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Quantification and Nominal Anaphora

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Abstract

Quantification and nominal anaphora is a rich and complex area of research, which has attracted many people; especially since the development of dynamic semantics in the early eighties. One way to obtain a systematic view on this field, is to focus on the quantificational resources employed to interpret anaphoric relationships. In this article, I will develop such a view as far as first-order quantification is concerned; i.e., generalized quantification over the individuals in a domain but not over sets of such individuals. With hindsight, this allows one to discern the three most prominent methods of interpretation:¹

- i) DRT: polyadic quantification over open expressions (Kamp 1981, Heim 1982);
- ii) DPL: principles of scope extension for quantificational antecedents (Groenendijk and Stokhof 1991);
- iii) ‘Q-types’: contextually restricted quantification (Neale 1990, Heim 1991).

In this article, I compare these approaches to determine the extent in which they are applicable in the first-order domain. In particular, which antecedents and which nominal anaphora are interpreted correctly by these methods? The main results describe precisely when polyadic quantification and principles of scope extension give the appropriate truth conditions, and when they fail to do so. Yet, a balanced judgement should include collective quantification, where higher-order variants of the quantifiers for which polyadic quantification and scope extension do work, could be used to give a general mechanism of interpretation after all. This would lead to the interesting observation that anaphora may enforce an increase in the expressive power of the logic needed to formalize natural language. However, the collective part of this research remains to be done (but see van den Berg 1996).

Overview The article is structured as follows. Section 1 starts with a study of DRT’s use of polyadic quantification to interpret anaphoric relationships. It recalls the proportion problem, which is then shown to be real: only a few quantifiers evade it. The topic of section 2 is DPL and the principles of scope extension it employs. I describe the familiar problems with these principles, first noted by Evans (1977, 1980), and determine how serious they are. The conclusion is that the principles can be sustained for singular but not for plural anaphora. I make some preliminary remarks on how the latter could be handled.

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¹I only mention the publications which started work on these methods. In the full paper I hope to do more justice to Dekker 1993 and his later work, and to Pagin and Westerstahl 1993.

In order to be able to study scope extension logically we have to interpret anaphorical relationships within quantifier theory. I do so by means of an offspring of E-type anaphora: contextually restricted quantifiers, here called ‘Q-type anaphora’. Although this approach to interpreting anaphorical dependencies appears to be promising, it has some shortcomings of its own. Perhaps the major problem is formalization. Cf. van der Does 1996a for the main puzzles. Using ideas from van den Berg 1996 and van der Does 1993a, section 2.5 quickly presents the first-order system QTL for singular Q-type anaphora. A proper presentation and study of QTL has to wait for the final version of this paper.²

In case the editors have decided to retain the appendix A, after all it has made the article way to long, it offers some basic facts on quantifier theory, especially on polyadic quantification and the interpretation of (in)definite NPs. Some of these facts can also be found elsewhere (e.g., van Benthem 1986, Westerståhl 1989, van der Does and van Eijck 1996).

1 DRT and polyadic quantification

Lewis 1975. argued that adverbs of quantification such as *once*, *twice*, ..., *sometimes*, *often*, *mostly* quantify over cases—i.e., tuples of objects,—rather than over situations, events, or similar ontological categories. On this view, (1a) is modeled by (1b). It means (1c) and (1d), where quantification is over pairs of a domain.

- (1) a. Mostly, a farmer owns a donkey
 b. $mostly\ xy(Fx \wedge Dy, Oxy)$
 c. $cQ^2(\mathbf{most})_E F \times D, O$
 d. $|F \times D \cap O| > |F \times D \cap \bar{O}|$

In order to enable *mostly* to bind x and y these variables should be left free by the indefinites *a farmer* and *a donkey*. Indeed, Lewis holds that indefinites are non-quantificational. They correspond to ‘restricted’ variables, as x and y in (1b) within the scope of *mostly*.

1.1 DRT

Lewis’ view on (in)definites as variables is developed further in the discourse representation theories of Kamp (1981) and Heim (1982, 1983). Table 1 gives first-order DRT in a nutshell.

TYPE	SYNTAX	SEMANTICS (= \llbracket SYNTAX \rrbracket)
cnd	$Rx_1 \dots x_n$	$\{g \mid \llbracket R \rrbracket g(x_1) \dots g(x_n)\}$
cnd	$x_1 = x_2$	$\{g \mid g(x_1) = g(x_2)\}$
cnd	$\neg\delta_1$	$\{g \mid \neg\exists h : g[\delta_1]h\}$
cnd	$\delta_1 \rightarrow \delta_2$	$\{g \mid \forall h[g[\delta_1]h \rightarrow \exists k : h[\delta_2]k]\}$
drs	$\langle x_1 \dots x_n; \bigwedge_{i=1}^m \chi_i \rangle$	$\{\langle g, h \rangle \mid h[x_1 \dots x_n]g \ \& \ h \in \llbracket \chi_i \rrbracket \ (1 \leq i \leq m)\}$

TABLE 1 DRT’s syntax and semantics.

A discourse δ is *true* (relative to \mathcal{M} and g) iff there is h such that: $g[\delta]h$.

Note that the syntax of DRT discerns between conditions (hence: cnd’s) and DRSs. Relative to a first-order model \mathcal{M} (= $\langle E, \llbracket - \rrbracket \rangle$), conditions denote a set of assignments for \mathcal{M} , whereas DRSs denote a relation between such assignments.³ In table 1, the $x_1 \dots x_n$ vary over distinct variables, the δ_j over DRSs, and the χ_i over

²This article is part of a larger project on quantification and anaphora. Besides new findings, it aims to give a unified presentation of the material in van der Does 1994a and van der Does 1996c. It has also borrowed a few paragraphs from van der Does 1996b.

³Note that DRT’s distinction between conditions and DRSs can be removed as soon as we lift the denotation of conditions from sets to relations by means of the function Δ :

$$\Delta(X) := \{(x, x) \mid x \in X\}$$

conditions. Also $h[x_1 \dots x_n]g$ means: $\exists d_1 \dots d_n : g = h_{d_1 \dots d_n}^{x_1 \dots x_n}$, with $h_{d_1 \dots d_n}^{x_1 \dots x_n}$ the assignment which is d_i at x_i ($1 \leq i \leq n$) but $h(y)$ otherwise.

In Kamp 1981 and many subsequent articles one finds algorithms to assign DRSs to sentences. For (2a) the anaphoric link between *a donkey* and *it* is resolved in the DRS (2b), which means the same as (2c) and (2d).

- (2) a. No farmer who owns a donkey beats it
 b. $\neg \langle x, y; Fx \wedge Dy \wedge Oxy \wedge Bxy \rangle$
 c. $no\ xy(Fx \wedge Dy \wedge Oxy, Bxy)$
 d. $F \times D \cap O \cap B = \emptyset$

Observe that *no* in (2c) is a case quantifier, as defined in appendix A.2. Similar polyadic treatments can be given in case of *every*, *some*, *not all*, or conditionals.

- (3) a. Every philosopher who reads a comic hates it.
 b. $\langle x, y; Px \wedge Cy \wedge Rxy \rangle \rightarrow \langle ; Hy \rangle$
 c. If a philosopher reads a comic he remains silent.
 d. $\langle x, y; Px \wedge Cy \wedge Rxy \rangle \rightarrow \langle ; Sx \rangle$

In general, which variables are to be bound is decided at a representational level where they are introduced as discourse referents.

The polyadic nature of DRT can be highlighted by means of a truth preserving translation into first-order logic with polyadic quantifiers.⁴

- a. $(Rx_1 \dots x_n)^* \equiv Rx_1 \dots x_n$;
 b. $(x_1 = x_2)^* \equiv x_1 = x_2$;
 c. $(\neg \delta)^* \equiv \neg(\delta)^*$;
 d. $(\langle x_1 \dots x_n; \bigwedge_{i=1}^m \chi_i \rangle \rightarrow \delta_2)^* \equiv \forall x_1 \dots x_n (\bigwedge_{i=1}^m \chi_i^* \rightarrow \delta_2^*)$;
 e. $(\langle x_1 \dots x_n; \bigwedge_{i=1}^m \chi_i \rangle)^* \equiv \exists x_1 \dots x_n \bigwedge_{i=1}^m \chi_i^*$

Fact 1 Let $\llbracket \varphi^* \rrbracket_{\mathcal{M}} := \{h : \mathcal{M}, h \models \varphi^*\}$. For each model \mathcal{M} , cnd χ , and DRS δ :

- i) $\llbracket \chi \rrbracket_{\mathcal{M}}^{\text{drt}} = \llbracket \chi^* \rrbracket_{\mathcal{M}}$
 ii) $\text{DOM}(\llbracket \delta \rrbracket_{\mathcal{M}}^{\text{drt}}) = \llbracket \delta^* \rrbracket_{\mathcal{M}}$ □

The translation shows clearly that the quantificational scheme underlying (3) can be reconstructed as:

$$(4) \quad Qx_1 \dots x_n [\varphi, \psi]$$

In (4), $x_1 \dots x_n [\varphi, \psi]$ is the part corresponding to a DRS $\langle x_1 \dots x_n; \varphi \wedge \psi \rangle$ with $x_1 \dots x_n$ its universe, and φ, ψ conditions. The quantifier Q is either seen as a discourse operator \exists , or as a device to turn DRSs into duplex conditions. The question we shall be concerned with is: for which Q does this scheme give defensible truth conditions?

1.2 Polyselective quantification

Case quantification as used by Lewis 1975, Kamp 1981, Heim 1982 among many others is sometimes called *unselective*. But strictly speaking this is a misnomer, because Q does not bind all the free variables within its scope. In (4) only the variables $x_1 \dots x_n$ provided by the DRS are bound, all other variables remain free. Since the number of variables may vary, such quantification is referred to as *polyselective* in what follows.

Semantically, the polyselective quantification underlying (3) is much like case quantification. But as opposed to $\text{CQ}^n(\mathbf{Q})$, the polyselective case quantifier \mathbf{Q}^{cq} has no finite type. It is, so to speak, a quantifier of indefinite arity:

$$\mathbf{Q}_E^{cq} = \bigcup_{n \in \mathbb{N}} \text{CQ}^n(\mathbf{Q})_E$$

At the relational level denotations of this form are called *tests*. This move toward the use of a uniform semantical type is made in DPL (cf. section 2).

⁴This formulation of first-order logic is of course equivalent to the standard monadic one. However, as Westerståhl 1994 shows, this equivalence is rare among quantifier logics.

Note that the finite case quantifier to which Q^{cq} reduces is fully determined by the arity of its arguments:

$$\mathbf{Q}_E^{cq} R^n S^n \Leftrightarrow \text{CQ}^n(\mathbf{Q})_E R S$$

for all n and all $R, S \subseteq E^n$. In terms of this notion, (5a) holds for $\mathcal{M} = \langle E, [-] \rangle$ iff (5b).

- (5) a. $\mathcal{M}, a \models Q^{cq} x_1 \dots x_n [\varphi, \psi]$
 b. $\mathbf{Q}_E^{cq} (\llbracket \lambda x_1 \dots x_n \cdot \varphi \rrbracket_a, \llbracket \lambda x_1 \dots x_n \cdot \psi \rrbracket_a)$

with $\llbracket \lambda x_1 \dots x_n \cdot \varphi \rrbracket_a := \{ \langle d_1 \dots d_n \rangle \mid \mathcal{M}, a_{d_1}^{x_1} \dots a_{d_n}^{x_n} \models \varphi \}$.

1.3 The proportion problem

Case and polyselective quantification give defensible truth conditions for sentences such as (6a), with an adverb of quantification, or for Kamp's (6b).

- (6) a. Mostly, if a farmer owns a donkey he beats it.
 b. Most lovers eventually become enemies.

However, in other contexts it leads to the well-known proportion problem. E.g., the truth conditions of (7a) as given by (7b) are correct if Q is *all*, *some*, *no*, or *not all*. But they are undesirable in quite a few other cases.

- (7) a. Q men who own a car wash it on Sunday.
 b. $Qxy [Mx \wedge Cy \wedge Oxy, Wxy]$

The familiar counterexample, due to Partee 1984, is that of *most*. Then, (7b) would be true incorrectly if there are two men, one of which owns two cars that he washes on Sunday while the other owns just one car which he doesn't wash on Sunday. Note, incidentally, that in this situation the case quantifier *two* leads to the same difficulties. This shows that the proportion problem is nothing to do with first-order definability (*two* is first-order definable, while *most* is not).

The proportion problem leads to a more logical question, namely: which quantifiers in sentences such as (7) evade it? Before this question can be answered, one should choose a sensible meaning of the sentences involved with which the case quantificational interpretation is to be compared. I study one option: reduction to weak and strong readings.

1.4 The scope of the proportion problem

The source of the proportion problem is that case quantifiers run over the wrong domain: sequences of individuals rather than just individuals. On this analysis, we should look for a meaning of anaphoric sentences, which involves monadic quantification. In particular, we may ask when a case quantifier is equivalent to its weak or strong reading (which are definable in terms of monadic quantifiers). Following up on a remark in Kanazawa 1994, I prove that only in a few instances case quantification gives the correct truth conditions for anaphoric sentences. This may explain why recently alternative proposals have been made to interpret nominal anaphora dependent on a quantifier, such as Q-types.

Kanazawa (1994) assumes—as most semanticist do,—that donkey sentences have either a weak or a strong reading. For instance, (8a) means either (8weak), or (8strong).

- (8) a. Q men who own a car wash it on Sunday
 w. $Qx_1 [Fx_1 \wedge \exists x_2 [Dx_2 \wedge O x_1 x_2], \exists x_2 [Dx_2 \wedge O x_1 x_2, Bx_1 x_2]]$
 s. $Qx_1 [Fx_1 \wedge \exists x_2 [Dx_2 \wedge O x_1 x_2], \forall x_2 [Dx_2 \wedge O x_1 x_2, Bx_1 x_2]]$

This suggests the following interpretation of the proportion problem: the case quantificational reading of a donkey sentences is correct iff it is equivalent to a weak or to a strong reading; it is incorrect otherwise. Kanazawa's remark concerns quantifiers whose 2-ary case quantifier yields the strong or the weak reading provided the beatings are consistent, i.e., that either all or none of the donkeys owned by farmers

are beaten. He states that under these circumstances the logical quantifiers which have this property are precisely the sixteen Boolean combinations of *some* and *all*.⁵ To flesh out this interesting fact, we study it without the assumption of consistent beatings, and for arbitrary polyselective case quantification.

First some terminology. Let $R \subseteq E^{n+1}$ be an $n + 1$ -ary relation. Define $\exists(R)$ and $\forall(R)$ by

$$\begin{aligned}\exists(R) &:= \{d \in E \mid \exists d_1 \dots \exists d_n. Rdd_1 \dots d_n\} \\ \forall(R) &:= \{d \in E \mid \forall d_1 \dots \forall d_n. Rdd_1 \dots d_n\}\end{aligned}$$

For instance, the set of farmers who own a donkey is $\exists(F \times D \cap O)$. The polyselective weak and strong readings of a quantifier \mathbf{Q} are defined by:

$$\begin{aligned}\bullet \quad \mathbf{Q}_E^{wr} RS &\Leftrightarrow \mathbf{Q}_E \exists(R) \exists(R \cap S) \\ \bullet \quad \mathbf{Q}_E^{sr} RS &\Leftrightarrow \mathbf{Q}_E \exists(R) \forall(R \Rightarrow S)\end{aligned}$$

for all $n \geq 2$ and $R, S \subseteq E^n$. Here, $R \Rightarrow S := \bar{R} \cup S$. As for case quantification, we have syntactic analogues Q^{wr} and Q^{sr} , too.

A case quantifier \mathbf{Q}^{cq} is equivalent to its weak reading iff $\mathbf{Q}_E^{cq} = \mathbf{Q}_E^{wr}$, for all E ; and similarly for the strong reading. Instances of case quantifiers which reduce to their weak reading are *some* and *no*. The quantifiers *all* and *not all* are equivalent to their strong readings. By way of example, we have a look at *all*. Although we could reason semantically, it is easier to use a syntactic argument. At the syntactical level (i–iv) are equivalent whenever $x_j \notin \text{FV}(\varphi_i)$ ($0 \leq i < j \leq n$).

- i) $\forall^{cq} x_1 \dots x_n [\bigwedge_{i=1}^n \varphi_i, \psi]$
- ii) $\forall^{sr} x_1 \dots x_n [\bigwedge_{i=1}^n \varphi_i, \psi]$
- iii) $\forall x_1 [\exists x_2 \dots x_n \bigwedge_{i=1}^n \varphi_i, \forall x_2 \dots x_n (\bigwedge_{i=2}^n \varphi_i, \psi)]$
- iv) $\forall x_1 [\varphi_1 \wedge \exists x_2 (\varphi_2, \dots \exists x_n \varphi_n), \forall x_2 (\varphi_2, \dots \forall x_n (\varphi_n, \psi) \dots)]$

For example, the sentences (9a,b) are synonyms:

- (9) a. $\forall^{cq} x_1 x_2 [Fx_1 \wedge Dx_2 \wedge Ox_1 x_2, Bx_1 x_2]$
- b. $\forall x_1 [Fx_1 \wedge \exists x_2 [Dx_2 \wedge Ox_1 x_2], \forall x_2 [Dx_2 \wedge Ox_1 x_2, Bx_1 x_2]]$

In this sense, the meaning of donkey sentences as given by case *all* is equivalent to its strong reading.

From the above equivalences we can generate some more by means of the following regularities:

- i) The quantifiers which are equivalent to their weak (strong) reading are closed under Boolean connectives. This closure does not extend to the mixed instances.
- ii) \mathbf{Q}^{cq} is equivalent to its weak (strong) reading iff $\mathbf{Q}^{cq \neg}$ is equivalent to its strong (weak) reading.⁶

To see that the mixed cases are not closed under conjunction, consider $\exists \wedge \neg \forall$. This quantifier is neither equivalent to its strong reading nor to its weak reading. For if two farmers own two donkeys of which each beats just one, (10a) is true while (10b,c) are false. There is no farmer who beats all of his donkeys, but each beats some of them.

- (10) a. $(\exists \wedge \neg \forall) xy [Fx \wedge Dy \wedge Oxy, Bxy]$
- b. $(\exists \wedge \neg \forall) x [Fx \wedge \exists y (Dy, Oxy), \forall y (Dy \wedge Oxy, Bxy)]$
- c. $(\exists \wedge \neg \forall) x [Fx \wedge \exists y (Dy, Oxy), \exists y (Dy \wedge Oxy, Bxy)]$

The mixed cases are only closed under Boolean connectives in the situations considered by Kanazawa 1994; if either all or none of the donkeys are beaten, the weak and strong readings collapse.

⁵Since quantifiers are elements of a power set they allow the familiar Boolean structure. For example, $\neg \mathbf{Q} := \wp(E) \times \wp(E) - \mathbf{Q}$, and $\mathbf{Q} \wedge \mathbf{Q}' := \mathbf{Q} \cap \mathbf{Q}'$, etc.

⁶The *internal negation* of a quantifier \mathbf{Q} is defined by: $\mathbf{Q} \neg(A, B) := \mathbf{Q}(A, \bar{B})$.

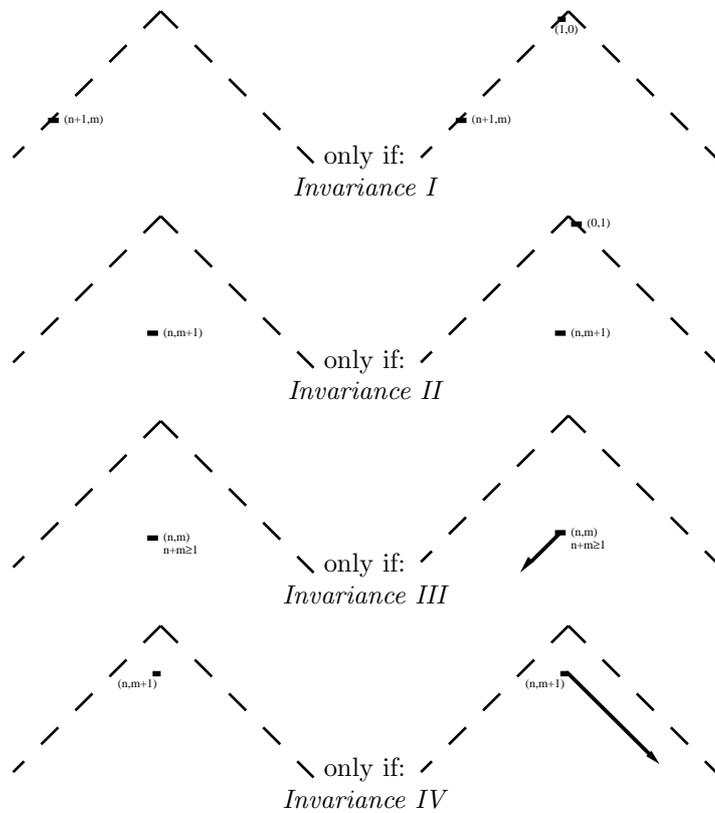
Theorem 2 *The only logical polyselective quantifiers which are equivalent to their weak reading are the Boolean compounds of **some** and **no**_e (i.e., **no** with the requirement that its first argument be non-empty).*

Corollary 3 *The only logical polyselective quantifiers which are equivalent to their strong reading are the Boolean compounds of **not all** and **all**_e (i.e., **all** with the requirement that its first argument be non-empty).*

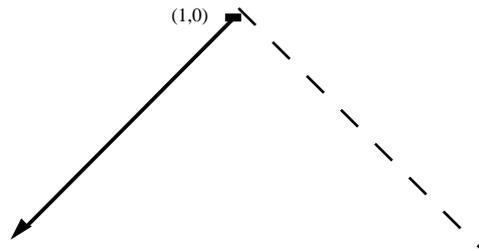
In both cases there are eight, not sixteen such compounds.

PROOF. A detailed, fully general proof of theorem 2 can be found in the next section. Here I assume FIN, and use a geometrical argument within Van Benthem trees.

That the Boolean compounds of **some** and **no**_e are equivalent to their weak readings is clear from the above observations. Conversely, assume that a quantifier **q** has this property. Then its tree exhibits the invariances I-IV:

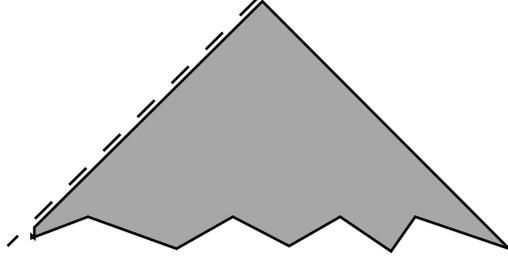


Given these invariances, one may reason by cases. First, if **q** is the empty tree it is **some** \wedge \neg **some**. So assume **q** is nonempty but does *not* contain the root $\langle 0, 0 \rangle$. Its points either do or do not lie on the column $\langle i, 0 \rangle$. In case **q** has no points outside this column, invariance I implies that **q** is **no**_e



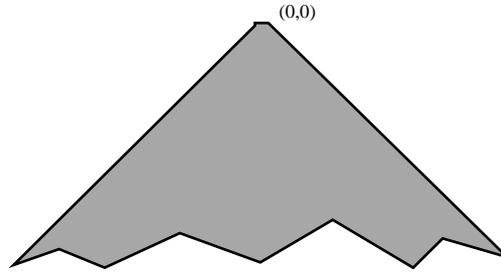
The quantifier \mathbf{no}_e

But if it only has points outside this column, we infer from invariance II, III, and IV that \mathbf{q} is **some**.



*The quantifier **some***

Finally, if \mathbf{q} has both points on and outside the column, it is **some** \vee \mathbf{no}_e .



*The quantifier **some** \vee \mathbf{no}_e*

The remaining cases where \mathbf{q} does contain the root $\langle 0, 0 \rangle$ are complements of the above four.

As to the proof of the corollary, it is enough to observe that \mathbf{Q} is equivalent to a weak reading iff $\mathbf{Q}\neg$ is equivalent to a strong reading. \square

It is worthwhile to notice that four of the eight quantifiers equivalent to a weak reading are either trivial (**true** and **false**) or almost trivial (**some** \vee \mathbf{no}_e and its negation). This means that only the quantifiers **some**, \mathbf{no}_e , **no**, and ‘**some** plus root’ are useful quantifiers that reduce to their weak readings.

Theorem 2 and corollary 3 imply Kanazawa’s result.

Corollary 4 *If $R, S \subseteq E^n$ satisfy: $\forall(R \Rightarrow S) \cup \forall(R \Rightarrow \bar{S})$ the only logical polyselective quantifiers which reduce to their weak reading are the Boolean compounds of **all** and **some**.*

PROOF. The constraint on R and S implies that $\exists(R \cap S) = \exists(R) \cap \forall(R \Rightarrow S)$. Due to conservativity, this means that the weak and the strong reading of a quantifier collapse. A case quantifier \mathbf{Q} reduces to its weak reading iff it reduces to its strong reading. Theorem 2 and its corollary 3 give that such \mathbf{Q} are equivalent to a Boolean combination BC_1 of *some* and \mathbf{no}_e , or equivalent to a Boolean combination BC_2 of *all_e* and *not all*. But both are Boolean combinations of *all* and *some*, for *all_e* is *some and all*, and \mathbf{no}_e is *not all and not some*. The converse is immediate. Due to the collapse of weak and strong readings there are no genuine mixed cases, and we already know that either of the readings is closed under Boolean compounds. \square

I now turn to a proof of the general case, which may be skipped by those not interested in such technicalities.

1.5 The general case

In this section I prove theorem 2 without assuming FIN (the corollaries 3 and 4 are obtained as before). I have to prove two things:

I Theorem 2 holds for all logical n -ary case quantifiers of \mathbf{Q} ($n \geq 2$);

II The characterization of \mathbf{Q} 's n -ary case quantifiers is uniform for all such n .

The result for the polyselective case is a direct consequence of I and II. For \mathbf{Q}^{cq} reduces to its weak reading, iff $\text{cQ}^n(\mathbf{Q})$ does for each $n \geq 2$.

Proof of I By definition an n -ary case quantifier reduces to its weak reading iff

$$(11) \quad \text{cQ}^n(\mathbf{Q})RS \Leftrightarrow \mathbf{Q}\exists(R)\exists(R \cap S)$$

That for each n the Boolean compounds of **some** and **no_e** have (11) follows from the observations in section 1.4. As to the other direction, fix an $n \geq 2$ for the rest of this proof. Intuitively it is clear that the $\text{cQ}^n(\mathbf{Q})$ with (11) are rather insensitive to the cardinality of the difference and intersection of their arguments. Adopting the view of quantifiers as relations between cardinalities, the following claims make this intuition precise.

- i) If $\text{cQ}^n(\mathbf{Q})\kappa 0$ and $\kappa \geq 1$, then $\text{cQ}^n(\mathbf{Q})10$.
- ii) If $\text{cQ}^n(\mathbf{Q})\kappa\lambda$ and $\lambda \geq 1$, then $\text{cQ}^n(\mathbf{Q})01$.
- iii) If $\text{cQ}^n(\mathbf{Q})\kappa\lambda$ and $\kappa + \lambda \geq 1$, then $\text{cQ}^n(\mathbf{Q})(\kappa + \mu)\lambda$ for all μ .
- iv) If $\text{cQ}^n(\mathbf{Q})\kappa\lambda$ and $\lambda \geq 1$, then $\text{cQ}^n(\mathbf{Q})\kappa(\lambda + \mu)$ for all μ .

Before proving (i–iv), I first show how they imply the result. Suppose that $\text{cQ}^n(\mathbf{Q})$ has an element different from $\langle 00 \rangle$. Then the antecedent of (i) or (ii) obtains. In case of (i), it follows from (iii) that $\text{cQ}^n(\mathbf{Q})\kappa 0$, for all $\kappa \geq 1$. That is, $\text{cQ}^n(\mathbf{Q})$ subsumes **no_e**. And in case of (ii), (iii) and (iv) imply that $\text{cQ}^n(\mathbf{Q})\kappa\mu$, for all κ and all $\mu \geq 1$. That is, $\text{cQ}^n(\mathbf{Q})$ subsumes **some**. All in all, this means that $\text{cQ}^n(\mathbf{Q})$ is fully determined by the eight possible combinations of the pairs $\langle 00 \rangle$, $\langle 10 \rangle$, and $\langle 01 \rangle$. Let **fe** be defined by: $\mathbf{fe}AB \Leftrightarrow A = \emptyset$. We have:

00	10	01	$\text{cQ}^n(\mathbf{Q})$	<i>simplified</i>
+	+	+	some \vee \neg some	true
+	+	–	\neg some	no
+	–	+	\neg no_e	some \vee fe
+	–	–	\neg some \wedge \neg no_e	fe
–	+	+	some \vee no_e	\neg fe
–	+	–	no_e	no_e
–	–	+	some	some
–	–	–	some \vee \neg some	false

Now for the proofs of (i–iv). They make ample use of the simple fact that the quantifiers with (11) also have:

$$(12) \quad \frac{\exists(R) = \exists(R') \text{ and } \exists(R \cap S) = \exists(R' \cap S')}{\text{cQ}^n(\mathbf{Q})RS \Leftrightarrow \text{cQ}^n(\mathbf{Q})R'S'}$$

The proofs of claim (i) and (ii) are similar. As to (ii), assume that $\text{cQ}^n(\mathbf{Q})\kappa\lambda$ with $\lambda \neq 0$. Let X and Y be disjoint $n - 1$ -ary relations such that $|X| = \kappa$ and $|Y| = \lambda$. Define $S = \{0\} \times Y$, and $R = S \cup \{0\} \times X$. Then, $|R \setminus S| = \kappa$ and $|R \cap S| = \lambda$, and so $\mathbf{Q}^{(n)}RS$. Now consider $R' = S' = \{0\}^n$. Since $\lambda \neq 0$, $R \neq 0$ and $S \neq 0$. Therefore: $\exists(R) = \exists(R')$ and $\exists(R \cap S) = \exists(R' \cap S')$. It follows from (12) that $\text{cQ}^n(\mathbf{Q})R'S'$. So, $\langle |R' \setminus S'|, |R' \cap S'| \rangle = \langle 0, 1 \rangle \in \text{cQ}^n(\mathbf{Q})$.

As to claim (iii), for n -ary R , an element a , and $n - 1$ -ary X define:

$$R +_a X := R \cup \{a\} \times X$$

Clearly, for all $a \in \exists(R)$: $\exists(R) = \exists(R +_a X)$. Combined with (12), this gives for all $a \in \exists(R)$:

$$(13) \quad \text{cQ}^n(\mathbf{Q})RS \Leftrightarrow \text{cQ}^n(\mathbf{Q})R +_a XR \cap S$$

Since $\text{cQ}^n(\mathbf{Q})$ is logical, the right-hand side of (13) only depends on the following cardinalities:

- $|(R +_a X) \cap R \cap S|$ ($= |R \cap S|$), and
- $|R +_a X \setminus R \cap S|$ ($= |(R \setminus S) +_a (X \setminus (R \cap S)_a)|$)

Since we are free to choose any X , (13) says: if $R \neq \emptyset$ $\text{cQ}^n(\mathbf{Q})$ is invariant under increase of the size of $|R \setminus S|$. This proves claim (iii).

Finally, note that whenever $a \in \exists(R \cap S)$ (13) can be refined to:

$$(14) \quad \text{cQ}^n(\mathbf{Q})RS \Leftrightarrow \mathbf{Q}R +_a X(R \cap S) +_a X$$

For such an a the right-hand side depends on the cardinalities:

- $|R +_a X \cap (R \cap S) +_a X|$ ($= |(R \cap S) +_a X|$), and
- $|R +_a X \setminus (R \cap S) +_a X|$ ($= |(R \setminus S) \setminus \{a\} \times X|$)

Again X is arbitrary, and in particular it can be chosen so that $\{a\} \times X$ is disjoint from $R \cap S$. So, (14) says that $\text{cQ}^n(\mathbf{Q})$ is invariant under increase of the size of non-empty intersections, as claim (iv) requires.

This ends the proof of I. Statement II remains: if \mathbf{Q} satisfies (11) for all its n -ary case quantifiers ($n \geq 2$), these case quantifiers are all equivalent to the same Boolean compound. I prove this by showing that under the assumption the case quantifiers are identical.

Proof of II. Assume that logical $\text{cQ}^n(\mathbf{Q})$ satisfies (11) for all $n \geq 2$. I have to prove that $\text{cQ}^n(\mathbf{Q}) = \text{cQ}^m(\mathbf{Q})$ for all m . So let κ and λ be arbitrary cardinals. Choose disjoint $n - 1$ -ary X, Y , and disjoint $m - 1$ -ary X', Y' such that: $|X| = |X'| = \kappa$ and $|Y| = |Y'| = \lambda$. Define: $S = \{0\} \times Y$, $S' = \{0\} \times Y'$, $R = S \cup \{0\} \times X$, and $R' = S' \cup \{0\} \times X'$. Clearly, $\exists(R) = \exists(R')$ and $\exists(R \cap S) = \exists(R' \cap S')$. Since $\text{cQ}^n(\mathbf{Q})$ and $\text{cQ}^m(\mathbf{Q})$ both satisfy (11), we have:

$$(15) \quad \text{cQ}^n(\mathbf{Q})RS \Leftrightarrow \mathbf{Q}^{(m)}R'S'$$

Since: $|R \setminus S| = |R' \setminus S'| = \kappa$ and $|R \cap S| = |R' \cap S'| = \lambda$, it follows from (15) that:

$$(16) \quad \text{cQ}^n(\mathbf{Q})\kappa\lambda \Leftrightarrow \text{cQ}^m(\mathbf{Q})\kappa\lambda$$

But the cardinals were chosen arbitrarily, so: $\text{cQ}^n(\mathbf{Q}) = \text{cQ}^m(\mathbf{Q})$, as required. \square

It is clear that first-order case quantification is too restrictive to give the general interpretation of nominal anaphora. In the next section, I study DPL as giving another way to interpret nominal anaphora: scope extension of the antecedent quantifier.

2 DPL and scope extension

In the previous section I have recalled how DRT's logic for singular anaphora can be seen as based on case quantification. An alternative would be to view DRT as exploiting the equivalence of (17a) and (17b).

- (17) a. A man walks in the park. He whistles.
 b. A man walks in the park and whistles.

Due to this equivalence, (17a) can be transformed into (18) without loss of meaning.

$$(18) \quad \exists x[\text{M}x \wedge \text{W}x \wedge \text{W}hx]$$

As is well-known, (18) is obtained as follows. The antecedent sentence of (17a) induces the DRS (19a), where x is a discourse referent restricted by the conditions in the body.

- (19) a. $x[\text{M}x \wedge \text{W}x]$
 b. $x[\text{M}x \wedge \text{W}x \wedge \text{W}hx]$

Next the anaphor sentence ‘Wh x ’ is merged with this DRS to give (19b). Finally, (19b) is interpreted as (18) by means of the discourse operator \exists . This strategy complies with Geach’s view that pronouns are bound variables. Since the meaning of the antecedent sentence on its own is $\exists x[Mx \wedge Wx]$, the process can also be viewed as extending the scope of $\exists x$ to include the anaphor sentence. That this is so, is seen even more clearly in DPL’s compositional reconstruction of DRT (Groenendijk and Stokhof 1991). A crucial feature of DPL is that the formalization (20a) of (17a) is equivalent to (20b), which has its standard meaning.

- (20) a. $\exists x(Mx \wedge Wx) \wedge Whx$
 b. $\exists x(Mx \wedge Wx \wedge Whx)$

A related principle of scope extension corresponds to the synonymy of (21a) and (21b).

- (21) a. If a man walks in the park, he whistles.
 b. Every man who walks in the park, whistles

In DPL, and *mut. mut.* in DRT, these sentences can be obtained from each other since (22a) and (22b) are equivalent.

- (22) a. $\exists x(Mx \wedge Wx) \rightarrow Whx$
 b. $\forall x(Mx \wedge Wx \rightarrow Whx)$

As in the case of DRT and polyadic quantification, I now use the translation of DPL into first-order logic to highlight its use of scope extension.

2.1 DPL

In contrast with DRT, DPL makes no difference in the type of semantical objects assigned to its formulas. Table 2 summarizes the syntax and semantics of DPL.

TYPE	SYNTAX	SEMANTICS (= $\llbracket \text{SYNTAX} \rrbracket$)
frm	$Rx_1 \dots x_n$	$\{\langle g, g \rangle \mid \llbracket R \rrbracket g(x_1) \dots g(x_n)\}$
frm	$x_1 = x_2$	$\{\langle g, g \rangle \mid g(x_1) = g(x_2)\}$
frm	$\neg\varphi$	$\{\langle g, g \rangle \mid \neg\exists h : g\llbracket \varphi \rrbracket h\}$
frm	$\varphi \wedge \psi$	$\{\langle g, h \rangle \mid \exists k : g\llbracket \varphi \rrbracket k \ \& \ k\llbracket \psi \rrbracket h\}$
frm	$\exists x\varphi$	$\{\langle g, h \rangle \mid \exists d \in E : g_d^x \llbracket \varphi \rrbracket h\}$

TABLE 2 DPL’s syntax and semantics.

In this logic $\varphi \rightarrow \psi$ can be defined by $\neg(\varphi \wedge \neg\psi)$, and $\forall x\varphi$ by $\neg\exists x\neg\varphi$. The following translation of DPL into first-order logic is abstracted from Groenendijk and Stokhof 1991.

- a. $(Rx_1 \dots x_n)^\circ \equiv Rx_1 \dots x_n$;
 b. $(x_1 = x_2)^\circ \equiv x_1 = x_2$;
 c. $(\neg\varphi)^\circ \equiv \neg\varphi^\circ$;
 d. $(\exists x\varphi)^\circ \equiv \exists x\varphi^\circ$
 e. $(Rx_1 \dots x_n \wedge \chi)^\circ \equiv Rx_1 \dots x_n \wedge \chi^\circ$
 $(\neg\varphi \wedge \chi)^\circ \equiv \neg\varphi^\circ \wedge \chi^\circ$
 $(\exists x\varphi \wedge \chi)^\circ \equiv \exists x(\varphi^\circ \wedge \chi^\circ)$
 $((\varphi \wedge \psi) \wedge \chi)^\circ \equiv (\varphi \wedge (\psi \wedge \chi))^\circ$

The clause for conjunction uses a subrecursion on the structure of its first conjunct. In combination with the clause for quantified formulas, this has the effect of giving an antecedent quantifier widest scope possible, either within the entire text or within the scope of a negation.

Fact 5 For each \mathcal{M} and φ : $\text{DOM}(\llbracket \varphi \rrbracket_{\mathcal{M}}^{\text{dpl}}) = \llbracket \varphi^\circ \rrbracket_{\mathcal{M}}$, with $\llbracket \varphi^\circ \rrbracket_{\mathcal{M}}$ as before. \square

We have seen that DRT and DPL have truth preserving translations into first-order logic. As to the converse, just note that for *rectified* formulas φ , in which

bound and free variables are disjoint and each quantifier comes with its unique variable

$$\text{DOM}(\llbracket \varphi \rrbracket_{\mathcal{M}}^{\text{dpl}}) = \llbracket \varphi \rrbracket_{\mathcal{M}}$$

via the identity translation. Since first-order equivalent rectified formulas when viewed as DPL-formulas have the same domain, we arrive at the familiar fact that DRT, DPL, and first-order logic are as expressives.

2.2 The scope extension problem

The generality of scope extension as a principle of anaphor resolution has already been challenged by Evans (1977,1980). In his polemics with Geach he held that not all pronouns with an antecedent are bound variables. His main observation was that pairs like (23a) and (23b) are unequivalent.

- (23) a. Quite a few men drove a Jaguar. They whistled.
 b. Quite a few men drove a Jaguar and whistled.

The difference between (23a) and (23b) cannot be explained if ‘they’ in (23a) is bound by ‘quite a few’ in the standard sense. For then both sentences would mean that about half of the men who drove a Jaguar whistled. Instead, (23a) implies (23b) but not conversely. In particular, (23b) does not imply the anaphor sentence of (23a), which means that each man who drove a Jaguar whistled.

Observations such as these evoke a basic question: how general is scope extension as a principle to interpret anaphor? Does it have a fully general version, or should we perhaps distinguish between the noun phrases which do and those which do not allow their scope to be extended (cf. Chierchia 1992)?

The logical results in section 2.3 imply that scope extension can be sustained for all singular but not for all plural anaphora. This is shown in detail in section 2.4 and 2.6, respectively. (Section 2.5 is a short interlude on a logic for singular Q-types.) This explains why it is non-trivial to incorporate dynamic generalized quantifiers within DRT and DPL. Section 2.6 has some preliminary remarks on the proposals made in this direction.

2.3 The scope of scope extension

As in case of the proportion problem, the proper setting for studying the problem of scope extension is generalized quantifier theory. But then we first have to specify how anaphoric relationships influence meaning within this framework. A convenient way is to view anaphoric noun phrases as contextually restricted quantifiers. Then the anaphoric element is a determiner \mathbf{A} , left unspecified for the moment, while the anaphoric link between \mathbf{D}^i and \mathbf{A}_i in (24a) means (24b).⁷

- (24) a. $\mathbf{D}^i AB. \mathbf{A}_i C$
 b. $\mathbf{D} AB. \mathbf{A} A \cap BC$

This treatment seems far removed from the dynamic semantics of DRT and DPL, but I shall indicate how the theories fit within this approach.

In view of (20) and (22), we want to know for which \mathbf{D} and \mathbf{A} (25a–25b) and (26a–26b) are equivalent:⁸

- (25) a. $\mathbf{D}^i AB \ \& \ \mathbf{A}_i C$
 b. $\mathbf{D} AB \cap C$

- (26) a. $\mathbf{D}^i AB \Rightarrow \widetilde{\mathbf{A}}_i C$
 b. $\widetilde{\mathbf{D}} A \cap BC$

The reason for using $\widetilde{\mathbf{A}}$ in (26a) is that the type $\langle 1 \rangle$ quantifiers in DRT and DPL leave some room in formalizing the principles of scope extension. Recall that DPL

⁷In this article I often speak of determiners, but *mut. mut.* the same remarks can be made for non-complex noun phrases.

⁸ $\widetilde{\mathbf{D}'}$ is the dual of \mathbf{D}' defined by: $\widetilde{\mathbf{D}'} = \neg \mathbf{D}' \neg$.

defines $P \rightarrow Q$ as $\neg(P \& \neg Q)$. This means that the equivalence of (26a–26b) reduces to the equivalence of (27a–27b).

- (27) a. $\mathbf{D}^i AB \& \mathbf{A}_i C$
 b. $\mathbf{D} A \cap BC$

Accordingly, we should study two related but quite different ways of scope extension: Roberts’ telescoping in (25) *versus* what might be called ‘absorption’ in (27).

The question concerning (25a–25b) and (26a–26b) is one of inverse logic: for which \mathbf{D} are the equivalences $\langle \mathbf{A} \rangle$ or $[\mathbf{A}]$ valid?⁹

$$\langle \mathbf{A} \rangle \quad \mathbf{D}^i AB \& \mathbf{A}_i C \Leftrightarrow \mathbf{D} AB \cap C$$

$$[\mathbf{A}] \quad (\mathbf{D}^i AB \Rightarrow \widetilde{\mathbf{A}}_i C) \Leftrightarrow \widetilde{\mathbf{D}} A \cap BC$$

In $\langle \mathbf{A} \rangle$ and $[\mathbf{A}]$, \mathbf{D} is called the antecedent determiner and \mathbf{A} the anaphoric determiner. The full paper studies $\langle \mathbf{A} \rangle$ and $[\mathbf{A}]$ separately. For now I just observe that they collapse in case of intersective \mathbf{D} : $\mathbf{D} AB \cap C \Leftrightarrow \mathbf{D} A \cap BC$. Section 2.3.2 has a formal treatment of $\langle \mathbf{A} \rangle$, but the examples in section 2.3.1 will indicate what we might expect.

2.3.1 Some examples

The most familiar pattern of Evans’ inference uses a pronoun for the anaphoric element; cf. the equivalence in (28).

- (28) All men walked in the park. They whistled.
 \Leftrightarrow All men walked in the park and whistled.

Here, ‘they’ is interpreted as a universal quantifier restricted to the context set ‘men walking in the park’. So, (28) is an instance of $\langle \mathbf{all} \rangle$. For conservative \mathbf{D} one direction of $\langle \mathbf{all} \rangle$ is always valid.

$$(29) \quad \mathbf{D} AB \quad A \cap B \subseteq C \\ \Rightarrow \mathbf{D} AB \cap C$$

But as we shall see, in almost all other cases the converse is false. For instance, (30) is invalid.

- (30) Some men walked in the park and whistled.
 $\not\Leftrightarrow$ Some men walked in the park. They whistled.

The first sentence of the conclusion is implied, which is the case iff \mathbf{D} in $\langle \mathbf{A} \rangle$ is $\text{MON}\uparrow$. But clearly the premise does not imply the second sentence which states that all men walking in the park whistle.

Once anaphora are viewed as contextually restricted quantifiers, or Q-types, there are quite a few variants of (28) without pronouns. Cf. Van Deemter 1991. Take (31), for example.¹⁰

- (31) Five men bought whiskey. Five got drunk.
 \Leftrightarrow Five men bought whiskey and got drunk.

As I already noted, the antecedent determiner in $\langle \mathbf{A} \rangle$ must be $\text{MON}\uparrow$. And indeed, (31) is valid if ‘five’ is read as the monotone increasing ‘at least five’, but invalid if read as the non-monotonic ‘exactly five’. The use of the monotone decreasing ‘at most five’ would result in invalidity as well. Moreover, (32) is invalid on each option for the numeral.

- (32) Five men bought whiskey. Some got drunk.
 $\not\Leftrightarrow$ Five men bought whiskey and got drunk.

⁹Cf. van Benthem 1984 who asked similar questions for syllogistic patterns of reasoning.

¹⁰In the discussion at the Tenth Amsterdam Colloquium after my talk, Barbara Partee said to disagree with the walk-in-the-park variant of (31) because it requires the second *five* to introduce a new quintet. According to Hans Kamp this variant would be acceptable after some reflection. Herman Hendriks suggested the present version which raised no linguistic objections.

The above examples suggest that as soon as the antecedent **D** and the anaphoric **A** are one and the same $\text{MON}\uparrow$ determiner a valid equivalence results. That this is not true is shown by (33).

- (33) Most men went to the party. Most whistled.
 $\not\Leftarrow$ Most men went to the party and whistled.

Although the top discourse follows from the sentence at bottom, the converse direction fails (consider four men of which three walk and two whistle). Therefore, we need to look for special $\text{MON}\uparrow$ determiners, and I shall give the relevant properties shortly.

2.3.2 The validity of $\langle \mathbf{A} \rangle$

In this section I characterize the logical determiners which validate $\langle \mathbf{A} \rangle$ for a certain choice of **A**. Relative to **A**, these are the determiners for which it does not make a difference whether they are linked anaphorically to **A** (the lefthand side of $\langle \mathbf{A} \rangle$), or whether they have their scope extended to include the descriptive material of the anaphor sentence (the righthand side of $\langle \mathbf{A} \rangle$).

$\langle \mathbf{A} \rangle$ \mathbf{DAB} & $\mathbf{AA} \cap \mathbf{BC}$, iff $\mathbf{DAB} \cap \mathbf{C}$.

As it happens, in this characterization the choice of **A** cannot be fully arbitrary, but it is sufficient to require that **A** is restrictive. This notion corresponds to the second of the three inferences which make up $\langle \mathbf{A} \rangle$ for the special case where **A** is **D**.¹¹

- (34) (a) $\frac{\mathbf{DAB} \cap \mathbf{C}}{\mathbf{DAB}}$ (b) $\frac{\mathbf{DAB} \cap \mathbf{C}}{\mathbf{DA} \cap \mathbf{BC}}$ (c) $\frac{\mathbf{DAB} \mathbf{DA} \cap \mathbf{BC}}{\mathbf{DAB} \cap \mathbf{C}}$

I leave the import of (34c) for the full paper. Property (34a) is familiar: **D** has it iff **D** is $\text{MON}\uparrow$. The most relevant property for the characterization we are after is (34b). A determiner **D** has (34b) iff it is *restrictive*.¹²

- (35) $\frac{\mathbf{DAB} \mathbf{B} \subseteq \mathbf{C}}{\mathbf{DA} \cap \mathbf{CB}}$

For instance, from right to left we have: $\mathbf{DAB} \cap \mathbf{C}$; (res) $\mathbf{DA} \cap \mathbf{BB} \cap \mathbf{C}$; (cons) $\mathbf{DA} \cap \mathbf{BC}$. Notice that restrictiveness is weaker than intersectivity and $\downarrow\text{MON}$. Prime cases such as the anaphoric use of ‘some’ and ‘all’ will therefore be covered by our result. An example of a restrictive quantifier which is neither intersective nor $\downarrow\text{MON}$ is ‘most’; witness the validity of (36):

- (36) $\frac{\text{Most logic books are yellow and expensive}}{\text{Most yellow logic books are expensive}}$

I now prove theorem 9, which states that logical **A** have $\langle \mathbf{A} \rangle$ iff the only determiners which validate $\langle \mathbf{A} \rangle$ are those obtained from **A** by restricting the cardinality of its first argument.

Lemma 6 *Let **A** be restrictive. For each **D** with $\langle \mathbf{A} \rangle$ we have for all A such that $\mathbf{DA} \neq \emptyset$: $\mathbf{AA} = \mathbf{DA}$.*

PROOF. Let **D** have $\langle \mathbf{A} \rangle$. Then $\mathbf{D} \subseteq \mathbf{A}$, as is shown by the following implications: \mathbf{DAB} ; (cons) $\mathbf{DAA} \cap \mathbf{B}$; $\langle \mathbf{A} \rangle$ $\mathbf{AA} \cap \mathbf{AB}$; \mathbf{AAB} . Conversely, let A be such that $\mathbf{DA} \neq \emptyset$, and assume \mathbf{AAB} . Choose an X with \mathbf{DAX} . Since **D** has $\langle \mathbf{A} \rangle$ it is $\text{MON}\uparrow$, whence $\mathbf{DAX} \cup \mathbf{B}$. Since **A** is restrictive: $\mathbf{AA} \cap (\mathbf{X} \cup \mathbf{B})\mathbf{B}$. Now $\langle \mathbf{A} \rangle$ implies: $\mathbf{DA}(\mathbf{X} \cup \mathbf{B}) \cap \mathbf{B}$; i.e., \mathbf{DAB} . \square

The proof of this lemma already gives some information on the kind of **D** which validate $\langle \mathbf{A} \rangle$.

¹¹Those who feel uncomfortable with classes and cardinals may assume FIN, and read ‘natural number’ for ‘cardinal’ and ‘set’ for ‘class’.

¹²In his study of conditional relations, van Benthem 1986 proposed this principle as part of *confirmation*. Cf. also Lapiere 1996.

Corollary 7 *If \mathbf{D} has $\langle \mathbf{A} \rangle$ then \mathbf{D} is $\text{MON}\uparrow$ and $\mathbf{D} \subseteq \mathbf{A}$* □

For logical determiners the restriction in lemma 6 to $\mathbf{D}A \neq \emptyset$ can be removed by means of the following concept.

Definition 1 For a class of cardinals Θ the determiner \mathbf{D}^Θ is defined by:

$$\mathbf{D}^\Theta AB \Leftrightarrow |A| \in \Theta \ \& \ \mathbf{D}AB$$

In case $\Theta = \{\kappa\}$ I write \mathbf{D}^κ for \mathbf{D}^Θ .

Lemma 8 *Let \mathbf{A} be logical and restrictive. If a logical determiner \mathbf{D} has $\langle \mathbf{A} \rangle$, then $\mathbf{D} = \mathbf{A}^\Theta$ for some class of cardinals Θ .*

PROOF. Let \mathbf{D} have $\langle \mathbf{A} \rangle$, and define:

$$\Theta := \{\kappa : \exists A(\mathbf{D}A \neq \emptyset \ \& \ |A| = \kappa)\}$$

To show $\mathbf{D} \subseteq \mathbf{A}^\Theta$, assume $\mathbf{D}AB$. Then, $|A| \in \Theta$, and by corollary 7: $\mathbf{A}^\Theta AB$. Conversely, to show $\mathbf{A}^\Theta \subseteq \mathbf{D}$ assume $\mathbf{A}^\Theta AB$. Then $|A| \in \Theta$ and $\mathbf{A}AB$. By definition of Θ , there is an A' with $|A'| = |A|$ and $\mathbf{D}A' \neq \emptyset$. Lemma 6 gives:

(#) For all B' : $\mathbf{D}A'B' \Leftrightarrow \mathbf{A}A'B'$.

Since $|A| = |A'|$, there is a bijection $g : A \rightarrow A'$. Clearly:

$$|A \cap \overline{A \cap B}| = |A' \cap \overline{g(A \cap B)}| \text{ and } |A \cap B| = |A' \cap g(A \cap B)|$$

We have the following equivalences: $\mathbf{A}AB$; (logicality) $\mathbf{A}A'g(A \cap B)$; (#) $\mathbf{D}A'g(A \cap B)$; (logicality) $\mathbf{D}AB$. □

Since (almost?) all important instances of \mathbf{A} are logical and restrictive, lemma 8 makes plain that for most antecedent determiners in $\langle \mathbf{A} \rangle$ scope extension does not preserve meaning. Lemma 8 also enables a characterization of the logical \mathbf{A} which have $\langle \mathbf{A} \rangle$.

Theorem 9 *\mathbf{A} is logical and $\langle \mathbf{A} \rangle$, iff for all logical \mathbf{D} : \mathbf{D} has $\langle \mathbf{A} \rangle$ iff $\mathbf{D} = \mathbf{A}^\Theta$ for some class of cardinals Θ .*

PROOF. First assume that \mathbf{A} is logical and $\langle \mathbf{A} \rangle$. I have already observed that the \mathbf{A} with $\langle \mathbf{A} \rangle$ are restrictive, and it is clear that \mathbf{A}^Θ is logical if \mathbf{A} is. So with a view to lemma 8 the only thing which remains to prove the equivalence for logical \mathbf{D} is that the \mathbf{A}^Θ have $\langle \mathbf{A} \rangle$. But this property of \mathbf{A} is preserved as well:

$$\begin{aligned} & \mathbf{A}^\Theta AB \ \& \ \mathbf{A}A \cap BC \\ \Leftrightarrow & |A| \in \Theta \ \& \ \mathbf{A}AB \ \& \ \mathbf{A}A \cap BC \\ \Leftrightarrow & |A| \in \Theta \ \& \ \mathbf{A}AB \cap C \\ \Leftrightarrow & \mathbf{A}^\Theta AB \cap C \end{aligned}$$

Conversely, if the logical determiners with $\langle \mathbf{A} \rangle$ are precisely the \mathbf{A}^Θ for some Θ , then \mathbf{A} must have $\langle \mathbf{A} \rangle$ too: $\mathbf{A} = \mathbf{A}^{\{\kappa: \kappa=\kappa\}}$ □

The next sections apply theorem 9 to show its impact for current discourse logics. In section 2.4 it is used to study the semantics of singular pronouns, and in section 2.6 to study plural definites and numerals.

2.4 Singular anaphora and their antecedents

Singular anaphora are perhaps the most puzzling of all, since their meaning may change with the place of occurrence. Nevertheless, I shall argue that they can be treated as Q-type contextualized quantifiers in a principled way. Besides, this section gives (i) a full description of the determiners which satisfy the principle of scope extension for singular pronouns (section 2.4.1), and (ii) a universal to single out the determiners which may be antecedent to a singular pronoun (section 2.4.2). A formalization of the Q-type approach is in section 2.5. The picture which emerges is that the semantics of singular anaphora can be given by means of scope extension or in terms of contextual restriction. Mainly plural anaphora distinguish the two.

2.4.1 Singular anaphora

Let us first make a few observations to show that if we interpret singular pronouns as contextually restricted existential quantifiers they behave much as in DRT and DPL. To this end recall that the singular version of ‘some’ is interpreted as in (37).

$$(37) \quad \begin{aligned} \mathbf{some}AB &\Leftrightarrow A \cap B \neq \emptyset \\ \mathbf{some}^sAB &\Leftrightarrow A \cap B \neq \emptyset \ \& \ |A| = 1 \end{aligned}$$

Similarly, on a Russellian view the semantics of ‘a’ is either **some** or **some**^s, where **some**^s is equivalent to a Russellian singular description **the**^s. Since **some** has ⟨**some**⟩ we know from theorem 9 that **some**^s has ⟨**some**⟩ as well. Moreover, **some**^s has ⟨**some**^s⟩. Thus we see that interpreting singular pronouns as existentials gives what is required.

$$(38) \quad \frac{\text{A man walks in the park. He whistles.}}{\text{A man walks in the park and whistles}}$$

To be precise, the equivalence (38) holds iff ‘a’ and ‘he’ are both interpreted as **some**; or both as **some**^s; or ‘a’ is interpreted as the description **some**^s and ‘he’ as the existential **some**. It is invalid if ‘a’ is an existential and ‘the’ a singular definite description.

Since **some** is intersective the remarks in section 2.3 make plain that it has [some], too. So, donkey-like sentences can be handled along these lines as well:¹³

$$(39) \quad \frac{\text{If a man walks in the park, he whistles.}}{\text{Every man who walks in the park, whistles.}}$$

It is worth to notice that the difference in quantificational force of the singulars—existential in (38), universal in (39)—is closely related to the polarity of the antecedent and the anaphor. By default we have that an occurrence of a singular pronoun is existential (*weak*) if its polarity and that of its antecedent ‘a’ are the same (both positive or both negative); it is universal (*strong*) otherwise. This generalizes Kanazawa’s observation concerning monotonic determiners to arbitrary constructions, and can be formalized by means of a monotonicity calculus. Cf. Van Benthem 1986, Sánchez Valencia 1990, Kanazawa 1994. See also the formal system in section 2.5. However, in case of other antecedents than ‘a’ it seems that singular pronouns favour a weak reading.

DRT and DPL The existential semantics for singular pronouns appears far removed from compositional versions of DRT, but it is not. A denotation for singular pronouns which is essentially the same as that in DPL can be phrased in terms of choice functions. This can be seen as follows. In DPL the pronoun ‘he’ in (17a) is a variable which is captured dynamically by the preceding existential quantifier.

(17a) A man walks in the park. He whistles.

The existential associated with the indefinite ‘a’ introduces an object d stored under x to give assignment a_x^d .

$$(17a') \quad a \xrightarrow{\exists x} a_x^d \xrightarrow{Mx} a_x^d \xrightarrow{Wx} a_x^d \xrightarrow{Whx} a_x^d$$

Next, the atomic sentences within the scope of the quantifier test one after the other whether d is a man and a walker. If so, d is picked up by the pronoun ‘he’. When d is a whistler as well, (17a) is true.

¹³It would be interesting to compare this view on singular descriptions with that in the philosophical literature. Cf. also Gawron, Nerbonne and Peter 1991. Neale (1990, chapter 5 & 6) shows that donkeys, sage plants, bathrooms and MIGs, among other anaphoric sentences, can be handled by interpreting singular E-type pronouns as numberless descriptions; i.e., universal quantifiers with a non-empty first argument. This works well in all the cases were a universal reading is required, but it fails for such texts as in (38).

A slightly different but logically equivalent approach would hold that ‘he’ chooses an object from the set of walking men. For the case at hand, this can be made precise as follows.¹⁴

Definition 2 A function $g : \wp(A) \rightarrow A$ is a *choice function* for A iff for all non-empty $X \subseteq A$: $g(X) \in X$. In terms of a choice function f the determiner $\langle f \rangle$ is defined by:

$$\langle f \rangle_E AB \Leftrightarrow A \neq \emptyset \wedge f_E(A) \in B$$

for all $A, B \subseteq E$. The dual $[f]$ of $\langle f \rangle$ is defined by: $[f]_E AB \Leftrightarrow \neg \langle f \rangle_E A\overline{B}$.

Using $\langle f \rangle$ the anaphoric link in (40a) can be interpreted as (40b).

- (40) a. $\mathbf{D}^i AB$ & $\mathbf{A}_i C$
 b. $\exists f : \mathbf{D}AB$ & $\langle f \rangle A \cap BC$

We should therefore ask for the \mathbf{D} which satisfy (cf).

- (cf) $\exists f : \mathbf{D}AB$ & $\langle f \rangle A \cap BC$, iff $\mathbf{D}AB \cap C$.

But since $\exists f : \langle f \rangle AB$ iff $A \cap B \neq \emptyset$, for all A, B , it is the same to ask for the \mathbf{D} with $\langle \text{some} \rangle$. We therefore know from theorem 9:

Corollary 10 *The logical determiners \mathbf{D} which have (cf) are precisely those with $\mathbf{D} = \text{some}^\Theta$ for some class of cardinals Θ .* \square

The result shows that as soon as pronouns are treated by means of existential quantification or choice functions DRT’s and DPL’s mechanism of extending the scope of **some** only carries over to the indicated subparts of this quantifier. To know whether or not this is too restrictive, we have to determine the class of possible antecedents of singular pronomina. This is done in section 2.4.2. Here it is shown that the only other possible antecedents are the analogous subparts of *one*. Hence, corollary 10 does not prohibit intuitively acceptable linkings, for such antecedents can be defined in terms of **some** in the familiar way. Consider for instance (41).

- (41) a. Just one man drove a Jaguar. He whistled.
 b. Just one man drove a Jaguar and whistled.

Like in case of (23), the difference between (41a) and (41b) cannot be explained if ‘he’ in (41a) is bound by ‘just one’ in the standard sense. However, this observation does not suffice to show that a bound variable analysis of (41a) is impossible. One could read the antecedent sentence as stating: $\exists x. 'x$ is the unique man driving a Jaguar’. This first-order definition of ‘just one’ splits the quantifier into an existential and a uniquely satisfied predicate, and scope extension would still hold for the first. However, in case of quantifiers like ‘quite a few’ and ‘most’ this resort to first-order definitions is impossible. Cf. Barwise and Cooper 1981, among others.

2.4.2 Antecedents for singular anaphora

According to corollary 10 the interpretation of singular pronouns by means of scope extension should be limited to certain subparts of ‘some’. So, adopting scope extension as the main principle of pronoun resolution one would like to know whether this gives us all we want. To this end we have to determine which noun phrases may figure as antecedent to singular anaphora (hence: s-antecedent). I answer this question by proposing constraints on their denotations. They imply that the only logical determiners which may be antecedent to a singular anaphor are those of

¹⁴We gloss over the fact that the choice function should be dependent on the variable. Cf. Van der Does 1994a. Another important issue is that the restricting domains may be dependent on each other. Cf. Van den Berg 1991, 1996, among other places. The idea to use choice functions to interpret singular pronouns is suggested frequently: Urs Egli, Meyer-Viol 1992, Chierchia 1992, Gawron, Nerbonne, Peters 1991, Von Heusinger & Peregrin, Van Eijck 1993. Also: Muskens 1989.

form: ‘some^s’, ‘a’, ‘just one’, ‘the’, ‘one of the’, ‘one...out of –’. These can all be interpreted as **some**^Θ or **one**^Θ.

A strong intuition concerning singular antecedents is that their denotation introduces a possible referent for the anaphor. The singular dynamicity constraint SDYN corresponds to this intuition. Given CONS, which is assumed throughout, the following two formulations are equivalent:

$$\begin{aligned} \text{SDYN1 } \mathbf{D}_E AB &\Rightarrow \exists b \in B : \mathbf{D}_E A\{b\} \\ \text{SDYN2 } \mathbf{D}_E AB &\Rightarrow \exists b \in A \cap B : \mathbf{D}_E A\{b\} \end{aligned}$$

I propose the following universal (of which the converse does not hold!).

Constraint 11 *All s-antecedents satisfy SDYN.*

The constraint can be taken locally relative to given E, A, B or globally relative to all E, A, B . This makes a difference for determiners such as ‘every’ and ‘at most one’ which for particular E, A, B may respectively be equivalent to the SDYN ‘the^s’ and ‘just one’, although they are clearly not SDYN on all E, A, B . In such situations the local variant would allow:

(42) Every/at most one man cried. He was unhappy.

But (42) is ruled out on the global variant. It has as an immediate consequence that no non-empty SDYN determiner is $\text{MON}\downarrow$. For if \mathbf{D} is globally SDYN then it is *positive* in the sense that: $\mathbf{D}_E AB$ always implies $A \neq \emptyset \neq B$. And of course each non-empty $\text{MON}\downarrow \mathbf{D}$ has $\mathbf{D}A\emptyset$ for some A . This corresponds to our judgement that there can be no link between the antecedents and ‘he’ in (43).

(43) No/at most one man walked his dog. *He is lazy.

From now on we work with the global version of SDYN.

What about logical s-antecedents with a different monotonicity behaviour? Proposition 12 answers the $\text{MON}\uparrow$ case. For simplicity I assume FIN. Let X be a set of natural numbers, and define as before: $\mathbf{D}^X AB \Leftrightarrow_{\text{def}} |A| \in X \ \& \ \mathbf{D}AB$.

Proposition 12 *The only $\text{MON}\uparrow$ s-antecedents are those of form **some**^X with X a set of natural numbers.*

PROOF. It is clear that **some**^X SDYN, and $\text{MON}\uparrow$. So, take a logical \mathbf{D} with these properties. Since \mathbf{D} is logical, it may be viewed as a relation \mathbf{d} between numbers. For such relations SDYN corresponds to: $\mathbf{d}nm \Rightarrow m \geq 1 \ \& \ \mathbf{d}(n+m-1)1$. E.g., if $\mathbf{d}03$ then also $\mathbf{d}21$. And according to $\text{MON}\uparrow$ all positions to the right of a + for \mathbf{d} are + as well: $\mathbf{d}nm \Rightarrow \forall k(m \leq k \leq n+m \Rightarrow \mathbf{d}n+m-k, k)$. Similarly for $\text{MON}\downarrow$ and +’s to the left. Using these regularities we see first that not: $\mathbf{d}m0$ for all m , due to SDYN.

$$\begin{array}{cccc} & & - & \\ & & - & ? \\ & - & ? & ? \\ - & ? & ? & ? \\ \vdots & \vdots & \vdots & \vdots \end{array}$$

Second, for each line $m+1$ at which \mathbf{d} has a ‘+’ SDYN gives $\mathbf{d}m1$. For example:

$$\begin{array}{cccc} & & - & \\ & & - & ? \\ & - & ? & ? \\ - & ? & ? & + \\ \vdots & \vdots & \vdots & \vdots \end{array} \quad \text{only if:} \quad \begin{array}{cccc} & & - & \\ & & - & ? \\ - & ? & ? & ? \\ - & + & ? & + \\ \vdots & \vdots & \vdots & \vdots \end{array}$$

In combination with $\text{MON}\uparrow$ this implies for that m and all $n \geq 1$: $\mathbf{d}mn$.

$$\begin{array}{cccc}
& & - & ? \\
& & - & ? & ? \\
- & + & + & + \\
\vdots & \vdots & \vdots & \vdots
\end{array}$$

Now, let $X = \{n : \mathbf{d}$ has a ‘+’ at row $n\}$. From the previous remarks it is clear that the resulting pattern is that of \mathbf{some}^X . \square

In principle, this simple result already gives many possible antecedents for singular antecedents. But presumably not all of them are realized in natural language: X should not be too complex to describe. E.g., $\mathbf{some}^{\mathbb{N}}$, \mathbb{N} the set of natural numbers, is the meaning of *some, a, at least one* (which are equivalent according to quantifier theory). Also, *(at least) one of the n* denotes: \mathbf{some}^n , which for $n = 1$ gives the meaning of *the^{sg}*, etc.

Proposition 12 is in harmony with corollary 10, which states that scope extension for singular pronouns is just possible for the \mathbf{some}^X . So, if there are any problematic SDYN antecedents they should be non-monotonic. The behaviour of such determiners can be quite chaotic. But as the previous proof has already shown, they will always subsume ‘exactly one ^{X} ’ for some set X of natural numbers.

Corollary 13 *For each logical SDYN \mathbf{D} : exactly one ^{X} $\subseteq \mathbf{D}$, X some set of numbers.*

PROOF. Just observe that \mathbf{D} is $\mathbf{D}^{\{n : \mathbf{d}$ has a ‘+’ at row $n\}}$, and reason as in the previous proof. \square

Manageable non-monotonic determiners are the indefinites, which are often presented as the prototypical antecedents that introduce discourse referents. Let P be a set of natural numbers, and let $\langle P \rangle AB \Leftrightarrow |A \cap B| \in P$, as in the appendix. We have the following characterization.

Proposition 14 *The only non-empty, logical, non-monotonic, weakly intersective, SDYN \mathbf{D} are those of form $\langle \{1\} \cup P \rangle^X$ with $X \neq \emptyset, P$ sets of numbers such that (i) $0 \notin P$, (ii) $\exists k > 1 : k \notin P$.*

PROOF. Let \mathbf{D} be of form $\langle \{1\} \cup P \rangle^X$. That \mathbf{D} is SDYN is immediate from the fact that it contains the relevant parts of the 1-diagonal (i.e., $\mathbf{d}m1$ for all $m+1 \in X$). It is also clear that \mathbf{D} is weakly intersective. The non-monotonicity of \mathbf{D} now follows from non-emptiness, (i) and (ii). For by (i) \mathbf{D} has no points $m0$ so that it is not $\text{MON}\downarrow$. And by (ii) each row $k+l$ at which \mathbf{D} has a ‘+’ lacks at least one point $\mathbf{d}kl$ with $l > 1$, which blocks $\text{MON}\uparrow$.

Conversely, let \mathbf{D} have the properties named. Define $P := \{k : \exists l.\mathbf{d}lk\}$, and $X := \{n : \mathbf{d}$ has a ‘+’ at row $n\}$. It is clear that $\mathbf{D} \subseteq \langle P \rangle^X$, so assume $\langle P \rangle^X AB$. Then $|A \cap B| \in P$ and $|A| \in X$. So $\exists l.\mathbf{d}l|A \cap B|$ and $|A - B| + |A \cap B| \in X$. By weak intersectivity: $\mathbf{d}|A - B||A \cap B|$, and hence $\mathbf{D}AB$. It remains to show that $1 \in P$, and that P has (i) and (ii). Since \mathbf{D} is non-empty, $1 \in P$ follows from SDYN, as does (i). Finally, \mathbf{D} cannot contain all m -diagonals with $m \geq 1$, for then it would be the monotonic \mathbf{some}^X . So (ii) is true as well. \square

There are of course many determiners $\langle \{1\} \cup P \rangle^X$ which do not figure as s-antecedents; the converse of constraint 1 is definitely false. For instance, *one or five* has this form, and (44) is marked.

(44) One or five boys sold records. *He was rich.

In fact it seems that the cases were $P = \emptyset$ are the only non-monotonic s-antecedents, besides the $\text{MON}\uparrow$ \mathbf{some}^X that comes from $P = \{n : n \geq 1\}$.

Constraint 15 *For each X , only the strongest and the weakest $\langle \{1\} \cup P \rangle^X$ with $0 \notin P$ figure as s-antecedents.*

Here, strength is measured by inclusion: $\langle\{1\}\rangle^X \subseteq \langle\{1\} \cup P\rangle^X \subseteq \mathbf{some}^X$. It is attractive that the constraint couples logical strength to simplicity of linguistic expression. Still, it is descriptive rather than explanatory.

In section 2.6 I consider distributive plural anaphora and their antecedents. But first I define a formal logic to interpret singular Q-type anaphora.

2.5 Q-types and contextualized quantification

In section 2.4.1 I gave an impression of how singular anaphora may be interpreted as contextually restricted quantifiers. Recently, this offspring of the E-type approach has gained appeal, but as yet a formal system comparable with DRT or DPL has not been given. Combining ideas of van den Berg 1996 and van der Does 1996a I propose to remedy this situation by means of the logic QTL.

Definition 3 Let \mathcal{M} be a first-order model. And for A a set of \mathcal{M} -assignments, let $A_x := \{a(x) : a \in A\}$. The notions:

- i) $\llbracket\varphi\rrbracket$: the *context change potential* of a formula φ ,
- ii) $\mathcal{M}, A, a \models \varphi$: the *satisfaction* of φ in \mathcal{M} relative to A and a ,

are defined simultaneously as follows:

- i)

$A(\llbracket R(x_1, \dots, x_n) \rrbracket)$	$:=$	A
$A(\llbracket \neg\varphi \rrbracket)$	$:=$	D^{VAR}
$A(\llbracket \varphi \wedge \psi \rrbracket)$	$:=$	$(A(\llbracket \varphi \rrbracket))(\llbracket \psi \rrbracket)$
$A(\llbracket \exists x\varphi \rrbracket)$	$:=$	$\{a \in A : \mathcal{M}, a \models \varphi\}$
- ii)

$\mathcal{M}, A, a \models R(x_1, \dots, x_n)$	\Leftrightarrow	$\llbracket R \rrbracket(\llbracket x_1 \rrbracket, \dots, \llbracket x_n \rrbracket)$
$\mathcal{M}, A, a \models \neg\varphi$	\Leftrightarrow	$\mathcal{M}, A, a \not\models \varphi$
$\mathcal{M}, A, a \models (\varphi \wedge \psi)$	\Leftrightarrow	$\mathcal{M}, A, a \models \varphi$ and $\mathcal{M}, A(\llbracket \varphi \rrbracket), a \models \psi$
$\mathcal{M}, A, a \models \exists x\varphi$	\Leftrightarrow	There is $d \in A_x : \mathcal{M}, A, a_d^x \models \varphi$

A formal studies of QTL has to be left for another occasion. Here I restrict myself to giving two examples of its workings. First, (45a) becomes (45b), which means (45c) and hence also (45d).

- (45) a. A man walks in the park, and he whistles.
 b. $\exists x[\text{M}x \wedge \text{W}x] \wedge \exists x.\text{W}hx$
 c. $\exists x[\text{M}x \wedge \text{W}x] \wedge \exists x[\text{M}x \wedge \text{W}x \wedge \text{W}hx]$
 d. $\exists x[\text{M}x \wedge \text{W}x \wedge \text{W}hx]$

Second, (46a) becomes (46b), with $\varphi \rightarrow \psi$ defined by $\neg(\varphi \wedge \neg\psi)$. Also, (46b) means (46c) and hence (46d).

- (46) a. If a man walks in the park, he whistles.
 b. $\exists x[\text{M}x \wedge \text{W}x] \rightarrow \forall x.\text{W}hx$
 c. $\exists x[\text{M}x \wedge \text{W}x] \rightarrow \forall x[\text{M}x \wedge \text{W}x \rightarrow \text{W}hx]$
 d. $\forall x[\text{M}x \wedge \text{W}x \rightarrow \text{W}hx]$

This finishes the discussion of singular anaphora. The article closes with some remarks on plural anaphora.

2.6 Plural anaphora and some of their antecedents

2.6.1 Definite anaphora

Definite noun phrases such as ‘they’, ‘the man’, ‘those four records’ are often interpreted by means of an anaphoric relation. In quantification theory—and in the type-shifted view on open expressions in the appendix,—a definite **D**A is interpreted as a principal filter of form $\{Y : A \cap X \subseteq Y\}$ (The context variable X is sometimes disregarded in what follows.) This means that each definite **D** is $\downarrow\text{MON}$. Therefore, each such **D** has $\langle\mathbf{D}\rangle$, since this holds for each $\downarrow\text{MON}$ determiner. Let us have a brief look at some examples.

There are numberless, singular, and plural forms of the quantifier **all**:

- (47) $\mathbf{all}AB \Leftrightarrow A \subseteq B$
 $\mathbf{all}^s AB \Leftrightarrow A \subseteq B \ \& \ |A| = 1$
 $\mathbf{all}^p AB \Leftrightarrow A \subseteq B \ \& \ |A| > 1$

The semantics of ‘the’ is \mathbf{all}^s (singular) or \mathbf{all}^p , (plural), and that of ‘all’ and ‘they’ is \mathbf{all} (numberless) or \mathbf{all}^p (plural). Each of these versions satisfy $\langle \mathbf{all} \rangle$. In fact we have for all Θ : \mathbf{all}^Θ has $\langle \mathbf{all}^\Theta \rangle$, even though these determiners need not be $\downarrow\text{MON}$. This accords with our intuitions. For instance, (48) is valid if ‘the’ and ‘they’ are both numberless, both plural, or ‘the’ plural and ‘they’ numberless:

- (48) The books are boring. They are dusty.
The books are boring and dusty.

In case ‘the’ is numberless and ‘they’ plural, (48) is invalid. For conservative antecedent \mathbf{D} the direction from top to bottom is always valid, but for (48) the converse fails (the bottom sentence need not imply that at least two boring books are dusty). Theorem 9 implies the following characterization for the numberless case.

Proposition 16 *Call a determiner \mathbf{D} maximal iff for all E : $\mathbf{D}_E \subseteq \mathbf{all}_E$. The only determiners with $\langle \mathbf{all} \rangle$ are the maximal $\text{MON}\uparrow$ ones.*

Notice that the maximal $\text{MON}\uparrow$ determiners are precisely those of form \mathbf{all}^Θ . Natural language examples of maximal $\text{MON}\uparrow$ determiners would be: *the four, the five or more, ...* And (49) is indeed valid.

- (49) The four statutes were expensive. They sold quickly.
The four statutes were expensive and sold quickly.

This would also be true on the plural form of ‘they’, for another consequence of theorem 9 is:

Proposition 17 *The only logical \mathbf{D} with $\langle \mathbf{all}^\Theta \rangle$ are those with $\mathbf{D} = \mathbf{all}^{\Theta'}$ for some $\Theta' \subseteq \Theta$.*

Finally I consider numerals, as main examples of antecedents in discourse logic.

2.6.2 Numerals

In quantifier theory indefinite noun phrases, such as the numeric ones, are often identified with intersectives (see the appendix). Call a determiner $\text{INT}\uparrow$ iff it is intersective and $\text{MON}\uparrow$. It is not hard to see that each $\text{INT}\uparrow \mathbf{A}$ has $\langle \mathbf{A} \rangle$. Well-known examples of $\text{INT}\uparrow$ determiners are ‘a’, ‘some’, ‘at least n ’, ‘infinitely many’, besides conjunctions and disjunctions of such determiners. For example, (50) is valid, iff ‘five’ means: at least five.

- (50) Five women went to a bar. Five had a beer.
Five women went to a bar and had a beer.

Theorem 9 shows that the only other antecedent determiners with this property are the parts of ‘five’ with the size of their first argument restricted. In English some of these determiners are realized in partitive constructions like ‘five of the ten’ ($\mathbf{5}^{10}$) or ‘five of the ten or more’ ($\mathbf{5}^{\{\kappa:10 \leq \kappa\}}$), as in (51).

- (51) Five of the ten or more women went to a bar. Five had a beer.
Five of the ten or more women went to a bar and had a beer.

Scope extension for plural anaphora is best studied by prototypical discourses such as (52).

- (52) Four women went to a bar. They had a beer.
Four women went to a bar and had a beer.

Is (52) valid? The answer varies with our view on numerals, which could be adjectival, referential, or quantificational. Cf. van der Does 1993b and 1994b for the issues at stake.

It is clear that (52) is invalid if ‘four’ is the quantifier **four**: it is not of form **all**^o. By contrast, (52) would hold on a referential treatment. For then the semantics of ‘four books appeared’ is:

$$(53) \quad \mathbf{all}^4 B \cap CA \Leftrightarrow B \cap C \subseteq A \ \& \ |B \cap C| = 4$$

with C some context set. This referential use of numerals is close to their adjectival use, where they also put a cardinal restriction on the noun; e.g., (54a) is true iff (54b) is.

- (54) a. Four books appeared.
 b. $\exists X[|X| = 4 \wedge X \subseteq B \wedge X \subseteq A]$

Yet, there is a crucial difference: (54b) would make the equivalence in (52) invalid. This is best seen from a slightly more general stance. Logically speaking any determiner **D** could be treated in an adjectival way (whether or not this is desirable empirically). The recipe in (55) is basic to the above proposal Cf. also Verkuyl 1981, Van Eijck 1985, Kamp and Reyle 1993.

$$(55) \quad \text{adj}_1(\mathbf{D})AB \Leftrightarrow \exists X \subseteq A[\mathbf{D}AX \wedge X \subseteq B]$$

Van Benthem (1986) observes that $\text{adj}_1(\mathbf{D})$ is always $\text{MON}\uparrow$. Indeed: $\text{adj}_1(\mathbf{D}) = \mathbf{D}$ iff **D** is conservative and $\text{MON}\uparrow$. Now, $\text{adj}_1(\mathbf{just\ n}) = \text{adj}_1(\mathbf{at\ least\ n}) = \mathbf{at\ least\ n}$, so we are back to the quantificational use which invalidates (52).

The semantics of numerals in second-order DRT can best be seen as a combination of the referential and the quantificational use. In DRT (54b) results from interpreting the DRS (56) by means of the discourse operator \exists .

$$(56) \quad X[|X| = 4 \wedge X \subseteq B \wedge X \subseteq A]$$

One could think of X as a discourse referent (cf. the contextually restricted noun in (53)). It lives during certain stages in the construction of a DRS, but becomes a bound variable as soon as a discourse operator is applied. In case of the discourse in (52) the DRS (56) is transformed into (57).

$$(57) \quad X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \wedge X \subseteq S]$$

Therefore, (52) should be valid, since (58a) is of course equivalent to (60).

- (58) a. $\exists X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \wedge X \subseteq S]$
 b. $\exists X[|X| = 4 \wedge X \subseteq D \wedge X \subseteq B \cap S]$

A problem for this approach is that does not explain why (59) is invalid.

- (59) Just four books appeared and sold like hot cakes.
Just four books appeared. They sold like hot cakes

In current versions of DRT ‘just four’ is either a quantifier or it is more like an adjective. In the first case it does not allow anaphoric links outside its scope. Such a link is perfectly in order in the conclusion of (59), so the use should be adjectival. But then we come accross the identity named earlier: $\text{adj}_1(\mathbf{just\ n}) = \text{adj}_1(\mathbf{at\ least\ n}) = \mathbf{at\ least\ n}$. The anaphoric link is in place but at the expense of altering the meaning of the premiss to: at least four books appeared and sold like hot cakes.

A proposal which circumvents these problems is in (60). It is a second-order analogue of the solution given for the dynamics of ‘just one’ in (41).

$$(60) \quad \text{adj}_2(\mathbf{D})AB \Leftrightarrow \exists X[\mathbf{D}AX \wedge X = A \cap B]$$

This gives $\text{adj}_2(\mathbf{D}) = \mathbf{D}$ iff **D** is conservative, and hence includes almost all natural language determiners. Following the same route as before we now arrive at the interpreted DRS in (61a).

- (61) a. $\exists X[|X| = 4 \wedge X = B \cap A \wedge X \subseteq S]$
 b. $|A \cap B| = 4 \wedge A \cap B \subseteq S$

Notice that (61a) is equivalent to the Q-type approach in (61b), which invalidates (59). It clearly shows that it is not the quantifier itself whose scope is extended

but rather that of the logically superfluous second-order discourse operator. Indeed, a DRT-like formalization of these anaphora is possible, and can be summarized in the slogan: plural anaphora are bound to be Q-types.

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A Quantifier theory: some basic facts

People reason; sometimes good, sometimes not so good. But what distinguishes a good reasoning from the others? In the early Greek attempts to answer this question, especially by Aristotle, the concept of *logic*, and the concept of a *quantifier* came to the fore. Aristotle saw quantifiers, such as *all*, *some*, *no*, as two place relations among properties. This view on quantification is intertwined with the logic of syllogisms, where it enables a description of the inferences possible, and a classification of the valid and the invalid ones.

Aristotle's concept of quantification turned out to be important for the development of logic and linguistics, among other disciplines. But before it could have such impact, it had to gain precision and generality in the hands of such logicians as Frege 1879, Peirce 1885, and Mostowski 1957. This process of refinement resulted in the definition of Lindström 1966.

Definition 4 A *quantifier type* τ is a sequence $\langle n_1 \dots n_k \rangle$ of natural numbers.

Definition 5 A *quantifier* \mathbf{Q} of type $\langle n_1 \dots n_k \rangle$ is a functor which assigns to each set E a subset \mathbf{Q}_E of $\wp(E^{n_1}) \times \dots \times \wp(E^{n_k})$ which is closed under bijections:¹⁵

$$\text{ISOM } \mathbf{Q}_E R^{n_1} \dots R^{n_k} \Leftrightarrow \mathbf{Q}_{E'} \pi(R^{n_1}) \dots \pi(R^{n_k}).$$

for each bijection $\pi : E \rightarrow E'$. Here,¹⁶

$$\pi(R^n) := \{ \langle \pi(d_1), \dots, \pi(d_n) \rangle : \langle d_1 \dots d_n \rangle \in R^n \}$$

A *local* quantifier is defined similarly by keeping a certain domain E fixed.

This abstract notion on quantification points to a rich field of pure potentiality which until recently awaited realization in mathematical logic and in semantics of natural language. However, as we shall see, not anything goes. Besides inspiration, quantifier theory should at the same time enable us to delimit the scope of otherwise fertile claims. In particular, quantification and anaphora is an area where this double function of quantifier theory can be put to work.

A.1 Monadic quantification

In the last decade it has become clear that natural languages like Dutch, English, and German, employ various forms of quantification (cf. Keenan and Westerstahl 1996 for an up-to-date overview). Still, in these languages the monadic quantifiers of type $\langle 1, 1 \rangle$ are perhaps the most prominent ones, since they figure as determiner denotations. For this reason, Partee et al. 1987 speak of 'D-quantification', which should be compared with 'A-quantification' in section A.2 (cf. also Partee 1995). Logically, the quantifiers corresponds to a kind of Lindström quantifier satisfying certain constraints.

Definition 6 A logical type $\langle 1, 1 \rangle$ quantifier is a functor \mathbf{D} which assigns to each non-empty domain E a two place relation among sets:

$$\mathbf{D}_E \in \wp(\wp(E) \times \wp(E))$$

Logical \mathbf{D} satisfy three constraints:

- Conservativity (CONS): $\mathbf{D}_E AB$ iff $\mathbf{D}_E AA \cap B$;
- Extension (EXT): $\mathbf{D}_E AB$ iff $\mathbf{D}_{E'} AB$, for all $A, B \subseteq E \subseteq E'$;
- Isomorphy (ISOM): for all bijections π from E onto E' :
 $\mathbf{D}_E AB$ iff $\mathbf{D}_{E'} \pi[A] \pi[B]$.

¹⁵Equivalently, a quantifier type τ can be viewed as the signature of a relational model $\mathcal{M} := \langle E, R^{n_1}, \dots, R^{n_k} \rangle$ with k relations of arity n_i ($1 \leq i \leq k$). For such \mathcal{M} , $\mathbf{Q}_E R^{n_1} \dots R^{n_k}$ could also be written as $\mathcal{M} \in \mathbf{Q}$. On this view, a global quantifier is a class of models of signature τ which is closed under isomorphic copies: if $\mathcal{M} \in \mathbf{Q}$ and $\mathcal{M} \cong \mathcal{N}$, then $\mathcal{N} \in \mathbf{Q}$. One may alternate between these concepts without much notice.

¹⁶The sign '=' means: identical by definition to.

Familiar examples of type $\langle 1, 1 \rangle$ D-quantifiers are in (62).

$$(62) \quad \begin{array}{ll} \text{some} & := \{ \langle E, X, Y \rangle : X \cap Y \neq \emptyset \} \\ \text{all} & := \{ \langle E, X, Y \rangle : X \subseteq Y \} \\ \text{no} & := \{ \langle E, X, Y \rangle : X \cap Y = \emptyset \} \\ \text{not all} & := \{ \langle E, X, Y \rangle : X \cap \bar{Y} \neq \emptyset \} \\ \text{at most } n & := \{ \langle E, X, Y \rangle : |X \cap Y| \leq n \} \\ \text{n} & := \{ \langle E, X, Y \rangle : |X \cap Y| = n \} \\ \text{at least } n & := \{ \langle E, X, Y \rangle : |X \cap Y| \geq n \} \\ \text{most} & := \{ \langle E, X, Y \rangle : |X \cap Y| > |X \cap \bar{Y}| \} \end{array}$$

Fact 18 *Conservativity and extension is equivalent to universality:*

$$\text{UNIV } \mathbf{D}_E AB \text{ iff } \mathbf{D}_A AA \cap B \quad \square$$

The combination of conservativity and extension makes the first argument of a quantifier set the stage. Indeed, we may forget about the domain E by putting: $\mathbf{D}AB$ iff for some E $\mathbf{D}_E AB$. Further, each UNIV type $\langle 1, 1 \rangle$ \mathbf{D} can be seen as the restriction of a type $\langle 1 \rangle$ \mathbf{D}' as follows.

Definition 7 Let \mathbf{D}' be a type $\langle 1 \rangle$ quantifier. Its *restriction* \mathbf{D}'^r of type $\langle 1, 1 \rangle$ is defined by:

$$\mathbf{D}'^r_E AB \text{ iff } \mathbf{D}'_A A \cap B$$

Fact 19 (Westerståhl 1984a) *A type $\langle 1, 1 \rangle$ determiner is UNIV iff there is type $\langle 1 \rangle$ \mathbf{D}' such that $(\mathbf{D}')^r = \mathbf{D}$.* \square

That quantifiers are closed under isomorphisms captures the idea that they are insensitive to particular individuals. An equivalent way to formalize this idea uses the notion of *strong equality* between sets. Relative to a domain E , the sets X and Y are strongly equal, $X =_s Y$, iff $|X| = |Y|$ and $|E - X| = |E - Y|$. Only on finite domains strong equality is identical to equicardinality. For example, the natural numbers have the same cardinality as the even numbers, but relative to the natural numbers these sets are not strongly equal. The quantifiers on a domain E can be characterized as the sets of sets which do not discern between strongly equal sets. For type $\langle 1 \rangle$ quantifiers, proposition 20 generalizes this characterization across universes.

Proposition 20 *A functor \mathbf{Q} such that for all E $\mathbf{Q}_E \subseteq \wp(E)$ is a quantifier iff for all E, E' and all $X \subseteq E, Y \subseteq E'$ with $|X| = |Y|$ and $|E - X| = |E' - Y|$:*

$$\mathbf{Q}_E(X) \Leftrightarrow \mathbf{Q}_{E'}(Y)$$

PROOF. [\Rightarrow :] Suppose \mathbf{Q} is a quantifier and $|X| = |Y|$ and $|E - X| = |E' - Y|$. Then the injections given by the identities can be joined to give a bijection $\pi : E \rightarrow E'$. Since \mathbf{Q} is a quantifier and $\pi(X) = Y$, we have: $\mathbf{Q}_E(X) \Leftrightarrow \mathbf{Q}_{E'}(Y)$. [\Leftarrow :] This is immediate, since for each bijection $\pi : E \rightarrow E'$ we have $|X| = |\pi(X)|$ and $|E - X| = |\pi(E - X)| = |E' - \pi(X)|$ for all $X \subseteq E$. \square

Proposition 20 says that the truth of a statement $\mathbf{Q}_E(X)$ depends entirely on the cardinals $|E - X|$ and $|X|$, not just on $|X|$. Indeed, Mostowski 1957 notes that a quantifier \mathbf{Q}_E can be represented as a relation \mathbf{q} between cardinals μ, κ as follows:

$$\mathbf{q}(\mu, \kappa) \text{ iff } \exists X, E[\mathbf{Q}_E(X) \ \& \ \mu = |E - X| \ \& \ \kappa = |X|]$$

This, in combination with the fact that UNIV type $\langle 1, 1 \rangle$ \mathbf{D} can be written as $(\downarrow \mathbf{D})_A(A \cap B)$, with $\downarrow \mathbf{D}$ type $\langle 1 \rangle$, gives for logical (and hence UNIV) \mathbf{D} the relation \mathbf{d} among cardinals $|A - B|$ and $|A \cap B|$:

$$\mathbf{d}|A - B||A \cap B| \text{ iff } \mathbf{D}AB.$$

On the further assumption of FIN, quantifiers can be represented in the tree of numbers.

FIN Only finite models are considered.¹⁷

The tree representation is used in van Benthem 1984 to introduce an attractive geometrical style of reasoning (examples will be given shortly). Figure 1 has the general format of the tree. If (i, j) is a number pair in the tree for Q , then $i + j$ is the

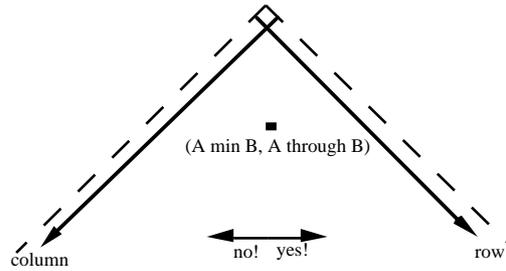
$ A = 0$								$0, 0$		
$ A = 1$							$1, 0$	$0, 1$		
$ A = 2$						$2, 0$	$1, 1$	$0, 2$		
$ A = 3$			$3, 0$		$2, 1$		$1, 2$	$0, 3$		
$ A = 4$		$4, 0$		$3, 1$		$2, 2$		$1, 3$	$0, 4$	
$ A = 5$	$5, 0$		$4, 1$		$3, 2$		$2, 3$		$1, 4$	$0, 5$
$ A = \dots$							\vdots			

FIGURE 1 General Format of a Numerical Tree

cardinality of the universe A , i is the cardinality of $A - B$, and j is the cardinality of $A \cap B$. A sequence (r, i) for a certain $r \in \mathbb{N}$ is called a ‘row’, and a sequence (j, c) for a certain $c \in \mathbb{N}$ is called a ‘column’.

Fact 21 offers a more schematic representation of the tree of numbers.

Fact 21 (FIN) *A logical quantifier \mathbf{D} can be identified with a subset \mathbf{d} in the tree of numbers:*



Van Benthem's Tree

□

The reader may wish to try for himself what the tree looks like for such quantifiers as *all*, *n*, *most*. Some of the trees will be given in the course of this article.

A.2 Case quantification

In the above examples quantification is essentially over objects. But it has been claimed that quantification over *cases*, i.e., tuples of objects, also occurs in the semantics of English. In particular, Lewis 1975 employs case quantification in his semantics for adverbs of quantification; hence, the term ‘A-quantification’ in Partee et al. 1987. DRT refines Lewis’ proposal to deal with anaphoric dependencies. Here, case quantifiers are used as so-called discourse operators.

There is a systematic way to turn object quantification into quantification over n -ary cases.¹⁸ For now it is sufficient to restrict ourselves to quantification of type $\langle n, n \rangle$.¹⁹

Definition 8 For each type $\langle 1, 1 \rangle$ quantifier \mathbf{Q} , the type $\langle n, n \rangle$ case quantifier $\text{CQ}^n(\mathbf{Q})$ is defined by

$$\text{CQ}^n(\mathbf{Q})_E(R_1, R_2) \Leftrightarrow \mathbf{Q}_{E^n}(R_1, R_2)$$

¹⁷In the studies of natural language quantification, FIN is often assumed. It makes sense for a considerable part of count quantification, but perhaps less so for mass quantification.

¹⁸Sometimes case quantification is termed ‘resumptive quantification’. But its different applications call for a more neutral name.

¹⁹See van der Does 1996b, p. 12, for a more general definition.

with E^n the n -ary Cartesian product of E and $R_i \subseteq E^n$.

Recall from definition (5) that \mathbf{Q}_{E^2} is a subset of $\wp(E^2) \times \wp(E^2)$. Therefore, $\text{CQ}^n(\mathbf{Q})$ combined with $2n$ place relations gives a truth value; $\text{CQ}^n(\mathbf{Q})$ is a quantifier of type $\langle n, n \rangle$.

A.3 (In)definites

In discourses, (in)definite noun phrases play a prominent rôle. Indefinites, such as *some man, two children*, are prime examples of antecedents, and definites such as *they, the man, those four records* of anaphora. There is an ongoing debate as to their formal semantical status. DRT treats (in)definites as open expressions, which are bound by means of polyadic operators. In quantification theory, the (in)definites are special kinds of quantifier. Keenan 1987 proposes to identify indefinites with the intersective determiners.²⁰

Definition 9 There are three equivalent ways to define the type $\langle 1, 1 \rangle$ intersectives. A quantifier is *intersective* iff it is invariant under identical intersections:

$$\text{INT } A \cap B = A' \cap B' \text{ implies: } \mathbf{D}AB \text{ iff } \mathbf{D}A'B'$$

iff it is conservative and co-conservative

$$\text{COCO } \mathbf{D}AB \text{ iff } \mathbf{D}A \cap BB$$

iff it is conservative and *symmetric*:

$$\text{SYM } \mathbf{D}AB \text{ iff } \mathbf{D}BA$$

The first two generalize to arbitrary monadic quantifiers (type $\langle 1, \dots, 1 \rangle$).

In Barwise and Cooper 1981 *definites* $\mathbf{D}A$ are identified with principal filters of form $\{Y : A \cap X \subseteq Y\}$ (the context set X is sometimes disregarded in what follows). Observe that in Partee's triangle (in)definites as open expression can also be seen as filters by setting $\{X : gx \in X\}$ (Partee 1987). The variable generating the filter is quantified by local context. Of course, on this approach the characterization of Barwise and Cooper can no longer be sustained.

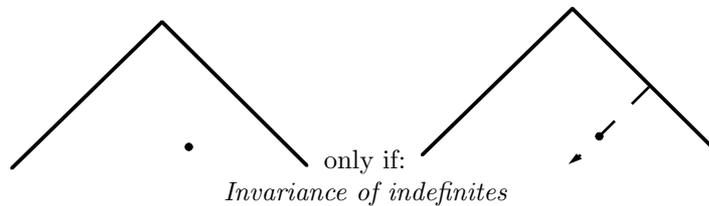
Since intersectives only depend on $A \cap B$, their general form is $\langle \Theta \rangle AB$, with Θ a class of cardinals and: $\langle \Theta \rangle AB$ iff $|A \cap B| \in \Theta$. Under FIN, this reduces to $\langle P \rangle$ with P the set of natural numbers $\Theta \cap \mathbb{N}$. For example, a singleton $\{n\}$ gives *just n* , and *at least m* corresponds to $\langle \{\kappa : m \leq \kappa\} \rangle$.

In a sense, definites are 'restricted' indefinites. That is, given Θ the definite $[\Theta]$ can be defined by: $[\Theta]AB$ iff: $|A \cap B| \in \Theta$ and $|A - B| = 0$. Alternatively:

$$[\Theta]A := \{B : A \subseteq B \wedge |A| \in \Theta\}$$

Thus we arrive at the definites as principal filters. For example, $[\{n\}]A$ is *the n A*, and $[\{\kappa : m \leq \kappa\}]A$ is *the m or more A*, and so forth...

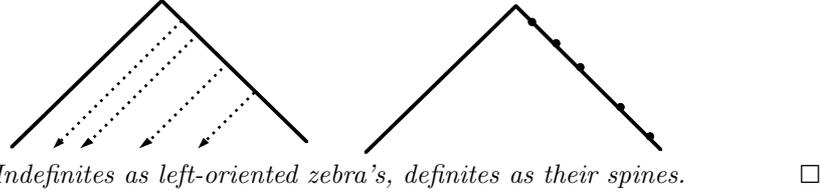
Viewed as subsets of Van Benthem's tree, indefinites are extremely well-behaved.



Given the above invariance, it is clear that the following theorem holds:

Theorem 22 *In Van Benthem's tree, the intersectives are just the left-oriented zebra's, and definites are their spines.*

²⁰This improves on the proposal in Barwise and Cooper 1981 in terms of weakness; i.e., neither reflexive nor irreflexive.



A.4 First-order (in)definites

It is clear from the general forms $\langle \Theta \rangle$ and $[\Theta]$ that there are many more (in)definites than are realized in natural language. For natural language quantifiers, the Θ should be ‘regular’. I shall not speculate on this issue here, but rather show that at least the first-order definable (in)tersectives are non-chaotic,

Definition 10 A type $\langle 1, 1 \rangle$ quantifier \mathbf{D} is first-order definable iff there is a first-order formula $\varphi(P_1, P_2)$ in the language with two one place predicates P_i , so that \mathbf{D} , viewed as a class of models, is of form:

$$\mathbf{D} := \{ \langle E, A, B \rangle : \langle E, A, B \rangle \models \varphi(P_1, P_2) \}$$

A quick cardinality argument shows that some intersective quantifiers are non-first-order definable, since there are only countably many of those. But it is possible to give a more informative characterization of the first order definable intersectives.²¹

Theorem 23 An intersective quantifier $\langle \Theta \rangle$ is first-order definable iff:

$$(*) \quad \exists n \in \mathbb{N} \cap \Theta \forall \kappa > n (\kappa \in \Theta) \text{ or } \exists n \in \mathbb{N} - \Theta \forall \kappa > n (\kappa \notin \Theta)$$

PROOF. From right to left, assume $\exists n \in \mathbb{N} \cap \Theta \forall \kappa > n (\kappa \in \Theta)$ (the other possibility is similar). Let n be minimal with the property $\forall \kappa \geq n (\kappa \in \Theta)$, and let $\{n_1, \dots, n_k\}$ be the remaining k elements in Θ smaller than n , if any. Then $\langle \Theta \rangle$ is defined by

$$\bigvee_{1 \leq i \leq k} \mathbf{exactly} \mathbf{n}_i(P_1, P_2) \vee \mathbf{at\ least} \mathbf{n}(P_1, P_2)$$

Conversely, we use compactness and Löwenheim-Skolem to prove that $\langle \Theta \rangle$ is non-first-order if $(*)$ is false. Assume

$$(**) \quad \forall n \in \mathbb{N} \cap \Theta \exists \kappa > n (\kappa \notin \Theta) \text{ and } \forall n \in \mathbb{N} - \Theta \exists \kappa > n (\kappa \in \Theta)$$

and suppose for a contradiction that $\langle \Theta \rangle$ is first-order definable, say, by $\varphi(P_1, P_2)$. I discern two cases.

Case (i): Θ contains all infinite cardinals. Consider

$$\Gamma := \{ \neg \varphi(P_1, P_2) \} \cup \{ \mathbf{at\ least} \mathbf{k}(P_1, P_2) : k \in \mathbb{N} \}$$

and let Δ be a finite part of this theory. For k maximal so that $\mathbf{at\ least} \mathbf{k}(P_1, P_2)$ is in Δ , choose $n \geq k$ smallest in Θ (whether or not $k \in \Theta$, n exists by assumption). Then $\langle D, A, B \rangle$ with $|A \cap B| = n$ satisfies Δ . By compactness Γ has a model $\mathcal{M} = \langle E, A, B \rangle$. In \mathcal{M} , $|A \cap B|$ is infinite, and $|A \cap B| \notin \Theta$. But this contradicts the fact that $|A \cap B| \notin \Theta$ implies that $A \cap B$ is finite.

Case (ii) There is an infinite $\kappa \notin \Theta$. Now consider

$$\Gamma^* := \{ \varphi(P_1, P_2) \} \cup \{ \forall x (P_1 x \wedge P_2 x) \} \cup \{ \mathbf{at\ least} \mathbf{k}(P_1, P_2) : k \in \mathbb{N} \}$$

By assumption $(**)$, Θ is nonempty. So, similar to the previous case we see that Γ is finitely satisfiable, and hence has an infinite model \mathcal{M} . Up- and downward Löwenheim-Skolem allow us to choose \mathcal{M} so that $|E| = \kappa$. But

$$\mathcal{M} \models \varphi(P_1, P_2) \wedge \forall x (P_1 x \wedge P_2 x)$$

implies $|E| \in \Theta$; contradicting $\kappa \notin \Theta$. □

In other words, the first-order definable intersectives are precisely the left-oriented zebra's with a finite number of stripes and with a tail which is either entirely

²¹These results also follow from more general ones in van Benthem 1984 and Westerståhl 1984b, but they are obtained directly with almost the same ease.

white or entirely black. These are the most common ones. Intersectives such as *an even number of*, *finitely many*, *at least* \aleph_α , which are non-first-order according to theorem 23, or not of this kind.

Corollary 24 *We have the following definability results;*

- i) \mathbf{D} is a first-order definable $\text{MON}\uparrow$ intersective quantifier iff \mathbf{D} is a first-order definable $\uparrow\text{MON}$ intersective quantifier iff \mathbf{D} is **at least** \mathbf{n} for some $n \in \mathbb{N}$.
- ii) \mathbf{D} is a first-order definable $\text{MON}\downarrow$ intersective quantifier iff \mathbf{D} is a first-order definable $\downarrow\text{MON}$ intersective quantifier iff \mathbf{D} is **at most** \mathbf{n} for some $n \in \mathbb{N}$.

PROOF. As to (i), let \mathbf{D} be $\text{MON}\uparrow$, intersective, and first-order definable. By SYM: \mathbf{D} is $\text{MON}\uparrow$ iff \mathbf{D} is $\uparrow\text{MON}$. Further, we may assume that $\mathbf{D} \neq \emptyset$. Then, (*) in theorem 23 implies that $\mathbf{D} = \langle \Theta \rangle$ with $\mathbb{N} \cap \Theta \neq \emptyset$ (for otherwise $\Theta = \emptyset$ and hence also \mathbf{D}). Let n be the smallest natural number in Θ . The invariance of $\text{MON}\uparrow$ says that if a point $\langle n, m \rangle$ is in \mathbf{D} , as a relation between cardinals, then so are all the points to its right. In combination with the invariance of intersectives this shows that $\mathbf{D} = \mathbf{at\ least\ } \mathbf{n}$. The other direction is clear.

Next, (ii) follows from (i): \mathbf{D} is a first-order definable $\text{MON}\downarrow$ intersective quantifier, iff $\neg\mathbf{D}$ is a first-order definable $\text{MON}\uparrow$ intersective quantifier, iff $\neg\mathbf{D}$ is **at least** \mathbf{n} for some $n \in \mathbb{N}$, iff \mathbf{D} is **at most** \mathbf{m} for some $m \in \mathbb{N}$. \square

The proof of corollary 25 gives a means to transfer part of corollary 24 to definites (but I leave the details to the reader).

Corollary 25 *A definite quantifier $[\Theta]$ is first-order definable iff:*

$$(*) \exists n \in \mathbb{N} \cap \Theta \forall \kappa > n (\kappa \in \Theta) \text{ or } \exists n \in \mathbb{N} - \Theta \forall \kappa > n (\kappa \notin \Theta)$$

PROOF. We have already noted that $[\Theta](A, B)$ is definable as $\langle \Theta \rangle(A, B) \wedge A - B = \emptyset$. So $[\Theta]$ is first-order definable if $\langle \Theta \rangle$ is. The remaining argument is as before. \square

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