Noninvasive Quantum Measurement of Arbitrary Operator Order by Engineered Non-Markovian Detectors

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The development of solid-state quantum technologies requires the understanding of quantum measurements in interacting, non-isolated quantum systems. In general, a permanent coupling of detectors to a quantum system leads to memory effects that have to be taken into account when interpreting the measurement results. We analyze a generic setup of two detectors coupled to a quantum system and derive a compact formula in the weak-measurement limit that interpolates between an instantaneous (text-book type) and almost continuous - detector dynamics-dependent - measurement. A quantum memory effect that we term system-mediated detector-detector interaction is crucial to observe non-commuting observables simultaneously. Finally, we propose a mesoscopic double-dot detector setup in which the memory effect is tunable and that can be used to explore the transition to non-Markovian quantum measurements experimentally.

Historically the interpretation of quantum measurement uses the projection postulate implying an instantaneous collapse of the wave function in the measurement process [1]. Whereas this scheme fits e.g. ideal detector detection very well, it is unsuitable in most other measurement procedures. For instance, in a measurement of a current in a solid-state environment, the system-detector interaction is much weaker and a collapse is avoided. A great deal of theoretical and experimental investigations have been carried out since von Neumann’s work [1-3]. In particular, the limit of noninvasive measurement processes has been studied theoretically [4] as well as experimentally [5, 6].

In mesoscopic physics, the question of current noise in the quantum regime has attracted a lot of interest over the years [7]. In particular, the question of measuring the current correlator was addressed early on [8, 12]. While most experiments address high-frequency correlations, in agreement with the symmetrized correlator [13, 14], with on-chip detectors the quantum non-symmetrized noise can be extracted [15, 16]. First theoretical strides towards the measurement and interpretation of general unsymmetrized operator orders have been reported [19, 24].

The goal of this Letter is to understand which system correlations are obtained in a generalized quantum measurement if two detectors coupled weakly and continuously to a system are read out independently. We note that the measurement outcomes can be expressed by noise and response functions of the system and the detectors. Hence, these outcomes depend crucially on the internal dynamics of the detectors and, therefore, by suitably engineered detectors the measurement can be tuned such that a specific operator order is measured. We propose a mesoscopic double-dot detector setup to explore the transition to non-Markovian quantum measurements experimentally. Going beyond the usually discussed bath-induced non-Markovian self-interaction [25], we identify the system-mediated detector-detector interaction as crucial ingredient to observe non-commuting observables simultaneously.

In the following, we will assume that two detectors are coupled to a common system by small interaction Hamiltonians, which allows us to linearize the time-evolution and obtain a microscopic expression for the observed quantum correlation. The measurement procedure is illustrated in Fig. 1. Besides the direct influence of the system onto the detector observables (green arrows), the detectors influence each other mediated by the system (blue and orange arrows). This influence depends on the time-dependent response of the system and will lead to interesting consequences. We start by deriving the second-order cross correlation for a non-Markovian weak quantum measurement from a microscopic model. We will show that the different contributions can be understood in terms of a linear-response interaction of the sub-

FIG. 1: Non-Markovian cross-correlation measurement: Two detectors a and b are coupled to a system for a finite time and read out at times $t_a > t_b$. Detector b collects information directly on the system and on detector a (which are transmitted through the system). The presence of detector b after its readout (light blue) also contributes to the information which detector a collects on detector b at a later time $t_a$.
systems involved. After making some general statements on the properties and relevance of terms beyond the textbook approach of symmetric operator order, we will illustrate our findings in several examples.

**Microscopic model.** — The Hamiltonian of the system to be measured is denoted as $H_0$, and we will consider two independent detectors described by $H_a$ and $H_b$ that are coupled to the system by $H_{\text{int}}$. The total Hamiltonian is given by

$$\hat{H} = \sum_{\alpha=a,b} \hat{H}_\alpha + H_{\text{int}}.$$  

The two detectors with detector variables $\hat{D}_a$ and $\hat{D}_b$ are weakly and linearly coupled to the system observables $\hat{A}$ and $\hat{B}$,

$$H_{\text{int}} = \eta_a \hat{D}_a \hat{A} + \eta_b \hat{D}_b \hat{B},$$  

where $\eta_a$ and $\eta_b$ denote small coupling parameters which in general can be modulated in time. There are no restrictions on the detector Hamiltonians $\hat{H}_\alpha$.

In the following we will study the correlation measurement scheme shown in Fig. 1. The density matrix of the total system is assumed to be in a product state $\hat{\rho}' = K_m U(t_a - t_0) \hat{\rho} U(t_0 - t_b) K_m^\dagger$. Its trace $\text{Tr}\{\hat{\rho}'\} = p(a, t_a | b, t_b)$ gives the conditional probability of finding $m_a$ if $m_b$ has been measured before; the unconditional probability is obtained by applying Bayes’ theorem $p(a, t_a | b, t_b) = p(a, t_a | m_b, t_b) p(m_b, t_b)$. To establish the relation between measurement results of the two detectors and operator order of the corresponding observables we consider

$$C(t_a, t_b) = \int \, \text{d}m_a \int \, \text{d}m_b \, m_a \, m_b \, p(m_a, t_a | m_b, t_b)$$

$$- \int \, \text{d}m_a \, m_a \, p(m_a, t_a) \int \, \text{d}m_b \, m_b \, p(m_b, t_b).$$  

Using the weak-coupling assumption, expanding $U_t$ to second order in $\eta$ and rescaling $C \to C/\eta_a \eta_b$, we find that the correlation function can be written as the sum of three contributions

$$C(t_a, t_b) = C_{\text{sym}}(t_a, t_b) + C_{\text{det}}^a(t_a, t_b) + C_{\text{det}}^b(t_a, t_b),$$  

with

$$C_{\text{sym}}(t_a, t_b) = \int \, \text{d}t \, \chi_{XY}(t, t') \langle [\hat{X}_a(t), \hat{Y}_a(t')] \rangle \chi_{XY}(t, t'),$$

and the symmetrized noise functions

$$S_{XY}(t, t') = \frac{1}{2} \langle [\delta \hat{X}_a(t), \delta \hat{Y}_a(t')] \rangle.$$  

The expectation values in Eqs. (1) and (7) are calculated with the initial system density matrix $\hat{\rho}_0$ respectively the density matrices $\hat{\rho}_a(t) = \theta(t_a - t - t_b) \hat{\rho}_a + \theta(t - t_a) \hat{\rho}_b'$ of the detectors ($\alpha = a, b$), which can incorporate a change of the detector state from $\hat{\rho}_a$ to $\hat{\rho}_b'$ due to its readout at time $t_a$. The detector observable is given by $\hat{M}_a = \int \, \text{d}m_a \, m_a \hat{K}_a^\dagger \hat{K}_a$.

Equations (1) and (5) are the main result of our Letter. The origin of the various terms is illustrated in Fig. 2. $C_{\text{sym}}$ describes the symmetrized correlation of the system observables $\hat{A}$ and $\hat{B}$ which is transmitted to both detectors via $\chi^a$ and $\chi^b$. $C_{\text{det}}^a$ corresponds to the internal correlation of $\hat{M}_a$ and $\hat{D}_a$ in detector $a$ where $\hat{D}_a$ is measured by detector $b$ via $\chi^0$ and $\chi^b$ (and similarly for $C_{\text{det}}^b$). The system takes a new role as a linear-response mediator here, since the commutators in the response
function Eq. (6) make information accessible that is not contained in the symmetrized noise, Eq. (7).

If system and detectors are initialized in stationary states, Eqs. (6) and (7) depend only on the time differences and Eq. (4) can be expressed in Fourier space as

\[ C(\omega) = \int dt_a - t_b e^{i\omega(t_a - t_b)} C(t_a, t_b) \]

with the result

\[
C_{\text{sym}}(\omega) = \chi_{MD}(\omega)\chi_{MD}^*(-\omega)S_{AB}^0(\omega),
\]

\[
C_{\alpha a}^{\text{det}}(\omega) = \chi_{MD}(\omega)\chi_{BA} S_{MD}^0(\omega),
\]

\[
C_{\beta b}^{\text{det}}(\omega) = \chi_{MD}^*(\omega)\chi_{AB} S_{MD}^0(-\omega).
\]

For simplicity we concentrate on the second-order cross-correlation here, the generalization is discussed in [20]. Hence, in the frequency domain and for equal detectors we have \( C_{\text{sym}}(\omega) = |\chi_{MD}(\omega)|^2 S_{AB}^0(\omega) \). Thus, the non-Markovian nature of our setup results in a simple frequency filter effect for the symmetric part of the measurement.

We will now focus on the system-mediated detector-detector interactions \( C_{\alpha a}^{\text{det}} \) and discuss their relevance and properties, especially their effect on the finally measured operator order. Since the detectors measure each other’s noise in linear response through the system, the presence of the commutator in the response functions leads to the appearance of antisymmetrically ordered system terms in addition to the symmetrized expressions which can combine in various ways. We find that (i) \( C_{\alpha a}^{\text{det}} \) is a non-Markovian quantity as it depends on the memory of the detector and the time history of the measurement process. (ii) the response–to–noise ratio \( \chi/S \) of a detector is a indicator of the relevance of \( C_{\alpha a}^{\text{det}} \) and therefore the measured operator order. (iii) \( C_{\alpha a}^{\text{det}} \) and \( C_{\beta b}^{\text{det}} \) can cancel with clever detector engineering. (iv) For certain systems, \( C_{\beta b}^{\text{det}} \) leads to observable effects even in the Markovian coupling limit, i.e. for instantaneous interaction of system and detectors. The observations (i)-(iv) are illustrated in the corresponding examples in the following.

(i) Ladder-operator measurement: We study a system that is assumed to be a harmonic oscillator with frequency \( \Omega \). The detectors are assumed to be identical harmonic oscillators with frequency \( \Omega' \), and their memory is controlled by the damping parameter \( \lambda \). This damping results from the dissipation to a bath, see [20]. The measurement is taken at coinciding times \( t_a = t_b = 0 \). We are interested in the correlation of the ladder operators \( \hat{a} \) and \( \hat{a}^\dagger \) of the system. Since they are not hermitian, we have to perform two measurements in which the detectors are coupled to the position in one measurement with the resulting correlation function \( C_{xx} \) and to the momentum in the other measurement resulting in \( C_{pp} \). The sum of these correlators \( C = (C_{xx} + C_{pp})/2 \) corresponds to the scalar relation \( \alpha \alpha^* = (x^2 + p^2)/2 \) with \( \alpha = (x + ip)/\sqrt{2} \) and is given by

\[
C = \xi_s(\{\hat{a}, \hat{a}^\dagger\}) - \xi_s(\{\hat{a}, \hat{a}^\dagger\}), \quad (9)
\]

where \( \xi_s = \frac{1}{2} \int dt s \chi_{MD}(t)\chi_{MD}^*(s) \cos(\Omega(t - s)) \) and \( \xi_s = \int dt s \theta(s - t) [\chi_{MD}^*(t)S_{MD}^0(s) + \chi_{MD}(t)S_{MD}^0(s)] \times \sin(\Omega(s - t)) \) are the weights of the symmetric and antisymmetric contributions. The antisymmetric contribution is essentially determined by the system-mediated detector-detector interaction \( C_{\beta b}^{\text{det}} \). Choosing the detector variable \( D = \hat{p} \) and the observable \( M = \hat{x} \) yields \( \chi_{MD}(t) = \theta(t)e^{-\lambda t} \cos(\Omega' t) \) and \( S_{MD}(t) = e^{-\lambda t} \sin(\Omega t) \cosh(\beta\Omega' t)/2 \) with the detectors inverse thermal energy \( \beta \). Figure 3 shows the ratio of the weights \( \xi_s \) and \( \xi_s \) as a function of \( \lambda \) and \( \Omega' \), i.e. have a long memory, the symmetric and antisymmetric contributions are of the same size and lead to normal order of the operators in the expression for \( C \propto \langle \hat{a}\hat{a}\rangle \). If the detector oscillators are overdamped \( (\lambda \ll \Omega') \), i.e. approach the Markovian limit, the symmetric contribution vanishes and the correlation function corresponds to symmetrized operator order \( C \propto \langle \{\hat{a}\hat{a}\} \rangle \).

(ii) Relevance of the system-mediated contributions: The response to noise ratio \( \chi/S \) of the detector characterizes the strength of the asymmetric system operator order, which can be illustrated e.g. with a monochromatic laser beam. \( S \) and \( \chi \) resemble a harmonic oscillator in many respects, with the difference that the noise is proportional to the photon number \( N \) whereas the response is unaffected by it. Thus we have \( \chi/S \approx 1/N \). A single photon behaves very much like a harmonic oscillator and \( C_{\text{sym}} \) and the \( C_{\beta b}^{\text{det}} \) contribute in the same order with a fine tuning possible by damping and resonance effects as shown in Fig. 3. Multiple photons will almost exclusively measure the system’s response function, i.e. neither symmetric nor normal ordering but the antisymmetric operator order, \( C_{\alpha a}^{\text{det}} \) and the direct symmetric system measurement is negligible.

(iii) Detector engineering: We now use the possibility to prepare the two detectors in different states. We consider a mesoscopic electronic realization with two \( (a = a, b) \) double quantum-dot detectors capacitively coupled to the system as depicted in Fig. 3(a). The two dots in each detector are connected via a tunnel cou-
pling $t_a$ and the energy difference of the left and right level $\epsilon_a$ can be tuned via a gate voltage. Both double dots are capacitively connected to a quantum point contact in which the current $I_a$ is measured. We assume the current correlation function $\langle I_a(t)I_b(t') \rangle$ to be proportional to the occupation number correlation function $\langle \sigma^a_{\tau}(t)\sigma^b_{\tau}(t') \rangle$ with $\sigma^a_\tau = n_{1,a} - n_{2,a}$.

The state of the double dots is controlled via a bias voltage, see Fig. 4. It can be characterized by the occupation difference of the energy eigenstates $\Delta n_a = n_a - n_a^+$, $\epsilon_a = 0$, no damping.

FIG. 4: Sketch of the setup. (a) Two double-dot detectors are capacitively coupled to a system and the occupation of one of the dots is recorded via a bypassing current. (b) The state of each detector can be set by additional contacts (not shown in (a)) which are biased to adjust the occupation difference of the high and low energy eigenstate $n^+$ and $n^-$. (c) Weights $\xi_s$ and $\xi_a$ as a function of the occupation difference of the energy eigenstates $\Delta n_a = n_a - n_a^+$, $\epsilon_a = 0$, no damping.

and the noise

$$S_{s,s}(t,t') = \frac{8t_s^2}{\omega_s^4} \cos(\omega_s(t-t')) + \frac{8\epsilon_s^2}{\omega_s^2} (1-(\Delta n_s)^2),$$

with $\omega_s = 2\sqrt{t_s^2 + \epsilon_s^2}$. For measuring the ladder operators of a harmonic oscillator we can again write the correlator in the form of Eq. (9). The results for $\xi_s$ and $\xi_a$ are shown in Fig. 4(c). When both detectors are set to the same occupation difference $\Delta n_a = n_a^+ - n_a^-$, the noise contributions of both detectors sum up and we obtain normal order (anti-normal if both detectors are switched to emission mode) near resonance and for sufficient low damping. This regime was already explored in Fig. 3.

Since $C^\text{det}_a(\epsilon = 0)$, tuning one of the detectors to level inversion of the other $\Delta n_a = -\Delta n_b$ results in a cancelation of the terms $C^\text{det}_a$ and we are able to measure the pure symmetrized noise/correlation of the system. If one of the $\Delta n_a$ equals zero, not only the corresponding $C^\text{det}_a$ but also $S^\text{sym}$ vanishes. In this case the only remaining contribution to $C$ is the linear-response measurement of this detector’s noise by the other detector via the system, i.e., $C$ only contains information about the antisymmetric operator order. Tuning the voltage that controls $\Delta n_a$ of any of these regimes can be selected.

A regime to measure the antisymmetric system contribution completely independent of the system state can be explored by utilizing the constant term in the noise. In the frequency domain $C^\text{det}_a(\omega = 0)$ will be directly proportional to the zero-frequency detector noise $S^\text{det}_{MD}(\omega = 0) = 2(\epsilon_a/\omega_a)^2(1-(\Delta n_a)^2)$ which is tunable over a wide range by the gate voltage that controls $\epsilon_a$. If we set a sufficiently large level difference $\epsilon$ the measurement outcome will be dominated by the detectors measuring each other’s noise through the system. In this way we obtain the pure antisymmetric system order incorporated in $\chi_{AB}^\text{MD}(\omega)$.

(iv) Markovian limit: We now consider the correlation in the limit of short coupling times, $\eta_a \to \delta(t_t-a)$ in Eq. 2, in a stationary setup

$$C^\text{sym}(t_a\hat{t}_b) = \chi^a_{MD}(0)\chi^b_{MD}(0)S^a_{AB}(t_a-t_b)$$

$$C^\text{det}_a(t_a\hat{t}_b) = \chi^a_{MD}(0)S^a_{MD}(0)^\text{sym}\chi^b_{MB}(t_b-t_a)$$

$$C^\text{det}_b(t_a\hat{t}_b) = \chi^a_{MD}(0)S^b_{MD}(0)^\text{sym}\chi^b_{AB}(t_a-t_b).$$

(10)

Note that only one of the terms $C^\text{det}_a$ will contribute depending on whether $t_a > t_b$ or $t_b > t_a$. For thermal detectors $\chi^a_{MD}(0)$ and $S^a_{MD}(0)$ are related via the fluctuation-dissipation theorem and one finds a general requirement on the detectors for measuring the symmetrized correlation. If $\chi^a_{MD}$ (without the Heaviside function) is symmetric in $t$, e.g. in example (i), $S^a_{MD}$ is antisymmetric in $t$ and therefore $C^\text{det}_a = 0$ and $C$ reduce to the symmetrized expression. This is independent of the frequency filter effect of the detector or any system properties. However, any antisymmetric contribution in $\chi^a_{MD}$ will cause
a finite value of $C^\text{det}_\alpha$. For non-thermal detector states there are even less restrictions for $C^\text{det}_\alpha$ to remain in the Markovian limit.

To be concrete, we consider two identical $(a = b)$ two-level detectors. The detector variable is set to $\hat{D} = \hat{\sigma}_z$ and the detector observable points in direction $\hat{M} = \vec{r}\hat{\sigma}$ with $\vec{r} = (\cos(\phi), \sin(\phi), 0)^T$. We obtain $\chi_{\text{MD}}(0) = -\sin(\phi)\langle \hat{\sigma}_z \rangle$ and $S_{\text{MD}}(0) = \cos(\phi) - \langle \vec{r}\hat{\sigma} \rangle \langle \hat{\sigma}_z \rangle$. Figure 5 shows that by tuning the angle $\phi$ we can realize the full range of operator orders from normal over symmetric to antisymmetric order with continuous transitions.

Conclusion and Outlook. — In contrast to theoretical models which assume instantaneous measurements, constantly coupled detectors appear naturally in many experiments. In our work we include the non-Markovian effects in such a setup and explore a variety of possible outcomes corresponding to non-symmetrized operator orders. Our results open the avenue to design tunable quantum detection systems which can observe tailored correlation functions.

We have proposed a quantum detector consisting of a pair of double quantum dots which realizes such a tunable scheme in a mesoscopic nanostructured circuit. Applying our analysis to develop quantum detector setups in the ultrafast optical domain or in the optical detection of coherence appears to be a promising research direction. Furthermore, a weak detection scheme is potentially useful to perform a weak quantum process tomography, which might present an alternative to the standard route to test quantum algorithms. Another interesting future challenge will be to explore the full statistics of quantum systems in a non-Markovian detection scheme, eventually even going beyond the weak-measurement limit.

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