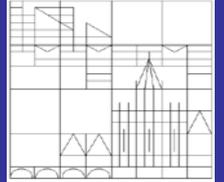




University of Konstanz  
Department of Economics



# Routinization and the Decline of the U.S. Minimum Wage

*Finn Martensen*

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# Routinisation and the Decline of the U.S. Minimum Wage

Finn Martensen\*  
University of Konstanz

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## Abstract

The U.S. minimum wage declined in real terms since the late 1970s. In the same time, the wage of the least skilled workers fell in real terms, while the wage of the highest skilled workers increased. To shed light on these issues, I use a simple model of routinisation. High-ability workers, after having received additional education, can substitute low-ability co-workers by machines. Technical progress results in more high-ability workers receiving additional education and in a declining wage for low-ability workers. A government opposes both unemployment and wage inequality. I calibrate the model and show that technical progress induces the government to lower the minimum wage. Hence, the model contributes to understand the decline in the U.S. minimum wage.

*Keywords:* Minimum wage, Routinisation, Education, Wage inequality, Unemployment

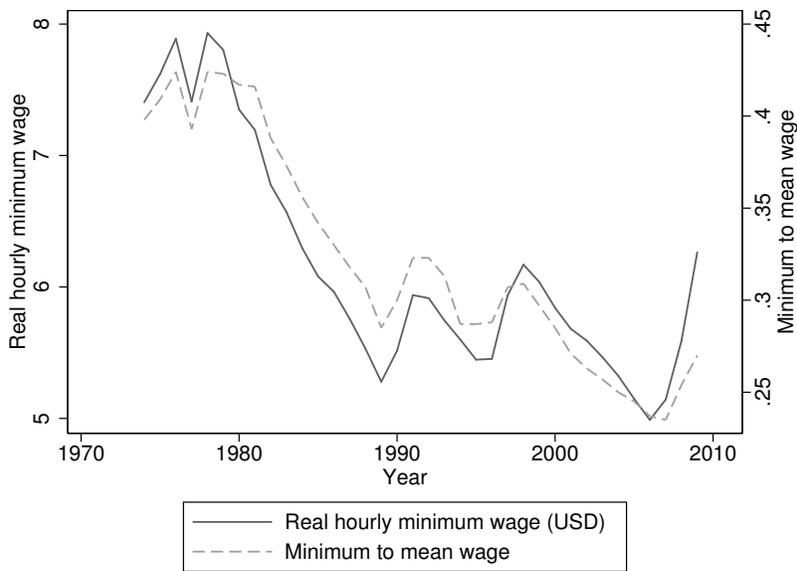
*JEL-Classification:* E24, I24, J31, J88

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\*E-Mail: [finn.martensen@uni-konstanz.de](mailto:finn.martensen@uni-konstanz.de). I would like to thank Wolf-Heimo Grieben, Volker Hahn, Matthias Hertweck, Leo Kaas, Georgi Kocharkov, Christian Manger, Almuth Scholl, Karsten Wasiluk and seminar participants at the University of Konstanz for helpful comments. All remaining errors are mine.

# 1 Introduction

Since the late 1970s, the U.S. legal minimum wage has decreased until the late 2000s in real terms, in particular during the 1980s and since the late 1990s for about a decade. It has also dropped in comparison to the mean wage of full-time workers, as shown in Figure 1. Comparing the minimum wage to the mean wage, however, hides that real wages have been diverging for workers with different education levels, as shown in Figure 2. In particular, the real wage of the lowest skilled workers, who did not finish high school, has decreased since around 1980. The mean wage of high school graduates, who mainly have vocational training, has decreased first and increased afterwards again, but remained relatively constant. The mean real wage of the highest skilled workers, who have at least a college degree, has increased.

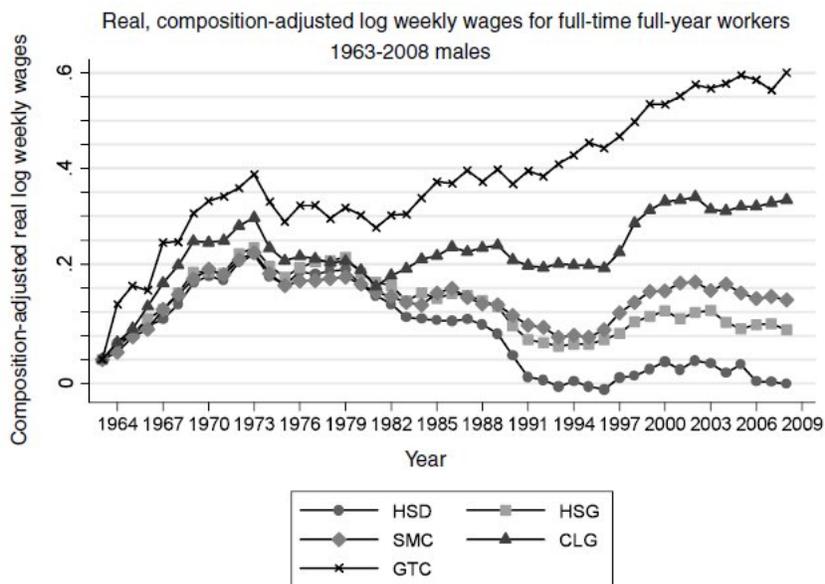


*Notes:* Real hourly minimum wage adjusted by CPI. Mean wage refers to full-time workers.  
*Source:* [OECD \(2011b\)](#).

**Figure 1.** Real minimum wage and minimum to mean wage in the U.S.

What is the link between the minimum wage and the diverging real wages of workers with different education levels – that is, how do they affect each other? Given that the college premium rises, how does the decline in the minimum wage affect the education decision? And how can we explain the decrease in the

real minimum wage?



Notes: HSD: High school dropouts; HSG: High school graduates; SMC: Some college; CLG: College graduates; GTC: Greater than college.  
Source: [Acemoglu and Autor \(2011\)](#).

**Figure 2.** Real wages by education level in the U.S.

The dominant explanation for changes in wage inequality is a nuanced version of skill-biased technical change, the so-called routinisation hypothesis ([Autor et al., 2003](#); [Spitz-Oener, 2006](#); [Goos and Manning, 2007](#); [Autor et al., 2008](#); [Reenen, 2011](#); [Goos et al., 2014](#)). It assumes that a production process consists of several tasks. Some of these tasks, referred to as routine tasks, are easy to code into a computer program and can be done by both workers and machines. As machines become better and cheaper, they replace workers. In other words, labour demand for routine tasks decreases, as capital substitutes workers doing routine tasks and complements workers doing non-routine tasks. Routine tasks are mostly done by low-skilled workers, while non-routine tasks are mostly done by high-skilled workers.

The increased computerisation leads to a polarisation of the labour market, where the demand for medium-skilled workers decreases relative to the demand of low- and high-skilled workers. [Goos et al. \(2009\)](#) show that job polarisation occurred also in most European countries between 1993-2006 and is mostly due

to routinisation. [Firpo et al. \(2011\)](#) show that technical change and deunionisation affected wage inequality in the 1980s and 1990s, and offshoring became important from the 1990s on. [Engelmann \(2014\)](#) presents U.K. evidence for effects of technical change on wage inequality. [Akcomak et al. \(2013\)](#) and [Goos et al. \(2014\)](#) argue that technical change is much more important than offshoring in explaining polarisation. They also support the view that routinisation is best modelled as capital-augmenting technical progress.

I use a variant of a model proposed by [Acemoglu \(2002\)](#) as a simple model of routinisation. I modify this model to have three types of skills and endogenous education. High-ability workers jointly work either with low-skilled workers or with capital after having received additional education, which means that they can substitute their low-ability co-workers by machines.

The different skill levels are a stylised version of the education groups that we observed in [Figure 2](#) of real wages. As such, the model includes a high school premium and a college premium. The model can account for diverging real wages of high, medium, and low skilled workers. It can also account for an increasing share of high-skilled workers and a rising skill premium. To do so, it does not require a task-based model, as suggested by [Acemoglu and Autor \(2011\)](#).

I add a minimum wage and show how it modifies the effect of technical change on education and on wage inequality. On top of that, I endogenise the minimum wage by a simple objective function of the government. The government opposes unemployment and wage inequality. I calibrate the model to U.S. data. Technical change suggests falling minimum wages.

For the U.S., there is some evidence that labour unions support and small enterprises oppose minimum wages ([Silberman and Durden, 1976](#); [Cox and Oaxaca, 1982](#); [Sobel, 1999](#)), while other authors emphasise that ideology is actually much more important ([Poole and Rosenthal, 1991](#); [Bartels, 2006](#)), and that the minimum wage is rarely decisive for voting behaviour ([Waltman, 2000](#)). Also, the decrease in unionisation and the decline of the minimum wage do not necessarily imply that unions lobby for minimum wages. A decline in unionisation is also correlated with technical change ([Dinlersoz and Greenwood, 2013](#)), which is the most important determinant of increasing wage inequality, as mentioned above.<sup>1</sup> Hence, instead of taking a political economy approach, I assume that

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<sup>1</sup>To explain cross-country differences, [Aghion et al. \(2011\)](#) focus on cultural differences and [Boeri \(2012\)](#) on different setting regimes, that is legislated or bargained minimum wages.

the government uses the minimum wage to reduce wage inequality.<sup>2</sup>

To formalise the decomposition of a production process into tasks that can be allocated to different production factors, [Acemoglu and Autor \(2011\)](#) set up a simple task-based model with three skill levels.<sup>3</sup> Yet, all production factors are  $q$ -complements in their model, which means that an increase in the productivity of one factor also increases the productivity of all other factors. Hence, technical progress in their model can not explain the declining real wage of low-skilled workers.

Interacting technical change with minimum wages, [Bárány \(forthcoming\)](#) also focuses on the interaction between the minimum wage, wage inequality, and education. Her interest is more on the effect of the minimum wage on other wages in a general equilibrium model with endogenous technical change. She has only two skill levels and focuses on skill-biased technical change, while I have three skill levels and routine-biased technical change.

## 2 The Model

I use a partial equilibrium model of the labour market similar to [Acemoglu \(2002, p. 48ff.\)](#) with some modifications. Using a partial equilibrium model allows to focus on the effect of changes in the production technology on labour demand.<sup>4</sup> There is a single good that can be produced by two different production technologies. I call them the traditional ( $T$ ) and the modern ( $M$ ) technology. There are two groups of workers, the low-ability ( $L$ ) and the high-ability ( $H$ ) workers. The total mass of workers is  $H + L = 1$ . Workers of both abilities can work with the traditional technology. The high-ability workers can

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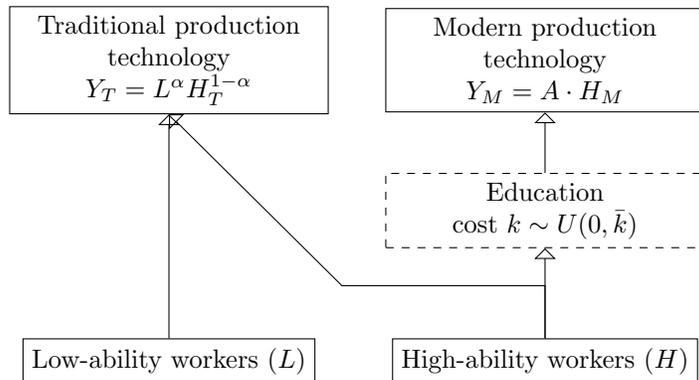
<sup>2</sup>On the normative side, the role of the minimum wage as a redistributive tool has been investigated by the optimal taxation literature. [Allen \(1987\)](#) argues that a minimum wage is desirable under linear income taxation, but never under non-linear income taxation. [Marceau and Boadway \(1994\)](#) show that a minimum wage, combined with unemployment insurance, can be welfare-improving, although a minimum wage increases unemployment. [Boadway and Cuff \(2001\)](#) show that a minimum wage can also be employment-enhancing. [Hungerbühler and Lehmann \(2009\)](#) show that, in a model with search frictions and non-linear income taxes, a minimum wage is desirable if the workers' bargaining power is too low. Even without taxation, a minimum wage is socially desirable if the government values redistribution from high- to low-skilled workers, and this result maintains to hold under non-linear income taxes ([Lee and Saez, 2012](#)). While they argue that it is welfare-improving to reduce the before-tax minimum wage and increase tax transfers, such that the after-tax minimum wage remains constant, my results point to a different aspect, such that the higher pressure on low-skilled labour markets led to a decrease in minimum wages.

<sup>3</sup>See [Autor \(2013\)](#) for a shorter presentation of the task-based model.

<sup>4</sup>A partial equilibrium analysis is also in line with, e.g., [Acemoglu et al. \(2001\)](#) and [Acemoglu and Autor \(2011\)](#).

choose to undertake costly education, which allows them to work with the modern technology. Low-ability workers can not work with the modern technology.<sup>5</sup> The model is similar to [Acemoglu \(2002, p. 48ff.\)](#), but I add education costs to have wage inequality between educated and uneducated high-ability workers. It will turn out that education costs make the effect of technical change on wage inequality ambiguous.

The cost of education  $k$  is distributed uniformly between 0 and  $\bar{k}$  among high-ability workers. Low-ability workers and the uneducated high-ability workers work together with the traditional technology with a Cobb-Douglas production function,  $Y_T = L^\alpha H_T^{1-\alpha}$ .<sup>6</sup> The production function for the modern technology is  $Y_M = A \cdot H_M$ , where  $A$  is an exogenous technology parameter. I assume a constant marginal productivity of educated high-ability workers for tractability. Total production is  $Y = Y_T + Y_M$ . [Figure 3](#) depicts the basic structure of the model.



**Figure 3.** Basic structure of the model.

The interpretation of the production technologies is that low-ability workers perform routine cognitive or manual tasks, while high-ability workers perform rather abstract cognitive or manual tasks. The model reflects the main findings by [Spitz-Oener \(2006\)](#): Occupations today require more complex skills, require education upgrading, and are much more computerised. By assuming two different production technologies, I omit the main weakness of a single pro-

<sup>5</sup>The real world interpretation of this might be that high-ability workers have access to academic education, while low-ability workers have no access to academic education, as their degree of schooling is too low.

<sup>6</sup>That means they are  $p$ -substitutes (demand increases with the *relative price* of the other factor) and  $q$ -complements (demand and relative wage increases with the *supply* of the other factor) ([Sato and Koizumi, 1973](#)).

duction technology, in which all production factors are  $q$ -complements as in, e.g., [Teulings \(2003\)](#) and [Acemoglu and Autor \(2011\)](#), such that an increase in the productivity of one factor increases the marginal productivity of all other factors.

## 2.1 Laissez-faire Equilibrium

Before I introduce a minimum wage, I analyse a laissez-faire equilibrium without a minimum wage. This will make clear how the minimum wage affects the model's predictions.

### 2.1.1 Equilibrium Wages

Labor markets are competitive, so the wage of high-ability workers working with the traditional technology is

$$w_T^H = (1 - \alpha) \left( \frac{L}{H_T} \right)^\alpha, \quad (1)$$

while the wage of low-ability workers is

$$w_T^L = \alpha \left( \frac{H_T}{L} \right)^{1-\alpha}. \quad (2)$$

Both wages depend on the ratio of uneducated high-ability to low-ability workers. If the ratio is higher, there are more high-ability co-workers per low-ability worker, such that their wage is higher, and less low-ability co-workers per high-ability worker, such that their wage is lower.

For educated high-ability workers, who work with the modern technology, the wage is

$$w_M = A. \quad (3)$$

It depends only on technology  $A$ .

### 2.1.2 Education Decision

High-ability workers choose education if the higher wage that they earn afterwards, net of education costs, is larger than the wage that they earn from

working with the traditional technology, i.e. if

$$A - k > (1 - \alpha) \left( \frac{L}{H_T} \right)^\alpha. \quad (4)$$

In the laissez-faire equilibrium, the threshold value of education costs,  $\tilde{k}_c$ , determines the worker who is just indifferent between education and staying uneducated. It follows that the share of high-ability workers who choose education is  $\frac{\tilde{k}_c}{\bar{k}}$ , while a fraction of  $1 - \frac{\tilde{k}_c}{\bar{k}}$  remains uneducated, as  $k$  is uniformly distributed between 0 and  $\bar{k}$ . So,  $H_T = H \left( 1 - \frac{\tilde{k}_c}{\bar{k}} \right)$ . Hence,  $\tilde{k}_c$  is given by the no-arbitrage equation,

$$A = \underbrace{\tilde{k}_c + (1 - \alpha) \left( \frac{L}{H \left( 1 - \frac{\tilde{k}_c}{\bar{k}} \right)} \right)^\alpha}_{\equiv \Phi}. \quad (5)$$

As  $\Phi$  increases monotonously with  $\tilde{k}_c < \bar{k}$ , and as  $\lim_{\tilde{k}_c \rightarrow \bar{k}} \Phi = \infty$ , a unique solution exists with  $0 < \tilde{k}_c < \bar{k}$  for any value of  $A$  under

**Assumption 1**

$$\lim_{\tilde{k}_c \rightarrow 0} \Phi < A \Rightarrow A > (1 - \alpha) \left( \frac{L}{H} \right)^\alpha.$$

### 2.1.3 Effect of Skill-Biased Technical Change

If  $A$  increases, the wage of educated high-ability workers,  $w_M$ , increases unambiguously. In a nutshell, more high-ability workers will choose education because the cost of education now also pays for more high-ability workers. This reduces  $\frac{H_T}{L}$  and hence the marginal productivity of low-ability workers and hence their wage  $w_T^L$ , but it increases the marginal productivity of high-ability workers who work with them and hence their wage  $w_T^H$ .

The share of educated high-ability workers  $\frac{\tilde{k}_c}{\bar{k}}$  increases with  $A$ , shown by applying the implicit function theorem to (5):

$$\frac{d\tilde{k}_c}{dA} \frac{1}{\bar{k}} = \frac{1}{1 + \alpha(1 - \alpha) \left( \frac{L}{H} \right)^\alpha \frac{1}{\bar{k}} \left( \frac{1}{1 - \frac{\tilde{k}_c}{\bar{k}}} \right)^{1+\alpha}} \frac{1}{\bar{k}} > 0. \quad (6)$$

What happens to wage inequality? The wage of uneducated high-ability workers

relative to low-ability workers is

$$\frac{w_T^H}{w_T^L} = \frac{1-\alpha}{\alpha} \frac{L}{H_T} = \frac{1-\alpha}{\alpha} \frac{L}{\left(1 - \frac{\bar{k}_c}{\bar{k}}\right) H}. \quad (7)$$

As is clear from the discussion above, the relative wage increases as  $A$  increases, because the real wage of high-ability workers increases, while the real wage of low-ability workers decreases.

Comparing the wages of educated to uneducated high-ability workers, we have

$$\frac{w_M}{w_T^H} = \frac{A}{(1-\alpha) \left( \frac{L}{\left(1 - \frac{\bar{k}_c}{\bar{k}}\right) H} \right)^\alpha}. \quad (8)$$

Wage inequality increases as long as

$$\frac{\bar{k}}{1+\alpha} > \tilde{k}_c, \quad (9)$$

and it decreases otherwise.<sup>7</sup> As  $\tilde{k}_c$  increases with  $A$ , top/median-wage inequality increases with  $A$  for low levels of technology  $A$ , while it decreases for high levels of  $A$ .

Finally, the relative wage of educated high-ability workers compared to low-ability workers is

$$\frac{w_M}{w_T^L} = \frac{A}{\alpha \left( \frac{\left(1 - \frac{\bar{k}_c}{\bar{k}}\right) H}{L} \right)^{1-\alpha}}, \quad (10)$$

which increases with  $A$ , as  $w_M$  increases and  $w_T^L$  decreases. We can summarise the results in

**Proposition 1** *An increase in  $A$*

- *increases the share of educated high-ability workers,  $H_M$ ,*
- *increases the wage educated high-ability workers,  $w_M$ ,*
- *increases the wage of uneducated high-ability workers,  $w_T^H$ ,*
- *decreases the wage of low-ability workers,  $w_T^L$ ,*
- *increases top-bottom wage inequality,  $\frac{w_M}{w_T^L}$*

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<sup>7</sup>See Appendix A.1 for a proof.

- increases median-bottom wage inequality,  $\frac{w_T^H}{w_T^L}$
- increases top-median wage inequality,  $\frac{w_M}{w_T^H}$ , for  $A < \tilde{A}$ , where  $\tilde{A}$  is implicitly defined as  $\frac{\tilde{k}}{1+\alpha} = \tilde{k}_c(\tilde{A})$ , and decreases otherwise.

#### 2.1.4 Effect of Workforce Composition

If  $h = \frac{H}{L}$  increases, the threshold value of education costs increases, which implies less high-ability workers in the traditional sector. Implicitly differentiating equation (5), I obtain

$$\frac{d\tilde{k}_c}{dh} = -\frac{\partial\Phi/\partial h}{\partial\Phi/\partial\tilde{k}_c} = \frac{\alpha(1-\alpha)h^{-\alpha-1}\left(1-\frac{\tilde{k}_c}{k}\right)^{-\alpha}}{1+\alpha(1-\alpha)h^{-\alpha}\frac{1}{k}\left(1-\frac{\tilde{k}_c}{k}\right)^{-\alpha-1}} > 0. \quad (11)$$

There is obviously no effect on  $w_M$ , as it only depends on technology  $A$ . The wage of uneducated high-ability workers decreases, as we have just seen that it now pays off for more high-ability workers to engage in education. The intuition is that for a given share of uneducated high-ability workers, there are now less low-ability workers. This reduces the marginal productivity of uneducated high-ability workers. Hence, for a given share  $H_T$ , their wage decreases. Therefore, it pays off for more  $H_T$  workers to engage in education.

The wage of low-ability workers,  $w_T^L$ , increases: The marginal productivity of uneducated high-ability workers decreases, as the ratio of uneducated high-ability to low-ability workers,  $\frac{H_T}{L}$ , increases. Hence, the marginal productivity of low-ability workers must increase, as it depends positively on  $\frac{H_T}{L}$ . We can also show this effect more formally: The derivative

$$\frac{\partial w_T^L}{\partial h} = \alpha(1-\alpha)h^{-\alpha}\left(1-\frac{\tilde{k}_c}{k}\right)^{1-\alpha} + \alpha(1-\alpha)h^{1-\alpha}\left(1-\frac{\tilde{k}_c}{k}\right)^{-\alpha}\left(-\frac{1}{k}\frac{\partial\tilde{k}_c}{\partial h}\right) \quad (12)$$

is positive as

$$h\frac{1}{k}\frac{\partial\tilde{k}_c}{\partial h} < \left(1-\frac{\tilde{k}_c}{k}\right). \quad (13)$$

The effects on wage inequality follow immediately:  $\frac{w_M}{w_T^H}$  increases as  $w_T^H$  decreases;  $\frac{w_M}{w_T^L}$  decreases as  $w_T^L$  increases, and  $\frac{w_T^H}{w_T^L}$  decreases, as  $w_T^H$  decreases and  $w_T^L$  increases.

We can summarise these results:

**Proposition 2** *An increase in the share of high-ability workers,  $h$ ,*

- *increases the share of educated high-ability workers,  $H_M$ ,*
- *decreases the wage of uneducated high-ability workers,  $w_T^H$ ,*
- *increases the wage of low-ability workers,  $w_T^L$ ,*
- *increases wage inequality between educated and uneducated high-ability workers,  $\frac{w_M}{w_T^H}$ ,*
- *decreases wage inequality between educated high-ability and low-ability workers,  $\frac{w_M}{w_T^L}$ ,*
- *decreases wage inequality between uneducated high-ability and low-ability workers,  $\frac{w_T^H}{w_T^L}$ .*

Having established the properties of the laissez-faire equilibrium, we can introduce an exogenous minimum wage.

### 3 Equilibrium with Minimum Wage

There is a minimum wage  $\bar{w}$  that applies to low-ability workers, such that  $w_T^L < \bar{w} < w_T^H$ . Hence, the wage of low-ability workers in the minimum-wage setting,  $w_{T,\bar{w}}^L$ , is just equal to the minimum wage,

$$w_{T,\bar{w}}^L = \bar{w}. \quad (14)$$

Thus, the minimum wage  $\bar{w}$  determines the marginal productivity of low-ability workers, which is

$$\bar{w} = \alpha \left( \frac{H_T}{(1 - u_L)L} \right)^{1-\alpha} = \alpha \left( \frac{\left(1 - \frac{\tilde{k}_{\bar{w}}}{k}\right) H}{(1 - u_L)L} \right)^{1-\alpha}, \quad (15)$$

where  $u_L$  is the unemployment rate of low-ability workers, and  $1 - \frac{\tilde{k}_{\bar{w}}}{k}$  is the share of uneducated high-ability workers in the minimum wage setting. Uneducated high-ability workers earn just their marginal productivity,

$$w_{T,\bar{w}}^H = (1 - \alpha) \left( \frac{(1 - u_L)L}{\left(1 - \frac{\tilde{k}_{\bar{w}}}{k}\right) H} \right)^\alpha. \quad (16)$$

Using (14) to substitute the unemployment rate,  $u_L$ , the wage  $w_{T,\bar{w}}^H$  can be rewritten as

$$w_{T,\bar{w}}^H = (1 - \alpha) \left( \frac{\alpha}{\bar{w}} \right)^{\frac{\alpha}{1-\alpha}}. \quad (17)$$

Thus, the wage of high-ability workers in the traditional production sector decreases with the minimum wage. This results from a decreasing ratio of uneducated high-ability to low-ability workers, which decreases the marginal productivity of uneducated high-ability workers, while it increases the marginal productivity of low-ability workers as a result of the higher minimum wage.

As the minimum wage should be binding for low-ability workers, not for high-ability workers, we need  $\bar{w} < w_{T,\bar{w}}^H$  which leads to

**Assumption 2**

$$\bar{w} < (1 - \alpha)^{1-\alpha} \alpha^\alpha. \quad (18)$$

The wage in the modern technology sector is still given by  $w_M = A$ . Hence, the threshold education cost at which workers are just indifferent between working with either technology,  $\tilde{k}_{\bar{w}}$ , is determined by

$$w_M - \tilde{k}_{\bar{w}} = w_{T,\bar{w}}^H \Leftrightarrow \tilde{k}_{\bar{w}} = A - (1 - \alpha) \left( \frac{\alpha}{\bar{w}} \right)^{\frac{\alpha}{1-\alpha}}, \quad (19)$$

which is obviously increasing with the minimum wage.<sup>8</sup>

To determine the share of employed low-ability workers, I substitute  $\tilde{k}_{\bar{w}}$  from (19) into the equilibrium equation for low-ability workers (15) and obtain

$$1 - u_L = \underbrace{\frac{H}{L} \left( \frac{\alpha}{\bar{w}} \right)^{\frac{1}{1-\alpha}} \left( 1 - \frac{A - (1 - \alpha) \left( \frac{\alpha}{\bar{w}} \right)^{\frac{\alpha}{1-\alpha}}}{\bar{k}} \right)}_{\equiv L^D / L}, \quad (21)$$

where  $L^D$  is the demand for low-ability workers.

### 3.1 Feasible Range of the Minimum Wage

The lower limit for the minimum wage to be binding,  $\bar{w}_{\min}$ , is of course the competitive wage, at which the unemployment rate is zero. So,  $\bar{w}_{\min}$  is defined

<sup>8</sup>To ensure  $0 < \tilde{k}_{\bar{w}} < \bar{k}$ , a necessary condition is

$$\alpha \left( \frac{1 - \alpha}{A} \right)^{\frac{1-\alpha}{\alpha}} < \bar{w} < \alpha \left( \frac{1 - \alpha}{A - \bar{k}} \right)^{\frac{1-\alpha}{\alpha}}, \quad (20)$$

but this does not determine the relevant range of minimum wages, as will be discussed in detail in Appendix B.1.

implicitly by

$$u_L = 0 \Leftrightarrow 1 = \frac{H}{L} \left( \frac{\alpha}{\bar{w}_{\min}} \right)^{\frac{1}{1-\alpha}} \left( 1 - \frac{A - (1-\alpha) \left( \frac{\alpha}{\bar{w}_{\min}} \right)^{\frac{\alpha}{1-\alpha}}}{\bar{k}} \right). \quad (22)$$

The upper bound for the minimum wage,  $\bar{w}_{\max}$ , is determined either by Assumption 2 which ensures that the minimum wage does not apply to high-ability workers, that is  $\bar{w} < w_T^H$ , or by the restriction that the minimum wage should be low enough that there are still some low-ability workers employed, that is  $1 - u_L > 0$ . The second restriction coincides with the restriction that there are high-ability workers working with the traditional technology, that is  $\tilde{k}_{\bar{w}} < \bar{k}$ .

It depends on the parameters which restriction applies. The first one prevails if  $A - \bar{k} < \alpha^\alpha (1 - \alpha)^{1-\alpha}$ , while the second one applies otherwise, such that the upper threshold  $\bar{w}_{\max}$  is

$$\bar{w}_{\max} \equiv \begin{cases} \alpha^\alpha (1 - \alpha)^{1-\alpha} & \text{if } A - \bar{k} < \alpha^\alpha (1 - \alpha)^{1-\alpha} \\ \alpha \left( \frac{1-\alpha}{A-\bar{k}} \right)^{\frac{1-\alpha}{\alpha}} & \text{otherwise} \end{cases}. \quad (23)$$

The second case applies only if  $A > \bar{k}$ , that is, for a sufficiently large state of technology.

### 3.2 Effect of Technical Change

If there is technical progress, the wage in the modern technology sector,  $w_M$ , increases. The wages that arise from the traditional technology do not change, as the minimum wage determines the ratio of low-ability to uneducated high-ability workers and hence the marginal productivity of these workers. Consequently, technical change has no effect on wage inequality between uneducated high-ability and low-ability workers, but it increases wage inequality between educated high-ability workers and the other two groups of workers.

Instead of entailing changes in the wages of workers in the traditional technology,  $w_{T,\bar{w}}^L$  and  $w_{T,\bar{w}}^H$ , technical progress changes the shares of low- and high-ability workers working with the traditional technology. As the higher wage from the traditional technology attracts more high-ability workers, the share of educated high-ability workers increases, as follows immediately from (19). As this would decrease the marginal productivity of low-ability workers, some of these become unemployed to balance the decline in the number of high-ability

workers: It also follows immediately from (21), that the unemployment rate  $u_L$  increases with  $A$ .

**Proposition 3** *In an equilibrium with a binding minimum wage, an increase in technology,  $A$ ,*

- *increases the wage of educated high-ability workers,  $w_M$ ,*
- *has no effect on  $w_{T,\bar{w}}^L$  and  $w_{T,\bar{w}}^H$ ,*
- *increase wage inequality between educated high-ability and low-ability workers,  $\frac{w_M}{\bar{w}}$ ,*
- *increase wage inequality between educated and uneducated high-ability workers,  $\frac{w_M}{w_{T,\bar{w}}^H}$ ,*
- *increases the share of educated high-ability workers,  $\frac{\bar{k}_{\bar{w}}}{k}$ ,*
- *increases unemployment of low-ability workers,  $u_L$ .*

Note that the wage of uneducated high-ability workers does not change, compared to the laissez-faire equilibrium. This implies that the effect of technical change on the share of educated high-ability workers is stronger than in the laissez-faire equilibrium, where the wage of uneducated high-ability workers would increase with a decreasing share of them.

Last, what is the effect on the relative range of minimum wages? As long as  $A - \bar{k} < \alpha^\alpha(1 - \alpha)^{1-\alpha}$ , it is obvious that  $\bar{w}_{\max}$  remains constant, and when  $A - \bar{k} > \alpha^\alpha(1 - \alpha)^{1-\alpha}$ ,  $\bar{w}_{\max}$  decreases. The lower limit  $\bar{w}_{\min}$  decreases, as  $L^D$  decreases with  $\bar{w}$  and also with  $A$ .<sup>9</sup>

### 3.3 Effect of a Higher Minimum Wage

As the minimum wage increases, this increases the share  $\frac{\bar{k}_{\bar{w}}}{k}$  of high-ability workers who choose education, which is obvious from equation (19) above. As the minimum wage pins down the ratio of uneducated high-ability to low-ability workers, the unemployment rate of low-ability workers has to increase. The unemployment rate is actually monotonously increasing in the minimum wage as long as  $u_L < 1$ .<sup>10</sup>

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<sup>9</sup>See Appendix B.2 for a proof.

<sup>10</sup>See Appendix B.2 for a proof.

The increase in the unemployment rate can be decomposed into two effects: A higher minimum wage requires a higher ratio of low-ability to uneducated high-ability workers. This is the compositional effect. As the share of uneducated high-ability workers decreases with a higher minimum wage, the unemployment rate increases even more. Differentiating the unemployment rate,  $u_L$ , with respect to the minimum wage,  $\bar{w}$ , yields The increase in the unemployment rate can be decomposed into two effects: A higher minimum wage requires a higher ratio of low-ability to uneducated high-ability workers. This is the compositional effect. As the share of uneducated high-ability workers decreases with a higher minimum wage, the unemployment rate increases even more. Differentiating the unemployment rate,  $u_L$ , with respect to the minimum wage,  $\bar{w}$ , yields

$$\frac{\partial u_L}{\partial \bar{w}} = \overbrace{\frac{H}{L} \frac{1}{\tilde{k}} \frac{\left(\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1-\alpha}}}{(1-\alpha)} \left(\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1-\alpha}}}^{\text{educational effect}} + \overbrace{\frac{H}{L} \left(1 - \frac{\tilde{k}_{\bar{w}}}{\tilde{k}}\right) \frac{\alpha^{\frac{1}{1-\alpha}} \bar{w}^{\frac{-2+\alpha}{1-\alpha}}}{1-\alpha}}^{\text{compositional effect}}. \quad (24)$$

The effect on wages is an increase, of course, in the wage of low-ability workers, which is given by the minimum wage, and a decrease in the wage of uneducated high-ability workers, given by (17). The wage of educated high-ability workers does not change, as it only depends on technology  $A$ .

Consider first the relative wage between high-ability workers:

$$\frac{w_M}{w_{T,\bar{w}}^H} = \frac{A}{(1-\alpha) \left(\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1-\alpha}}} \quad (25)$$

It is obvious that the relative wage increases with the minimum wage  $\bar{w}$ , as  $w_{T,\bar{w}}^H$  decreases with  $\bar{w}$ .

Second, the wage of uneducated high-ability workers relative to the wage of employed low-ability workers,

$$\frac{w_{T,\bar{w}}^H}{w_{T,\bar{w}}^L} = \frac{(1-\alpha) \left(\frac{\alpha}{\bar{w}}\right)^{\frac{\alpha}{1-\alpha}}}{\bar{w}}, \quad (26)$$

decreases with the minimum wage. This is obvious as  $w_{T,\bar{w}}^H$  decreases and  $w_{T,\bar{w}}^L = \bar{w}$  increases. This theoretical finding is also in line with Autor et al. (2008), who find for the U.S. that a decreasing minimum wage contributed to increasing wage inequality at the lower tail, that is at the median/bottom-ratio.

Third, the wage of educated high-ability workers relative to the wage of employed low-ability workers decreases as well with the minimum wage, which is also obvious:

$$\frac{w_M}{w_{T,\bar{w}}^L} = \frac{A}{\bar{w}} \quad (27)$$

The relative wage of median wage earners to the poorest workers does not change with the improving technology. However, it changes the overall wage distribution, as the share of workers earning  $w_{T,\bar{w}}^H$  shrinks. As the minimum wage pins down the marginal productivity of both low and uneducated high-ability workers, their ratio has to remain constant. Hence, whenever the number of uneducated high-ability workers decreases, the number of employed low-ability workers must decrease as well to keep the ratio of these workers constant. So the unemployment rate increases. This also reshapes the income distribution.

**Proposition 4** *An increase in the minimum wage,  $\bar{w}$ ,*

- *increase the share of educated high-ability workers,  $H_M$ ,*
- *increases the unemployment rate of low-ability workers,  $u_L$ ,*
- *decreases the wage of uneducated high-ability workers,  $w_T^L$ ,*
- *decreases wage inequality between educated high-ability and low-ability workers,  $\frac{w_M}{w_{T,\bar{w}}^L}$*
- *decreases wage inequality between uneducated high-ability and low-ability workers,  $\frac{w_{T,\bar{w}}^H}{w_{T,\bar{w}}^L}$ ,*
- *increases wage inequality between educated and uneducated high-ability workers,  $\frac{w_M}{w_{T,\bar{w}}^H}$ ,*

### 3.4 Effect of Workforce Composition

If  $h = \frac{H}{L}$  increases, there are no effects on wages, as these are either determined by technology  $A$  or by the minimum wage  $\bar{w}$ . Also, there is no effect on the share  $\frac{k_{\bar{w}}}{k}$  of uneducated high-ability workers, as it is only determined by wages.

The only effect of a higher  $h$  is a lower unemployment rate  $u_L$ , as the ratio of uneducated high-ability to low-ability workers in the traditional sector increases. This raises the share of employed low-ability workers to readjust the marginal productivity of these back to the level given by the minimum wage.

## 4 Setting the Minimum Wage

As discussed in the Introduction, minimum wages are rarely decisive for the voting behaviour of individuals (Waltman, 2000). So, using a politico-economic approach that connects political decisions to voting behaviour, e.g. to majority voting, might overstate the relevance of minimum wages for elections, and vice versa. Instead, I represent the political decision by an objective function of the government.

The government has a quadratic loss function with two components: the total unemployment rate  $Lu_L$  and wage inequality between the top earners and the workers receiving the lowest wage, that is  $\frac{A}{\bar{w}}$ . Opposition to unemployment should be without discussion, and wage inequality is another topic in the political discussion, see e.g. OECD (2011a). For wage inequality, I take the wage of educated high-ability workers relative to the wage of low-ability workers. This is most closely related to the income gap between rich and poor.

The government takes into account all equilibrium effects of the minimum wage, and its objective function is

$$\max_{\bar{w}} -\gamma (Lu_L(\bar{w}))^2 - (1 - \gamma) \left(\frac{A}{\bar{w}}\right)^2, \quad (28)$$

where  $0 < \gamma < 1$  is the weight for the total unemployment rate  $Lu_L(\bar{w})$ , and where  $u_L(\bar{w})$  is given by equation (21). So, the maximisation problem writes

$$\max_{\bar{w}} -\gamma L^2 \left(1 - \frac{H}{L} \left(\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1-\alpha}} \left(1 - \frac{A - (1 - \alpha) \left(\frac{\alpha}{\bar{w}}\right)^{\frac{\alpha}{1-\alpha}}}{k}\right)\right)^2 - (1 - \gamma) \left(\frac{A}{\bar{w}}\right)^2. \quad (29)$$

The first order condition is

$$\gamma u_L(\bar{w}) L^2 \left(\frac{\partial u_L}{\partial \bar{w}}\right) \stackrel{!}{=} (1 - \gamma) \frac{A^2}{\bar{w}^3}$$

and implies that the marginal loss from the increase in unemployment is equated with the marginal gain from the decrease in wage inequality.

The effect of a change in  $A$  on the government's decision about the minimum wage is analytically intractable. To get a reliable answer, I calibrate the model. I compute statistical counterparts for the shares of workers  $H_M$ ,  $H_T$ , and  $L$ , the unemployment rate  $u_L$ , and the wages  $w_M$ ,  $w_T^H$ ,  $w_T^L = \bar{w}$  for five years. Then I take the averages of the yearly values, and solve for the model's parameters,

that is the elasticity of production with respect to low-ability workers  $\alpha$ , the upper bound of education costs  $\bar{k}$ , and the political preference parameter  $\gamma$ .

To compute the statistical counterparts, I use data from the years 1979-1983, as the minimum wage started to decline in this period. For the minimum wage and the consumer price index, I use data from the OECD, and for all other variables, I use population data from the U.S. March CPS 1980-1984, compiled by IPUMS International (King et al., 2010).<sup>11</sup> A detailed description of the construction of the statistical counterparts from the U.S. CPS is in Appendix C.

For the population data, I consider males and females aged 24 to 65 who belong to the active labour force, except for the armed forces.<sup>12</sup> According to the model, I divide the remaining sample into three groups. For the share of educated high ability workers  $H_M$ , I take the share of full-year fully-employed college graduates. For the share of uneducated high-ability workers  $H_T$ , I take full-year fully-employed high-school finishers without college degree. I only take full-year fully-employed workers as I assume that there is no unemployment in these groups of workers. Fully employed workers are those who work at least 40 hours per week.

For the share of low-ability workers  $L$ , I take all fully-employed workers who did not finish secondary school.<sup>13</sup> As I assume unemployment in this group of workers, I also include those individuals who are not full-year employed. I compute the unemployment rate of low-ability workers as the average share of the year that each individual was unemployed.

To compute wages, I consider only wage and salary income and compute the hourly wage by dividing the annual wage by weeks worked and hours worked per week.<sup>14</sup> I deflate nominal wages to 1979 values using the CPI from the OECD (2011b).

Table 1 shows the statistical counterparts that I use as input and the computed parameters of the model.<sup>15</sup> Figure 4 depicts the statistical counterparts of

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<sup>11</sup>The CPS collects data for the previous calendar year, such that, e.g., the CPS 1980 contains data about income and work for 1979.

<sup>12</sup>Workers are distinguished as being employed, unemployed, or not in the labour force, and I drop all individuals not in the labour force. The corresponding question in the census refers to the last week before the interview.

<sup>13</sup>Duffy et al. (2004) find evidence for a very low threshold for skilled workers that is just above primary school.

<sup>14</sup>Self-employment income also contains negative values, which are losses. Also, self-employment income is not subject to minimum wages.

<sup>15</sup>Although the value of  $\gamma$  might at first sight put little weight on wage inequality, one should keep in mind that the unemployment rate is below one, while the measure of wage inequality

| Input statistics  |           |         |
|---|-----------|---------|
| Target  | Symbol    | Value   |
| Average wage of college graduates                               | $w_M$     | \$9.143 |
| Average wage of workers with at most secondary education        | $w_T^H$   | \$6.700 |
| Minimum wage  | $\bar{w}$ | \$2.800 |
| Share of workers with at most primary education                 | $L$       | 0.231   |
| Share of fully employed workers with high school graduation     | $H_T$     | 0.525   |
| Share of fully employed workers with college degree             | $H_M$     | 0.244   |
| Unemployment rate of workers with less than secondary education | $u_L$     | 0.163   |
| Computed parameters   |           |         |
| Elasticity of traditional production w.r.t. low ability workers | $\alpha$  | 0.134   |
| Upper bound of education costs                                  | $\bar{k}$ | 0.870   |
| Government's weight of unemployment rate                        | $\gamma$  | 0.999   |

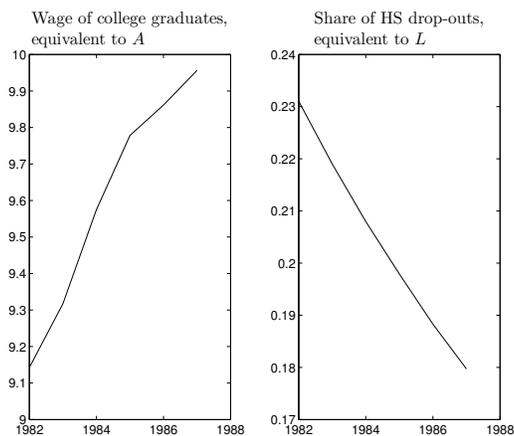
*Notes:* Determination of the model's parameters using statistics from the OECD (minimum wage) and from the U.S. CPS 1980-1984 (all others).

**Table 1.** Calibration of the model

the model's parameters for a period of ten years from the U.S. CPS 1980-1989 in five-year moving averages. The average wage of college graduates proxies the technology parameter  $A$ , and the share of high-school drop-outs proxies the share of low-ability workers  $L$ . There is clearly an increase in the average wage of college graduates, which motivates to simulate an increase in the technology parameter  $A$ . The share of high-school drop-outs steadily declined in this period. I therefore consider a decline in the share of low-ability workers  $L$ .

Given the model's parameters  $\alpha$ ,  $\gamma$ , and  $\bar{k}$ , I change the value of technology  $A$ , which is equal to the wage of educated high-ability workers, and I change the share of unskilled workers  $L$ . Therefore, I take the five-year averages from CPS 1983-1987 as input, as this is the period that is the most distant period from the base period CPS 1980-1984 for which the parameter restrictions of the model still hold. Given these input values, I calculate the minimum wage set by the government. Table 2 shows the simulation results. For comparability, I also state the model's outcomes for an exogenous minimum wage. Note that in this case the wage of uneducated high-ability workers,  $w_T^H$ , does not change, as it is determined by the constant minimum wage  $\bar{w}$ .

An increase in  $A$  leads, *ceteris paribus*, to a higher unemployment rate among low-ability workers and to higher wage inequality. Endogenising the minimum wage, the government chooses to lower the minimum wage. Therefore, the increase in the unemployment rate is less excessive, but wage inequality even is not restricted to this value. The high value of  $\gamma$  sort of corrects for these imbalances.



Notes: 5-year moving averages of statistical counterparts of the model's variables  $A$  and  $L$  from the U.S. CPS 1980-1989. Years give CPS years. Wages in 1979 values, adjusted by CPI.

Figure 4. Average wage of college graduates and share of high-school drop-outs.

| Symbol            | Data averages |           | Simulation for 1983-1987 averages |                |                            |              |                |
|-------------------|---------------|-----------|-----------------------------------|----------------|----------------------------|--------------|----------------|
|                   | 1980-1984     | 1983-1987 | Endog. MW                         |                |                            | Exog. MW     |                |
| $w_M = A$         | \$9.143       | \$9.778   |                                   |                |                            |              |                |
| $L$               | 0.231         | 0.198     | $A \uparrow$                      | $L \downarrow$ | $A \uparrow, L \downarrow$ | $A \uparrow$ | $L \downarrow$ |
| $\bar{w}$         | \$2.800       | \$2.589   | \$2.669                           | \$3.174        | \$3.032                    | -            | -              |
| $w_{T,\bar{w}}^H$ | \$6.700       | \$6.888   | \$6.750                           | \$6.572        | \$6.619                    | -            | -              |
| $u_L$             | 0.163         | 0.177     | 0.214                             | 0.175          | 0.230                      | 0.264        | 0.022          |
| $L \cdot u_L$     | 0.038         | 0.035     | 0.049                             | 0.035          | 0.046                      | 0.061        | 0.004          |
| $H_M$             | 0.244         | 0.265     | 0.303                             | 0.268          | 0.330                      | 0.308        | 0.255          |
| $H_M/H$           | 0.318         | 0.331     | 0.394                             | 0.334          | 0.411                      | 0.400        | 0.318          |

Notes: The simulation takes the parameters from Table 1 as given, computed for the baseline period 1980-1984. It computes the minimum wage that is set by the government and other relevant variables, changing either  $A$  or  $L$  or both to their respective values for 1983-1987. Years refer to U.S. CPS years.

Table 2. Simulation of the minimum wage decision.

increases. The share of educated high-ability workers still increases, but less than in the case of an exogenous minimum wage. The lower minimum wage leads to a higher wage of uneducated high-ability workers.

A decrease in  $L$  leads, *ceteris paribus*, to a lower unemployment rate. By consequence, the government increases the minimum wage to reduce wage inequality and increase the unemployment rate again to achieve equality in equation (30). In this case, it is interesting to note that the unemployment rate  $u_L$  increases compared to the baseline calibration. The government focuses instead on the total unemployment rate  $L \cdot u_L$ , which decreases. Therefore, the government is able to decrease both the unemployment rate and wage inequality. The higher minimum wage also decreases the wage of uneducated high-ability workers.

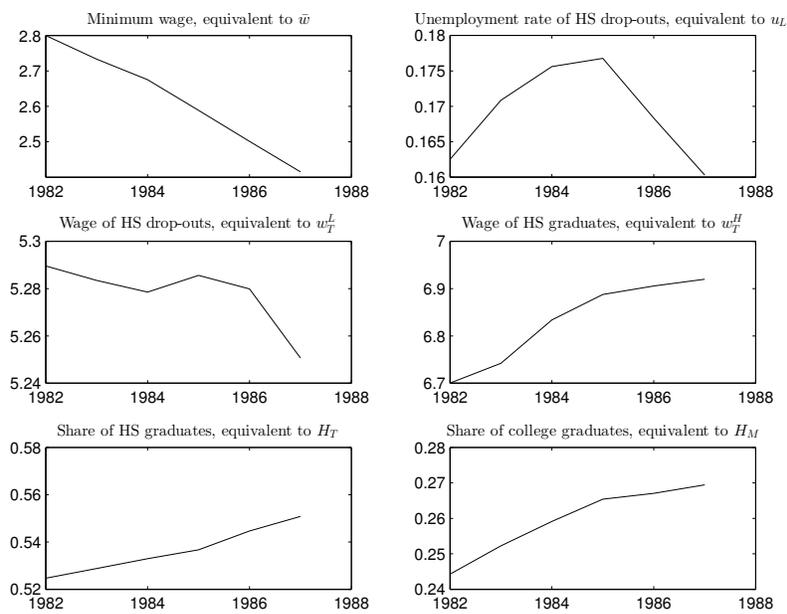
Changing both  $A$  and  $L$  to the CPS 1983-1987 values, the decline in  $L$  dominates the decision and the government sets a higher minimum wage, which is at odds with the data. The higher minimum wage results in a decline in the wage of uneducated high-ability workers, in an increase in the unemployment rate of low-ability workers, and in an increase in the share of educated high-ability workers.

## 5 Discussion

Given the predictions of this model, what can it explain and where does it fail? Figure 5 depicts the development of the statistical counterparts of the model's endogenous variables.

Let us first consider the model in the *laissez-faire* equilibrium. The CPS data reveal a decline in the share of high-school drop-outs and an increase in the share of high-school graduates without a college degree. If the model was correct, we would observe an increase in the wage of high-school drop-outs,  $w_T^L$ , and a fall in the wage of high-school graduates,  $w_T^H$ . Yet the opposite is the case. The wage of high-school drop-outs declined slightly, while the wage of high-school graduates increased. Thus, this fact does not support the view that both types of workers are  $q$ -complements in a joint production function.

Also, viewing all high-school graduates as a homogeneous group of workers is questionable. Instead, if these workers have very differentiated skills, it would be possible to view them as complements in the production function, such that adding an additional worker with a new skill increases the productivity of all



Notes: 5-year moving averages of statistical counterparts of the model's endogenous variables from the U.S. CPS 1980-1989. Wages in 1979 values, adjusted by CPI.

Figure 5. Wages, unemployment rate and shares of workers.

other workers. This might explain better the increase in the wage of high-school graduates, yet it still does not explain the decline in the wage of high-school drop-outs.

In the equilibrium with a minimum wage, the model predicts an increase in the unemployment rate of low-ability workers,  $u_L$ , in case of an increase in the technology parameter  $A$ , *ceteris paribus*. Yet also the share of high-school drop-outs declined as well as the minimum wage. Both a decrease in the share of low-ability workers,  $L$ , and a decrease in the minimum wage reduce the unemployment rate of low-ability workers. The long-term effect on the unemployment rate is unclear. The data show an up-and-down of the unemployment rate of high-school drop-outs. Thus, it is not necessarily at odds with the data to associate increased technical progress with a higher unemployment rate among the lowest skilled workers. Yet the detailed mechanisms are not clear.

Thus, the failure of the model to predict correctly the changes in the minimum wage is one thing, but the failure to predict correctly market wages is even more important. Thus, for future research it would be promising to reconsider the production functions and the complementarity and substitutability of the production factors to get a better understanding of the changes in real wages, as well as the allocation of factors to different sectors.

## 6 Conclusion

I set up a simple partial-equilibrium model of routine-biased technical change, similar to a simple model by [Acemoglu \(2002\)](#). My model features heterogeneous workers with three skill levels. Education, which is necessary to move from medium to high skill levels, is endogenous. This reflects the increasing complexity of occupations which require skill upgrading ([Spitz-Oener, 2006](#)).

In the *laissez-faire* equilibrium without a minimum wage, technical progress leads to diverging real wages, in particular to decreasing real wages of the least skilled workers, and to increasing real wages of medium- and high-skilled workers. Technical progress also increases the share of high-skilled workers. This results from a reallocation of high-ability workers to a modern production technology, where the work previously done by low-ability workers is now done by machines. Thus, less high-ability workers work with low-ability workers, whose marginal productivity declines in consequence.

In the equilibrium with a minimum wage, there are important lessons to be

learned from this model. Technical progress also leads to more education, that is a higher share of high-skilled workers. The effect is stronger than in the laissez-faire equilibrium. But technical progress also leads to more unemployment among low-ability workers, as high-ability workers replace them by machines. The government opposes both unemployment and wage inequality. I calibrate the model to U.S. data. As technology improves, the government lowers the minimum wage. It thus gives up wage inequality for more employment among low-ability workers. As also the share of high-school drop-outs decreased, I also analysed the change in the minimum wage resulting from a lower share of low-ability workers. In this case, the government sets a higher minimum wage. Simulating the government's decision by taking into account the changes in both variables for both variables, the model predicts an increase in the minimum wage, which is at odds with the U.S. data.

For future research, several modifications and extensions are interesting. The source of the increased pressure on the labour market of low-ability workers does not seem to stem from changed input shares of workers with different education levels. Therefore, we need to understand better the decrease in the demand for the lowest-skilled workers. Instead of viewing production factors as complements, it might make more sense to focus on the allocation of production factors to different sector. This has so far been treated as exogenous in the literature, see e.g. [Autor and Dorn \(2013\)](#). Yet an endogenous explanation is missing. A model with workers being heterogeneous along a continuous dimension would allow for a continuous distribution of wages. Also, the wages of workers who work with the traditional technology depend solely on the ratio of uneducated high-ability to low-ability workers. It would be appealing to have a general equilibrium framework with different sectors, such that changes in real wages can also be attributed to changes in demand, as the relative price of high-tech consumptions goods decreases with technical progress. The literature also still lacks a task-based theory in which not all production factors are complements, but some are also substitutes, or a task-based theory with multiple sectors that explains not only the allocation of factors to tasks, but also to sectors, such that the effects we observe do not depend on arbitrary assumptions about the production functions.

## A Appendix for Laissez-faire Equilibrium

### A.1 Change in Wage Inequality Between High-Ability Workers

The derivative of the relative wage  $\frac{w_M}{w_T}$  with respect to  $A$  is

$$\frac{\partial \frac{w_M}{w_T}}{\partial A} = \frac{1 - A\alpha \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right) \frac{1}{k} \frac{\partial \tilde{k}_c}{\partial A}}{(1 - \alpha) \left( \frac{L}{H} \right)^\alpha \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right)^\alpha}. \quad (\text{A-1})$$

As the denominator is positive, the sign depends only on the numerator. It is positive if

$$1 > A\alpha \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right) \frac{1}{k} \frac{\partial \tilde{k}_c}{\partial A}, \quad (\text{A-2})$$

and using (6)

$$1 > A\alpha \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right) \frac{1}{k} \frac{1}{1 + \alpha(1 - \alpha) \left( \frac{L}{H} \right)^\alpha \frac{1}{k} \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right)^{1+\alpha}}, \quad (\text{A-3})$$

which can be rewritten<sup>16</sup> as

$$1 > \alpha \frac{1}{k} \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right) \left[ A - (1 - \alpha) \left( \frac{L}{H} \right)^\alpha \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right)^\alpha \right]. \quad (\text{A-4})$$

Since the term in brackets is equal to  $\tilde{k}_c$  from equation (5), wage inequality increases if and only if

$$1 > \alpha \frac{\tilde{k}_c}{k} \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right) \Leftrightarrow \frac{\bar{k}}{1 + \alpha} > \tilde{k}_c \quad (\text{A-5})$$

and decreases otherwise.

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<sup>16</sup>Multiply by  $1 + \alpha(1 - \alpha) \left( \frac{L}{H} \right)^\alpha \frac{1}{k} \left( \frac{1}{1 - \frac{\tilde{k}_c}{k}} \right)^{1+\alpha}$  and rearrange.

## B Appendix for Equilibrium With Minimum Wage

### B.1 Consistency of Technical Assumptions

There are several restrictions on the minimum wage:

1.  $\bar{w} < (1 - \alpha)^{1-\alpha} \alpha^\alpha$  such that the minimum wage is not binding for high ability workers ( $\bar{w} < w_{T,\bar{w}}^H$ ),
  2.  $\bar{w} < \alpha \left( \frac{1-\alpha}{A-\bar{k}} \right)^{\frac{1-\alpha}{\alpha}}$  such that the unemployment rate is lower than 1 ( $\tilde{k}_{\bar{w}} < \bar{k}$  or  $u_L < 1$ ),
  3.  $1 > \frac{H}{L} \left( \frac{\alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} \left( 1 - \frac{A-(1-\alpha)\left(\frac{\alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}}}{\bar{k}} \right)$  such that the unemployment rate is positive ( $u_L > 0$ ),
  4.  $\bar{w} > \alpha \left( \frac{1-\alpha}{A} \right)^{\frac{1-\alpha}{\alpha}}$  for  $\tilde{k}_{\bar{w}} > 0$ .
- The first and the fourth assumption are consistent, as long as

$$(1 - \alpha)^{1-\alpha} \alpha^\alpha < A, \quad (\text{B-6})$$

which is derived from rearranging

$$\alpha \left( \frac{1-\alpha}{A} \right)^{\frac{1-\alpha}{\alpha}} < (1 - \alpha)^{1-\alpha} \alpha^\alpha. \quad (\text{B-7})$$

- The first and the second give upper bounds for the feasible range of minimum wages. It depends on the state of technology,  $A$ , whether the first or the second is more restrictive. The highest value of the minimum wage that is feasible, i.e. at which  $u_L = 1$ , decreases with the state of technology and will just be equal to the value at which  $\bar{w} = w_T^H$  if

$$A = \bar{k} + (1 - \alpha)^{1-\alpha} \alpha^\alpha. \quad (\text{B-8})$$

So, the upper limit is given by  $(1 - \alpha)^{1-\alpha} \alpha^\alpha$  if  $A < \bar{k} + (1 - \alpha)^{1-\alpha} \alpha^\alpha$ , and it is given by  $\alpha \left( \frac{1-\alpha}{A-\bar{k}} \right)^{\frac{1-\alpha}{\alpha}}$  otherwise.

- The third and the fourth state lower bounds for the minimum wage. The third is actually the competitive wage  $w_T^L$ , and it is possible to show that it is more restrictive than the fourth. If the fourth were more binding than the

competitive wage, it would then be that  $u_L > 0$ . I show by contradiction that this is not possible: Assume that the minimum wage is just equal to  $\bar{w} = \alpha \left(\frac{1-\alpha}{A}\right)^{\frac{1-\alpha}{\alpha}}$ . Since this is derived from the restriction that  $\tilde{k}_{\bar{w}} = 0$ , this implies that all high ability workers work in the service sector, since the wage in the tech sector and in the service sector are just the same. Plugging this wage in the right hand side of assumption three, which gives the demand for low ability workers, we get, if we assume  $u_L > 0$ ,

$$\frac{L}{H} > \left(\frac{A}{1-\alpha}\right)^{\frac{1}{\alpha}} \quad (\text{B-9})$$

which can be rewritten as

$$\left(\frac{L}{H}\right)^{\alpha} (1-\alpha) > A. \quad (\text{B-10})$$

This is obviously at odds with the assumption that  $A > (1-\alpha) \left(\frac{L}{H}\right)^{\alpha}$  which is necessary for  $\tilde{k}_c > 0$  in the competitive case. So, the assumption that  $u_L > 0$  at  $\bar{w} = \alpha \left(\frac{1-\alpha}{A}\right)^{\frac{1-\alpha}{\alpha}}$  is wrong, and it must be that  $u_L < 0$  if  $\bar{w}$  is just equal to  $\alpha \left(\frac{1-\alpha}{A}\right)^{\frac{1-\alpha}{\alpha}}$ . Since at the competitive wage, we have  $u_L = 0$ , and since the unemployment rate is increasing in  $\bar{w}$ , the competitive wage is more restrictive than the minimum wage which is needed for  $\tilde{k}_{\bar{w}} = 0$ .

- The second and the fourth are consistent, as  $\alpha \left(\frac{1-\alpha}{A}\right)^{\frac{1-\alpha}{\alpha}} < \alpha \left(\frac{1-\alpha}{A-\bar{k}}\right)^{\frac{1-\alpha}{\alpha}}$ .

## B.2 Effect of Minimum Wage on Unemployment Rate

The unemployment rate (21) increases with the minimum wage  $\bar{w}$  if  $L^D$  on the right hand side of (21) decreases with  $\bar{w}$ . This is true if

$$\bar{w} < \alpha \left(\frac{(1-\alpha)(1+\alpha)}{A-\bar{k}}\right)^{\frac{1-\alpha}{\alpha}} \quad (\text{B-11})$$

Since  $(1+\alpha)^{\frac{1-\alpha}{\alpha}} > 1$ , the value of  $\bar{w}$  at which the sign of the derivative changes is larger than the upper limit of the minimum wage  $\bar{w}_{\max}$  defined by (23). So for the range of minimum wages that we consider here, the unemployment rate increases monotonously with the minimum wage.

## C Calibration Details

|  | CPS Variable |                    |
|--|--------------|--------------------|
|  | Name         | Values             |
| <b>Sample selection</b>                |              |                    |
| CPS years                              | Year         | [1980, 1984]       |
| Age                                    | Age          | [24, 65]           |
| Age                                    | Age          | [24, 65]           |
| Employment status                      | Empstat      | [10, 29]           |
| <b>Skill groups</b>                    |              |                    |
| <i>Low-ability workers</i>             |              |                    |
| Education level                        | Educ         | $\leq 71$          |
| Weeks worked per year                  | Wkswork1     | $\in [0, 52]$      |
| Hours worked per week                  | Uhrswork     | $= 0$ or $\geq 40$ |
| <i>Uneducated high-ability workers</i> |              |                    |
| Education level                        | Educ         | [72, 100]          |
| Weeks worked per year                  | Wkswork1     | $= 52$             |
| Hours worked per week                  | Uhrswork     | $\geq 40$          |
| <i>Educated high-ability workers</i>   |              |                    |
| Education level                        | Educ         | $\geq 110$         |
| Weeks worked per year                  | Wkswork1     | $= 52$             |
| Hours worked per week                  | Uhrswork     | $\geq 40$          |
| <b>Other variables used</b>            |              |                    |
| Wage income                            | Incwage      | all values         |

**Table 3.** Sample selection from U.S. CPS for the model’s statistical counterparts

The unemployment rate  $u_L$  is calculated as

$$u_L = \frac{\sum_i \frac{52 - Wkswork1_i}{52} \mathbf{1}_{\leq 71}(Educ_i)}{\sum_i \mathbf{1}_{\leq 71}(Educ_i)}, \quad (\text{C-12})$$

where the subscript  $i$  denotes an observation in the CPS.

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