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Control of electron-hole pair generation by biharmonic voltage drive of a quantum point contact

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A time-dependent electromagnetic field creates electron-hole excitations in a Fermi sea at low temperature. We show that the electron-hole pairs can be generated in a controlled way using harmonic and biharmonic time-dependent voltages applied to a quantum contact, and we obtain the probabilities of the pair creations. For a biharmonic voltage drive, we find that the probability of a pair creation decreases in the presence of an in-phase second harmonic. This accounts for the suppression of the excess noise observed experimentally (Gabelli and Reulet, arXiv:1205.3638), proving that dynamic control and detection of elementary excitations in quantum conductors are within the reach of the present technology.

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The controllable generation and manipulation of single- to few-particle excitations in mesoscopic conductors has attracted much attention recently.[1–9] The interest in the subject stems from the emerging field of electron quantum optics, which has the goal of developing coherent electronic devices suitable for quantum communication and quantum computation. This is based on the analogy with photon quantum optics where pairs of photons in an entangled polarization state are used for transmission and processing of quantum information. In a solid-state system, a similar functionality can be achieved using pairs of quasiparticles with entangled spin or orbital degrees of freedom.[10,11] An important building block of electronic devices aimed at quantum computation is a tunable electron source capable of creating quasiparticles in a coherent way. An on-demand coherent electron source has been realized recently using a localized electronic level in a quantum dot, weakly coupled to a conductor, which is populated and emptied by a modulation of its energy via a periodic gate voltage.[3] This results in a sequence of quantized single-electron current pulses whose properties can be inferred from the measurements of the current noise spectrum.[6,12]

An altogether different route which does not require electron confinement has been suggested theoretically by Keeling, Klich, and Levitov.[13] They proposed making use of time-dependent voltage pulses to create excitations from a degenerate Fermi sea in a mesoscopic conductor. It has been found that Lorentzian-shaped pulses $V(t)$ of a quantized area $\int eV(t)dt/\hbar = 2\pi N$ (N is an integer, e is the electron charge, and $\hbar$ is the reduced Planck constant, hereafter $\hbar = 1$) create exactly N electrons above the Fermi level with no additional excitations. The many-body quantum state created by these pulses is a product state of N particles added to an unperturbed Fermi sea, independent of the relative position of the pulses, their duration, or overlap. Conversely, the charge-transfer statistics in this case is binomial, which supports the picture of independent charge quanta created. The single-particle character of excitations can be probed by noise measurements: it manifests itself as a reduction of the current noise power which assumes its minimal value set by the dc voltage offset.[14] In general, however, a time-dependent field does create additional excitations in a Fermi sea. These excitations give rise to excess photon-assisted noise, exceeding the minimal dc noise level.[15,16] The photon-assisted noise was observed experimentally in diffusive phase-coherent metallic conductors,[17] normal-metal–superconductor junctions,[18] and quantum point contacts[19] with harmonic ac voltage applied. Noise spectroscopy of a quantum tunnel junction with a more complex biharmonic voltage drive has been carried out recently.[8]

The physical picture behind photon-assisted noise has been revealed in Ref. 9 by an analysis of the full counting statistics: The elementary excitations in the system at low temperature are the *electron-hole pairs* created by the ac voltage component, in addition to electrons injected by the dc voltage offset. This picture is valid beyond noise measurements and pertains to the full statistics of the transferred charge. Since electrons and holes from a pair are created with the same probability, the pairs give rise only to the excess noise and higher even-order current correlators, whereas they give no contribution to the average current and odd-order correlators. The number of created electron-hole pairs depends on the shape and the amplitude of the ac voltage applied. This opens a route toward dynamic control of elementary excitations in quantum conductors which, if proved feasible, could be used in the electron-hole sources to produce quasiparticles with entangled spin or orbital degrees of freedom.[7,20–27] Alternatively, such control can also be used to minimize excitations present in the system and approach the limit of an ideal single-electron source using realistic voltage pulses.

In this paper, we investigate the feasibility of a dynamic control of elementary electron-hole pair excitations. We analyze experimental data on the photon-assisted noise in quantum conductors subject to time-dependent drive,[8,19] identify elementary excitations generated in the experiments, and show how the measured excess noise is composed of the contributions of electron-hole pairs created. For a quantum contact with harmonic voltage drive studied in Ref. 19, we find that a single electron-hole pair is created per period with a certain probability, in addition to the electrons that are injected by dc voltage offset. We find how the probability of pair creation depends on the amplitude of the ac voltage drive, which is in agreement with the observed excess noise.

In a recent experiment, Gabelli and Reulet[8] studied photon-assisted noise in a quantum tunnel junction subject to a biharmonic time-dependent voltage, where the dc offset, the ac amplitudes, and the relative phase are tunable. They observed a reduction of noise when an in-phase second harmonic is present in the drive. We find that the statistics of the transferred charge is the simplest when the dc offset is an integer multiple of the driving frequency. In that case, only a few (one or two) electron-hole pairs are created per period with probabilities that depend on the shape and the amplitude of the biharmonic voltage component. In addition, we relate the reduction of the noise in the presence of an in-phase second harmonic to the suppressed probability of the electron-hole pair creation as the drive voltage approaches the shape of optimal Lorentzian pulses. The agreement with the observed excess noise corroborates that the dynamic control of elementary excitations in quantum conductors has been achieved experimentally.

The system we study is a coherent mesoscopic conductor characterized by transmission eigenvalues $\{T_n\}$, where n labels spin-degenerate transport channels. The conductance of the conductor is $G = (e^2/\pi)\sum_n T_n$ and the Fano factor $F = (\sum_n T_n R_n)/\sum_n T_n$, where $R_n = 1 - T_n$ are reflection probabilities. The conductor is subject to a periodic voltage drive $V(t) = \bar{V} + V_{\rm ac}(t)$, where $\bar{V}$ is the dc voltage offset and $V_{\rm ac}(t)$ is the ac voltage component with zero average and period $\tau = 2\pi/\omega$. At low temperature $T \ll \omega$, the cumulant generating function $\mathcal{S}(\chi)$ of the charge-transfer statistics can be cast in a form that manifestly reveals what are the elementary processes in the system:[9] $\mathcal{S}(\chi) = \mathcal{S}_{\rm dc}(\chi) + \mathcal{S}_{\rm ac}(\chi)$, where $\mathcal{S}_{\rm dc} = (t_0|e\bar{V}|/\pi)\sum_n \ln[1 + T_n(e^{-i\kappa\chi} - 1)]$ and

$$\mathcal{S}_{\rm ac} = \frac{2t_0}{\tau}\sum_{n,k}\Big((1-\bar{v})\ln\big[1 + p_k^{(N)}T_nR_n(e^{i\chi} + e^{-i\chi} - 2)\big] + \bar{v}\,\ln\big[1 + p_k^{(N+1)}T_nR_n(e^{i\chi} + e^{-i\chi} - 2)\big]\Big). \tag{1}$$

Here, t_0 is the measurement time, which is much larger than the characteristic time scale on which current fluctuations are correlated, $\kappa = 1$ ($\kappa = -1$) for $e\bar{V} > 0$ ($e\bar{V} < 0$) is related to the direction of the charge transfer, and $N = \lfloor e\bar{V}/\omega \rfloor$ ($\bar{v} = e\bar{V}/\omega - N$) is the integer (fractional) part of $e\bar{V}/\omega$. Coefficients $p_k^{(N)}$ ($0 \leqslant p_k^{(N)} \leqslant 1$) in $\mathcal{S}_{\rm ac}$ are the probabilities of electron-hole pair creations which depend on the details of the time-dependent voltage applied; see Fig. 1. They are given by $p_k^{(N)} = \sin^2(\alpha_k/2)$, where $e^{\pm i\alpha_k}$ are the *pairs* of complex-conjugate eigenvalues of the matrix $M_{nm} = {\rm sgn}(n + 0^+)\sum_{k=-\infty}^{\infty} a_{n+k}a_{m+k}^*\,{\rm sgn}(0^+ - k - N)$ (cf. Ref. 9 for details). The Fourier coefficients $a_n = \tau^{-1}\int_0^\tau dt\, e^{-i\phi(t)}e^{in\omega t}$ characterize the drive, where $\phi(t) = \int_0^t dt'\, eV_{\rm ac}(t')$ is the phase acquired.

The physical interpretation of $\mathcal{S}(\chi)$ is as follows. The dc part $\mathcal{S}_{\rm dc}(\chi)$ describes unidirectional single-electron transfers due to a finite voltage offset $\bar{V}$. The attempt frequency is $|e\bar{V}|/\pi$ and the transfer probability T_n per transport channel. The ac part of the cumulant generating function $\mathcal{S}_{\rm ac}(\chi)$ describes the events of electron-hole pair creations. The electron-hole pairs (labeled by k) are created by the ac component of the drive with probability p_k per voltage cycle. The charge transfer occurs with probability $p_kT_nR_n$ when one particle, e.g., an electron, is transmitted and the hole is reflected, or vice versa. Since the electron and hole from a pair are transmitted with equal probabilities, the electron-hole pairs give no contribution to the average current, but they do contribute to the current noise power. We note that the total number of attempts to create an electron-hole pair is t_0/τ (per spin-split transport channel), which gives one attempt per voltage cycle. These attempts are shared between processes $p_k^{(N)}$ and $p_k^{(N+1)}$ that correspond to the nearest integer values of $e\bar{V}/\omega$. The simplest statistics—determined by one type of the processes $p_k^{(N)}$ only—is obtained for the voltage offset $e\bar{V} = N\omega$, which is an integer multiple of the driving frequency. For a nonzero dc component, the created electron-hole pairs coexist with N electrons injected toward the contact per period.

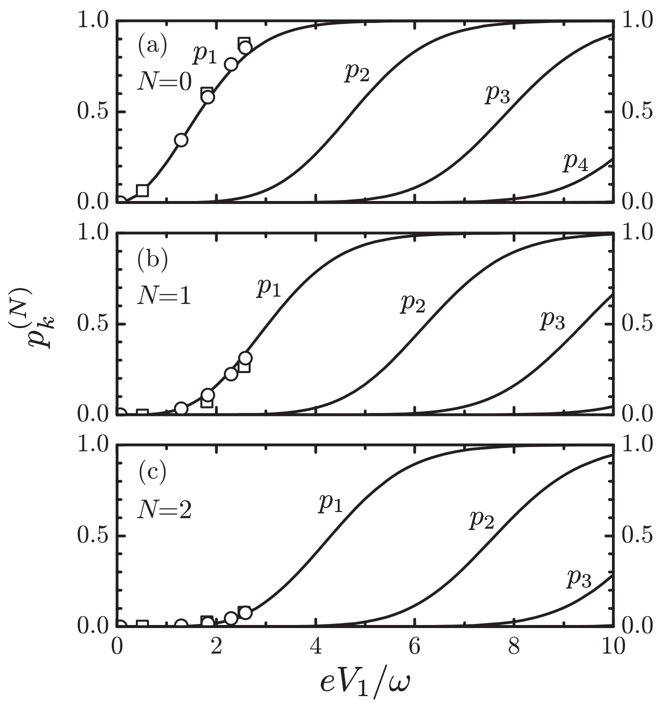

FIG. 1. Probabilities p_k of electron-hole pair creations for harmonic drive $V(t) = \bar{V} + V_1\cos(\omega t)$ as a function of the ac amplitude V_1 and for different dc voltage offsets $e\bar{V}/\omega = N$: (a) $N = 0$, (b) $N = 1$, and (c) $N = 2$. dc offset $\bar{V}$ sets the number of electrons injected toward the contact while V_1 controls the electron-hole pair generation. Symbols denote experimental data for the excess noise $S_{\rm ac}$ in units $2GF\omega$ taken from Ref. 19 for a quantum contact with $F = 1/2$ and (◯) $\omega/2\pi = 17.32$ GHz, (□) $\omega/2\pi = 8.73$ GHz.

From the cumulant generating function, we obtain the current noise power $S = (e^2/t_0)\partial_{i\chi}^2\mathcal{S}|_{\chi=0} = S_{\rm dc} + S_{\rm ac}$ with $S_{\rm dc} = GF|e\bar{V}|$ and $S_{\rm ac} = 2GF\omega\sum_k[(1-\bar{v})p_k^{(N)} + \bar{v}p_k^{(N+1)}]$. Therefore, the excess noise of the electron-hole pairs is a piecewise linear function of a dc voltage offset. At $e\bar{V}/\omega = N$ (N is integer), $S_{\rm ac}$ is given by the sum of the probabilities of the pair creations

$$S_{\rm ac}|_N = 2GF\omega\sum_k p_k^{(N)} \tag{2}$$

and linearly interpolates in-between for $e\bar{V}/\omega$ noninteger. The experimentally observed excess noise[8,19] and its decomposition into contributions of the electron-hole pairs are shown in Figs. 1–3.

In Ref. 19, the authors measured photon-assisted noise in a quantum contact ($F = 1/2$) subject to harmonic time-dependent drive $V(t) = \bar{V} + V_1\cos(\omega t)$. For this drive,

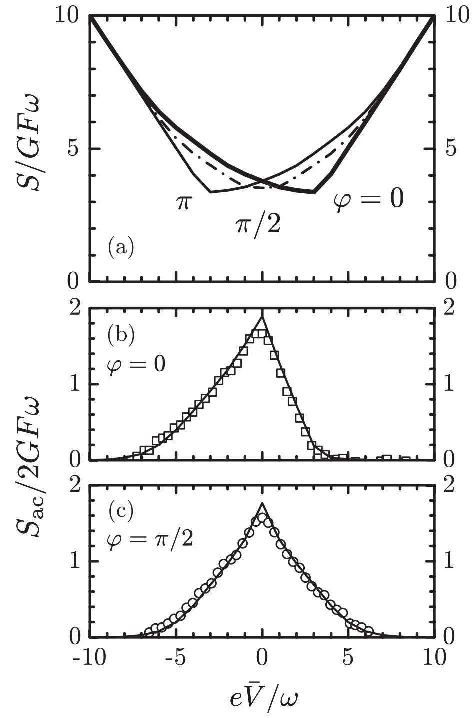

FIG. 2. (a) Current noise S as a function of dc voltage $\bar{V}$ for biharmonic time-dependent drive $V(t) = \bar{V} + V_1\cos(\omega t) + V_2\cos(2\omega t + \varphi)$ with $eV_1/\omega = 5.4$, $eV_2/\omega = 2.7$, and $\varphi = 0$ (thick solid line), $\pi/2$ (dash-dotted line), and π (thin solid line). The excess noise S_{ac} is shown as a function of $\bar{V}$ for the same voltage drive with (b) $\varphi = 0$ and (c) $\varphi = \pi/2$. Symbols in (b) and (c) denote experimental data taken from Ref. 8 for a tunnel junction ($F = 1$) at low temperature ($T = 0.14\omega$).

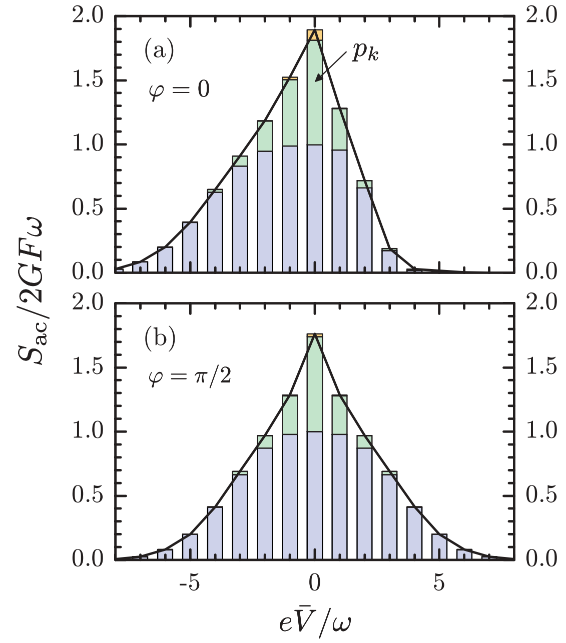

FIG. 3. (Color online) Decomposition of the excess noise S_{ac} shown in Fig. 2 into elementary electron-hole pair excitations [see Eq. (2)]. Probabilities $p_k^{(N)}$ of the pair creations are denoted by stacked bars at dc voltages $e\bar{V}/\omega = N$ with N integer: $k = 1$ (blue), $k = 2$ (green), and $k = 3$ (red bars). The elementary excitations comprise a few (one or two) electron-hole pairs generated with probabilities $p_{1,2}^{(N)}$ per voltage cycle.

$a_n = J_n(eV_1/\omega)$, where $J_n(x)$ are Bessel functions of the first kind. The predicted probabilities $p_k^{(N)}$ of the electron-hole pair creations are shown in Fig. 1 as a function of the ac amplitude V_1 and for different values $e\bar{V}/\omega = N$ of the dc component. As the drive amplitude V_1 is increased, the probability of the pair creation also increases and more pairs are created per period. The symbols in Fig. 1 denote experimental data for the excess noise $S_{ac}/2GF\omega$ at dc offsets $e\bar{V}/\omega = N$ ($N = 0,1,2$), measured for driving frequencies $\omega/2\pi = 17.32$ and 8.73 GHz. The experimental results are in agreement with Eq. (2). We find that for the amplitudes used in the experiment, only a single electron-hole pair is created per period with probability p_1. The interval eV_1/ω where a single pair is excited is broad and is not restricted to the weak driving limit $eV_1/\omega \ll 1$; see Fig. 1(a). For a nonzero dc offset, the pair creation is also accompanied by the single-electron transfers. When the ac amplitude is smaller than the dc offset, the creation of electron-hole pairs is strongly suppressed; see Figs. 1(b) and 1(c). In the situation in which the probability of a pair creation is small, the noise is dominated by the dc shot noise $S_{dc} \gg S_{ac}$. In that case, generated electron-hole pairs can be probed more directly in a beam splitter geometry where current *cross correlation* occurs due to processes in which the pairs are split and the two particles enter different outgoing terminals.[9,20]

In a recent experiment, Gabelli and Reulet[8] studied the noise in a quantum tunnel junction ($F = 1$) with a biharmonic voltage applied, $V(t) = \bar{V} + V_1\cos(\omega t) + V_2\cos(2\omega t + \varphi)$, where the dc offset $\bar{V}$, the ac amplitudes V_1 and V_2, and the phase shift φ were tunable. The coefficients a_n in this case are given by $a_n = \sum_{m=-\infty}^{\infty} J_{n-2m}(eV_1/\omega)J_m(eV_2/2\omega)e^{-im\varphi}$. The observed current noise power and the decomposition of the excess noise into contributions of the electron-hole pairs are shown in Figs. 2 and 3. The current noise power as a function of $\bar{V}$ and for different phase shifts φ is shown in Fig. 2(a), where we use the experimental values of the parameters $eV_1 = 5.4\omega$ and $eV_2 = 2.7\omega$. At low temperature $T \ll \omega$, the current noise power reads[15] $S = GF\sum_{n=-\infty}^{\infty} |e\bar{V} + n\omega|\,|a_n|^2$, which is a piecewise linear function of the dc voltage offset $e\bar{V}$ with kinks at integer multiples of ω. This can be seen in the differential noise $S' = \partial S/\partial(e\bar{V})$ which consists of a series of steps at $e\bar{V}/\omega = N$ (N is an integer) with the step size $S'|_{N+0} - S'|_{N-0} = 2GF|a_{-N}|^2$ (cf. Fig. 3 in Ref. 8). The excess noise is $S_{ac}|_N = 2GF\omega\sum_{n=1}^{\infty} n|a_{-N\mp n}|^2$, where the upper (lower) sign is taken for $N \geqslant 0$ ($N < 0$). This analysis quantitatively accounts for the photon-assisted noise observed experimentally; see Figs. 2(b) and 2(c). However, the information on elementary excitations present in the system is implicit and encoded in the Fourier components a_n characterizing the drive.

The decomposition of S_{ac} into contributions of electron-hole pair excitations given by Eq. (2) is shown in Fig. 3. The probabilities $p_k^{(N)}$ of the pair creations are depicted as stacked bars. We observe that the excess noise consists of a small number (one or two) of the electron-hole pairs excited per period with certain probabilities. The pairs are accompanied with $e\bar{V}/\omega = N$ electrons injected toward the contact per

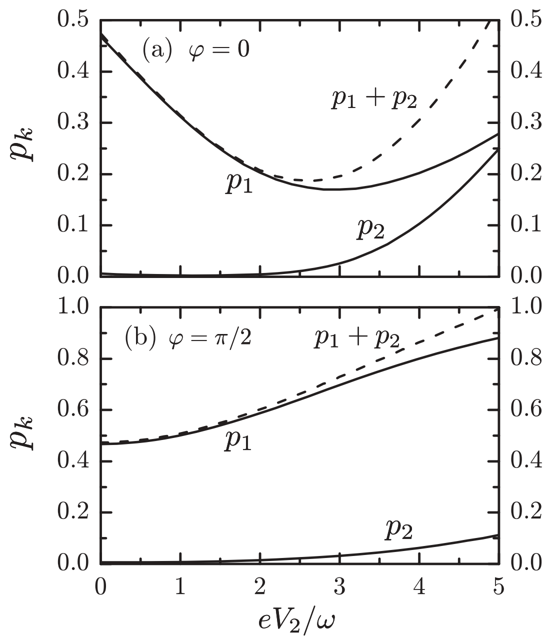

FIG. 4. Probabilities p_k of the electron-hole pair creations as a function of the amplitude V_2 of the second harmonic for the biharmonic voltage drive with $e\bar{V}/\omega = 3$, $eV_1/\omega = 5.4$, and the phase difference (a) $\varphi = 0$ and (b) $\varphi = \pi/2$. Dashed lines indicate the excess noise $S_{\rm ac}$ in units $2GF\omega$. The presence of an in-phase second harmonic decreases the probability of the electron-hole pair creation leading to suppression of $S_{\rm ac}$ observed experimentally.[8]

period. Therefore, for small offset voltages, the number of electron-hole pairs is significant as compared to the number of electrons injected per period. As the dc offset is increased, the probability of pair creation decreases and the excess electrons start to dominate. The probability of pair creation can further be tuned by changing the shape of the ac voltage component.

In Ref. 8, the authors have used a setup where the dc offset and the amplitude of the first harmonic are kept fixed and the amplitude of the second harmonic V_2 is varied to minimize the noise. In Fig. 4, we show the probabilities of the electron-hole pair generation as a function of V_2 for a dc offset $e\bar{V}/\omega = 3$ and the amplitude of the first harmonic $eV_1/\omega = 5.4$. For a simple harmonic drive ($V_2 = 0$), there is only one electron-hole pair generated per period with probability p_1. When the second harmonic is in phase with the first one ($\varphi = 0$), the probability p_1 of the pair generation *decreases* as the amplitude V_2 is increased; see Fig. 4(a). As V_2 is increased further, the second electron-hole pair is generated with the increasing probability p_2 per period. For the amplitude $eV_2/\omega \approx 2.6$, the total probability $p_1 + p_2$ of the electron-hole pair creation exhibits a minimum. This leads to the minimal excess noise in Eq. (2) which has been observed in Ref. 8. On the other hand, when the phase difference between the first and the second harmonic is $\varphi = \pi/2$, the shape of the drive deviates from the optimal one as V_2 is increased. The probability of the electron-hole pair generation grows monotonically with V_2 [see Fig. 4(b)] and the excess noise increases.[8]

In conclusion, we have studied the available experimental data on quantum noise in mesoscopic conductors with applied time-dependent voltages and provided an interpretation in terms of independent electrons and the electron-hole pairs created by the drive. This interpretation is valid for a generic quantum junction and to all orders in the statistics of the transferred charge. We have shown how the excess photon-assisted noise is composed of the contributions of the electron-hole pairs, whose number and probability of creation can be controlled by changing the shape and amplitude of the applied voltage. The agreement of the predicted pair creation probabilities and the observed photon-assisted noise corroborates that the dynamic control of elementary excitations has been achieved experimentally. This can be utilized in electron-hole sources that emit quasiparticles with entangled spin or orbital degrees of freedom for use in mesoscopic electronics and electron quantum optics.

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