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Immigration, Education and Labour Market Institutions

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Zusammenfassung:
In this paper we analyse the effect of immigration on the labour market conditions for different skill groups. We develop a model of endogenous labour supply in which immigration affects educational decisions. We show how the skill premium changes with immigration of low skilled labour under flexible and rigid wages. Depending on the labour market institutions, immigration of low skilled labour is absorbed either by an increased skill premium or increased unemployment of low skilled labour. Thus, we extend the existing explanations of changing skill premia and unemployment of low skilled labour experienced during the last decades. Our model gives a rationale for the debate on immigration of high skilled workers as a way to reduce unemployment of low skilled workers in highly rigid labour markets.

JEL Klassifikation: F22, J31
Schlüsselwörter: Immigration, Skill premium, Labour supply
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In this paper we analyse the effect of immigration on the labour market conditions for different skill groups. We develop a model of endogenous labour supply in which immigration affects educational decisions. We show how the skill premium changes with immigration of low skilled labour under flexible and rigid wages. Depending on the labour market institutions, immigration of low skilled labour is absorbed either by an increased skill premium or increased unemployment of low skilled labour. Thus, we extend the existing explanations of changing skill premia and unemployment of low skilled labour experienced during the last decades. Our model gives a rationale for the debate on immigration of high skilled workers as a way to reduce unemployment of low skilled workers in highly rigid labour markets.

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1 Introduction

During the last decades, we observed changing labour market conditions of low skilled workers in the US and Western Europe. In the United States wage dispersion between educational groups increased dramatically from the 1960s to the 1990s.¹ In contrast, we find a secular rise in unskilled unemployment rates instead of a larger wage differential by skill in Continental Europe.² The main reason for the different labour market outcome is the different institutional framework characterising most European labour markets. These two developments – higher wage differentials and lower unemployment of unskilled workers in the US and lower wage differentials and higher unemployment of unskilled workers in Continental Europe – are vividly summarized by (Krugman, 1994) as different sides of the same coin.

Several competing explanations for these labour market developments are debated in the literature. Globalisation as an argument was put forward by Wood (1995, 1998). Within the Heckscher-Ohlin framework increased trade with less-developed countries mostly hurts unskilled workers in the industrialised countries. Another strand of the literature emphasises the rapid technological change in the workplace experienced during the last three decades. This technological change results in a demand shift favouring skilled labour. Immigration as a supply side explanation was proposed by e.g. Borjas (1994); Borjas et al. (1997). Large and steady inflows of legal and illegal immigrants into the US and Western Europe are a striking fact of the last four decades. Borjas et al. (1997) show for the US that immigration of low skilled workers explains a substantial share of the deteriorating labour market position of low skilled workers. Therefore immigration – especially immigration of low skilled workers – has a direct effect on the wages of low- and high skilled natives. This effect might be even stronger if the influence

¹The wages of workers in the 90th percentile have risen by nearly 40 percent relative to those in the 10th percentile (c.f. Katz and Autor (1999)).
²The unemployment rate of unskilled workers, e.g. in Germany, rose from 6.1 percent (1975) to 20 percent (1995) (c.f. Bundesagentur für Arbeit).
of changing wages on the human capital investment is accounted for. We will present a model of immigration analysing this feedback effect.

The early literature on immigration (c.f. Berry and Soligo (1969)) analysed immigration with a competitive labour market. The result is that immigration is beneficial for the host country but it will not fully apply to distorted labour markets where wages are set by unions. Schmidt et al. (1994) show that both – higher unemployment or increased employment – might arise which crucially depends if unskilled and skilled workers are either substitutes or complements. In contrast, Fuest and Thum (2001) consider an efficient bargaining model in which the relative labour supply is endogenous. They show that the reaction of labour supply to the expected future immigration internalises the negative effect of immigration on the labour market outcome of workers. Since their result depends on the incorporated bargaining process, our results are different concerning unemployment and the evolution of wage dispersion.

The objective of this paper is to extend the existing literature on immigration by analysing the effect of immigration on the native’s educational decision and its feedback on unemployment and the wage distribution. We introduce a general equilibrium model with heterogeneous individuals whose decision over education forms the labour supply. They decide whether to invest in post-secondary education and work as skilled workers or enter the labour market immediately as unskilled workers without any further education. In this economic environment, the decision of an individual to become skilled will solely depend on the relative wage and his inherent ability. Thereby the skill premium between the educational groups depends on the relative wage and on the ability composition of both groups. Hence, immigration influences the labour market through two channels: firstly, by altering the existing relative labour supply, immigration directly changes the relative wage. We call this channel the direct wage effect of immigration. Secondly,

3The model is an extension of Meckl and Weigert (2003).
4The theoretical literature on immigration mostly concentrates on this direct effect on wages. See Borjas (1999) for a detailed overview. Beside that Berry and Soligo (1969) already consider the feedback effect for capital accumulation due to changing factor prices resulting from immigration.
a changing relative wage will induce natives to revise their educational decisions. This will modify the composition of the educational groups and constitutes the indirect effect of immigration. We present a measure of skill premium to analyse whether immigration of low skilled labour can account for the observed pattern of changing skill premia experienced in the US. We show that under reasonable conditions immigration of low skilled labour can magnify the direct wage effect resulting in an even stronger reaction of the measured skill premium. If we introduce rigid wages – e.g. minimum wage legislation or union power which is true for Continental Europe – this result will change significantly. The skill premium changes slightly at the cost of increased unemployment of low skilled workers.\(^5\) Furthermore we show that with rigid wages immigration of high skilled labour could be an opportunity to decrease unemployment of low skilled labour.

The remainder of the paper is organised as follows. In Section 2, we present the basic model, discuss the labour market equilibrium and our measure of skill premium. Immigration and its influence on the economy with flexible and rigid wages will be analysed in section 3. Section 4 concludes.

# 2 The model

## 2.1 Technology

Consider an economy in which competitive firms produce a homogeneous consumption good \(Y\) with two different factors of production: skilled labour \(H\) and unskilled labour \(L\) measured in efficiency units.\(^6\) The production technology \(Y(H, L)\) is assumed to be

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\(^5\)This mechanism with respect to unemployment is discussed in Schmidt et al. (1994) but they make no further statement about the wage distribution.

\(^6\)Capital as a third factor can be ignored as long as capital is assumed as perfectly mobile internationally with an exogenously given global interest rate. Otherwise the capital income of natives also depends on the relative inflow of immigrants and one has to analyse the wealth- and the wage distribution simultaneously.
Competition firms maximise profits and the first order conditions are then given by:

\[
\frac{\partial Y(H, L)}{\partial H} = w_H, \quad \frac{\partial Y(H, L)}{\partial L} = w_L,
\]

where we have normalised the price of the final product to one. The marginal product for both types of labour equals the factor prices per efficiency unit of labour. The first order conditions (2) and (3) define the aggregated relative labour demand \((H/L)_{LD}\) as a function of the relative factor price \(\omega = w_H/w_L\):

\[
\left(\frac{H}{L}\right)_{LD} = g(\omega).
\]

Given our assumption on the production technology, the relative labour demand depends negatively on the relative wage \(\omega\): \(g'(\omega) < 0\).

### 2.2 Households and preferences

Households are assumed to be heterogeneous with respect to their inherent abilities \(a\) which are continuously distributed according to the density function \(f(a)\). Without a loss of generality we restrict the support of \(a\) on the interval \([0, 1]\) and normalise the total native population to one.

Given his ability \(a\), an agent decides whether to train or not. This decision is irreversible and trained and untrained worker are no substitutes in production.\(^8\) An untrained
worker with ability $a$ supplies $(1 + a)$ efficiency units of unskilled labour. Therefore he earns a total wage income of $w(a) = (1 + a)w_L$. Alternatively he can spend an exogenously given fraction $\lambda$ of time in training to supply $(1 + ba)(1 - \lambda)$ efficiency units of skilled labour. The parameter $b > 1$ measures the gross effect of education on marginal efficiency units of a trained worker with ability $a$. Hence, a trained worker earns a total wage income of $w(a) = (1 + ba)(1 - \lambda)w_H$. The wage function for each skill group as a function of the ability $a$ is depicted in figure 1.

Preferences $u(c)$ are defined over the consumption of the homogeneous good $Y$ and are identical for all agents. An individual will become a skilled worker if utility of his post training income is higher than his utility as an unskilled worker. He will choose to invest in training if his ability is higher than some threshold value $t$ implicitly defined by:

$$u((1 + bt)(1 - \lambda)w_H) = u((1 + t)w_L).$$

(5)

Workers with ability $t$ are indifferent between investing in training or not. The threshold value depends on the relative factor price $\omega$ as well as on the exogenous parameter $b$ and $\lambda$. Assuming risk neutral individuals,$^9$ the threshold is just the intersection of the two nominal wage functions (see figure 1) and can be calculated as:

$$t = \frac{(1 - \lambda)\omega - 1}{1 - b(1 - \lambda)\omega}.$$

(6)

The parameters $b$, $\lambda$ and the relative wage $\omega$ have to satisfy the following condition: $2/(1 + b) \leq (1 - \lambda)\omega \leq 1$, such that $t$ lies in the interval $[0, 1]$. For the remainder of the paper we assume this condition to be fulfilled. If the relative wage changes, the threshold

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$^9$The assumption of risk-averse agents does not alter the qualitative results.
value changes according to:

\[ t'(\omega) = \frac{(1 - \lambda)(1 + tb)}{1 - b(1 - \lambda)\omega} < 0. \]  \hspace{1cm} (7)

The rationale for the negative sign is that a higher relative wage makes it favourable for agents with lower ability to invest in training. Even a small change in \( \omega \) might result in a large reaction of \( t \) if the denominator is very small.

The economy’s total supply of high skilled labour and low skilled labour equals the weighted sum of efficiency units of the respective group and therefore depends directly on the training decisions made by households:

\[ L(t) = \int_0^{t(\omega)} (1 + a)f(a)da, \quad H(t) = \int_{t(\omega)}^1 (1 - \lambda)(1 + ba)f(a)da. \]  \hspace{1cm} (8)

Apparently \( L'(t) > 0 \) and \( H'(t) < 0 \) since a higher threshold value expands the ability interval of the less skilled workers and narrows that of the high skilled workers. Relative labour supply \( (H/L)_{LS} \) can be written as an implicit function of the relative factor reward \( \omega \):

\[ \left( \frac{H}{L} \right)_{LS} = \frac{\int_{t(\omega)}^1 (1 - \lambda)(1 + ba)f(a)da}{\int_0^{t(\omega)} (1 + a)f(a)da}, \]  \hspace{1cm} (9)

which has a positive slope: \( d(H/L)_{LS}/d\omega > 0 \). Formerly low skilled workers will invest in training and therefore reduce the supply of low skilled labour while at the same time raise the supply of high skilled labour.

2.3 Labour market equilibrium and the skill premium

As we have shown above the relative labour demand is negatively sloped and the relative labour supply is positively sloped. An unique labour market equilibrium \((\omega^*, (H/L)^*)\) exists and determines a specific threshold value \( t^*(\omega^*, (H/L)^*) \). The labour market is depicted in figure 2, where the point \( A \) constitutes the market equilibrium without
Immigration.

In order to discuss the influence of immigration on the skill premium we need to define a measure of the skill premium. Because the economy is populated by heterogeneous agents, an unique wage does not exist for every skill group but a wage distribution for both groups. An apparent measure for the respective group wage would be the group’s mean wage. But there is a major drawback in using the mean wage alone because the wage distribution of each group is a linear transformation of the assumed skill distribution, which turns out to be a skewed distribution. Using the mean as a representative wage alone might over- or understate the reaction of the skill premium since it is sensitive to outliers. To address the problem, we also use the median wage as a representative wage. We define the native’s skill premium \( x \) as the ratio of the representative wage of high and low skilled workers \( m_H \) and \( m_L \):

\[
x = \frac{m_H(t)}{m_L(t)} = \frac{m(w \mid a \geq t)}{m(w \mid a < t)} = \frac{1 + bm(a \mid a \geq t)}{1 + m(a \mid a < t)} (1 - \lambda)\omega.
\] (10)

There are two channels which influence the skill premium in equilibrium. A change in labour supply by immigration will change the equilibrium wage ratio \( \omega \) and the equilibrium relative labour used in production \( H/L \). Both, the relative wage and the relative labour influence the skill premium through a change in \( t \).

3 Immigration and the labour market

3.1 Immigration under flexible wages

We consider a one-shot immigration in terms of efficiency units \( H_I, L_I \). We are neither interested in the total number of immigrants nor in the ability distribution of immigrants because we assume that they are already educated. If the immigrants are not educated,
they are not allowed to invest in training in the host country.

The influence of immigration on labour supply can be summarised by the immigration augmented labour supply function:

\[
\left( \frac{H}{L} \right)_{LS} = \frac{H_I + \int_{\omega_I}^1 (1 - \lambda)(1 + ba)f(a)da}{L_I + \int_0^{\omega_I} (1 + a)f(a)da}
\]

(11)

Three different cases can arise: \( H_I, L_I > 0, H_I = 0, L_I > 0 \) and \( H_I > 0, L_I = 0 \). With exclusive low (high) skilled immigrants, the supply curve shifts to the right (left). But with both, positive low and high skilled immigrants – which is the typical case –, the labour supply curve will rotate clockwise. To characterise the clockwise rotation, consider relative labour supply of immigrants which differs from the equilibrium labour supply: \( H_I/L_I \neq (H/L)^* \). Given the equilibrium relative wage \( \omega^* \), the relative supply increases (decreases) if the relative inflow of immigrants is higher (lower) than the equilibrium relative labour supply (see equation (11) and figure 2). The rotation point is then given by the point \( (\omega', H_I/L_I) \) because at this relative wage the initial relative labour supply is the same as the potential inflow of relative labour and as the immigration augmented relative labour supply (11).

Now we are looking how the relative factor price changes in equilibrium if we introduce immigration. We take the total differential of (11) and of (4) at the equilibrium relative wage \( \omega^* \) and we get:

\[
d \left( \frac{H}{L} \right)^*_{LS} = \frac{1}{L_I + \int_0^{\omega_I} (1 + a)f(a)da} \left( dH_I - \left( \frac{H}{L} \right)^* dL_I \right) + \frac{\partial (H/L)}{\partial \omega^*} d\omega^*,
\]

and

\[
d \left( \frac{H}{L} \right)^*_{LD} = g'(\omega^*)d\omega^*.
\]

In equilibrium, we have \( d \left( \frac{H}{L} \right)^*_{LS} = d \left( \frac{H}{L} \right)^*_{LD} \) and we calculate then the following expres-
sion for the equilibrium relative wage rate:

\[ d\omega^* = \frac{1}{L_I + \int_0^{g(\omega)} (1 + a)f(a)da} \frac{dH_I - (H/L)_I^* dL_I}{\partial (H/L)_I \partial \omega}. \] (12)

The equilibrium relative wage rate increases (decreases) if the immigration includes relatively less (more) high skilled workers in comparison to the existing equilibrium relative labour supply in the host country. This result is the same as in the immigration literature summarised in Borjas (1999, 1995). Throughout the rest of the paper, we consider – by setting \( H_I \) and \( L_I \) to zero in (12) – the equilibrium change for a marginal first time immigration. If relative immigration is equal to the initial equilibrium relative labour \( (dH_I/dL_I) = (H/L)_I^* \) then the equilibrium coincides with the rotation point described above. So the equilibrium stays the same but the labour supply curve has changed its position. The case \( dH_I/dL_I > (H/L)_I^* \) leads to excess relative labour supply at the initial equilibrium wage rate \( \omega^* \). The relative wage falls resulting in less education, a rise in \( t \), and in the new equilibrium we have a lower relative wage and higher relative labour in efficiency units. The case \( dH_I/dL_I < (H/L)_I^* \) leads to the opposite equilibrium outcome with a higher fraction of natives investing in training, a fall in \( t \), and a higher relative wage and lower relative labour in efficiency units. For the rest of our paper, we are only looking at immigration of relatively low skilled labour: \( dH_I/dL_I < (H/L)_I^* \). This assumption seems to be realistic for the US and Western Europe (c.f. Borjas (1994) ) and under the consideration of illegal immigration, which is mainly unskilled, the evidence is even stronger. 10

Now we consider the change of the measured skill premium due to the relative wage change caused by immigration of low skilled labour. As we will show, we can divide the change of the skill premium into a direct wage effect and into a compositional effect of the respective educational group.

10The size of the population of illegal immigrants is estimated by 3 Mio. in the beginning nineties for the US. Most of this population is of Mexican origin, see Espenshade (1995) and Warren and Passel (1987). For estimates of the population of illegal immigrants in Europe see Entorf (2000).
The elasticity of the skill premium with respect to the relative wage change can be computed as:

$$
\varepsilon_{x, \omega} = x'(\omega^*) \frac{\omega^*}{x(\omega^*)} = 1 - |t'(\omega^*)| \omega^* G(t^*),
$$

(13)

with $G(t) = \left( \frac{b m'_H(t)}{1 + b m_H(t)} - \frac{m'_L(t)}{1 + m_L(t)} \right)$.

The first term measures the equilibrium change of the skill premium with respect to relative wage changes caused by immigration. We call this term the direct wage effect.\(^{11}\)

On the contrary, the function $G(t)$ in the second term measures the difference in the rate of change of representative labour supply of the two educational groups (see Figure 3). The sign of the function $G(t)$ is ambiguous in general. Hence the effect of immigration (resulting in a change of the wage ratio) on the measured skill premium is also ambiguous.

As the threshold value is only another way of expressing relative labour supply, there are cases in which $G(t) < 0$ and immigration will magnify the direct effect of relative wages. Depending on the initial relative labour supply of the host country, the measured relative wage might react stronger. This compositional effect or indirect effect is empirically shown by Borjas (1994) and Borjas et al. (1997): with a rise in the quantity of low skilled immigrants, the natives show a growing number of college graduates and a falling number of high school drop-outs.

We presume an economy with initially high relative labour supply experiencing an immigration in efficiency units: $d H_I / d L_I < (H/L)^*$. The relative wage $\omega$ rises and the threshold value $t$ falls. Therefore more natives will invest in human capital (the low skilled immigrants “push” them into education). The mean/median high and low skilled efficiency units deteriorate: the most able of the low skilled group change to the high skilled group through their decision for education. At the same time the high skilled

\(^{11}\)The measured direct wage effect differs from study to study. The empirical evidence shows that we have only a small effect on the native’s labour market outcome (c.f.Borjas (2003)) but neglecting the second term in (13).
group becomes less skilled because more low skilled natives decide to train. Magnification occurs if the low skilled mean/median deteriorates faster. Then the impact of immigration on the measured skill premium is magnified by the changing composition of the respective groups.

We show in the following that whether magnification $G(t) < 0$ or compensation $G(t) > 0$ occurs, will depend on the initial relative labour supply and on the distribution of abilities $f(a)$.

**The skill premium with symmetric and unimodal distribution of abilities**

**Assumption 1:** The abilities are distributed according to a symmetric and unimodal distribution with mean, median and modus at 1/2. The following boundary conditions are also imposed:

$$f(0) = f(1) = c \geq 0 \quad \text{and} \quad \lim_{a \to 0} f'(a) = -\lim_{a \to 1} f'(a) > 0.$$ 

We will proceed as follows: Firstly, we calculate the compositional effect $G(t)$ for our two measures of the representative wage and show under which conditions the direct wage effect is magnified or compensated. Secondly, we discuss the difference between both measures and illustrate our findings.

The median ability of the low skilled workers $a_L(t)$ and the high skilled workers $a_H(t)$ are defined by:

$$a_L(t) = F^{-1} \left[ \frac{F(t)}{2} \right], \quad a_H(t) = F^{-1} \left[ \frac{1 - F(t)}{2} \right].$$

Considering the derived formula for $G(t)$ in (13), we need to compute the change of the

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12 The median for the respective groups is calculated using the conditional ability distribution: $1/F(t) \int_0^a f(a)da$ and $1/1 - F(t) \int_1^a f(a)da$. 

12
median abilities due to a change in the threshold value $t$:

$$\frac{da_L(t)}{dt} = \frac{1}{2} \frac{f(t)}{f(a_L(t))}, \quad \frac{da_H(t)}{dt} = \frac{1}{2} \frac{f(t)}{f(a_H(t))}.$$  

Using those derived formulas above in the definition of $G(t)$ (13) gives us the condition for magnification:

$$G(t) = \frac{1}{2} \left[ \frac{f(t)}{f(a_H(t))} \frac{b}{1 + ba_H(t)} - \frac{f(t)}{f(a_L(t))} \frac{1}{1 + a_L(t)} \right] < 0,$$

and we arrive at the following proposition:

**Proposition 1:** Under assumption 1 and the requirement that $f(0) = f(1) = c \in [0, 1/2)$, the effect of a change of the relative wage on the skill premium – measured by median wages – through immigration is magnified by endogenous labour-supply reaction, iff the relative labour supply before immigration is sufficiently high. For a detailed proof, please consult the appendix.

The requirement that $f(0) = f(1) = c \in [0, 1/2)$ is not very restrictive because even a boundary weight of less than $1/2$ is very inplausible. If one thinks of abilities as some kind of measurable IQ the usual distribution found in IQ studies is the normal distribution with mean 100 and a standard deviation of 10 to 15. This would give us an approximate weight of zero for the lower and upper bound of the ability interval.

Next to the considered median wage, we analyse the change of the skill premium using the mean as a representative wage. The mean wage of the low and high skilled group are defined by:

$$E_L(t) = \frac{1}{F(t)} \int_0^t af(a)da \quad \text{and} \quad E_H(t) = \frac{1}{1 - F(t)} \int_t^1 af(a)da.$$  

Taking the derivative of the mean wage with respect to the ability threshold $t$ leads to:
\[ E'_L(t) = \frac{f(t)}{F(t)} (t - E_L(t)) \quad \text{and} \quad E'_H(t) = \frac{f(t)}{1 - F(t)} [E_H(t) - t]. \]

Using the mean and its derivative in the expression for \( G(t) \) gives the following condition for magnification:

\[
G(t) = \left[ \frac{bE'_H(t)}{1 + bE_H(t)} - \frac{E'_L(t)}{1 + E_L(t)} \right] < 0,
\]

and we get the following proposition:

**Proposition 2:** Under assumption 1 and the requirement that \( f(0) = f(1) = c \in [0, 1/2) \), the effect of a change of the relative wage on the skill premium – measured by mean wages – through immigration is magnified by endogenous labour-supply reaction, iff the relative labour supply before immigration is sufficiently high. For a detailed proof, please consult the appendix.

As proposition 1 and 2 show there is no qualitative difference in using the median or the mean wage as the representative wage. But as illustrated in figure 3, where we simulated the function \( G(t) \) for the triangle distribution for \( b = 1.4 \), the magnification (compensation) effect is even stronger by using the median.

What both propositions indicate is quite surprising because the occurrence of magnification (compensation) under a flexible wage regime does not depend on structural parameters of the model but shows some kind of “path dependence”: The existing (equilibrium) relative labour supply before immigration is the main cause for magnification or compensation. Hence countries with a high relative labour supply are very prone to magnification of relative price changes caused by immigration of low skilled labour. As we argue in the next section these findings are very sensitive to labour market institutions. The results might not hold if we replace the undistorted labour market by a
labour market with a binding minimum wage as it may be the case under union power or fair-wage considerations.

### 3.2 Immigration under rigid wages

In this subsection we analyse the influence of immigration on the measured skill premium with a minimum nominal wage. By introducing a minimum wage we are aiming at the experiences made in Continental Europe. We argue that immigration of low skilled workers, which is characteristic for these countries (c.f. Zimmermann (1995)), can lead to labour market conditions compatible with the observation in Continental Europe for the last decades. Two different scenarios are possible: a binding minimum wage before and after immigration or an initially non binding minimum wage which then becomes a binding one after immigration of low skilled labour. We analyse only the first scenario since the second is just the transition from the flexible wage case to the minimum wage case.

Consider a nominal minimum wage $w$ which is binding only for the group of low skilled workers. Then there exists an ability threshold $a(w_L; w)$ representing the least employable ability:

$$a(w_L; w) = \{ a : w = (1 + a) w_L, w > w_L \}.$$  

(14)

The native unemployment rate resulting from such a binding minimum wage is given by all workers with abilities lower than the threshold value $a(w_L; w)$:

$$U = \int_0^{a(w_L; w)} f(a) da.$$  

(15)

A lower wage rate in efficiency units for low skilled workers drives the ability threshold $a$ up and leaves more unskilled workers unemployed: $da/dw_L < 0$. The main difference to the flexible wage case is that a relative labour supply does not exist for all relative wages
We have to differentiate between the relative labour supply resulting from educational decisions of individuals at a given relative wage $\omega$ and the employable relative labour supply at the given low skilled wage rate $w_L$. To illustrate this point consider the first order condition of firms for low skilled labour (3) which can be rewritten as a function of relative labour $\left(\frac{H}{L}\right)$ used in production.

The relative labour used in production determines the wage rate in efficiency units paid for low skilled labour. Figure 4 combines the relative labour supply (right part, labeled $LS$) and the low skilled wage rate per efficiency unit for a given relative labour used in production (left part). With a minimum wage $w$ there exists a specific relative labour $\left(\frac{H}{L}\right)$ which is the lowest possible relative labour compatible without unemployment of low skilled workers ($a\left(w_L, w\right) = 0$; dashed line). The equilibrium point $A$ depicted in figure 4 cannot be supported as a market equilibrium, because all workers with abilities lower than $a$ will not be hired due to the minimum wage legislation. Therefore one has to differentiate between the supplied labour resulting from human-capital investments and the effective labour (labeled $eLS$ in the diagram) used in production. The effective relative labour used in production will be higher than the supplied labour leaving the least able unemployed. The effective relative labour supply with a minimum wage can be written as:

$$\left(\frac{H}{L}\right)_{eLS} = \frac{\int_{1}^{\omega(1-\lambda)(1+ba)f(a)da}}{\int_{a(w_L, w)}^{(1+a)f(a)da}} \text{ if } w \geq w_L.$$ (16)

With a binding minimum wage, the equilibrium relative wage rate will be lower than in the flexible wage case but the relative skilled labour used in production is higher. This situation is illustrated as point $B$ in figure 4. Compared to a country with flexible wages, we observe less education and therefore a lower relative labour supply (but higher relative labour used in production) in the country with a binding minimum wage.
When it comes to our measure of the skill premium nothing changes for the representative wage of the high skilled group but we have to revise the representative wage of the low skilled group:

\[ m_L = m(w \mid a \leq a < t) = 1 + m(a \mid a \leq a < t). \] \hspace*{1cm} (17)

Due to the minimum wage, the wage distribution is truncated at \( w \). If we use the representative wage in equation (17), we end up with the measured skill premium with a binding minimum wage:

\[ x = \frac{1 + bm(a \mid a \geq t)}{1 + m(a \mid a \leq a < t)}(1 - \lambda)\omega. \] \hspace*{1cm} (18)

Now the measured wage differential by education does not only depend directly on \( \omega \) and the educational threshold \( t(\omega) \) but also on the minimum wage \( w \) via the least employable ability \( a \). Note that there might be significant differences in the skill premium among countries with rigid wages depending on the absolute value of minimum wages. In comparison to the flexible wage case the skill premium under rigid wages is always lower at the same threshold \( t \) which is independent of the used wage measure.

Consider an immigration of low skilled workers \( dH_I/dL_I < (H/L)^* \) into a country with a binding minimum wage, where we assume that a part of the low skilled immigrants have abilities high enough to be employed in the pre-immigration economy.\(^{13}\) Figure 4 illustrates the impact of immigration of low skilled workers on the labour market.\(^{14}\) The initial equilibrium is given by point \( B \). Immigration of low skilled labour leads to a clockwise rotation of the labour supply curve (from \( LS \) to \( LS' \)). Therefore the effective labour supply curve also changes its position (from \( eLS \) to \( eLS' \)) but still originating

\(^{13}\)If all immigrants have abilities below \( a \) nothing changes in the economy because they are not employable. We abstract from a social security system transferring resources from the employed to the unemployed which might distort the educational decision of individuals. Therefore we drop this case.

\(^{14}\)The formal derivation can be found in the appendix.
from $H/L$. The new equilibrium is represented by point $C$. The equilibrium relative wage $\omega^*$ has increased and the employed relative labour has deteriorated even though more natives invested into training (resulting in a lower threshold $t$). But even more low skilled workers will be pushed into unemployment due to decreased wages for the low skilled workers thereby driving up the least employable abilities $a$.

When it comes to the change in the measured skill premium, results differ from the flexible wage case. With binding minimum wages the percentage change of the skill premium resulting from a one percent change of the relative wage can be calculated as:

$$\varepsilon^R_{x,\omega} = 1 - \left| t'(\omega^*) \right| \frac{\omega^* G_R(t^*) - \omega^*}{\frac{1}{1 + m_L(a^*, t^*)} \frac{\partial m_L(a^*, t^*)}{\partial a} \frac{dw_L}{d\omega}}$$

with $G_R(t) = \left( \frac{b}{1 + bm_H(t)} \frac{\partial m_H(t)}{\partial t} - \frac{1}{1 + m_L(a^*, t^*)} \frac{\partial m_L(a^*, t^*)}{\partial t} \right)$,

where $R$ stands for the rigid wage regime. In comparison to the flexible wage case, the change in the skill premium with a binding minimum wage is augmented by the third term. This term measures the change of the representative wage of low skilled workers due to a change in the least employable abilities $a$. This term has a positive sign as long as $dw_L/d\omega < 0$ which is fulfilled given our assumption about the production technology. Therefore the change in the unemployment of low skilled workers always counteracts the direct wage effect. A direct comparison with the flexible wage case reveals that for the same threshold $t$ the difference of the elasticities crucially depends on the term $G_R(t) - G(t)$:

$$\varepsilon^R_{x,\omega} - \varepsilon^x_{x,\omega} = - \left| t'(\omega) \right| \omega \left( G_R(t) - G(t) \right) - \omega \frac{1}{1 + m_L(a, t)} \frac{\partial m_L(a, t)}{\partial a} \frac{d\omega}{dw_L} \frac{dw_L}{d\omega}.$$  

**Proposition 3:** Under assumption 1 and at the same educational threshold $t$ the compensation (magnification) of the direct wage effect of immigration is stronger (smaller or even overcompensated) in an economy with a binding minimum wage regime than in
a country with a flexible wage regime if the minimum wage is not too high.

The claim of proposition 3 is that the experiences made by two countries being identical except of the minimum wage might be totally different. The country with flexible wages might experience magnification of the skill premium while the country with rigid wages experiences a smaller increase or even a decreasing skill premium. Thus the rigid wage regime has a higher chance for compensation to arise even under the condition of a high relative labour supply before immigration. This stems from the fact, that the ability interval \([a, t]\) of the employed low skilled workers changes both from below and from above, the change of the mean or median position will slow down. To illustrate what is meant by “a minimum wage not too high” in proposition 3 consider equation (20) using the median wage and using the symbol \(A\) for the third term:

\[
\varepsilon^R_{x,\omega} - \varepsilon^L_{x,\omega} = - \left| t'(\omega) \right| \omega \frac{f(t)}{2} \left( \frac{1}{1 + a_L} \frac{1}{f(a_L)} - \frac{1}{1 + a_R} \frac{1}{f(a_R)} \right) - \omega A.
\]

Whenever the term in brackets is positive, the elasticity under rigid wages is smaller meaning either less magnification or more compensation. If \(a_R \leq 1/2\), this condition is satisfied because \(a_L < a_R\) and \(f'(a) \geq 0\) if \(0 \leq a \leq 1/2\). If the minimum wage is very high and therefore \(a_R > 1/2\), cases might arise in which the difference \(\varepsilon^R_{x,\omega} - \varepsilon^L_{x,\omega}\) is positive. But this scenario stands for an unrealistic high unemployment rate and a small effective relative labour supply, which we do not need to consider for our analysis.

Our model can explain the different experiences made in the US and Continental Europe which have quite the same endowment of relative labour. Under rigid wages, immigration of low skilled workers leads to less dramatic changes of the skill premium at cost of higher unemployment of low skilled workers. This result is in line with experiences in Europe where those countries with a highly rigid wage system experienced a secular rise in unemployment of low skilled labour but only moderate changes in the skill premium. Fitzenberger (1999) even finds a slight deterioration of the skill premium.
for Germany from 1975-1990. Other European countries – as the Netherlands – show a stable dispersion, a nearly stable dispersion (France) or a small increase in skill differentials (Austria) (c.f. Siebert (1997)).

4 Discussion and Conclusion

We argued that immigration not only generates direct wage effects by altering the labour supply, but also influences the decision of heterogeneous agents to invest in education. Taking this indirect effect into account, we can show that under reasonable conditions the direct effect is either magnified or compensated depending on the initial relative labour employed in production. The magnification results from the revised human capital investment decision of the native population. A higher fraction of natives invests in education making the group of high skilled more heterogeneous and at the same time the group of low skilled more homogeneous. Considering that US immigration of the past 20 years was on average more low skilled compared to the native labour force, our model can give an additional explanation for the observed wage change and increased supply of college graduates. Our model also gives some results on inequality within labour groups. By changing the composition of the group of high skilled labour, the residual wage inequality rises in that group and falls in the group of low skilled labour. Although the first result is observed in the US, the second is contrary to the fact. But one has to remember that empirical findings on low skilled residual wage inequality are less strong.

With the introduction of a minimum wage regime in our model, we try to evaluate the other side of the coin. Minimum wage legislation is able to stabilise or even decrease the skill premium at the cost of higher unemployment of low skilled workers. Furthermore we show that for these countries an immigration policy favouring immigration of high skilled labour can be a tool to decrease unemployment of low skilled labour. Immigration of high skilled labour will lead to a lower relative wage $\omega^*$ but at the same time a higher
relative labour \( (H/L)^* \) in the host country. As a result, unemployment of low skilled labour will decrease because the “skill upgrading” means a higher low skilled wage \( w_L \) in equilibrium and therefore a lower least employable ability \( a \). At the same time politicians have to accept that less natives will decide to invest in further education. As educational policy tends to foster investments in further training both policies together shall be hard to implement.
Appendix

Slope of relative labour supply with and without immigration

We calculated the slope of the relative labour supply curve \((H/L)_{LS}\) as the derivative of the immigration augmented labour supply curve in which \(H_I, L_I = 0\) covers the case with no immigration:

\[
\frac{d(H/L)_{LS}}{d\omega} = \frac{|t'(\omega)|}{L_I + \int_0^{t(\omega)} (1 + a) f(a) da} \left((1 + bt(\omega)) (1 - \lambda) + (1 + t(\omega)) h(\omega; H_I, L_I)\right) > 0
\]

(21)

Comparative statics of \(\omega^*\) under minimum wages

Next, we derive the comparative statics of the labour market for binding and nonbinding minimum wage legislation. Labour market equilibrium \((\omega^*, (H/L)^*)\) with immigration is given by:

\[\left(\frac{H}{L}\right)_{LD}^* = g(\omega^*)\]  
\[\left(\frac{H}{L}\right)_{LS}^* = \frac{H_I + (1 - \lambda) \int_0^{\omega^*} (1 + ba) f(a) da}{L_I + \int_0^{\omega^*} (1 + a) f(a) da}\]  
\[a^*(w_L^*; w) = \begin{cases} \frac{w - w_L^*}{w_L^*} & \text{if } w > w_L^* \\ 0 & \text{otherwise} \end{cases}\]  
\[w_L^* = f((H/L)^*) - f'((H/L)^*) (H/L)^*\]

(22), (23), (24), (25)

in which the first and second equation constitute the labour supply and demand which are equal in equilibrium \((H/L)_{LD}^* = (H/L)_{LS}^*\). The third and the fourth equation give the lowest employable ability at the given minimum wage \(w\) and the resulting low skilled wage per efficiency units at the equilibrium ratio of high to low skilled labour. Since we are interested in the equilibrium change of the skill premium and of the relative physical
labour supply through immigration, we take the total differential of labour demand and supply:

\[
d\left(\frac{H}{L}\right)^*_{LD} = g'(\omega^*)d\omega^* \\
d\left(\frac{H}{L}\right)^*_{LS} = \frac{\partial (H/L)_{LS}}{\partial \omega}d\omega^* + \frac{\partial (H/L)_{LS}}{\partial a} \frac{da}{dw_L}d\omega^* + \frac{\partial (H/L)_{LS}}{\partial H_I} dH_I + \frac{\partial (H/L)_{LS}}{\partial L_I} dL_I \\
dw_L^* = -f''((H/L)^*)(H/L)^*d\left(\frac{H}{L}\right)^* \\
\]

(26)

(27)

(28)

Since the equilibrium requires equality of the labour supply and demand, the comparative statics of the equilibrium requires:

\[
d\left(\frac{H}{L}\right)^*_{LS} = d\left(\frac{H}{L}\right)^*_{LD}. \\
\]

The term \( dw_L^* \) in (27) can be substituted with (28). After substitution of \( d(H/L)^* \) with the demand relation (26) we arrive at:

\[
\left(1 - \frac{\partial (H/L)_{LS}}{\partial \omega} \frac{da}{dw_L} \right) f''((H/L)^*) (H/L)^* g'(\omega^*) d\omega^* = \frac{\partial (H/L)_{LS}}{\partial \omega} d\omega^* + \frac{\partial (H/L)_{LS}}{\partial H_I} dH_I + \frac{\partial (H/L)_{LS}}{\partial L_I} dL_I \\
\]

(29)

We use the results from the text for the partial effects of immigration of low and high skilled labour to solve for \( d\omega^* \):

\[
d\omega^* = \frac{1}{L_I + f''((H/L)^*) (H/L)^* g'(\omega^*) - \frac{\partial (H/L)_{LS}}{\partial \omega}} \left( dH_I - \left(\frac{H}{L}\right)^* dL_I \right) \\
\]

(30)

The relative wage change coincides with the flexible wage case derived in the text by setting \( \frac{\partial (H/L)_S}{\partial a} \) equal to zero. All results from the flexible wage regime apply because there is no principal difference between the minimum wage regime and the flexible wage regime. The main difference is that with binding minimum wage for the group of low skilled workers, there exists no competitive relative labour supply. But
there exists a supply relation which already incorporates demand effects.

**Proof of Proposition 1**

First, we proof that the limit of the function $G(t)$ at the lower (upper) bound is always negative (positive): $\lim_{t \to 0} G(t) < 0, \lim_{t \to 1} G(t) > 0$. Then we show that $G(t)$ has at most one root. Taking the limit of $G(t)$ at the lower and upper bound of the ability intervall gives the following expressions:

$$\lim_{t \to 0} G(t) = \frac{1}{2} \left( \frac{f(0)}{f(a_H(0))} \frac{b}{1 + ba_H(0)} - \frac{f(0)}{f(a_L(0))} \frac{1}{1 + a_L(0)} \right) = \frac{1}{2} \left( \frac{c}{f(1/2)} \frac{b}{1 + b/2} - 1 \right)$$

$$\lim_{t \to 1} G(t) = \frac{1}{2} \left( \frac{f(1)}{f(a_H(1))} \frac{b}{1 + ba_H(1)} - \frac{f(1)}{f(a_L(1))} \frac{1}{1 + a_L(1)} \right) = \frac{1}{2} \left( \frac{b}{1 + b} - \frac{c}{f(1/2)} \frac{2}{3} \right)$$

Because of $f(1/2) > 1$, the term $c/f(1/2) \in [0, c)$ and therefore $c < 1/2$ is a necessary and sufficient condition for $\lim_{t \to 0} G(t) < 0$ and $\lim_{t \to 1} G(t) > 0$ to hold independent of the value of $b$. Note that $a b \ll \infty$ also allows for $c > 1/2$. To get the unambiguous result, that only magnification occurs with relative labour sufficiently high ($t$ small), we need to rule out more than one root. We define the root of $G(t)$ as the value $t^*$ leading to: $G(t^*) = 0$. Simplifying the function of $G(t)$ leads to:

$$\tilde{G}(t) = \frac{f(a_L(t))}{f(a_H(t))} - \frac{1 + ba_H(t)}{b(1 + a_L(t))}.$$  

The sign of $G(t)$ and $\tilde{G}(t)$ are the same, therefore it is sufficient to show that $\tilde{G}(t)$ has at most one root because this result will also apply to $G(t)$. Taking the derivative of $\tilde{G}(t)$ gives us:

$$\tilde{G}'(t) = \frac{df(a_L(t))}{dt} / f(a_H(t)) - \frac{d(1 + ba_H(t))}{dt} / b(1 + a_L(t)).$$  

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The first term in (33) is positive because we assumed single peakness and symmetry of the distribution. The second term can be further calculated as:

\[
\frac{d\left(1 + ba_H(t)\right)}{b(1 + a_L(t))} = \frac{f(t)}{2(1 + a_L(t))f(a_L(t))} \left( \frac{f(a_L(t))}{f(a_H(t))} - \frac{1 + ba_H(t)}{b(1 + a_L(t))} \right),
\]

implying that the sign of the second term in (33) is given by:

\[
\text{sgn} \left( \frac{d\left(1 + ba_H(t)\right)}{b(1 + a_L(t))} \right) = \text{sgn} \left( \tilde{G}(t) \right).
\]

As a consequence we get that whenever \( \tilde{G}(t) \leq 0 \) holds, we know that \( \tilde{G}'(t) > 0 \) and therefore the function \( \tilde{G}'(t) \) – and also \( G(t) \) – can have at most one root. Together with the result of \( \lim_{t \to 0} G(t) < 0 \) and \( \lim_{t \to 1} G(t) > 0 \), we establish the result that \( G(t) \) has one unique root. Furthermore we have proven for \( t \) sufficiently low that magnification occurs \( (G(t) < 0) \).

**Proof of Proposition 2**

We calculate the limit of the first derivatives of the mean wage for the respective educational groups with the following properties of the truncated mean:

\[
\lim_{t \to 0} E'_L(t) = \lim_{t \to 1} E'_H(t),
\]

\[
\lim_{t \to 1} E'_L(t) = \lim_{t \to 0} E'_H(t).
\]

First, we derive \( \lim_{t \to 0} E'_L(t) \):

\[
\lim_{t \to 0} E'_L(t) = \lim_{t \to 0} \frac{f(t)}{F(t)} - \lim_{t \to 0} \frac{f(t)}{F(t)^2} \int_0^t af(a)da. \quad (34)
\]
Applying L’Hôpital’s rule on the first term of the equation (34) gives:

$$\lim_{t \to 0} t \frac{f(t)}{F(t)} = \lim_{t \to 0} \frac{f(t) + tf'(t)}{f(t)}. $$

Applying L’Hôpital’s rule a second time for $c = 0$:

$$\lim_{t \to 0} \frac{f(t)}{F(t)} = \lim_{t \to 0} \frac{f(t) + tf'(t)}{f(t)} = \lim_{t \to 0} \frac{2f'(t) + tf''(t)}{f(t)} = 2. $$

We arrive then at the following result for the first term:

$$\lim_{t \to 0} t \frac{f(t)}{F(t)} = \begin{cases} 2 & \text{for } c = 0 \\ 1 & \text{for } c > 0 \end{cases}. \quad (35) $$

We are now looking at the second term of equation (34):

$$\lim_{t \to 0} \frac{f(t)}{F(t)} \int_0^t a f(a) da = \lim_{t \to 0} \frac{f'(t) \int_0^t a f(a) da}{2F(t)f(t)} + \frac{1}{2} \lim_{t \to 0} t \frac{f(t)}{F(t)}. \quad (36) $$

The second term of equation (36) has the same limit as equation (35) but the first term is still undecidable ($c = 0$).

Applying l’Hôpital’s rule twice and with $t = 0$, $f(t) = 0$, $F(t) = 0$, we get the following expression:

$$\lim_{t \to 0} \frac{2f'(t)^2}{6f''(t)^2} = \frac{1}{3}. $$

The limit of the second term of (34) is therefore:

$$\lim_{t \to 0} \frac{f(t)}{F(t)^2} \int_0^t a f(a) da = \begin{cases} \frac{4}{3} & \text{for } c = 0 \\ \frac{1}{2} & \text{for } c > 0 \end{cases}. \quad (37)$$
We can derive $\lim_{t \to 0} E'_L(t)$ from the equations (34) and (37):

$$\lim_{t \to 0} E'_L(t) = \begin{cases} \frac{2}{3} & \text{for } c = 0 \\ \frac{1}{2} & \text{for } c > 0 \end{cases},$$

(38)

and with the relation $\lim_{t \to 1} E'_L(t) = \lim_{t \to 1} \frac{f(t)}{F(t)}(t - E_L)$, we get:

$$\lim_{t \to 1} E'_L(t) = \begin{cases} 0 & \text{for } c = 0 \\ \frac{c}{2} & \text{for } c > 0 \end{cases}.$$

Taking again the limit of $G(t)$ at the lower and upper bound of the ability intervall gives the following expressions:

$$\lim_{t \to 0} G(t) = \left[ \frac{bE'_H(0)}{1 + bE_H(0)} - \frac{E'_L(0)}{1 + E_L(0)} \right] = \left[ \frac{b}{1 + b/2} \frac{c}{2} - E'_L(0) \right],$$

$$\lim_{t \to 1} G(t) = \left[ \frac{bE'_H(1)}{1 + bE_H(1)} - \frac{E'_L(1)}{1 + E_L(1)} \right] = \left[ \frac{b}{1 + b} E'_H(1) - \frac{c}{3} \right].$$

As we have shown in equation (38), $E'(0) = E'(1)$ have the value $2/3$ for $c = 0$ and $1/2$ for $c > 0$. Therefore, we conclude that the limit of the function $G(t)$ at the lower (upper) bound is always negative (positive): $\lim_{t \to 0} G(t) < 0, \lim_{t \to 1} G(t) > 0$ for $c \in [0, 1/2)$. Furthermore this is a necessary and sufficient condition for $\lim_{t \to 0} G(t) < 0$ and $\lim_{t \to 1} G(t) > 0$ to hold independently of the value of $b$. 

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Figure 1: Determination of the threshold value $t(\omega)$
Figure 2: The labour market equilibrium with relatively low skilled immigration
Figure 3: The function $G(t)$ assuming a triangle distribution and $b = 1.4$. The dashed line shows the median and the solid line is the mean.
Figure 4: The labour market equilibrium under a binding minimum wage $w$. 

$H/L$ $w_L$ $w$ $LS$ $LS'$ $LD$ $eLS$ $eLS'$ $A$ $B$ $C$
References


