, 2, \cdots, J$, where $J$ denotes the maximum age. Individuals survive from age $j$ to $j+1$ with probability $\psi_{j+1}$. For simplicity, we assume that the survival rate before retirement is equal to one; agents face death hazard once they retire, i.e., $\psi_j \in [0,1)$ for $j \geq j_r$, where $j_r$ denotes the retirement age. Let $N_t$ denote the initial size of the cohort that enters the economy in period $t$; $N_t$ grows at a constant rate $n$, i.e., $N_t = (1+n)N_{t-1}$. Since the growth rate of population is constant and the age-specific survival rates are time-invariant, the relative share of population of each age cohort is constant over time. To ease aggregation later on, we define $m_j$ as the size of population of age cohort $j$ relative to the youngest cohort alive in the current period:

$$m_j := \frac{N_{t-j+1} \left( \prod_{i=0}^{j-1} \psi_i \right)}{N_t}.$$  

3.3.2 Firms and Production

Firms hire labor and capital on competitive spot markets to produce a final good. Workers come in two skill types, indexed by $s = \{c,n\}$, where we refer to college-educated workers as skilled workers ($s = c$), and to those without college education as unskilled workers ($s = n$). Within each skill type, labor is perfectly substitutable across ages. Labor is imperfectly substitutable across skill types in the tradition of Katz and Murphy [1992] and Borjas [2003]. Let $L_{t,c}$ and $L_{t,n}$ denote aggregate labor–in terms of efficiency units– of skilled workers and unskilled workers, respectively. Then aggregate labor across skill types is given by

$$L_t := \left( L_{t,c}^\xi + L_{t,n}^\xi \right)^{1/\xi},$$  

(3.1)
where \( \frac{1}{1-\zeta} \) is the elasticity of substitution between skilled and unskilled labor.

Final output is produced according to the Cobb-Douglas production function,

\[ Y_t = AK_t^\alpha L_t^{1-\alpha}, \]

where \( A \) denotes total factor productivity and \( \alpha \) is a parameter that governs the elasticity of output with respect to capital. With perfect competition and a constant return to scale production function, the size distribution of firms is indeterminate; without loss of generality, we assume the existence of a representative firm. The representative firm takes the wage rates of skilled and unskilled labor, \( w_{t,c} \) and \( w_{t,n} \), and the interest rate \( r_t \) as given.

### 3.3.3 Endowments, Labor Productivity and Preferences

When a new generation enters the economy, individuals are heterogeneous in their learning ability \( e \) and initial wealth \( a \). Initial wealth endowments are education-contingent. An individual will receive an actual initial wealth of \( a_c \) if he decides to attend college. Otherwise, the individual will receive an initial wealth of \( a_n \) with \( a_n < a_c \). The rationale behind this assumption is that inter vivos transfers from parents to their college age children are mainly motivated by parents’ preference for their children to attend college. In addition, each individual is endowed with one unit of labor in each period of its life.

There are three elements to labor productivity \( h_{s,j} \) for each skill type: a deterministic life-cycle productivity profile \( \epsilon_{s,j} \), a fixed productivity effect \( \theta_s \in \Theta_s \), and a stochastic component \( \eta_s \) that evolves according to a Markov process \( \pi_{\eta_s} \). Labor productivity is given by

\[
h_{s,j} = \begin{cases} 
\epsilon_{s,j} \cdot \exp (\theta_s + \eta_{s,j}), & \text{if } j < j_r \\
0, & \text{otherwise}
\end{cases}
\]

\(^4\)We use heterogeneity in initial wealth to capture the family income effect on college attendance. For example, Belley and Lochner [2007] show that family income has significant effects on educational achievement in the early 2000s.
where we assume that labor productivity drops to zero at the retirement age $j_r$.

Individuals have preferences over streams of consumption $c_j$ and leisure $\tilde{l}_j$. Except for college students, $\tilde{l}_j = 1 - l_j$, where $l_j$ denotes labor supply. In the case of college students, $\tilde{l}_j = 1 - \xi(e) - l_j$, where $\xi(e)$ is the time cost of college. More precisely, individuals’ preferences are given by

$$u(c_1, 1 - 1_s \cdot \xi(e) - l_1) + \beta \mathbb{E}_1 \sum_{j=2}^{J} \beta^{j-2} \left( \prod_{i=0}^{j} \psi_i \right) u(c_j, 1 - l_j),$$

where $1_s$ is a variable indicating an individual’s college decision, equal to one if the individual decides to attend college, and zero otherwise.

3.3.4 College Education

At age one, after having drawn her learning ability $e$ and education-contingent initial wealth $a$, an individual decides whether to attend college. Attending college takes one period. Moreover, going to college entails a resource cost that is a fraction or multiple, $\iota$, of the wage rate of skilled labor $w_{t,c}$, and a time cost $\xi(e) \in (0, 1)$ that depends on learning ability.\footnote{Ability-based time cost is intended to capture the notion that students of higher ability take less time to reach the same academic achievement. Clearly, this involves a high degree of simplification since high-ability students may aspire to achieve more than their peers. Thus we homogenize college education in this sense.} To accommodate the case where college education is, at least partially, funded by government subsidies, let $z$ denote the fraction of resource cost borne by the government. Upon deciding to go to college, financially constrained individuals can work part-time and/or take out student loans subject to a borrowing constraint.

3.3.5 Market Structure

Financial markets are incomplete in that there is no insurance against idiosyncratic labor productivity shocks and mortality risks. Individuals can self-insure against those risks by accumulating risk-free assets in the form of capital and government bonds.
Borrowing is allowed only for financing college education. We further assume that student loans are fully paid back before retirement when early death hazard sets in so that we rule out student loan defaults.

3.3.6 Government

The government runs the tax system and the pension system. First, the government collects taxes on individual consumption, capital income, labor income and issues one-period government bonds so as to finance public expenditure $G_t$, debt service payment on outstanding government bonds $B_t$ and education subsidies. Taxes on consumption and interest income are flat with a tax rate of $\tau_c$ for consumption expenditures and $\tau_k$ for interest income. Following Heathcote, Storesletten, and G. Violante 2010, we consider a potentially progressive labor income tax function,

$$\tau_l(y) = 1 - \lambda y^{-m},$$

where $\tau_l(y)$ is the tax rate at income level $y$, $m > 0$ is a measure of progressivity of the tax schedule and $\lambda$ is a parameter that governs the average tax rate (for a given $m$).

In addition, the government collects accidental bequests and redistributes them to the generation that has just entered the economy. To come up with a plausible calibration of the initial wealth distribution, we assume that the government fills the gap between accidental bequests and actual transfers in each period.

The pension system operates on a pay-as-you-go basis: it collects contributions from current workers and distributes the revenues directly to current pensioners. In period $t$, current workers of skill type $s \in \{c, n\}$ contribute a fraction, $\tau_{p,s}$, of their labor income to the pension fund, and current retirees receive a pension benefit that is a fraction, $\kappa_s$, of the average income of the working age population with characteristics $(s, \theta_s)$:

$$\text{pen}_t(s, \theta_s) = \kappa_s w_t(s, \theta_s) \bar{L}_t(s, \theta_s),$$

where $\bar{L}_t(s, \theta_s)$ is the average labor supply, in efficiency unit terms, of that group. The budget constraint of the pension system
then reads
\[
\sum_{s \in \{c,n\}} \tau_{p,s} w_{t,s} L_{t,s} = \sum_{s \in \{c,n\}} \sum_{\theta_s \in \Theta_s} \sum_{j=1}^J \text{pen}_t(s, \theta_s) m_j(s, \theta_s),
\]
(3.2)
where \(m_j(s, \theta_s)\) is the relative size of age cohort \(j\) that falls into the skill category \(s\) and has a fixed productivity component \(\theta_s\).

3.3.7 Life Cycle

In this subsection, we set out the individuals’ life-cycle decisions more precisely.

Decisions at age 1: At age one, individuals make their decisions in two steps. First, an individual decides whether to attend college. Second, after college decision has been made, the individual chooses consumption and labor supply for that period.

Upon entering the economy, individuals with learning ability \(e\) and education-contingent initial wealth \(\{a_c, a_n\}\) decide whether to attend college. The college decision is formally defined as

\[
1_s(e, a) = \begin{cases} 
1, & \text{if } W(e, a_c, c) > W(e, a_n, n) \\
0, & \text{otherwise,}
\end{cases}
\]

where \(W(e, a_s, s)\) is the expected present value of life-time utility of an individual with college decision \(s\), learning ability \(e\) and an initial wealth endowment of \(a_s\). It is formally defined as

\[
W(e, a_s, s) = \sum_{\theta \in \Theta_s} \sum_{\eta \in \mathcal{H}_n} \pi_{\theta_n}(\theta) \pi_{\eta_n}^*(\eta) V(1, e, s, a_s, \theta, \eta)
\]

where \(\pi_{\theta_n}(\theta)\) and \(\pi_{\eta_n}^*(\eta)\) are the distributions from which individuals draw their fixed productivity effect \(\theta\) and stochastic productivity component \(\eta\), \(\pi_{\eta_n}^*(\eta)\) is the stationary distribution of \(\pi_{\eta_n}(\eta' \mid \eta)\), and \(V(1, e, s, a_s, \theta, \eta)\) is the expected present value of life-time utility of an individual with state vector \((1, e, s, a_s, \theta, \eta)\). While initial wealth is drawn from an education-dependent distribution, initial fixed productivity effect
θ and idiosyncratic productivity shock η are drawn from distributions for unskilled workers, whether the individual is college-bound or not. Upon finishing college education, skilled workers, those with a college degree, will redraw the fixed productivity effect θ and idiosyncratic shock η from distributions for skilled workers. Thereafter, the stochastic productivity component evolves over time according to πηs(η′ | η).

Given its initial wealth a, college decision s, fixed labor productivity effect θ and its initial draw of stochastic productivity η, each individual then chooses consumption and labor supply so as to maximize its expected present value of lifetime utility. Formulated recursively, each individual solves the following Bellman equation

\[
V(1, e, s, a, \theta, \eta) = \max_{c, l} \left\{ u(c, 1 - 1_s(e, a)\xi(e) - l) + \beta \psi^2 \sum_{\eta'} \pi_{\eta_s}(\eta'|\eta)V(2, e, s, a', \theta, \eta') \right\}
\]

subject to the budget constraint

\[
(1 + \tau_c)c + a' + 1_s(1-z)\xi_{t,c} = (1 + (1 - \tau_k)r_t)a + (1 - \tau_{p,s})w_{t,s}h_{s,1}l - y\xi_t(y) + Tr,
\]

where \( y = (1 - \tau_{p,s})w_{t,s}h_{s,1}l \), and the borrowing constraint

\[
a' \geq -A_1.
\]

For college-bound individuals, the Bellman equation is slightly different because they will redraw \( \theta \) and \( \eta \) at the beginning of period two from \( \pi_{\theta_s}(\theta) \) and \( \pi_{\eta_s}^*(\eta) \), where \( \pi_{\eta_s}^*(\eta) \) is the stationary distribution of \( \pi_{\eta_s}(\eta'|\eta) \).

**Decisions at age \( j = 2, ..., j_r - 1 \):** While decisions at age one may differ for college-bound and non-college bound individuals, decisions during the normal working life are pretty standard: given the current situation \((j, e, s, a, \theta, \eta)\), each individual chooses consumption and labor supply so as to maximize its expected present value of utility.\(^6\) The

\(^6\)From period 2 onward, learning ability \( e \) becomes a redundant state variable. We nonetheless keep it in the state vector for consistency. The same is true for \( \theta \) and \( \eta \) for retirees.
Bellman equation reads

\[ V(j, e, s, a, \theta, \eta) = \max_{c, l} \left\{ u(c, 1 - l) + \beta \psi_{j+1} \sum_{\eta'} \pi_{\eta s}(\eta' | \eta) V(j + 1, e, s, a', \theta, \eta') \right\} \]  

(3.6)

subject to

\[ (1 + \tau_c)c + a' = (1 + (1 - \tau_k)r_t)a + (1 - \tau_{p,j})w_{t, s}h_{s, j}l - y\tau_l(y) + Tr, \]  

(3.7)

and

\[ a' \geq -A_j. \]  

(3.8)

**Decisions at age \( j_r, ..., J \):** After retirement, individuals’ labor productivity drops to zero, and they live on capital income and pension benefits. The associated Bellman equation is given by

\[ V(j, e, s, a, \theta, \eta) = \max_c \left\{ u(c, 1) + \beta \psi_{j+1} V(j + 1, e, s, a', \theta, \eta) \right\} \]  

(3.9)

subject to

\[ (1 + \tau_c)c + a' = (1 + (1 - \tau_k)r_t)a + pen_t(s, \theta) + Tr. \]  

(3.10)

### 3.3.8 Competitive Equilibrium

To define the general equilibrium of the model economy, it is useful to introduce some additional notation. In particular, we need to define the distribution of individuals on the state space. Let \( \mathcal{E}, \mathcal{J} = \{1, 2, ..., J\}, \mathcal{S} = \{c, n\}, \mathcal{A} = \mathbb{R}, \mathcal{F} \) and \( \mathcal{H} \) denote, respectively, the support for ability \( e \), age \( j \), education level \( s \), wealth \( a \), fixed productivity effect \( \theta \) and the stochastic productivity component \( \eta \), where \( \mathcal{E} \) is a countable subset of the unit interval, and \( \mathcal{F} \) and \( \mathcal{H} \) are countable subsets of the real line. And let \( \Sigma \) denote the Borel \( \sigma \)-algebra defined on the product space \( \mathcal{X} = \mathcal{E} \times \mathcal{J} \times \mathcal{S} \times \mathcal{A} \times \mathcal{F} \times \mathcal{H} \). Then for any \( X \in \mathcal{X} \), a measure \( \phi(X) \) can be properly defined.

With this preparation, we now define the stationary recursive competitive equilibrium as follows.
Definition 4 A stationary recursive competitive equilibrium is a collection of: (i) decision rules of individuals \{1_s(e,a), c(j,e,s,a,\theta,\eta), l(j,e,s,a,\theta,\eta)\}; (ii) aggregate capital and labor inputs, \{K_t, L_{t,c}, L_{t,n}\}, on the part of firms; (iii) value functions \(V(j,e,s,a,\theta,\eta)\); (iv) government policies \{\tau_c, \tau_k, \tau_p, s, \eta(p, s, \theta_s), \kappa_s, z, Tr\}; (v) prices \{r_t, w_{t,c}, w_{t,n}\}; (vi) education system characterized by \{t, \xi(e)\}; and (vii) a vector of measures \(\phi\), such that:

1 The decision rules of individuals solve their respective life-cycle problems, and \(V(j,e,s,a,\theta,\eta)\) is the associated value function.

2 Aggregate capital and labor inputs, \{K_t, L_{t,c}, L_{t,n}\}, solve the representative firm’s profit maximization problem, which is fully characterized by the following first order conditions:

\[
r_t = \alpha A k_t^{\alpha - 1} - \delta, \tag{3.11}
\]
\[
w_{t,c} = (1 - \alpha) k_t^{\alpha} \left( \frac{L_t}{L_{t,c}} \right)^{1-\zeta} = w_t \left( \frac{L_t}{L_{t,c}} \right)^{1-\zeta}, \tag{3.12}
\]
and
\[
w_{t,n} = (1 - \alpha) k_t^{\alpha} \left( \frac{L_t}{L_{t,n}} \right)^{1-\zeta} = w_t \left( \frac{L_t}{L_{t,n}} \right)^{1-\zeta}, \tag{3.13}
\]
where \(k_t = \frac{K_t}{L_t}\), \(w_t = (1 - \alpha) k_t^{\alpha}\), and the college wage premium is given by

\[
\frac{w_{t,c}}{w_{t,n}} = \left( \frac{L_{t,n}}{L_{t,c}} \right)^{1-\zeta}. \tag{3.14}
\]

3 The labor market for each skill type clears:

\[
L_{t,s} = \sum_{j=1}^{J} \int \int \int_{X(j,s)} h_{s,j}(\theta,\eta) l(j,s,a,\theta,\eta) \phi(j,s,a,\theta,\eta) da \, d\theta \, d\eta, \tag{3.15}
\]
where \(X(j,s)\) is the subset of the state space \(X\) corresponding to age \(j\) and skill type \(s\).
4 The capital market clears:

\[ K_{t+1} + B_{t+1} = A_{t+1}, \]  

where \( B_{t+1} \) is the supply of government bonds and

\[ A_{t+1} = \sum_{s \in \{c,n\}} \sum_{j=1}^{J} \int \int \int_{X(j,s)} a'(j, s, a, \theta, \eta) \phi(j, s, a, \theta, \eta) \, da \, d\theta \, d\eta. \]

5 The good market clears:

\[ Y_t = C_t + G_t + E_t + I_t, \]  

where

\[ C_t = \sum_{s \in \{c,n\}} \sum_{j=1}^{J} \int \int \int_{X(j,s)} c(j, s, a, \theta, \eta) \phi(j, s, a, \theta, \eta) \, da \, d\theta \, d\eta, \]

\[ E_t = \int \int \int_{X(1,c)} \omega_{t,c} \phi(1, c, a, \theta, \eta) \, da \, d\theta \, d\eta, \]

\( G_t \) is government spending, and \( I_t = (n + \delta)K_t \) is gross investment.

6 The government budget constraint holds:

\[ \tau_c C_t + \tau_k r_t A_t + T_{L,t} + (1 + r_t) A_b,t + (1 + n) B_{t+1} = G_t + (1 + r_t) B_t + (1 + r_t) A_{init,t} + Z_t + T r_t, \]  

where \( T_{L,t} \) denotes labor income tax revenues, as given by

\[ T_{L,t} = \sum_{s \in \{c,n\}} \sum_{j=1}^{J} \int \int \int_{X(j,s)} \tau_l(y) y \phi(j, s, a, \theta, \eta) \, da \, d\theta \, d\eta, \]
with \( y = (1 - \tau_t)w_{t,s}h_{s,j}(\theta, \eta)l(j, s, a, \theta, \eta) \), \( A_{b,t} \) denotes accidental bequest:

\[
A_{b,t} = \sum_{s \in \{c,n\}} \sum_{j=1}^J \int \int \alpha'(j, s, a, \theta, \eta)(1 - \psi_{j+1})\phi(j, s, a, \theta, \eta) \, da \, d\theta \, d\eta,
\]

(3.20)

\( A_{\text{init},t} \) is the aggregate wealth transfer to the newly arrived generation:

\[
A_{\text{init},t} = \sum_{s \in \{c,n\}} \phi(1, s) \int_A a f(s, a) \, da,
\]

(3.21)

where \( \phi(1, s) \) is the measure of individuals at age one with college decision \( s \) and \( f(s, a) \) is the distribution from which initial wealth is drawn, and \( Z_t \) is the aggregate education subsidies

\[
Z_t = \int \int_{E \times A} 1_s(e, a)z_iw_{t,c}(e, a) \, da \, de.
\]

(3.22)

7 The pension budget (3.2) holds.

8 Individual behaviors are consistent with aggregate behavior: measure \( \phi \) is a fixed point of \( \phi(X) = \Pi(X, \phi) \), for any \( X \in X \), where \( \Pi(X, \cdot) \) is the transition function generated by the decision rules of individuals, the process of exogenous states, and the survival probabilities.

9 All aggregate variables are constant over time.

### 3.4 Calibration

This section discusses our parameter choices. We calibrate the model to the US economy. The majority of parameters are either estimated directly from the data or calibrated internally by matching certain aggregate moments in the US data. The rest of the parameters are taken from the literature. In addition, parameters that pertain to the education system and government policy are also estimated for Germany—these counterfactual values are then used in the policy experiments to assess the roles of
various policy dimensions in shaping wage inequality.

3.4.1 Demographics

A period in the model corresponds to four years. New generations enter the economy at the age of 18. It takes four years to complete college. Individuals retire at the age of 66 and the maximum age is 94. In addition, the size of the population grows at a constant rate \( n = 1\% \) annually, which is roughly consistent with the long run population growth rate of the US for the period between 2005 and 2014. Survival probabilities \( \{\psi_j\} \) are computed from the actuarial life tables compiled by the US Social Security Administration. Consistent with our focus on full time male workers, we consider survival probabilities for male workers. The reference year is 2011.

3.4.2 Preferences

We consider a fairly standard utility function

\[
u(c, 1 - 1_s \xi(e) - l) = \frac{[c^\nu(1 - 1_s \xi(e) - l)^{1-\nu}]^{\frac{1}{1-\frac{1}{\gamma}}}}{1 - \frac{1}{\gamma}},
\]

where \( \nu \) is a taste parameter for consumption, \( \frac{1}{\gamma} \) is risk aversion parameter. The two parameters \( \nu \) and \( \gamma \) jointly determine (i) the average labor supply, (ii) the intertemporal elasticity of substitution of consumption, and (iii) the Frisch labor supply elasticity. \( \gamma \) is set to 0.5 (see, for example, Krueger and Ludwig 2016). \( \nu \) is chosen such that individuals on average work one-third of their time endowment. The subjective discount factor \( \beta \) is set so as to target a capital-output ratio of around 2.41, which falls in the range commonly used in the literature.

3.4.3 Technology

The aggregate production function is of Cobb-Douglas form. The capital share \( \alpha \) is set to 0.33. Total factor productivity \( A \) is set to one. We set the elasticity of substitution between skilled labor and unskilled labor such that in equilibrium the
college wage premium is in line with the data for the period between 2010 and 2015 (see, for example, Valletta 2017). This leads to an elasticity of substitution of 1.67, which is slightly higher than the estimate in Katz and Murphy 1992 and Ciccone and Peri 2005. In addition, we set the annual depreciation rate to 7.55%, as in Krueger and Ludwig 2016.

3.4.4 Labor Productivity

Recall that the labor productivity of workers with education $s$ and of age $j$ is given by

$$h_{s,j} = \begin{cases} 
\epsilon_{s,j} \cdot \exp (\theta_s + \eta_{s,j}), & \text{if } j < j_r \\
0, & \text{otherwise}
\end{cases}$$

To estimate the processes that govern labor productivity, we first run cross-sectional regressions of log earnings on education $s$ and age $j$:

$$\ln(w_j) = f(X_j; x) + \tilde{w}_j,$$  \hspace{1cm} (3.23)

where $f(X_j; x)$, a function of age and education, captures the life-cycle productivity profile, $X_j$ is a vector of observables including education dummies and a cubic polynomial in age, and $\tilde{w}_j$ is a residual term. Estimates for $\epsilon_{s,j}$ are then obtained by normalizing $f(X_j; x)$ such that the mean labor productivity of skilled workers at age two–when college education is completed–is normalized to one. The residual term $\tilde{w}_{t,s}$ captures the fixed effect and stochastic component of the labor productivity process. More precisely, we consider the following process:

$$\tilde{w}_j = \theta + \eta_j,$$

$$\eta_j = \rho \eta_{j-1} + \epsilon_{\eta,j}.$$

We estimate this process using the Panel Study of Income Dynamics data under the assumption that $\theta$ and $\epsilon_{\eta,j}$ are normally distributed with mean zero and variance $\sigma_\theta$
Our estimation strategy is similar to Guvenen, Kuruscu, and Ozkan 2014, Karahan and Ozkan 2013, and Krueger and Ludwig 2016. The estimates are reported in Table 3.2. Then we approximate the AR(1) process that governs the evolution of $\eta$ using a two-state Markov chain with transition matrix

$$
\pi_{\eta,s} = \begin{bmatrix}
p_s & 1 - p_s \\
1 - p_s & p_s
\end{bmatrix}
$$

The estimated Markov process is reported in Table 3.2.

This leaves us with the fixed effect component of labor productivity. We assume that the skill-specific fixed effect $\theta_s$ takes on two values \(\{\theta_{s,l}, \theta_{s,h}\}\) and that the probability of drawing a high fixed effect $\theta_{s,h}$ depends on one’s ability. More precisely, we assume that

$$
\pi(\theta_e = \theta_{e,h} | e) = e
$$

and

$$
\pi(\theta_n = \theta_{n,h} | e) = \omega e,
$$

where $\omega$ is a parameter to be calibrated. Finally, we choose the free parameter $\omega$ such that the steady state of the model economy matches the college earnings premium of marginal students, which is primarily determined by $\omega$, see, e.g., Findeisen and Sachs 2015 and Krueger and Ludwig 2016.

---

We restrict the sample to male heads of households, aged 18-66, over the period 1978-2015. As of 1997, variables are available biennially. To minimize the impact of changes in hours worked, we consider only full-time full-year workers.

We use the Rouwenhorst method. See Kopecky and Suen 2010 for a discussion of the Rouwenhorst method.
3.4.5 Education Costs and Subsidies

For the benchmark calibration, we assume that the fraction of educational costs borne by the government is the same for all individuals, regardless of their learning ability and financial situation.\(^9\) To pin down the two parameters concerning the resource costs of college and government subsidies, \(\iota\) and \(z\), we rely on the data in Education at a Glance (OECD 2014, 2015). The year of reference is 2011. Table A7.3a(b) and Table A7.4a(b) (Education at a Glance, 2015) report, respectively, the private and public costs for a person attaining tertiary education. The private costs for a person attaining tertiary education are 55,000 US dollars, while public costs stand at 55,900 US dollars (in 2011 dollars).\(^{10}\) From this, we can infer the fraction of resource costs borne by the US government: 

\[
\begin{align*}
    z &= \frac{55,900}{55,000 + 55,900} = 0.504.
\end{align*}
\]

In addition, Table B1.3a. (Education at a Glance, 2014) reports that the average duration of tertiary studies is 3.17 years. Furthermore, GDP per capita in 2011 is 49,791 dollars. Combining all information, we have:

\[
\frac{\iota w_{t,c}}{\hat{Y}} = \frac{(55,000 + 55,900)/3.17}{49,791} = 0.703,
\]

where \(\hat{Y}\) denotes GDP per capita. We calibrate the cost parameter \(\iota\) such that in the benchmark model the ratio of the resource costs of college to GDP per capita is 0.703.\(^{11}\)

3.4.6 Initial Wealth Endowment

Heterogeneity in initial wealth in this paper is intended to capture the family income effects on college attendance. An appropriate calibration requires micro-level data on college expenses. To calibrate the initial wealth distribution, we use the Consumer Expenditure Survey (2015) public-use microdata (PUMD) collected by the US

---

\(^9\)Need-based grants and merit-based scholarships are considered later on as extensions.

\(^{10}\)The cost of higher education is slightly lower in Germany—the private and public costs for a person attaining tertiary education are 52,000 US dollars and 87,500 US dollars, respectively.

\(^{11}\)Our estimate of \(\frac{\iota w_{t,c}}{\hat{Y}} = 0.703\) is slightly higher than the estimate (0.694) in Krueger and Ludwig 2016.
Bureau of Labor Statistics. The public-use microdata contains detailed data on college expenses such as tuition and fees. After consolidating the data, we estimate the distribution of college expenses. This distribution is then used in the model for college-bound individuals to draw their initial wealth. Since our focus is on initial transfers for educational purposes, those who do not go to college receive zero initial wealth.

### 3.4.7 Learning Ability and Time Costs of College

We assume that learning ability follows a truncated normal distribution over the unit interval. The probability density function is given by

\[ \pi_e(e) = \frac{\psi_{sn} \left( \frac{e - \mu_e}{\sigma_e} \right)}{\sigma_e \left( \Psi \left( \frac{1 - \mu_e}{\sigma_e} \right) - \Psi \left( \frac{-\mu_e}{\sigma_e} \right) \right)}, \]

(3.24)

where \( \psi_{sn} \) and \( \Psi \) denote, respectively, the probability density function and the cumulative distribution function of standard normal distribution, \( \mu_e \) and \( \sigma_e \) denote the mean and the standard deviation of the un-truncated distribution. Without loss of generality, we set \( \mu_e \) to 0. The standard deviation, \( \sigma_e \), is chosen such that 95% of the probability mass falls in the unit interval.

In addition, we model the time cost of attending college as a parametric function of learning ability:

\[ \xi(e) = \exp(-\lambda_e e), \]

(3.25)

where \( \lambda_e \) is a parameter. Given the specification for the distribution of learning ability, \( \lambda_e \) is chosen in an attempt to match tertiary education attainment in the data (see OECD 2018).
3.4.8 Government Policy

We choose the government debt level and government spending to target a debt-to-GDP ratio of 60% and a government spending to GDP ratio of 17%, as in Krueger and Ludwig 2016. The consumption tax rate is estimated from the U.S. National Income and Product Accounts data set, which gives rise to an estimate of \( \tau_c = 7.3\% \). The capital income tax rate is taken from Chari and Kehoe 2006. Consistent with the current social security configuration, pension benefits are set to be 45% of the average income of each skill group, i.e., \( \kappa_s = 45\% \). Payroll tax rate \( \tau_{p,s} \) is then set to balance the budget. To estimate the two parameters associated with the labor income tax function \( m \) and \( \lambda \), note that \( m \) is a natural measure of progressivity and \( \lambda \) governs the average tax rate (for a given \( m \)). More precisely, let \( \tilde{y} \) denote the post-tax wage earnings, then \( \tilde{y} = \lambda y^{1-m} \), where \( y \) is the pretax wage earnings. The elasticity of post-tax earnings to pre-tax earnings is then given by

\[
\frac{d\tilde{y}/\tilde{y}}{dy/y} = 1 - m. \tag{3.26}
\]

We rely on the estimates for \( m \) of OECD (see, Taxing Wages (2013), Table I.8.). \( \lambda \) is then chosen such that the average tax rate in the model is in line with the data.\[14\]

3.4.9 Borrowing Constraints

The borrowing constraint for age one is set such that college-bound individuals can finance up to a fraction, \( \Phi \), of their college tuition and fees with student loans:

\[
A_j = \Phi(1 - z(c))tw_{t,c}.
\]

Borrowing constraints for \( j \geq 2 \) are set such that borrowers repay at least a minimum amount, \( P \), in each period, and that the loan is fully paid back by the age of retirement. More precisely, for \( j = 2, 3, \cdots, j_r \):

\[
A_j = (1 + r_t)A_{j-1} - P,
\]

\[14\]The average tax rate is 17.1% for the US.
Table 3.3: Baseline parameterization: parameter values in parentheses are estimated from German data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated externally</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographics</td>
<td>Population growth rate (annually)</td>
<td>1%</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Survival probabilities</td>
<td>Actuarial Life Tables</td>
</tr>
<tr>
<td>(j_r)</td>
<td>Retirement age (age 66)</td>
<td>13</td>
</tr>
<tr>
<td>(J)</td>
<td>Maximum age (94)</td>
<td>20</td>
</tr>
<tr>
<td>Preferences</td>
<td>Risk aversion parameter</td>
<td>0.5</td>
</tr>
<tr>
<td>Technology</td>
<td>Capital share of output</td>
<td>0.33</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Total factor productivity</td>
<td>1</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Depreciation rate (annually)</td>
<td>7.55%</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>Life-cycle productivity profile</td>
<td>Supplementary data</td>
</tr>
<tr>
<td>(\epsilon_{s,j})</td>
<td>Fixed productivity effect</td>
<td>Table 3.2</td>
</tr>
<tr>
<td>(\eta_s)</td>
<td>Idiosyncratic productivity shocks</td>
<td>Table 3.2</td>
</tr>
<tr>
<td>(\rho_s)</td>
<td>Persistence parameter</td>
<td>Table 3.2</td>
</tr>
<tr>
<td>(p_s)</td>
<td>Transition probability</td>
<td>Table 3.2</td>
</tr>
<tr>
<td>Edu. costs and subsidies</td>
<td>Subsidy rate</td>
<td>0.504 (0.944)</td>
</tr>
<tr>
<td>Ability and time cost of college</td>
<td>Mean of learning ability</td>
<td>0.5</td>
</tr>
<tr>
<td>(\mu_e)</td>
<td>Standard deviation of learning ability</td>
<td>0.255</td>
</tr>
<tr>
<td>Government policy</td>
<td>Consumption tax</td>
<td>7.3% (19%)</td>
</tr>
<tr>
<td>(\gamma_c)</td>
<td>Capital income tax</td>
<td>28.3% (30.5%)</td>
</tr>
<tr>
<td>(\tau_{p,s})</td>
<td>Payroll tax</td>
<td>14% (33.7%)</td>
</tr>
<tr>
<td>(\kappa_s)</td>
<td>Pension benefits</td>
<td>45% (50%)</td>
</tr>
<tr>
<td>(m)</td>
<td>Progressivity</td>
<td>0.1 (0.27)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Labor tax parameter</td>
<td>0.857</td>
</tr>
<tr>
<td><strong>Calibrated internally (target)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferences</td>
<td>Parameter for leisure (hours worked)</td>
<td>0.374</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Discount factor ((K/Y))</td>
<td>0.906</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Elasticity of substitution (college premium)</td>
<td>0.400</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>Scale parameter (see Section 3.4.4)</td>
<td>0.850</td>
</tr>
<tr>
<td>Ability and time cost of college</td>
<td>Time cost parameter (college attainment)</td>
<td>1.23</td>
</tr>
<tr>
<td>(\lambda_e)</td>
<td>Edu. costs and subsidies</td>
<td>Cost parameter ((\hat{\eta}_{w,c}))</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Borrowing constraints</td>
<td>Student loan parameter</td>
</tr>
</tbody>
</table>
and $A_{ij} = 0$.

To calibrate $\Phi$, we rely on the National Postsecondary Student Aid Survey (2011-2012) from which we estimate an average loan size of $21,300$. We then choose $\Phi$ such that in equilibrium an average loan covers $\frac{21300}{55000}$ of the college costs borne by individuals.

### 3.5 Model Dynamics

Before conducting policy experiments and drawing policy conclusions, it is useful to first examine the model dynamics. In this section, we examine the life-cycle profiles of average earnings, consumption and labor supply by education, investigate the role of initial wealth in college attendance and look at the ability-composition of college students.

#### 3.5.1 Life-Cycle Profiles

Figure 3.1 plots the life cycle profile of average consumption, asset holdings, hours worked and earnings for skilled workers and unskilled workers, respectively.

Consumption rises steadily over the working life for both skilled and unskilled workers. It dips slightly when workers retire because pension benefits are only a fraction of average earnings. Then consumption decreases gradually as they age and face rising death hazards.

Wealth exhibits the typical hump-shaped profile. For young individuals who are not attending college, initial wealth is slightly positive. College students, on the other hand, take out loans and therefore have negative wealth. On average, it takes about eight years to pay off student loans.

Labor supply does not differ much across skill types. At age one, when the young generation enters the economy, labor supply is low not only for college-bound individuals but also for non-college-bound individuals. For college students, the reason is obvious because attending college takes time. For those who are not attending college, the relatively low level of labor supply reflects their low productivity, which
is in line with the data.

Finally, the life cycle earnings profile also matches the data pretty well. Earnings are relatively low for young individuals as they enter the economy. College students do not have much time to work; those who are not attending college have low productivity. Earnings rise more rapidly for college-educated workers than unskilled workers. Average earnings decline gradually before retirement and drop to zero at the time of retirement. Tough the wage rate for college students is the same as unskilled workers, college students earn more than their non-college attending peers. This is because college students have higher probabilities of drawing a high fixed productivity effect \( \theta \) (see Section 3.4.4).

3.5.2 Initial Wealth, Ability and College Attendance

Heterogeneity in initial wealth endowment is a shortcut to capture the family income effect on higher education attendance. It is calibrated to reflect heterogeneity in family contribution to higher education expenditures. From a policy maker’s point of view, it is particularly important to know whether there are still high ability individuals left behind merely because they were born into low income families. For this reason, we plot the college decisions of individuals by learning ability and initial wealth endowment and the fraction of individuals in college by ability.

Figure 3.2 plots the expected lifetime utility of an individual if it decides to attend college and receives an initial endowment at the 10th (yellow), 50th (red) and 90th (blue) percentile of the initial wealth distribution and the expected lifetime utility of the individual if it decides not to attend college (dashed line). Since initial wealth endowment for non-college bound individuals is zero, they differ only in terms of learning ability. An individual will attend college if and only if attending college delivers a higher expected lifetime utility. Figure 3.2 shows that even relatively low ability individuals can benefit from a college education if they can afford to do so.\(^{15}\)

\(^{15}\)It is important to note that this is because college wage premium is high in the benchmark model. If medium and high ability individuals who cannot afford to go to college were enrolled
On the other hand, some medium-high ability individuals with low initial wealth are unable to reap the benefits of college education. Figure 3.3 plots the fraction of college-age individuals in college by ability. It shows that a substantial fraction of medium- to high-ability individuals cannot afford to attend college although they would benefit from a college education.

Finally, to get a sense of the ability composition of college students, we also plot (see Figure ??) the cumulative fraction of college age individuals in college. It shows into college through increased government financial aid, college wage premium would drop and low ability individuals would find it not optimal to go to college.
that, in the benchmark model, college students consist of mainly high and medium ability agents (consistent with the data), and that low ability individuals constitute only a small fraction of college students. The reason is twofold. First, assuming that ability follows a truncated normal distribution, low ability individuals constitute only a small fraction of the population. Second, for those low ability individuals, only a small fraction is rich enough to attend college. While high ability individuals can pay their way through college by working part time, this option is not viable for low ability individuals because they face higher time cost of college and earn less when working part time.

Later we will show that changes in government policy in general and education policy in particular would affect earnings inequality primarily through their effects on the ability composition of college students. We will also show that a well designed education policy can reduce earnings inequality without compromising efficiency.

3.5.3 Earnings Inequality: Model vs Data

Since earnings inequality is a major variable of interest of this paper. It is therefore important to know how far the model can go in generating realistic earnings distribution. For this reason, we report two different measures of earnings inequality, wage ratios and Gini coefficients for wage income, and compare them with before tax earnings inequality observed in the data (Table 3.4)\textsuperscript{16}

The benchmark model generates a Gini coefficient slightly lower than that observed in the data. On the other hand, the model slightly overshoots overall earnings inequality when it is measured by the wage ratio between the 90th percentile and 10th percentile of the wage income distribution. But overall, the model does a reasonably good job in matching earnings inequality in the data.

\textsuperscript{16}Both the Gini index and wage ratios are used in the literature to measure earnings inequality. The Gini index is typically a more robust measure since wage differentials are based on only two data points across the wage income distribution. Moreover, survey data on earnings are typically top-coded. Nonetheless we report both measures. See Burkhauser, Feng, and Jenkins\textsuperscript{2009} for more discussions on different measures of inequality.
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Figure 3.2: Expected lifetime utility of an individual if it decides to attend college and receives an initial endowment at the 10th (yellow), 50th (red) and 90th (blue) percentile of the initial wealth distribution and the expected lifetime utility of the individual if its decides not to attend college (dashed line)

Table 3.4: Earnings inequality: model vs data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>P90/P10</td>
<td>5.16</td>
<td>6.18</td>
</tr>
<tr>
<td>P90/P50</td>
<td>2.36</td>
<td>2.45</td>
</tr>
<tr>
<td>P50/P10</td>
<td>2.18</td>
<td>2.52</td>
</tr>
</tbody>
</table>

3.6 Policy Experiments

To evaluate the roles of the various aspects of government policy in shaping earnings inequality, we conduct several policy experiments. First, we replace the benchmark education subsidy rate with the one estimated from German data. To allow for
the crowding-out effect of increased government subsidies for higher education, we consider the extreme case that initial wealth transfers completely vanish. Relaxing this assumption can only strengthen the role of education policy. Second, keeping everything else unchanged, we replace the benchmark labor income tax progressivity parameter with the one estimated for Germany. This experiment allows us to evaluate the role of tax progressivity. Finally, we also consider deviations from the benchmark along policy dimensions other than education policy and tax progressivity. In each of these experiments, the government budget is balanced through lump-sum taxes. We report the results of these policy experiments in this section.

3.6.1 Education Inequality and Earnings Inequality

In this subsection, we examine the roles of various policy dimensions in driving college attendance and consequently in shaping earnings inequality. In Table 3.5 we
Figure 3.4: Cumulative fraction of college age individuals in college report college attendance and various measures of earnings inequality for the benchmark model as well as for model variants that deviate from the benchmark in various policy dimensions. We first report the results for the benchmark model (2nd column). Then we report the results for the education policy experiment (3rd column), where we replace the status quo subsidy rate \( z \) with its counterpart estimated from German data. In the fourth column, we report results for the model variant where we only change the labor income tax progressivity parameter \( m \) and keep other policy dimensions the same as in the benchmark. Note that since the average labor income tax rate also depends on the progressivity parameter, when conducting this experiment, we adjust \( \lambda \) such that the average tax rate remains the same as in the benchmark. In the fifth column, we report results for the experiment in which we change both the college subsidy rate and labor tax progressivity. Finally, we report results for the experiment where we change other policy dimensions other than education policy and labor income tax, including capital income tax and consumption
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Table 3.5: Policy experiment results

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Education</th>
<th>Progressivity</th>
<th>Edu. &amp; Prog.</th>
<th>Other taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini</td>
<td>0.370</td>
<td>0.310</td>
<td>0.342</td>
<td>0.304</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>16.2%</td>
<td>7.6%</td>
<td>18.1%</td>
<td>3.6%</td>
<td></td>
</tr>
<tr>
<td>P90/P10</td>
<td>6.184</td>
<td>4.371</td>
<td>5.486</td>
<td>4.363</td>
<td>5.696</td>
</tr>
<tr>
<td></td>
<td>29.3%</td>
<td>11.3%</td>
<td>29.5%</td>
<td>5.7%</td>
<td></td>
</tr>
<tr>
<td>P90/P50</td>
<td>2.440</td>
<td>2.126</td>
<td>2.381</td>
<td>2.099</td>
<td>2.340</td>
</tr>
<tr>
<td></td>
<td>13.2%</td>
<td>2.79%</td>
<td>14.3%</td>
<td>2.3%</td>
<td></td>
</tr>
<tr>
<td>P50/P10</td>
<td>2.524</td>
<td>2.056</td>
<td>2.303</td>
<td>2.078</td>
<td>2.435</td>
</tr>
<tr>
<td></td>
<td>18.6%</td>
<td>8.75%</td>
<td>17.7%</td>
<td>2.4%</td>
<td></td>
</tr>
<tr>
<td>% in college</td>
<td>41.5</td>
<td>59.6</td>
<td>46.1</td>
<td>59.9</td>
<td>47.6</td>
</tr>
</tbody>
</table>

The results suggest that raising college subsidy rate from the US status quo to

Figure 3.5: Lorenz curve under the benchmark model (solid blue line) and the education policy experiment (dashed line).
the German level would reduce earnings inequality, as measured by the Gini index, by as much as 16.2% (see also Figure 3.4). It would greatly boost college attendance—from 41.5% to 59.6%. Besides, such a move would shift the ability composition of college students. Figure 3.5 shows the fraction of college-age individuals in college in the benchmark model (solid line) and that in the counterfactual experiment (dashed line) where education subsidy is raised to the German level. Interestingly, such a reform would not only have distributional gains, in the next section, we will show that it would also lead to more efficient aggregate outcomes.

On the other hand, increasing labor tax progressivity has only a modest impact on inequality. For example, replacing the US status quo progressivity with its German counterpart can reduce earnings inequality by 7.6%, about half of the reduction that can be achieved by increased government subsidies for higher education. This finding is in stark contrast to Guvenen, Kuruscu, and Ozkan 2014, who single out labor tax progressivity as the most important determinant of earnings inequality. Moreover, increased labor tax progressivity necessarily leads to lower economic efficiency, a topic we will address in the next subsection. In addition, our results suggest more progressive labor income tax may actually increase college attendance. The reason is that the additional government revenues are transferred to individuals in a lump-sum manner, which can be used by credit-constrained young individuals to finance college education.

In line with the literature (see, e.g., Krueger and Ludwig 2016), our results also suggest that education policy and labor tax progressivity constitute a policy substitute for each other in terms of reducing earnings inequality. In the next subsection, we show that education policy can partially offset the distortion of increased labor tax progressivity.

Finally, our results suggest that the consumption tax and the capital income tax only have a minimal effect on earnings inequality.

\footnote{Guvenen, Kuruscu, and Ozkan 2014 consider a model with complete markets. Thus they are silent on the role of education affordability.}
CHAPTER 3. EDUCATION AFFORDABILITY AND WAGE INEQUALITY

3.6.2 Equality, Efficiency and Welfare

In the previous subsection, we focus on the importance of various policies in shaping earnings inequality. In this subsection, we bring the other component of social welfare, efficiency, into the picture, and examine the welfare gains of each policy reform.

Table 3.6 reports a few key macroeconomic variables under the benchmark and deviations from it along the education policy dimension and the labor income tax dimension. Three observations can be made. First, raising government spending on education not only reduces earnings inequality, it also increases macroeconomic efficiency: output, capital stock and aggregate labor (in efficiency units) are higher than those in the benchmark. Second, not surprisingly, more progressive labor income taxes lower earnings inequality but also lead to greater distortions: workers work fewer hours, and output, capital stock and aggregate labor (in efficiency units) are lower.
compared with the benchmark. Third, higher subsidies for higher education partially offset the distortions induced by progressive labor taxation. This is not because higher subsidies provide workers better incentives to work, but rather because higher subsidies for education boost labor productivity through increased college attendance.

Table 3.7 reports the welfare gains of each hypothetical policy reform relative to the benchmark. It shows that, in line with Krueger and Ludwig 2016, a combination of more generous subsidies for college and more progressive labor income taxation is most desirable in terms of social welfare. Specifically, replacing the college subsidy rate and labor tax progressivity with their German counterparts would increase social welfare by an amount equivalent to a 21.25% increase in consumption.

Table 3.6: Macroeconomic variables under the benchmark and deviations from it along the education policy dimension and the labor income tax dimension. The number below each variable is the percentage increase of that variable relative to the benchmark.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark Education Policy Progressivity Edu. &amp; Prog.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}$</td>
<td>1.241</td>
</tr>
<tr>
<td>$\hat{K}$</td>
<td>0.748</td>
</tr>
<tr>
<td>$\hat{C}$</td>
<td>0.726</td>
</tr>
<tr>
<td>$\hat{L}$</td>
<td>1.650</td>
</tr>
<tr>
<td>$\hat{h}$</td>
<td>0.349</td>
</tr>
<tr>
<td>$w$</td>
<td>0.481</td>
</tr>
<tr>
<td>$r$</td>
<td>0.295</td>
</tr>
<tr>
<td>$\frac{w_c}{w_n}$</td>
<td>1.747</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.7: Social Welfare. The first row shows the average life-time utility. The second row reports the welfare gains, in consumption-equivalent terms, of each hypothetical policy reform compared with the benchmark.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Education Policy</th>
<th>Progressivity</th>
<th>Edu. &amp; Prog.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{V}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.74%</td>
<td>8.53%</td>
<td>21.25%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 3.6.3 Winners and Losers

In this subsection, we examine the winners and losers of increased government subsidies for college. For this purpose, we plot the utility gains, in consumption-equivalent terms, from the hypothetical education policy reform (see Figure 3.6). It shows that poor individuals with medium and medium-high ability would benefit the most from increased government subsidies for college: this group would enjoy an increase in life-time utility equivalent to an increase in consumption by as much as 80%. Those are the individuals that cannot afford to go to college in the benchmark, although they would benefit greatly from a college education. In addition, low ability individuals would also benefit from the hypothetical education policy reform, albeit for different reasons. They do not go to college anyway. But when more individuals attend college, the college wage premium decreases. Unskilled workers enjoy a higher wage rate. Finally, our results suggest that a small fraction of rich and high-ability individuals would lose out from the hypothetical education policy reform due to compressed college wage premium. For example, an individual with an initial wealth at the 90th percentile of the initial wealth distribution and a learning ability at the 99th percentile of the ability distribution would suffer a utility loss equivalent to a 17.5% decrease in consumption.

### 3.7 Conclusion

We study the role of education affordability in shaping earnings inequality in the context of an overlapping generations model. We find that earnings inequality, as measured by the Gini coefficient for before-tax gross earnings, would decrease by as
much as 16.2 percent if the current education policy, the share of higher education costs borne by the government, were replaced with its German counterpart. Two groups of individuals would benefit from the hypothetical policy reform. First, poor individuals with medium and medium-high ability would benefit the most from increased government subsidies for college. Those are the individuals that cannot afford to go to college in the benchmark, although they would benefit greatly from a college education. Second, individuals of low ability would also benefit from the hypothetical education policy reform. They do not go to college anyway. But when more individuals attend college, the college wage premium decreases. As a consequence, unskilled workers enjoy a higher wage rate. On the other hand, our results suggest that a
small fraction of rich and high-ability individuals would lose out from the hypotheti-
cal education policy reform due to compressed college wage premium. For example, an individual with an initial wealth at the 90th percentile of the initial wealth distri-
bution and a learning ability at the 99th percentile of the ability distribution would 
suffer a utility loss equivalent to a 17.5% decrease in consumption. In line with the 
literature, we find that a combination of more generous subsidies for college and more 
progressive labor income taxation is most desirable in terms of social welfare. Finally, in contrast with the existing literature, we find that labor tax progressivity plays a less significant role in explaining earnings inequality.
Bibliography


[59] OECD. Educational attainment and labour-force status. 2018. DOI: http://dx.doi.org/10.1787/889e8641-en

Contributions

Chapter 1 is a joint work with Prof. Dr. Volker Hahn. Both of us were actively involved in the project. Chapter 2 is a single-authored paper. I am responsible for carrying out the research and writing the paper. That being said, the research idea of chapter 2 was inspired by discussions with my supervisor Prof. Dr. Volker Hahn. Chapter 3 is coauthored with Oliko Vardishvili. We contributed equally to the paper.
Appendix A

A.1 Derivation of the Conditions Describing Optimal Central Bank Behavior

To solve the policy planning problem under the policy variant where the central bank maximizes social welfare, we form the Lagrangian:

\[
L(\Pi_t, Y_t, R_t, \lambda_{1,t}, \lambda_{2,t}) = \Psi_t^{1-\sigma}Y_t^{1-\sigma} - 1 \left(1 - \sigma \right) - \frac{\chi Y_t^{1+\eta}}{1 + \eta} - \lambda_{1,t}\left\{ \frac{\Psi_t^{-\sigma}Y_t^{-\sigma}}{R_t} - \beta \mathbb{E}_t \left[ \frac{\Psi_{t+1}^{-\sigma}Y_{t+1}^{-\sigma}}{\Pi_{t+1}} \right] \right\} \\
- \lambda_{2,t}\left\{ \Psi_t^{-\sigma}Y_t^{1-\sigma} \left[ \frac{\psi}{\theta}(\Pi_t - 1)\Pi_t + \frac{\theta - 1}{\theta} - \chi \Psi_t^{\sigma}Y_t^{\sigma+\eta} \right] \\
- \beta \mathbb{E}_t \left[ \Psi_t^{-\sigma}Y_t^{1-\sigma} \frac{\psi}{\theta}(\Pi_{t+1} - 1)\Pi_{t+1} \right] \right\} + \mu_t(R_t - 1)
\]

The Karush-Kuhn-Tucker conditions include the first order condition with respect to \(Y_t\)

\[
\Psi_t + \frac{\sigma \lambda_{1,t}}{Y_t R_t} + (\sigma - 1)\lambda_{2,t}\left[ \frac{\theta - 1}{\theta} + \frac{\psi}{\theta}(\Pi_t - 1)\Pi_t \right] = \chi Y_t^{\sigma+\eta}\Psi_t^{\sigma}[1 - (1 + \eta)\lambda_{2,t}], \tag{A.1}
\]

the first order condition with respect to \(\Pi_t\)

\[
\left( \Psi_t + \frac{\sigma \lambda_{1,t}}{Y_t R_t} \right) \psi(\Pi_t - 1) + \lambda_{2,t}\sigma\psi(\Pi_t - 1) \left( \frac{\theta - 1}{\theta} + \frac{\psi}{\theta}(\Pi_t - 1)\Pi_t \right) \\
+ \lambda_{2,t}\frac{\psi}{\theta}(2\Pi_t - 1) = 0, \tag{A.2}
\]
the first order condition with respect to $R_t$
\[- \lambda_{1,t} \frac{C_t^{-\sigma}}{R_t^\sigma} + \mu_t = 0, \quad (A.3)\]
the complementary slackness condition
\[\mu_t (R_t - 1), \quad (A.4)\]
as well as the IS curve \((1.15)\), the Phillips curve \((1.16)\), the ZLB condition \((1.18)\) and
\[\mu_t \geq 0. \quad (A.5)\]

Similarly, we form the Lagrangian for the policy problem under inflation targeting as follows.
\[L^{IT}(\Pi_t, Y_t, R_t, \lambda_{1,t}, \lambda_{2,t}) = \left\{- (\Pi_t - \Pi^*)^2 + \alpha (Y_t - Y^*)^2\right\} - \lambda_{1,t} \left\{ \Psi_t^{-\sigma} \frac{Y_t^{-\sigma}}{R_t} - \beta \mathbb{E}_t \left[ \frac{\Psi_{t+1}^{-\sigma} Y_{t+1}^{-\sigma}}{\Pi_{t+1}} \right] \right\} - \lambda_{2,t} \left\{ \Psi_t^{-\sigma} Y_t^{1-\sigma} \left[ \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t + \frac{\theta - 1}{\theta} - \chi \Psi_t^\sigma Y_t^{\sigma+\eta} \right] \right\} - \beta \mathbb{E}_t \left[ \Psi_{t+1}^{-\sigma} Y_{t+1}^{1-\sigma} \frac{\psi}{\theta} (\Pi_{t+1} - 1) \Pi_{t+1} \right] + \mu_t (R_t - 1) \]

The Karush-Kuhn-Tucker conditions include the first order condition with respect to $\Pi_t$
\[- 2 (\Pi_t - \Pi^*) + \lambda_{1,t} \sigma \psi \frac{Y_t}{R_t} C_t^{-\sigma-1}(\Pi_t - 1) + \lambda_{2,t} \left\{ \sigma \psi \Psi_t^{-\sigma} Y_t^{1-\sigma} (\Pi_t - 1) \Psi (\Pi_t - 1) \Pi_t + \theta_t - 1 \right\} + \psi (2 \Pi_t - 1) \Psi_t^{-\sigma} Y_t^{1-\sigma} = 0, \quad (A.6)\]
the first order condition with respect to $Y_t$

$$-2\alpha (Y_t - Y^*) - \lambda_{1,t}\sigma \frac{\Psi_t}{C_t^{\sigma+1} R_t} + \lambda_{2,t} \left\{ (1 - \sigma)(\Psi_t Y_t)^{-\sigma} \left[ \psi(\Pi_t - 1)\Pi_t + \theta_t - 1 \right] - \chi(1 + \eta)\theta_t Y_t^{\eta} \right\} = 0,$$  

(A.7)

the first order condition with respect to $R_t$

$$- \lambda_{1,t} \frac{C_t^{-\sigma}}{R_t^\sigma} + \mu_t = 0,$$  

(A.8)

the complementary slackness condition

$$\mu_t (R_t - 1) = 0,$$  

(A.9)

as well as the IS curve (1.15), the Phillips curve (1.16), the ZLB condition (1.18) and

$$\mu_t \geq 0.$$  

(A.10)

A.2 Identification of historical markup shocks

To identify the history markup shocks, we take the other model parameters as given. Particularly, we assume that there were no major discount factor shocks during the period from 1983:1 to 2007:3. The identification procedure consists of four steps:

1. Fitting a VAR model in terms of output $Y_t$, inflation $\Pi_t$ and the nominal interest rate $R_t$

$$x_t = \nu + A_1 x_{t-1} + \ldots + A_p x_{t-p}$$  

(A.11)

where $x_t = (Y_t, \Pi_t, R_t)'$. Output is calculated as detrended real GDP using the HP filter with a conventional smoothing parameter of 1600, inflation is computed using the GDP implicit price deflator and the nominal interest rate is measured by the quarterly average of the federal funds rate. The Schwarz criterion (SIC) suggests an optimal lag order of three. In addition, it is important to note that detrended real GDP cannot be directly used as output because the
2. We use the VAR one-period-ahead forecasts at period $t$ to determine the conditional expectations in the Phillips curve (1.16).

3. Plugging the actual values of output $Y_t$, inflation $\Pi_t$ as well as the conditional expectations obtained in the previous step into the Phillips curve. Solving the resulting equation, we obtain $\theta_t$.

4. Repeating step 2 – 3, we obtain the estimated time series of the historical markup shocks $\theta_1, \theta_2, \ldots, \theta_T$. The resulting series is shown in A.1.

Figure A.1: Historical markup shocks
A.3 Solution Method and Algorithm

Markov perfect rational expectations equilibrium under discretion consists of a monetary policy strategy $R(\beta, \theta)$ and private sector responses $Y(\beta, \theta)$ and $\Pi(\beta, \theta)$. This appendix describes how we solve for these policy functions. First, we discretize the state space by assigning $N$ interpolation nodes, within three standard deviations of the mean, along the dimension of $\theta$. Then the problem reduces to one of finding the optimal monetary policy and private sector responses at each node of the discretized state space. For convenience, we stack the optimal monetary policy $R(\beta, \theta)$ and private sector responses $Y(\beta, \theta)$ and $\Pi(\beta, \theta)$ into a two-dimensional array $P$. At each interpolation node, the values of the policy functions are fully determined by the equilibrium conditions, as derived in Appendix A. Some of those equilibrium conditions involve expectations of future variables. We approximate the values of those expectations using $M$-node Gauss-Hermite quadrature.\

Our numerical algorithm consists of the following steps:

Step 1: Choose the degree of approximation $N$ and $M$; Assign the interpolation nodes and Gauss-Hermite quadrature nodes. Make an initial guess for the policy function $P$.

Step 2: For each interpolation node, evaluate the expectations in the IS curve (1.15) and Phillips curve (1.16) based on the current guess $P^i$. Then, taking the expected values as given, solve the system of equilibrium conditions for a new guess $P^{i+1}$.

Step 3: Stop if $\max |P^i - P^{i+1}| < \tau$, where $\cdot |_{\text{max}}$ returns the maximum absolute value of an array and $\tau > 0$ denotes the convergence threshold value; otherwise, repeat step 2.

We choose $N = 269$, $M = 8$ and $\tau = 1.49 \times 10^{-8}$.

\footnote{See Judd 1998 (p. 261) for more details.}
Appendix B

B.1 Derivation of the Policy Maker’s First Order Conditions

To solve the policy planning problem under inflation targeting, we form the Lagrangian as follows.

\[
L^{IT}(\Pi_t, Y_t, R_t, \lambda_{1,t}, \lambda_{2,t}) = \left\{ - (\Pi_t - \Pi^*)^2 - \alpha (Y_t - Y^*)^2 \right\} - \lambda_{1,t} \left\{ \frac{\Psi_t^{-\sigma} Y_t^{-\sigma}}{R_t} - \beta \mathbb{E}_t \left[ \frac{\Psi_{t+1}^{-\sigma} Y_{t+1}^{-\sigma}}{\Pi_{t+1}} \right] \right\} - \lambda_{2,t} \left\{ \Psi_t^{-\sigma} Y_t^{1-\sigma} \left[ \frac{\psi}{\theta} (\Pi_t - 1) \Pi_t + \frac{\theta - 1}{\theta} - \chi \Psi_t^{-\sigma} Y_t^{\sigma + \eta} \right] - \beta \mathbb{E}_t \left[ \Psi_{t+1}^{-\sigma} Y_{t+1}^{1-\sigma} \frac{\psi}{\theta} (\Pi_{t+1} - 1) \Pi_{t+1} \right] \right\} + \mu_t (R_t - 1)
\]

The Karush-Kuhn-Tucker conditions include the first order condition with respect to \( \Pi_t \)

\[
-2 (\Pi_t - \Pi^*) + \lambda_{1,t} \sigma \psi \frac{Y_t}{R_t} C_t^{-\sigma - 1} (\Pi_t - 1) + \lambda_{2,t} \left\{ \sigma \psi \Psi_t^{-\sigma - 1} Y_t^{1-\sigma} (\Pi_t - 1) [\psi (\Pi_t - 1) \Pi_t + \theta_t - 1] + \psi (2\Pi_t - 1) \Psi_t^{-\sigma} Y_t^{1-\sigma} \right\} = 0,
\]

\[\text{(B.1)}\]
the first order condition with respect to $Y_t$

$$-2\alpha (Y_t - Y^*) - \lambda_{1,t}\sigma \frac{\Psi_t}{C_t^{\sigma+1} R_t} + \lambda_{2,t} \left\{ (1 - \sigma)(\Psi_t Y_t)^{\sigma} [\psi(\Pi_t - 1)\Pi_t + \theta_t - 1] - \chi(1 + \eta)\theta_t Y_t^m \right\} = 0$$

(B.2)

the first order condition with respect to $R_t$

$$- \lambda_{1,t} \frac{C_t^{\sigma-\sigma}}{R_t} + \mu_t = 0,$$

(B.3)

the complementary slackness condition

$$\mu_t (R_t - 1),$$

(B.4)

as well as the IS curve (2.1), the Phillips curve (2.3), the ZLB condition (2.7) and

$$\mu_t \geq 0.$$  

(B.5)

The KKT conditions for inflation targeting with a temporarily higher inflation target are similar except for the fact that, depending on the state, the inflation target may vary, and that expectations are formed under a different policy regime.
B.2 Two Opposing Forces at Work: Analytic Results

Combining (2.10) and (2.11), we obtain the nominal interest rate under the high target

\[ i_t^{**} = \sigma E_t y_{t+1} + \left(1 + \frac{\sigma \beta}{\kappa}\right) E_t \pi_{t+1} - \frac{\sigma}{\kappa} \pi^{**} \]

\[ = \sigma E_t y_{t+1} + \left(1 + \frac{\sigma \beta}{\kappa}\right) [p \pi^* + (1-p)\pi^{**}] - \frac{\sigma}{\kappa} \pi^{**} \]

\[ = \sigma [py^* + (1-p)y^{**}] + \left(1 + \frac{\sigma \beta}{\kappa}\right) [p \pi^* + (1-p)\pi^{**}] - \frac{\sigma}{\kappa} \pi^{**} \]

\[ = \sigma \left\{ \frac{1 - \beta}{\kappa} \pi^* + (1 - p) \left[ 1 - \beta (1 - p) \right] \pi^{**} - \beta p \pi^* \right\} \]

\[ + \left(1 + \frac{\sigma \beta}{\kappa}\right) [p \pi^* + (1-p)\pi^{**}] - \frac{\sigma}{\kappa} \pi^{**}, \]

(B.6)

where \( y^* \) and \( y^{**} \) denote the output gap under the low target and the high target, respectively.

B.3 Additional Graphical Results
Figure B.1: Standard inflation targeting: $\Pi^* = 2\%$ vs $\Pi^* = 4\%$. 
Figure B.2: Hybrid IT with $p = 0.5$ (upper panels), $p = 0.25$ (middle panels), and $p = 0.05$ (lower panels).
Bibliography


[59] OECD. *Educational attainment and labour-force status*. 2018. DOI: [http://dx.doi.org/10.1787/889e8641-en](http://dx.doi.org/10.1787/889e8641-en)


