

Shall We Tax the Risk Premium?

Bodo Hilgers and Dirk Schindler*

Universität Konstanz[†]

October 10, 2002

Should the realized risk premium be taxed – or not? In a simple two asset portfolio model we analyze the optimal taxation rule when the economy faces aggregate risk. We show in an appropriate designed tax system, that the risk premium of the risky asset should be fully taxed if the households are risk neutral in public consumption. If they are risk averse in public consumption, too, a positive tax rate below 100 % is optimal. We show further, that an efficient risk allocation between public and private consumption can be achieved without any distortion costs.

JEL-Classification: **H21, G11, D10**

Keywords: Risk-Taking, Risk Premium, Optimal Taxation, Aggregate Risk

*This paper has benefited from discussions with Wolfgang Eggert, Bernd Genser, Martin Kolmar, Jürgen Meckl, Stefan Zink and the participants of the post-graduate seminar at the University of Constance in summer 2001. The paper was presented at the annual meeting of the Verein fuer Socialpolitik in Innsbruck in September 2002. Thanks apply also to all participants of this session.

[†]Corresponding author: Dirk Schindler, Universität Konstanz, Fach D 133, 78457 Konstanz, Germany; Email: Dirk.Schindler@uni-konstanz.de; Phone +49-7531-883691, Fax +49-7531-884101.

1 Introduction

In most OECD countries like all G 7 members the concept of comprehensive income as tax base is applied. In case of capital income real-world tax systems treat interest, dividends, and capital gains asymmetrically. They are not only taxed with different rates, with the tax rate on capital income being usually much lower,¹ but capital gains are also taxed at the point of time of their realization whereas interest taxes must be paid by accrual. This asymmetric taxation in combination with an increasing diffusion and use of new financial derivative instruments generates serious tax arbitrage problems (see i.e. Plambeck, Rosenbloom and Ring, 1995, Alworth, 1998, Mintz, 2000). For example, the realization principle can be used by constructing tax straddles and realizing losses at once, receiving tax credits and postponing gains of the same amount (Constantinides, 1983, Stiglitz, 1983). In order to postpone capital gains derivatives like future and forward contracts or options are well suited and enable the tax arbitrageur to avoid conflicts with wash-sales restrictions existing in some tax codes. Another problem of the realization principle is the lock-in effect which results in suboptimal capital allocations (see i.e. Feldstein and Yitzhaki, 1978).

One possibility to avoid these problems is to abolish capital taxation and introduce a consumption-based tax system. Another possibility is to maintain the aim of comprehensive income and correct the deficiencies of real-world tax codes. This could be achieved by the imputed-interest method proposed by Auerbach (1991, 1992) and Bradford (2000). The retrospective taxation imputes a tax burden on the ex-post hypothetically received interest and compound interest during the holding period. Therefore the timing option of the realization principle vanishes. This method only needs information about the realized capital amount (not the capital gain), the holding period and the riskless interest rate for each year of the holding period. The two proposals differ in the treating of risky returns. Bradford (2000) develops a tax formula which taxes the received riskless interest and incorporates a tax on the risky excess return. He supposes that the tax rate could

¹In Germany the tax rate on capital gains is even zero if they aren't realized within one year.

therefore be set arbitrarily. Due to Auerbach (1991), the risk premium generates no utility and therefore should not be taxed. He restricts the imputed-interest method on taxing the hypothetically accrued riskless interest. In a joint work, Auerbach and Bradford (2001) present a generalized version of their retrospective taxation method.

But the question which still remains open is, which method is the better one? The problem can also be generalized and formulated as: shall the risk premium be taxed and if so is there any optimal tax rate by which this could be done? This question is also relevant in case of an interest adjusted income tax which taxes only the lifetime consumption (see i.e. Rose 1999 or Zodrow 1995, who calls this method "individual tax prepayment ITP"). In spite of a huge positive literature on portfolio effects of taxation on risk-taking there are surprisingly only very little contributions to the normative questions of optimal risk taxation. Especially the question of taxing risk premia or not is still unresolved up to date.

The starting point of the literature on risk-taking effects and taxation are the seminal papers of Mossin (1968), Stiglitz (1969), and Sandmo (1969). They isolated for the two-asset case in a partial model a risk-inducing substitution effect and a wealth-elasticity dependent income effect. Sandmo (1977) generalized some important results to the case of several risky assets. An excellent survey at the risk-taking literature can be found in Sandmo (1985). Buchholz and Konrad (2000) examine also the case of decreasing returns to scale and information asymmetries. Konrad (1991) considers risk, taxes, and induced risk-taking effects in a general-equilibrium model. A survey on earlier approaches to optimal taxation delivers Allingham (1972). The most important articles in this normative area are Richter (1992) and Christiansen (1993, 1995). Richter (1992) establishes in a two-asset world an elasticity rule for the optimal taxation of the risky asset. Christiansen (1993) develops a very similar rule and shows that the uniform taxation of the risky and the safe asset is optimal only for very special assumptions. Christiansen (1995) characterizes the first-best solution for state-contingent lump sum taxes.

The first two papers imply at least implicitly in their analysis that the risk premium should be taxed at a positive tax rate. Unfortunately all these results are derived in models which are not appropriate to answer the question, because the risk premium is always taxed *inter alia* with the safe return on the risky asset by the same tax rate. But as we know there are at least as much independent instruments necessary as independent goals should be realized. In the most simple two-period two-asset portfolio models, tax income is used to finance the provision of a public good which allows to shift risk from the risk averse private sector to the risk-neutral public sector and thus *ceteris paribus* increases welfare. Otherwise the tax destroys the Pareto-efficiency of the allocation and thus *ceteris paribus* decreases welfare. The optimum is characterized by an allocation where both effects are balanced. Thus, the tax rate on the risky asset and therefore on the risk premium combines the efficiency and risk shifting effect on the expense of an answer if the risk premium should or should not be taxed.

In what follows we present a simple two-period two-asset portfolio model which is due to the use of a slightly different tax system than in other papers appropriate to answer the question if risk premium should be taxed or not. Therefore we will use the Richter-Christiansen model and introduce a new tax system. We are able to reproduce some of the risk-taking results as well as some results of Richter (1992) and Christiansen (1993).

The proceeding of the paper is as follows. In Section 2 we establish the model, characterize in Section 3 the household decision and examine the welfare-maximization problem the government has to solve in Section 4. Finally we analyze the optimal tax rate on the risk premium. The paper closes with some conclusions.

2 The model

We apply a two-period model with homogenous investors (or one representative investor) and only two assets in order to concentrate on the efficiency considerations and on the risk shifting effect of taxing the risk premium. One asset yields a safe return of $r > 0$ which is assumed to be given exogenously. The other asset's return, $\tilde{x} \in [-1, \infty]$, is state dependent with a probability distribution $F(\tilde{x})$. We abstract for uncertain inflation that renders the real rate of return of asset 1 uncertain. This assumption seems reasonable in order to analyze the effects of taxing assets in different risk classes.² To keep things as simple as possible, we suppose that Fisher separability is fulfilled, and hence savings can be determined independently of the portfolio choice decision.³

As we are only interested in the latter, we do not model the saving decision of the household. Instead, the investor has an exogenously given initial wealth which she completely invests in one or in both assets available in the first period. Private consumption in the second period is financed by the principal and the return of the former investment. There is also a public good g provided by the government and financed out of tax revenue in Period 1. As we suppose that all households are small and the overall population size is normalized to 1 in the aggregate, they take the provision of the public good as given and independent of their individual behavior. Thus all prices are treated parametrically.

Denote initial wealth as W_0 and the investment in the risky asset as a . The household budget constraint in Period 1 is then

$$\tilde{W}_1 = (\tilde{x} - r) \cdot a + (1 + r) \cdot W_0 - \tilde{g}. \quad (1)$$

The household maximizes a von Neumann-Morgenstern utility function that is supposed to be additive separable in the consumption of the public good. This

²The safe asset can be interpreted as an indexed government bond. Nowadays for example indexed bonds are available on the capital market.

³This assumption requires special assumptions concerning the utility function. See i.e. Drèze and Modigliani (1972) or Sandmo (1974).

assumption is crucial because the public good affects the investment decision only by the way of the amount necessary to finance it. The utility function is written as

$$\Omega = U(\tilde{W}_1) + V(\tilde{g}). \quad (2)$$

Partial derivatives are supposed to fulfill $U_W > 0$, $V_g > 0$, $U_{WW} < 0$, $V_{gg} < 0$. So, the investors are risk averse in their wealth of private and public consumption.

So far, our model is in line with the analysis of Richter (1992) and Christiansen (1993). But the tax system we implement is different from the ones formerly used. All preceding papers either tax the safe return with one tax rate and the complete return of the risky asset with another tax rate or concentrate on a net tax, which corresponds to a consumption tax (see for example Sandmo, 1977, Richter, 1992). In our model we divide the random return of the risky asset in a safe part which yields a rate of r and the risk premium $(\tilde{x} - r)$ paid by the market. The safe return of both assets is then taxed at rate t_0 and a tax rate t_1 is applied to the risk premium. Thereby we assume full loss offset.

In the next section we show that from the point of view of a representative investor the tax on the market risk premium is equivalent to taxing the preference dependent risk premium of an investor. The advantage of this tax system is that we have one tax rate for safe income and another for the income resulting from incurring risk. So we are able to calculate an optimal tax rate that applies solely to the risk premium. We can therefore answer the question if the risk premium should be taxed – or not. Furthermore the tax rate t_0 on the save return in both assets is in fact a wealth tax and is equivalent to a lump-sum tax, here.

The budget constraint of the government can then be written as

$$\tilde{g} = t_1 \cdot (\tilde{x} - r) \cdot a + t_0 \cdot r \cdot W_0. \quad (3)$$

As long as $t_1 \neq 0$, \tilde{g} is a stochastic variable.

Inserting the government budget constraint (3) in the budget constraint of the household (1) gives

$$\tilde{W}_1 = (1 - t_1) \cdot (\tilde{x} - r) \cdot a + (1 + (1 - t_0) \cdot r) \cdot W_0. \quad (4)$$

Private consumption in $t = 1$ is financed completely out of \tilde{W}_1 .

This is a three stage game and the timing of events is as follows. At first, the government sets the optimal tax rates for a given probability distribution of risky return \tilde{x} and considers the behavior of the households. Second, for the resulting expected after-tax returns, the households maximize utility by choosing their optimal risky investment a . Finally, the uncertainty vanishes and \tilde{x} realizes. We can solve the problem backwards. First, we focus on the household maximization problem for given tax rates, and then use these results for welfare maximization by setting optimal tax rates at the government stage.

3 Household Decision

The household chooses a to maximize her expected utility for a given value of \bar{g} and given tax rates. So her maximization problem is:

$$\max_a \left\{ E [U(\tilde{W})] = \int_{-1}^{\infty} U((1-t_1) \cdot (\tilde{x}-r) \cdot a + (1+(1-t_0)r) \cdot W_0) \cdot f(\tilde{x}) \cdot d\tilde{x} + V(\bar{g}) \right\} \quad (5)$$

The first order condition simplifies to

$$E [U_W \cdot (\tilde{x} - r)] = 0, \quad (6)$$

and the second order condition gives

$$(1-t_1)^2 \cdot E [U_{WW} \cdot (\tilde{x} - r)^2] < 0 \quad (7)$$

in case of an interior optimum. By the assumption $E[\tilde{x}] > r$, an interior solution is guaranteed (see Arrow, 1970).

From (6) it follows that the representative investor increases a as long as the expected marginal utility of wealth evaluated with the risk premium is positive. In the optimum the investor balances risk and reward of the risky asset. To make this point clearer we show that the tax on the market risk premium equals a tax on

the risk premium of the representative investor. The FOC can also be written as

$$E[U_W \cdot \tilde{x}] = E[U_W] \cdot r.$$

Applying $E[x \cdot y] = E[x] \cdot E[y] + \text{Cov}(x, y)$ gives

$$E[(\tilde{x} - r)] = -\frac{\text{Cov}(U_W, \tilde{x})}{E[U_W]}. \quad (8)$$

The right hand side of equation (8) is the risk premium of the representative investor.⁴ In the optimum, this risk premium must be equal to the risk premium paid by the capital market. This result is not surprising, but it is clear that taxing the market risk premium taxes the individual preference-dependent risk premium of an investor.

We can now derive some results of the positive theory of taxation and risk-taking.

Proposition 1:

- (a) *The tax t_1 on the risk premium generates only a substitution effect with respect to the investment in the risky asset, and we get $\frac{\partial a}{\partial t_1} = \frac{a}{1-t_1}$.*
- (b) *The tax t_0 on the safe return exhibits an income effect which depends on the risk – wealth elasticity η and takes the form $\frac{da}{dt_0} = -\frac{r \cdot a}{1+(1-t_0) \cdot r} \cdot \eta$.*

Proof: Part (a) follows immediately. Differentiating the FOC w.r.t. t_1 gives $(-a) \cdot E[U_{WW} \cdot (\tilde{x} - r)^2]$. Dividing this expression by the SOC and multiplying by (-1) we arrive at $\frac{\partial a}{\partial t_1} = \frac{a}{1-t_1}$.

The proof of part (b) is somewhat more difficult. Implicit differentiation of the FOC concerning t_0 delivers $\frac{da}{dt_0} = -\frac{E[U_{WW} \cdot (\tilde{x} - r) \cdot (-rW_0)]}{E[U_{WW} \cdot (1-t_1) \cdot (\tilde{x} - r)^2]}$. The implicit differential of the FOC concerning initial wealth W_0 gives $\frac{da}{dW_0} = -\frac{E[U_{WW} \cdot (\tilde{x} - r) \cdot (1+(1-t_0) \cdot r)]}{E[U_{WW} \cdot (1-t_1) \cdot (\tilde{x} - r)^2]}$. Rearranging this expression and inserting results in $\frac{da}{dt_0} = -\frac{r \cdot W_0}{1+(1-t_0) \cdot r} \cdot \frac{da}{dW_0}$. Now, define the risk – wealth elasticity as $\eta = \frac{W_0}{a} \cdot \frac{da}{dW_0}$ and we get $\frac{da}{dt_0} = -\frac{r \cdot a}{1+(1-t_0) \cdot r} \cdot \eta$. \square

⁴See also Christiansen (1993), S. 59f.

Proposition 1 (a) is counterintuitive on first sight. Taxing the risk premium more heavily leads to higher investment into the risky asset. This effect follows from the preferences of the investor for a certain risk position. Increasing the tax rate on the risk premium distorts this risk position by shifting risk from the private sector to the public sector. To establish her desired risk position again the investor raises her investment in the risky asset.⁵ Thus, the representative investors holds pre- and post-tax the same revenue-risk-position and $a^* = \frac{a}{1-t_1}$. The tax on the risk premium is almost the same as an income tax in case of a safe return rate of zero. As this tax also does not alter the budget constraint, we get the same result as in the standard analysis for this special case. The social risk-taking always increases with the tax rate t_1 , whereas private risk-taking keeps constant. The effect of t_0 on absolute private risk-taking is qualitatively very similar to the case of a tax applying onto initial wealth (see Stiglitz, 1969, proposition 1b).⁶

4 Welfare Maximization and Optimal Income Taxation

We characterize the optimal income tax policy by maximizing a social welfare function for a balanced budget tax reform which seeks to keep the level of the provision of the public good constant in expected values. This problem can be motivated pictorially: The government (or a single secretary) sets exogenously an arbitrary level of public expenditure for social policy measures. The secretary of finance then tries to finance this requirements as efficient as possible in expected values.

We start by characterizing a first best welfare optimum using state dependent

⁵Obviously in this context the assumption of \bar{x} being independent of a and of unconstrained risky investment is only reasonable for tax rates much smaller than 100 %. But in case of short sale restrictions and $a \leq W_0$ $\frac{da}{dt_1}$ will be zero if the risky investment a reaches W_0 .

⁶Stiglitz uses the concept of absolute risk aversion. Increasing (decreasing) risk aversion corresponds to a risk wealth elasticity smaller (greater) than one.

lump sum taxes T_i .⁷ Therefore, we differentiate the welfare function totally, and set it equal to zero. Thus the optimum is characterized by a situation where no infinitesimal change of endogenous variables can increase welfare any further.

According to (2) the social welfare function is defined as follows:

$$\begin{aligned}\Omega &= E [U (\tilde{W}) + V (\tilde{g})] \\ &= E [U ((\tilde{x} - r) \cdot a + (1 + r) \cdot W_0 - T_i)] + E [V (\tilde{g})]\end{aligned}\quad (9)$$

Also the budget constraint has to be taken into account:

$$\tilde{g} = T_i. \quad (10)$$

Differentiating totally, we get:

$$d\Omega = E [U_W \cdot ((\tilde{x} - r) \cdot da - dT_i)] + E [V_g \cdot d\tilde{g}] \quad (11)$$

$$d\tilde{g} = dT_i \quad (12)$$

Substituting (12) in (11) gives

$$d\Omega = E [U_W \cdot (-d\tilde{g} + (\tilde{x} - r) \cdot a)] + E [V_g \cdot d\tilde{g}] \quad (13)$$

and by using the FOC of an household (6) we get for an optimum

$$\begin{aligned}d\Omega &= E [(V_g - U_W) \cdot d\tilde{g}] \\ &= (E [V_g] - E [U_W]) \cdot E [d\tilde{g}] + \text{Cov}[V_g, d\tilde{g}] - \text{Cov}[U_W, d\tilde{g}] = 0.\end{aligned}\quad (14)$$

Proposition 2:

- (a) *The marginal social net revenue of an optimal tax equals the difference between the expected marginal utility of public and private consumption:*

$$(E [V_g] - E [U_W]) \cdot E [d\tilde{g}]$$

⁷Using state dependent lump-sum taxes means, the government sets ex ante for each possible realization x_i of \tilde{x} a conditional lump-sum tax T_i . The subscript i indicates then different states. So, the tax revenue also depends on \tilde{x} . See also Christiansen (1993), p. 73f.

(b) *Taxes affect the allocation of public and private consumption in every state. Thus a risk-shifting effect between private and public consumption occurs. The net-effect is represented by the difference between the covariances: $\text{Cov}(V_g, d\tilde{g}) - \text{Cov}(U_W, d\tilde{g})$*

In contrast to the uniform taxation of interest revenues per asset, tax policy has no welfare effect through variation in portfolio for state dependent lump sum taxes. If there are portfolio distortion costs, the optimal taxation is a trade off between efficient risk allocation and the welfare loss stemming from the portfolio effect.

Now we are able to analyze the optimal income tax policy. We assume a balanced budget tax reform where the government keeps the provision of the public good constant in expected terms. As the budget must be balanced in every state of nature, the realization of \tilde{g} varies and depends still on the realization of \tilde{x} .

Total differentiation of the government's budget condition gives

$$d\tilde{g} = r \cdot W_0 \cdot dt_0 + (\tilde{x} - r) \cdot a \cdot dt_1 + t_1 \cdot (\tilde{x} - r) \cdot da. \quad (15)$$

From the balanced budget condition we get that (15) must be equal to zero in expected values. Therefore, we get

$$E[d\tilde{g}] = r \cdot W_0 \cdot dt_0 + E[(\tilde{x} - r)] \cdot a \cdot dt_1 + t_1 \cdot E[(\tilde{x} - r)] \cdot da = 0. \quad (16)$$

Solving (16) for $\frac{dt_0}{dt_1}$, we derive

$$\frac{dt_0}{dt_1} = -\frac{E[(\tilde{x} - r)] \cdot a}{r \cdot W_0} \cdot \left(1 + \frac{\partial a}{\partial t_1} \cdot \frac{t_1}{a}\right). \quad (17)$$

Using $a = a(t_1)$ as the optimal response of an household to a tax rate t_1 (see Proposition 1) and $t_0 = t_0(t_1)$ from the balanced budget condition (17) we can state the social maximization problem as follows:

$$\begin{aligned} \max_{t_1} \{ \Omega &= E[U((1 - t_1) \cdot (\tilde{x} - r) \cdot a(t_1) + (1 + (1 - t_0(t_1)) \cdot r) \cdot W_0)] + E[V(\tilde{g})] \\ \text{s.t. } &\tilde{g} = t_1 \cdot (\tilde{x} - r) \cdot a(t_1) + t_0(t_1) \cdot r \cdot W_0 \} \end{aligned} \quad (18)$$

Therefore, we get the following FOC:

$$\begin{aligned} \frac{d\Omega}{dt_1} &= \frac{\partial E[U(\tilde{W}_1)]}{\partial a} \partial a \cdot \frac{\partial a}{\partial t_1} - E[U_W \cdot (\tilde{x} - r)] \cdot a - E[U_W] \cdot r \cdot W_0 \cdot \frac{dt_0}{dt_1} \\ &+ E[V_g \cdot (\tilde{x} - r)] \cdot \left(a + t_1 \cdot \frac{da}{dt_1} \right) + E[V_g] \cdot \frac{\partial t_0}{\partial t_1} = 0 \end{aligned} \quad (19)$$

Using $\frac{\partial E[U(\tilde{W}_1)]}{\partial a} = 0$ from household choice (6) and collecting terms gives:

$$(E[U_W] - E[V_g]) \cdot r \cdot W_0 \cdot \frac{dt_0}{dt_1} = E[V_g \cdot (\tilde{x} - r)] \cdot a \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right) \quad (20)$$

By substituting (17) into (20), we get:

$$(E[U_W] - E[V_g]) \cdot E[(\tilde{x} - r)] \cdot a \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right) = E[V_g \cdot (\tilde{x} - r)] \cdot a \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right)$$

Using (8) and again $E[x \cdot y] = E[x] \cdot E[y] + \text{Cov}(x, y)$ this can be written as

$$-E[U_W] \cdot E[(\tilde{x} - r)] \cdot a \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right) = \text{Cov}(V_g, (\tilde{x} - r)) \cdot a \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right)$$

or

$$\text{Cov}(U_W, (\tilde{x} - r)) \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right) = \text{Cov}(V_g, (\tilde{x} - r)) \cdot \left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right). \quad (21)$$

As $\frac{\partial a}{\partial t_1} = \frac{a}{1-t_1}$ from Proposition 1, $\left(1 + \frac{t_1}{a} \frac{da}{dt_1} \right) = 0$ only if $t_1 \rightarrow \infty$ which generates a minimum. So, social welfare is maximized, if

$$\text{Cov}(U_W, (\tilde{x} - r)) = \text{Cov}(V_g, (\tilde{x} - r)). \quad (22)$$

We are now able to conclude:

Proposition 3:

It is never optimal to use solely the lump sum tax t_0 to finance the public good and not to tax the risk premium.

Proof:

Suppose the case $t_1 = 0$. Then, the governmental budget constraint does not depend on the risky return \tilde{x} . So, g is deterministic and V_g is fixed. Therefore, $\text{Cov}(V_g, (\tilde{x} - r)) = 0$. From (22) then follows $\text{Cov}(U_W, (\tilde{x} - r)) = 0$. This is only possible for either $U_W = \text{constant}$ and the households being risk neutral in private consumption, which conflicts with our general assumptions, or for $t_1 = 1$, which contradicts the initial assumption $t_1 = 0$. \square

The intuition behind this is straightforward. If we use only the lump sum tax we have a fixed level of g and get ex ante $U_W = V_g$ in *expected terms*. Ex post the actual marginal utility of private consumption depends on the realization of \tilde{x} . Then, it is optimal to have in bad states a lower level of g and in good states a higher one. But this can be efficiently⁸ reached by linking public expenditure to the realization of \tilde{x} by taxing the risk premium $(\tilde{x} - r)$ with $t_1 > 0$.

In the special case of households being risk neutral in public consumption is $V_g = \text{constant}$ and (22) simplifies to

$$\text{Cov}(U_W, (\tilde{x} - r)) = 0. \quad (23)$$

Proposition 4:

If the households are risk neutral in public consumption the optimal tax rate on the risk premium is $t_1 = 1$.

Proof:

For $t_1 = 1$:

$$\tilde{W} = 1 + r(1 - t_0) \cdot W_0 \quad \forall \tilde{x} \implies U_W = \text{const.} \forall \tilde{x} \implies \text{Cov}(U_W, (\tilde{x} - r)) = 0$$

\square

The intuition behind Proposition 4 is as simple as surprising. By using the tax on the risk premium with $t_1 = 1$ and full loss offset all the aggregate risk is concentrated in the public consumption. As the households are risk neutral in \tilde{g}

⁸Here, efficiently means in accordance with optimal risk allocation.

they do not worry about this risk. As further the portfolio choice is not distorted there are no costs in risk shifting. The lump sum tax t_0 is used only to balance the budget and if \tilde{g} is set optimally to equate the expected values of marginal utilities in private and public consumption.

If the households are risk averse in public consumption, too, V_g will depend on realization of \tilde{x} and so, $\text{Cov}(V_g, (\tilde{x} - r)) < 0$ as $V_{gg} < 0$. Therefore, in the optimum

$$\text{Cov}(U_W, (\tilde{x} - r)) = \text{Cov}(V_g, (\tilde{x} - r)) \quad (24)$$

must still hold. This is only possible, if $t_1 \in (0; 1)$.

Proposition 5:

If households are risk averse in public consumption, there is an optimal tax rate t_1 with $0 < t_1 < 1$.

As the households are risk averse both in private and public consumption, the risk must be diversified between both aspects in an optimum. This diversification depends on the relative strength of the risk aversion in private consumption compared to the one in public consumption. Therefore, the tax rate t_1 depends on this relative risk aversion. The higher the relative strength of risk aversion in private consumption, the higher will be the tax rate on the risk premium. Thus, only if the households are risk neutral in private consumption and risk averse in public consumption the risk premium should not be taxed.

Compared with the first best optimum using state dependent lump sum taxes our tax system delivers the same condition for optimal risk allocation as in Proposition 2. To see this, recognize, that $E[d\tilde{g}] = 0$ and by using (15), (16)

$$d\tilde{g} = d\tilde{g} - E[d\tilde{g}] = (a \cdot dt_1 + t_1 \cdot da)(\tilde{x} - \bar{x}) \quad (25)$$

with $\bar{x} = E[\tilde{x}]$. Then,

$$\text{Cov}(U_W, d\tilde{g}) = \text{Cov}(U_W, (a \cdot dt_1 + t_1 \cdot da)(\tilde{x} - \bar{x})) \quad (26)$$

and the same applies to $\text{Cov}(V_g, d\tilde{g})$. Further, $\text{Cov}(U_W, (\tilde{x} - r)) = \text{Cov}(U_W, \tilde{x})$.

Using some basic covariance rules, it follows

$$\begin{aligned}
\text{Cov}(V_g, d\tilde{g}) &= \text{Cov}(U_W, d\tilde{g}) & (27) \\
\Leftrightarrow \text{Cov}(U_W, (a \cdot dt_1 + t_1 \cdot da)(\tilde{x} - \bar{x})) &= \text{Cov}(V_g, (a \cdot dt_1 + t_1 \cdot da)(\tilde{x} - \bar{x})) \\
\Leftrightarrow (a \cdot dt_1 + t_1 \cdot da) \cdot \text{Cov}(U_W, (\tilde{x} - \bar{x})) &= (a \cdot dt_1 + t_1 \cdot da) \cdot \text{Cov}(V_g, (\tilde{x} - \bar{x})) \\
&\Leftrightarrow \text{Cov}(U_W, \tilde{x}) = \text{Cov}(V_g, \tilde{x}) \\
&\Leftrightarrow \text{Cov}(U_W, (\tilde{x} - r)) = \text{Cov}(V_g, (\tilde{x} - r)). & (28)
\end{aligned}$$

So, if we assume that the public good g is set optimally and the lump sum tax on safe interest income t_0 is used to equate the expected marginal utilities, we get the same result as in a first best optimum. The reason is that we have no portfolio distortion effect and achieve optimal risk allocation by using the tax on the risk premium. Furthermore, it should be clear that equalizing tax rates in manner of a comprehensive income taxation can lead only in special cases to an optimum.

5 Conclusion

As we are not able to use state contingent lump-sum taxes, we are in a second-best world. In such a world we showed for a balanced budget tax reform that a positive taxation of the risk premium is optimal and the resulting risk shifting can be done without any portfolio distortion costs by using a positive tax rate on the risk premium. Therefore, the risk allocation is efficient. The tax on the safe market rate of return in both assets is used solely in order to balance the budget and to equate the expected marginal utilities of private and public consumption. In a nutshell, we can mimic a first best solution given the assumptions of our model. If the households are risk neutral in public consumption it is even optimal to tax the risk premium fully. So, private consumption is deterministic in this case and all risk is optimally concentrated at the public good.

Taxing the risk premium has consequences for the design of a timing-neutral capital gains tax system à la Auerbach-Bradford, too. The first proposal was taxing solely the imputed safe rate of return (Auerbach, 1991) then they developed

a generalized tax system incorporated some risk tax (Auerbach and Bradford, 2001). As the risk premium should be taxed in an optimum the suggestion of Auerbach that the risk premium should not be taxed can't be verified. We are able to state now, that the second and, I'm afraid, more complicated proposal for a retrospective capital gains taxation is potentially optimal. Our results further support the suggestion that in an interest adjusted tax system the extraordinary gains $\tilde{x} - r$ should be taxed. The tax rate hereby need not be equal to the tax rate on wage income. If they are identical we have a comprehensive income tax, but, as shown, in an optimal tax scenario this can happen only by pure chance.

Due to the simplicity of our model further research is needed. Obviously a more sophisticated model should take account of an intertemporal consumption decision. Furthermore the interest rates should be endogenized in a next step. Supposing the returns as fixed is acceptable in a small open economy and an integrated perfect world capital market. But, we are then not in a position to make conclusions concerning over- or under-investment in the risky asset.

Other possibilities to expand the paper are using a model with more than two periods and integrating more than two assets. But we think that at least expanding the amount of assets would not alter our result.

6 References

- Allingham, M. G. (1972). Risk Taking and Taxation. *Zeitschrift für Nationalökonomie* 32, 203-224.
- Alworth, J. (1998). Taxation and Integrated Financial Markets: The Challenges of Derivatives and Other Financial Innovations. *International Tax and Public Finance* 5, 507-534.
- Arrow, K. J. (1970). *Essays in the Theory of Risk Bearing*. North-Holland, Amsterdam.

- Auerbach, A. J. (1991). Retrospective Capital Gains Taxation. *American Economic Review* 81, 167-178.
- Auerbach, A. J. (1992). On the Design and Reform of Capital-Gains Taxation. *American Economic Review Papers and Proceedings*, 263-267.
- Auerbach, A. J. and D. F. Bradford (2001). Generalized Cash Flow Taxation. CESifo Working Paper Series No. 425, München.
- Bradford, D. F. (2000). Fixing Realization Accounting: Symmetry, Consistency and Correctness in the Taxation of Financial Instruments. In: Bradford, David F. (ed.), *Taxation, Wealth and Saving*, MIT-Press, Cambridge, Mass., 371-428.
- Buchholz, W. and K. Konrad (2000). Risiko und Steuern. In: Andel, Norbert (Hrsg.), *Probleme der Besteuerung III*. Schriften des Vereins für Socialpolitik, Gesellschaft für Wirtschafts- und Sozialwissenschaften, N.F. Bd. 259/III, Duncker und Humblot, Berlin, 63-139.
- Constantinides, G. M. (1983). Capital Market Equilibrium with Personal Tax. *Econometrica* 51, 611-636.
- Christiansen, V. (1993). A Normative Analysis of Capital Income Taxes in the Presence of Aggregate Risk. *The Geneva Papers on Risk and Insurance Theory* 18, 55-75.
- Christiansen, V. (1995). Normative Aspects of Differential, State-Contingent Capital Income Taxation. *Oxford Economic Papers* 47, 286-301.
- Drèze, J. H. and F. Modigliani (1972). Consumption Decision under Uncertainty. *Journal of Economic Theory* 5, 308-335.
- Feldstein, M. and S. Yitzhaki (1978). The Effects of the Capital Gains Tax on the Selling and Switching of Common Stock. *Journal of Public Economics* 9, 17-36.

- Konrad, K. (1991). Risk Taking and Taxation in Complete Markets. *The Geneva Papers on Risk and Insurance Theory* 16, 167-177.
- Mintz, J. (2000). Taxation of Investment and Finance in an International Setting: Implications for Tax Competition. CoFE Diskussion Paper 00/33, Universität Konstanz, Konstanz.
- Mossin, J. (1968). Taxation and Risk Taking: An Expected Utility Approach. *Economica* 35, 74-82.
- Plambeck, C. T., H. D. Rosenbloom and D. M. Ring (1995). General Report. Tax aspects of derivative financial instruments, *Cahier de Droit Fiscal International*, 653-690.
- Richter, W. F. (1992). The Optimal Taxation of Risky Capital Income: An Elasticity Rule. *Journal of Economics* 55, 101-112.
- Rose, M. (1999). Recommendations on Taxing Income for Countries in Transition to Market Economies. M. Rose (ed.), *Tax Reform for Countries in Transition to Market Economies*. Lucius & Lucius, Stuttgart.
- Sandmo, A. (1969). Capital Risk, Consumption and Portfolio Choice. *Econometrica* 37, 586-599.
- Sandmo, A. (1974). Two-Period Models of Consumption Decisions under Uncertainty: a survey. In: Drèze, Jacques H. (ed.), *Allocation under Uncertainty: Equilibrium and Optimum*, Macmillian Press, London.
- Sandmo, A. (1977). Portfolio Theory, Asset Demand and Taxation: Comparative Statics with Many Assets. *Review of Economic Studies* 49, 517-528.
- Sandmo, A. (1985). The effects of Taxation on Savings and Risk Taking. In: Auerbach, Alan J. and Martin Feldstein (eds.), *Handbook of Public Economics Vol. I*, Elsevier Science Publishers B.V. 265-311.
- Stiglitz, J. E. (1969). The Effects of Income, Wealth and Capital Gains Taxation on Risk-taking. *Quarterly Journal of Economics* 83, 262-283.

Stiglitz, J. E. (1983). Some aspects of the taxation of capital gains, *Journal of Public Economics* 21, pp. 257-294.

Zodrow, G. R., (1995). Taxation, uncertainty and the choice of a consumption tax base. *Journal of Public Economics* 58, 257-265.